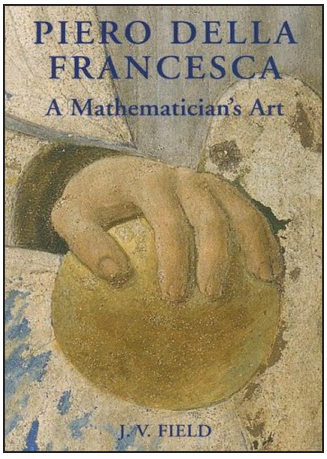
Book Review



Piero della Francesca. A Mathematician's Art

Reviewed by Michele Emmer

Piero della Francesca. A Mathematician's Art
J. V. Field
Yale University Press, 2005
256 pages, US$55
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Piero della Francesca (c. 1420–1492) was the painter who "set forth the mathematical principles of perspective in fairly complete form... Piero was the painter-mathematician and the scientific artist *par excellence*, and his contemporaries so regarded him. He was also the best geometer of his time." So writes Morris Kline in his monumental work *Mathematical Thought from Ancient to Modern Times* [11]. Clearly, the title of J. V. Field's book, *Piero della Francesca. A Mathematician's Art*, is amply justified.

The Rediscovery of Piero della Francesca

> Giotto and Paolo Uccello, Signorelli and Piero della Francesca practically did not exist in the first decade of this century. They returned to public attention due to the critical taste characteristic of Italian research, diametrically opposed to solid German philology. Would Lionello Venturi's book (1926) [18] have come to life if the ex-futurist Carlo Carrá had not written of Giotto and Paolo Uccello ten years earlier in *La Voce* (1916)? And could Roberto Longhi [14] have discerned the "problem" of Piero without his curiosity about the avant-garde (I am thinking of his great text on Boccioni's sculptures and on Severini's *pailettes*)? And could the academic historians have ever felt it legitimate to re-engage the 15th century (the pre-Raphaelists, as they were called at the time) without the ongoing debate fostered by the painters' discoveries and the contributions of the *letterati* (Bontempelli, for example)? The reason is that these artists of ours were clearly neither nostalgic nor interested by antiquity: simply, antiquity did not exist and they were perhaps constructing it themselves, searching the territory of a new and unpredictable avant-garde. De Chirico did not look for museographic painting: he himself underlined its value for individuals in the 1920s. No one would have been interested in Luca Pacioli, had it not been for the research of another *pentito* futurist painter, Gino Severini. All in all, Art History itself would have been different without the contribution of these painters and technicians of image.

So wrote Maurizio Fagiolo dell'Arco in the catalogue of the exhibition *Piero della Francesca e il Novecento (Piero della Francesca and the 20th*

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Century) [4]. The 500th anniversary of the artist's death (1991–92) was the occasion of several important conferences and exhibitions. Four volumes worth mentioning were published: *Piero and the 20th Century*, the catalogue of the 1991 exhibition in San Sepolcro (Piero's birthplace) [12]; *Piero and Urbino, Piero and the Renaissance Courts*, the catalogue of the 1992 Urbino exhibition [3]; Milton Glaser's catalogue *Piero della Francesca*, for the exhibition held at Arezzo in 1991 [16]; and finally *Through Piero's Eyes: Clothing and Jewelry in Piero della Francesca's Works* [1]. These constituted an important celebration of a great Italian Renaissance artist who influenced twentieth-century art, as witnessed by De Chirico's metaphysical period (1910–20), with his Ferrara and Modena cathedrals, his trains, those empty piazzas that convey a mysterious and perturbing motionlessness, an enigmatic stillness. Giorgio Morandi and so many others are also clear examples.

Dell'Arco added, in his chapter entitled "Styles of the return to order: perspective and space, light, geometry":

> Right after the [First World] war, research resumes in exactly the opposite way from the past. Painting returns to being precise, mentality to being ancient, structures meditated and color orderly. Giotto and Paolo Uccello were evoked by a master of futurism, Carlo Carrá. Frescoes became the subject of meditation, *cassoni* and *predelle* were reread, and Carrá and De Chirico point a clear finger to Milano's neoclassicism, Mastrini to Etruria (ancient Tuscany) and Oppi to Giovanni Bellini. All of these are clear sources. The most secret, albeit the most fertile, will continue to be the meditation on Piero della Francesca, the name which Roberto Longhi brought to the fore starting in 1914.[1] Many artists welcomed Longhi's book on Piero with memorable texts. In fact, when the topics of proportion, number, color-light, perspective, regular bodies are referred to, Piero della Francesca is always implicit. [4]

