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June/July 2007
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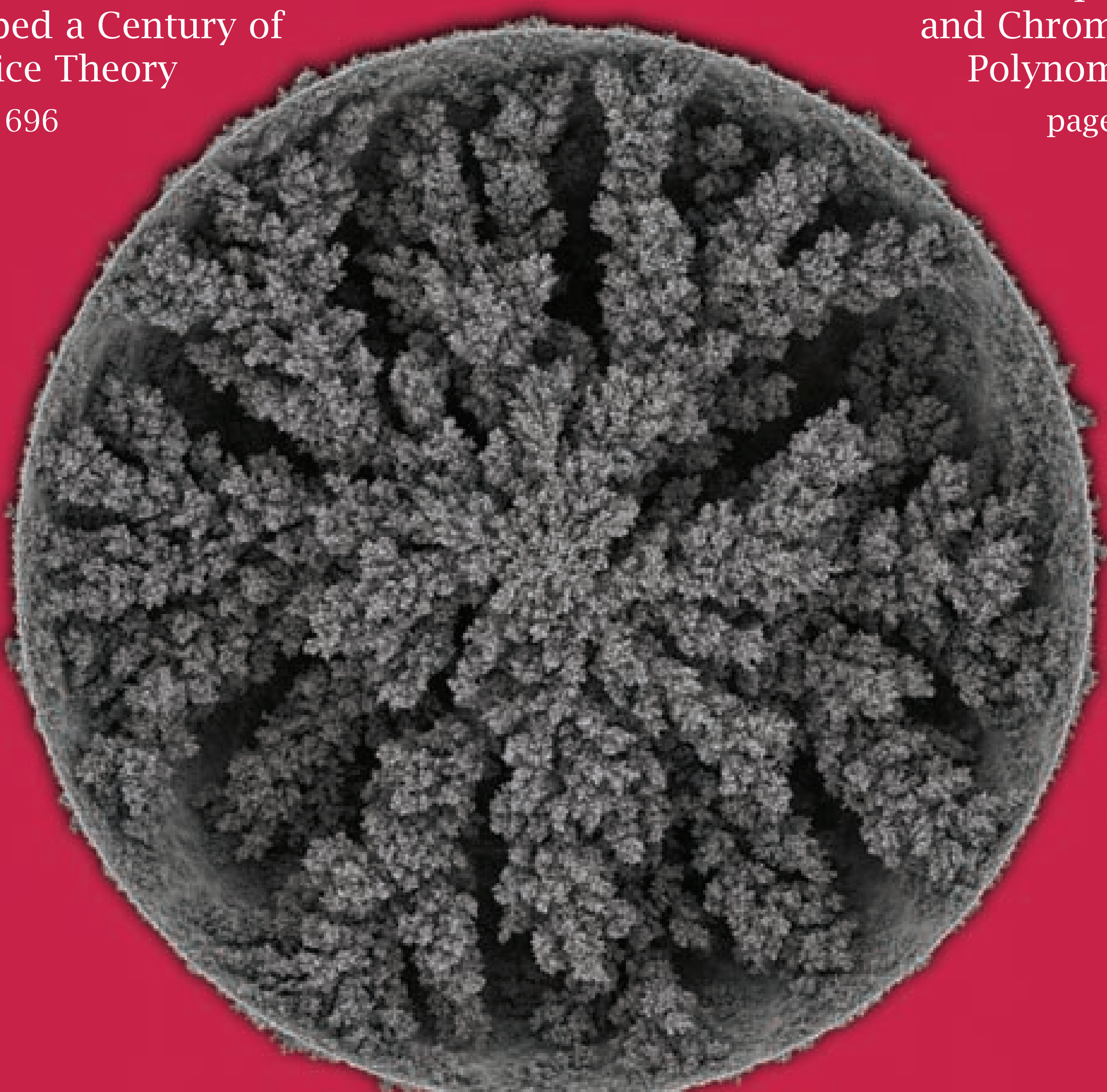
Two Problems That Shaped a Century of Lattice Theory

page 696

Sudoku Squares and Chromatic Polynomials

page 708



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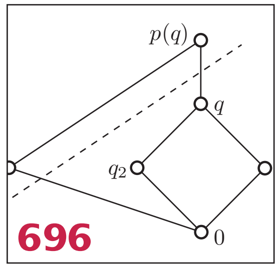

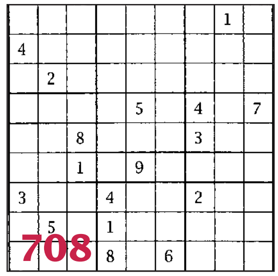

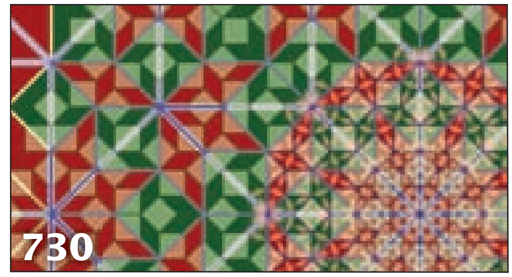

Communications

718 WHAT IS...a Galois Representation?
Mark Kisin

727 IIT Bombay Is Looking for Faculty
Bill Casselman

730 Bridges London, 2006
Mike Field

733 Sum of Its Parts Results in AMS Award for UCLA Math
Lisa Mohan

738 Varadhan Receives 2007 Abel Prize

740 The Mathematical Work of Jon Kleinberg
Gert-Martin Greuel, John E. Hopcroft, Margaret H. Wright

744 The Work of Kyosi Itô
Philip Protter

746 Popping Bubbles
Christopher Tuffley

748 NSF Budget Request for Fiscal Year 2008
Allyn Jackson

752 Cyber-enabled Discovery and Innovation
Allyn Jackson

Commentary

693 Letter from the Editor: Paper *Notices* Shot Down?
Andy Magid

722 *Yearning for the Impossible*—A Book Review
Reviewed by Daniel Biss

724 *John von Neumann: Selected Letters*—A Book Review
Reviewed by George Dyson

Features

696 Two Problems That Shaped a Century of Lattice Theory

George Grätzer

Lattices are ordered sets for which every pair of elements has an upper and a lower bound. That natural examples of lattices, like the power set of a set, have additional properties which may not be consequences of the lattice axioms was recognized early in the twentieth century. As the author shows, basic questions about the relations among these properties have inspired research in the subject down to the present day.

708 Sudoku Squares and Chromatic Polynomials

Agnes M. Herzberg and M. Ram Murty

The authors interpret the nine by nine Sudoku puzzle as a graph coloring problem. Viewed this way, they can generalize the puzzle to arbitrary size. They compute the chromatic number of the resulting graph, and they estimate the number of "Sudoku squares" of any given size.

Notices
of the American Mathematical Society

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Departments

About the Cover 800

Mathematics People 755

2007–2008 AMS Centennial Fellowship Awarded, Allen Receives ACM Turing Award, Ferran Sunyer i Balaguer Prize Awarded, Phùng Awarded von Kaven Prize, Polchinski and Maldacena Awarded Heineman Prize, Shu Receives Computational Science and Engineering Award, 2006 John von Neumann Theory Prize Awarded, Lynch Awarded ACM Knuth Prize, Avila and Petermichl Awarded Salem Prize, Van der Hofstad Awarded Rollo Davidson Prize, Maggioni Awarded Popov Prize, 2007 Clay Research Awards Announced, 2007 CMS Prizes Awarded, Minasyan Awarded Emil Artin Junior Prize, Sloan Fellows Announced, NSF Graduate Research Fellowships Announced, Guggenheim Fellowships Awarded, Fulbright Awards Announced, Putnam Prizes Awarded, Intel Science Talent Search Winners Announced.

Mathematics Opportunities 760

NSF CAREER Program Guidelines Available, Computational Science Training for Undergraduates in the Mathematical Sciences, Call for Nominations for Sloan Fellowships, Call for Nominations for ICTP Ramanujan Prize, Call for Nominations for SASTRA Ramanujan Prize, Call for Nominations for Heineman Prize, News from the CRM Montreal.

For Your Information 762

Statement on E_8 *Read in Congress, Lenstra Appointed ICM-2010 Program Committee Chair, New Director for CRM Barcelona,* NUMB3RS *Receives Award.*

Inside the AMS 763

Epsilon Awards for 2007, Deaths of AMS Members.

Reference and Book List 765

Mathematics Calendar 771

New Publications Offered by the AMS 785

Classified Advertising 799

Meetings and Conferences Table of Contents 815

From the AMS Secretary

Applications and Recommendations Invited for Position of Mathematical Reviews Associate Editor 770

Paper *Notices* Shot Down?

A major party candidate for State Superintendent of Public Instruction last fall here in Oklahoma generated a certain amount of local comment with a campaign proposal to have students in public schools trained to use textbooks as bullet shields in the event of a school shooting incident. The campaign even released a video, which was played on local television news, of the candidate and aides shooting a variety of books with a variety of weapons, including an "AK-47 Kalashnikov" shooting at and through a calculus book. The proposal, which in an Associated Press story the candidate acknowledged could be thought of as "weird, crazy", and the video, generated some light-hearted press coverage, especially about the bullets making it all the way through the calculus book, not many journalists having gone that far. In the candidate's defense, it should be pointed out that the apparent source of the proposal was a child in a school shooting who had been protected by text books in his backpack acting as body armor. Anyway the candidate, who did get 344,000 votes, was soundly defeated.

But naturally I wondered how many math books would be required to stop "AK-47" bullets and decided to experiment, with the help of a local gun club. First a technical point: an AK-47 is a machine gun. It was obvious in the candidate's video that the weapon being used was not a machine gun, but a semi-automatic version of the AK-47. So that's what was used in the experiment. Also, even though expendable calculus books should be easy to come by, and some no doubt deserve blasting, I decided to use another resource. I have accumulated a certain number of surplus copies of the *Notices* for various reasons, and decided to use those instead. For the record, the copies employed had partially water damaged covers. No archival copies of the *Notices* were harmed in this experiment.

Here's the results: at a distance of 20 yards, (military surplus, full metal jacket) bullets from an "AK-47" penetrated to a maximum depth of 4 and 5/32 inches in a stack of *Notices*. I can vouch for the experimental results, but of course the *Notices* are not intended for use as protection against bullets and nothing in this report should be regarded as suggesting or implying such usage.

Target practice or other extreme use aside, deciding what to do with one's *Notices* back issues is not a trivial problem. I've always kept mine, a collection which now runs from January 1969 to the present. Others discard them according to various formulae, for example after a fixed time such as a few years, or a few months. Automatic pre-discarding is apparently not an option, at least according to a friend, an applied mathematician who travels a lot. He was unhappy enough about receiving the *Notices* at all ("filling up his mailbox") that he let his membership in the Society lapse. When contacted by membership services about rejoining, he agreed on the condition that he would not receive any *Notices*, even though his dues included a subscription. And then upon reinstatement we (automatically, I trust) promptly shipped him all the back issues of the *Notices* he missed! Despite this, he remains a Society member, and a friend.

I see my *Notices* collection, like my parallel *Bulletin* collection, sort of like tree rings, visually marking linearly my years as a mathematician. Whenever I actually need to look up something in an old *Notices*, like the rest of the world I turn to the *Notices* area on the AMS website. Here one finds a portable document format file of every *Notices* article. These pdf files are produced from the same files sent to the printer from which the paper *Notices* is produced. So they look exactly the same as the printed article. An issue of the *Notices* is, however, more than just the set of the pdf files of the articles and related editorial matter that appear in it (there are the advertisements, for example). Because the *Notices* is the "journal of record" of the Society, and the Society's bylaws require such a journal, the printed *Notices* has a certain official status.

That status may be about to change. The Society plans to make complete issues of the *Notices*, exact replicas of the printed version, available online as pdf files. Such files then could become the Society's journal of record. Of course there are no plans to discontinue the printed *Notices*. There are some implications in having the official *Notices* online. For example, occasionally we have some color images which we print in black and white because cost and other issues limit the number of color pages. In the official pdf *Notices* online, could those be kept in color? Or what about url references in articles: those could be active links in the online *Notices*. And if minor enhancements like color and active links are reasonable, what about taking serious advantage of having the journal of record *Notices* online, such as animations and hyperlinks? There are technical questions here about what can be done, and financial ones about who is going to do it and what it would cost, but also policy questions, about how things are to be preserved and made accessible to all members. The *Notices* Editorial Board started this discussion at its January 2007 meeting. Although final decisions will properly be made by Society leadership and governance, reader comments to the *Notices* are welcome.

—Andy Magid

A First Course in Operations Research

The lead March feature article about George Dantzig, on page 351, lists early courses in Operation Research. But in January–March 1956, I took a course in OR at Caltech, given by Samuel Karlin. He told us on the first day (January 4) that he thought it was probably the first-ever undergraduate course in OR.

Linear programming was one of the topics covered.

It is peculiar that Karlin's name never appears in the article—for one thing, he left Caltech for Stanford in 1956, ten years before Dantzig (and Cottle) went to Stanford. I suppose that they knew one another, and I wonder why Karlin was omitted.

—Martin C. Tangora
University of Illinois at Chicago
tangora@uic.edu

(Received February 26, 2007)

Is *JAMS* Area-blind?

Ordinarily the solution to an important long-open problem is an occasion for celebration. One of the most famous problems in lattice theory is Dilworth's half-century-old Congruence Lattice Problem, whether the congruence lattices of lattices are exactly the distributive algebraic lattices. In January 2006 Friedrich Wehrung submitted his 14-page solution to the *Journal of the AMS*. At a recent meeting of the full board the editors acknowledged the referees' highest praise but rejected the paper for lack of "interaction with other areas of mathematics".

Lattices arise naturally in many areas of mathematics and have been widely applied in computer science and elsewhere. The congruence lattices of algebras are algebraic (Birkhoff-Frink 1948), and all algebraic lattices so arise (Grätzer-Schmidt 1963). The congruence lattices of lattices are furthermore distributive (Funayama-Nakayama 1942); Dilworth showed in the 1940s that all finite distributive lattices so arise, subsequently extended by Huhn in 1985 to distributive algebraic lattices with $\aleph_1$ compact generators.

Wehrung refuted the general case with an application of Kuratowski's little-known Free Set Theorem. In earlier work he had applied it to measure theory and K-theory, reminiscent of the versatility of Cohen's forcing counterexamples in logic.

Judging from this rejection and the areas represented in recent *JAMS* volumes, the flagship journal of the AMS would appear to specialize in some areas at the expense of others. Whereas fully a quarter of its papers since its 1988 inception have been in algebraic geometry and number theory, some areas including lattice theory aren't even on *JAMS*'s radar.

Yet *JAMS*'s masthead mission statement, "This journal is devoted to research articles of the highest quality in all areas of pure and applied mathematics," implies that it is area-blind. *JAMS* could change the statement, but then what would the AMS be without a journal in which the leading results in all areas can compete on a level playing field?

On behalf of the area of lattice theory, the undersigned therefore petition the AMS to encourage *JAMS* to live up to its mission statement.

More information about the Congruence Lattice Problem and its solution can be found at http://clp.stanford.edu.

—Brian Davey
La Trobe University
b.davey@latrobe.edu.au

—Melvin Henriksen
Harvey Mudd College
henriksen@hmc.edu

—Petar Marković
University of Novi Sad
pera@im.ns.ac.yu

—Vaughan Pratt
Stanford University
pratt@cs.stanford.edu

(Received April 1, 2007)

Reply to Davey, Henriksen, Marković and Pratt

Submissions to *JAMS* are initially handled by individual editors, and only about 15 percent of the most promising manuscripts go to the full editorial board for a final decision. Wehrung's paper was one of these, and the board—consisting at the time of the undersigned—certainly recognized the importance of his work. However we had to make some hard choices, even involving short papers like Wehrung's. After considering the matter quite carefully, we finally decided not to accept the paper.

We would caution against trying to read too much into a single editorial decision. *JAMS* gets substantially more first-rate submissions than we are able to accept, and we end up declining many top-notch papers (often with glowing referee reports) in all areas of mathematics. We appreciate that there can be disagreement about the decisions involved in selecting among outstanding manuscripts. But we reaffirm that *JAMS* is committed to publishing highest-quality research across the full spectrum of mathematics.

—Ingrid Daubechies
Princeton University
ingrid@math.princeton.edu

—Robert Lazarsfeld
University of Michigan
rlaz@umich.edu

—John Morgan
Columbia University
jm@cpw.math.columbia.edu

—Andrei Okounkov
Princeton University
okounkov@math.princeton.edu

—Terence Tao
UCLA
tao@math.ucla.edu

Correction

In the feature article on Oswald Veblen (*Notices*, May 2007), lines 4, 5, and 6 on page 617, column one, should read "Over the summer the Carnegie Corporation and Rockefeller Foundation awarded grants of US$60,000 and US$12,000, respectively", not "Rockefeller Foundation and Carnegie Corporation" as printed.

—Steve Batterson

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Two Problems That Shaped a Century of Lattice Theory

George Grätzer

A century of lattice theory was shaped to a large extent by two problems. This introductory article defines the basic concepts, introduces these two problems, and describes their effect on lattice theory.

In Parts 1 and 3 there is a very brief introduction of the basic concepts. The reader may find a more detailed introduction in Part 1 of my 2006 book, *The Congruences of a Finite Lattice* [8], and complete coverage of the topic in my 1998 book, *General Lattice Theory,* second edition [7].

The two areas we discuss are

Uniquely complemented lattices: discussed in Part 2.

Congruence lattices of lattices: discussed in Part 4.

The two problems, the personalities, and the times are completely different for the solution of these two problems. But they also share a lot. Each problem has been around for half a century. It is not clear—or known—who first proposed the problem. Nevertheless, everybody knows about it. Everybody expects a positive solution. And then somebody overcomes the psychology of the problem and pushes really hard for a negative solution. However, the negative solution turns out to require groundbreaking new ideas and is technically very complicated.

George Grätzer is Distinguished Professor of Mathematics at the University of Manitoba. His email address is `gratzer@ms.umanitoba.ca`*.* *The research of the author was supported by the NSERC of Canada.*

Acknowledgements

I received manuscripts, lecture notes, and corrections from a number of individuals, including H. Lakser, W. A. Lampe, R. Padmanabhan, V. Pratt, R. W. Quackenbush, and F. Wehrung. On matters historical, I was advised by J. B. Nation.

Part 1—Lattice Theory 101

Basic Concepts

Orders. An *order* $\mathbf{A} = \langle A, \leq \rangle$ (or A if $\leq$ is understood) consists of a nonempty set A and a binary relation $\leq$ on A (that is, a subset of A^2)—called an *ordering*—such that the relation $\leq$ is reflexive ($a \leq a$, for all $a \in L$), antisymmetric ($a \leq b$ and $b \leq a$ imply that $a = b$, for all a, $b \in L$), and transitive ($a \leq b$ and $b \leq c$ imply that $a \leq c$, for all a, b, $c \in L$). An order that is *linear* ($a \leq b$ or $b \leq a$, for all a, $b \in L$) is called a *chain*.

In an order P, the element u is an *upper bound* of $H \subseteq P$ iff $h \leq u$, for all $h \in H$. An upper bound u of H is the *least upper bound* of H iff, for any upper bound v of H, we have $u \leq v$. We shall write $u = \bigvee H$. The concepts of *lower bound* and *greatest lower bound* (denoted by $\bigwedge H$) are similarly defined. We use the notation $a \wedge b = \bigwedge\{a, b\}$ and $a \vee b = \bigvee\{a, b\}$ and call $\wedge$ the *meet* and $\vee$ the *join* of the elements a and b.

Lattices. An *order* $\mathbf{L}$ (or L if the $\wedge$ and $\vee$ are understood) *is a lattice* iff $a \wedge b$ and $a \vee b$ always exist. In lattices, the join and meet are both *binary operations*, which means that they can be applied to a pair of elements a, b of L to yield again an element of L. They are *idempotent* ($a \wedge a = a$, $a \vee a = a$, for all $a \in L$), *commutative*

($a \wedge b = b \wedge a$, $a \vee b = b \vee a$, for all $a, b \in L$), *associative* ($(a \wedge b) \wedge c = a \wedge (b \wedge c)$, $(a \vee b) \vee c = a \vee (b \vee c)$, for all $a, b, c \in L$), and together satisfy the *absorption identities* ($a \wedge (a \vee b) = a$, $a \vee (a \wedge b) = a$, for all $a, b \in L$).

An *algebra* $\mathbf{L} = \langle L, \wedge, \vee \rangle$ *is a lattice* iff L is a nonempty set; $\wedge$ and $\vee$ are binary operations on L; both $\wedge$ and $\vee$ are idempotent, commutative, and associative; and they jointly satisfy the two absorption identities.

It is the most intriguing aspect of lattice theory that lattices can be viewed as orders, so we can use order-theoretic concepts (such as completeness; see Part 3); and they are also algebras, so we can use algebraic concepts (such as free lattices).

It is easy to see that a lattice as an algebra and a lattice as an order are "equivalent" concepts. Starting with a poset $\mathbf{L} = \langle L, \leq \rangle$ which is a lattice, set $\mathbf{L}^a = \langle L, \wedge, \vee \rangle$; then $\mathbf{L}^a$ is a lattice. Starting with an algebra $\mathbf{L} = \langle L, \wedge, \vee \rangle$ which is a lattice, set $a \leq b$ iff $a \wedge b = a$; then $\mathbf{L}^p = \langle L, \leq \rangle$ is an order, and the order $\mathbf{L}^p$ is a lattice. In fact, for an order $\mathbf{L} = \langle L, \leq \rangle$ which is a lattice, $(\mathbf{L}^a)^p = \mathbf{L}$; and for an algebra $\mathbf{L} = \langle L, \wedge, \vee \rangle$ which is a lattice, $(\mathbf{L}^p)^a = \mathbf{L}$.

If K and L are lattices as algebras and φ maps K into L, then we call φ a *homomorphism* if $(a \vee b)\varphi = a\varphi \vee b\varphi$ and $(a \wedge b)\varphi = a\varphi \wedge b\varphi$. If the map is one-to-one and onto, it is called an *isomorphism*. If φ is one-to-one, it is called an *embedding*. If K and L are lattices as orders and φ maps K into L, then we call φ an *isomorphism* if it is one-to-one and onto and $a \leq b$ iff $a\varphi \leq b\varphi$. Note that the two isomorphism concepts are equivalent.

Semilattices. An *algebra* $\langle L, \wedge \rangle$ *is a meet-semilattice* iff L is a nonempty set; $\wedge$ is a binary operation on L; and $\wedge$ is idempotent, commutative, and associative. We can introduce meet-semilattices as orders and establish the equivalence of the two approaches as we did for lattices. Similarly, we can define a join-semilattice. A lattice is a meet-semilattice and a join-semilattice defined on the same set that jointly satisfy the two absorption identities.

Examples.

- All subsets of a set, ordered under inclusion; meet is intersection, and join is union.
- All closed subspaces of a topological space, ordered under inclusion; meet is intersection, and join is the closure of the union.
- All continuous functions on the real $[0, 1]$ interval, ordered componentwise.
- All subgroups of a group, ordered under inclusion; meet is intersection, and join the subgroup generated by the union. Similarly, for normal subgroups of a group, ideals of a ring.
- All subspaces of a geometry ordered under inclusion; meet is intersection, and join is the subspace spanned by the union.

And of course everybody knows Boolean algebras (lattices) from logic.

Let me give one more example. An *equivalence relation* ε on a set X is a reflexive, *symmetric* ($a \leq b$ iff $b \leq a$, for all $a, b \in L$), and transitive binary relation. If x and y are in relation ε, that is, $\langle x, y \rangle \in \varepsilon$, we write $x \mathrel{\varepsilon} y$ or $x \equiv y \ (\varepsilon)$. On the set $\operatorname{Part} X$ of all equivalence relations on X, we can introduce an ordering: $\varepsilon_1 \leq \varepsilon_2$ if ε_1 is a refinement of ε_2; that is, $x \mathrel{\varepsilon_1} y$ implies that $x \mathrel{\varepsilon_2} y$. Then $\operatorname{Part} X$ is a lattice, called the *partition lattice* of X. Clearly, $\langle x, y \rangle \in \varepsilon_1 \wedge \varepsilon_2$ iff $\langle x, y \rangle \in \varepsilon_1$ and $\langle x, y \rangle \in \varepsilon_2$. The join, however, is more complicated to describe.

Diagrams. In the order P, *a is covered by b* (in notation, $a \prec b$) iff $a < b$ and $a < x < b$ holds for no x. The covering relation, $\prec$, determines the ordering in a finite order.

The *diagram* of an order P represents the elements with small circles. The circles representing two elements x, y are connected by a straight line iff one covers the other: if x is covered by y, then the circle representing y is higher than the circle representing x. Three examples are shown in Figure 1: the two-element chain, the four-element Boolean lattice, and the partition lattice on a four-element set.

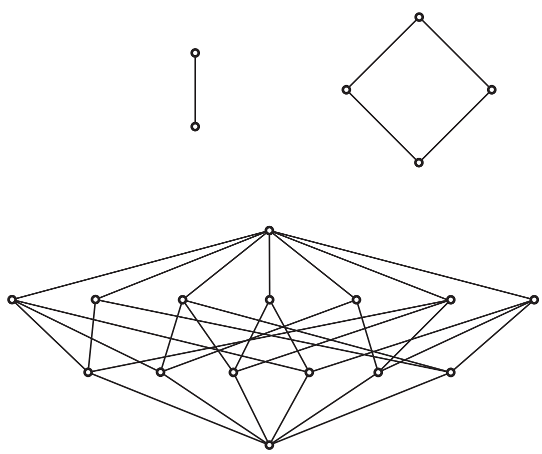

Figure 1. Three lattice diagrams.

Distributive Lattices. A lattice L is *distributive* if

$$x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$$

holds for all $x, y, z \in L$; that is, $\wedge$ distributes over the $\vee$. It is an equivalent condition that $\vee$ distributes over the $\wedge$. *Modularity* is the same, except that we require the identity to hold only for $x \geq z$.

Distributive lattices are easy to construct. Take a set A and a nonempty collection L of subsets of A with the property that if $X, Y \in L$, then $X \cap Y$, $X \cup Y \in L$ (*a ring of sets*). Then L is a distributive lattice, since $\cap$ distributes over $\cup$. It is a result of G. Birkhoff, 1933, that the converse holds: *every distributive lattice is isomorphic to a ring of sets.*

A lattice has a *zero element*, 0, if $0 \leq x$, for all $x \in L$. A lattice has a *unit element,* 1, if $x \leq 1$, for all $x \in L$. A lattice is *bounded* if it has a zero and a unit. A bounded lattice L is *complemented* if for every $x \in L$, there is a $y \in L$ with $x \wedge y = 0$ and $x \vee y = 1$. A *Boolean lattice* is a distributive complemented lattice. Of course, all subsets of a set, ordered under inclusion, make up a Boolean lattice; the complement is the set complement.

Now let B be a Boolean lattice, and let y and z both be complements of the element x. Then $y = y \wedge 1 = y \wedge (x \vee z) = (y \wedge x) \vee (y \wedge z) = 0 \vee (y \wedge z) = y \wedge z$. Similarly, $z = y \wedge z$, and so $y = z$. A Boolean lattice is *uniquely complemented.* So we can consider a Boolean lattice B a *Boolean algebra* $\langle B, \wedge, \vee, ' \rangle$, where $'$ is a unary operation and a' is the (unique) complement of a.

How Lattice Theory Started

Garrett Birkhoff was the founder of modern lattice theory with his 1940 book *Lattice Theory* [1] and with the most influential second edition [2] in 1948 and the third edition [3] in 1967. The first edition was built on a large body of work in the mid- and late 1930s published by him, R. P. Dilworth, O. Frink, J. von Neumann, O. Ore, S. Mac Lane, and others.

The concept of a lattice comes from two sources. The first source is usually cited as R. Dedekind's two classic papers, 1897 and 1900. However, by tracing back the references in these, one can see that R. Dedekind was thinking (modular) lattice-theoretically for at least twenty years prior to that. R. Dedekind took notes at Dirichlet's lectures on number theory and wrote them up as a book with eleven "Supplements", which went through various revisions in the editions of 1863, 1871, 1879, and 1893. Section 169 in Supplement XI of the 1893 edition is about lattices, including the axioms, modular law, duality, and the free modular and distributive lattices on three generators—all developed as properties of modules and ideals. Furthermore, R. Dedekind points out that the lattice terminology (and the modular law) were already in an 1877 paper.

In his 1897 paper, R. Dedekind notes that general lattices were treated by E. Schröder in his famous book *Algebra der Logik* (1880, reprinted in English in 1966) and that this had led him to consider nonmodular lattices.

E. Schröder introduced—but did not name—lattices as orders exactly as we did above, of course, with different notation. There was a well-publicized debate in which C. S. Peirce claimed that all lattices were distributive, but counterexamples were provided by A. Korselt, 1894, R. Dedekind, and E. Schröder. Finally C. S. Peirce explained in a footnote in E. V. Huntington, 1904, that by a lattice he meant something somewhat different.

This debate had a profound effect. While discussing one of E. Schröder's axiom systems for Boolean algebras, E. V. Huntington, 1904, reproduced C. S. Peirce's proof, showing that distributivity can be derived from Schröder's axioms. Then he added the problem which we will state in the next section.

Part 2—Uniquely Complemented Lattices

The Problem

E. V. Huntington, 1904, stated the following problem:

Problem. *Is every uniquely complemented lattice distributive?*

Two series of papers appeared which strengthened the belief that the answer is a resounding YES. The first series was published by a reasonably large group of mathematicians interested in the axiomatics of Boolean algebras. They proved theorems of the type that if we make the complementation just a bit special, we get Boolean algebras. For instance, E. V. Huntington, 1904, published the result that if $\langle B, \wedge, \vee, ' \rangle$ is a uniquely complemented lattice with the property:

$$x \wedge y = 0 \text{ implies that } y \leq x',$$

then $\langle B, \wedge, \vee, ' \rangle$ is a Boolean algebra.

The second series of papers added a condition (P) to the lattices under consideration and concluded that a uniquely complemented lattice satisfying (P) is distributive. We call such properties *Huntington Properties.* The first may have been stated by J. von Neumann and G. Birkhoff with "P = modular". So the result is:

Theorem 1. *A uniquely complemented modular lattice is distributive.*

Here are some examples up to 1945:

- G. Bergman, 1929, "P = uniquely relatively complemented" (for all $a \leq b \leq c$, there is a unique d with $b \wedge d = a$ and $b \vee d = c$).
- G. Birkhoff and M. Ward, 1939, "P = complete and atomic" (a lattice L is *complete* if $\bigwedge X$ and $\bigvee X$ exist for any subset X of L; a lattice L is *atomic* if L has a 0 and for every $a \in L$, $0 < a$, there exists an element p such that $p \leq a$ and p is an atom, that is, if $0 \prec p$).
- R. P. Dilworth, 1940, re-proving G. Birkhoff and M. Ward for "P = finite dimensional".

Dilworth's Bombshell. In 1945, R. P. Dilworth announced a negative solution: there is a nondistributive uniquely complemented lattice. But what he published was so much more:

Theorem 2 (The Dilworth Theorem for Uniquely Complemented Lattices)**.** *Every lattice can be embedded in a uniquely complemented lattice.*

Dilworth was not very clear about the origin of the problem: "For several years one of the outstanding problems of lattice theory has been..." G. Birkhoff in *Mathematical Reviews* wrote: "It has been widely conjectured that..." Neither gives any references as to the origin of the problem. G. Birkhoff and M. Ward, 1933, reference E. V. Huntington, 1904, for the lattice axioms, which Huntington stated as being due to E. Schröder, but not for the problem. If the reader is surprised, I suggest he try to read the original paper of E. V. Huntington, and there he may find the clue. In my earlier papers on the subject, I reference only R. P. Dilworth, 1945, but in my lattice books (e.g., [7]) I give the correct reference. But I have no recollection of reading E. V. Huntington, 1904, until the preparation for this article.

The Solutions

Dilworth's Solution. After a year of attempting to obtain an affirmative solution, R. P. Dilworth decided to construct a counterexample. His paper, providing the result, has four sections.

Section 1 describes the free lattice generated by an order. To illustrate the concept, let us start with the order $P = \{a, b\}$, where a and b are *incomparable* (both $a \leq b$ and $b \leq a$ fail). Then we form $a \wedge b$ and $a \vee b$ and get the first lattice of Figure 2. There are some things to prove: for instance, why is it that we did not have to add the element $a \vee ((a \vee b) \wedge (a \wedge b))$? Answer: because the lattice axioms force that $a \vee ((a \vee b) \wedge (a \wedge b)) = a$.

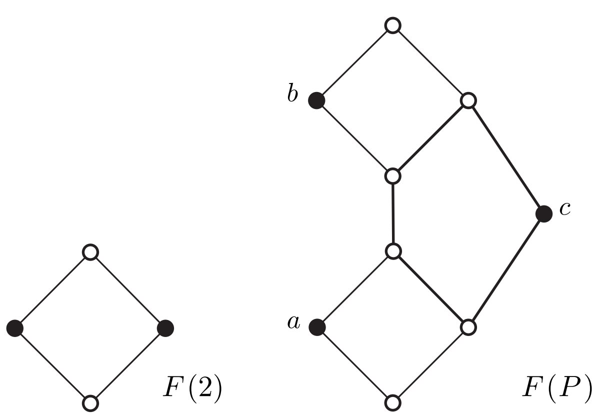

Figure 2. Two free lattices generated by orders.

