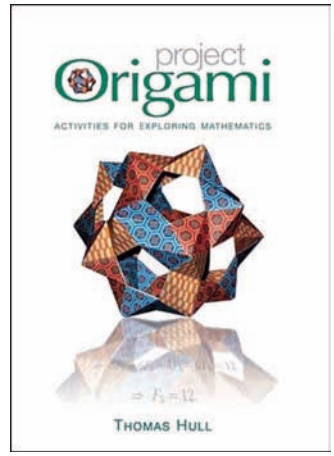
Book Review

Project Origami: Activities for Exploring Mathematics

Reviewed by Helena Verrill



Project Origami: Activities for Exploring Mathematics
Thomas Hull
A K Peters, Ltd., 2006
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Origami often starts with folding a boat or a hat. Traditionally one starts with a single square of paper, which is folded to create a finished model, without the use of scissors or glue. The end result could equally well be a bird or a hexagon; in either case the process of folding involves a latent understanding of simple geometrical concepts. It is not surprising that origami is quite popular with teachers and students in elementary schools. Is it possible to use origami in the higher level mathematics classroom?

An affirmative answer is given in Thomas Hull's book *Project Origami: Activities for Exploring Mathematics*. Based on Hull's extensive experience of combining origami and mathematics teaching over the last fifteen years, it aims to help the teacher bring origami into the mathematics classroom, at the high school, college, and university level. Although there are quite a number of books covering the mathematics of origami, most are more elementary, or not oriented for classroom use. Hull's book is very much intended for classroom use; it is a book of lesson plans, aimed at the teacher. Twenty-two classroom "activities" are presented. Each comes with detailed descriptions, including expected time taken for folding, where to expect students to experience difficulty, how to help them, and a discussion of pedagogy. There are plenty of diagrams and handouts, which can also be downloaded from the publisher's website. Most teachers will not have the luxury of being able to teach a whole course devoted to the mathematics of origami, but an appendix lists which activities might fit into which courses. However, it is inconvenient that the book has no index.

The book gives a broad overview of mathematics applied to origami, presented at a level appropriate for undergraduates, with references to more advanced works. The activities are roughly ordered according to the following sequence of topics:

(1) Constructibility,
(2) Polyhedra,
(3) Flat foldable crease patterns.

Constructibility is covered by the first eight activities. This concerns problems such as determining what points can be constructed as the intersection of two creases, and what shapes can be folded, when only a certain set of folding procedures are allowed. The starting point is the activity of folding an equilateral triangle from a square, followed by successively more and more complex constructions. For example, activity 6 shows a method of combining folding and pencil marks to plot points on a singular cubic curve. Most of these activities would fit nicely into a course on Euclidean geometry in the plane, along with straight-edge and compass constructions. Both origami constructions and straight-edge and compass constructions allow students to learn about theorems and proof; different materials and methods are used but can be applied to the same problems, such as "produce an equilateral triangle". Part of the attraction of activity 5, on the

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simple origami construction of trisection of an angle, is that we know this cannot be done with straight-edge and compass.

Many of these activities could be developed much further and would be ideal starting points for undergraduate projects. A detailed discussion of Galois theory, the usual tool for dealing with geometric constructibility, is beyond the scope of the book, though some indications are given at the end of activity 6. An important issue to be addressed before attempting to prove what can and cannot be constructed in origami, is to determine what exactly are the rules of folding. These are much harder to pin down than the rules of straight-edge and compass constructions. This is discussed in the pedagogy sections of activities 5 and 6. Hull gives a more extensive discussion on his Web pages [Hull], where he describes folding rules known as the "Huzita origami axioms". More recently, a construction called the "multifold" has been investigated by Robert Lang and others, who find that including this fold allows even more possible constructions [Chow and Fan].

Activity 8, with enough material for four classes, introduces "Haga's origamics". Kazuo Haga has developed a series of educational origami activities, aimed at high school students in Japan, and has written two books on the topic, currently available only in Japanese. These activities also involve Euclidean geometry but have a different flavor from the first few activities in the book and are less comparable with straight-edge and compass constructions. A typical problem asks the student to investigate what kinds of polygons can be formed when all four corners of a square of paper are folded to a single point on the paper. The aim of Haga's origamics is to develop scientific reasoning skills, and these problems could be used in courses on an introduction to proof.

Activity 2, on an approximate method of dividing strips into N equal pieces using a limiting process, involves little Euclidean geometry and would fit better into the beginning of a calculus course.