I was very happy to have received J. V. Field's book on Piero della Francesca for review, because I too have been influenced by reading Longhi's essays, by de Chirico's metaphysical works, and by the paintings of Morandi and Carrá. Piero has long been one of my favorite masters. I have also been so lucky as to own a house in Senigallia, in the Marche region in the center of Italy, a few hundred meters away from the church where Piero's famous *Madonna di Senigallia* was housed until the beginning of the Second World War, when it was moved to the Palazzo Ducale in Urbino. It has never been returned. But Urbino is quite close to Senigallia, and there, in addition to the Madonna, one can see *The Flagellation of Christ*. I have therefore seen these works by Piero dozens and dozens of times. In my own work, I have cited Longhi on several occasions. As Dell'Arco says, that is where, for the most part, everything started.

Longhi was born in 1890. In 1911 he graduated from the University of Turin with a thesis on Caravaggio. His interest was not only in ancient art but also in the avant-garde, specifically futurism and Boccioni in particular. In 1914 he published a long article on "Piero dei Franceschi [i.e., della Francesca] and the Development of Venetian Painting" in the journal[2] published by Adolfo Venturi, who had been his professor in *Scuola di perfezionamento* (school for advanced studies). In 1927 he revised and republished this article in the journal *Valori Plastici* under the title "Piero della Francesca" [14]. Longhi proposed a look with fresh eyes at the experiments in painting carried out by part of the artistic culture of Tuscany at the beginning of the fifteenth century, "when some artists, too much in love with space to come back to a superficial *spazialitá* (sense of space), too much in love with color to rely on a *chiaroscuro* intended to create an illusion of space, understood that there was only one way to express in a picture shape and color at the same time: perspective." Longhi also wrote: "By constructing, through an effort at synthesis, shapes according to their countable and measurable surfaces, [perspective] succeeded in presenting them all as a projection upon a plane, ready to be clothed in calm, broad areas of color" [14, p. 11]. Giacomo Agosti added, in his essay[3] "From Piero dei Franceschi to Piero della Francesca": "This interpretation of perspective as the art of synthesizing 'shape and color at the same time' allowed him (Longhi) to begin by identifying in Paolo Uccello and Domenico Veneziano the Florentine models closest to Piero della Francesca; and this interpretation brought him to recognize the most evident manifestations of the visual influence of Piero in the works of Antonello da Messina, 'The most *disperato prospettico* (enthusiastic expert in perspective) of the human figure' and those of Giovanni Bellini,

[1] *R. Longhi, Piero dei Franceschi e lo sviluppo della pittura veneziana, L'Arte, XVII (1914), pp. 198–221 and 241–256. Reprinted in [14].*

[2] *L'Arte; see footnote 1.*

[3] *G. Agosti, Da Piero dei Franceschi a Piero della Francesca (qualche avvertenza per la lettura di due saggi longhiani), in [12], p. 199.*

'who, without rejecting the problem of form, considers it as taking to higher levels the problem of color.'" Longhi wrote: "Here is what constitutes the unity-distinction of Antonello and Giovanni: their synthesis a priori is Piero" [14, p. 39]. And regarding *The Flagellation*, he added "the perfect union between architecture and painting that emerges should be understood as a mysterious combination of mathematics and painting". Piero is a great painter; Piero is a great mathematician.

Piero as a Painter, Piero as a Mathematician

In the introduction to the book under review, Field writes (p. 1): "Piero della Francesca is now best remembered as a painter. He has a secure position in the history of Italian Renaissance Art. However in his own lifetime, and for some time thereafter, he was also known as a mathematician. In his detailed scholarly monograph on Piero's painting, Eugenio Battisti went so far as to say that there should be a further volume to consider Piero's activity as a mathematician." Field concludes that since the mathematician and the painter were one person, it seems clear that the mathematics and the painting should be taken together, to see what one may have to tell us about the other. "As the title of the present study indicates, I am inclined to think Piero's mathematics does have a recognizable relationship with his painting. Of course there is no avoiding the recognition that Piero is likely to remain a more significant figure in the history of art than in the history of science" (p. 2). It is important to underscore, and Field does so right away, that there is an inevitable shift in style between the historical study of art and that of mathematics. In mathematics the course of history can reasonably be considered in terms of progress. In art this is clearly not the case. Piero's art was forgotten for many centuries because of a change in aesthetic taste.

