

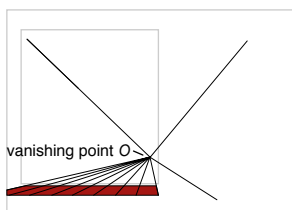


Courtesy of Yale University Press.

***The Flagellation of Christ.* (59 x 81.5 cm) It was famous in the Renaissance as a demonstration of Piero's technical skills in perspective. Unusual choice of low view point, striking colors, strange interior lighting, and depiction of contemporary figures—all contribute to what is even now a somewhat disturbing picture.**

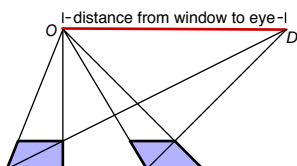
The Mathematics of Perspective

There are two principal theorems concerned with practical perspective. The first is well known: *All lines in a parallel pencil, when viewed through that window, are seen to meet at a single point, said to be at infinity.* The proof is a simple argument involving intersecting planes. If the pencil is made up of lines perpendicular to the view plane, the point at infinity is called the **vanishing point** O . All the points at infinity make up a single horizontal line, the **horizon**. The SW-NE diagonals of orthogonally oriented squares form a particular pencil intersecting the horizon in a single point D .

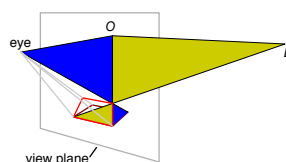


The second is not so well known, and its proof is not so straightforward.

The point D is at the same distance from the vanishing point O as the eye is from the view plane.



This theorem is equivalent to the **distance point construction**, apparently first described—if briefly—by Piero della Francesca. A closely related construction was described by Alberti much earlier in the fifteenth century. This older and clumsier method was presumably that discovered by Brunelleschi, who was as far as we know the first to apply strict perspective in drawing. As for the proof of the Theorem, the following diagram shows that it can be seen easily by picking a particular square, and then rotating part of the diagram out of the view plane.



Reference

J. V. FIELD, *The Discovery of Infinity*, Oxford University Press, 2000.

ERWIN PANOFSKY, *Perspective as Symbolic Form*, Zone Books, 1997.

TONY PHILLIPS, *Alberti's perspective construction*, the January, 2002, AMS Feature Column: <http://www.ams.org/featurecolumn/archive/alberti1.html>.

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—Bill Casselman