Next take the order $P = \{a, b, c\}$, where $a < b$. Let us start by adding joins. Since $a \vee b = b$, we add only $b \vee c$ and $a \vee c$; since $\vee$ is idempotent, we do not have to bother with $x \vee x$. So now we have five elements: $a, b, c, a \vee c, b \vee c$, and we start forming meets; we get three new elements: $b \wedge (a \vee c)$, $b \wedge c$, and $a \wedge c$. Now we are back to joins, and we find that we get only one new element: $a \vee (b \wedge c)$. It is now an easy computation to show that the lattice axioms imply that this set of elements is closed under join and meet. We obtain the second lattice of Figure 2, the *free lattice generated by P*. A typical step in the verification is to prove that $((a \vee (b \wedge c)) \vee c = a \vee c$.

R. P. Dilworth starts with an order P, forms the *lattice polynomials*, and then describes when $p \leq q$ is forced by the lattice axioms, for the lattice polynomials p and q. Then introducing $p \equiv q$ if $p \leq q$ and $q \leq p$, he shows the equivalence classes form the free lattice over P.

To freely generate a uniquely complemented lattice, Dilworth needs a unary operation. He denotes it by *, and instead of lattice polynomials, he has to form *operator polynomials,* such as $((a \vee (b^* \wedge c)) \vee b$. He then describes when the equivalence of two operator polynomials p and q is forced by the lattice axioms. This is the hardest part of the paper: Section 2, the main result, needs twenty-three steps, some really technical and ingenious.

So now we have the free lattice with an operator, but of course it is no good, because $(p^*)^*$ is never (almost never) p, as it would be in a uniquely complemented lattice. So in Section 3 Dilworth comes up with a brilliant idea. Let N be the set of all operator polynomials that have *no subpolynomial of the form* $(p^*)^*$. It is easily seen that N defines a sublattice of the free operator lattice, but clearly N is not closed under *. So here is the idea: define a unary operation, $'$, on N. If $p, p^* \in N$, then $p' = p^*$. If $p^* \notin N$, then he gives a clever inductive definition of p' so that $p' \in N$. The main part of this section is this idea. To prove that it works is not that difficult. The heavy lifting was done in Section 2.

Section 4 deals with the free uniquely complemented lattices. The problem is that in the free algebra constructed in Section 3, we do not have $p \wedge p' = 0$ and $p \vee p' = 1$. The bad polynomials are those that contain (or are contained in) a p and p'. So here is the final idea: take those polynomials that have no such bad subpolynomials. It then takes only two pages to compute that these polynomials along with 0 and 1 define the free uniquely complemented lattice over P.

Newer Solutions. Dean's Theorem, 1964, extends Dilworth's free lattice generated by an order P to the free lattice generated by an order P *with any number of designated joins and meets;* we require that they be preserved. Dean's proof is the same complicated induction as the one in R. P. Dilworth, 1945. A greatly simplified proof can be found in H. Lakser's Ph.D. thesis, 1968; see also H. Lakser, 2007, manuscript.

It was around 1966 when I finished my book on universal algebra (it was published in 1968) and I started working on a book on lattice theory. It was clear to me that such a book should contain a proof of the Dilworth Theorem for Uniquely Complemented Lattices, but it was also clear that I would need a lattice-theoretic proof. And the most substantial parts of the original proof (Sections 2 and 3) are well beyond the reach of a book on lattices.

So D. Kelly, H. Lakser, C. Platt, J. Sichler, and I started systematically to work on free lattices and free products. The most important influence of R. P. Dilworth's paper came from the motivation and the ideas of Section 1, in particular the concept of covering. So when C. C. Chen came to Winnipeg in 1967, we had already developed a good understanding of this field.

With C. C. Chen our goal was to produce a proof of the Dilworth Theorem for Uniquely Complemented Lattices for my book. This we accomplished in C. C. Chen and G. Grätzer, 1969. We proved that the Dilworth Theorem for Uniquely Complemented Lattices can be proved with the ideas of Sections 1 and 4 of Dilworth's paper, completely eliminating Sections 2 and 3, which are very complicated and not lattice-theoretic. The proof is in two steps: the first is based on Section 1, and the second is based on Section 4 of R. P. Dilworth, 1945.

Actually, the result we proved is much stronger than the Dilworth Theorem for Uniquely Complemented Lattices. Let us call a lattice L *almost uniquely complemented* if it is bounded and every element has at most one complement. A $\{0,1\}$-*embedding* is an embedding that maps the 0 to 0 and the 1 to 1.

Theorem 3. *Let L be an almost uniquely complemented lattice. Then L can be $\{0,1\}$-embedded into a uniquely complemented lattice.*

R. P. Dilworth's embedding preserves no existing complement.

The method employed in C. C. Chen and G. Grätzer, 1969, was generalized to ($\mathcal{C}$-reduced) free products in G. Grätzer, 1971 and 1973, with some interesting applications in G. Grätzer and J. Sichler, 1970, 1974, 2000.

A completely different approach was taken by M. Adams and J. Sichler, 1978. They introduced *testing lattices* that allowed them to construct continuumly many such varieties (classes defined by identities) in which the Dilworth Theorem for Uniquely Complemented Lattices holds. See also V. Koubek, 1984.

Newest Solution. As opposed to the two steps of C. C. Chen and G. Grätzer, 1969, in G. Grätzer and H. Lakser, 2006, we provide a one-step solution.

Let K be a bounded lattice. Let $a \in K - \{0,1\}$, and let u be an element not in K. We extend the partial ordering $\leq$ of K to $Q = K \cup \{u\}$ by $0 \leq u \leq 1$. We extend the lattice operations $\wedge$ and $\vee$ to Q as *commutative partial meet and join operations.* For $x \leq y$ in Q, define $x \wedge y = x$ and $x \vee y = y$. In addition, let $a \wedge u = 0$ and $a \vee u = 1$; see Figure 3.

To construct and describe the lattice $F(Q)$ freely generated by Q, we repeatedly form joins and meets of elements of Q, obtaining the *polynomials* over Q, which will represent elements of $F(Q)$. For the polynomials A and B over Q, let $A \leq B$ denote the relation forced by the lattice axioms and the structure of Q. We observe that given any polynomial A, there is a largest element A_* of K with $A_* \leq A$ and a smallest element A^* of K with $A^* \geq A$.

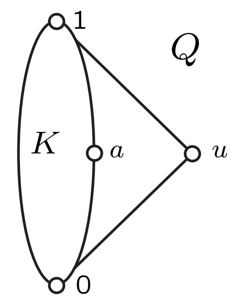

Figure 3. The partial lattice Q.

An easy computation (less than a page) shows that the following statements hold:

(i) *$u \leq u$. If $x \in K$, then $u \leq x$ iff $x = 1$.*
(ii) *$u_* = 0$. If $x \in K$, then $x_* = x$.*
(iii) *$u \leq A \wedge B$ iff $u \leq A$ and $u \leq B$.*
(iv) *$(A \wedge B)_* = A_* \wedge B_*$.*
(v) *$u \leq A \vee B$ iff either $u \leq A$ or $u \leq B$ or $A_* \vee B_* = 1$.*
(vi)

$$(A \vee B)_* = \begin{cases} 1 & \text{if } a \leq A_* \vee B_* \text{ and} \\ & \quad \text{either } u \leq A \text{ or } u \leq B; \\ A_* \vee B_* & \text{otherwise.} \end{cases}$$

The only complement of u is a.

Let A be a polynomial that defines a complement of u. Then

$$1 = (A \vee u)_* = \begin{cases} 1 & \text{if } a \leq A_* \vee u_* = A_*; \\ A_* & \text{otherwise.} \end{cases}$$

So either $a \leq A_*$ or $1 = A_*$; in either case, $a \leq A_*$. Dually, $a \geq A^*$. Thus

$$A \leq A^* \leq a \leq A_* \leq A,$$

and so $A \equiv a$.

Isn't this easy?

It is also easy to show that if K is almost uniquely complemented, then the only other complemented pairs in $F(Q)$ are the complemented pairs in K. Thus if a does not have a complement in K, we get an almost uniquely complemented $\{0,1\}$-extension in which a has a complement. By transfinite induction on the set of noncomplemented elements of K, we get an almost uniquely complemented $\{0,1\}$-extension K_1 of $K_0 = K$, where each element of K_0 has a complement. Then, by a countable induction, we get a uniquely complemented $\{0,1\}$-extension K_ω of $K_0 = K$.

This method has applications that previous techniques could not give (previous techniques can be used only to construct *complements*, not *relative complements*).

Theorem 4. *Let $[a,c] = \{x \in K \mid a \le x \le c\}$ be an interval of a lattice K. Let us assume further that every element in $[a,c]$ has at most one relative complement. Then K has an extension L such that $[a,c]$ in L, as a lattice, is uniquely complemented.*

See G. Grätzer and H. Lakser, 2005. There are many variants of the stronger results; we give only one more example. Let us say that a *bounded lattice* has *transitive complementation* if whenever b is a complement of a and c is a complement of b, either $a = c$ or c is a complement of a. Let us call a lattice L *n-complemented* if every element $a \ne 0,1$ has *exactly n* complements. Similarly, a lattice L is *at most n-complemented* if every element $a \ne 0,1$ has *at most n* complements; example: any at most uniquely complemented lattice. For instance, in a transitively 2-complemented lattice L, every element $a \ne 0,1$ belongs to a (unique) sublattice M_3 (see Figure 4) so that every complement of a is in this sublattice. Then we have (G. Grätzer and H. Lakser, manuscript):

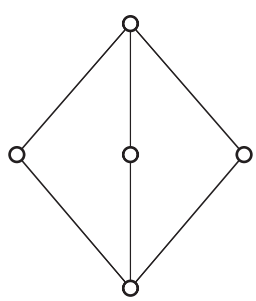

Figure 4. The lattice M_3.

Theorem 5. *Let K be an at most n-complemented and transitively complemented lattice. Then K can be $\{0,1\}$-embedded into a transitively n-complemented lattice.*

For $n = 1$, this gives the C. C. Chen and G. Grätzer, 1969, result and so, in turn, the Dilworth Theorem for Uniquely Complemented Lattices.

Huntington Properties and Varieties

We have seen a few examples of Huntington Properties. Among the structural properties, one of the nicest is from H.-J. Bandelt and R. Padmanabhan, 1979: *every interval contains a covering pair.*

A *Huntington Variety* is a lattice variety in which every uniquely complemented lattice is distributive. W. McCune, R. Padmanabhan, and B. Veroff, 2007, observed that there are continuumly many such varieties.

The forthcoming book of R. Padmanabhan and S. Rudeanu [9] has a chapter on Huntington Varieties. Here is an intriguing result from the book:

Theorem 6. *The variety $\mathbf{M} \vee \mathbf{N}_5$ is Huntington.*

$\mathbf{M} \vee \mathbf{N}_5$ is the smallest variety containing all modular lattices and the 5-element nonmodular lattice N_5.

We conclude this section with one more Huntington Property, an identity, from W. McCune, R. Padmanabhan, and B. Veroff, 2007:

Theorem 7. *Every uniquely complemented lattice satisfying*

$$x \wedge ((y \wedge (x \vee z)) \vee (z \wedge (x \vee y))) = (x \wedge y) \vee (x \wedge z)$$

is distributive.

Concluding Comments

It is interesting how little we know about a subject on which we have published so many papers. Ask any question and probably we do not know the answer. Let me mention a (very) few of my favorite ones.

All known examples of nondistributive uniquely complemented lattices are freely generated, one way or another. Is there a construction of a nondistributive uniquely complemented lattice that is different?

In the same vein, is there a "natural" example of a nondistributive uniquely complemented lattice from geometry, topology, or whatever else?

Is there a complete example of a nondistributive uniquely complemented lattice?

Part 3—Lattice Theory 201

Subalgebra Lattices of Algebras

Let $\mathbf{A} = \langle A, F\rangle$ be an *algebra*; that is, A is a nonempty set and every $f \in F$ is a finitary operation on A. If F is understood, we denote the algebra by A. In the case of lattices as algebras, $F = \{\wedge, \vee\}$, and both operations are binary. Boolean algebras are usually defined as algebras with $F = \{\wedge, \vee, ', 0, 1\}$, where $\wedge$ and $\vee$ are binary operations, $'$ is a unary operation, while 0 and 1 are nullary operations.

A *subalgebra* $B \subseteq A$ is a nonempty subset closed under all the operations in F: that is, if $f \in F$ is n-ary and $b_1, \dots, b_n \in B$, then $f(b_1, \dots, b_n)$ formed in A is in B. So we can regard $\mathbf{B} = \langle B, F\rangle$ as an algebra, a subalgebra of $\mathbf{A}$. The intersection of any set of subalgebras is a subalgebra again, provided that it is nonempty. This leads us to the following notation: Sub A is the set of subalgebras of A;

if the intersection of all subalgebras of A is the empty set, then we add $\emptyset$ to Sub A.

Sub A under containment is an order, in fact, a *complete lattice*. Sub A is *meet-continuous*; that is,

$$X \wedge \bigvee \mathcal{C} = \bigvee (X \wedge C \mid C \in \mathcal{C}),$$

for any chain $\mathcal{C}$ in Sub A.

For any nonempty subset X of A, there is a smallest subalgebra $[X]$ containing X, called the subalgebra generated by X. If a subalgebra B is of the form $[X]$, for a finite X, we call it a *finitely generated subalgebra*.

Following G. Birkhoff and O. Frink, 1948, in a complete lattice L, we call an element x *join-inaccessible* if whenever $x = \bigvee \mathcal{C}$ for a chain $\mathcal{C}$ in L, $x \in \mathcal{C}$. It is easy to see that finitely generated subalgebras are exactly the *join-inaccessible* elements of Sub A. We thus arrive at the breakthrough definition of G. Birkhoff and O. Frink, 1948.

A lattice L is called *compactly generated* if it is complete, meet-continuous, and every element is a (complete) join of join-inaccessible elements. G. Birkhoff and O. Frink, 1948, proved the following result:

Theorem 8 (The Birkhoff-Frink Theorem)**.** *A lattice L is isomorphic to the subalgebra lattice of a finitary algebra iff it is compactly generated.*

Congruence Lattices of Algebras

Let A (that is, $\mathbf{A} = \langle A, F \rangle$) be an algebra. An equivalence relation Θ is called a *congruence relation* if it has the *Substitution Property* for all $f \in F$. If f is n-ary, the Substitution Property for f is the following:

$$a_1 \equiv b_1(\Theta),\ \ldots, a_n \equiv b_n(\Theta)$$
$$\text{imply that} \quad f(a_1, \ldots, a_n) \equiv f(b_1, \ldots, b_n)(\Theta).$$

For $a \in A$, we denote by a/Θ the Θ-class containing a; that is,

$$a/\Theta = \{\, b \in A \mid a \equiv b(\Theta) \,\}$$

and A/Θ is the set of all a/Θ, $a \in A$. On A/Θ we can define the operations $f \in F$ by

$$f(a_1/\Theta, \ldots, a_n/\Theta) = f(a_1, \ldots, a_n)/\Theta,$$

and we get the *quotient algebra* A/Θ (that is, $\mathbf{A}/\Theta = \langle A/\Theta, F \rangle$). This is how we construct quotient groups, quotient rings, quotient lattices, and so on.

Let A be an algebra and let a, $b \in A$. Since the intersection of congruences is a congruence again, there is a smallest congruence $\operatorname{con}(a, b)$ such that $a \equiv b$. We call $\operatorname{con}(a, b)$ a *principal congruence*; they correspond to one-generated subalgebras. Finite joins of principal congruences were called by G. Birkhoff and O. Frink *finitely generated congruences* (today, we call them *compact congruences*); they are like the finitely generated subalgebras.

G. Birkhoff and O. Frink, 1948, observed:

Theorem 9. Con A*, the congruences of the algebra A, form a compactly generated lattice.*

They raised the problem whether the converse is true. The problem is also raised in [2]. Interestingly, neither references G. Birkhoff, 1945, where the problem is first raised for algebras finitary or infinitary.

I remember when we first started thinking about this problem with E. T. Schmidt. Suppose for a compactly generated lattice L that we construct the algebra A. It really bothered me that I did not know how to utilize the meet-continuity of L in proving that L is isomorphic to Con A.

Variants of Compactly Generated Lattices

In the late 1950s it became clear to a number of mathematicians that there are two important variants of the definition of compactly generated lattices. To state them, we need some elementary concepts.

Let L be a complete lattice and $c \in L$. Let us call c *compact* (L. Nachbin, 1959) if whenever $c \leq \bigvee X$, for some $X \subseteq L$, then $c \leq \bigvee X_1$, for some *finite* $X_1 \subseteq X$. We define an *algebraic lattice* as a complete lattice in which every element is a join of compact elements.

Let S be a join-semilattice with zero. An ideal I of S is a subset with three properties:

(i) $0 \in L$.
(ii) If $a, b \in L$, then $a \vee b \in L$.
(iii) If $a \in L$ and $x \leq a$, then $x \in L$.

The set Id S of all ideals of S is an order under containment. It is a compactly generated lattice in which the join-inaccessible elements are the principal ideals: $\downarrow a = \{\, x \in S \mid x \leq a \,\}$, for $a \in S$.

Theorem 10. *The following conditions on a lattice L are equivalent:*

(i) *L is a compactly generated lattice.*
(ii) *L is an algebraic lattice.*
(iii) *L can be represented as the ideal lattice of a join-semilattice with zero.*

The equivalence of the first two conditions was observed by G. Birkhoff [3], while the equivalence of the last two conditions is in L. Nachbin, 1959. Further references: A. Komatu, 1943; R. Büchi, 1952; and G. Grätzer, 1965.

Theorem 10 is quite trivial, but note that the Birkhoff-Frink Theorem can, as a result, be proved in a few lines. Let L be compactly generated. Then $L = \operatorname{Id} S$ for a join-semilattice S with zero. Define on S the $\vee$, and for all a, $b \in S$ define the unary operation $f_{a,b}$ as follows:

$$f_{a,b}(x) = \begin{cases} a \wedge b, & \text{for } x = a; \\ 0 & \text{otherwise.} \end{cases}$$

Set $F = \{\vee\} \cup \{\, f_{a,b} \mid a, b \in S \,\}$. Then the subalgebras of $\langle S, F \rangle$ are exactly the ideals of S;

hence $\mathrm{Sub}\langle S, F\rangle = L$, proving the Birkhoff-Frink Theorem.

Part 4—Congruence Lattices of Lattices

Congruence Lattices of Algebras

G. Birkhoff and O. Frink, 1948, raised the question whether congruence lattices of algebras can be characterized as compactly generated lattices. This problem was earlier raised by G. Birkhoff in a 1945 lecture and again in [2] as Problem 50. This problem was solved in G. Grätzer and E. T. Schmidt, 1963. (In this part, we use the acronym CLCT for the mouthful "Congruence Lattice Characterization Theorem".)

Theorem 11 (CLCT for Algebras). *The congruence lattice of an algebra can be characterized as an algebraic lattice.*

An equivalent formulation is: *Every join-semilattice with zero can be represented as* $\mathrm{Con_c}\,\mathbf{A}$*, the join-semilattice with zero of compact congruences of an algebra* $\mathbf{A}$.

For a long time I had the nightmare that somebody would come along and present a brief construction of the algebra and a few-lines-long proof the way we did for the Birkhoff-Frink theorem. Any takers?

More polished versions of the original proof appeared in G. Grätzer [6]; W. A. Lampe, 1968, 1973; E. T. Schmidt, 1969; E. Nelson, 1980; and P. Pudlák, 1985. Other variants by B. Jónsson and R. N. McKenzie were written up, circulated, but not published.

All these proofs, for larger algebraic lattices L, construct algebras $\langle A, F\rangle$ with more and more operations. This cannot be avoided. R. Freese, W. A. Lampe, and W. Taylor, 1979, proved that if we take the algebraic lattice of all subspaces of an infinite-dimensional vector space over an uncountable field of cardinality $\mathfrak{m}$ and represent it as the congruence lattice of an algebra $\langle A, F\rangle$, then $\mathfrak{m} \leq |F|$.

G. Birkhoff, in his 1945 lecture, raised the question of how we can characterize congruence lattices of not necessarily finitary algebras. The congruence lattice is obviously complete, but we no longer have meet-continuity or compact elements. This was solved in G. Grätzer and W. A. Lampe, 1979:

Theorem 12 (CLCT for Infinitary Algebras). *The congruence lattice of an infinitary algebra can be characterized as a complete lattice.*

In the early 1980s, R. Wille raised a related question (mentioned in K. Reuter and R. Wille, 1987). How can we characterize the lattice of *complete congruences* of a complete lattice? Again, this lattice is obviously complete.

I resolved Wille's problem in G. Grätzer, 1989:

Theorem 13 (CLCT for Complete Lattices). *The lattice of complete congruences of a complete lattice can be characterized as a complete lattice.*

Note that Theorem 12 immediately follows from this result.

A large number of papers (G. Grätzer, 1989, 1990; G. Grätzer and H. Lakser, 1991, 1992; R. Freese, G. Grätzer, and E. T. Schmidt, 1991; G. Grätzer and E. T. Schmidt, 1993 (three papers), 1995 (three papers)) obtained better results. I quote just one from G. Grätzer and E. T. Schmidt, 1993:

Theorem 14 (CLCT for Complete and Distributive Lattices). *The lattice of complete congruences of a complete and distributive lattice can be characterized as a complete lattice.*

I believe that the complete and distributive lattice constructed to prove this theorem is a candidate for the most complicated complete and distributive lattice ever constructed.

Congruence Lattices of Finite Lattices

For a lattice L, the congruence lattice is distributive. This remarkable—but easy to prove—fact was published only in N. Funayama and T. Nakayama, 1942. About the same time R. P. Dilworth discovered—but did not publish—the even more remarkable converse:

Theorem 15 (The Dilworth Theorem for Finite Congruence Lattices). *Every finite distributive lattice can be represented as the congruence lattice of a finite lattice.*

This result was made into an exercise (one with an asterisk, meaning difficult) in [2]. E. T. Schmidt and I got really interested in the result and inquired from G. Birkhoff where the result came from, but he did not know and encouraged us to write to R. P. Dilworth. Unfortunately, Dilworth was busy editing the proceedings of a lattice theory meeting, but eventually we got a response. Yes, he proved the result, and the proof was in his lecture notes. No, copies of his lecture notes were no longer available.

So we published a proof, G. Grätzer and E. T. Schmidt, 1963. In fact, we proved something much stronger. Let us call a lattice L with zero *sectionally complemented* if for every $a \leq b \in L$, there is a $c \in L$ with $a \wedge c = 0$ and $a \vee c = b$. This is what we proved:

Theorem 16 (CLCT for Finite Sectionally Complemented Lattices). *Every finite distributive lattice can be represented as the congruence lattice of a finite sectionally complemented lattice.*

Proof Outline. An element a of a lattice D is *join-irreducible* if $a \neq 0$ and $a = x \vee y$ implies that $a = x$ or $a = y$. The join-irreducible elements of

D form an order $\mathrm{J}(D)$. If D is distributive, $\mathrm{J}(D)$ determines D and every finite order P is representable as the $\mathrm{J}(D)$ of a finite distributive lattice D. For a finite lattice L, let $\mathrm{Con_J}\, L$ denote the order of join-irreducible congruences.

A *chopped lattice* $\langle M, \wedge, \vee \rangle$ is a finite meet-semilattice $\langle M, \wedge \rangle$ in which $\vee$ is a *partial operation* so that whenever $a \vee b$ is defined it is $\bigvee\{a, b\}$. We can define ideals just as for join-semilattices with zero except that the second condition reads: *If a, $b \in I$ and $a \vee b$ exists, then $a \vee b \in I$.* The set $\mathrm{Id}\, M$ of all ideals of M is a lattice, and we can view M as a part of $\mathrm{Id}\, M$ by identifying $a \in M$ with $\downarrow a$.

Then we have (G. Grätzer and H. Lakser, 1968):

Theorem 17. *Every congruence of M has one and only one extension to* $\mathrm{Id}\, M$*. In particular, the congruence lattice of M is isomorphic to the congruence lattice of* $\mathrm{Id}\, M$*.*

So now we can rewrite the theorem we want to prove:

CLCT for Finite Lattices—order and chopped lattice version. *Every finite order P can be represented as* $\mathrm{Con_J}\, M$ *for a chopped lattice M.*

The basic gadget for the construction is the lattice $N_6 = N(p, q)$ of Figure 5. The lattice $N(p, q)$ has only one nontrivial congruence relation, Θ, where Θ is the congruence relation with congruence classes $\{0, q_1, q_2, q\}$ and $\{p_1, p(q)\}$, indicated by the dashed line. In other words, $p_1 \equiv 0$ "implies" that $q_1 \equiv 0$, but $q_1 \equiv 0$ "does not imply" that $p_1 \equiv 0$. We will use the $N_6 = N(p, q)$ to achieve such "congruence-forcing".

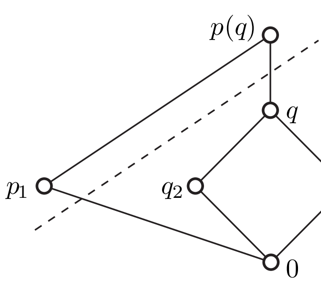

Figure 5. The lattice $N_6 = N(p, q)$ and the congruence Θ.

To convey the idea of the proof, we present two small examples in which we construct the chopped lattice M from copies of $N(p, q)$.

Let $P = \{a, b, c\}$ with $c \prec b \prec a$. We take two copies of the gadget, $N(a, b)$ and $N(b, c)$, and "merge" them to form the chopped lattice M, as shown in Figure 6.

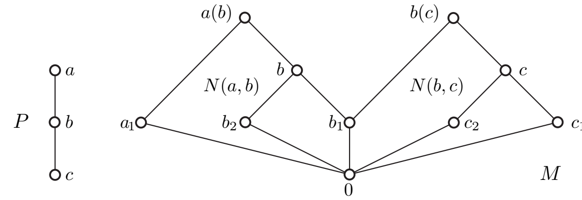

Figure 6. The chopped lattice M for $P = C_3$.

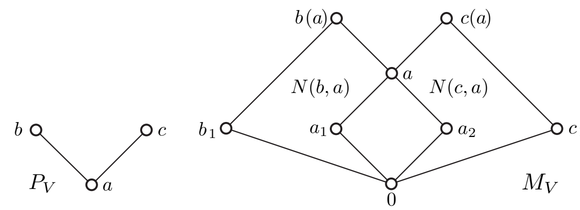

Figure 7. The chopped lattice for the order P_V.

Next consider the three-element order P_V of Figure 7. We take two copies of the gadget, $N(b, a)$ and $N(c, a)$, and "merge" them to form the chopped lattice M_V; see Figure 7.

The reader should now be able to picture the general proof. Instead of the few atoms in these examples, we start with M_0, which has enough atoms to reflect the structure of P. Whenever $b \prec a$ in P, we build a copy of $N(a, b)$ to obtain the chopped lattice M; see Figure 8.

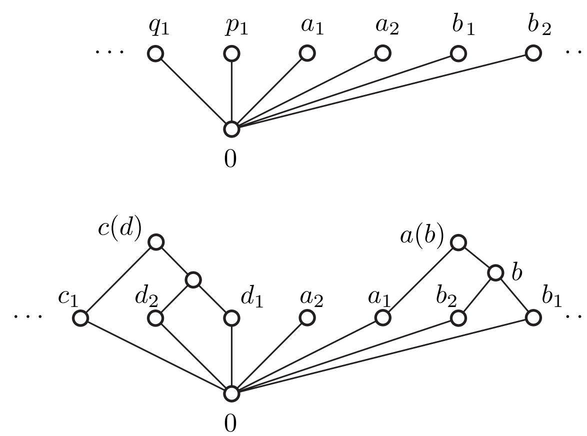

Figure 8. The chopped lattice M_0 and M.

Sectionally Complemented Lattices. Actually, the chopped lattices we construct are sectionally complemented. Unfortunately, the ideal lattice of a sectionally complemented chopped lattice is not necessarily sectionally complemented. We are not going to give the reason why it is sectionally complemented for this construction, but refer the reader to G. Grätzer and E. T. Schmidt, 1962; to the books [7] and [8]; and to the recent series of papers: G. Grätzer and H. Lakser, 2005 (two papers); G. Grätzer, H. Lakser, and M. Roddy, 2005; G. Grätzer and M. Roddy, 2007.

Other Special Properties for Finite Lattices. Let's see which properties (P) of finite lattices are such that

Every finite distributive lattice can be represented as the congruence lattice of a finite lattice with property (P).

So

P = sectionally complemented

is such. I refer the reader to my book [8] for a thorough survey of this field. Here is just one example. Let L be a finite lattice and let Θ be a congruence. We call Θ *isoform* if any two congruence classes of Θ are *isomorphic* as lattices. A lattice is *isoform* if all of its congruences are isoform. It was proved in G. Grätzer and E. T. Schmidt, 2003, that

P = isoform

is such a property. For stronger results on this property see G. Grätzer, R. W. Quackenbush, and E. T. Schmidt, 2004, and G. Grätzer and H. Lakser, two manuscripts.

The General Problem

Anybody familiar with the papers N. Funayama and T. Nakayama, 1942, and G. Birkhoff and O. Frink, 1948, would naturally raise the question:

Problem. *Can every distributive algebraic lattice L be represented as the congruence lattice of a lattice K?*

Surprisingly, this did not make it into G. Birkhoff and O. Frink, 1948, or [2]. When asked, Birkhoff and Frink in 1961 called it an oversight (personal communication). Certainly, R. P. Dilworth was aware of this problem. The first time it appeared in print was in G. Grätzer and E. T. Schmidt, 1962, but already in G. Grätzer and E. T. Schmidt, 1958, a partial positive solution is given. For sure, the second question raised in G. Grätzer and E. T. Schmidt, 1962, was new:

Problem. *Are further conditions on L necessary if we require K to be sectionally complemented?*

A join-semilattice with zero S is

(i) *distributive* if for all $a, b, c \in S$ with $c \leq a \vee b$, there are $x \leq a$ and $y \leq b$ such that $c = x \vee y$;
(ii) *representable* if it is isomorphic to $\mathrm{Con}_c\, L$, for some lattice L.

Now we can state the semilattice formulation of the general problem:

Problem. *Is every distributive join-semilattice with zero representable?*

Positive Results. The first group of positive results started with two papers of E. T. Schmidt, 1968, 1981. To state Schmidt's results, we need some concepts.

A congruence Θ of a join-semilattice with zero S is *monomial* if any Θ-equivalence class has a largest element. A congruence of S is *distributive* if it is a join of monomial congruences.

A *generalized Boolean semilattice* is defined as the underlying join-semilattice of a sectionally complemented distributive lattice with zero. A join-semilattice with zero satisfies *Schmidt's Condition* if it is isomorphic to B/Θ for some distributive congruence Θ of a generalized Boolean semilattice B. One of the best results about the representability of distributive semilattices with zero is E. T. Schmidt, 1968:

Theorem 18. *Any semilattice with zero satisfying Schmidt's Condition is representable.*

Using this result, E. T. Schmidt, 1981, proved:

Theorem 19. *Every distributive lattice with zero is representable.*

In a recent paper, F. Wehrung, 2003, extended Schmidt's result:

Theorem 20. *Every countable union of distributive lattices with zero is representable.*

Surprisingly, Wehrung had to use methods inspired by set theory and forcing (Boolean-valued models) to prove this result. No elementary proof is known.

The second group of positive results is phrased in terms of the *cardinality* of the join-semilattice with zero. A. Huhn mentions in the introduction of his first 1989 paper (both Huhn papers of 1989 were prepared for publication by H. Dobbertin after A. Huhn passed away in 1985) that around 1980, H. Bauer proved a result (unpublished) implying the following:

Theorem 21. *Every countable distributive join-semilattice with zero is representable.*

An extended version of this result is proved by H. Dobbertin, 1986:

Theorem 22. *Every distributive join-semilattice with zero in which any principal ideal is countable is representable.*

A. Huhn, 1989, second paper, uses Schmidt's result to obtain:

Theorem 23. *Every distributive join-semilattice with zero of cardinality at most $\aleph_1$ is representable.*

An elementary proof of this result was published in G. Grätzer, H. Lakser, and F. Wehrung, 2000.

Wehrung's Bombshell

Last year, F. Wehrung announced that

Theorem 24. *There exists a distributive join-semilattice with zero of cardinality* $\aleph_{\omega+1}$ *that is not representable.*

The manuscript was widely circulated and is regarded by the experts as correct. In this section I outline how this semilattice is constructed. The reader should have no difficulty supplying the missing details. In the following two sections I discuss the background for this construction and provide some hints about the proof. These sections are a bit more technical than the rest of this exposition.

In this section we work with *partial join-semilattices*, defined as $\mathbf{P} = \langle P, \le, 0, \vee \rangle$ such that $\langle P, \le \rangle$ is an order; $0 \le a$, for all $a \in P$; and $\vee$ is a partial binary operation such that if $a \vee b$ is defined, then $a \vee b$ is the least upper bound of a and b in the order $\langle P, \le \rangle$; and if $a \le b$ in $\langle P, \le \rangle$, then $a \vee b = b = b \vee a$. The following is easy:

A partial join-semilattice P has a free extension F(P) to a join-semilattice which contains P as a subsemilattice.

We start with the following partial join-semilattice. Let Ω and Ω' be disjoint sets, and let $': \Omega \to \Omega'$ be a bijection. Let P_Ω be the disjoint union of $\{0, 1\}$, Ω, and Ω'. Define $0 < a$, $a' < 1$, for all $a \in \Omega$. Let $\mathbf{P}_\Omega = \langle P_\Omega, \le, 0, \vee \rangle$ be the partial join-semilattice induced by defining $a \vee a' = 1$, for all $a \in \Omega$. Then $F(P_\Omega)$ is isomorphic to

$$\{\emptyset, \Omega \cup \Omega'\} \bigcup \{\, A \cup B' \mid A, B \subseteq \Omega, \; |A|, |B| < \omega, \; A \cap B = \emptyset \,\},$$

ordered by containment. So now we have a join-semilattice and we have to make it distributive. It is easy to describe this process. For each $c \le a \vee b$, add two elements a^+, b^+ with $a^+ \le a$, $b^+ \le b$ and declare that $a^+ \vee b^+ = c$; this yields a partial join-semilattice. Form the join-semilattice it generates. Now iterate this countably many times and take the union $\mathcal{G}(\Omega)$ of the join-semilattices formed.