Field begins by asserting that "since Piero was known to have written on the mathematics of perspective, art historians have generally [following Longhi, I may add—M. E.] been ready to describe his pictures as in some sense 'mathematical', though the quality most often remarked upon is a 'stillness' that is rather hard to define in any precise way. The assertion that Piero's pictures are 'mathematical' is usually so vague that it is understandable that some art historians have preferred largely to ignore it" (pp. 3–4). Even if the greater part of Piero's mathematics is rather elementary, I am not shocked at all as a mathematician by the loose usage of the word "mathematics". We are in the field of art history, where opinions and preferences (such as, for example, the rediscovery of Piero) are important. Field adds that "there has also been a most unfortunate fashion for drawing lines over Piero's pictures with the purpose of exposing their alleged underlying geometrical structure." More frequently than not, these geometric constructions are carried out using reproductions that are evidently quite different from the original, at least in size. Field comments, and one cannot disagree, that "trying to find perspective schemes seems to me like trying to extract the sunbeams from cucumbers." The book is full of imaginative comments, mostly in the notes. I will give a few examples. At the risk of seeming unscholarly, Field forgoes correcting errors in the "apparently relevant literature rather than stir my own and my readers' unhappy memories of maths homework that came back covered with a teacher's comments in red" (p. 5).

In order to have a deeper understanding of Piero's mathematics, the author refers to Luca Pacioli (1445–1517), the mathematician and author of the famous book *De divina proportione* (On the Divine Proportion) [15], with drawings by Leonardo da Vinci, where of course *divina proportione* refers to the so-called Golden Section. The most recent research on his life seems to show that Pacioli was not a pupil of Piero, although both were born in Borgo San Sepolcro. In any case, what is sure is that Pacioli knew the mathematical works of Piero, at least after Piero's death in 1492. They were very likely acquainted: Field argues that in the *Madonna dell'Ovo*, now at the Brera in Milan, there is a portrait of Pacioli done by Piero.

In fact, in Piero's mathematics there is no evidence of an interest in *divina proportione*. So one should first look elsewhere in seeking to identify traces of mathematics in Piero's pictures: "The present study will accordingly look much more carefully at Piero's mathematics than previous studies have done. By doing so it will be possible to offer an explanation of that famous 'stillness'. It is not possible to give mathematical proof, but from my failure to find serious deviations from mathematically correct perspective I strongly suspect that the reason why Piero's pictures look mathematically correct is because they are indeed correct. Thus everything shown is represented as seen from the single ideal viewpoint, though, thanks to the tolerance of the eye, it will of course look convincing even when seen from points at a considerable distance from the ideal viewpoint" (p. 7). The main thesis of Field is that Piero's very considerable mathematical talent does seem to have contributed to his personality as a painter.

Field's Book

The book starts with the background: the training of an artist at the time of Piero, including the study of the abacus. Probably Piero was sent outside of Borgo San Sepolcro to study in an abacus school. "Since Piero never actually taught mathematics, his own

interest in the subject has something of the same purely intellectual motivation in it. Piero must have enjoyed mathematics." Piero himself wrote a *Trattato d'abaco* (a treatise on the abacus).

The second chapter is a history of perspective, starting naturally with Filippo Brunelleschi (1377–1446) and Alberti's treatise *De pictura* (1413–1472). Field reconstructs through drawings the perspective constructions of Alberti, later called *costruzione legittima* (correct construction). Investigating the work of Donatello (1386–1466) and Masaccio (1401–1428), Field remarks that "in any picture it is architectural elements that are most likely to provide sets of orthogonals and transversals" and points to Masaccio's fresco of the Trinity in the church of Santa Maria Novella in Florence as providing "a good example in which to search for indications of the method by which the perspective scheme was constructed." Nevertheless, "the process of extending lines is in general to be avoided and its results treated with caution." Finally, as was known to Masaccio, "mathematical correctness was not of the essence in obtaining the required illusion of the third dimension" (p. 50).

