2007 Robbins Prize

The David P. Robbins Prize prize was awarded at the 113th Annual Meeting of the AMS in New Orleans in January 2007.

The Robbins Prize was established in 2005 in memory of David P. Robbins by members of his family. Robbins, who died in 2003, received his Ph.D. in 1970 from the Massachusetts Institute of Technology. He was a long-time member of the Institute for Defense Analysis Center for Communications Research and a prolific mathematician whose work (much of it classified) was in discrete mathematics. The prize is given for a paper that (1) reports on novel research in algebra, combinatorics, or discrete mathematics, (2) has a significant experimental component, (3) is on a topic broadly accessible, and (4) provides a simple statement of the problem and clear exposition of the work. The US$5,000 prize is awarded every three years. This is the first time the prize was awarded.

The Robbins Prize is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2007 prize, the members of the selection committee were: Jonathan M. Borwein, Jeffrey C. Lagarias (chair), David I. Lieberman, Richard P. Stanley, and Robin Thomas.

The 2007 Robbins Prize was awarded to Samuel P. Ferguson and Thomas C. Hales. The text that follows presents the selection committee's citation, brief biographical sketches, and the awardees' responses upon receiving the prize.

Citation

This Robbins Prize is presented to Thomas C. Hales and Samuel P. Ferguson for the paper: Thomas C. Hales, "A proof of the Kepler conjecture", *Ann. Math.* **162** (2005), 1065–1185. Section 5 of this paper is jointly authored with Samuel P. Ferguson.

The Kepler conjecture asserts that the densest three-dimensional sphere packing is attained by the cannonball packing. This 400-year-old problem, going back to Kepler in 1611, was mentioned as part of Hilbert's eighteenth problem. The proof of this result is a landmark achievement.

These two authors used experimental methods to formulate a local density inequality that would both establish the result and be provable by a computation of feasible length. Laszlo Fejes Tóth suggested in the 1950s that it might be possible to prove Kepler's conjecture by establishing a local inequality that would simultaneously be maximized at every sphere center in the cannonball packing. Local inequalities at a single sphere center can in principle be proved by maximizing a nonlinear function over a compact set, but in practice the resulting problems are too large to be computationally feasible. One of the contributions of this work was to find a way to obtain a computationally feasible problem. In the early 1990s Hales began an approach to formulating suitable local density inequalities that combined information from both the Voronoi and Delaunay triangulations associated to the sphere centers. A very delicate balance is needed between their contributions to obtain a suitable inequality, which was arrived at by computer experiments. Although Samuel Ferguson is credited only with one section of the cited paper, he made essential contributions on the theoretical and experimental side, in formulating the local density inequality used and in proving the most difficult special case of it.

The cited paper elegantly describes the main theoretical structure of the proof. It formulates a novel and complicated local density inequality and shows that its proof would establish Kepler's conjecture. The proof of the local inequality reduces to a very large nonlinear optimization problem of minimizing a function over a compact region consisting of many connected components of high dimension. The authors introduce decomposition methods that simplify the optimization. The optimization is checked analytically in neighborhoods of the two global minima, and after many reductions the remainder is checked by computer; there are thousands of cases. The cited paper presents an extensive road map of this proof and includes motivation for the truth of the given local inequality. A more detailed version appears in six papers published in *Discrete and Computational Geometry* **36** (2006), 5–265, four authored by T. C. Hales, one by S. P. Ferguson alone, and one joint paper which formulates the precise local inequality.

Some controversy has surrounded this proof, with its large computer component, concerning its reliable checkability by humans. Addressing this issue, Hales has an ongoing project, called the "Flyspeck" project, whose object is to construct a "second-generation" proof which is entirely checkable by computer in a formal logic system.

Biographical Sketch: Thomas C. Hales

Thomas C. Hales received his master's degree from Stanford University in the School of Engineering and his Ph.D. from Princeton University in mathematics in 1986 under Robert Langlands. He has held positions at Harvard University, the University of Chicago, and the University of Michigan. He is currently the Andrew Mellon Professor of Mathematics at the University of Pittsburgh. His honors include the Chauvenet Prize (2003) of the Mathematical Association of America and the R. E. Moore Prize (2004) for applications of interval analysis. His research interests include representation theory, motivic integration, discrete geometry, and formal proof theory.

Response: Thomas C. Hales

It is an honor to be a recipient of the David P. Robbins Prize. Without the fundamental contributions of my collaborator, Samuel P. Ferguson, the Kepler conjecture would still be unsolved. He made essential contributions to the formulation of the local density inequality and to the computer algorithms that were used. He solved the most difficult case that arises in the proof. I am proud to share the prize with him.

The solution to the Kepler conjecture relies on fundamental advances by many researchers in several domains. It is a pleasure to acknowledge the many researchers who developed algorithms that permit the rapid solution of largescale linear programs, those who developed the tools of interval computations, and L. Fejes Tóth, who had the original vision about how the Kepler conjecture might be solved by computer. Finally, I wish to thank my colleagues in the formal theorem-proving community for elevating computer proofs to unprecedented levels of mathematical rigor.

Thomas C. Hales

Samuel Ferguson

Biographical Sketch: Samuel Ferguson

Samuel Ferguson earned a B.S. in mathematics at Brigham Young University in 1991. A Research Experience for Undergraduates program at the College of William and Mary provided support for his interest in pursuing graduate studies in mathematics. He earned his Ph.D. in 1997 at the University of Michigan, working with Tom Hales. He is currently employed by the National Security Agency.

Response: Samuel Ferguson

I am honored to have been selected for this award. Having met David Robbins and being familiar with some of his remarkable work makes this award all the more meaningful. I wish to express my gratitude to everyone who has helped me along the way, from my parents and siblings, to teachers, mentors, and friends. Thank you.