2007 Steele Prizes

The 2007 Leroy P. Steele Prizes were awarded at the 113th Annual Meeting of the AMS in New Orleans in January 2007.

The Steele Prizes were established in 1970 in honor of George David Birkhoff, William Fogg Osgood, and William Caspar Graustein. Osgood was president of the AMS during 1905–1906, and Birkhoff served in that capacity during 1925–1926. The prizes are endowed under the terms of a bequest from Leroy P. Steele. Up to three prizes are awarded each year in the following categories: (1) Lifetime Achievement: for the cumulative influence of the total mathematical work of the recipient, high level of research over a period of time, particular influence on the development of a field, and influence on mathematics through Ph.D. students; (2) Mathematical Exposition: for a book or substantial survey or expository-research paper; (3) Seminal Contribution to Research: for a paper, whether recent or not, that has proved to be of fundamental or lasting importance in its field, or a model of important research. Each Steele Prize carries a cash award of US$5,000.

The Steele Prizes are awarded by the AMS Council acting on the recommendation of a selection committee. For the 2007 prizes, the members of the selection committee were: Rodrigo Banuelos, Daniel S. Freed, John B. Garnett, Victor W. Guillemin, Nicholas M. Katz, Linda P. Rothschild (chair), Donald G. Saari, Julius L. Shaneson, and David A. Vogan.

The list of previous recipients of the Steele Prize may be found on the AMS website at `http://www.ams.org/prizes-awards`.

The 2007 Steele Prizes were awarded to David B. Mumford for Mathematical Exposition, to Karen K. Uhlenbeck for a Seminal Contribution to Research, and to Henry P. McKean for Lifetime Achievement. The text that follows presents, for each awardee, the selection committee's citation, a brief biographical sketch, and the awardee's response upon receiving the prize.

Mathemamatical Exposition: David B. Mumford

Citation

The Leroy P. Steele Prize for Mathematical Exposition is awarded to David Mumford in recognition of his beautiful expository accounts of a host of aspects of algebraic geometry. His *Red Book of Varieties and Schemes*, which began life over forty years ago, introduced successive generations of beginning students to "modern" algebraic geometry and to how the "modern" theory clarifies classical problems. Students could then go on to his 1970 book *Abelian Varieties*, where the whole theory is developed "without the crutch of the Jacobian", and which remains the definitive account of the subject. Here again the classical theory is beautifully intertwined with the modern theory, in a way which sharply illuminates both. Students who wanted to learn about the crutch of the Jacobian had to wait for his 1974 Michigan lectures *Curves and their Jacobians*, now reprinted with the latest reedition of the *Red Book*. Two years later saw the appearance of *Complex Projective Varieties*. And the years 1983–1991 saw the appearance of his three-volume *Tata Lectures on Theta Functions*. In all of these books, there is constant interaction between modern methods and classical problems, leading the reader to a deeper appreciation of both. This modern-classical interaction also underlies, at the more abstruse level, his 1965 book *Geometric Invariant Theory* and his 1966 book *Lectures on Curves on an Algebraic Surface*, a pair of books which provided many advanced readers their baptism by fire into the world of moduli spaces. All of these books are, and will remain for the foreseeable future, classics to which the reader returns over and over.

Biographical Sketch

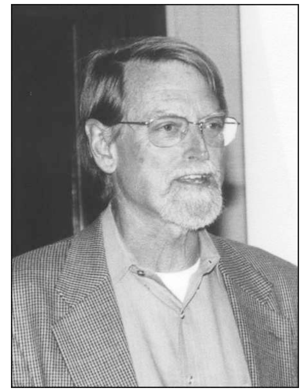
David Mumford was born in Sussex, England, in 1937, but grew up in the U.S. from 1940 on. He went to Harvard University as a freshman in 1953 and stayed there until 1996, working up through the ranks. He was awarded a Fields Medal in 1974, was chair of the Department of Mathematics in 1981–84, and became also a member of the Division of Applied Sciences in 1985. In 1996, he moved to the Division of Applied Mathematics at Brown University, joining its strong interdisciplinary program. He delivered the AMS Gibbs Lecture entitled "The shape of objects in two and three dimensions" in 2003.