The third chapter describes the early life of Piero—his family, his education, and his training as a painter—and refers to the problem of dating Piero's works. The book is full of illustrations, many of them in color. Most of the reproductions are of good quality, with the exception of a few that are shown so small that details are impossible to read. On page 90 Field writes: "Certain elements in his style apparently owe a debt to the occasionally fussy prettiness of detailing found in, say, Pisanello (1395–1455c.) and Gentile da Fabriano (c.1370–1427)." Fabriano is a small town in the region of Umbria in the center of Italy, very famous for its ancient paper industry. Just two weeks before writing this review I visited in Fabriano an exceptional exhibition of the works of Gentile and other artists of the same period, including Masaccio and Donatello. I agree with the opinion [13] of the curator of the exhibition that Gentile, who spent part of his life in Venice, was the binding ring between the painting of Florence, Umbria, and Venice. He was one of the most important and brilliant painters that Italian civilization has ever produced, not just a painter of "occasionally fussy prettiness of detailing".

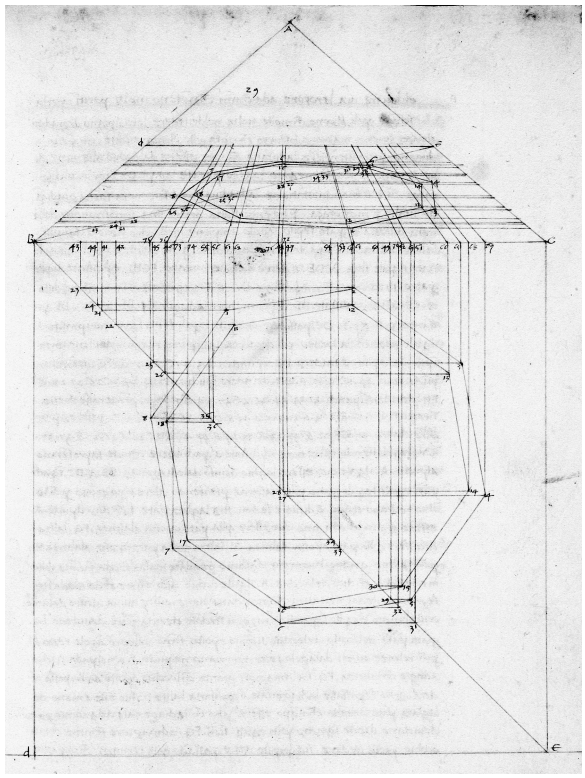
Chapter 4 bears the title: "The Sense of Space". "In Piero della Francesca's time, space did not exist" (p. 95) is how the chapter begins. "In saying that his paintings convey a sense of space, we mean that each picture conveys a sense of its own pictorial space, that is a particular space. But Piero himself would not have thought in these terms. In his day, space was not considered to be an entity with a separate existence. Space was defined as extension, and measured by body." As an example Field recalls one of Piero's most famous masterpieces, *The Baptism of Christ* (National Gallery, London). She carefully examines the proportions of the painting, measuring from photographs by means of an ordinary ruler, without any claim to precision and taking no notice of the scale of the actual picture. This is not the only occurrence of this practice in the book. But Field reminds readers that the *Baptism* is often and entirely reasonably cited as an example of Piero's beautiful rendering of light. Piero's abacus treatise, quite unusually, treated three-dimensional geometry, as did his famous *Libellus de quinque corporibus regularibus*, incorporated by Pacioli in *De divina proportione* (for which incorporation Vasari accused Pacioli of plagiarism). Piero's mathematical thinking about three-dimensional geometry surely influenced his way of rendering bodies in space. Nevertheless, this need for perspective is clearly not manifest in all of Piero's works. In many of them there is "no perspective": figures stand out against the background landscapes; color and light are used to make bodies seem corporeal. This is the case in the absolute masterpiece I have already spoken about, the *Madonna di Senigallia*. Regarding this work, Field writes (p. 115): "For reasons that may be to do with its delicate color and exquisite degree of finish, reproductions of this picture manage to look what passes for reasonably like it without looking sufficiently like it to be beautiful." I can confirm, as I have seen the painting dozens of times, that the original is always better than any reproductions, given that the picture's texture and its purity are difficult to get across through a book or a postcard. It becomes necessary to immerse oneself in the painting and to feel part of it in order to capture the sense of space that Piero's color makes the viewer perceive. That magical stillness. Little does it matter that "no great subtlety is required to note that the composition presents us with figures against a background from which they are essentially disjoint."

