2007 Veblen Prize

The 2007 Oswald Veblen Prize in Geometry was awarded at the 113th Annual Meeting of the AMS in New Orleans in January 2007.

The Veblen Prize is awarded every three years for a notable research memoir in geometry or topology that has appeared during the previous five years in a recognized North American journal (until 2001 the prize was usually awarded every five years). Established in 1964, the prize honors the memory of Oswald Veblen (1880–1960), who served as president of the AMS during 1923–1924. It carries a cash award of US$5,000.

The Veblen Prize is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2007 prize, the members of the selection committee were: Cameron M. Gordon, Michael J. Hopkins (chair), and Ronald J. Stern.

Previous recipients of the Veblen Prize are: Christos D. Papakyriakopoulos (1964), Raoul H. Bott (1964), Stephen Smale (1966), Morton Brown and Barry Mazur (1966), Robion C. Kirby (1971), Dennis P. Sullivan (1971), William P. Thurston (1976), James Simons (1976), Mikhael Gromov (1981), Shing-Tung Yau (1981), Michael H. Freedman (1986), Andrew Casson (1991), Clifford H. Taubes (1991), Richard Hamilton (1996), Gang Tian (1996), Jeff Cheeger (2001), Yakov Eliashberg (2001), Michael J. Hopkins (2001), and David Gabai (2004).

The 2007 Veblen Prize was awarded to two pairs of collaborators: to Peter Kronheimer and Tomasz Mrowka, and to Peter Ozsváth and Zoltán Szabó. The text that follows presents the selection committee's citations, brief biographical sketches, and the awardees' responses upon receiving the prize.

Peter Kronheimer

Tomasz Mrowka

Citation: Peter Kronheimer and Tomasz Mrowka

The 2007 Veblen Prize in Geometry is awarded to Peter Kronheimer and Tomasz Mrowka for their joint contributions to both three- and four-dimensional topology through the development of deep analytical techniques and applications. In particular the prize is awarded for their seminal papers:

"Embedded surfaces and the structure of Donaldson's polynomial invariants", *J. Differential Geom.* **41** (1995), no. 3, 573–734.

Since 1982, most of the progress in four-dimensional differential topology has arisen from the applications of gauge theory pioneered by S. K. Donaldson. In particular, Donaldson's polynomial invariants have been used to prove a variety of results about the topology and geometry of four-manifolds. This paper is one of the pinnacles of this development. It gives a conceptual framework and an organizing principle for some of the disparate observations and calculations of Donaldson invariants that had been made earlier, it reveals a deep structure encoded in the Donaldson invariants which is related to embedded surfaces in four-manifolds, and it has been the point of departure and the motivating example for important further developments, most spectacularly for Witten's introduction of the so-called Seiberg-Witten invariants.

"The genus of embedded surfaces in the projective plane", *Math. Res. Lett.* **1** (1994), no. 6, 797–808.

This paper proves the Thom conjecture, which claims that if C is a smooth holomorphic curve in $\mathbf{C}P^2$, and C' is a smoothly embedded oriented two-manifold representing the same homology class as C, then the genus of C' satisfies $g(C') \geq g(C)$.

"Witten's conjecture and property P", *Geometry and Topology* **8** (2004), 295–310.

Here the authors use their earlier development of Seiberg-Witten monopole Floer homology to prove the Property P conjecture for knots. In other words, if $K \subset S^3$ is a nontrivial knot, and $K_{p/q}$ is the three-manifold obtained by p/q Dehn surgery along K with $q \neq 0$, then $\pi_1(K_{p/q})$ must be nontrivial. The proof is a beautiful work of synthesis which draws upon advances made in the fields of gauge theory, symplectic and contact geometry, and foliations over the past twenty years.

Biographical Sketch: Peter Kronheimer

Born in London, Peter Kronheimer was educated at the City of London School and Merton College, Oxford. He obtained his B.A. in 1984 and his D.Phil. in 1987 under the supervision of Michael Atiyah. After a year as a Junior Research Fellow at Balliol and two years at the Institute for Advanced Study, he returned to Merton as Fellow and Tutor in Mathematics. In 1995 he moved to Harvard University, where he is now William Caspar Graustein Professor of Mathematics. A recipient of the Förderpreis from the Mathematisches Forschungsinstitut Oberwolfach, and a Whitehead Prize from the London Mathematical Society, he was elected a Fellow of the Royal Society in 1997.