The distributive join-semilattice with zero, $\mathcal{G}(\Omega)$, thus formed is F. Wehrung's bombshell: it is not representable provided that $\aleph_{\omega+1} \le |\Omega|$.

Uniform Refinement Properties. Weakly distributive homomorphisms of join-semilattices with zero are defined in E. T. Schmidt, 1968. The following is a variant by F. Wehrung.

A $\{\vee, 0\}$-homomorphism $\mu: S \to T$ of the join-semilattices with zero is *weakly distributive* if for all $a, b \in S$ and all $c \in T$, $\mu(c) \le a \vee b$ implies that there are $x, y \in S$ such that $c \le x \vee y$, $\mu(x) \le a$, and $\mu(y) \le b$.

In most related works, the following "uniform refinement property" is used. It was introduced in F. Wehrung, 1998, 1999, and modified in M. Ploščica, J. Tůma, and F. Wehrung, 1998.

Let e be an element in a join-semilattice S with zero. We say that the *weak uniform refinement property*, WURP, holds at e if for all families $(a_i \mid i \in I)$ and $(b_i \mid i \in I)$ of elements of S such that $a_i \vee b_i = e$, for all $i \in I$, there exists a family $(c_{i,j} \mid \langle i, j \rangle \in I \times I)$ of elements of S such that the relations

(1) $c_{i,j} \le a_i, b_j$,
(2) $c_{i,j} \vee a_j \vee b_i = e$,
(3) $c_{i,k} \le c_{i,j} \vee c_{j,k}$

hold, for all i, j, $k \in I$. We say that S satisfies WURP if WURP holds at every element of S.

In M. Ploščica and J. Tůma, 1998, it is proved that WURP does not hold in $\mathcal{G}(\Omega)$, for any set Ω of cardinality at least $\aleph_2$. Hence $\mathcal{G}(\Omega)$ does not satisfy Schmidt's Condition. A similar result is proved in F. Wehrung, 1999.

However, the join-semilattices with zero used in these negative results are complicated. The following result, proved in M. Ploščica, J. Tůma, and F. Wehrung, 1998, is more striking, because it shows that a very well-known lattice, $F(\aleph_2)$, produces a *representable* semilattice that does not satisfy Schmidt's Condition.

Theorem 25. *Let* $F(\aleph_2)$ *be the free lattice on* $\aleph_2$ *generators. The join-semilattice with zero,* $\mathrm{Con_c}\, F(\aleph_2)$*, does not satisfy* WURP*. Consequently,* $\mathrm{Con_c}\, F(\aleph_2)$ *does not satisfy Schmidt's Condition.*

In fact, they prove a lot more. Let $\mathrm{F}_{\mathbf{V}}(\Omega)$ denote the free lattice on Ω in $\mathbf{V}$ for any nondistributive variety $\mathbf{V}$ of lattices.

The join-semilattice with zero, $\mathrm{Con_c}\, \mathrm{F}_{\mathbf{V}}(\Omega)$*, does not satisfy* WURP *for any set* Ω *of cardinality at least* $\aleph_2$*. Consequently,* $\mathrm{Con_c}\, \mathrm{F}_{\mathbf{V}}(\Omega)$ *does not satisfy Schmidt's Condition.*

It is proved in J. Tůma and F. Wehrung, 2001, that $\mathrm{Con_c}\, \mathrm{F}_{\mathbf{V}}(\Omega)$ *is not isomorphic to* $\mathrm{Con_c}\, L$*, for any lattice L with permutable congruences.* This is an important contribution to the second problem of G. Grätzer and E. T. Schmidt, 1963. By using a slight weakening of WURP, this result is extended to arbitrary *algebras* with permutable congruences in P. Růžička, J. Tůma, and F. Wehrung (to appear in *J. Algebra*). Hence, for example, *if* Ω *has at least* $\aleph_2$ *elements, then* $\mathrm{Con}\, \mathrm{F}_{\mathbf{V}}(\Omega)$ *is not isomorphic to the normal subgroup lattice of any group or the submodule lattice of any module.*

(Actually, to keep the exposition at an elementary level, I omitted a great deal. This work started with F. Wehrung, 1998, in which he attacked a problem of H. Dobbertin, 1983, on measure theory—Are there any nonmeasurable refinement monoids?—and a ring-theoretic problem of K. R. Goodearl, 1991—Is it the case that the positive cone of every dimension group with order-unit is isomorphic to $\mathcal{V}(R)$, for some regular ring R?

Unfortunately, I do not know how to present these topics in an introductory exposition within the space constraints I have.)

Solving CLP; the Erosion Lemma. Let us restate F. Wehrung's bombshell:

Theorem 26. *The join-semilattice with zero* $\mathcal{G}(\Omega)$ *is not isomorphic to* $\mathrm{Con}_c\, L$*, for any lattice* L*, whenever the set* Ω *has at least* $\aleph_{\omega+1}$ *elements.*

So the lattice that is a counterexample to CLP had been known for nearly ten years. All prior results about this lattice made use of some form of permutability of congruences. The novelty in F. Wehrung's approach was to find structure in congruence lattices of non-congruence-permutable lattices.

We shall denote by ε the "parity function" on the natural numbers, defined by the rule

$$\varepsilon(n) = \begin{cases} 0 & \text{if } n \text{ is even,} \\ 1 & \text{if } n \text{ is odd,} \end{cases} \quad \text{for any natural number } n.$$

Let L be an algebra possessing a structure of a semilattice $\langle L, \vee \rangle$ such that every congruence of L is also a congruence for $\vee$. We put

$$U \vee V = \{\, u \vee v \mid \langle u, v \rangle \in U \times V \,\}, \quad \text{for all } U,\ V \subseteq L,$$

and we denote by $\mathrm{Con}_c^U\, L$ the $\langle \vee, 0 \rangle$-subsemilattice of $\mathrm{Con}_c\, L$ generated by all principal congruences $\Theta_L(u, v)$, where $\langle u, v \rangle \in U \times U$. We put $\mathrm{con}_L^+(x, y) = \mathrm{con}_L(x \vee y, y)$, for any $x, y \in L$.

The Erosion Lemma. *Let* $x_0,\ x_1 \in L$*, and let* $Z = \{\, z_i \mid 0 \le i \le n \,\}$*, for a positive integer* n*, be a finite subset of* L *with* $\bigvee_{i<n} z_i \le z_n$*. Put*

$$\alpha_j = \bigvee (\mathrm{con}_L(z_i, z_{i+1}) \mid i < n,\ \varepsilon(i) = j), \quad \text{for all } j < 2.$$

Then there are congruences $\Theta_j \in \mathrm{Con}_c^{\{x_j\} \vee Z}\, L$*, for* $j < 2$*, such that*

$$z_0 \vee x_0 \vee x_1 \equiv z_n \vee x_0 \vee x_1 \ (\Theta_0 \vee \Theta_1) \quad \text{and}$$
$$\Theta_j \subseteq \alpha_j \cap \mathrm{con}_L^+(z_n, x_j), \text{ for all } j < 2.$$

The proof of Theorem 26 proceeds by establishing a "structure" theorem for congruence lattices of semilattices, namely, the Erosion Lemma, against "nonstructure" theorems for free distributive extensions $\mathcal{G}(\Omega)$, the main one being called the "Evaporation Lemma". While these are technically difficult, they are, in some sense, "predictable". In contrast, the proof of the Erosion Lemma is much easier.

The cardinality bound $\aleph_{\omega+1}$ has been improved to the optimal bound $\aleph_2$ by P. Růžička (manuscript).

Theorem 27. *The semilattice* $\mathcal{G}(\Omega)$ *is not isomorphic to* $\mathrm{Con}_c\, L$*, for any lattice* L*, whenever the set* Ω *has at least* $\aleph_2$ *elements.*

A key part in F. Wehrung's proof is a combinatorial argument of K. Kuratowski, 1951. Let $[X]^{<\omega}$ denote the set of all finite subsets of X and $[X]^n$ (for a positive integer n) the set of all n-element subsets of X. For a map $\Phi\colon [X]^n \to [X]^{<\omega}$, an $(n+1)$-element subset U of X is *free with respect to* Φ if $x \notin \Phi(U - \{x\})$ for all $x \in U$.

Theorem 28 (Kuratowski's Free Set Theorem)**.** *Let* n *be a natural number, and let* X *be a set of cardinality at least* $\aleph_n$*. For every map* $\Phi\colon [X]^n \to [X]^{<\omega}$*, there exists an* $(n+1)$*-element free subset of* X *with respect to* Φ*.*

See the book P. Erdös, A. Hajnal, A. Máté, and R. Rado [5], especially Chapter 10, for this result.

P. Růžička's proof follows the main lines of F. Wehrung's proof, except that it introduces an enhancement of Kuratowski's Free Set Theorem, called there *the existence of free trees*, which it uses in the final argument involving the Erosion Lemma.

A Short List of Books

[1] G. Birkhoff, *Lattice Theory*, first edition, Amer. Math. Soc., Providence, RI, 1940.

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[4] K. P. Bogart, R. Freese, and J. P. S. Kung (editors), *The Dilworth Theorems. Selected Papers of Robert P. Dilworth*, Birkhäuser-Verlag, Basel–Boston–Berlin, 1990.

[5] P. Erdös, A. Hajnal, A. Máté, and R. Rado, *Combinatorial Set Theory: Partition Relations for Cardinals*, Studies in Logic and the Foundations of Mathematics, vol. 106, North-Holland Publishing Co., Amsterdam, 1984.

[6] G. Grätzer, *Universal Algebra*, second edition, Springer-Verlag, New York–Heidelberg, 1979.

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[8] ______, *The Congruences of a Finite Lattice: A Proof-by-Picture Approach*, Birkhäuser Boston, 2006.

[9] R. Padmanabhan and S. Rudeanu, *Axioms for Lattices and Boolean Algebras*, manuscript.

[10] V. N. Saliĭ, *Lattices with Unique Complements*, translated from the Russian by G. A. Kandall, Translations of Mathematical Monographs, vol. 69, Amer. Math. Soc., Providence, RI, 1988.

Sudoku Squares and Chromatic Polynomials

Agnes M. Herzberg and M. Ram Murty

The Sudoku puzzle has become a very popular puzzle that many newspapers carry as a daily feature. The puzzle consists of a 9×9 grid in which some of the entries of the grid have a number from 1 to 9. One is then required to complete the grid in such a way that every row, every column, and every one of the nine 3×3 sub-grids contain the digits from 1 to 9 exactly once. The sub-grids are shown in Figure 1.

Figure 1. A Sudoku grid.

Agnes M. Herzberg is professor emeritus of mathematics at Queen's University, Canada. Her email address is `herzberg@post.queensu.ca`.
M. Ram Murty is professor of mathematics at Queen's University, Canada. His email address is `murty@mast.queensu.ca`.
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Recall that a Latin square of rank n is an $n \times n$ array consisting of the numbers such that each row and column has all the numbers from 1 to n. In particular, every Sudoku square is a Latin square of rank 9, but not conversely because of the condition on the nine 3×3 sub-grids. Figure 2 (taken from [6]) shows one such puzzle with seventeen entries given.

							1	
4								
	2							
				5		4		7
		8				3		
		1		9				
3			4			2		
	5		1					
			8		6			

Figure 2. A Sudoku puzzle with 17 entries.

For anyone trying to solve a Sudoku puzzle, several questions arise naturally. For a given puzzle, does a solution exist? If the solution exists, is it unique? If the solution is not unique, how many solutions are there? Moreover, is there a systematic way of determining all the solutions? How many puzzles are there with a unique solution? What is the minimum number of entries that can be specified in a single puzzle in order to ensure a unique solution? For instance, Figure 2 shows that the minimum is at most 17. (We leave it to the reader that the puzzle in Figure 2 has a unique solution.) It is unknown at present if a puzzle with 16 specified entries exists that yields a unique solution. Gordon Royle [6] has collected 36,628 distinct Sudoku puzzles with 17 given entries.

We will reformulate many of these questions in a mathematical context and attempt to answer them. More precisely, we reinterpret the Sudoku puzzle as a vertex coloring problem in graph theory. This enables us to generalize the questions and view them from a broader framework. We will also discuss the relationship between Latin squares and Sudoku squares and show that the set of Sudoku squares is substantially smaller than the set of Latin squares.

Chromatic Polynomials

For the convenience of the reader, we recall the notion of proper coloring of a graph. A λ-coloring of a graph G is a map f from the vertex set of G to $\{1, 2, ..., \lambda\}$. Such a map is called a *proper coloring* if $f(x) \neq f(y)$ whenever x and y are adjacent in G. The minimal number of colors required to properly color the vertices of a graph G is called the *chromatic number* of G and denoted $\chi(G)$. It is then not difficult to see that the Sudoku puzzle is really a coloring problem. Indeed, we associate a graph with the 9×9 Sudoku grid as follows. The graph will have 81 vertices with each vertex corresponding to a cell in the grid. Two distinct vertices will be adjacent if and only if the corresponding cells in the grid are either in the same row, or same column, or the same sub-grid. Each completed Sudoku square then corresponds to a proper coloring of this graph. We put this in a slightly more general context. Consider an $n^2 \times n^2$ grid. To each cell in the grid, we associate a vertex labeled (i, j) with $1 \le i, j \le n^2$. We will say that (i, j) and (i', j') are *adjacent* if $i = i'$ or $j = j'$ or $\lceil i/n \rceil = \lceil i'/n \rceil$ *and* $\lceil j/n \rceil = \lceil j'/n \rceil$. (Here, the notation $\lceil \cdot \rceil$ means that we round to the nearest greater integer.) We will denote this graph by X_n and call it the *Sudoku graph* of rank n. A Sudoku square of rank n will be a proper coloring of this graph using n^2 colors. A Sudoku puzzle corresponds to a partial coloring and the question is whether this partial coloring can be completed to a total proper coloring of the graph.

We remark that, sometimes, it is more convenient to label the vertices of a Sudoku graph of rank n using (i, j) with $0 \le i, j \le n^2 - 1$. Then, (i, j) and (i', j') are adjacent if $i = i'$ or $j = j'$ or $[i/n] = [i'/n]$ and $[j/n] = [j'/n]$, where now $[\cdot]$ indicates the greatest integer function. That is, $[x]$ means the greatest integer which is less than or equal to x.

Recall that a graph is called *regular* if the degree of every vertex is the same. An easy computation shows that X_n is a regular graph with each vertex having degree $3n^2 - 2n - 1 = (3n + 1)(n - 1)$. In the case $n = 3$, X_3 is 20-regular and in case $n = 2$, X_2 is 7-regular.

The number of ways of coloring a graph G with λ colors is well known to be a polynomial in λ of degree equal to the number of vertices of G. Our first theorem is that given a partial coloring C of G, the number of ways of completing the coloring to obtain a proper coloring using λ colors is also a polynomial in λ, provided that λ is greater than or equal to the number of colors used in C. More precisely, this is stated as Theorem 1.

Theorem 1. *Let G be a finite graph with v vertices. Let C be a partial proper coloring of t vertices of G using d_0 colors. Let $p_{G,C}(\lambda)$ be the number of ways of completing this coloring using λ colors to obtain a proper coloring of G. Then, $p_{G,C}(\lambda)$ is a monic polynomial (in λ) with integer coefficients of degree $v - t$ for $\lambda \ge d_0$.*

We will give two proofs of this theorem. The most direct proof uses the theory of partially ordered sets and Möbius functions, which we briefly review. A partially ordered set (or poset, for short) is a set P together with a partial ordering denoted by $\le$ that satisfies the following conditions: (a) $x \le x$ for all $x \in P$; (b) $x \le y$ and $y \le x$ implies $x = y$; (c) $x \le y$ and $y \le z$ implies $x \le z$.

We will consider only finite posets. Familiar examples of posets include the collection of subgroups of a finite group partially ordered by set inclusion and the collection of positive divisors of a fixed natural number n partially ordered by divisibility. A less familiar example is given by the following construction.

Let G be a finite graph and e an edge of G. The graph obtained from G by identifying the two vertices joined by e (and removing any resulting multiple edges) is denoted G/e and is called the contraction of G by e. In general, we say that the graph G' is a contraction of G if G' is obtained from G by a series of contractions. The set of all contractions of a finite graph G can now be partially ordered by defining that $A \le B$ if A is a contraction of B.

Given a finite poset P with partial ordering $\le$, we define the *Möbius function* $\mu : P \times P \to \mathbb{Z}$ recursively by setting

$$\mu(x, x) = 1, \quad \sum_{x \le y \le z} \mu(x, y) = 0, \quad \text{if } x \neq z.$$

The main theorem in the theory of Möbius functions is the following. If $f : P \to \mathbb{C}$ is any complex valued function and we define

$$g(y) = \sum_{x \le y} f(x),$$

then

$$f(y) = \sum_{x \le y} \mu(x, y) g(x),$$

and conversely. We refer the reader to [5] for amplification of these ideas.

Proof of Theorem 1. We will use the theory of Möbius functions outlined to prove Theorem 1. Let (G, C) be given with G a graph and C a proper coloring of some of the vertices. We will say (G', C) is a subgraph of (G, C) if G' is obtained by contracting some edges of G with at most one end-point in C. This gives us a partially ordered set. The minimum contraction would be the vertices of C with the adjacencies amongst them preserved. We will also use the letter C to denote this minimum graph in our poset. For each subgraph (G', C) of this poset, let $p_{G',C}(\lambda)$ be the number of ways of properly coloring G' with λ colors with the specified colorings for C. Let $q_{G',C}(\lambda)$ be the number of ways of coloring (not necessarily proper) the vertices of G' using λ colors, with the specified colorings for C. Clearly, $q_{G',C}(\lambda) = \lambda^{v'-t}$, where v' is the number of vertices of G' and t is the number of vertices of C. If $\lambda \geq d_0$, then given any λ-coloring of (G, C), we may derive a proper coloring of a unique subgraph (G', C) simply by contracting the edges whenever two adjacent vertices of (G, C) have the same color assigned. In this way, we obtain the relation

$$q_{G,C}(\lambda) = \lambda^{v-t} = \sum_{C \leq G'} p_{G',C}(\lambda).$$

By Möbius inversion, we deduce that

$$p_{G,C}(\lambda) = \sum_{C \leq G'} \mu(C, G')\lambda^{v'-t},$$

and the right-hand side is a monic polynomial with integer coefficients, of degree $v - t$, as stated. □

We can also prove Theorem 1 without the use of Möbius functions. We apply induction on the number of edges of the graph (G, C). We consider three cases:

(1) Let us suppose that e is an edge connecting two vertices of G at most one of which is contained in C. We will use the following notation. $G - e$ will denote the graph obtained from G by deleting the edge e, but not its end-points. The graph obtained from G by identifying the two vertices joined by e (and removing any multiple edges) will be denoted G/e. With this notation, we have

$$p_{G,C}(\lambda) = p_{G-e,C}(\lambda) - p_{G/e,C}(\lambda)$$

because each proper coloring of G is also a proper coloring of $G - e$ and a proper coloring of $G - e$ gives a proper coloring of G if and only if it gives distinct colors to the end-points x, y of e. Thus, the number of proper colorings $p_{G,C}(\lambda)$ can be obtained from $p_{G-e,C}(\lambda)$ by subtracting those colorings that assign the same color to both x and y, and this number corresponds to $p_{G/e,C}(\lambda)$. Each of $G - e$ and G/e have fewer edges than G. Thus, we may apply induction and complete the proof in this case.

(2) Now suppose that G has one vertex v_0 not contained in C. If this vertex is not adjacent to any vertex of C, then $G = C \cup v_0$, which is the disjoint union of C and the vertex v_0. Thus, we can color v_0 using any of the λ colors. Thus, $p_{G,C}(\lambda) = \lambda$ in this case. If this vertex is adjacent to d vertices of C, and these vertices use d_0 colors, then, $p_{G,C}(\lambda) = \max(\lambda - d_0, 0)$.

(3) Every vertex of G is contained in C. In this case, we already have a coloring of G and $p_{G,C}(\lambda) = 1$.

Thus, by induction on the number of edges of the graph, the theorem is proved. □

In a later section, we will examine the implications of this theorem for the Sudoku puzzle. For now, we remark that given a Sudoku puzzle, the number of ways of completing the graph is given by $p_{X_3,C}(9)$. A given Sudoku puzzle (X_3, C), has a unique solution if and only if this number $p_{X_3,C}(9) = 1$. It would be extremely interesting to determine under what conditions a partial coloring can be extended to a unique coloring. In this direction, we have the following general result.

Theorem 2. *Let G be a graph with chromatic number $\chi(G)$ and C be a partial coloring of G using only $\chi(G) - 2$ colors. If the partial coloring can be completed to a total proper coloring of G, then there are at least two ways of extending the coloring.*

Proof. Since two colors have not been used in the initial partial coloring, these two colors can be interchanged in the final proper coloring to get another proper extension. □

Theorem 2 implies that if C is a partial coloring of G that can be completed *uniquely* to a total coloring of G, then C must use at least $\chi(G) - 1$ colors. In particular, we have that in any 9×9 Sudoku puzzle, at least eight of the colors must be used in the "given" cells. In general, for the $n^2 \times n^2$ Sudoku puzzle, at least $n^2 - 1$ colors must be used in the "given" partial coloring in order that the puzzle has a unique solution.

Scheduling and Partial Colorings

Given a graph G with a partial coloring, we may ask if this can be extended to a full coloring of the graph. It is well known that coloring problems of graphs encode scheduling problems in real life. The extension from a partial coloring to a total coloring corresponds to a scheduling problem with additional constraints, where, for example, we may want to schedule meetings of various committees in time slots, with some committees already pinned down to certain time slots. Of course, the corresponding adjacency relation is that two committees are joined by an edge if they have a member in common. This is a question that is of interest in its own right, and it seems difficult to determine criteria for when a partial proper coloring can be extended to a proper coloring of the whole graph.

A similar situation arises for frequency channel assignments. Suppose there are television transmitters in a given region and they need to be assigned a frequency channel for transmission. Two transmitters within 100 miles of each other are to be assigned different channels, for otherwise there will be interference in the signal. It is often the case that some transmitters have already been assigned their frequency channels and new transmitters are to be assigned new channels with these constraints. This is again a problem of completing a partial coloring of a graph to a proper coloring. Indeed, we associate a vertex to each transmitter and join two of them if they are within 100 miles of each other. A channel assignment corresponds to a "color" assigned to that vertex.

We can multiply our examples to many different "real life" contexts. In each case, the problem of completing a partial coloring of a graph to a proper coloring emerges as the archetypal theme.

Latin squares and Sudoku squares are then only special cases of this theme. However, they can also be studied independently of this context. The explicit construction of Latin squares is well known to have applications to the design of statistical experiments. In agricultural studies, for example, one would like to plant v varieties of plants in v rows and v columns so that the pecularities of the soil in which they are planted do not have bearing on the experiment. Agriculturists have always used a $v \times v$ Latin square to design such an experiment. This serves to balance the treatments of the experiment before randomization takes place.

In this context, if one were interested in also testing the role of various fertilizers on the growth of these plants, a Sudoku square might be used, where each sub-grid (or each band) has a different fertilizer applied to it, thus having each fertilizer on each treatment.

Explicit Coloring for X_n

In this section, we will show how one may properly color the Sudoku graph X_n. Recall that the *chromatic number* of a graph is the minimal number of colors needed to properly color its vertices. Thus, the complete graph K_n consisting of n vertices in which every vertex is adjacent to every other vertex has chromatic number n.

Theorem 3. *For every natural number n, there is a proper coloring of the Sudoku graph X_n using n^2 colors. The chromatic number of X_n is n^2.*

Proof. Let us first note that all the cells of the upper left corner $n \times n$ grid are adjacent to each other and this forms a complete graph isomorphic to K_{n^2}. The chromatic number of K_{n^2} is n^2 and thus, X_n would need at least n^2 colors for a proper coloring. Now, we will show that it can be colored using n^2 colors. As remarked earlier, it is convenient to label the vertices (i, j) with $0 \le i, j \le n^2 - 1$. Consider the residue classes mod n^2. For $0 \le i \le n^2 - 1$, we write $i = t_i n + d_i$ with $0 \le d_i \le n - 1$ and $0 \le t_i \le n - 1$ and similarly for $0 \le j \le n^2 - 1$, as well. We assign the "color" $c(i, j) = d_i n + t_i + n t_j + d_j$, reduced modulo n^2, to the (i, j)-th position in the $n^2 \times n^2$ grid. We claim that this is a proper coloring. To see this, we should show that any two adjacent coordinates (i, j) and (i', j') have distinct colors. Indeed, if $i = i'$, then we must show $c(i, j) \ne c(i, j')$ unless $j = j'$. If $c(i, j) = c(i, j')$, then $n t_j + d_j = n t_{j'} + d_{j'}$ which means $j = j'$. Similarly, if $j = j'$, then $c(i, j) \ne c(i', j)$ unless $i = i'$. If now, $[i/n] = [i'/n]$ and $[j/n] = [j'/n]$, then $d_i = d_{i'}$ and $d_j = d_{j'}$. If $c(i, j) = c(i', j')$, then

$$t_i + n t_j = t_{i'} + n t_{j'}.$$

Reducing this mod n gives $t_i = t_{i'}$. Hence, $t_j = t_{j'}$ so that $(i, j) = (i', j')$ in this case also. Therefore, this is a proper coloring. □

Counting Sudoku Solutions

We will address briefly the question of uniqueness of solution for a given Sudoku puzzle. It is not always clear at the outset if a given puzzle has a solution. In this section, we derive some necessary conditions for there to be a unique solution.

Figure 3 gives an example of a Sudoku puzzle which affords precisely two solutions.

9		6		7		4		3
			4			2		
	7			2	3		1	
5						1		
	4		2		8		6	
		3						5
	3		7				5	
		7			5			
4		5		1		7		8

Figure 3. A Sudoku puzzle with exactly two solutions.

We leave it to the reader to show that the puzzle in Figure 3 leads to the configuration in Figure 4.

9	2	6	5	7	1	4	8	3
3	5	1	4	8	6	2	7	9
8	7	4	9	2	3	5	1	6
5	8	2	3	6	7	1	9	4
1	4	9	2	5	8	3	6	7
7	6	3	1			8	2	5
2	3	8	7			6	5	1
6	1	7	8	3	5	9	4	2
4	9	5	6	1	2	7	3	8

Figure 4. The "solution" to the puzzle in Figure 3.

Clearly, one can insert either of the arrangements in Figure 5 to complete the grid. Thus, we have two solutions.

9	4
4	9

4	9
9	4

Figure 5. Two ways of completing the puzzle in Figure 4.

This observation leads to the following remark. If in the solution to a Sudoku puzzle, we have a configuration of the type indicated in Figure 6 in the same vertical stack, then at least one of these entries must be included as a "given" in the initial puzzle, for otherwise, we would have two possible solutions to the initial puzzle simply by interchanging a and b in the configuration.

a	b
b	a

Figure 6. A configuration leading to two solutions.

As remarked earlier, if the distinct number of "colors" used in a given Sudoku puzzle is at most seven, then there are at least two solutions to the puzzle. We noted that this was so because we could interchange the two unused colors and still get a valid solution. The multiplicity of solutions can also be seen from the chromatic polynomial. If d_0 is the number of distinct colors used, we have seen that $p_{X_3,C}(\lambda)$ is a polynomial in λ provided $\lambda \geq d_0$. Since the chromatic number of X_3 is 9, we must have $p_{X_3,C}(\lambda) = 0$ for $\lambda = d_0, d_0 + 1, ..., 8$. As $p_{X_3,C}(\lambda)$ is a monic polynomial with integer coefficients, we can write

$$p_{X_3,C}(\lambda) = (\lambda - d_0)(\lambda - (d_0 + 1)) \cdots (\lambda - 8)q(\lambda),$$

for some polynomial $q(\lambda)$ with integer coefficients. Putting $\lambda = 9$ gives

$$p_{X_3,C}(9) = (9 - d_0)!q(9)$$

and the right hand side is greater than or equal to 2 if $d_0 \leq 7$. This gives us the stated necessary condition for there to be a unique solution, provided that there is a solution, which is a tacit assumption in every given Sudoku puzzle. In the last section, we give a Sudoku puzzle that uses only eight colors and has 17 given entries.

Counting Sudoku Squares of Rank 2

In a recent paper [3], Felgenhauer and Jarvis computed the number of Sudoku squares by a brute force calculation. There are

$$6,670,903,752,021,072,936,960$$

valid Sudoku squares. This is approximately 6.671×10^{21}, a very tiny proportion of the total number of 9×9 Latin squares which is (see [1])

$$5,524,751,496,156,892,842,531,225,600$$
$$\approx 5.525 \times 10^{27}.$$

This mammoth number of Sudoku squares can be cut down to size if we make a few simple observations. First, beginning with any Sudoku square, we can create $9! = 362880$ new Sudoku squares simply by relabelling. More precisely, if a_{ij} represents the (i, j)-th entry of a Sudoku square, and σ is a permutation of the set $\{1, 2, ..., 9\}$, then the square whose (i, j)-th entry is $\sigma(a_{ij})$ is also a valid Sudoku square. There are other symmetries one can take into account. For example, the transpose of a Sudoku square is also a Sudoku square. We can also permute any of the three bands, or the three stacks and the rows within a band, or the columns within a stack. When all these symmetries are taken into account, the number of essentially different Sudoku squares is the more manageable number

$$5,472,730,538,$$

approximately 5.47×10^9, as was shown in [4].

These calculations can be better understood if we consider the case of the number of distinct 4×4 Sudoku squares. Without loss of generality, we may as well label the entries in the first 2×2 block as in Figure 7.

It is not hard to see that there are at most 2^4 ways of completing this grid. However, we can be a bit more precise. One can show that taking into account the obvious symmetries already indicated, there are essentially only two 4×4 Sudoku squares! Indeed, since permuting the last two columns is an allowable symmetry, we may suppose without

1	2		
3	4		

Figure 7. A 4×4 Sudoku grid.

loss of generality that the last two entries in the first row are $(3, 4)$ in this order. Similarly, since we may interchange the last two rows, we may suppose that our square is as shown in Figure 8.

1	2	3	4
3	4		
2			
4			

Figure 8. A 4×4 Sudoku puzzle.

Then, 1 and 4 are the only possible entries in the diagonal position (3,3) and it is easily seen that the choice of 1 leads to a contradiction. This leads to the following arrangements.

1	2	3	4
3	4		
2		4	
4			1

1	2	3	4
3	4		
2		4	
4			2

1	2	3	4
3	4		
2		4	
4			3

Figure 9. Three 4×4 Sudoku puzzles.

The squares can be completed easily as shown in Figure 10.

However, the last configuration is equivalent to the second one upon taking the transpose and interchanging 2 and 3. Thus, there are really only two nonequivalent solutions for the 4×4 Sudoku square. From this reasoning, we also see that without taking into account the symmetries, we get a total of $4! \times 2 \times 2 \times 3 = 288$ Sudoku squares of rank 2.

In this context, it is interesting to determine the minimum number of cells filled in a 4×4 Sudoku puzzle that leads to a unique solution. Figure 11 gives one with four entries. Is there one with three? We will indicate a proof that there is not.

1	2	3	4
3	4	1	2
2	1	4	3
4	3	2	1

1	2	3	4
3	4	2	1
2	1	4	3
4	3	1	2

1	2	3	4
3	4	1	2
2	3	4	1
4	1	2	3

Figure 10. Three 4×4 Sudoku squares determined from Figure 9.

1			
			2
		4	
	3		

Figure 11. A 4×4 Sudoku puzzle with 4 cells filled in.

As we remarked earlier, the number of "colors" used in any partial coloring of the Sudoku graph of rank n must be at least $n^2 - 1$ in order that there be a unique solution. Thus, in the puzzle in Figure 11, we must use at least 3 colors. To prove that the minimum number for the rank 2 Sudoku graph is four, we must show if only three distinct "colors" are filled in, we do not have a unique solution. This is easily done by looking at the two inequivalent 4×4 Sudoku grids given in Figure 10 and checking the cases that arise one by one.

Permanents and Systems of Distinct Representatives

In this section, we will review several theorems on permanents and systems of distinct representatives that will be used in the next section in our enumeration of Sudoku squares. For further details, we refer the reader to [5].

If A is an $n \times n$ matrix, with the (i, j)-th entry given by a_{ij}, the *permanent* of A, denoted per A, is by definition

$$\sum_{\sigma \in \mathfrak{S}_n} a_{1\sigma(1)} a_{2\sigma(2)} \cdots a_{n\sigma(n)},$$

where $\mathfrak{S}_n$ denotes the symmetric group on the n symbols $\{1, 2, ..., n\}$. The matrix A is said to be *doubly stochastic* if both the row sums and column sums are equal to 1.

In 1926 B. L. van der Waerden posed the problem of determining the minimal permanent among all $n \times n$ doubly stochastic matrices. It was felt that the minimum is attained by the constant matrix all of whose entries are equal to $1/n$. Over the years, this feeling changed into the conjecture that

$$\text{(1)} \qquad \operatorname{per} A \geq \frac{n!}{n^n},$$

for any doubly stochastic matrix A and was then referred to as the van der Waerden conjecture. By 1981 two different proofs of the conjecture appeared, one by D. I. Falikman and another by G. P. Egoritsjev. As for upper bounds, H. Minc conjectured in 1967 that if A is a $(0,1)$ matrix with row sums r_i, then

$$\text{(2)} \qquad \operatorname{per} A \leq \prod_{i=1}^{n} r_i!^{1/r_i}.$$

This conjecture was proved in 1973 by L. M. Brégman (see p. 82 of [5]). We will utilize both the upper and lower bounds for the permanents in our enumeration of Sudoku squares. The application will be via the theorem of Phillip Hall (sometimes called the *marriage theorem*), which we now describe.