As she examines Piero's geometrical constructions, Field notes his paintings' symmetrical properties. She adds in a note (p. 126, footnote 38) that I am sure will be of interest to mathematicians, "The experts on the mathematics of symmetry in the real world tend to be crystallographers. A crystallographer and a specialist in the relevant branch of mathematics have now written an excellently clear and well-illustrated book on the subject: I. Hargittai and M. Hargittai, *Symmetry: A unifying concept* [8]." I must say, knowing the Hargittais quite well, that István is a chemist and Magdolna is not a mathematician. The book [8] is an elementary one, meant for students. I suggest that the reader look at Hargittai's other books [9] and [10] on symmetry and at what is considered the bible in this area, B. Grünbaum and G. C. Shepard, *Tilings and Patterns* [7]. Being the author of several books and films

on M. C. Escher, I do not wish to comment on the last sentence of Field's note: "In addition to its other virtues this book shows a highly commendable tendency to avoid the work of M. C. Escher." In the other books of Hargittai, however, there are plenty of works by Escher.

In Chapter 5 Field goes into the details of *De prospectiva pingendi* [2], Piero's treatise on perspective. "Perspective was a form of study in which it was acknowledged as legitimate to use mathematics in the pursuit of natural philosophy. The use of mathematical methods to give the illusion of depth in pictures clearly belonged to the mathematical end of the subject" (p. 129). The starting point of Piero is the definition of the entities: "A point is that which has no parts, accordingly the geometers say it is imagined; the line they say has length without width" (p. 134). Field carefully examines the results on geometry enunciated and proven by Piero. She reconstructs in detail the diagrams, reproducing whenever possible the artist's original drawings. His drawings become more and more complicated; for example, he shows the construction of the perspective image of an octagonal building. The author states (p. 155) that "the mathematics concerned need not to go beyond the methods we find in Piero's *Trattato d'abaco*." Why not, if it can help us understand?

Piero defends the knowledge of perspective that is necessary for painting: "Therefore it seems to me that I should show how much this science is necessary to painting. I say that perspective literally means things seen at distance, represented as enclosed within given limits [that is, on the picture plane] and in proportion, according to the quantity of the distances, without which nothing can be degraded correctly [Piero calls degradations the deformations performed on the figures]... I say it is necessary [to employ] perspective, which distinguishes all quantities proportionately, as a true science, demonstrating the degradation and magnification of all quantities by means of lines" (p. 163).



Courtesy of Yale University Press.

Perspective drawing of an octagon building plan. The horizontal-vertical-diagonal-horizontal line segments from P to P' implement the geometric construction of the perspective transformation. From the Parma MS of *De Prospectiva Pingendi*.

The Flagellation of Christ

When talking about the true science of perspective, one cannot overlook *The Flagellation of Christ*. Field writes (p. 174): "It would be perhaps more accurate to describe Piero's *Flagellation* not as an example of correct perspective but as *the* example. Despite longstanding disputes about the overall significance of the picture, and in particular the identification of the three figures in the right foreground, the *Flagellation* is almost invariably chosen as an illustration to elementary lectures on perspective and has had a long career as the subject of reconstructions." An essay exploring new hypotheses on the meaning of this painting, and in particular regarding the identity and the significance of the characters, was recently published in Italy by Ronchey: *L'enigma di Piero* [17] (following the big success of books like *The Da Vinci Code*, the words "code" and "enigma" have become very appreciated by publishers). Pilate, the character on the left with purple boots, would have been John VIII Palaeologos, the next to last emperor of Byzantium. The character with his back turned to the viewer would have been the Turkish sultan. Jesus represents Western Christianity flagellated by the Turks. One of the characters to the right would represent Johannes Bessarion, who later became a cardinal; the gentleman in brocade is Nicolò III d'Este of Ferrara, the site of the 1439 Council, in which John VIII participated; the young man between them is Thomas Palaeologos, the last legitimate heir of the Eastern Empire. Thus the scene, painted in 1459, would have referred to events that took place in 1439. Field's volume also reproduces an image of a three-dimensional reconstruction of Piero's painting carried out by Antonio Criminisi. Field comments (p. 177), "Apart from perspective,

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The Montefeltro Altarpiece. **(251 x 172.5 cm) It has been conjectured that the figure second from the right is Luca Pacioli. He was the author of the book *Summa de Arithmetica, Geometrica, Proportioni et Proportionialitate* (1478), best known for its account of double entry book-keeping, and of *De Divina Proportione* (1509), in which are found images of regular polyhedra attributed to Leonardo.**

there is another good reason for using *The Flagellation* as an illustration: its wonderful colour. Even by the standards of the fifteenth century, this is a very pretty picture. It seems that one was meant to go close to the picture in order to admire such details in its finish." This I can confirm. Looking very closely at this picture always stirs strong emotions.