Next to mathematics, his main pastime has often been music—the horn in particular, which he studied as a pupil of Ifor James. Peter Kronheimer lives in Newton, Massachusetts, with his wife Jenny and two sons, Matthew and Jonathan.

Biographical Sketch: Tomasz Mrowka

Tom Mrowka is a professor at the Massachusetts Institute of Technology. He received his undergraduate degree from MIT in 1983 and attended graduate school at the University of California at Berkeley, receiving his Ph.D. under the direction of Clifford H. Taubes in 1989. After graduate school he held postdoctoral positions at the Mathematical Sciences Research Institute in Berkeley (1988–89), Stanford (1989–91) and Caltech (1991–92). He held a professorship at Caltech from 1992 until 1996 and was a visiting professor at Harvard (spring of 1995) and at MIT (fall of 1995) before returning to MIT permanently in the fall of 1996.

He received the National Young Investigator Grant of the NSF in 1993 and was a Sloan Foundation Fellow from 1993 to 1995. He gave an invited lecture in the topology section of the 1994 International Congress of Mathematicians in Zurich, the Marston Morse lectures at the Institute for Advanced Study in Princeton in 1999, the Stanford Distinguished Visiting Lecture Series in 2000, the Joseph Fels Ritt Lectures at Columbia University in 2004, and the 23rd Friends of Mathematics Lecture at Kansas State University in 2005.

He works mainly on the analytic aspects of gauge theories and applications of gauge theory to problems in low-dimensional topology.

Response: Peter Kronheimer and Tomasz Mrowka

We are honored, surprised, and delighted to be selected, together with Peter Ozsváth and Zoltán Szabó, as recipients of the Oswald Veblen Prize in Geometry.

The Thom conjecture, and other related questions concerning the genus of embedded 2-manifolds in 4-manifolds, are natural and central questions in 4-dimensional differential topology. After Simon Donaldson's work in gauge theory opened up this field, these problems became tempting targets for the newly available techniques. In the summer of 1989, we were both at MSRI and discussed the idea of using "singular instantons" to prove such conjectures. But it was not until two years later, when we spent a month together at Oberwolfach, that a proof began to emerge of a version of the Thom conjecture for embedded 2-manifolds in *K3* surfaces. This theorem and its proof filled our first two joint papers, and provided the first truly sharp results for the genus problem.

In March 1993, we met at Columbia, at the invitation of John Morgan. We understood that the singular instanton techniques that we had used for the genus problem should lead to universal relations among the values of Donaldson's polynomial invariants for 4-manifolds. At the time, calculating Donaldson's invariants in special cases was a challenging occupation. Although we could see how to prove relations, no coherent picture was emerging; and at the end of this visit, Peter headed to LaGuardia with the forest still not visible for the trees. New York's "Blizzard of the Century" closed the airport, and we worked together for another day, during which we noticed that our relations implied a simple linear recurrence relation for certain values of Donaldson's invariants. This soon led to a beautiful structure theorem for the polynomial invariants in terms of "basic classes", intricately entwined with the genus question through an "adjunction inequality". These developments provided a proof of the Thom conjecture for a large class of algebraic surfaces, though the original version for the complex projective plane had to wait until 1994 and the introduction of the Seiberg-Witten equations.