Suppose that we have subsets $A_1, A_2, \ldots, A_n$ of the set $\{1, 2, \ldots, n\}$. We would like to select distinct elements $a_i \in A_i$. Such a selection is called a *system of distinct representatives*. The theorem gives necessary and sufficient conditions for when this can be done (see [5]). If for every subset S of $\{1, 2, \ldots, n\}$, we let

$$N(S) = \cup_{j \in S} A_j,$$

then a little reflection shows that it is necessary that $|N(S)| \geq |S|$ for there to exist a selection. (Here, $|S|$ indicates the cardinality of the set S.) Hall's theorem states that this is also sufficient. The number of ways this can be done is given by the permanent of the $n \times n$ matrix A defined as follows. It is a $(0,1)$ matrix whose (i,j)-th entry is 1 if and only if $i \in A_j$. We will refer to A as the *Hall matrix* associated with the sets $A_1, \ldots, A_n$.

These results can be used to obtain upper and lower bounds for the number of Latin squares of order n (as in [5]). Since we will need the lower bound in the next section, we give the details of this calculation.

For an $n \times n$ Latin square, the number of ways of filling in the first row is clearly $n!$. Suppose we have completed k rows of the Latin square. We now want to fill in the $(k+1)$-st row. For each cell i of the $(k+1)$-st row, we let A_i be the set of numbers not yet used in the i-th column. The size of A_i is therefore $n-k$. To fill in the $(k+1)$-st row of our Latin square is tantamount to finding a set of distinct representatives of the sets $A_1, \ldots, A_n$. The number of ways this can be done is the permanent of the corresponding Hall matrix A. Since $(n-k)^{-1}A$ is doubly stochastic, equation (1) shows that there are at least

$$\frac{(n-k)^n n!}{n^n}$$

ways of doing this. Taking the product over k ranging from 0 to $n-1$ gives:

Lemma 4. *The number of Latin squares of order n is at least $n!^{2n}/n^{n^2}$.*

Corollary 5. *The number of Latin squares of order n^2 is at least*

$$n^{2n^4} e^{-2n^4 + O(n^2 \log n)}.$$

Proof. By Lemma 4, the number of Latin squares of order n^2 is at least

$$n^2!^{2n^2}/n^{2n^4}.$$

Using Stirling's formula,

$$\log n! = n \log n - n + \frac{1}{2} \log n + O(1),$$

we obtain the stated lower bound. □

Latin Squares and Sudoku Squares

We can now prove:

Theorem 6. *The number of Sudoku squares of rank n is bounded by*

$$n^{2n^4} e^{-2.5n^4 + O(n^3 \log n)}$$

for n sufficiently large.

Proof. The $n^2 \times n^2$ Sudoku square is composed of n^2 sub-grids of size $n \times n$. The entries in each sub-grid can be viewed as a permutation of $\{1, 2, \ldots, n^2\}$. Thus, a crude upper bound for the number of Sudoku squares is given by

$$[(n^2)!]^{n^2}.$$

We begin by observing that there are n "bands" in the Sudoku grid. ("Bands" are groups of the n successive rows.) The number of ways of completing the first band can be estimated as follows. We have $n^2!$ choices for the first row. The number of ways of filling the second row is calculated by evaluating the permanent of the following matrix. We have an $n^2 \times n^2$ matrix whose rows parametrize the cells of the second row. Let us note that for each cell, we have $n^2 - n$ possibilities since we already used n colors in the corresponding $n \times n$ sub-grid. The number of ways of filling in the second row is the number of ways of choosing a system of distinct representatives from this list of possibilities. More precisely, we consider the $n^2 \times n^2$ matrix whose rows parametrize the cells of the second row, and whose columns parametrize the numbers from 1 to n^2, and we put a 1 in the (i,j)-th entry if j is a permissible value for cell i and zero otherwise. This gives us a $(0,1)$ matrix whose permanent is the number of ways of choosing the set of distinct representatives (see [5]). By equation (2), this quantity is bounded by

$$(n^2 - n)!^{\frac{n}{n-1}}.$$

Proceeding similarly gives the number of possibilities for the third row as

$$(n^2-2n)!^{\frac{n}{n-2}}.$$

In this way, we obtain a final estimate of

$$\prod_{k=0}^{n-1}(n^2-kn)!^{\frac{n}{n-k}}$$

for the number of ways of filling in the first band consisting of n rows of a Sudoku square of rank n. Suppose now that $(i-1)$ of the n bands have been completed. We calculate the number of ways of completing the i-th band. Here, we will give an upper bound. The number of possible entries for the first cell of the i-th band is

$$n^2-(i-1)n.$$

Thus, as before, the number of ways of completing the first row of the i-th band is bounded by

$$(n^2-(i-1)n)!^{\frac{n^2}{n^2-(i-1)n}},$$

since for each cell, we must exclude the entries already entered in the column to which the cell belongs. Similarly, the number of ways of completing the second row of the i-th band is at most

$$(n^2-((i-1)n+1))!^{\frac{n^2}{n^2-((i-1)n+1)}}.$$

We proceed in this way until the i-th row of the i-th band to get an estimate of

$$(n^2-((i-1)n+i))!^{\frac{n^2}{n^2-((i-1)n+i)}}.$$

For the $(i+1)$-th row, we change our strategy and exclude the numbers already entered in the sub-grid in which the cell belongs. This means we have entered in entries already in the subgrid and we must exclude these to get an estimate of

$$(n^2-in)!^{\frac{n^2}{n^2-in}},$$

for the number of ways of filling in the i-th row of the i-th band. Proceeding in this manner, we get that the number S_n of Sudoku squares of rank n is bounded by

$$\prod_{i=1}^{n}(n^2-(i-1)n)!^{\frac{n^2}{n^2-(i-1)n}}$$

$$(n^2-[(i-1)n+1])!^{\frac{n^2}{n^2-[(i-1)n+1]}}\cdots$$

$$\cdots(n^2-[(i-1)n+i])!^{\frac{n^2}{n^2-[(i-1)n+i]}}$$

$$(n^2-in)!^{\frac{n^2}{n^2-in}}\cdots(n^2-(n-1)n)!^{\frac{n^2}{n^2-(n-1)n}}.$$

Thus,

$$\frac{\log S_n}{n^2}\le\sum_{i=1}^{n}\left(\sum_{j=0}^{i}\frac{\log(n^2-[(i-1)n+j])!}{n^2-(i-1)n+j}\right.$$
$$\left.+\sum_{k=i}^{n-1}\frac{\log(n^2-kn)}{(n^2-kn)}\right).$$

We estimate the sum using Stirling's formula. It is

$$\le\sum_{i=1}^{n}\left(\sum_{j=0}^{i}\log(n^2-[(i-1)n+j])+\sum_{k=i}^{n-1}\log(n^2-kn)\right)$$
$$-n^2+O(\log^2 n).$$

This is easily seen to be

$$\le\sum_{i=1}^{n}\left(i\log(n^2-(i-1)n)+\sum_{k=i}^{n-1}\log(n^2-kn)\right)$$
$$-n^2+O(\log^2 n).$$

Thus, $(\log S_n)/n^2+n^2+O(n\log n)$ is

$$\le\frac{1}{2}n^2\log n+$$
$$\sum_{i=1}^{n}\left(i\log(n-i+1)+(n-i)\log n+\sum_{k=i}^{n-1}\log(n-k)\right).$$

The summation over k is

$$\log(n-i)!=(n-i)\log(n-i)-(n-i)+O(\log n),$$

by Stirling's formula. Combining all of this gives

$$\frac{\log S_n}{n^2}\le 2n^2\log n-2.5n^2+O(n\log n),$$

from which the theorem is easily derived. □

We have the following corollary.

Corollary 7. *Let p_n be the probability that a randomly chosen Latin square of order n^2 is also a Sudoku square. Then*

$$p_n\le e^{-0.5n^4+O(n^3\log n)}.$$

In particular, $p_n\to 0$ as n tends to infinity.

Proof. By Theorem 6, the number of Sudoku squares of rank n is at most

$$n^{2n^4}e^{-2.5n^4+O(n^3\log n)}.$$

The number of Latin squares of rank n^2 is by Corollary 5, at least

$$n^{2n^4}e^{-2n^4+O(n^2\log n)}.$$

Thus, the probability that a random Latin square of order n^2 is also a Sudoku square is bounded by

$$e^{-0.5n^4+O(n^3\log n)},$$

which goes to zero as n tends to infinity. □

In fact, the theorem shows that the number of Sudoku squares is substantially smaller than the number of Latin squares.

Concluding Remarks

It is interesting to note that the Sudoku puzzle is extremely popular for a variety of reasons. First, it is sufficiently difficult to pose a serious mental challenge for anyone attempting to do the puzzle. Secondly, simply by scanning rows and columns, it is easy to enter the "missing colors", and this gives the solver some encouragement to persist. The novice is usually stumped after some time. However, the puzzle can be systematically solved by keeping track of the unused colors in each row, in each column, and in each sub-grid. A simple process of elimination often leads one to complete the puzzle. Some of the puzzles classified under the "fiendish" category involve a slightly more refined version of this elimination process, but the general strategy is the same. One could argue that the Sudoku puzzle develops logical skills necessary for mathematical thought.

What is noteworthy is that this simple puzzle has given rise to several problems of a mathematical nature that have yet to be resolved. We have already mentioned the "minimum Sudoku puzzle" problem, where we ask if there is a Sudoku puzzle with 16 or fewer entries that admits a unique solution.

We have already commented that if only 7 or fewer colors are used, the puzzle does not admit a unique solution. One may ask if there is a puzzle using only 8 colors with a minimum number of entries. In Figure 12, we give such a puzzle that again has only 17 given entries (taken from [6]).

							1	2
				3	5			
			6				7	
7						3		
			4			8		
1								
			1	2				
	8						4	
	5					6		

Figure 12. A Sudoku puzzle using only 8 colors.

These questions suggest the more general question of determining the "minimum Sudoku" for the general puzzle of rank n. It would be interesting to determine the asymptotic growth of this minimum as a function of n. Our discussion shows that this minimum is at least $n^2 - 1$. Is it true that the minimum is $o(n^4)$?

Another interesting problem is to determine the number of Sudoku squares of rank n. More precisely, if $p_{X_n}(\lambda)$ is the chromatic polynomial of the Sudoku graph of rank n, what is the asymptotic behavior of $p_{X_n}(n^2)$? If S_n is the number of Sudoku squares of rank n, it seems reasonable to conjecture that

$$\lim_{n\to\infty} \frac{\log S_n}{2n^4 \log n} = 1.$$

It is clear that this limit, if it exists, is less than or equal to 1. We have already noted the various symmetries of the Sudoku square. For example, applying a permutation to the elements $\{1, 2, ..., n^2\}$, gives us a new Sudoku square. Thus, starting from one such square, we may produce $n^2!$ new Sudoku squares. There are also the band permutations, and these are $n!$ in number, as well as the stack permutations, which are also $n!$ in number. We may permute the rows within a band as well as columns within a stack and each of these account for $n!^n$ symmetries. In addition, we can take the transpose of the square. In total, this generates a group of symmetries, which can be viewed as a subgroup of $\mathfrak{S}_{n^4}$. It would be interesting to determine the size and structure of this subgroup. If we agree that two Sudoku squares are *equivalent* if one can be transformed into the other by performing a subset of these symmetries, then an interesting question is the asymptotic behavior of the number S_n^*, of the number of inequivalent Sudoku squares of rank n.

The question of determining the asymptotic behavior of S_n and S_n^* seems to be as difficult as the enumeration of the number of Latin squares of order n. In this context, there are some partial results. A $k \times n$ Latin rectangle is a $k \times n$ matrix with entries from $\{1, 2, ..., n\}$ such that no entry occurs more than once in any row or column. Godsil and McKay [2] have determined an asymptotic formula for the number of $k \times n$ Latin rectangles for $k = o(n^{6/7})$. This suggests that we consider the notion of a *Sudoku rectangle* of rank (k, n) with n^2 columns and kn rows with entries from $\{1, 2, ..., n^2\}$ such that no entry occurs more than once in any row or column or each $n \times n$ sub-grid. It may be possible to extend the methods of [2] to study the asymptotic behavior of the Sudoku rectangles of rank (k, n) for k in certain ranges.

Acknowledgements. We thank Cameron Franc, David Gregory, and Akiko Manada for their helpful comments on an earlier version of this paper. We also thank Jessica Teves for assistance in the literature search.

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WHAT IS... a Galois Representation?

Mark Kisin

Let $\overline{\mathbb{Q}}$ be the field of algebraic numbers. The Galois group $G_{\mathbb{Q}} = \mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ is the group of automorphisms of the field $\overline{\mathbb{Q}}$. A *Galois representation* is simply a representation of this group, or indeed of any Galois group.

Since $G_{\mathbb{Q}}$ is a profinite group—the projective limit of the finite groups $\mathrm{Gal}(K/\mathbb{Q})$ where K is a finite Galois extension of $\mathbb{Q}$—any continuous representation of $G_{\mathbb{Q}}$ on a complex vector space V acts through a finite quotient. We get a richer theory if we consider the action of $G_{\mathbb{Q}}$ on vector spaces over the p-adic numbers $\mathbb{Q}_p$.

In this case a continuous representation may have infinite image. Moreover, very interesting examples of p-adic Galois representations arise from geometry. An algebraic variety X over $\mathbb{Q}$ is an object defined by finitely many algebraic equations with rational coefficients. Grothendieck's *p-adic étale cohomology* attaches to such an X a collection of finite dimensional $\mathbb{Q}_p$-vector spaces with a continuous action of $G_{\mathbb{Q}}$. We will denote these by $H^i(X, \mathbb{Q}_p)$, where i is a non-negative integer.

The vector spaces $H^i(X, \mathbb{Q}_p)$ have a simple description in terms of the complex solutions of the equations defining X. These form a topological space $X(\mathbb{C})$, and the $H^i(X, \mathbb{Q}_p)$ are obtained from the singular cohomology groups $H^i(X(\mathbb{C}), \mathbb{Z})$ by tensoring by $\mathbb{Q}_p$. Unfortunately, one cannot see the action of $G_{\mathbb{Q}}$ with this definition!

At this point it is natural to ask the following question:

> *Which p-adic representations of $G_{\mathbb{Q}}$ occur as an $H^i(X, \mathbb{Q}_p)$ for some X?*

Mark Kisin is professor of mathematics at the University of Chicago. His email address is kisin@math.uchicago.edu.

There is a remarkable conjecture due to Fontaine and Mazur which predicts the answer. To explain this we need to say something about the anatomy of $G_{\mathbb{Q}}$. If ℓ is a prime, let $\overline{\mathbb{Q}}_\ell$ be an algebraic closure of $\mathbb{Q}_\ell$ and fix an embedding $\overline{\mathbb{Q}} \subset \overline{\mathbb{Q}}_\ell$. The elements of $G_{\mathbb{Q}}$ which admit a continuous extension to $\overline{\mathbb{Q}}_\ell$ form a subgroup $D_\ell \subset G_{\mathbb{Q}}$, which is isomorphic to $\mathrm{Gal}(\overline{\mathbb{Q}}_\ell/\mathbb{Q}_\ell)$ and depends on the chosen embedding $\overline{\mathbb{Q}} \subset \overline{\mathbb{Q}}_\ell$ only up to conjugation by elements of $G_{\mathbb{Q}}$. There is a short exact sequence

$$0 \to I_\ell \to D_\ell \to \mathrm{Gal}(\overline{\mathbb{F}}_\ell/\mathbb{F}_\ell) \to 0$$

where $\overline{\mathbb{F}}_\ell$ is an algebraic closure of the finite field $\mathbb{F}_\ell$ of ℓ elements. The quotient $\mathrm{Gal}(\overline{\mathbb{F}}_\ell/\mathbb{F}_\ell)$ is pro-free and topologically generated by the Frobenius automorphism $\mathrm{Frob}_\ell : x \mapsto x^\ell$ of $\overline{\mathbb{F}}_\ell$. A representation of $G_{\mathbb{Q}}$ is said to be *unramified* at ℓ if I_ℓ acts trivially.

There is an analogy between this picture and the fundamental group of a punctured Riemann surface. The analogues of the groups I_ℓ are the subgroups generated by a loop around a puncture. The latter subgroups are of course isomorphic to $\mathbb{Z}$, but the groups I_ℓ are much more complicated than this. Moreover in the topological picture the analogue of the quotient $\mathrm{Gal}(\overline{\mathbb{F}}_\ell/\mathbb{F}_\ell)$ is trivial.

The conjecture of Fontaine-Mazur says that a continuous representation $\rho : G_{\mathbb{Q}} \to \mathrm{GL}(V)$ on a finite dimensional $\mathbb{Q}_p$-vector space V is a subquotient of some $H^i(X, \mathbb{Q}_p)$—we will say that ρ *comes from geometry*—if and only if it satisfies the following two conditions

(1) ρ is unramified at all but finitely many primes
(2) $\rho|_{D_p}$ is *potentially semi-stable*

(More precisely one should consider not just $H^i(X, \mathbb{Q}_p)$ but all its twists by a power of the cyclotomic character.)

The first condition is very natural because if X has good reduction at a prime ℓ then $H^i(X, \mathbb{Q}_\ell)$ will be unramified at ℓ. The second condition is more subtle. Although we have not explained what it means, it depends only on the restriction of ρ to D_p. This is rather remarkable, because if ρ comes from geometry, then results of Deligne and de Jong imply that for a prime ℓ at which ρ is unramified, the eigenvalues of $\rho(\mathrm{Frob}_\ell)$ are *Weil numbers.* This means that they are algebraic and their complex absolute values all have the form $\ell^{w/2}$, where w belongs to a finite collection of integers depending only on X. A priori these eigenvalues are just p-adic numbers, and have no reason to be algebraic. A ρ which comes from geometry is also conjectured to be part of a compatible system of ℓ-adic representations, as well as have an associated complex L-function. It seems incredible that a condition at only one prime p could imply all this, and yet there is mounting evidence for the truth of the conjecture! A representation ρ satisfying (1) and (2) is said to be *geometric.*

That a ρ which comes from geometry satisfies (2) is a consequence of the work of many people, beginning with a paper of Fontaine-Messing. In fact, this is a local result—any variety $X/\mathbb{Q}_p$ gives rise to representations of $\mathrm{Gal}(\overline{\mathbb{Q}}_p/\mathbb{Q}_p)$ which are potentially semi-stable. However the local version of the converse is completely false—a potentially semi-stable representation of $\mathrm{Gal}(\overline{\mathbb{Q}}_p/\mathbb{Q}_p)$ need not come from geometry.

Most of the work on the Fontaine-Mazur conjecture has exploited a connection between Galois representations and *automorphic forms.* The most classical example is that of modular forms. If k, N are non-negative integers, a *modular form* of weight k on $\Gamma_1(N)$ is a holomorphic function f on the complex upper half plane, which satisfies $f(\gamma(z)) = (cz+d)^k f(z)$ for elements $\gamma = \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right)$ in $\mathrm{SL}_2(\mathbb{Z})$, whose reduction modulo N has the form $\left(\begin{smallmatrix} 1 & * \\ 0 & 1 \end{smallmatrix}\right)$. In particular, $f(z+1) = f(z)$ so that f has a Fourier expansion $f = \sum_{n\in\mathbb{Z}} a_n q^n$, where $q = e^{2\pi i z}$. Modular forms are also required to satisfy certain growth conditions, which imply that $a_n = 0$ for $n < 0$.

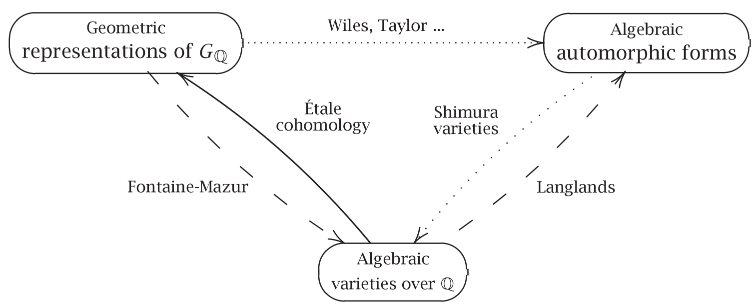
The space of modular forms on $\Gamma_1(N)$ of weight k is finite dimensional, and comes equipped with a collection of commuting operators T_n, $n \geq 1$. If $f = \sum_{n=0}^{\infty} a_n q^n$ is a simultaneous eigenvector for these operators, then $a_1 \neq 0$, and T_n has eigenvalue $\lambda_n = \frac{a_n}{a_1}$.

A theorem of Shimura, Deligne, and Deligne-Serre asserts that for such an f, the field $\mathbb{Q}(\lambda_n)_{n\geq 1}$ is a number field E_f, and that if λ is a finite prime of E_f then there is a continuous representation

$$\rho_{f,\lambda} : G_{\mathbb{Q}} \to \mathrm{GL}_2(E_{f,\lambda})$$

which is unramified at primes ℓ not dividing λN. Moreover at such primes the trace of the representation is given by the Fourier coefficients: $\mathrm{tr}(\rho_{f,\lambda}(\mathrm{Frob}_\ell)) = a_\ell$. The *Cebotarev density theorem* asserts that the elements Frob_ℓ are dense in the group $G_{\mathbb{Q}}$, so the existence of the representation $\rho_{f,\lambda}$ implies that the a_ℓ satisfy a plethora of λ-adic congruences, and even do so simultaneously for all possible λ. This is quite remarkable, given their definition as Fourier coefficients.

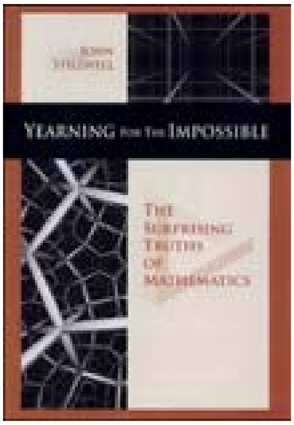
Starting with the spectacular work of Wiles and Taylor-Wiles on the modularity of elliptic curves and Fermat's Last Theorem, there has been significant progress, due to many people, toward establishing the Fontaine-Mazur conjecture for 2-dimensional representations. The basic idea is to show that a geometric representation ρ is equivalent to one of the representations $\rho_{f,\lambda}$, the latter coming from geometry by construction. This relationship between modular forms and geometric objects is an instance of a philosophy of Langlands that algebraic geometry over $\mathbb{Q}$ should be related to certain (so called algebraic) automorphic forms. Together with the Fontaine-Mazur conjecture it suggests that three, apparently completely different, kinds of objects should be intimately related. The situation can be summarized in the following diagram.



Here the dashed arrows indicate a conjecture, while the dotted ones indicate partial progress. Sadly there is only one completely solid arrow! Getting from one bubble to another is usually a highly nontrivial exercise, however success often carries enormous rewards. For example, Deligne was able to pass from modular forms to algebraic geometry and thereby prove the Ramanujan conjecture (and construct the $\rho_{f,\lambda}$ by moving up the solid arrow). Wiles (and his students) was able to pass from Galois representations to modular forms, and thereby prove (via the solid arrow) that elliptic curves were modular, and that their L-functions were therefore entire, as well as proving Fermat's Last Theorem!

Despite some partial results, the basic mechanism linking these three worlds is still largely a mystery. Attempts to understand it are sure to be a rich source of mathematics for decades to come.

Book Review



Yearning for the Impossible

Reviewed by Daniel Biss

Yearning for the Impossible
John Stillwell
A K Peters, Ltd., 2006
$29.95, 244 pages, ISBN 978-1-56881-254-0

John Stillwell's *Yearning for the Impossible* is a new book that seeks to teach some mathematics to an audience of nonspecialists. Its title refers to the idea that mathematical advances tend to come about when researchers manage to understand ideas or constructions once thought to be impossible. So, for example, there are chapters about irrational numbers and imaginary numbers (after all, nobody used to think such things existed), chapters about 4-dimensional geometries and different orders of infinity (because nobody really knew whether one could make sense of such things), and a chapter about unique factorization via prime ideals (because unique factorization without prime ideals is, um, impossible).

The first thing I noticed when perusing this list is that the word "impossible" is being used awfully flexibly. The idea that irrational numbers were impossible was simply the incorrect statement that all lengths constructible via straightedge and compass would be expressible as fractions. In other words, there existed two notions of "number", namely, the set of rational numbers and the set of constructible numbers. Everybody assumed they were the same, but they aren't, and the impossibility lies in the difference between these two concepts. The idea that imaginary numbers were impossible was, by contrast, the perfectly consistent aesthetic decision that going beyond the real numbers and introducing i was simply absurd. The impossibility of unique factorization in arbitrary rings, of course, is a true mathematical theorem.

So, to recapitulate Stillwell, mathematics happens when we grapple with the impossible. The "impossible", in turn, is anything that is either hard to think about, hard to visualize, hard to interpret, or else literally undoable without changing some of the rules. In other words, mathematics is what happens when we're forced to be creative. What began as an intriguing and possibly even provocative theme has now revealed itself to be somewhat more pedestrian. Mathematics, Stillwell turns out to be saying, is sort of like art. And science. And technology. And snowboarding. And pretty much any other activity that requires its practitioners to do something they didn't already know how to do when they began.

Now, to be fair, Stillwell essentially admits this in his first paragraph:

> I hoped to show a general audience that mathematics is a discipline that demands imagination, perhaps even fantasy.

In other words, for all the fancy talk about yearning for the impossible, what's really being asserted is that you need to be imaginative to do math. While this is an important point to make, I do not believe that it is robust enough to structure a book around. Here's why: mathematics has become a highly jargonized discipline, and a book for nonmathematicians needs to circumvent a lot of that jargon and still get at the meaning. If you believe that at least some of the jargon is useful in communicating something, then the jargon-free model must find a communicative replacement for it. Consequently, it's essential that a prospective author of a math-book-for-nonmathematicians devise a useful strategy or theme around which to structure the book.

Daniel Biss is assistant professor of mathematics at the University of Chicago and a Research Fellow at the Clay Mathematics Institute. His email address is `daniel@math.uchicago.edu`.

What should such a theme accomplish? Well, its primary task is to focus the reader's attention and energy in a way that facilitates the communication of some mathematics. In other words, it must be some description or characterization of our subject that primes the reader to view mathematics in a new and useful way. Simply put, it must articulate some unique and telling quality of mathematics.

For this reason, the impossibility theme feels like a bit of a bait-and-switch. I wanted the book to have a theme that helped illustrate the fact that mathematics is an imaginative enterprise, but when examined closely, the purported theme turned out to be a restatement of this fact rather than an illustration of it.

Right. So Stillwell's titular "impossibility" gambit is a little hollow. What's left to the book? Well, its table of contents reveals a fairly typical list of math-that-mathematicians-can't-figure-out-why-nonmathematicians-don't-know-and-love. In addition to the topics mentioned above, it touches on infinitesimals, curvature, projective geometry, and "periodic spaces" (things like tori and cylinders and so forth). As a mathematician, I can't figure out why nonmathematicians don't know and love these topics; this has always been a source of frustration for me. Consequently, I'm invariably eager to see whether a new attempt at explicating them will pass the Mother Test—in other words, whether I will send a copy of the book to my mother. (In recent months, the Mother Test has been threatened in prominence by the newly minted Wife Test, but it turns out that my wife has more than enough stuff to read already.) This book fails that test, but just barely. In other words, it rises nobly to the challenge of describing these topics to a genuine novice; some of the chapters, I think, are very clear, and others are less so. In some sense, a mathematician is the least qualified reviewer of the ultimate success of such a book, but I feel confident in saying that there is much to admire in Stillwell's attempt.

Of course, the task Stillwell sets for himself varies tremendously in difficulty from chapter to chapter. A discussion of irrational numbers, in addition to being entirely elementary, might even be familiar to some veterans of high school mathematics. Ideals and unique factorization, on the other hand, are totally new to practically everyone, and, moreover, they're pretty confusing and weird. (Come to think of it, I wonder if Stillwell really meant "weird" when he said "impossible".)

So let's discuss the chapter on ideals and factorization, remembering that it's probably one of the hardest to write. Stillwell begins by describing prime factorization of integers (not addressing uniqueness or anything, just saying what a prime factorization is). He then discusses division, remainders, and the Euclidean algorithm, which of course leads right into uniqueness of factorizations. He then, at some pains, introduces Gaussian integers and proves that they also have unique factorization. Finally, it's time to drop the bombshell, namely, the failure of unique factorization in $Z[\sqrt{(-5)}]$.

Now comes the hard part. Stillwell needs to introduce ideals, explain why they might be construed to constitute a "reasonable generalization" of the notion of number, and then address their unique factorization properties. He does a reasonable job, going on and on about the mind-blowing qualities of the equation $2 \times 3 = 6 = (1 + \sqrt{(-5)}) \times (1 - \sqrt{(-5)})$ for long enough that the reader begins to genuinely wish for some sort of resolution of this apparently nonunique factorization of the number 6, if only to pacify Stillwell. He then argues that such a resolution can be accomplished only if 2 and $1 + \sqrt{(-5)}$ share some sort of "ideal" common factor. This is good stuff, and I found myself wishing that I'd been taught the definition of an ideal by Stillwell; he's accomplishing something very important and difficult here in demonstrating that there's some real struggle present in the process of mathematical discovery.

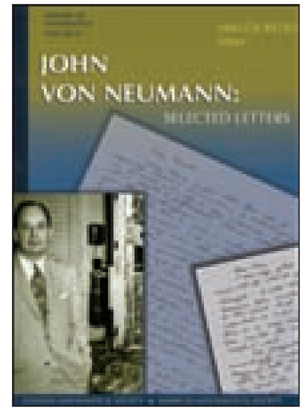
The magic still hasn't happened, though; that takes place when one decides to give the set of all numbers of the form $2M + (1 + \sqrt{(-5)})N$, where M and N are integers, the status of an "ideal number" (or ideal) and then plays the common factorization game with these objects. It's a remarkable intellectual leap and a confusing one at that, and Stillwell labors mightily to make it seem natural. He nearly succeeds, but the truth is that it's almost a hopeless task, because of course the leap isn't natural at all—that's why it's so brilliant.

This brilliance, built mostly out of a childlike faith in the notion of uniquely factorable "ideal numbers", is the source of the real mathematics here; everything else is a combination of arithmetic and hot air. This, I suspect, is what Stillwell is getting at with his fixation on the impossible. And if the theme were a truly useful one and the book effective, then we'd be ready for the moment of brilliance. We'd realize that the impossible moment, that hallmark of mathematical achievement, was just around the corner, and we'd be eager to ride its wave to the promised shores.

Instead, the appearance of ideals 170-odd pages into the book just left me anxious and confused, full of questions. Is this as clear as it could be? Does it pass the Mother Test? Do I understand what an ideal is as well as I did before I read this chapter?

I don't know. If truth be told, I can't figure out why nonmathematicians don't know and love this topic, but I'm starting to wonder if it's just too hard to explain it to them. Maybe even impossible.

Book Review



John von Neumann: Selected Letters

Reviewed by George Dyson

John von Neumann: Selected Letters
Miklós Rédei, editor
AMS/LMS History of Mathematics, 2005
$59.00, 301 pages
ISBN 0-8218-3776-1

"If influence of a scientist is interpreted broadly enough to include impact on fields beyond science proper, then John von Neumann was probably the most influential mathematician who ever lived," states Miklós Rédei in *John von Neumann: Selected Letters*, introducing a convincing body of evidence, much of it published for the first time. The 121 letters (organized by correspondent) are lightly annotated, and despite the absence of an index (or even a chronological listing) this will be an enduring reference work. A detailed mathematical and biographical introduction occupies the first forty pages of the book.

John von Neumann (1903–1957) roamed freely among all branches of science and mathematics, and, in his spare time, helped create several new fields in the gaps where disciplines did not yet exist. He was a prolific correspondent, delivering penetrating commentary in handwritten letters that were often composed in hotel rooms, aboard ships, in airplanes, on trains, and in between meetings that kept him from ever settling down to uninterrupted work. On every available scrap of paper, he left a trail of ideas. There is an astonishing density of expression, due, in this reviewer's opinion, not only to von Neumann's intellect but to the medium of *handwriting* as a process for distilling the essentials into a minimum of words.

Von Neumann's correspondence provides a much different picture from his published work. "I write rather freely and fast if a subject is 'mature' in my mind," he explained to F. B. Silsbee (2 July 1945), apologizing for an undelivered manuscript, "but develop the worst traits of pedantism and inefficiency if I attempt to give a preliminary account of a subject which I do not have yet in what I can believe to be its final form." His hesitancy to discuss unfinished work in print is marvelously absent from his informal correspondence, leaving a record both of the development of many of his well-known results and the germs of many still-undeveloped avenues of research. He was fond of the postscript and with a simple "P.S." would often sketch out some completely new idea.

The present selection is rich in von Neumann's contributions to many fields of mathematics ("every part of it except number theory and topology", in the assessment of his colleague Eugene Wigner) and despite inevitable omissions delivers an impressive sampling of the breadth and influence of von Neumann's work. The book opens with a brief but eloquent foreword by Peter Lax, followed by a detailed and exceptionally informative introduction by Marina von Neumann Whitman (only child of von Neumann, and an important figure in her own right as an economist and advisor to four presidents of the United States). Rédei then presents a concise introductory survey of von Neumann's mathematical contributions, divided into the following subject areas: logic and foundations of mathematics, operator algebras, unbounded operators, quantum mechanics, quantum logic, ergodic theory, computer science, and game theory.

