About the Cover … and a Bit More

Alan Turing invented the abstract Turing machine in the mid-1930s, one presumably capable of performing—albeit slowly—any calculation that could be performed by anyone at any time. But a bit later in the same decade he designed two rather more physical machines, ones that were by no means universal. The first was a project of his while he was in Princeton, in 1937, and was assembled from relays that he built himself in the workshop of the Princeton physics department. This was to some extent a collaboration with a Canadian graduate student in physics named Malcolm MacPhail, who wrote in a letter to Turing's biographer, Andrew Hodges, "my small contribution to the project was to lend Turing the key to the shop, which was probably against all the regulations, and to show him how to use the lathe, drill press, (etc.) without chopping off his fingers. And so, he wound the relays; and to our surprise and delight the calculator worked." The machine apparently did only one task—it multiplied two integers in binary format, which at that time, when adding machines all used decimal format, would have been an innovation.

The second machine was the one whose blueprint is on the cover and the preceding pages. Its purpose was to assist in verifying the Riemann hypothesis. It was designed in 1939, when Turing was back in Cambridge, with the assistance of Malcolm MacPhail's brother Donald, then a graduate student in engineering. Turing applied for and received a small but helpful grant from the Royal Society to cover the cost of construction, and on the application form he admitted that "Apparatus would be of little permanent value". It was necessarily inaccurate, but the idea was that it would give likely locations for zeroes which could then be checked by more traditional methods.

The Mathematics

From Turing's application to the Royal Society:

It is proposed to make calculations of the Riemann zeta-function on the critical line for $1,450 < t < 6,000$ *with a view to discovering whether all the zeros of the function in this range of* t *lie on the critical line. An investigation for* $0 < t < 1,464$ *has already been made by Titchmarsh. The most laborious part of such calculations consists in the evaluation of certain trigonometrical sums*

$$\sum_{r=1}^{m} \frac{1}{\sqrt{r}} \cos(t \log r - \vartheta) \quad m = \left\lfloor \sqrt{\frac{t}{2\pi}} \right\rfloor$$

In the present calculation it is intended to evaluate these sums approximately in most cases by the use of apparatus somewhat similar to what is used for tide prediction. When this method does not give sufficient accuracy it will be necessary to revert to the straightforward calculation of the trigonometric sums, but this should be only rarely necessary. I am hoping that the use of the tide-predicting machine will reduce the amount of such calculation necessary in a ratio of $50:1$ *or better. It will not be feasible to use already existing tide predictors because the frequencies occurring in the tide problems are entirely different from those occurring in the zeta-function problem. I shall be working in collaboration with D. C. MacPhail, a research student who is an engineer. We propose to do most of the machine-shop work ourselves, and are therefore applying only for the cost of materials, and some preliminary computation.*

The formula referred to is that of Riemann and Siegel, a recent discovery. The function ϑ is defined by the formula

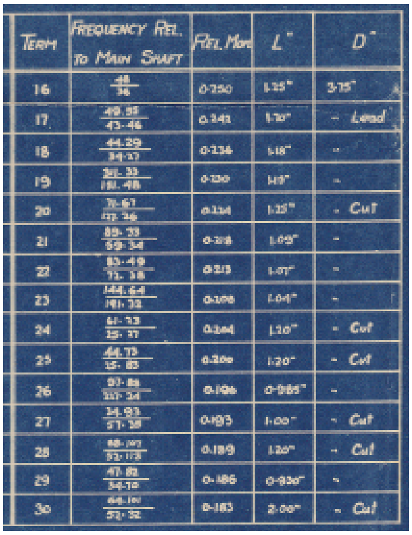
$$\vartheta(t) = \arg \gamma(1/2 + it), \gamma(s) = \pi^{-s/2}\Gamma(s/2).$$

Because of Stirling's formula, it has a simple approximation in the range for large values of t.

In the table on the blueprint, r is in the first column, $\log_8 n$ in the second but expressed as a ratio of products of integers. For example, $\log_8 16 = 4/3 = 48/36$. There seem to be two errors in the table, for $n = 24$ and 30. In the third column is $1/\sqrt{n}$. I do not know what the fourth column L'' is.

Gears

In a letter to Turing's mother after his death, Malcom MacPhail wrote, "Alan's zeta function computer was a device for adding up a large number of sines and cosines of various periods and amplitudes… The gears, of which there were to be