The mathematics used in *De prospectiva pingendi* has been analyzed using mathematical tools that are clearly much more modern than those used by Piero. In a short paper[4] published with the critical edition of *De prospectiva pingendi* edited by Nicco-Fasola, the mathematician Ghione writes "To consider Piero's treatise as a manual, however complete, in which for the first time the rules of drawing with perspective are given through systematically correlated graphic schemes, would be to underestimate not only Piero della Francesca's ideas, which are clearly expressed, but also mathematical ideas that, after following a long and winding road, gave life to the first streams that would end up in the 19th century's great flow of modern Projective Geometry." Field is in agreement with this view. Three examples from Piero's treatises are considered in another essay [6] recently published in Italy, "The mathematics of Piero della Francesca": the volume of a pavilion vault, the surface of a cross vault, and the solution of algebraic equations of degree greater than two.[5] The authors conclude that Piero "is a specialist in both painting and of mathematics. We believe that it is more true to historic reality to consider Piero as one of the very first craftsmen who became scientists, a situation that constituted one of the main engines of the scientific revolution."

In Chapter 6 ("Optics and Illusion") Field examines another important masterpiece of Piero's, *La Madonna dell'Ovo*, also called the *The Montefeltro Alterpiece* (Luca Pacioli probably appears among the characters on the right). Then in Chapter 7 she asks the crucial question, "But is it art?" "The present chapter considers his work as a whole in the context of the intellectual life of his time, and in particular its relation to the learned arts taught in universities." There is a long section on the history of mathematical sciences, considering in particular Nicolaus Cusanus and Johannes Regiomontanus, and the importance of the works of Piero in this context.

Towards Galileo Galilei

In the last chapter, "From Piero della Francesca to Galileo Galilei", Field addresses scientific developments that took place in the period that followed and the role played by Piero's legacy. She concludes (p. 324):

> This description of a revival of mathematics from the fifteenth to the seventeenth centuries is not a complete story in itself, and still less a complete account of the origins of the Scientific revolution. Nor is the sketch of part of a complicated story—a sketch that does no more than trace some strands in a tangled skein—intended to argue for the recognition of a direct debt owned by, say, Galileo to Piero della Francesca. Though Piero's reference to the "force of lines" in his introduction to the

[4] *F. Ghione, Breve introduzione sul contenuto matematico del De prospectiva pingendi di Piero della Francesca, in [2], pp. xxix–xlii.*

[5] *See also M. D. Davis,* Piero della Francesca's Mathematical Treatises, *Longo editore, Ravenna 1977.*

third book of *De prospectiva pingendi* does indeed seem like a pre-echo of Galileo's later advocacy for mathematics, Piero proposed this "force" as establishing the truth of artificial perspective, that is he was concerned with a matter in which the use of mathematics was entirely accepted. Galileo was extending the use of mathematical methods into areas where they had not been used by his predecessors and were not generally accepted by contemporaries.

My purpose is not to attempt to establish a direct connection between such later work and Piero, but rather to show what developing story Piero's work as a whole, both his paintings and his writings, taken together as a contribution to the cultural history of his time, should be seen to belong to. Piero is a good example of the learned craftsman, and the activities of his intellectual kin were to make notable contributions not only to the development of mathematics but also to the emergence of a mathematical natural philosophy in the two centuries following Piero's death.

I believe that Field's long essay has reached the objective it set for itself, and I believe that it can make mathematicians become interested in Piero's scientific works. But above all, it attests to the strong aesthetic and emotional impact of Piero the artist. I would like to conclude by inviting readers to go and see Piero's original works, to go to Urbino to admire at least the *The Flagellation of Christ* and *La Madonna di Senigallia*. No reproduction (in Piero's case this is absolutely true) can do justice to the physical beauty of the actual works and to the unexplainable "stillness".

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[2] Piero della Francesca, *De prospectiva pingendi*, critical edition by G. Nicco-Fasola, Casa editrice Le lettere, Firenze, 1984.