Subjects that are less well represented include: hydrodynamics (especially turbulence, and the behavior of shock waves), statistics (especially the "Monte Carlo" method and its repercussions), cellular automata, meteorology, and theoretical biology. Von Neumann's manifold contributions to nuclear weapons design, weapons effects, aerospace technology, military strategy, and the area once known as operations research, though

George Dyson is a research associate at Western Washington University. His email address is gdyson@ias.edu.

mathematical in nature, are understandably largely absent here.

The difficulty with von Neumann as a subject is that he defied categorization from one minute to the next. "No matter which way you looked he always seemed to belong somewhere else," explained Klari, his second wife who survived him by six years. "The pure mathematicians claimed that he had become a theoretical physicist; the theoretical physicists looked at him as a great help and advisor in applied mathematics; the applied mathematician was awed that such a pure and ivory-towerish mathematician would show so much interest in his applied problems and, I suspect, in certain government circles they may have thought of him as an experimental physicist, or even an engineer."

The unifying theme in von Neumann's life and work—abundantly demonstrated by his correspondence—was an insatiable urge to be absorbing information and solving problems all the time. "Johnny's most characteristic trait was his boundless curiosity about everything and anything, his compulsive ambition to know, to understand any problem, no matter on what level," wrote Klari after his death. "Anything that would tickle his curiosity with a question mark, he could not leave alone; he would sulk, pout, and be generally impossible until, at least to his own satisfaction, he had found the right answer."

To John von Neumann, the normal barriers between pure and applied mathematics did not exist. "A certain contact with the strivings and problems of the world that surrounds us is desirable and even necessary," he wrote to J. Robert Oppenheimer (19 February 1948) defending his right, as a professor at the Institute for Advanced Study, to be distracted (and remunerated) by outside work. As explained by his collaborator Stanislaw Ulam, he had an ability, "perhaps somewhat rare among mathematicians, to commune with the physicists, understand their language, and to transform it almost instantly into a mathematician's schemes and expressions. Then, after following the problems as such, he could translate them back into expressions in common use among physicists."

And not only physicists. Von Neumann was able to commune with anyone, translate their problems (even problems that they did not recognize as problems) into the language of mathematics, find a solution, and translate this back into their language, bringing the results down to earth.

All the letters in this volume are historically or mathematically important, can be read on multiple levels, and are technically precise. To identify favorites is impossible, but I will exercise the reviewer's privilege by singling out one example that shows the von Neumann mind at work. This letter was written on April 9, 1953, to T. V. Moore of the Standard Oil Development Company, for whom von Neumann was working as a consultant, supposedly on questions of computer modeling for oil and gas exploration, but evidently also on other things. "This is the letter that I promised you," begins von Neumann, "restating the proposal that I made regarding the 'La Salina Operations Problem'…that we discussed when we last met in New York." Von Neumann then precisely describes the landscape: "This problem…deals with the operations of 18 tankers between La Salina and Las Piedras and Aruba. The comings and goings of the tankers are described in statistical terms only, i.e., they are subject to fluctuations which depend on fortuitous events like weather, conditions in the ports of call, etc."

John von Neumann, December 28, 1903–February 8, 1957.

Photograph used with permission of Marina von Neumann Whitman.

Then he becomes specific, demonstrating that he is not about to ignore the details: "Each one of the tankers has a separate and characteristic mean roundtrip time for its assigned run, which may be La Salina–Las Piedras–La Salina (for 6 tankers), or La Salina–Aruba–La Salina (for 12 tankers). The times for the…actual trip of a specific tanker…may be presumed to have a purely statistical distribution, to the extent to which it is due to weather and to similar factors. Under certain conditions, the captains or the crews of all ships heading for a certain port may desire to make port at a definite moment. (You mentioned the effects of a good picture or amusement in port.)"

Suddenly, things don't look so good: "My impression is that this problem…is one of considerable difficulty from the point of view of a strict analytical-mathematical treatment. That is, I think that it will be very difficult to derive complete formulas for the probabilities and means involved, and to penetrate to the ultimate mean that is desired—namely, to the 'mean turnover' of the entire fleet…as a function of the number of berths."

Now comes the light at the end of the tunnel: "I would, therefore, suggest that the problem be treated as a 'statistical experiment'…The procedure would have to be somewhat like this: Represent each tanker by some suitable form of record, e.g., by a punch card. Program calculations which will develop the further history of this tanker, always deriving those quantities which depend on

Photograph used with permission of Nicholas Vonneumann.

Budapest, 1915, at age 11, with cousin Katalin (Lily) Alcsuti. "She greatly admired, but didn't understand what John was writing," explains von Neumanns's brother Nicholas Vonneumann. "He used such graphics as the letter sigma and so on."

chance…with the use of suitable tables of random numbers…to control the behavior of the fictitious tankers on the punch card."

Of course, we now see immediately that von Neumann is describing a Monte Carlo simulation, the now-ubiquitous method of statistical approximation that he, Stan Ulam, Nick Metropolis, and others developed at Los Alamos for designing bombs. By replacing "neutron" with "tanker" and "cross section" with "number of berths", von Neumann has captured the essence of the problem and explains to Standard Oil, in plain English, exactly what he proposes to do:

> In this scheme, then, each punch card represents a tanker and has to be used to develop the tanker's further history in its dependence on various chance events and their interaction with each other. It is thus possible to trace the history of as many days of operation as desired (say, a few years). I would only like to emphasize that the data handling and computing equipment to process these punch cards at an adequately high speed exists, and that one could in this way obtain good statistical averages in quite short times.

Similarly delightful insights abound throughout this book, on subjects as diverse as the priority dispute between von Neumann and G. D. Birkhoff over the ergodic theorem (handled with masterly diplomacy) to von Neumann's first impression of Alan Turing ("I think that he is quite promising") and the attempt to win an overdue professorship for Kurt Gödel ("he may easily do more work in mathematics proper. In fact…his probability of doing some is no worse than that of most mathematicians past 35").

Von Neumann gives advice on computer hardware (to Lewis Strauss, 20 October 1945): "If we devote…several years to experimentation with such a machine, without a need for immediate applications, we shall be much better off at the end of that period in every respect, including the applications." And six years later he has advice for software engineers (to Marston Morse, 23 April 1952): "The difficulty is that most people who have been active in this field seem to believe that it is easier to write a new code than to understand an old one." Either one of these insights are worth the price of the book.

On the ever-recurring question—whether mathematicians should offer their services to the military or not—von Neumann's answer (to Saunders Mac Lane, 17 May 1948) is as pragmatic as it gets: "The Army is a large and nonhomogeneous organization. I think that we have a chance to do some work which is useful, both to the Army and to the mathematical community. We can do this in one particular sector of the Army where the authorities have 'seen the light'. I don't think that we should be influenced too much by the inadequacies in other sectors."

It was generous, and perhaps risky, for the *Notices* to assign this review to someone who is not a mathematician but has spent many years tracking down von Neumann's correspondence, which could fill a shelf of volumes such as this. This reviewer has assumed the technical accuracy of Miklós Rédei's mathematical introduction, and resisted the temptation to mention documents not included in the book.

There are two possible approaches to compiling a selection of von Neumann's letters. One approach is to select brief excerpts from as many letters as possible. The other approach, taken by Miklós Rédei, is to present a more limited number of letters in their entirety. This gives, in effect, a Monte Carlo sampling of the mind that produced the letters, based on the random walk of a limited subset of its unbounded store of ideas. The resulting statistical approximation is good. This approach is less likely to reflect the bias of the editor, and the result is one with which I believe the author of the letters would agree.

Photograph used with permission of Marina von Neumann Whitman.

Von Neumann in the Florida Everglades, January 1938, on his first American road trip with Klari von Neumann, who notes that "between 1946 and 1955 we crossed the country twenty-eight times by car."

IIT Bombay Is Looking for Faculty

Bill Casselman

In this issue of the *Notices,* as well as in last month's, can be found a classified advertisement by the Indian Institute of Technology Bombay (IITB) for several open faculty positions. The Institute is located in Powai, just to the northeast of central Mumbai, which is the financial capital of India and now in a fascinating period of transition. Mumbai is perhaps known to mathematicians primarily as the home of the Tata Institute for Fundamental Research.

These advertisements are a small part of a long story.

The Indian Institutes of Technology

There are currently seven Indian Institutes of Technology distributed around the country, and they are without qualification among the best undergraduate schools in the world. The number is about to be enlarged to ten, and the enrollment of the existing ones is to be expanded.

It was in 1946 that a governmental committee recommended the establishment of several institutes of technology in India, apparently with the model of the Massachusetts Institute of Technology in mind. The very first was the one at Kharagpur, in the vicinity of what was then Calcutta. This was housed in a vacant building known as the Hijli Detention Center, originally built by the British in 1930 to hold some of the rapidly growing population of political prisoners demanding independence for India. This irony was not left unnoticed by Prime Minister Nehru in his inauguration of the Institute.

Other Institutes followed—IITB, founded in 1958, was the second. It is about to celebrate its Golden Jubilee.

A degree from one of the IITs has always been a valuable possession, but in the current economic boom it has become almost priceless. Graduates of IIT occupy important positions in academia, industry, and government in India, and they are also among the founders and executives of major Silicon Valley companies. Competition for admission is conceivably the most intense anywhere in the world—in each of recent years roughly 300,000 students took exams for 5,000 positions at the seven schools. The ones who get in are both intelligent and willing to work hard—astonishingly hard, compared to the level of effort most North American mathematics faculty see in their students. Admission to these elite institutions by no means leads to a relaxing time, as it does in some countries—course loads are very heavy, and nearly all students lead a tough if rewarding life.

The original Institute building—the former Hijli detention center—at Kharagpur.

The Current Expansion

The current expansion plans contrast strongly with the pace of growth of the IITs up to now—it took sixty years to establish seven campuses, and in the next few years three more will be set up. Enrollment at the existing ones will be increased dramatically, and the number of faculty positions in many departments will be roughly doubled. This reflects a general concern of the government to improve higher education, and particularly technical education, around the country—this year's national budget for technical education is twice what it has been in previous years.

Bill Casselman is professor of mathematics at the University of British Columbia and graphics editor of the Notices.

A view of IITB from the east.

The current expansion plans have several causes. On the one hand, the economy of India is doing spectacularly well, and there is a great need for more skilled workers. But there is also a political demand to improve accessibility to higher education for that part of the population currently at the bottom of the ladder, which in India includes in particular those who once occupied the lower castes. In parts of the country this is a huge percentage of the population. Although the importance of caste classification has diminished in recent years, it still plays a significant role. There is currently in place at the IITs a quota of 22.5% for certain disadvantaged groups, and the new proposal is to add to this a quota of 27% for what are called officially Other Backward Classes (OBCs). The total enrollment at IITB would then rise by 54%. This will be an interesting experiment in affirmative action. As I write, a suit to reject the government's plans for these quotas (called officially *reservations*) has begun in the Supreme Court of India, but it seems almost certain that they will be carried through in some form.

Curiously, the political pressure to increase enrollment of OBCs does not seem to have led to similar pressure for enrollment of women.

The Role of Mathematics in the IITs

The IITs are primarily schools of engineering. Although much of the teaching effort of their mathematics departments goes into service courses, they offer a wide variety of programs aimed at both undergraduate and graduate students. At the moment there are about forty students in the Ph.D. program at IITB, and the current chairman of the IITB Mathematics Department, Jugal Verma, tells me that he considers the department now to be primarily a research department. There is currently no possibility to major in mathematics in IITB as part of the standard undergraduate program, although advanced degrees are offered in many fields of mathematics. The most popular option for undergraduates these days is computer science, and for understandable reasons—only the very best of those admitted to the IITs have the choice of majoring in CS.

At the center of the IITB campus.

Many of the faculty members at IITs who are best known internationally are in CS. This includes some who are known for their mathematical accomplishments. One is Manindra Agrawal of IIT Kanpur, who, along with some undergraduate research students, found an algorithm of polynomial complexity for primality testing a few years ago. Another is Narendra Karmarkar, well known for finding a polynomial time algorithm to solve linear programming problems, who was an undergraduate in CS at IITB.

One proposal among those currently on the table at IITB is a joint major in computer science and mathematics. For someone considering working at IITB, the prospect of teaching some of the world's very brightest undergraduates would seem a strong attraction.

The Mathematics Department at IITB is among the best in research in all of India, in addition to enjoying an extremely pleasant teaching environment. There are currently twenty-seven faculty members, and the plan is to add in the next few years thirty-seven new positions; this is proportionally more than the expected increase in student enrollment, and the intention is to bring back an enviable faculty:student ratio of 1:9 seen in former

The Hindi word for mathematics is *ganith*, displayed on the side of the mathematics building. Instruction in IITB is in English.

years. The general expansion of higher education in India, as well as a migration of talent out of the country, has contributed to a relative scarcity of mathematicians capable of teaching at the level the IITs expect. For this reason, IITB has received official permission to advertise abroad, and hence the advertisements. The difficulties of acquiring enough good faculty in the next few years has also led to consideration of an arrangement of visitor teaching programs.

The Environment of IITB

IITB is located in the Mumbai suburb of Powai, a bit to the north east of the city. The campus is fairly self-contained, physically isolated from its neighborhood and relatively attractive. Just to its north lies Sanjay Gandhi National Park, a very large and interesting place to visit—with a couple of large lakes, an ancient temple at Kanheri, and an extremely varied wildlife population. Until a new fence was recently constructed there was a problem on campus with leopards raiding the resident pet population. (This is different in degree but not in kind, I suppose, from the problems with the persistent coyote populations of some North American universities.) There is one large and pleasant lake on the edge of campus, as visitors to the official guest house will recall fondly.

Most faculty as well as staff live on the campus itself; the current plans for expansion include plans for much building construction and renovation, of both offices and residences.

References

The photograph of the Hijli detention center has been taken from the Wikipedia site. `http://en.wikipedia.org/wiki/Hijli`

Wikipedia also has a good article about the IIT campuses in general:

`http://en.wikipedia.org/wiki/Indian_Institutes_of_Technology`

The official website for the mathematics department at IITB is

`http://www.math.iitb.ac.in/`

A heron in flight over the lake at the western edge of the IITB campus.

Bridges London, 2006

Mike Field

What Is Bridges?

Bridges London was the ninth meeting of the annual Bridges conference. The common thread running through Bridges meetings has been mathematics and how it intertwines with art, sculpture, architecture, music, poetry.… A Bridges meeting is structured so that there can be fruitful and stimulating interaction between people from very diverse fields. For a mathematician, Bridges can be a fascinating experience: just listening and talking to sculptors (who often have no formal mathematical background) and seeing the astonishing range and power of their mathematical visualization. Educational outreach is an important component of Bridges, and all Bridges meetings have had programs and workshops for teachers, usually with a geometric theme (London did not disappoint in this respect). Bridges meetings also include live performed music and theater as well as excursions, often to locations with a strongly geometric theme.

The seeds for Bridges were most probably sown during the "Art and Mathematics" (AM) conferences organized by Nat Friedman at the State University of New York at Albany from 1992 to 1998 (the last meeting being the AM 98 meeting held at Berkeley and coorganized with Carlo Séquin of the University of California, Berkeley). These meetings had a focus on art, especially sculpture, and architecture, and later evolved into ISAMA (International Society of the Arts, Mathematics, and Architecture). Reza Sarhangi (then of Southwestern College, Winfield, Kansas) participated in many of these meetings and concurrently was developing interdisciplinary courses on art and mathematics at Southwestern College that were an inspirational mix of art, mathematics, and education.

The first Bridges conference was organized by Reza Sarhangi and held in 1998 at Southwestern College, with the title "Bridges: Mathematical Connections in Art, Music, and Science". Under his leadership and direction, Bridges has been an annual event ever since. For several years the conference was held in Winfield, but after Reza Sarhangi's move to Towson University, Bridges has been held at Towson (2002), Granada (2003, joint with ISAMA), Winfield (2004), Banff (2005), and London (2006). In 2007 the meeting will be held July 24–27 at the University of the Basque Country, San Sebastian, Spain.

As the subtitle "Mathematical Connections in Art, Music, and Science" suggests, Bridges conferences are eclectic affairs. One gets some of the feel by reading through the list of contents of a Bridges proceedings. For example, in the 1998 proceedings there are papers ranging from "A Symmetry Classification of Columns" (by mathematicians Martin Golubitsky and Ian Melbourne) through "Continuum, Broken Symmetry and More" (by sculptor Charles Perry) to "The Mathematics of Steve Reich's Clapping Music" (by mathematician Joel Haack). However, the proceedings by themselves do not give the full sense of the depth, range, interest, or atmosphere of a Bridges conference. At the Winfield conferences, the international virtuoso violinist Corey Cerovsek played for the audience after plenary sessions in the morning, often preceding his performance with an extempore talk on a topic from physics or mathematics (concurrently with his musical studies, Corey completed all the coursework for a Ph.D. in mathematics when he was about sixteen). Aside from the music, there would usually be theater shows as well as teacher workshops, held at the end of the conference. Participation in these early meetings was about sixty. I had the pleasure of being at the 1999 and 2000 Winfield meetings and can testify to the great atmosphere of these meetings, where artists, mathematicians, computer scientists, and educators would talk into the early hours. As the years passed, the meetings have become larger, the number of parallel sessions in the afternoons greater (four parallel sessions as well as teacher workshops ran in London), but the underlying themes and dynamics have stayed the same.

Some Highlights of Bridges London

The Format and Organization

The 2006 meeting of Bridges was hosted by the Institute of Education, University of London. The

Mike Field is professor of mathematics at the University of Houston. His email address is `mf@uh.edu`*.* *Supported in part by NSF Grant DMS-0600927.*

local organizers were John Sharp and Philip Kent (both of the London Knowledge Lab, part of the Institute of Education). Overall coordination and finances were handled as usual by Reza Sarhangi.

The conference ran August 4–9, and all the lectures and talks were held in the superb Institute of Education main building in Bloomsbury. The conference comprised 8 one-hour plenary sessions held in the mornings; about 90 twenty-minute talks held in four parallel sessions in the afternoons; a series of teacher workshops, "Bridges for teachers, teachers for Bridges", held in the afternoons; and a (mathematics) musical evening. The conference concluded with a very well-attended Bridges family day, which included mathematical master classes, a Zometool workshop, and a mathematics activities event for children. There were also a mathematics and art exhibition organized by Robert Fauthner and several major excursions. I attempt below to convey at least some of the flavor of this very successful and interesting meeting.

The Talks

The eight plenary talks were given in the morning sessions and covered topics ranging from architecture through sculpture and computer science to more mathematical (and visual) presentations such as "Non-Euclidean Symmetry and Indra's Pearls" (Caroline Series, Warwick). I was particularly fascinated by the talk given by Brady Peters and Xavier DeKestellier from the Specialist Modeling Group (SMG) of the internationally renowned London-based architects Foster and Partners. Some of the more than one hundred major projects to which the SMG has contributed include the new Beijing Airport and the current Smithsonian Institution project in Washington, DC. Anyone who has visited London recently will have seen the "Gherkin" (more formally, 30 St Mary Axe or the Swiss Re building) and perhaps also the new London City Hall, situated close to the Tower of London. Both buildings were designed by Foster and Partners with input from the SMG. They are visually very striking and incorporate complex geometric forms (see the pictures accompanying this article and also the Foster and Partners website, `http://www.fosterandpartners.com/`). Both buildings were designed with energy conservation and ecological factors to the fore. The unusual deformed sphere shape of London City Hall optimizes energy conservation, and the building was designed using advanced computer modeling techniques. The "lean back" of the building provides shading for naturally ventilated offices, and the building's cooling systems utilize ground water pumped up from boreholes. Overall, the building uses only a quarter of the energy of a typical air-conditioned office building. As a mathematician, I found it particularly interesting to see the ways in which geometric form entered into the design process of these buildings: for example, through minimization of the generation of vortices and turbulence caused by a tall building like the Gherkin and by choosing geometries that optimize energy conservation and air flow within the building.

Approximately ninety talks were given in the afternoon sessions together with ten teacher workshops. The talks ranged over a wide area including music, architecture, tilings, polyhedra, puzzles, and Islamic art, all with a mathematical connection. Rather than attempt to describe a subset of the talks, I recommend consulting the more than 650-page *Bridges London Proceedings* (2006, published by Tarquin Press in Europe and available from `http://www.mathartfun.com` in the USA).

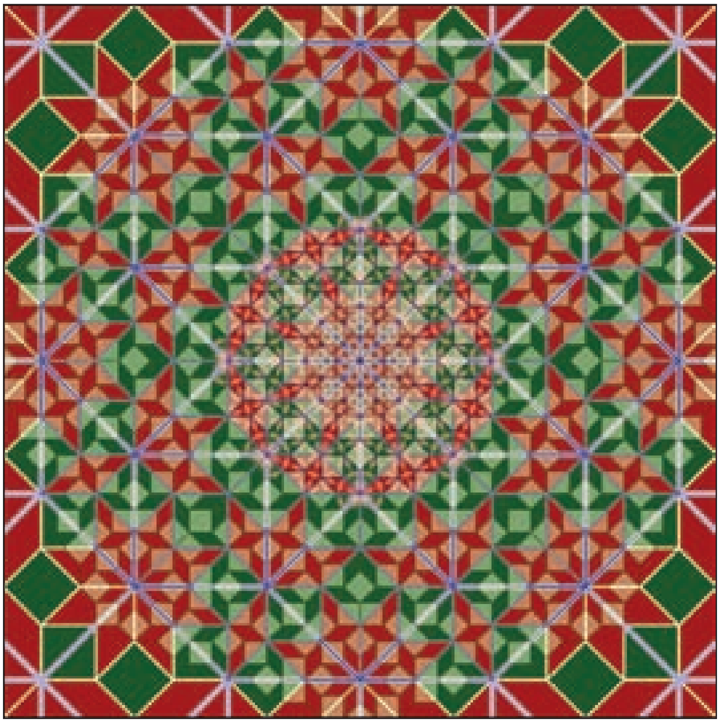
Top, "The Gherkin", 30 St Mary Axe, London. Bottom, London City Hall. Both buildings designed by Foster and Partners.

Workshops and Geometry Master Classes

Bridges has always included workshops for teachers. Bridges London introduced a new innovation: mathematical master classes, organized in conjunction with the Royal Institution. These were held during the morning of the final family day and included master classes on "Mathematics and Perspective" (Christopher Zeeman, FRS), "Anamorphic Art" (Alan Davies), "Mathematics and Juggling" (Colin Wright), and "Celtic and African Art" (Christopher Budd). The meeting concluded with an afternoon of mathematical activities for "children from 5 to 95", which ranged from mathematical origami to Zometool. More than 350 people, including many families, took part in the family day activities, and the event was sponsored by the Clothworkers' Foundation and the London Mathematical Society.

Excursions

August 5 and 8 were reserved for excursions. I took the opportunity of visiting the Ismaili Centre



Ammann Scaling, by Edmund Harriss, Imperial College, London.

in Kensington with a group of about twenty conferees led by John Sharp. (Although I had spent the previous year visiting Imperial College—just up the road—and had heard much about the Ismaili Centre, I never actually got to see inside.) The Ismaili Centre in London is a religious, social, and cultural meeting place for the Ismaili community in the United Kingdom and was the first center to be built for the Ismaili community in the West. The center has been open to the public since 2005 but only on certain days or for organized group visits. As might be expected, the geometry and design used in the center are exquisite. A very notable theme is the evolution of the complex carpet design as one moves through the building upwards towards the meeting and prayer areas (I wish I could show some pictures but no photography was allowed).

The Art Exhibit

In parallel with the conference, there was a mathematical art exhibit in the Institute of Education Building. There were more than fifty exhibitors covering a wide range of mathematically influenced sculpture, polyhedral models, and two-dimensional and fractal art. All of the works shown (and many others) can be viewed on the Bridges website, `http://www.bridgesmathart.org`. Rather than make a selection or discuss any of these works, I have included a picture by one of the younger exhibitors, Edmund Harriss from Imperial College London (his field of research is, not surprisingly, substitution tilings and quasi-crystals). The image shown, titled *Ammann Scaling*, is an Ammann-Beenker tiling and is an eightfold version of the famous Penrose tiling. Like the Penrose tiling, it can be generated by a cut-and-replace or substitution rule. In the image, several layers of the generation of the tiling have been placed on top of each other so as to show the construction and the resulting scaling symmetry.

Some Thoughts and Conclusions

There is a longstanding debate about the nature of art in mathematics and mathematics in art. In its purest form this is expressed by G. H. Hardy's comment in *A Mathematician's Apology* (Cambridge University Press, 1993) that "A mathematician, like a painter or poet, is a maker of patterns" (interestingly, his next sentence "A painting may embody an idea but the idea is usually commonplace and unimportant…," gets less attention in the mathematics press). My own view is that there is much scope for the use of mathematical forms in art, especially sculpture, but that the forms in themselves do not constitute art—they need to be transformed into art through the efforts, skill, and inspiration of the artist. In this respect it was refreshing to see so much innovation and new, mathematically inspired, art at the London meeting of Bridges.

Bridges has much to offer those interested in mathematics education. Everywhere in a Bridges conference there are examples and suggestions of how mathematics not only plays a role in everyday life but is an essential tool in understanding, interpreting, and indeed even *seeing* the world around us. The ideas discussed at a Bridges meeting seem to me to be quite central to the problem of developing successful general mathematical education as well as in overcoming the increasing alienation and hostility of many students and members of the general public towards mathematics and science.

Acknowledgments and Links. Thanks to Reza Sarhangi and John Sharp for providing information about Bridges and the organization of Bridges London. More information about Bridges may be found at `http://www.bridgesmathart.org`. Information about the forthcoming 2007 Bridges meeting in San Sebastian may also be found on the Bridges website. Thanks to Edmund Harriss for providing details about his work *Ammann Scaling*. The photographs of 30 St Mary Axe and City Hall, London, were taken by Nigel Young and are kindly reproduced with the permission of the copyright holders, Foster and Partners, London.

Last but not least, a very recent addition and resource for the art and mathematics community is JMA, the *Journal of Mathematics and the Arts* (editor-in-chief, Gary Greenfield, University of Richmond; publisher, Taylor and Francis). Many of the associate editors of JMA have a long association with Bridges.

Sum of Its Parts Results in AMS Award for UCLA Math

Lisa Mohan

The UCLA mathematics department received the 2007 AMS Award for Exemplary Achievement in a Mathematics Department. The award citation is in the May 2007 *Notices*, page 633.

Already a large department by any standards, UCLA mathematics continues to chart a path of extraordinary growth. Under the leadership of its newest and youngest chair, Christoph Thiele, the department is positioning itself as the largest pipeline into mathematical careers in the United States. This fall, the projected arrival of six new faculty to an already consummate group of fifty-two accomplished and dedicated professors will reinforce the department's position among the top twelve mathematics programs in the country (most recent National Research Council ranking) and one of the top four in applied mathematics (*U.S. News & World Report*). Its partner institute, the interdisciplinary Institute for Pure and Applied Mathematics (IPAM), recently won a five-year renewal grant from the National Science Foundation (NSF) with a substantial increase in funding as a result of its far-reaching, quality programming. The department's pioneering math education group, serving kindergarten through twelfth graders and their teachers, is finding innovative ways to increase mathematics competency in California public schools. Despite its size, indeed perhaps because of it, the UCLA mathematics department has managed to create a unique synergy among these complementary training programs. For this success, the department has garnered the second annual AMS Award for an Exemplary Program or Achievement in a Mathematics Department.

Undergraduate Education

The department's undergraduate program has experienced tremendous growth in the past decade, attracting over 850 mathematics majors in 2006. Degrees are up 56 percent from 160 in 1996 to 249 in 2006. What is the big appeal? In a word: choice. The department is a pioneer of the broad-based mathematics major. Understanding the importance that math brings to other disciplines, faculty have sought to forge partner programs with other departments, resulting in an array of undergraduate degrees impressive in scope. The department's popular joint program in mathematics and computer science in the 1980s later evolved into a host of different programs and joint majors for students. Importantly, the growth and emphasis on applied math did not happen at the expense of pure math. Indeed, in 1997 when Tony Chan (currently NSF assistant director for Mathematics and Physical Sciences) became the first applied mathematician to serve as chair, the department's two groups had forged a harmonious relationship, in part because the boundary between pure and applied mathematics is fluid and constantly shifting.

Projected 2007–2008 Academic Year

Faculty	57
Postdocs	34
Graduate students	195
Undergraduate majors	850
IPAM participants (annual estimate)	1,000

Research for Undergrads

Research Experiences for Undergraduates (REU) is an integral part of the department's undergraduate mission, involving a select subset of majors, typically twenty to thirty students per year, who are interested in research. The program has the active participation of senior faculty and postdoctoral mentors and was broadened in the last two years to include the applied mathematics laboratory. This environment provides students the opportunity to participate in real physical experiments, most recently with robotics and fluid flow. Testifying to the success of the program, Andrea Bertozzi, current director of applied mathematics, spearheaded a collaborative proposal with Harvey Mudd College for a US$1.3 million NSF research training grant to advance the careers of undergraduate students in math, engineering, and computer science. Utilizing this award, granted in July 2006, Bertozzi plans to

Lisa Mohan is the communications coordinator for the mathematics department at the University of California, Los Angeles. Her email address is `lisam@math.ucla.edu`.

Undergraduate Degrees

- Pure Mathematics
- Applied Mathematics (science/engineering)
- Math/Applied Science:
  - –Actuarial Plan
  - –Management/Accounting
  - –Medical and Life Sciences Mathematics
  - –History of Science
  - –Individual Plan (for students who have equal interest in two or three allied fields, i.e., Math/Chemistry, Math/Physics, Math/Statistics/Management)
- Mathematics of Computation
- Mathematics for Teaching (teaching careers)
- Mathematics/Economics
- Atmospheric Sciences/Mathematics

keep up to sixty undergraduates from both schools engaged in research over the next five years. Bertozzi points to the inclusive nature of research in the department, "Everyone has something to do. The trick is to identify those tasks for undergrads and show them how their work and involvement matter."

Graduate Education

The year 2000 was a turning point in the department's graduate program, with the awarding of a US$5 million NSF VIGRE (Vertical Integration of Research and Education) grant under the direction of Robert Greene. The goal of the program is to initiate changes in the way professional mathematicians are trained, specifically by promoting interaction between mathematics and other fields and by increasing the number of U.S. citizens and permanent residents in math and science. The program has expanded from 112 in its first year to a projected 195 students in fall 2007.

In 2005 the NSF recognized the success of UCLA's efforts by renewing the department's VIGRE grant. The continuing success of the program is attributable to collective faculty efforts to redevelop the program into one where students learn in a research group environment, starting early in their studies. Today the program supports the majority of the department's Ph.D. students, as well as several postdocs. In addition, funding is provided for new students to attend a summer preparatory course prior to the start of the fall semester. Over half of these students pass one of three qualifying exams before taking any courses. After the first year, Ph.D. students take special graduate seminars to streamline them into research projects. All research faculty are involved in running these seminars, with fifteen to twenty offered in every subdiscipline during any given quarter. 2006 Fields Medalist Terence Tao explains how the department's large size works to its advantage in this environment. "In our analysis group, for instance, there are seven or eight faculty and twenty graduate students who are interested in the field, so we came up with a graduate seminar where the students themselves present material. In a smaller place, we couldn't do this kind of seminar."

Graduate Student Summer Internship Program

The department sought to further broaden graduate students' research experiences in 2005, initiating a summer internship program in which students work outside the department with faculty from other disciplines or industry. The key is to draw in outside mentors, who, with no initial financial investment, gain the expertise of a young mathematician. A fruitful experience may lead to other opportunities down the road. Since the ratio of graduate students to tenure-track faculty is very high in the department compared to mathematics departments of comparable standing, the cross-collaboration is a huge benefit. Explains Bertozzi, "We have the organizational structure to leverage the time of senior faculty while providing enhanced learning experiences for students and postdocs. Everybody wins." Summer 2006 saw the participation of sixteen students working with departmental mentors in computer science, neuroscience, and electrical engineering; in the medical school's neuroimaging and neurology specialties; and at the Los Alamos National Laboratory. Industry and partner university mentors included the National Geospatial Intelligence Agency, Xerox, Digital Domain, Finantix Inc., MIT Mechanical Engineering, and the Department of Informatics at the University of Athens.

Postdoctoral Program

The department's postdoctoral program is one of the largest in the country. This year's hiring campaign will result in the addition in fall 2007 of twelve bright young researchers from all over the world. Currently, thirty-four postdocs work under various research faculty in the department, sometimes across disciplines. This integration of research interests and the opportunity to train under diverse faculty is a huge draw. Many postdocs go on to secure prestigious positions at world-renowned institutions, and some advance to faculty positions within the department, including former postdoc Terence Tao. Department chair Thiele believes that in addition to a first-rate research experience, the program provides mentoring opportunities for postdocs at the undergraduate and graduate levels that enhance their experience and are also vital to the successful synergy of the department's training programs. Postdocs are routinely encouraged to design courses for undergraduates in tandem with their research interests. Former postdoc Chad Topaz, now at the University of Southern California, assisted in the development of the

Mary Jo Robertson, photographer.

UCLA math faculty and postdocs (Chair Christoph Thiele, standing in front row, green jacket) under congratulatory banner outside UCLA's Institute for Pure and Applied Mathematics (IPAM).

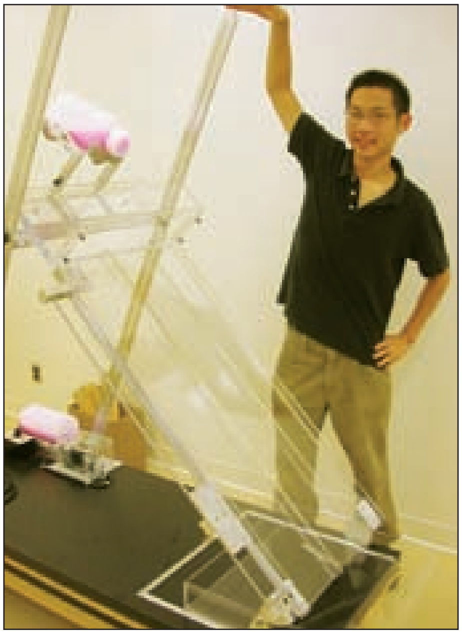
applied mathematics curriculum, designing a class on nonlinear dynamical systems. Another postdoc, Hedrick Fellow Raanan Schul, is passionate about bringing his research to undergraduates and plans to introduce a course on wavelets. Nathan Ryan, a fellow in the department's Program in Computing, has designed two undergraduate courses in mathematical cryptology and applied cryptology.