His research was in algebraic geometry from roughly 1960 to 1983. His focus was the construction and analysis of moduli spaces, especially that of curves and abelian varieties. From 1984 to the present, his research has concerned the construction of mathematical models for the understanding of perception, a field called Pattern Theory by its founder Ulf Grenander. Mumford's focus here has been the modeling of vision by computer and in the animal brain, especially statistical models.

Response

I am very honored by receiving this prize and also, as it is many years since I worked in algebraic geometry, very surprised to hear that people still read my books on the subject. The subject has grown in so many exciting and unexpected ways in the last few decades. It may be of some interest to recall what the state of that field was when I was a graduate student in the 1950s. Firstly, it was said that, between them, Zariski and Weil knew everything about the field and, if neither of them knew some fact, it was probably wrong or unimportant. But one thing they were both struggling with was finding a language in which they could express both characteristic p geometry and the arithmetic structures which bound it with characteristic 0, yet retain the geometric intuition which had so often driven the field. When pressed, all of his students had seen Zariski draw a small lemniscate on the corner of the blackboard, away from the mass of algebraic formulas, to revive his geometric intuition. Then Grothendieck arrived on the scene and with a simplicity that was pure genius defined the concept "spec", saying that all prime ideals were to be treated as points. About this time, I was reading, in Klein's history of nineteenth century mathematics, how Kronecker had started on the same road of integrating number theory and geometry—"*Es bietet sich da ein ungeheurer Ausblick auf ein rein theoretisches Gebiet* [It offered an enormous view of a purely theoretical area]". Well, that was what we grad students thought too!

But I loved pictures. I drew cartoons like those in the accompanying figure in my *Red Book* showing how everyone probably thought about schemes. I was amused when a book on *Five Centuries of French Mathematics* asked to include these cartoons with the description: "*Par nature, la notion de schema est trop abstraite pour être reellement 'desinée'. Ces dessins sont dus à l'auteur d'un des rares livres de géométrie algébrique qui osa se lancer dans telle aventure* [By its nature, the notion of scheme is too abstract to really be 'drawn'. These drawings are by an author of one of those rare books of algebraic geometry that dared to fling itself into such an adventure]". After all, it was the French who started impressionist painting and isn't this just an impressionist scheme for rendering geometry?



David B. Mumford

The connections between traditional Italian algebraic geometry and Grothendieck's ideas continued to fascinate me. My book *Lectures on Curves on an Algebraic Surface* was written to show how wonderfully Grothendieck's ideas had completed one of the great quests of the Italian geometers. That was the problem of relating two ways of measuring the "irregularity" of an algebraic surface: could you find algebraic but not linear families of divisors whose dimension was H^1 of the structure sheaf (they called it $p_a - p_g$)? Over the complex numbers, the theory of harmonic forms had come to the rescue and proved this, but they sought an algebraic proof. Grothendieck, by representing functors defined on arbitrary schemes, had, *in passing*, solved this. All you needed at the end was the simple fact that characteristic 0 group schemes were reduced and out it pops. What a triumph for the great abstraction with which he formulated mathematics.

One of the most moving sequels for me was that these books were translated into Russian—several of them by Manin himself—and reached what was then the isolated school of Russian algebraic geometers. I want to thank both Manin and my many co-authors, Ash, Bergman, Fogarty, Kempf, Knudsen, Kirwan, Nori, Norman, Ramanujam, Rapaport, Saint-Donat, and Tai, who have added wonderful material. Writing books is often a team effort and working with all these collaborators has been a major stimulus for me. I am deeply grateful to them all and to the prize committee for this recognition.

Seminal Contribution to Research: Karen K. Uhlenbeck

Citation

The 2007 Steele Prize for a Seminal Contribution to Mathematical Research is awarded to Karen Uhlenbeck for her foundational contributions in analytic aspects of mathematical gauge theory. These results appeared in the two papers:

1) "Removable singularities in Yang-Mills fields", *Comm. Math. Phys.* **83** (1982), 11–29; and

2) "Connections with bounds on curvature", *Comm. Math. Phys.* **83** (1982), 31–42.