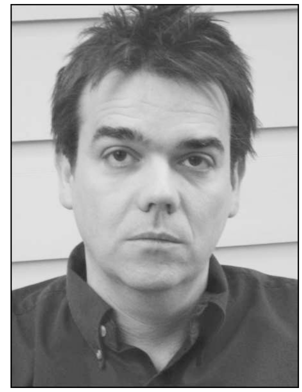
While techniques from gauge theory revolutionized the field of 4-manifolds, providing answers to many important questions, these ideas had no comparable impact in 3-dimensional topology, where the questions and tools remained very different. Around 1986, Andreas Floer used Yang-Mills gauge theory to define his "instanton homology" groups for 3-manifolds. The Euler number of these homology groups recaptured an integer invariant introduced by Andrew Casson which had already been seen to imply results about surgery on 3-manifolds, including partial results in the direction of the "Property P" conjecture. It seems likely that Floer himself foresaw the possibility of using his homology theory to prove stronger results in the same direction; in particular, he established an "exact triangle" relating the Floer homology groups of the 3-manifolds obtained by three different surgeries on a knot. But a missing ingredient at this time was any very general result stating that these homology groups were not trivial. In 1995 Yasha Eliashberg visited Harvard and lectured on his work with Bill Thurston on foliations and contact structures. It was apparent that these results could be combined with work of Cliff Taubes and Dave Gabai to give a non-vanishing theorem for a version of Floer's homology groups defined using the Seiberg-Witten equations, and to show, for example, that these versions of Floer homology encode sharp information about the genus of embedded surfaces in 3-manifolds. This was the first strong indication that, by combining non-vanishing theorems with surgery exact triangles, one would be able to use Floer groups to obtain significant new results about Dehn surgery. This hope was eventually realized in our joint paper with the other co-recipients of this prize, Peter Ozsváth and Zoltán Szabó, on lens space surgeries, and in the eventual resolution of the Property P conjecture using instanton homology. In the meantime, Ozsváth and Szabó's "Heegaard Floer theory" has transformed the field once again: it has led to a wealth of new results, and open problems to attract a new generation of researchers.

Peter would like to thank his wife Jenny and all his family for their love and support; and his mathematical mentors, Michael Atiyah, Simon Donaldson, and Dominic Welsh, for their guidance.

Tom thanks Gigliola, Mario, and Sofia for the joy they bring. Tom also thanks his teachers Victor Guillemin, Richard Melrose, Raoul Bott, Stephen Smale, John Stallings, and Rob Kirby for lighting up very different directions and ways of thinking at the beginning of his mathematical journey. Special thanks to his advisor Cliff Taubes and to John Morgan whose confidence and interest at the beginning of his career were crucial.

Finally we would both like to thank the American Mathematical Society for recognizing the field of gauge theory and low-dimensional topology with this year's Veblen Prize. We feel privileged to be chosen, together with our co-recipients, as representatives for an area of research that has seen so many exciting developments.

Peter Ozsváth



Zoltán Szabó

Citation: Peter Ozsváth and Zoltán Szabó

The 2007 Veblen Prize is awarded to Peter Ozsváth and Zoltán Szabó in recognition of the contributions they have made to 3- and 4-dimensional topology through their Heegaard Floer homology theory.

Ozsváth and Szabó have developed this theory in a highly influential series of over twenty papers produced in the last five years, and in doing so have generated a remarkable amount of activity in 3- and 4-dimensional topology. Specifically, they are cited for their papers

"Holomorphic disks and topological invariants for closed three-manifolds", *Ann. of Math.* (2) **159** (2004), 1027-1158;

"Holomorphic disks and three-manifold invariants: properties and applications", *Ann. of Math.* (2) **159** (2004), 1159-1245;

"Holomorphic disks and genus bounds", *Geometry and Topology* **8** (2004), 311-334.

The Heegaard Floer homology of a 3-manifold plays a role, in the context of the Seiberg-Witten invariants of 4-manifolds, analogous to that played by Lagrangian Floer homology in the context of the Donaldson invariants. There is also a version for knots, whose Euler characteristic is the Alexander polynomial. It detects the genus of a knot and also whether or not a knot is fibered. The combinatorial nature of these invariants has led to many deep applications in 3-dimensional topology. Among these are results about Dehn surgery on knots, such as the Dehn surgery characterization of the unknot, strong restrictions on lens space and other Seifert fiber space surgeries, and dramatic new results on unknotting numbers. Ozsváth and Szabó have used Heegaard Floer homology to define a contact structure invariant, which has led to new results in 3-dimensional contact topology. They have also defined a new concordance invariant of knots, which gives a lower bound on the 4-ball genus. The 4-dimensional version of Heegaard Floer homology has enabled them to give gauge-theory-free proofs of many of the results in 4-dimensional topology obtained in the last decade using Donaldson and Seiberg-Witten theory, such as the Thom Conjecture on the minimal genus of a smooth representative of the homology class of a curve of degree d in CP^2, and the Milnor Conjecture on the unknotting number of a torus knot.