Institute for Pure and Applied Mathematics

In perhaps the most high-profile, synergistic move of the department, five math faculty—Tony Chan, Eitan Tadmor (now at the University of Maryland), Bjorn Engquist, John Garnett, and Mark Green—capitalized on UCLA's nascent culture of cross-disciplinary collaboration by creating an institute for pure and applied mathematics, or IPAM. Selected from approximately twenty competing proposals in NSF's 1998 competition, IPAM was the only new national institute created and was funded with a US$12.75 million grant. The grant was renewed in 2005 with a 36 percent increase in funding. Green, who became the institute's director in 2001, attributes its success to the size and strength of UCLA's science departments, including a first-rate medical school, together with forward-thinking mathematics faculty who are actively involved in IPAM.

As Green sees it, IPAM's mission is to bring together enterprising risk-takers in math and science, often working together for the first time, to create breakthroughs in their fields. Green remembers one of the first programs, which is exemplary of the way IPAM works: "I was trying to make contacts around campus, asking, 'Who don't I know that I should know?' One of the people who was suggested to me was Art Toga." A neuroscientist, Toga works in brain mapping and neuroimaging at UCLA's medical school. With Chan, Tadmor, and fellow faculty member Stanley Osher (who was recently elected to the National Academy of Sciences and is IPAM's director of special projects), Green met with Toga, leading to IPAM's first imaging program, Imaging in Medicine and Neurosciences. This was only the beginning of cross-fertilization of math and medicine on the UCLA campus. Chan, Osher, and Toga went on to collaborate on a major grant application, resulting in the Center for Computational Biology (CCB), one of four road map institutes funded by the National Institutes of Health. This would be only one of several large-scale initiatives IPAM helped to inspire. Department mathematicians are essential to the success of these interdisciplinary endeavors. Explains Green, "We have put computation and mathematics on everybody's radar and demonstrated by example how useful we can be."

Well over one thousand participants a year now come through IPAM's doors, interacting in programs that are designed to create visionary, interdisciplinary collaboration between mathematicians and scientists from biology, medicine, engineering, and other disciplines, as well as industry and national laboratories. The institute's mission is nationally focused and geared toward assembling a broad research community, ranging from distinguished professors to young ladder faculty, postdocs, graduate students, and undergraduates.



UCLA math undergraduate (Chi Wey) working with slurry flow experiments in Andrea Bertozzi's Applied Math Lab.

IPAM hosts two 3-month-long programs in the fall and spring, a series of short programs in the winter, a Graduate Summer School, and Research in Industrial Projects for Students (RIPS) in the summer. In a recent integrative move, the department's graduate seminar series was expanded to include participation in long programs at the institute.

Because IPAM is actively engaged with the campus at large and the wider scientific community, Green views it as an opportunity to change the culture of math—to help mathematics get where it needs to be in the twenty-first century and capitalize on the huge number of niches for mathematicians that are evolving in other fields. Critical to this goal is a strong foundation of harmonious relations between pure and applied mathematics. One of the more dramatic events that brought pure and applied together was IPAM's 2004 program, Multiscale Geometry and Analysis in High Dimensions. Together, the department's most high-profile pure math star, Terence Tao, and his Caltech colleague Emmanuel Candès, who works in applied math, used a variety of pure math ideas in harmonic analysis to create startling new ways of dealing with data. The 2007 AMS von Neumann Symposium, to be held this summer, is devoted to this topic.

RIPS

Bringing together people who would not ordinarily find themselves in the same program extends to the institute's ambitious RIPS program. Created in 2001 and inspired by Robert Borrelli's math clinic at Harvey Mudd, RIPS matches undergraduates with industry partners to work on real-world projects for nine weeks in the summer. The program has grown from twelve students working on four projects in 2001 to thirty-six students working on nine projects in 2007. Starting this summer, the program will expand to Beijing in a new partnership with Microsoft Research in Asia. Program applicants hail from a wide range of institutions, many of which are nationally prominent and some of which are abroad. The result has been an unusually diverse mix of young talent, and typically 35 to 40 percent are women.

RIPS students work in teams of four on projects initiated by industry sponsors or national laboratories. A faculty mentor, usually a postdoc, identifies the cutting-edge mathematics the students will need in order to tackle the problem. The group stays in weekly contact with their industry mentor to keep the project on track. The program provides a unique experience for students and sponsors, including working on a team, writing a group project report, and giving a team project presentation to the other project groups as well as to the industry partner. In some cases, they present their projects at conferences. Says Green, "The thing the sponsors are always amazed by is what these kids can do."

Since the program has been so successful and the institute's facilities are at capacity, IPAM has forged a partnership with Microsoft Research in Asia to bring RIPS students to Beijing this summer. Green and Director of Microsoft Research in Asia Harry Shum will assemble twenty students—ten Americans and ten Chinese—to team up on five projects. In keeping with the collaborative spirit of IPAM, each team of four will be half Chinese and half American. Green initially floated the concept of RIPS Beijing with his own kids, who validated the idea of doing really interesting research at a cool place abroad. After Beijing is up and running, the institute hopes to expand the program internationally. But as Green is quick to note, IPAM is "not out to conquer the world. We are out to benefit it."

UCLA's Math Education Programs

Like IPAM, the department's highly successful K–12 math education programs sprang from and continue to be nurtured by the efforts of innovative department educators. Professor emeritus Phil Curtis, who will turn eighty next year, is credited with inspiring all of the math education programs beginning with the Visiting High School Mathematics Teacher Program in 1979 under the leadership of then department chair Ted Gamelin. It was the first step in a decades-long relationship with California public schools and the beginning of a multipronged effort to develop teacher-leaders in math. A major step was taken in 1987 when Curtis spearheaded the establishment of the Joint Mathematics Education Program, which puts undergraduate math majors on a fast track to obtain a teaching credential through a joint program with UCLA's Graduate School of Education and Information Studies.

Gamelin currently serves in the leadership role in the math education program. In addition to his considerable expertise, the department provides office space and administrative services, such as computing and accounting. Gamelin describes the investment the department has made this way: "They trust that we're running a quality operation, and the fact that the trust is reciprocal has made

K–12 Math Education Programs

California Mathematics Project (CMP)
Executive Director: Susie Håkansson

Dedicated to the professional development needs of K–12 math teachers statewide

CMP consists of nineteen sites on University of California (UC) and California State University (CSU) campuses led by a statewide office in the UCLA Department of Mathematics. In the summer of 2007 CMP was awarded a five-year, US$5.25 million grant by the California Postsecondary Education Commission (CPEC) to focus on math teacher retention at ten of the sites.

UCLA Mathematics Content Programs for Teachers (MCPT)
Director: Shelley Kriegler

An entrepreneurial venture creating services and materials for California schools

MCPT contracts for professional development and sales of materials to client school districts, teachers, and students. Currently in development is a new textbook and program for algebra readiness in California middle schools.

Mathematics Diagnostic Testing Project (MDTP)
Site Director and Relations with Schools Coordinator:
Heather Calahan

Preparing students for high school math by identifying the key topics and skills needed for success

Founded to assess the preparedness of California freshmen for university calculus in the 1990s, MDTP develops diagnostic tests that measure student readiness for courses ranging from prealgebra to calculus. Its success led to an expansion to diagnostic tests for grades 6 through 12 and offices in ten sites.

it work." He also points out that "In math education the work often involves issues that a research mathematician would not give a second thought to but that are quite challenging and require the insight and expertise of math education professionals to address."

Gamelin brought on three educators at the start of the new century to manage three unique and independent programs that substantially support the state's K–12 math students and teachers: Susie Håkansson, Shelley Kriegler, and Heather Calahan. Their visionary direction has created a renaissance of sorts for math education, culminating this year in the UCLA Mathematics and Teaching Conference, collectively hosted by the department and the three math education groups, with the participation of over one hundred fifty local mathematics teachers and other professionals.

In addition to their professional jobs, these three math educators assist the department with curriculum design and conduct seminars and courses for preservice teachers, further contributing to the collaboration between math education and the department. Today the multifaceted program is the most extensive outreach program to the K–12 community housed in a research mathematics department.

The synergy of its comprehensive training programs is at the heart of the department's acknowledged success in educating current and future mathematicians. The faculty make the most of the department's large size to maximize students' learning experiences. The southern California weather might have something to do with the natural harmony in the department, and certainly the large concentration of scientific capital in the southern California area is an advantage. UCLA's culture of interdisciplinary collaboration that made IPAM possible is also a big factor. Bottom line, the department is a collaborative bunch that sees itself at the forefront of raising the profile of math in society at large and preparing the way for the unprecedented opportunities for mathematicians in the twenty-first century.

Varadhan Receives 2007 Abel Prize

Srinivasa S. R. Varadhan

The Norwegian Academy of Science and Letters has decided to award the Abel Prize for 2007 to Srinivasa S. R. Varadhan of the Courant Institute of Mathematical Sciences, New York University, "for his fundamental contributions to probability theory and in particular for creating a unified theory of large deviations."

The Abel Prize carries a cash award of NOK 6,000,000 (US$875,000). Previous recipients are Jean-Pierre Serre (2003), Michael Atiyah and I. M. Singer (2004), Peter D. Lax (2005), and Lennart Carleson (2006).

Citation

Probability theory is the mathematical tool for analyzing situations governed by chance. The law of large numbers, discovered by Jacob Bernoulli in the eighteenth century, shows that the average outcome of a long sequence of coin tosses is usually close to the expected value. Yet the unexpected happens, and the question is: how? The theory of large deviations studies the occurrence of rare events. This subject has concrete applications to fields as diverse as physics, biology, economics, statistics, computer science, and engineering.

The law of large numbers states that the probability of a deviation beyond a given level goes to zero. However, for practical applications, it is crucial to know how fast it vanishes. For example, what capital reserves are needed to keep the probability of default of an insurance company below acceptable levels? In analyzing such actuarial "ruin problems", Harald Cramér discovered in 1937 that standard approximations based on the Central Limit Theorem (as visualized by the bell curve) are actually misleading. He then found the first precise estimates of large deviations for a sequence of independent random variables. It took thirty years before Varadhan discovered the underlying general principles and began to demonstrate their tremendous scope, far beyond the classical setting of independent trials.

In his landmark paper "Asymptotic probabilities and differential equations" in 1966 and his surprising solution of the polaron problem of Euclidean quantum field theory in 1969, Varadhan began to shape a general theory of large deviations that was much more than a quantitative improvement of convergence rates. It addresses a fundamental question: what is the qualitative behavior of a stochastic system if it deviates from the ergodic behavior predicted by some law of large numbers or if it arises as a small perturbation of a deterministic system? The key to the answer is a powerful variational principle that describes the unexpected behavior in terms of a new probabilistic model minimizing a suitable entropy distance to the initial probability measure. In a series of joint papers with Monroe D. Donsker exploring the hierarchy of large deviations in the context of Markov processes, Varadhan demonstrated the relevance and the power of this new approach. A striking application is their solution of a conjecture of Mark Kac concerning large time asymptotics of a tubular neighborhood of the Brownian motion path, the so-called "Wiener sausage".

Varadhan's theory of large deviations provides a unifying and efficient method for clarifying a rich variety of phenomena arising in complex stochastic systems, in fields as diverse as quantum field theory, statistical physics, population dynamics, econometrics and finance, and traffic engineering. It has also greatly expanded our ability to use computers to simulate and analyze the occurrence of rare events. Over the last four decades, the theory of large deviations has become a cornerstone of modern probability, both pure and applied.

Varadhan has made key contributions in several other areas of probability. In joint work with Daniel W. Stroock, he developed a martingale method for characterizing diffusion processes, such as solutions of stochastic differential equations. This new approach turned out to be an extremely powerful way of constructing new Markov processes, for example infinite-dimensional diffusions arising in population genetics. Another major theme is the analysis of hydrodynamical limits describing the macroscopic behavior of very large systems of interacting particles. A first breakthrough came in joint work with Maozheng Guo and George C. Papanicolaou on gradient models. Varadhan went even further by showing how to handle non-gradient models, greatly extending the scope of the theory. His ideas also had a strong influence on the analysis of random walks in a random environment. His name is now attached to the method of "viewing the environment from the travelling particle", one of the few general tools in the field.

Varadhan's work has great conceptual strength and ageless beauty. His ideas have been hugely influential and will continue to stimulate further research for a long time.

Biographical Sketch

Srinivasa S. R. Varadhan was born January 2, 1940, in Madras (Chennai), India. He is currently professor of mathematics and Frank J. Gould Professor of Science at the Courant Institute of Mathematical Sciences, New York University.

Varadhan received his B.Sc. honors degree in 1959 and his M.A. the following year, both from Madras University. In 1963 he received his Ph.D. from the Indian Statistical Institute, Calcutta, with the distinguished Indian statistician C. R. Rao as his thesis advisor. It is reported that during his thesis defense Varadhan noticed a visitor in the room whom he did not know and who asked many penetrating questions. After the exam he discovered that it was the famous Russian mathematician and probabilist A. N. Kolmogorov. Apparently Rao arranged the date of the exam knowing that Kolmogorov would be visiting India then in order to show off his star student, and Kolmogorov was not disappointed.

Srinivasa Varadhan began his academic career at the Courant Institute of Mathematical Sciences as a postdoctoral fellow (1963–66), strongly recommended by Monroe Donsker. Here he met Daniel Stroock, who became a close colleague and co-author.

In an article in the *Notices of the American Mathematical Society* Stroock recalls these early years: "Varadhan, whom everyone calls Raghu, came to these shores from his native India in the fall of 1963. He arrived by plane at Idlewild Airport and proceeded to Manhattan by bus. His destination was that famous institution with the modest name, the Courant Institute of Mathematical Sciences, where he had been given a postdoctoral fellowship." Varadhan was assigned to one of the many windowless offices in the Courant building, which used to be a hat factory. Yet despite the somewhat humble surroundings, as Stroock puts it, "from these offices flowed a remarkably large fraction of the postwar mathematics of which America is justly proud."

Srinivasa Varadhan has stayed loyal to Courant, where he served as assistant professor (1966–68), associate professor (1968–72), and became a full professor in 1972. When he and Stroock were awarded the AMS Steele Prize in 1996, Varadhan did not fail to mention that "The Courant Institute provided us with an ideal intellectual environment, active encouragement and the support of our senior colleagues, particularly Louis Nirenberg and Monroe Donsker."

Varadhan must have lived up to the high expectations he was met with as a postdoctoral fellow. In 1965 Louis Nirenberg wrote to Monroe Donsker recommending Varadhan for a faculty appointment at Courant: "I think very highly indeed of Varadhan and predict a great future for him. He is very young, and I think in many ways he might be the best appointment as assistant professor in probability we could make." Fifteen years later Srinivasa Varadhan was appointed director of Courant (1980–84), following Peter Lax.

In a letter of recommendation addressed to the president of New York University, Lax wrote, "We feel that now, when the Courant Institute is full of renewed vigor and is facing the future with confidence, is the time to pass on the leadership to a new generation." Thus, Srinivasa S. R. Varadhan followed Peter Lax both as director of Courant and now also as an Abel Laureate. Varadhan came back to serve a second period as director of Courant (1992–94).

Varadhan has held visiting positions at Stanford University (1976–77), the Mittag-Leffler Institute (1972), and the Institute for Advanced Study (1991–92). Varadhan was an Alfred P. Sloan Fellow (1970–72) and a Guggenheim Fellow (1984–85). His awards and honors include the AMS Birkhoff Prize (1994), the Margaret and Herman Sokol Award of the Faculty of Arts and Sciences of New York University (1995), and the AMS Steele Prize (1996). He also has two honorary degrees from Université Pierre et Marie Curie in Paris (2003) and from the Indian Statistical Institute in Kolkata, India (2004).

Varadhan was an invited speaker (1978) and a plenary speaker (1994) at the International Congress of Mathematicians (ICM) in 1978 and 1994. He was elected a member of the American Academy of Arts and Sciences (1988), the Third World Academy of Sciences (1988), and the National Academy of Sciences (1995). He was elected a Fellow of the Institute of Mathematical Statistics (1991), the Royal Society (1998), and the Indian Academy of Sciences (2004).

Srinivasa Varadhan is married to Vasundra Varadhan, who is a professor at New York University. They have one son, Ashok. Their eldest son, Gopal, was one of the victims of the 9/11 terrorist attacks on the World Trade Center Twin Towers.

—From Norwegian Academy of Science and Letters announcements

The Mathematical Work of Jon Kleinberg

Gert-Martin Greuel, John E. Hopcroft, and Margaret H. Wright

The *Notices* solicited the following article describing the work of Jon Kleinberg, recipient of the 2006 Nevanlinna Prize. The International Mathematical Union also issued a news release, which appeared in the October 2006 issue of the *Notices*.

The Rolf Nevanlinna Prize is awarded by the International Mathematical Union for "outstanding contributions in mathematical aspects of information sciences". Jon Kleinberg, the 2006 recipient of the Nevanlinna Prize, was cited by the prize committee for his "deep, creative, and insightful contributions to the mathematical theory of the global information environment".

Our purpose is to present an overall perspective on Kleinberg's work as well as a brief summary of three of his results:

- the "hubs and authorities" algorithm, based on structural analysis of link topology, for locating high-quality information on the World Wide Web;
- methods for discovering short chains in large social networks; and
- techniques for modeling, identifying, and analyzing bursts in data streams.

Kleinberg's Nevanlinna citation includes two other areas in which he has made important contributions: theoretical models of community growth in social networks and the mathematical theory of clustering. Readers interested in learning more about his work on these topics should consult the list of papers given on his home page [8].

Gert-Martin Greuel is professor of mathematics at the Universität Kaiserslautern, Germany, and director of the Mathematical Research Institute Oberwolfach. His email address is `greuel@mathematik.uni-kl.de`.

John E. Hopcroft is IBM Professor of Engineering and Applied Mathematics in Computer Science at Cornell University. His email address is `jeh@cs.cornell.edu`.

Margaret H. Wright is professor of computer science and mathematics and chair of the computer science department at the Courant Institute of Mathematical Sciences, New York University. Her email address is `mhw@cs.nyu.edu`.

Find the Mathematics; Then Solve the Problem

Kleinberg broadly describes his research [8] as centering "around algorithmic issues at the interface of networks and information, with emphasis on the social and information networks that underpin the Web and other on-line media". The phenomena that Kleinberg has investigated arise from neither the physical laws of science and engineering nor the abstractions of mathematics. Furthermore, the networks motivating much of his work lack both deliberate design and central control, as distinct from (for example) telephone networks, which were built and directed to achieve specific goals. He has focused instead on networks that feature very large numbers of unregulated interactions between decentralized, human-initiated actions and structures.

A striking motif in Kleinberg's research is his ability to discern and formulate plausible mathematical structures to describe problems that represent vague, even elusive, human goals. Some of his most brilliant work has begun by asking questions that might seem initially to have no clear answers—"What do people really want from a Web query?", "How can individuals find short paths in a social network using only local information?"—and then coming up with mathematical insights that illuminate the important features of reality. Once he has the mathematical definitions in hand, Kleinberg goes on to create powerful solution techniques that are both mathematically elegant and successful in practice.

It is difficult to overstate the impact of Kleinberg's work on several major real-world problems, and for this reason alone his work is well known to the broad scientific and technological community.

Two additional reasons for Kleinberg's high visibility are that his two best known results can be grasped intuitively without invoking details of the underlying mathematics and that he is a master expositor, with a much-admired ability to motivate and explain his work so that readers can follow his logic every step of the way.

Hubs and Authorities

There is no better way to appreciate Kleinberg's signature style than reading the journal version of his most famous paper, "Authoritative sources in a hyperlinked environment" [2]. The paper opens with a discussion of an important but imprecisely defined problem—finding the "most relevant" webpages in response to a given broad query. Beyond the quandary of how to define "most relevant" in a meaningful way, Kleinberg notes that the difficulty with broad queries is the vast overabundance of possibly relevant hits, so that what is needed is an automated way to filter out the most "definitive" pages.

Starting with what seem at first to be the obvious solutions, he carefully explains the impossibility of using purely internal features of a page to rate its authority, as well as the flaw in relying on the query words themselves. The next part of the paper motivates and proposes the nonobvious concept of using a link-based model of the graph representing the Web to create an algorithm for deciding which pages are "authoritative".

His method for finding these pages begins by constructing a small, focused subgraph G_σ of the Web, where σ is the query string, using link structure to identify its strong authorities. Along the way, Kleinberg explains how, in addition to finding highly authoritative pages, we would expect to find *hub pages*, i.e., those that contain links to many relevant authoritative pages and thereby allow unrelated pages to be discarded. Based on a natural equilibrium between hubs and authorities, a novel iterative algorithm is defined that updates numerical weights for each page until a fixed point is reached. Happily, Kleinberg also shows that a similar process and algorithm can be adapted when seeking "similar" webpages.

A feature of Kleinberg's algorithm that brought joy to fans of linear algebra is that the desired authority-weight and hub-weight vectors form, respectively, the principal eigenvectors of A^TA and AA^T, where A is the adjacency matrix of the graph of a collection of linked pages. As well, the nonprincipal eigenvectors of these matrices can be used to extract additional densely linked collections of hubs and authorities.

Always a careful scholar, Kleinberg provides a summary of previous approaches to a variety of related problems: measuring "standing" in social networks, "impact" in scientific citations, ranking of Web pages, hypertext document retrieval, clustering of explicitly linked moderate-size structures, and spectral graph partitioning. A fascinating historical note is his discussion of the then recently published Brin–Page page-rank algorithm [1], soon thereafter to become the basis of Google.

Near the end of the paper (which was published before the rise of Google), Kleinberg surveys three user studies designed to evaluate the effectiveness of his algorithm. These reported favorable results concerning Web users' perception of improved quality in their query results, but he cautions that such an evaluation is a challenging task because individual judgments of relevance are inherently subjective.

How Can It Be a Small World?

Stanley Milgram's social psychology experiments in the 1960s (see, for example, [6]) reported on and popularized the idea of a "small world"—that any two individuals who are apparently far apart in a social network can find "short paths" to reach one another. To model the mathematical properties required to ensure the existence of short paths, Watts and Strogatz [7] proposed a "superposition" structure in which a relatively small number of random long-range links are added to a high-diameter network with edges at each node representing local social links. The long-range links provide the opportunity for a short chain through the entire network. Informally, a "small-world" network is an n-node graph such that almost all pairs of nodes are connected by chains whose length is a polynomial in $\log n$, i.e., the number of links traversed to reach one node from another is likely to be exponentially smaller than the number of nodes.

In his fundamental paper "The small-world phenomenon: an algorithmic perspective" [3], Kleinberg noted that Milgram's findings not only indicated the surprising existence of short chains, but also revealed an equally surprising result about the existence of *algorithms* that would enable an arbitrary person, knowing only information about the locations of his/her individual acquaintances, to construct a short communication path to a target stranger.

To begin his analysis of small-world models, Kleinberg first proved a negative result—that, using the Watts-Strogatz model, no *decentralized* algorithm, meaning one whose decisions are based solely on local information, can produce paths of small expected length relative to the diameter of the network.

Next, he generalized the Watts-Strogatz model into an infinite family of random network models. Starting with two parameters characterizing a node's local and long-range contacts, the model can be simplified to a one-parameter family with an associated clustering exponent α that represents the probability of a long-range connection between two nodes as a function of their lattice distance.

When there is a uniform distribution over long-range contacts (corresponding to $\alpha = 0$), Kleinberg showed that, although short paths exist with high probability, a decentralized algorithm cannot find them efficiently, since the expected time is exponential in the expected minimum path length. In effect, the long-range links are "too random" to be useful to a decentralized algorithm. Although larger values of α allow a decentralized algorithm to take advantage of the structure of the long-range contacts, they become less useful in transmitting the message to an arbitrary far-away node.

The central result is that there is a unique value of α ($\alpha = 2$) for which a decentralized algorithm (using a greedy heuristic) will find a target node in a number of steps bounded by a polynomial in $\log n$. Furthermore, no efficient decentralized algorithm exists for any other value of α. These results generalize from two-dimensional to d-dimensional lattices, with the critical value of α equal to the dimension d. A later paper by Kleinberg [4] includes an extension of these results to other network models, including hierarchical models or models based on set systems.

His work on small-world phenomena has had a direct effect on the design of peer-to-peer systems and focused Web crawling techniques, and it has also inspired numerous papers by other authors. Using one of Kleinberg's contributions to illustrate the influence of the other, submission of the combination of "small world" and "Kleinberg" to Google on February 11, 2007, produced nearly 52,000 hits!

Word Bursts and Temporal Analysis

In his 2002 paper [5], Kleinberg considers the problem of extracting meaningful information from document streams (such as email messages or news articles) that arrive continuously over time—in particular, spotting the "burst of activity" that signals the first appearance of a new topic. His stated scientific goal in this work was to devise a mathematical model that allows such bursts to be identified efficiently, with a further aim of analyzing the content via the associated organizational framework. But, as he amusingly relates in the paper's introduction, his personal motivation (one we can all share) was a wish to find an organizing principle based on *time* rather than topic for his ever-increasing volume of accumulated email.

The model proposed by Kleinberg—an infinite-state automaton in which bursts are state transitions—is conceptually related to queueing theory models of bursty traffic. In the most basic form of his model, the gap in time between two consecutive events (messages) is given by an exponential density function such that the expected gap between messages is $1/\alpha$ for some $\alpha > 0$, where α can be interpreted as the rate of message arrivals. Bursts can be added by allowing the model to include interleaved periods with lower and higher rates; these correspond to different states for which the rate depends on the state. In the ultimate model analyzed by Kleinberg in detail, there are an infinite number of states, each denoted by q_i. The sequence $q_0, q_1, \ldots$ models inter-arrival times that decrease geometrically, and there is a cost $\tau(i, j)$ corresponding to the transition from state i to state j. Given this model, Kleinberg shows that an optimal (cost-minimizing) state sequence can be found efficiently by adapting a standard forward dynamic programming algorithm for hidden Markov models. From the results of running this algorithm, one can then define the hierarchical structure that is implicit in a sequence of bursts.

Several fascinating conclusions emerge from this work, including the fact that use of a state-transition model means that bursts are characterized by unambiguous beginnings and endings. Kleinberg notes that, when the document stream consists of email messages, the initial message at which the state transition took place can be seen as a "landmark" in subsequent extended message sequences. Later related work by Kleinberg and others has addressed traffic-based feedback on the Web and the temporal dynamics of online information streams (see [8]).

Summary

Jon Kleinberg's work perfectly fits the Nevanlinna Prize specification since, as we have seen, his mathematical insights have had wide application to multiple elements of information science—the effectiveness of advanced Web search engines, Internet routing, data mining, and the sociology of the World Wide Web. We refer the interested reader to his website [8] for further pointers to papers on these and other topics, including network analysis and management, gossip algorithms, clustering, data mining, comparative genomics, and geometric pattern matching.

Returning to our opening theme, Kleinberg's much-lauded work on information networks is characterized by (i) identifying and formulating fundamental mathematical structures in questions about the real world, (ii) defining meaningful mathematical models that represent crucial features of real-world phenomena, and finally (iii) creating effective algorithms that solve the resulting mathematical problems.

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The Work of Kyosi Itô

Philip Protter

The *Notices* solicited the following article describing the work of Kiyosi Itô, recipient of the 2006 Gauss Prize. The International Mathematical Union also issued a news release, which appeared in the November 2006 issue of the *Notices*.

Photo used with permission of Junko Itô.

Kyosi Itô, 1954, when he was a Fellow at the Institute for Advanced Study, Princeton.

On August 22, 2006, the International Mathematical Union awarded the Carl Friedrich Gauss Prize at the opening ceremonies of the International Congress of Mathematicians in Madrid, Spain. The prizewinner is Kyosi Itô. The Gauss prize was created to honor mathematicians whose research has had a profound impact not just on mathematics itself but also on other disciplines.

To understand the achievements of Itô, it is helpful to understand the context in which they were developed. Bachelier in 1900, and Einstein in 1905, proposed mathematical models for the phenomenon known as Brownian motion. These models represent the random motion of a very small particle in a liquid suspension. Norbert Wiener and collaborators showed in the 1920s that Einstein's model exists as a stochastic process, using the then-new ideas of Lebesgue measure theory. Many properties of the process were established in the 1930s, the most germane for this article being that its sample paths are of infinite variation on any compact time interval, no matter how small. This made the Riemann-Stieltjes integration theory inapplicable. Wiener wanted to use such integrals to study filtering theory and signal detection, important during the second world war. Despite these problems he developed a theory of integrals, known today as Wiener integrals, where the integrands are nonrandom functions. This served his purpose but was unsatisfying because it ruled out the study of stochastic differential equations, among other things.

The problem in essence is the following: how can one define a stochastic integral of the form $\int_0^t H_s dW_s$, where H has continuous sample paths and W is a Wiener process (another name for Brownian motion), as the limit of Riemann-style sums? That is, to define an integral as the limit of sums such as $\sum_{1\le i\le n} H_{\xi_i}(W_{t_{i+1}} - W_{t_i})$, with convergence for all such H. Unfortunately as a consequence of the Banach-Steinhaus theorem, W must then have sample paths of finite variation on compact time intervals. What Itô saw, and Wiener missed, was that if one restricts the class of potential integrands H to those that are adapted to the underlying filtration of sigma algebras generated by the Wiener process, and if one restricts the choice of $\xi_i \in [t_i, t_{i+1})$ to t_i, then one can use the independence of the increments of the Wiener process in a clever way to obtain the convergence of the sums to a limit. This became the stochastic integral of Itô. One should note that Itô did this in the mathematical isolation of Japan during the second world war and was one of the pioneers (along with G. Maruyama) of modern probability in Japan, which has since spawned some of the world's leading probabilists. Moreover since Jean Ville had named martingales as such only in 1939, and J. L. Doob had started developing his theory of martingales only in the 1940s, Itô was unaware of the spectacular developments in this area that were happening in the U.S., France, and the Soviet Union. Thus modern tools such as Doob's martingale inequalities were unavailable to Itô, and his

Philip Protter is professor of operations research at Cornell University. His email address is pep4@cornell.edu.

creativity in the proofs, looked at today, is impressive. But the key result related to the stochastic integral was Itô's change of variables formula.

Indeed, one can argue that most of applied mathematics traditionally comes down to changes of variable and Taylor-type expansions. The classical Riemann-Stieltjes change of variables, for a stochastic process A with continuous paths of finite variation on compacts, and $f \in C^1$ is of course

$$f(A_t) = f(A_0) + \int_0^t f'(A_s)dA_s.$$

With the Itô integral it is different and contains a "correction term". Indeed, for $f \in C^2$ Itô proved

$$f(W_t) = f(W_0) + \int_0^t f'(W_s)dW_s + \frac{1}{2}\int_0^t f''(W_s)ds.$$

This theorem has become ubiquitous in modern probability theory and is astonishingly useful. Moreover Itô used this formula to show the existence and uniqueness of solutions of stochastic ordinary differential equations:

$$dX_t = \sigma(X_t)dW_t + b(X_t)dt; \quad X_0 = x_0,$$

when σ and b are Lipschitz continuous. This approach provided methods with an alternative intuition to the semigroup/partial differential equations approaches of Kolmogorov and Feller, for the study of continuous strong Markov processes, known as diffusions. These equations found applications without much delay: for example as approximations of complicated Markov chains arising in population and ecology models in biology (W. Feller), in electrical engineering where dW models white noise (N. Wiener, I. Gelfand, T. Kailath), in chemical reactions (e.g., L. Arnold), in quantum physics (P. A. Meyer, L. Accardi, etc.), in differential geometry (K. Elworthy, M. Emery), in mathematics (harmonic analysis (Doob), potential theory (G. Hunt, R. Getoor, P. A. Meyer), PDEs, complex analysis, etc.), and, more recently and famously, in mathematical finance (P. Samuelson, F. Black, R. Merton, and M. Scholes).

When Wiener was developing his Wiener integral, his idea was to study random noise, through sums of iterated integrals, creating what is now known as "Wiener chaos". However his papers on this were a mess, and the true architect of Wiener chaos was (of course) K. Itô, who also gave it the name "Wiener chaos". This has led to a key example of Fock spaces in physics, as well as in filtering theory, and more recently to a fruitful interpretation of the Malliavin derivative and its adjoint, the Skorohod integral.

Itô also turned his talents to understanding what are now known as Lévy processes, after the renowned French probabilist Paul Lévy. He was able to establish a decomposition of a Lévy process into a drift, a Wiener process, and an integral mixture of compensated compound Poisson processes, thus revealing the structure of such processes in a more profound way than does the Lévy-Khintchine formula.

In the late 1950s Itô collaborated with Feller's student H. P. McKean Jr. Together Itô and McKean published a complete description of one-dimensional diffusion processes in their classic tome, *Diffusion Processes and Their Sample Paths* (Springer-Verlag, 1965). This book was full of original research and permanently changed our understanding of Markov processes. It developed in detail such notions as local times and described essentially all of the different kinds of behavior the sample paths of diffusions could manifest. The importance of Markov processes for applications, and especially that of continuous Markov processes (diffusions), is hard to overestimate. Indeed, if one is studying random phenomena evolving through time, relating it to a Markov process is key to understanding it, proving properties of it, and making predictions about its future behavior.