Connections are local objects in differential geometry, just as functions are local. But there are two crucial differences. First, connections admit automorphisms, called gauge transformations. Thus there are several different local representations of a connection. Second, the basic elliptic equation on functions—the Laplace equation—is linear whereas its counterpart on connections—the Yang-Mills equation—is nonlinear and not even elliptic as it stands. Its nonellipticity is tied up with the existence of automorphisms. One of Uhlenbeck's fundamental results proves the existence of good local representatives for connections, called Coulomb gauges. The Yang-Mills equations become elliptic when restricted to Coulomb gauges, and so Uhlenbeck deduces many basic theorems: smoothness of solutions, compactness of solutions with bounds on the curvature, etc. Uhlenbeck also proves that solutions to the Yang-Mills equations defined on a punctured ball with suitable boundedness on the curvature extend over the puncture. (Compare the much easier Riemann removable singularities theorem in complex analysis.) These theorems and the techniques Uhlenbeck introduced to prove them are the analytic foundation underlying the many applications of gauge theory to geometry and topology. The most immediate was Donaldson's work in the 1980s on smooth structures on 4-manifolds through invariants of the anti-self-dual equations, a system of first-order partial differential equations closely related to the Yang-Mills equations. Recall that Donaldson proved the existence of topological 4-manifolds which admit no smooth structure and topological 4-manifolds which admit inequivalent smooth structures. These equations have also advanced the theory of stable vector bundles in algebraic geometry. The analysis of various dimensional reductions of the anti-self-dual equations—the monopole and vortex equations and other closely related equations of gauge theory—begins with Uhlenbeck's theorems. More recently, these gauge theoretic ideas have yielded new insights in symplectic and contact geometry.

Karen K. Uhlenbeck

Biographical Sketch

Karen K. Uhlenbeck spent her early years in New Jersey, after which she attended the University of Michigan. She received her Ph.D. in 1968 under the direction of Richard Palais at Brandeis University. She has held posts at the Massachusetts Institute of Technology, the University of California at Berkeley, the University of Illinois in both Champaign-Urbana and Chicago, and the University of Chicago. Since 1988 she has held the Sid W. Richardson Foundation Regents Chair in Mathematics at the University of Texas in Austin.

Uhlenbeck is a member of the National Academy of Sciences and the American Academy of Arts and Sciences. She received a MacArthur Prize Fellowship in 1983, the Commonwealth Award for Science and Technology in 1995, and the National Medal of Science in 2000. Uhlenbeck is a co-founder of the IAS/Park City Mathematics Institute and the program for Women and Mathematics in Princeton.

Response

I thank the American Mathematical Society, its members and the Steele Prize committee for the honor and the award of the Steele Prize.

This honor confirms what I have been suspecting for quite some time. I am becoming an old mathematician, if I am not already there. It gives me cause to look back at my research and teaching. All in all, I have found great delight and pleasure in the pursuit of mathematics. Along the way I have made great friends and worked with a number of creative and interesting people. I have been saved from boredom, dourness, and self-absorption. One cannot ask for more.

My mathematical career has intersected some exciting mathematical changes. My thesis, written under Richard Palais, was written in the thick of the days of "Global Analysis", a period in which the tools and methods of differential topology were applied to analysis problems. This fell into disfavor, but it must be admitted that these ideas are today taken as a matter of course as part of the subject of analysis. During my days as an analyst, I wrote a paper on the regularity of elliptic systems, which I still think is the hardest paper I ever wrote.