Biographical Sketch: Peter Ozsváth

Peter Ozsváth was born on October 20, 1967, in Dallas, Texas. He received his B.S. from Stanford University (1989) and his Ph.D. from Princeton University (1994) under the direction of John W. Morgan. He held postdoctoral positions at Caltech, the Max-Planck-Institut in Bonn, the Mathematical Sciences Research Institute in Berkeley, and the Institute for Advanced Study in Princeton. He held faculty positions at Princeton University, Michigan State University, and the University of California, Berkeley. He has been on the faculty at Columbia University since 2002.

Ozsváth received a National Science Foundation Postdoctoral Fellowship and an Alfred P. Sloan Research Fellowship. His invited lectures include an Abraham Robinson Lecture at Yale University (2003), a William H. Roever Lecture at Washington University St. Louis (2004), a Kuwait Foundation Lecture at Cambridge University (2006), and a lecture in the topology section of the International Congress of Mathematicians (2006).

Response: Peter Ozsváth

I am greatly honored to be a co-recipient of of the Oswald Veblen Prize, along with my long-time collaborator Zoltán Szabó, and also Peter Kronheimer and Tomasz Mrowka, whose work has always been a profound source of inspiration for me.

Heegaard Floer homology grew out of our efforts at understanding gauge theory from a more combinatorial point of view. The mathematical starting point was Yang-Mills theory and then Seiberg-Witten theory, which started with the work of S. K. Donaldson, A. Floer, and C. H. Taubes. But we have also been fortunate to be able to draw on the work of many interlocking neighboring fields, including contact and symplectic geometry, where at various times the work of Y. Eliashberg and E. Giroux provided answers to fundamental questions, and also three-manifold topology, where the questions raised by C. Gordon serve as a guiding light and the work of D. Gabai provides powerful tools which fit very neatly into the context of Floer homology.

I would like to thank my family and friends for their support throughout the years, and I also owe a great debt of gratitude to my teachers, coauthors, and students. In particular, I thank Zoltán for those many caffeinated mathematical sessions. I also thank my thesis advisor J. W. Morgan for introducing me to gauge theory and T. S. Mrowka, to whom I have turned many times for insight and advice. I would also like to thank R. Fintushel, R. Kirby, and R. Stern for helping to make the field so pleasant socially and so rich mathematically. I am deeply grateful to my undergraduate teachers P. J. Cohen, R. L. Cohen, R. J. Milgram, and fellow student D. B. Karagueuzian, for introducing me to mathematics. I would also like to thank my more recent collaborators C. Manolescu, S. Sarkar, A. Stipsicz, D. P. Thurston, and my former students E. Grigsby, M. Hedden, P. Sepanski, and the many additional members of the Columbia mathematics department who are constantly bringing new insights to an exciting and ever-developing field.

Biographical Sketch: Zoltán Szabó

Zoltán Szabó was born in Budapest, Hungary, in 1965. He did his undergraduate studies at Eötvös Loránd University in Budapest, and then his graduate studies at Rutgers University with John Morgan and Ted Petrie. He has worked at Princeton University since graduating in 1994. He also spent a year at the University of Michigan in 1999–2000. He has been a professor at Princeton University since 2002. He received a Sloan Research Fellowship and a Packard Fellowship. He was an invited lecturer at the 2006 International Congress of Mathematicians in Madrid, and a plenary speaker at the 2004 European Congress of Mathematics in Stockholm. Szabó's main research interests are smooth 4-manifolds, 3-manifolds, knots, Heegaard Floer homology, symplectic geometry, and gauge theory.

Response: Zoltán Szabó

I am greatly honored to be named, along with Peter Kronheimer, Tom Mrowka, and Peter Ozsváth, as a recipient of the Oswald Veblen Prize. The joint work with Peter Ozsváth which is noted here grew out of our attempts to understand Seiberg-Witten moduli spaces over three-manifolds where the metric degenerates along a surface. This led to the construction of Heegaard Floer homology that involved both topological tools, such as Heegaard diagrams, and tools from symplectic geometry, such as holomorphic disks with Lagrangian boundary constraints. The time spent on investigating Heegaard Floer homology and its relationship with problems in low-dimensional topology was rather interesting. I am very glad that this effort was rewarded by the prize committee.

First of all I would like to thank my wife, Piroska, for her support over the years. I also owe a lot to my co-author Peter Ozsváth whose boundless energy made this work possible, and to my thesis advisor, John Morgan, who introduced me to the world of gauge theory.