Later in life, when conventional wisdom holds that mathematicians are no longer so spectacular, Itô embraced the semimartingale-based theory of stochastic integration, developed by H. Kunita, S. Watanabe, and principally P. A. Meyer and his school in France. This permitted him to integrate certain processes that were no longer adapted to the underlying filtration. Of course, this is a delicate business, due to the sword of Damocles Banach-Steinhaus theorem. In doing this, Itô began the theory of expansion of filtrations with a seminal paper and then left it to the work of Meyer's French school of the 1980s (Jeulin, Yor, etc.). The area became known as *grossissements de filtrations*, or in English as "the expansions of filtrations". This theory has recently undergone a revival, due to applications in finance to insider trading models, for example.

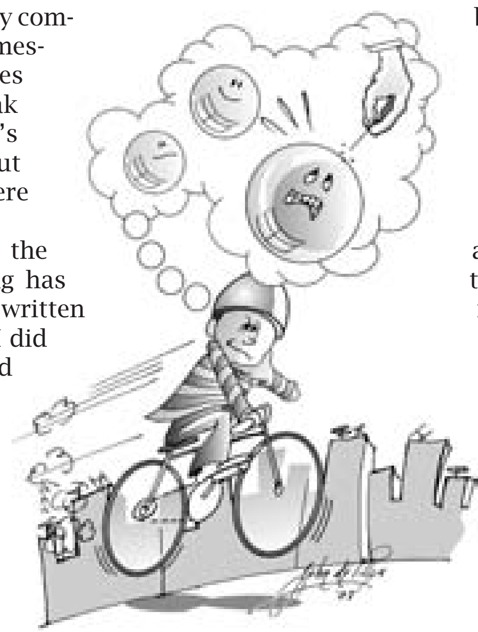
A much maligned version of the Itô integral is due to Stratonovich. While others were ridiculing this integral, Itô saw its potential for explaining parallel transport and for constructing Brownian motion on a sphere (which he did with D. Stroock), and his work helped to inspire the successful use of the integral in differential geometry, where it behaves nicely when one changes coordinate maps. These ideas have also found their way into other domains, for example in physics, in the analysis of diamagnetic inequalities involving Schrödinger operators (D. Hundertmark, B. Simon).

It is hard to imagine a mathematician whose work has touched so many different areas of applications, other than Isaac Newton and Gottfried Leibniz. The legacy of Kyosi Itô will live on a long, long time.

Popping Bubbles

Christopher Tuffley

It's Sunday evening on the first weekend of spring break, on the second of nine straight days without classes. Any other year I would have been home packing for the mountains, if not in them already, but this year I am not; I am in my office instead. In a little over a month I want to have a draft of my dissertation ready for my committee, and in any case this semester I am neither taking classes nor teaching, so spring break isn't really a holiday. There's work to be done, and guilt about work to be done, and I am here trying to do it.



Much as I enjoy teaching, the semester's break from TA'ing has been a real boon. I've finally written down carefully a calculation I did almost a year ago, and an old abandoned line of thought has come to life again. Yes, the technical difficulty that led me to drop it is still there, but only in three or more dimensions. The idea *does* work when $n = 2$, and I have finally broken the barrier between "one" and "many" that's plagued me for so long: where once I could speak only on circles and 1-complexes, now I can speak on surfaces and 2-complexes. The conjecture I'd hoped the idea would prove is true in two dimensions, and I'm one step closer to proving it in general—one step of infinitely many, it's true, but a step closer nonetheless.

The words, though, are slow to catch up with the ideas, and I am struggling to find the best ones to pin my thoughts to the page. Around 8:30 p.m. I call it an evening and pack up to head down the hill for home and dinner. I take my bike from its usual spot beneath the blackboard, brush the chalk dust from the seat, and start down the hall to the lifts. Michael's light is off, so I don't bother knocking on his door and simply push the down button.

As I wait, my mind, now free to wander, reminds me that I already knew the conjecture was true for surfaces. By puncturing its 2-cell, popping it like a bubble, I can shrink a surface down onto its one-dimensional skeleton. Doing this in $k + 1$ places lets me use the one-dimensional case I proved in a tiny corner of a paper already submitted for publication. As the lift descends, I realise it doesn't have to be a surface; the argument works as well for any 2-complex with a single 2-cell. Turning the corner from Hearst onto Sacramento, I realise I'm not limited to a single 2-cell either; if I have more, I just need more punctures. Over dinner it occurs to me that I can pop a 3-, 4-, 5- or n-cell as easily as a 2-cell, and doesn't an exercise I did for homework my first year of grad school give me the rest of what I need to climb two, three, four, five, all the way up?

It's 10:30 p.m., and I am back on my bike up the hill. I want to know what that homework exercise said. If it says what I want it to, I'll have my conjecture in its full generality, and the proof

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Christopher Tuffley is a lecturer in mathematics at Massey University, New Zealand. His email address is `C.Tuffley@massey.ac.nz`. *Written at BIRS Workshop 04w5555, Creative Scientific Writing.*

NSF Budget Request for Fiscal Year 2008

This article is the 35th in a series of annual reports outlining the president's request to Congress for the budget of the National Science Foundation. Last year's report appeared in the May 2006 issue of the *Notices*, pages 580–2.

After a disappointing outcome for the current fiscal year, National Science Foundation (NSF) support for mathematics research is slated to receive a boost for next year. In February 2007, President Bush sent to Congress his budget request for fiscal year 2008, which begins on October 1 this year. The request for the NSF contains an 8.5% increase for the Division of Mathematical Sciences (DMS), the main funder of academic mathematics research in the United States. This increase is comparable to the 8.7% increase for the NSF overall.

Fiscal year 2007 started in October 2006, but appropriations for most of the government had to wait not only until after the elections in November but until after the start of the new Congressional session. From October 2006 until mid-February 2007, most federal agencies were operating on continuing resolutions that kept their budgets at fiscal 2006 levels. In fact, the President's fiscal 2008 budget request arrived ten days before Congress finally passed appropriations bills for fiscal 2007. The appropriation for the NSF hewed closely to the president's original request and accorded the DMS only a meager 3.3% increase, about level with inflation. This small increase is all the more disappointing when one notes that the appropriation gave the NSF overall an increase of 4.8%.

According to an analysis by the American Association for the Advancement of Science (`http://www.aaas.org/spp/rd/`), the president's 2008 budget request mirrors the main priorities set by the Bush Administration, putting constraints on domestic spending and calling for increases for the military. These priorities can be seen in the request's treatment of research and development: funding for basic and applied research would fall

Table 1: National Science Foundation (Millions of Dollars)

	2004 Actual	Change	2005 Actual	Change	2006 Actual	Change	2007 Estimate*	Change	2008 Request
(1) Mathematical Sciences Research Support	$ 200.3	0.0%	$ 200.2	-0.3%	$ 199.5	3.3%	$ 206.0	8.5%	$ 223.5
(2) Other Research Support (Note a)	4277.0	-1.8%	4199.7	6.8%	4483.5	3.7%	4650.8	10.8%	5153.0
(3) Education and Human Resources (Note b)	944.1	-10.7%	843.5	-17.0%	700.3	13.8%	796.7	-5.8%	750.6
(4) Salaries and Expenses (Note c)	230.6	2.9%	237.3	10.6%	262.5	0.4%	263.6	14.6%	302.0
(5) Totals	$5652.0	-3.0%	$5480.8	3.0%	$5645.8	4.8%	$5917.2	8.7%	$6429.0
(6) (1) as a % of the sum of (1) and (2)	4.47%		4.55%		4.26%		4.24%		4.16%
(7) (1) as a % of (5)	3.54%		3.65%		3.53%		3.48%		3.48%

Tables prepared by Notices *staff. Totals may not add up due to rounding.* **Note a:** Support for research and related activities in areas other than the mathematical sciences. Includes scientific research facilities and instrumentation. **Note b:** Support for education in all fields, including the mathematical sciences. **Note c:** Administrative expenses of operating the NSF, including the National Science Board and the Office of the Inspector General. *Estimate of 2007 appropriation from the National Science Foundation and from the R and D Budget and Policy Program, American Association for the Advancement of Science.

2% across the government, while funding for development would rise 3%, mainly due to weapons development in the Department of Defense and spacecraft development at the National Aeronautics and Space Administration.

When it comes to basic research, the President's American Competitiveness Initiative (ACI) figures prominently in the fiscal 2008 request. The idea behind the ACI is that investments in research in physical sciences and engineering will spur innovations needed to keep the United States competitive in the global economy. The three agencies taking the lead on the ACI are the NSF, the Office of Science in the Department of Energy, and the National Institute of Standards and Technology. Each of these agencies is slated for a healthy increase in 2008. The ACI is the main reason for the substantial 9.0% requested increase for the NSF's Mathematical and Physical Sciences (MPS) directorate. The ACI was launched in fiscal 2007, and while it did not produce gains for the DMS in that year, it is one of the drivers behind the requested 8.5% increase for the division for fiscal 2008.

Samuel M. Rankin III, director of the AMS Washington Office, is optimistic that the increases for fiscal 2008 will be appropriated, partly because Congress is now controlled by the Democrats. He said that the Democrats are backing the idea of an innovation initiative more strongly than are the Republicans, despite President Bush pushing for the ACI since last year. "The Democratic leaders are very vocal about supporting innovation, especially [House speaker] Nancy Pelosi," Rankin remarked. The Democrats have also expressed concern over the fate of the NSF's Education and Human Resources (EHR) directorate, whose budget has sustained cuts in recent years. While the 2007 appropriation provided a substantial 13.8% increase to EHR, the 2008 request would bring the EHR budget below the 2005 level.

In fiscal 2008 the NSF will devote US$52 million to a new foundation-wide initiative called Cyber-enabled Discovery and Innovation (CDI). According

Table 2: Directorate for Mathematical and Physical Sciences (Millions of Dollars)

	2004		2005		2006		2007		2008	
	Actual	% of Total	Actual	% of Total	Actual	% of Total	Estimate*	% of Total	Request	% of Total
(1) Mathematical Sciences	$ 200.3	18.3%	$ 200.2	18.7%	$ 199.5	18.4%	$ 206.0	17.9%	$ 223.5	17.8%
(2) Astronomical Sciences	196.6	18.0%	195.1	18.2%	199.7	18.4%	215.0	18.7%	233.0	18.6%
(3) Physics	227.8	20.9%	224.9	21.0%	234.1	21.5%	249.0	21.6%	296.1	23.6%
(4) Chemistry	185.1	17.0%	179.3	16.8%	180.7	16.6%	191.0	16.6%	210.5	16.8%
(5) Materials Research	250.6	23.0%	240.1	22.4%	242.6	22.3%	257.0	22.4%	282.6	22.5%
(6) Office of Multidisciplinary Activities	31.1	2.8%	29.8	2.8%	29.9	2.7%	32.0	2.8%	34.4	2.7%
(7) Totals	$1091.6	100.0%	$1069.4	100.0%	$1086.6	100.0%	$1150.0	100.0%	$1253.0	100.0%

Table 3: Compilation of NSF Budget, 2002–2008 (Millions of Dollars)

	2002 Actual	2003 Actual	2004 Actual	2005 Actual	2006 Actual	2007 Estimate*	2008 Request	2002–2006 Change	2002–2008 Change
(1) Mathematical Sciences Research Support	$ 151.5	$ 178.8	$ 200.3	$ 200.2	$ 199.5	$ 206.0	$ 223.5	31.7%	47.5%
Constant Dollars	*84.2*	*97.2*	*106.0*	*102.5*	*99.0*			*17.6%*	
(2) Other Research Support (Note a)	3579.8	4054.7	4277.0	4199.7	4483.5	4650.8	5153.0	25.2%	43.9%
Constant Dollars	*1989.9*	*2203.6*	*2264.2*	*2150.4*	*2224.0*			*11.8%*	
(3) Education and Human Resources (Note b)	866.1	934.9	944.1	843.5	700.3	796.7	750.6	-19.1%	-13.3%
Constant Dollars	*481.4*	*508.1*	*499.8*	*431.9*	*347.4*			*-27.8%*	
(4) Salaries and Expenses (Note c)	176.6	201.0	230.6	237.3	262.5	263.6	302.0	48.6%	71.0%
Constant Dollars	*98.2*	*109.2*	*122.1*	*121.5*	*130.2*			*32.6%*	
(5) Totals	$4774.1	$5369.3	$5652.0	$5480.8	$5645.8	$5917.2	$6429.0	18.2%	34.7%
Constant Dollars	*2653.8*	*2918.1*	*2992.0*	*2806.3*	*2800.5*			*5.6%*	

Current dollars are converted to constant dollars using the Consumer Price Index (based on prices during 1982–84).
For Notes a, b, and c, see Table 1.

to the NSF budget request, the purpose of CDI is to develop "a new generation of computationally based discovery concepts and tools to deal with complex, data-rich, and interacting systems." (See the accompanying article about the CDI in this issue of the *Notices*.) DMS director Peter March said that the NSF is still working out the details of the funding mechanisms for the CDI, but he expects that a wide spectrum, from individual grants to large group grants, will be used. Under the terms of the request, the DMS would receive an increase of US$5.2 million to fund CDI activities. This amount represents 10% of the CDI funds for NSF overall, and 50% of the CDI funds for the MPS directorate. "So the DMS is playing in some sense a central role," March said.

Science Beyond Moore's Law (SBML) is another initiative in which the DMS is participating. The name of this MPS-based initiative refers to an empirical observation made in 1965 by Intel cofounder Gordon Moore, who predicted that the power of computing hardware would double approximately every eighteen months. This prediction has proven to be very accurate. However, such increases cannot continue forever; eventually one reaches physical limits in the number of transistors that can be packed onto a silicon chip. SBML will address the question of how to get around this limitation and to continue the upward trend of computing speed and power. "If you look at the algorithms marshaled to solve standard computational problems, the algorithmic speed-up has also been exponential, exactly paralleling Moore's law," March noted. "Developing the computational and algorithmic side of things in order to take advantage of new forms of computing is the way Science Beyond Moore's Law will play out in DMS." Under the terms of the 2008 request, the DMS would invest US$1.5 million in SBML.

In addition, the request calls for the DMS to spend US$1.0 million on discovery-based experiences for undergraduate students and to increase funding for research networks and for the mathematical sciences institutes by US$2.7 million. But the biggest share of the division's requested US$17.5 million increase—US$7.3 million, or just over 40% of the increase—would go into the core disciplinary programs: Algebra, Number Theory and Combinatorics; Analysis; Applied Mathematics; Computational Mathematics; Foundations; Geometric Analysis; Mathematical Biology; Probability; Statistics; and Topology.

While the requested increase for core areas of mathematics is encouraging, a perennial question arises: Will the emphasis on initiatives compromise the DMS investment in the core? March does not think so. "There is more than one priority at NSF as a whole, but in particular in DMS," he said. The division must hold fast to its basic mission of funding a wide spectrum of mathematics research—a mission that is more important than ever, as support for core mathematics research has declined greatly in other federal agencies. At the same time, March explained, "Mathematics and statistics have a lot to contribute to the whole scientific enterprise, and having healthy, appropriately funded research projects that cut across the boundaries of science and engineering is absolutely crucial. It's not 'either/or', it's 'both/and'. We have to have both. So the question is one of balance. How do you balance these priorities with a fixed budget?" Achieving such a balance is the perpetual challenge the DMS faces, in fiscal 2008 and beyond.

—*Allyn Jackson*

Note: Consult the webpage `http://www.nsf.gov/about/budget` for further information on the fiscal 2008 budget request for the NSF.

Cyber-enabled Discovery and Innovation

Allyn Jackson

The National Science Foundation (NSF) is launching a new initiative called Cyber-enabled Discovery and Innovation (CDI) which will begin at the start of the new fiscal year in October 2007. A budget of US$52 million has been requested for the first year of the CDI, and the NSF is planning substantial additional investments over the coming five years to bring the program up to a level of US$250 million by 2012. The NSF's Division of Mathematical Sciences (DMS) is playing a prominent role in CDI: In fiscal 2008 the DMS is slated to receive US$5.2 million for CDI activities, which is 10 percent of the CDI total for the NSF overall.

At the time of this writing the program had not been fully formulated and a program announcement was not yet available, so exactly what kinds of research CDI will support remains to be seen. How much mathematics gets funded as CDI grows in the coming years depends in part on how researchers in the mathematical sciences interpret the initiative and what kinds of proposals they submit.

Emphases of CDI

"There are some frontiers in science that are only now accessible because of advances in computation, both on the hardware and on the algorithmic side," explained DMS director Peter March. "The impetus behind Cyber-enabled Discovery and Innovation is to get at these frontiers now in a systematic way." To accomplish this, the NSF has identified three main scientific emphases. One is "Knowledge Extraction", which comprises a range of concepts and techniques for organizing, analyzing, and visualizing the massive data sets that have become ubiquitous in scientific research. A second is "Complex Interactions", which refers to systems characterized by many scales or components that interact. The interactions could be nonlinear and could produce different kinds of emergent phenomena at different scales. Examples include modeling of protein folding or the flow of information across the Internet. The third theme is "Computational Experimentation", which encompasses simulation and modeling of phenomena or processes that cannot be analyzed through any other means. Examples include modeling of weather and of cosmological phenomena such as the "big bang".

There are two other principal components of CDI that focus on the infrastructure needed to advance the scientific themes of the initiative. One is called "Virtual Environments", which refers to the distributed environments that are needed to allow researchers to collaborate across geographic boundaries and to share data and tools. The other component focuses on education and training of students and researchers to build a coterie of workers conversant with the methods and tools needed for the cyber-based research of the future.

The role of mathematics in CDI is "central", said Tony Chan, who heads the NSF's Mathematical and Physical Sciences Directorate. He pointed out that half of the directorate's CDI funds are going to the DMS. "Many of the intellectual ideas behind CDI are mathematical in nature: computational science, modeling, extracting knowledge from data, etc.," he said. "I hope that the mathematical sciences community will see this as an opportunity to expand its engagement with other disciplines and play a leadership role in this important scientific area. Breakthroughs in this area will depend on fundamental contributions from mathematical scientists." Chan also noted that CDI would be open to supporting theoretical research that has

Allyn Jackson is senior writer and deputy editor of the Notices. *Her email address is* `axj@ams.org`.

a potential for a cyber-based application but has not yet been applied in that way.

Examples in Mathematics

Asked what kind of research might be funded under CDI, March pointed to three quite different examples. The first is work by Gunnar Carlsson of Stanford University and his collaborators, who have used methods of algebraic topology to analyze large data sets. One of the recent projects of this group examined data from a collection of digital pictures of nature scenes. The data came from David Mumford of Brown University and his collaborators, who had developed a method for identifying high-contrast patches in the pictures. If these patches are 3 pixels by 3 pixels, with each pixel represented by a grayscale value, then the data forms a subset of 9-dimensional Euclidean space. A bit of data-massaging reduces the dimension so that the data lies in the 7-dimensional unit sphere in R^8. They then used a parametrized family of density estimators to obtain a subspace of "frequently occurring", or highly dense, patches. What does this subspace look like topologically?

Using one value of the estimation parameter, the work by Carlsson et al. revealed that the subspace has first Betti number 1; in other words, it contains a loop. Going back to the data, they found what the loop corresponds to. Some 3 by 3 patches are half dark and half light, with a line separating the light and dark regions. The loop corresponds to the rotation through 360 degrees of the line separating light and dark. Using a smaller value of the estimation parameter, they found two secondary loops, which again could be traced back to the data: One corresponds to patches with a dark band across a light background and the other to patches with a light band across a dark background. Reasoning topologically, they concluded that the subspace of high-contrast patches is well approximated by a two-dimensional Klein bottle sitting in S^7. This work could have important applications to image-compression, which is something Carlsson said they are actively working on. "We are looking for ways to apply this more precise understanding of the density situation to actual problems in compression," he said.

The driving principle behind this work, Carlsson noted, is that "topology should be viewed as a tool for geometric pattern recognition. ... Holes and things like that are patterns in the geometry [of a data set] that take it away from just being a Gaussian blob." Carlsson and collaborators are also using some of the techniques they have developed to analyze neuroscience data, specifically, data from the visual cortex of monkeys. The work on data sets is now opening up some new and interesting questions in algebraic topology. (The work of Carlsson and his collaborators is supported by the NSF and by the Defense Advanced Research Projects Agency.)

The second example also centers on large data sets but takes a completely different approach to address a different kind of problem. In 2004 Emmanuel Candès of the California Institute of Technology, together with his postdoc Justin Romberg and with Terence Tao of the University of California at Los Angeles, published a paper establishing a result that went completely against received wisdom and sparked a revolution in signal processing. The received wisdom is contained in theorems of Harry Nyquist and Claude Shannon, which dictate the rate at which one must sample a signal in order to fully reconstruct it. But this approach does not take into account the structure that is present in most data of scientific interest. Such data is typically compressible: Assuming a suitable representation of the data, such as local cosines or wavelets, one usually finds that many coefficients in the representation are close to zero. The sparsity of such a representation permits efficient signal processing and data compression. More surprisingly, what Candès, Romberg, and Tao found is that the compressibility of the signal can be exploited to make the data acquisition process itself more efficient.

In working with doctors on magnetic resonance imaging data, Candès came up with an algorithm intended to help reduce artifacts in the data. But in tests of the algorithm, a strange thing happened: the algorithm reconstructed the image exactly every time. When an algorithm "always spits out the truth, you have to wonder what's going on," Candès said. This was the starting point of his paper with Romberg and Tao. For their method to work, the data must be sparse when represented in a basis using one kind of waveforms (say, wavelets). Then, by sampling using a different kind of waveform (say, the Fourier transform) that are incoherent with the initial waveforms, one can sample in an extremely parsimonious way—meaning that very few samples are needed to reconstruct the original image. Their paper used optimization theory, combinatorics, and a good deal of probability theory, particularly random matrix theory. Indeed, there is an analogy between their result and results in theoretical computer science on randomized algorithms that can be used to solve—with errors as small as one likes—problems that are thought to be in NP.

The notion that one could reconstruct a signal from such minimal sampling was a big surprise. The new field of "compressive sampling" or "compressed sensing" has attracted huge attention within pure and applied mathematics, signal processing, and computer science and has even reached the engineering level, where it is influencing the design of MRI systems and analog-to-digital converters. One group of engineers at Rice University pushed the idea of compressive sampling to the limit and developed a digital camera that has just one pixel. This innovation was widely reported in the media and was highlighted in the annual report "Ten Emerging Technologies" in the March/April 2007 issue of the Massachusetts Institute of Technology magazine *Technology Review*. Remarked Candès, "It's very surprising to see this transfer from theorems... to hardware so quickly."

The third example of ongoing mathematical work that could be funded under CDI is that of the Atlas team, headed by Jeffrey Adams at the University of Maryland (the other principal investigators are Dan Barbasch, John Stembridge, Peter Trapa, and David Vogan). The team's goal is to solve a major open problem in Lie group theory, namely, the question of classifying the irreducible unitary representations of a semisimple Lie group. Although research over the past several decades has not produced a conceptual picture that would answer this question, enough theory has been built up that it is thought to be possible, for any given Lie group, to compute its unitary representations. The main purpose of the Atlas project is to create a computer program that will take a Lie group G as input and return a list of its unitary representations; a secondary purpose is to make available software that can be used for group theory research. A program that would compute the unitary dual of G, or even decide whether a given representation is unitary, would be extremely useful in a variety of mathematical areas where unitary representations arise, particularly number theory and quantum mechanics.

The work of the Atlas team became known worldwide when the American Institute of Mathematics, which provides support for the project and serves as the sponsoring institution for the project's NSF grant, mounted a publicity campaign to bring media attention to an Atlas milestone that focused on the exceptional Lie group E_8. This milestone consisted of the calculation of the irreducible admissible representations of the split real form of E_8 and of its Kazhdan-Lusztig-Vogan polynomials. The algorithms and software for carrying out this feat were created by the mathematician-computer scientist Fokko du Cloux (who passed away in November 2006) and bring the use of computing in mathematics research to a new level of sophistication. The work on E_8 is one step along the way to the ultimate goal of understanding which representations of Lie groups are unitary. The Atlas team found that the E_8 calculation deepened their mathematical understanding and also provided a crucial test-bed for computational methods.

Mathematics institutes in the U.S. have in recent years organized many events that reflect the themes of CDI. One example is the graduate summer school "Intelligent Extraction of Information from Graphs and High Dimensional Data", held in July 2005 at the Institute for Pure and Applied Mathematics at the University of California at Los Angeles. The school focused on mathematics problems arising in such applications as automated feature extraction, face and shape recognition, image analysis, graph mining, social and transactional networks, and robust network design. A workshop called "Mathematical Issues in Stochastic Approaches for Multiscale Modeling" was held in May of this year as part of a longer program in dynamical systems at the Mathematical Sciences Research Institute in Berkeley. This workshop centered on computational and analytical techniques for studying complex physical, biological, geophysical, and environmental systems, in which stochastic methods are playing a prominent role. The work of Candès and others is the topic of the short course "Compressive Sampling and Frontiers in Signal Processing", held in June of this year at the Institute for Mathematics and its Applications at the University of Minnesota. This same area of research is also the focus of this summer's AMS von Neumann Symposium, "Sparse Representation and High Dimensional Geometry".

It's Not Just Hardware

The examples given above are by no means the only ones, March pointed out. But they do illustrate some essential ways in which mathematics plays a role in cyber-enabled research: in creating new approaches to interpreting data, in fashioning new tools for handling and visualizing data, and in attacking theoretical problems that are inaccessible by conventional means. In these examples "the mathematics comes through in the way that mathematics usually does: in a clear conception of the problem, clear organization of the line of attack, and also innovation in the algorithmic side of things," March explained. "It's not just an issue of hardware, although one needs that. The issue bears on the algorithmic side. How the computation is conceived and organized plays a crucial role. This is an essentially mathematical issue, independent of whatever the goal of the computation is aimed at. So I think the role that mathematics and statistics are going to play in CDI is on the algorithmic side, helping to organize essential calculations so that they can be done effectively."

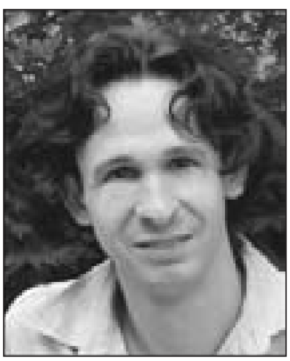
Information about CDI can be found in the NSF's fiscal year 2008 budget request at the URL `http://www.nsf.gov/about/budget/fy2008/pdf/39_fy2008.pdf`.

Mathematics People

2007–2008 AMS Centennial Fellowship Awarded

The AMS has awarded a Centennial Fellowship for 2007–2008 to Martin Kassabov of Cornell University. The fellowship carries a stipend of US$66,000, an expense allowance of US$3,500, and a complimentary Society membership for one year.

Martin Kassabov received his Ph.D. in 2003 from Yale University under the supervision of Efim Zelmanov. He was a postdoctoral fellow at the University of Alberta (2003–2004) and an H. C. Wang Assistant Professor at Cornell University (2004–2006). He is currently an assistant professor at Cornell University.



Martin Kassabov

Kassabov's main interests are in combinatorial algebra and its applications to group theory. A big part of his work involves studying representation theory of finite groups and constructions of expander graphs. His work also has applications to geometric group theory.

Kassabov plans to use the fellowship to visit his collaborators at Imperial College London, Hebrew University Jerusalem, and Alfréd Rényi Mathematics Institute in Budapest.

Please note: Information about the competition for the 2008–2009 AMS Centennial Fellowships will be published in the "Mathematics Opportunities" section of an upcoming issue of the *Notices*.

—Allyn Jackson

Allen Receives ACM Turing Award

Frances E. Allen, fellow emerita of the T. J. Watson Research Center, has been named the recipient of the 2006 A. M. Turing Award, given by the Association for Computing Machinery (ACM). She was honored for "contributions that fundamentally improved the performance of computer programs in solving problems, and accelerated the use of high performance computing."

According to the prize citation, Allen has "made fundamental contributions to the theory and practice of program optimization, which translates the users' problem-solving language statements into more efficient sequences of computer instructions. Her contributions also greatly extended earlier work in automatic program parallelization, which enables programs to use multiple processors simultaneously in order to obtain faster results. These techniques have made it possible to achieve high performance from computers while programming them in languages suitable to applications. They have contributed to advances in the use of high-performance computers for solving problems such as weather forecasting, DNA matching, and national security functions."

Allen is the first woman to be honored with the Turing Award, named for British mathematician Alan M. Turing. The award is considered the "Nobel Prize in Computing". It carries a US$100,000 prize, with financial support provided by Intel Corporation.

—From an ACM news release

Ferran Sunyer i Balaguer Prize Awarded

The Ferran Sunyer i Balaguer Foundation has awarded the Ferran Sunyer i Balaguer Prize for 2007 to Rosa M. Miró-Roig of the University of Barcelona for her monograph *Lectures on Determinantal Ideals.* According to the prize citation, the monograph "solves three central problems in the theory of determinantal ideals: the determination of the CI-liaison class and G-liaison determinantal ideals, the conjecture of multiplicity for determinantal ideals, and the non-obstruction and dimension of families of determinantal ideals."

The Ferran Sunyer i Balaguer Foundation (`http://www.crm.es/FerranSunyerBalaguer/ffsb.htm`) of the Institut d'Estudis Catalans awards this international prize every year to honor the memory of Ferran Sunyer i Balaguer (1912–1967), a self-taught Catalan mathematician who gained international recognition for his research in mathematical analysis despite the serious physical disabilities with which he was born. The prize carries a cash award of €12,000 (approximately US$16,300). The winning monographs are published by Birkhäuser-Verlag.

—From a Ferran Sunyer i Balaguer Foundation announcement

Phùng Awarded von Kaven Prize

Hô Hai Phùng of the University of Duisburg-Essen has been awarded the von Kaven Prize in Mathematics "in recognition of his outstanding work on quantum groups." The prize carries a cash award of €10,000 (approximately US$13,600). The von Kaven Foundation was founded in 2004 by Herbert von Kaven and the German Research Foundation (DFG).

—From a DFG news release

Polchinski and Maldacena Awarded Heineman Prize

Joseph Polchinski of the University of California, Santa Barbara, and Juan Maldacena of the Institute for Advanced Study have been awarded the 2007 Dannie Heineman Prize for Mathematical Physics "for profound developments in mathematical physics that have illuminated interconnections and launched major research areas in quantum field theory, string theory, and gravity."

The prize carries a cash award of US$7,500 and is presented in recognition of outstanding publications in the field of mathematical physics. The prize was established in 1959 by the Heineman Foundation for Research, Educational, Charitable, and Scientific Purposes, Inc., and is administered jointly by the American Institute of Physics (AIP) and the American Physical Society (APS). The prize is presented annually.

—From an APS announcement

Shu Receives Computational Science and Engineering Award

Chi Wang Shu of Brown University has been named the recipient of the 2007 SIAM/ACM Prize in Computational Science and Engineering. According to the prize citation, Shu was honored "for the development of numerical methods that have had a great impact on scientific computing, including TVD temporal discretizations, ENO and WENO finite difference schemes, discontinuous Galerkin methods, and spectral methods."

The prize is awarded every two years by the Society for Industrial and Applied Mathematics (SIAM) and the Association for Computing Machinery (ACM) in recognition of outstanding research contributions to the development and use of mathematical and computational tools and methods for the solution of science and engineering problems. The prize consists of a hand-calligraphed certificate with the citation and a cash prize of US$5,000.

On the selection committee were John B. Bell (chair), Anthony Ralston, and Mary F. Wheeler.

—From a SIAM/ACM announcement

2006 John von Neumann Theory Prize Awarded

The 2006 John von Neumann Theory Prize, the highest prize given in the field of operations research and management science, has been awarded to Martin Grötschel of the Technical University of Berlin, László Lovász of Eötvös Loránd University (Budapest), and Alexander Schrijver of the University of Amsterdam and CWI, the national mathematics and computer science institute in the Netherlands, "for their fundamental path-breaking work in combinatorial optimization." The award, which is presented by the Institute for Operations Research and the Management Sciences (INFORMS), carries a cash award of US$5,000.

The prize citation reads in part: "jointly and individually, they have made basic contributions to the analysis and solution of hard discrete optimization problems. In particular, their joint work on geometric algorithms based on the ellipsoid method of Yudin-Nemirovski and Shor showed the great power of cutting-plane approaches to such problems and provided a theoretical justification for the very active field of polyhedral combinatorics."

—From an INFORMS announcement

Lynch Awarded ACM Knuth Prize

Nancy Lynch of the Massachusetts Institute of Technology (MIT) has been awarded the Knuth Prize by the Association for Computing Machinery (ACM) Special Interest Group on Algorithms and Computation Theory (SIGACT). Lynch was selected "for her influential contributions to the theory of distributed systems, which solve problems using multiple processes or computers connected through a shared memory or network" and for her "seminal impact on the reliability of distributed computing systems, which are used to power traditional wired networks, modern mobile communications systems, and systems with embedded computers, including factory machinery, vehicles, robots, and other real-world devices." She is the first woman to receive the award.

The Knuth Prize is named in honor of Donald Knuth, professor emeritus at Stanford University, who is best known for his ongoing multivolume series *The Art of Computer Programming,* which played a critical role in establishing and defining computer science as a rigorous intellectual discipline. The prize carries a cash award of US$5,000 and is given by ACM SIGACT and the Institute of Electrical and Electronics Engineers (IEEE) Technical Committee on the Mathematical Foundations of Computer Science.

—From an ACM announcement

Avila and Petermichl Awarded Salem Prize

Artur Avila of Centre National de la Recherche Scientifique/Instituto Nacional de Matemática Pura e Aplicada (CNRS/IMPA) and Stephanie Petermichl of the University of Texas, Austin, have been awarded the Salem Prize for 2006.