The next revolution was single-handedly sponsored and spearheaded by S. T. Yau, who introduced techniques of analysis into the problems of topology, differential geometry, and algebraic geometry. Mind you, S. T. Yau was quite something in his younger days! I am quite proud of the paper I wrote with Jonathan Sachs on minimal spheres. Next we come to the introduction of gauge theory into topology, where I did the work which is cited in the award. I had started work on the analysis of gauge theory after hearing a lecture by Michael Atiyah on gauge theory at the University of Chicago, and was fully prepared to understand the thesis of his student Simon Donaldson, which used the two papers cited in this award. The work of Donaldson and Cliff

Taubes, whom I met when he was still a graduate student, was the start of a new era in four-manifold topology. Finally, due to what was now an addiction to intellectual excitement, I tried to follow the influence of physics on geometry which is associated with the name of Ed Witten. My work in integrable systems grew out of this connection with physics. This part of my career was not entirely successful. The more physics I learned, the less algebraic geometry I seemed to know.

Given that I started my academic career in the late 1960s at the University of California, Berkeley, during the Vietnam War, where protests and tear gas were commonplace, it must be said that I rarely found mathematics and the academic life boring.

The accomplishments of which I am most proud are not exactly mathematical theorems. One does mathematics because one has to, and if it is appreciated, all the better! However, encouraged by my young and enthusiastic colleague Dan Freed, I became involved in educational issues. We were among the founders of the IAS/Park City Mathematics Institute. The original intent was to bring mathematics researchers, students, and high school teachers together. This is now an ongoing institution with a yearly summer school, overseen by the Institute for Advanced Study in Princeton. The Women and Mathematics Program at IAS is an outgrowth of the Park City Institute. Founded by my collaborator Chuu-Lian Terng and me, the original purpose was to encourage and prepare more women to take part in the Park City Summer School. It has now grown to a self-sufficient two-week yearly program sponsored by IAS. I watch with real delight the emergence of our graduates into prominence in the mathematics community.

Another outcome of this involvement with education is our Saturday Morning Math Group at the University of Texas. We started this in conjunction with the beginnings of Park City. It is now an ongoing program which our graduate students organize for local high school students. It is often cited and much boasted of by our university. Finally, I would like to boast further of my department at the University of Texas. During the years that I have held an endowed chair in this department, we have become one of the leading departments of mathematics, admittedly below the top ranked, but still quite respectable. Certainly this is due mostly to my colleagues but I take a little credit. Our primary benefactor is also due some praise. We used to "thank Peter" after a particularly enjoyable colloquium talk and dinner and I do again now.

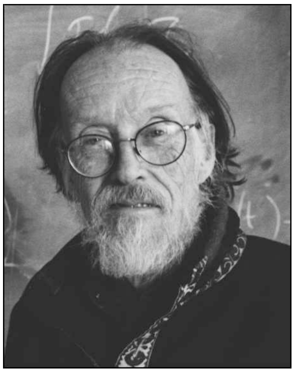
Starting from my days in Berkeley, the issue of women has never been far from my thoughts. I have undergone wide swings of feeling and opinion on the matter. I remain quite disappointed at the numbers of women doing mathematics and in leadership positions. This is, to my mind, primarily due to the culture of the mathematical community as well as harsh societal pressures from outside. Changing the culture is a momentous task in comparison to the other minor accomplishments I have mentioned.

I want to end by thanking my thesis advisor, Richard Palais, my two present collaborators Chuu-Lian Terng and Andrea Nahmod, my longtime friend and supporter, S. T. Yau, my colleagues, particularly Dan Freed and Lorenzo Sadun as well as all my collaborators, Ph.D. students, and assistants. My husband, Bob Williams, is due a share in this award.

Lifetime Achievement: Henry P. McKean

Citation

McKean launched his rich and magnificent mathematical career as an analytically oriented probabilist. After completing his thesis, which is motivated by, but makes essentially no explicit use of, probability theory, he began his collaboration with K. Itô. Together, he and Itô transformed Feller's analytic theory of one dimensional diffusions into probability theory, a heroic effort that is recorded in their famous treatise *Diffusion Processes and Their Sample Paths* [Die Grundlehren der mathematischen Wissenschaften, Band 125, Springer-Verlag, 1974]. After several years during which he delved into a variety of topics with probabilistic origins, spanning both Gaussian and Markov processes and including the first mathematically sound treatment of "American options", I. M. Singer deflected McKean's attention from probability and persuaded him to turn his powerful computational skills on a problem coming from Riemannian geometry. The resulting paper remains a milestone in the development of index theory.