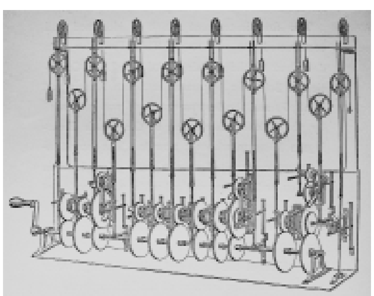
Term	Frequency Rel. to Main Shaft	Rel. Mod.	L"	D"
16	48 / 36	0.250	1.25"	3.75"
17	49.55 / 43.46	0.241	1.70"	- Lead
18	44.29 / 34.27	0.234	1.18"	-
19	341.33 / 181.48	0.230	1.19"	-
20	71.67 / 177.26	0.224	1.25"	- Cut
21	89.33 / 59.34	0.218	1.05"	-
22	83.49 / 71.38	0.213	1.07"	-
23	144.64 / 191.32	0.208	1.04"	-
24	61.73 / 35.37	0.204	1.20"	- Cut
25	44.73 / 15.83	0.200	1.20"	- Cut
26	97.88 / 217.34	0.196	0.985"	-
27	34.93 / 57.38	0.193	1.00"	- Cut
28	88.197 / 52.173	0.189	1.20"	- Cut
29	47.82 / 34.70	0.186	0.930"	-
30	64.101 / 53.32	0.183	2.00"	- Cut

Part of the table of gear specs.

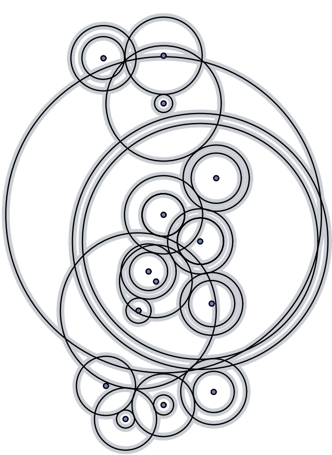
hundreds, were to provide an approximation to the required periods… Alan obtained rational approximations to [the periods]… by the method of continued fractions. A pair of gears having the number of teeth indicated respectively by the numerator and denominator of the fraction would then rotate at speeds having approximately the desired ratio."

The tide-predicting machines referred to by Turing in his application were part of a family designed much earlier by Lord Kelvin. It seems that he had seen one then still in use in Liverpool.

In correspondence with Andrew Hodges, Donald MacPhail mentioned something about a variation in the tide-predicting scheme involving balanced weights, but exactly how this would work is not apparent. The idea of using fractions to approximate real numbers in calculations involving periodic functions, and of using gears to sum such functions is very old. The blueprint doesn't look so much like one of Kelvin's machines as it does the design of the oldest extant computing device, the Antikythera mechanism (from the first century B.C.), a kind of orrery.



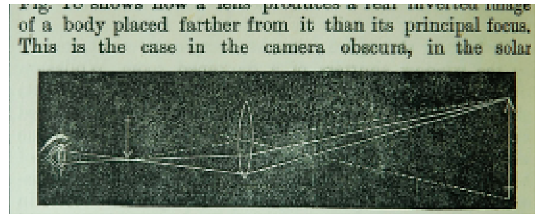
One of Kelvin's tide-predicting machines.



A schematic drawing of the Antikythera mechanism.

Blueprints

Blueprints have disappeared in my lifetime, although their memory (and the word itself in the name of many modern reprographic companies) lingers on. They were invented by the astronomer Sir John Herschel (son of the man who discovered Uranus) around 1840. Paper is coated with a solution, the parts which are to remain white are covered, and subsequent exposure to light turns the uncovered parts a dark, rather attractive, shade of blue ("Prussian" blue presumably because the solution is a compound of cyanide or Prussic acid). They are stable, accurate, and reproducible. They were used occasionally for very high quality book production in the nineteenth century, but I am not aware that the process was ever used for mathematical drawing. About the middle of the nineteenth century, however, traditional wood cuts were frequently replaced by ones with an inverted coloring scheme, and I am tempted to think this was partly influenced by blueprints.



From the article on Light in the 9th edition of the *Encyclopaedia Britannica*.

References

- The blueprint is AMT/C/2 in the Turing Archive of King's College, Cambridge, and the photograph we used was made by the Imaging Services of the Cambridge University Library. Copyright of the image is owned by P. N. Furbank, whom we wish to thank for permission to publish it. Professor Furbank was a friend of Turing as well as executor of his will, and is also the author of many books on literary topics. We also wish to thank Patricia McGuire, archivist of King's College, for much help in assembling material.
- Much of Chapter 4 of Andrew Hodges' definitive biography *The Enigma*, discusses Turing's machines. Hodges gave me much help in writing this, and in particular sent me scans of Turing's application for the Royal Society grant.
- The letter from Malcolm MacPhail to Mrs. Turing is AMT/A/21.
- More information on tide-predicting machines and the Antikythera mechanism can be found in the archives of the AMS Feature Column at `http://www.ams.org/featurecolumn/archive/index.html`.
- The letter from Titchmarsh to Turing is AMT/D/5 in the King's College archive.
- There is much information on blueprints on the Web. A good place to start (naturally) is `http://en.wikipedia.org/wiki/Blueprint`.
- A short obituary of D. C. MacPhail can be found at `http://www.homebiz.ca/News/Archives/011700.htm`.

—Bill Casselman, Graphics Editor
(`notices-covers@ams.org`*)*