[3] P. del Poggetto, ed., *Piero e Urbino, Piero e le Corti rinascimentali*, Marsilio, Venezia, 1992.

[4] M. Fagiolo dell'Arco, La grande ombra: Piero della Francesca e il novecento, in *Piero della Francesca e il novecento*, M. L. Lamberti and M. Fagiolo dell'Arco, eds., Marsilio, Venezia, 1991, p. 35.

[5] J. V. Field, *Piero della Francesca. A Mathematician's Art*, Yale University Press, New Haven, 2005.

[6] E. Gamba, V. Montebelli, and P. Piccinetti, La matematica di Piero della Francesca, *Lettera Matematica* **59** (2006), pp. 49–59.

[7] Milton Glaser, *Piero della Francesca* (A. Rauch, ed.), Marsilio, Venezia, 1991.

[8] B. Grünbaum and G. C. Shephard, *Tilings and Patterns*, W. H. Freeman and Co., New York, 1987.

[9] I. and M. Hargittai, *Symmetry: A Unifying Concept*, Shelter Publ. Inc., Bolinas, CA, 1994.

[10] I. Hargittai, ed., *Symmetry: Unifying Human Understanding*, Pergamon Press, New York, 1986.

[11] _____, ed., *Symmetry 2: Unifying Human Understanding*, Pergamon Press, New York, 1989.

[12] M. Kline, *Mathematical Thought from Ancient to Modern Times*, Oxford University Press, New York, 1972.

[13] M. L. Lamberti and M. Fagiolo dell'Arco, eds., *Piero della Francesca e il novecento*, Marsilio, Venezia, 1991.

[14] L. Laureati and L. Mochi Onori, eds., *Gentile da Fabriano e l'altro Rinascimento*, Electa, Milano, 2006.

[15] R. Longhi, *Piero della Francesca* (1927), reprinted by Sansoni, Firenze, 1963, pp. 9–111. English quotations from the translation by David Tabbat, Sheep Meadow Press, Riverdale-on-Hudson, NY, 2002.

[16] L. Pacioli, *De divina proportione*, Venezia, 1509.

[17] S. Ronchey, *L'enigma di Piero*, Rizzoli, Milano, 2006.

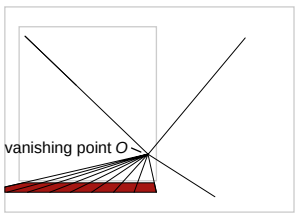
[18] L. Venturi, *Il gusto dei primitivi* (1926), reprinted Einaudi, Torino, 1981.

Courtesy of Yale University Press.

The Flagellation of Christ. **(59 x 81.5 cm) It was famous in the Renaissance as a demonstration of Piero's technical skills in perspective. Unusual choice of low view point, striking colors, strange interior lighting, and depiction of contemporary figures—all contribute to what is even now a somewhat disturbing picture.**

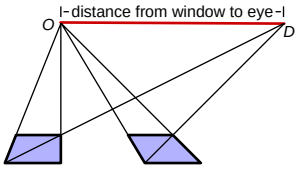
The Mathematics of Perspective

There are two principal theorems concerned with practical perspective. The first is well known: *All lines in a parallel pencil, when viewed through that window, are seen to meet at a single point, said to be at infinity.* The proof is a simple argument involving intersecting planes. If the pencil is made up of lines perpendicular to the view plane, the point at infinity is called the **vanishing point** O. All the points at infinity make up a single horizontal line, **the horizon** The SW-NE diagonals of orthogonally oriented squares form a particular pencil intersecting the horizon in a single point D.

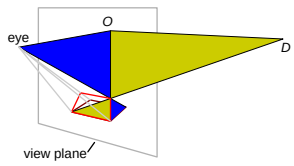


The second is not so well known, and its proof is not so straightforward.

The point D is at the same distance from the vanishing point O as the eye is from the view plane.



This theorem is equivalent to the **distance point construction**, apparently first described—if briefly—by Piero della Francesca. A closely related construction was described by Alberti much earlier in the fifteenth century. This older and clumsier method was presumably that discovered by Brunelleschi, who was as far as we know the first to apply strict perspective in drawing. As for the proof of the Theorem, the following diagram shows that it can be seen easily by picking a particular square, and then rotating part of the diagram out of the view plane.



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—Bill Casselman