Henry P. McKean

After moving to the Courant Institute, McKean played a central role in the creation of the analytic ideas which underpin our understanding of the KdV and related nonlinear evolution equations, and here again his computational prowess came to the fore. In recent years, McKean has returned to his probabilistic past, studying measures in pathspace, which are the "Gibbs" state for various nonlinear evolutions.

McKean has had profound influence on his own and succeeding generations of mathematicians. In addition to his papers and his book with Itô, he has authored several books that are simultaneously erudite and gems of mathematical exposition. Of particular importance is the little monograph in which he introduced Itô's theory of stochastic integration to a wide audience. As his long list of students attests, he has also had enormous impact on the careers of people who have been fortunate enough to study under his direction.

Biographical Sketch

Henry McKean was born in Wenham, Massachusetts, in 1930. He graduated from Dartmouth College (A.B. 1952), spent a year at Cambridge University (1952–53), then went to Princeton University (Ph.D. 1955). He worked at the Massachusetts Institute of Technology (1958–66), at Rockefeller University (1966–70), and since then at the Courant Institute, of which he was director (1988–92). In the year 1979–80 he was George Eastman Professor at Balliol College, Oxford. He is a member of the American Academy of Arts and Sciences and of the National Academy of Sciences and received a Doctor Honoris Causa from the University of Paris in 2002.

Introduced to probability by M. Kac (summer school, MIT, 1949), McKean continued in this subject for some twenty-five years with W. Feller (1953–56) and in a long collaboration with K. Itô (1952–65), including a visit to Kyoto (1957–58). In 1974 his interests shifted to Hamiltonian mechanics, in particular, to the application of infinite-genus projective curves to KdV, on which he spoke at the International Congress of Mathematicians in Helsinki in 1978, in parallel to S. P. Novikoff's report on the same topic. Now he alternates between "KdV and all that" and his old affection for Brownian motion.

Response

I have been lucky in my mathematical life. Now comes this new piece of luck, the Steele Prize, something I never imagined I might receive and am very grateful for. In school, I really disliked mathematics with its tiresome triangles and its unintelligible x until I began to learn calculus from the amiable Dr. Conwell. Then I saw that you could do something with it, and that was exciting. Besides, I was better at it than the other kids and I liked that *very* much. Coming to Dartmouth (not so much as to learn anything particular, but to ski) I knew I liked mathematics pretty well but decided on it only little by little, thinking I might be an oceanographer. (A skiing accident helped concentrate my mind.) There I started to read P. Levy on Brownian motion, the first love of my mathematical life, and I worked on that and related things with Kac, Feller, Itô, and Levinson who taught me so much, and not just in mathematics. This went on from 1949 to 1972 or so when I began to look for something else to do.

One morning in 1974 Pierre Van Moerbeke came and told me that KdV could be solved by an elliptic function, and being an amateur of these, I sat up, took notice, and made a 90-degree turn into Hamiltonian mechanics and the (to me) very surprising use of infinite-genus projective curves for solving mechanical problems with an infinite number of commuting constants of motion. This led to delightful collaborations with van Moerbeke, Trubowitz, Moser and Airault, Ercolani, and others, building on Peter Lax's deep understanding of the question and paralleling the work of S. P. Novikoff and his school. At the beginning, we knew nothing of algebraic geometry. I remember a private seminar with Sarnak, Trubowitz, Varadhan, and others: how we would get the giggles at how little we understood—except for Sarnak, who was way ahead. Anyhow, a beautiful picture slowly emerged, though it is still a mystery to me what projective curves have to do with all those constants of motion. I mean, why is complex structure hidden there? I suppose it must come from the fact that the "n choose 2" system of vanishing brackets for n constants of motion is vastly over-determined. But that is just one of the queer things about "KdV and all that".

Well then: I have been lucky in my teachers, in my collaborations, and in my students. A few of those last are named above. The others know who they are. My thanks to them all: to those still present and to those present only to memory, of whom I count myself merely the representative in the receipt of this generous prize.