Polyhedra are covered in activities 9 to 14. These are usually constructed by linking together many identically folded units, as in the model shown on the book's cover. This method of construction, modular origami, is an excellent way to produce polyhedra. Many modular origami polyhedra are sturdy, elegant, and often easier to make than gluing together cardboard polygons.

However, in spite of the polyhedron on the front cover, polyhedra are not a central theme. The overall emphasis is on the mathematics of folding, not on how to fold any specific model. Those looking for instructions on how to fold all platonic solids would be better off with one of the many books devoted to origami polyhedra. Of the models that are given, some, such as the five intersecting tetrahedra, look spectacular, but would be infeasible to carry out to completion in a single class; they might be better for a math club activity. Although activity 10 on business card modulars does give a way of producing triangular-faced polyhedra, the results are not very esthetic or stable in general.

It's great to have polyhedra to hand when teaching about symmetries, and the cover dodecahedron model is very nice. Many students do not know what a dodecahedron is, and being introduced to one by folding may be an effective way of interesting a student in these objects. One would not want to spend too much time on folding at the expense of explaining the mathematics of symmetries, so it might be necessary to just start students on the folding, giving them instructions for how to complete the model at home. Although there are alternative methods of producing a dodecahedron, Hull's edge unit has the advantage that it can also be used to create buckyballs and tori. The activities covered using this model introduce several concepts from graph theory, including Hamiltonian cycles and Euler's theorem. Here origami is used as a means to illustrate ideas from other areas of mathematics, which it does nicely, but the origami has no more relation to the mathematical concepts than knitting, for example.

Flat foldability and crease patterns are the central theme of the remaining activities. If you fold an origami model, which is "flat" when completed, i.e., it all lies in one plane, as much as possible allowing for the paper thickness, and then open it out again to a flat square of paper, you'll have a pattern of creases. The main question of activities 15 to 22 is whether a given set of crease lines can be obtained in this way. Whereas constructibility questions are concerned with what can be folded using a certain precise set of rules, for these questions the method of folding is largely irrelevant. Activity 16 leads the student through some theorems about necessary conditions for a crease pattern to be flat-foldable, whereas activity 17 covers impossible crease patterns. An omission from this sequence of activities is the map folding problem of finding how many ways a map can be folded up, only folding along the given grid pattern of crease lines.

Whereas the "constructibility" and "polyhedra" activities could be used as peripheral activities in courses on Euclidean geometry, group theory, or combinatorics, in the foldability activities, origami is central. These activities tend to use more advanced mathematics than most of the rest of the book, such as matrices and Gaussian curvature. They also perhaps take the student closest to actual origami research, which generally seems to be carried out by computer scientists and engineers rather than mathematicians.

Drawbacks of folding in a mathematics class. This is not an elementary origami book; some familiarity with folding is assumed. There is no uniform convention for lines used to indicate mountain or valley folds. For inexperienced folders, this could cause problems. A teacher wanting to use origami models in the mathematics classroom would probably want to have at least one more-elementary book for getting some experience with folding. Hull gives suggestions in the introduction, and the book also has an extensive bibliography.

Folding a successful model usually needs precise folds; making corrections can be difficult and leaves unwanted creases, weakening the structure. Students unfamiliar with origami probably won't be aware of these problems until too late and may find it frustrating and off-putting to end up with a crumpled piece of paper rather than the beautiful model they expected. In any case, folding complete models is often too time-consuming to fit into a class, especially since there will likely be some students requiring one-on-one help with folding. Whether or not these issues are a problem may depend on how much models are used in the classes; most activities are not essentially about building models, but about understanding the mathematics of flat-folding origami. However, the finished models are probably the motivation for many students' interest in folding, and students who have not been exposed to origami may be unmotivated in a class on the mathematics of origami that does not cover making some models.

Conclusions. There are various reasons for using this book in a class; one is the concept "origami is fun, let's use it to make mathematics fun!" For some students, particularly mathematics majors, "fun" may be motivation enough to study mathematics; however, such students probably don't need to be lured into mathematics by origami. For others, usefulness and applications are more motivating. Origami does have real world applications, from paper cup designs to airbag folding [Cipra], which would be important to mention in a course devoted solely to the mathematics of origami. Probably this book will appeal most to those teaching mathematics to liberal arts students. Perhaps some students otherwise uninterested in mathematics can be drawn into active participation in mathematics via the medium of origami. Whatever the reason for including origami in a mathematics class, anyone wishing to do so will find many invaluable ideas in this book and will probably discover that there are more possibilities than first imagined.

References

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[Hull] T. Hull, `http://www.merrimack.edu/~thull/OrigamiMath.html`.