Avila was selected "for his work on Lyapounov exponents and quasiperiodic behavior in unimodal maps, Schrödinger-like cocycles, interval exchange maps, and Teichmüller flows." Petermichl was honored "for her work on several crucial impacts to the theory of vector valued singular operators." The prize committee for the 2006 prize consisted of J. Bourgain, C. Fefferman, P. Jones, N. Nikolski, P. Sarnak, and J.-C. Yoccoz.

The Salem Prize is awarded every year to a young mathematician judged to have done outstanding work in the field of interest of Raphael Salem, primarily the theory of Fourier series.

—Jean Bourgain, Institute for Advanced Study, Princeton

Van der Hofstad Awarded Rollo Davidson Prize

Remco van der Hofstad of the Eindhoven University of Technology has been awarded the 2007 Rollo Davidson Prize. Van der Hofstad was honored for his work in probability and statistical mechanics. The Rollo Davidson Trust was founded in 1975 and awards an annual prize to young mathematicians working in the field of probability.

—From a Rollo Davidson Trust announcement

Maggioni Awarded Popov Prize

Mauro Maggioni of Duke University has been awarded the fifth Vasil Popov Prize "for his contributions to harmonic analysis on graphs, in particular for his work on diffusion geometry and the construction of multiscale analysis and wavelets based on diffusion processes on graphs." According to the prize citation, he "has introduced novel ideas and powerful new techniques which allow him to seamlessly integrate empirical applied mathematics with the deepest theoretical tools in pure mathematics. His work has already had a seminal impact in the fields of information organization, machine learning, spectral graph theory, image analysis, and medical diagnostics."

The Popov Prize honors the memory of Vasil A. Popov (1942–1990), the Bulgarian analyst best known for his work in nonlinear approximation. The prize is awarded every three years to a young mathematician who has made outstanding research contributions in approximation theory and/or related areas.

—From a Popov Prize Committee announcement

2007 Clay Research Awards Announced

The Clay Mathematics Institute (CMI) has announced the recipients of the 2007 Clay Research Awards.

Alex Eskin of the University of Chicago was honored "for his work on rational billiards and geometric group theory, in particular, his crucial contribution to joint work with David Fisher and Kevin Whyte establishing the quasi-isometric rigidity of Sol." Christopher Hacon of the University of Utah and James McKernan of the University of California, Santa Barbara, were chosen "for their work in advancing our understanding of the birational geometry of algebraic varieties in dimension greater than three, in particular, for their inductive proof of the existence of flips." Michael Harris of the Université de Paris VII and Richard Taylor of Harvard University were selected "for their work on local and global Galois representation…culminating in the solution of the Sato-Tate conjecture for elliptic curves with non-integral j-invariants."

—From a CMI announcement

2007 CMS Prizes Awarded

The Canadian Mathematical Society (CMS) has announced the awarding of several major prizes, all to members of the University of British Columbia.

Martin Barlow has been awarded the 2008 Jeffery-Williams Prize, which recognizes mathematicians who have made outstanding contributions to mathematical research. According to the citation, "Barlow is the leading international expert in the study of diffusions on fractals and other disordered media. He has made a number of profound contributions to a variety of fields, including probabilistic methods in partial differential equations, stochastic differential equations, filtration enlargement, local times, measure-valued diffusions and mathematical finance."

Izabella Laba has been awarded the 2008 Krieger-Nelson Prize, which recognizes outstanding research by a woman mathematician. According to the citation, she has "established a position as one of Canada's leading harmonic analysts. She has made major contributions to the Kakeya problem and to the study of translational tilings and distance sets."

Vinayak Vastal has been named the recipient of the 2007 Coxeter-James Prize, which recognizes young mathematicians who have made outstanding contributions to mathematical research. According to the citation, he "has made fundamental contributions to the Iwasawa theory of elliptic curves, introducing profound techniques from ergodic theory into the subject and obtaining startling theorems on the non-vanishing of p-adic L-functions and μ-invariants that had previously been unobtainable by more orthodox analytic methods."

—From a CMS announcement

Minasyan Awarded Emil Artin Junior Prize

Ashot Minasyan of the University of Geneva, Switzerland, has been awarded the 2007 Emil Artin Junior Prize in Mathematics. Minasyan was chosen for his paper "Separable subsets of GFERF negatively curved groups", published in the *Journal of Algebra* **304** (2006), 1090–1100.

Established in 2001, the Emil Artin Junior Prize in Mathematics carries a cash award of US$500 and is presented usually every year to a student or former student of an Armenian university who is under the age of thirty-five for outstanding contributions to algebra, geometry, topology, and number theory—the fields in which Emil Artin made major contributions. Previous awardees were V. Mikaelian (2001), A. Barkhudaryan (2002), G. Asatryan (2004), and M. Papikian (2005). The prize committee consisted of A. Basmajian, Y. Movsisyan, and V. Pambuccian.

—Artin Prize Committee announcement

Sloan Fellows Announced

The Alfred P. Sloan Foundation has announced the names of the recipients of the 2007 Sloan Research Fellowships. Each year the foundation awards 118 fellowships in the fields of mathematics, chemistry, computational and evolutionary molecular biology, computer science, economics, neuroscience, and physics. Grants of US$45,000 for a two-year period are administered by each fellow's institution. Once chosen, fellows are free to pursue whatever lines of inquiry most interest them, and they are permitted to employ fellowship funds in a wide variety of ways to further their research aims.

Following are the names and institutions of the 2007 awardees in mathematics: Mark Behrens, Massachusetts Institute of Technology; Sourav Chatterjee, University of California, Berkeley; Selim Esedoglu, University of Michigan; Alexander Gamburd, University of California, Santa Cruz; Benjamin Howard, Boston College; Xiantao Li, Pennsylvania State University; Chiu-Chiu Melissa Liu, Columbia University; David Nadler, Northwestern University; Jacob A. Rasmussen, Princeton University; Weiqing Ren, New York University; Ovidiu Savin, Columbia University; Scott Sheffield, New York University; Juan Souto, University of Chicago; Jared W. Tanner, University of Utah; Euguent Tevelev, University of Massachusetts, Amherst; Jacques Verstraete, McGill University; Akshay Venkatesh, New York University; Simone Warzel, Princeton University; Katrin Wehrheim, Massachusetts Institute of Technology; and Lexing Ying, University of Texas, Austin.

The mathematicians on the Sloan Fellowship program committee are Ingrid Daubechies of Princeton University, Benedict Gross of Harvard University, and Dusa McDuff of Stony Brook University.

—From a Sloan Foundation announcement

NSF Graduate Research Fellowships Announced

The National Science Foundation (NSF) has awarded its Graduate Research Fellowships for fiscal year 2007. This program supports students pursuing doctoral study in all areas of science and engineering and provides a stipend of US$30,000 per year for a maximum of three years of full-time graduate study. Following are the names of the awardees in the mathematical sciences for 2007, followed by their undergraduate institutions (in parentheses) and the institutions at which they plan to pursue graduate work.

Tamara Broderick (Princeton University), Carnegie-Mellon University; Melody Chan (Yale University), Princeton University; Atoshi Chowdhury (Princeton University), Harvard University; Yaim Cooper (Massachusetts Institute of Technology), Princeton University; Mariel Finucane (Smith College), Harvard University; Wushi Goldring (University of California, Los Angeles), Harvard University; Luis Guerrero (University of California, San Diego), University of California, Berkeley; Heather Harrington (University of Massachusetts, Amherst), Imperial College, London; Benjamin Harris (Brown University), Massachusetts Institute of Technology; Jack W. Huizenga (University of Chicago), Massachusetts Institute of Technology; Yunjiang Jiang (University of Georgia), Harvard University; Daniel Kane (Massachusetts Institute of Technology), Princeton University; Nathan Kaplan (Princeton University), University of Chicago; George A. Khachatryan (University of Chicago), Princeton University; Thomas M. Koberda (University of Chicago), Harvard University; Ian T. Le (Harvard University), Massachusetts Institute of Technology; Alexander Levin (Harvard University), Massachusetts Institute of Technology; Tianhui Li (Princeton University), Cambridge University; Po-Ru Loh (California Institute of Technology), Harvard University; Michael J. McCourt (Illinois Institute of Technology), Brown University; Stefan T. Patrikis (Harvard University), Princeton University; George J. Schaeffer (Carnegie-Mellon University), University of California, Berkeley; Zachary L. Scherr (Cornell University), Massachusetts Institute of Technology; Gwen M. Spencer (Harvey Mudd College), Cornell University; Ethan J. Street (University of Michigan, Ann Arbor), Stanford University; Daniel B. Walton (Harvey Mudd College), University of California, Berkeley; Sherry X. Wu (Cornell University), Princeton

University; James Y. Zou (Duke University), Massachusetts Institute of Technology.

—From an NSF announcement

Guggenheim Fellowships Awarded

The John Simon Guggenheim Memorial Foundation has announced the names of 189 United States and Canadian artists, scholars, and scientists who were selected as Guggenheim Fellows for 2007. Guggenheim Fellows are appointed on the basis of distinguished achievement in the past and exceptional promise for future accomplishment.

Following are the names of the awardees in the mathematical sciences, together with their affiliations and areas of research interest: Jeffrey F. Brock, Brown University: models, bounds, and effective rigidity in hyperbolic geometry; Michel X. Goemans, Massachusetts Institute of Technology: the traveling salesman problem; Michael Goldstein, University of Toronto: Anderson localization of eigenfunctions; Eric Urban, Columbia University: p-adic automorphic forms and p-adic L-functions; and Salil Vadhan, Harvard University: the complexity of zero-knowledge proofs.

—From a Guggenheim Foundation news release

Fulbright Awards Announced

The J. William Fulbright Foundation and the United States Department of State, Bureau of Educational and Cultural Affairs, have announced the names of the recipients of the Fulbright Foreign Scholarships for 2006–2007. Following are the U.S. scholars in the mathematical sciences who have been awarded Fulbright scholarships to lecture or conduct research, together with their home institutions and the countries in which they plan to use the awards.

Wayne W. Barrett (Brigham Young University), Israel; Thomas E. Gilsdorf (University of North Dakota, Grand Forks), Mexico; Albert J. Milani (University of Wisconsin, Milwaukee), Chile; Rupa Mitra (Minnesota State University, Moorhead), Bangladesh; Timothy E. O'Brien (Loyola University, Chicago), Thailand; and Carol A. Shubin (California State University, Northridge), Rwanda.

—From a Fulbright Awards announcement

Putnam Prizes Awarded

The winners of the sixty-seventh William Lowell Putnam Mathematical Competition have been announced. The Putnam Competition is administered by the Mathematical Association of America and consists of an examination containing mathematical problems that are designed to test both originality and technical competence. Prizes are awarded to both individuals and teams.

The five highest ranking individuals, listed in alphabetical order, were: Hansheng Diao, Massachusetts Institute of Technology; Daniel M. Kane, Massachusetts Institute of Technology; Tiankai Liu, Harvard University; Po-Ru Loh, California Institute of Technology; and Yufei Zhao, Massachusetts Institute of Technology.

Institutions with at least three registered participants obtain a team ranking in the competition based on the rankings of three designated individual participants. The five top-ranked teams (with team members listed in alphabetical order) were: Princeton University (Ana Caraiani, Andrei Negut, Aaron C. Pixton); Harvard University (Tiankai Liu, Alison B. Miller, Tong Zhang); Massachusetts Institute of Technology (Oleg Golberg, Daniel M. Kane, Kuat T. Yessenov); University of Toronto (Tianyi David Han, János Kramár, Viktoriya Krakovna); and University of Chicago (David Coley, Junehyuk Jung, Zhiwei Calvin Lin).

The top five individuals in the competition received cash awards of US$2,500. The first-place team was awarded US$25,000, with each member receiving US$1,000. The team awards for second place were US$20,000 and US$800; for third place, US$15,000 and US$600; for fourth place, US$10,000 and US$400; and for fifth place, US$5,000 and US$200.

The Elizabeth Lowell Putnam Prize is awarded periodically to a woman whose participation in the Putnam Competition is deemed particularly meritorious. This prize was awarded to Alison B. Miller of Harvard University for the second year in a row. The prize carries a cash award of US$1,000.

—Elaine Kehoe

Intel Science Talent Search Winners Announced

Three high school students working in mathematics have been awarded Intel Science Talent Search Scholarships for 2007. John Pardon, a seventeen-year-old student at Durham Academy in Chapel Hill, North Carolina, was awarded second place and a US$75,000 scholarship for a project that solved a classical open problem in differential geometry, showing that a finite-length closed curve in the plane can be made convex in a continuous manner without bringing any two points of the curve closer together. Dmitry Vaintrob, an eighteen-year-old student at South Eugene High School in Eugene, Oregon, won third place and a US$50,000 scholarship for his investigation of ways to associate algebraic structures to topological spaces, proving that loop homology and Hochschild cohomology coincide for an important class of spaces. Gregory Brockman, an eighteen-year-old student at Red River High School in Grand Forks, North Dakota, received the sixth-place scholarship of US$25,000 for his mathematics project that provided a thorough analysis of Ducci sequences, also known as the "four number game".

—From an Intel Corporation announcement

Mathematics Opportunities

NSF CAREER Program Guidelines Available

The guidelines for the Faculty Early Career Development (CAREER) Program of the National Science Foundation (NSF) are now available on the World Wide Web. The program solicitation number is 05-579. Information is available at `http://www.nsf.gov/pubsys/ods/getpub.cfm?nsf05579`. The deadline for submission of proposals is **July 19, 2007**.

—From an NSF announcement

Computational Science Training for Undergraduates in the Mathematical Sciences

The Computational Science Training for Undergraduates in the Mathematical Sciences (CSUMS) program of the National Science Foundation (NSF) is intended to enhance computational aspects of the education and training of undergraduate students in mathematics and statistics and to better prepare these students to pursue careers and graduate study in fields that require integrated strengths in computation and the mathematical sciences. The core of the activity is long-term research experiences for cohorts of at least six undergraduates. Projects must focus on research topics that require interplay between computation and mathematics or statistics. Proposals are welcome for projects that create models for education in the mathematical sciences and that influence the direction of academic programs for a broad range of students.

Deadline for full proposals is **October 17, 2007**. For further information, see `http://www.nsf.gov/publications/pub_summ.jsp?ods_key=nsf06559`.

—From an NSF announcement

Call for Nominations for Sloan Fellowships

Nominations for candidates for Sloan Research Fellowships, sponsored by the Alfred P. Sloan Foundation, are due by **September 15, 2007**. A candidate must be a member of the regular faculty at a college or university in the United States or Canada and must have received the Ph.D. or equivalent within the six years prior to the nomination. For information, write to: Sloan Research Fellowships, Alfred P. Sloan Foundation, 630 Fifth Avenue, Suite 2550, New York, NY 10111-0242; or consult the foundation's website: `http://www.sloan.org/programs/fellowship_brochure.shtml`.

—From a Sloan Foundation announcement

Call for Nominations for ICTP Ramanujan Prize

The Abdus Salam International Centre for Theoretical Physics (ICTP) invites nominations for the 2007 Ramanujan Prize for young mathematicians from developing countries. The prize, funded by the Niels Henrik Abel Memorial Fund, carries a cash award of US$10,000 and an allowance to visit ICTP to deliver a lecture.

The prize is awarded annually to a researcher from a developing country who is under forty-five years of age on December 31 of the year of the award and who has conducted outstanding research in a developing country. Researchers working in any branch of the mathematical sciences are eligible. The deadline for receipt of nominations is **July 31, 2007**. For further information, see the website `http://www.ictp.it/`.

—From an ICTP announcement

Call for Nominations for SASTRA Ramanujan Prize

The Shanmugha Arts, Science, Technology Research Academy (SASTRA) invites nominations for the 2007 SASTRA Ramanujan Prize. The prize carries a cash award of US$10,000, and the winner will be invited to give a talk at the SASTRA conference in December 2007. The deadline for nominations is **August 15, 2007**. For more information email `sastraprize@math.ufl.edu` or see the website `http://www.math.ufl.edu/sastra-prize/`.

—Krishnaswami Alladi, University of Florida

Call for Nominations for Heineman Prize

The American Physical Society (APS) and the American Institute of Physics (AIP) are seeking nominations for the 2008 Dannie Heineman Prize for Mathematical Physics. The prize recognizes outstanding publications in the field of mathematical physics. The prize carries a cash award of US$7,500, an award certificate, and travel expenses to the meeting at which the prize is given. The deadline for nominations is **July 1, 2007**. For more information see the APS website at `http://www.aps.org/praw/heineman/index.cfm`.

—From an APS announcement

News from the CRM Montreal

The Centre de Recherches Mathématiques (CRM) in Montreal, Canada, has announced its thematic program for the semester running from June to December 2007. Organized under the chairmanship of A. R. Humphries (McGill University), it will be devoted to applied dynamical systems.

The semester will begin (July 2–7, 2007) with the workshop Advanced Algorithms and Numerical Software for the Bifurcation Analysis of Dynamical Systems, preceded by a minicourse (June 30–July 1, 2007), both organized by E. Doedel (Concordia University) and H. Osinga (Bristol University). The miniconference Path and Boundary Value Problems: A Continuing Influence in Dynamics, to be held July 6–7, 2007, will celebrate Eusebius Doedel's sixtieth birthday.

The following workshops will be held:

September 16–19, 2007: Mathematical Neuroscience. Organizers: S. Coombes (University of Nottingham), A. Longtin (University of Ottawa), and J. Rubin (University of Pittsburgh).

September 22–23, 2007: Minicourse entitled Quantitative Biology.

September 24–28, 2007: Deconstructing Biochemical Networks. Organizers: P. S. Swain (McGill University), B. P. Ingalls (University of Waterloo), and M. C. Mackey (McGill University).

November 1–5, 2007: Joint AARMS-CRM workshop Recent Advances in Functional and Delay Differential Equations at Dalhousie University, Halifax, Nova Scotia. Organizers: J. Appleby (Dublin City University), H. Brunner (Memorial University), A. R. Humphries (McGill University), P. Keast (Dalhousie University), P. Muir (St. Mary's University), and D. E. Pelinovsky (McMaster University).

November 14–16, 2007: Dynamical Systems and Continuum Physics. Organizer: L. Tuckerman (PMMH-ESPCI, France).

December 11–14, 2007: Chaos and Ergodicity of Realistic Hamiltonian Systems. Organizers: H. Broer (Gröningen University) and P. Tupper (McGill University).

The Aisenstadt Lecturers will be John Rinzel (Courant Institute of the Mathematical Sciences, New York University) and John Tyson (Virginia Polytechnic Institute and State University). Moreover, two advanced graduate courses associated with the semester will be offered during the fall of 2007: A Practical Introduction to SDEs at McGill University, with P. Tupper, and Numerical Analysis of Nonlinear Equations at Concordia University, with E. Doedel.

Besides the thematic program, a number of activities will take place at the CRM during this period.

The SMS-NATO Advanced Study Institute 2007 Summer School will deal with Hamiltonian Dynamical Systems and Applications, June 18–29, 2007. The scientific directors are W. Craig (McMaster University) and A. I. Neistadt (Space Research Institute, Moscow).

A workshop, Mixed Integer Programming, will be held June 30–August 2, 2007. The organizers of the scientific program are O. Gunluk (IBM), M. Koeppe (Magdeburg University), A. Miller (University of Wisconsin, Madison), and J.-P. Richard (Purdue University); the local organizers are O. Marcotte (CRM) and Jacques Desrosiers (HEC Montreal).

A workshop, Nonlinear Integral Transforms: Fourier-Mukai and Nahm, will be held August 27–31, 2007, organized by B. Charbonneau (McGill University), J. Hurtubise (McGill University), M. B. Jardim (University of Campinas), and E. Markman (University of Massachusetts, Amherst).

The First Montreal Industrial Problem Solving Workshop will take place August 20–24, 2007, organized by J.-M. Rousseau (CIRANO and ncm_2).

The CRM is a joint sponsor of the symposium Probability and Stochastic Processes to be held at Carleton University, June 5–8, 2007, in honor of the seventieth birthday of D. A. Dawson. Organizers are Miklós Csörgö, Antal A. Járai, and Yiqiang Q. Zhao (Carleton University); other joint sponsors are the Fields Institute, the Laboratory for Research in Statistics and Probability, and Carleton University. The CRM and the Mathematics Research Center (MRC) at Stanford University are joint sponsors of a conference, New Perspectives and Challenges in Symplectic Field Theory, to be held at Stanford University June 25–29, 2007, in honor of the sixtieth birthday of Y. Eliashberg. The organizers are M. Abreu, R. Cohen, S. Givental, F. Lalonde, R. Lipshitz, L. Polterovich, and R. Schoen.

For more information on the lecturers at various events and on the support available for visitors, graduate students, and postdoctoral fellows, see `http://www.crm.umontreal.ca`.

—CRM announcement

For Your Information

Statement on E_8 Read in Congress

On March 27, 2007, Rep. Jerry McNerney (D-Calif.) read a statement to Congress about the mapping of the exceptional Lie group E_8. The text of the statement follows.

"The American Institute of Mathematics, MIT, Cornell University, University of Michigan, University of Utah, and University of Maryland together created a mathematical breakthrough this week made possible by congressional support of the National Science Foundation. The breakthrough involves defining the detailed structure of a geometric object called E_8, the largest of the exceptional Lie Groups used to study symmetry. E_8, one of the most complicated structures ever studied, is a 248-dimensional Lie Group used to explore the symmetries of a 57-dimensional object. Mapping out such an object is a magnificent achievement of the human mind. Connections between E_8 and string theory indicate that physical applications of E_8 will eventually emerge. The participants are to be commended for their work that has expanded the limits of human knowledge and brings hitherto unknown beauty and power to our human condition."

McNerney, who was elected to Congress in the fall of 2006, holds a Ph.D. in mathematics. His remarks came after the mapping of E_8 made headlines worldwide. For a sampling of the media coverage, visit the AMS Math Digest, `http://www.ams.org/mathmedia/mathdigest`, and go to the archive for March 2007. For more information on E_8, visit the website `http://aimath.org/E8/` at the American Institute of Mathematics.

—Allyn Jackson

Lenstra Appointed ICM-2010 Program Committee Chair

Hendrik W. Lenstra has been appointed chair of the Program Committee of the International Congress of Mathematicians in Hyderabad, India, August 19–27, 2010, by L. Lovasz, president of the International Mathematical Union. His coordinates are as follows: Hendrik W. Lenstra, Mathematisch Instituut, Universiteit Leiden, Postbus 9512, 2300 RA Leiden, The Netherlands; email: `hwlicm@math.leidenuniv.nl`. Proposals concerning the scientific program of ICM 2010 may be sent to Lenstra by email.

—International Mathematical Union announcement

New Director for CRM Barcelona

At its March 2007 meeting, the governing board of the Centre de Recerca Matemàtica (CRM) appointed Joaquim Bruna as CRM director. Bruna, a professor at the Universitat Autònoma de Barcelona, succeeds Manuel Castellet, who now holds the post of honorary director. The CRM was founded in 1984 by Castellet, who had served as director until Bruna's appointment.

Sponsored by the Institut d'Estudis Catalans and the Catalan government, the CRM is located on the campus of the Universitat Autònoma de Barcelona, which is in Bellaterra, a suburb of Barcelona. The CRM invites outstanding mathematicians for research visits; facilitates scientific contacts between visitors and young local researchers; carries out research programs; and organizes talks, conferences, advanced courses, seminars, and other scientific meetings. It also makes available research results through its publications. For further information visit the CRM website at `http://www.crm.cat`.

—Allyn Jackson

NUMB3RS Receives Award

The popular television drama series *NUMB3RS*, about an FBI agent whose mathematician brother helps solve crimes in the Los Angeles area by using mathematical problem-solving techniques, will receive a National Science Board group Public Service Award for 2007, along with the program's cocreators, Nick Falacci and Cheryl Heuton. The series producers have consulted with mathematicians to bring realism and accuracy to the show's stories. Cryptanalysis, probability theory, game theory, decision theory, principal components analysis, multivariate time series analysis, and astrophysics are among the disciplines employed in the series thus far. The National Science Board, the governing body of the National Science Foundation, presents its Public Service Award annually to recognize individuals and organizations for their extraordinary contributions to increase public understanding of science.

—From an NSF news release

Inside the AMS

Epsilon Awards for 2007

The AMS Epsilon Fund for Young Scholars was established in 1999 to provide financial assistance to summer programs for mathematically talented high school students in the United States. For many years these programs have provided mathematically talented youngsters with their first serious mathematical experiences. The name for the fund was chosen in remembrance of the late Paul Erdős, who was fond of calling children "epsilons".

The AMS has chosen six summer mathematics programs to receive Epsilon grants for activities in the summer of 2007. The grants will support program expenses and student scholarships and, in some cases, scholarships only. The programs were chosen on the basis of mathematical excellence and enthusiasm. Award amounts were governed by the varying financial needs of each program and totaled US$80,000.

The programs receiving grants are: Hampshire College Summer Studies in Mathematics, Amherst, Massachusetts; Michigan Math and Science Scholars Summer Program, University of Michigan, Ann Arbor; PROMYS, Boston University; Ross Mathematics Program, Ohio State University, Columbus; Summer Explorations and Research Collaborations for High School Girls (SEARCH), Mount Holyoke College, South Hadley, Massachusetts; and Texas State University Honors Summer Math Camp, Texas State University, San Marcos.

The grants for summer 2007 are paid for by the AMS Epsilon Fund for Young Scholars (supplemented by the AMS Program Development Fund). The AMS is continuing to build the endowment for the Epsilon Fund, with a goal of raising US$2 million through individual donations and grants. Once the Epsilon Fund endowment has reached the targeted amount, the AMS intends to award a total of US$100,000 in Epsilon grants each year.

For further information about the Epsilon Fund for Young Scholars, visit the website `http://www.ams.org/giving-to-ams/`, or contact `development@ams.org`. Information about how to apply for Epsilon grants is available at `http://www.ams.org/outreach/epsilon.html`. A fairly comprehensive listing of summer programs for mathematically talented high school students (including those with and without Epsilon grants) is available at `http://www.ams.org/outreach/mathcamps.html`.

—AMS Development Office

Deaths of AMS Members

Karen A. Ames, professor, University of Alabama in Huntsville, died on September 28, 2006. Born on November 5, 1953, she was a member of the Society for 25 years.

John Backus, IBM fellow, San Jose, CA, died on March 17, 2007. Born on December 3, 1924, he was a member of the Society for 35 years.

Lev M. Berkovich, professor, Samara State University, Russia, died on March 17, 2007. Born on September 22, 1935, he was a member of the Society for 17 years.

Joseph Blum, professor emeritus, from Rockville, MD, died on April 10, 2002. Born on March 7, 1919, he was a member of the Society for 55 years.

Frank E. Burk, professor, California State University, Chico, died on March 17, 2007. Born on September 29, 1942, he was a member of the Society for 13 years.

Charles A. Chapman, professor, Cedar Crest College, Allentown, PA, died on May 28, 2006. Born on July 12, 1938, he was a member of the Society for 9 years.

Gustave Choquet, professor, University of Paris VI, died on November 14, 2006. Born on March 1, 1915, he was a member of the Society for 58 years.

Paul J. Cohen, professor, Stanford University, died on March 23, 2007. Born on April 2, 1934, he was a member of the Society for 51 years.

Fokko du Cloux, professor, from Villeurbanne, France, died on November 10, 2006. Born on December 20, 1954, he was a member of the Society for 22 years.

James Eells, professor emeritus, from Cambridge, England, died on February 14, 2007. Born on October 25, 1926, he was a member of the Society for 57 years.

Steve Galovich, professor, Lake Forest College, IL, died on December 14, 2006. He was a member of the Society for 37 years.

Casper Goffman, professor emeritus, from West Lafayette, IN, died on September 25, 2006. Born on June 1, 1913, he was a member of the Society for 63 years.

Herbert Greenberg, professor, University of Denver, died on January 1, 2007. Born on November 28, 1921, he was a member of the Society for 65 years.

Peter L. Hammer, professor, Rutgers University, Piscataway, died on December 27, 2006. Born in December 1936, he was a member of the Society for 36 years.

Gisbert Hasenjaeger, from Plettenberg, Germany, died on September 2, 2006. Born on June 1, 1919, he was a member of the Society for 41 years.

Ralph Henstock, professor, University of Ulster, Ireland, died on January 6, 2007. Born on June 2, 1923, he was a member of the Society for 42 years.

Anthony Horsley, professor, London School of Economics, died on May 26, 2006. Born on April 5, 1939, he was a member of the Society for 19 years.

Ishaq Murad Khabaza, retired, Queen Mary University of London, died on September 17, 2005. Born on May 30, 1928, he was a member of the Society for a few months.

Alexander Khoury, professor emeritus, from Ottawa, Canada, died on August 21, 2006. Born on January 30, 1933, he was a member of the Society for 40 years.

Marvin J. Kohn, professor, Brooklyn College, CUNY, died on January 2, 2007. Born on January 1, 1944, he was a member of the Society for 39 years.

Wladyslaw Eliovich Lyantse, professor, Ivan Franko National University of Lviv, Ukraine, died on March 29, 2007. Born on November 19, 1920, he was a member of the Society for 11 years.

John S. MacNaughton, from Inverness, Scotland, died on February 9, 2006. Born on September 5, 1931, he was a member of the Society for 23 years.

Marvin G. Moore, professor emeritus, Bradley University, Peoria, IL, died on January 12, 2007. Born on December 8, 1908, he was a member of the Society for 69 years.

Gert H. Muller, professor emeritus, University of Heidelberg, Germany, died on September 9, 2006. Born on May 29, 1923, he was a member of the Society for 39 years.

Theresa L. Podmele, of Annandale, Virginia, died in December 1974. Born on September 22, 1898, she was a member of the Society for 38 years.

L. N. Rao Gandikota, retired, from Nagpur, India, died on August 30, 2006. Born on November 4, 1928, he was a member of the Society for 26 years.

Jack Zev Reichman, from West Hempstead, NY, died on December 25, 2006. Born on December 13, 1950, he was a member of the Society for 30 years.

Johann Schroeder, from Tensbuettel-Roest, Germany, died on January 3, 2007. Born on April 4, 1925, he was a member of the Society for 46 years.

Thomas Sigman, from Berkeley, CA, died on January 3, 2007. He was a member of the Society for 8 years.

Elbridge P. Vance, retired, Oberlin College, died on February 18, 2007. Born on February 7, 1915, he was a member of the Society for 69 years.

Oswald Wyler, professor emeritus, from Brunswick, ME, died on July 12, 2006. He was a member of the Society for 55 years.

Reference and Book List

*The **Reference** section of the* Notices *is intended to provide the reader with frequently sought information in an easily accessible manner. New information is printed as it becomes available and is referenced after the first printing. As soon as information is updated or otherwise changed, it will be noted in this section.*

Contacting the *Notices*

The preferred method for contacting the *Notices* is electronic mail. The editor is the person to whom to send articles and letters for consideration. Articles include feature articles, memorial articles, communications, opinion pieces, and book reviews. The editor is also the person to whom to send news of unusual interest about other people's mathematics research.

The managing editor is the person to whom to send items for "Mathematics People", "Mathematics Opportunities", "For Your Information", "Reference and Book List", and "Mathematics Calendar". Requests for permissions, as well as all other inquiries, go to the managing editor.

The electronic-mail addresses are `notices@math.ou.edu` in the case of the editor and `notices@ams.org` in the case of the managing editor. The fax numbers are 405-325-7484 for the editor and 401-331-3842 for the managing editor. Postal addresses may be found in the masthead.

Information for *Notices* Authors

The *Notices* welcomes unsolicited articles for consideration for publication, as well as proposals for such articles. The following provides general guidelines for writing *Notices* articles and preparing them for submission.

***Notices* readership.** The *Notices* goes to about 30,000 subscribers worldwide, of whom about 20,000 are in North America. Approximately 8,000 of the 20,000 in North America are graduate students who have completed at least one year of graduate school. All readers may be assumed to be interested in mathematics research, but they are not all active researchers.

***Notices* feature articles.** Feature articles may address mathematics, mathematical news and developments, mathematics history, issues affecting the profession, mathematics education at any level, the AMS and its activities, and other such topics of interest to *Notices* readers. Each

Where to Find It

A brief index to information that appears in this and previous issues of the *Notices.*

AMS Bylaws—*November 2005, p. 1239*

AMS Email Addresses—*February 2007, p. 271*

AMS Ethical Guidelines—*June/July 2006, p. 701*

AMS Officers 2006 and 2007 (Council, Executive Committee, Publications Committees, Board of Trustees)—*May 2007, p. 657*

AMS Officers and Committee Members—*October 2006, p. 1076*

Conference Board of the Mathematical Sciences—*September 2006, p. 911*

Information for *Notices* Authors—*June/July 2007, p. 765*

Mathematics Research Institutes Contact Information—*August 2006, p. 798*

National Science Board—*January 2007, p. 57*

New Journals for 2005, 2006—*June/July 2007, p. 767*

NRC Board on Mathematical Sciences and Their Applications—*March 2007, p. 426*

NRC Mathematical Sciences Education Board—*April 2006, p. 546*

NSF Mathematical and Physical Sciences Advisory Committee—*February 2007, p. 274*

Program Officers for Federal Funding Agencies—*October 2006, p. 1072 (DoD, DoE); December 2006, p. 1369 (NSF)*

Stipends for Study and Travel—*September 2006, p. 913*

article is expected to have a large target audience of readers, perhaps 5,000 of the 30,000 subscribers. Authors must therefore write their articles for nonexperts rather than for experts or would-be experts. In particular, the mathematics articles in the *Notices* are expository. The language of the *Notices* is English.

Most feature articles, including those on mathematics, are expected to be of long-term value and should be written as such. Ideally each article should put its topic in a context, providing some history and other orientation for the reader and, as necessary, relating the subject matter to things that readers are likely to understand. In most cases, articles should progress to dealing with contemporary matters, not giving only historical material. The articles that are received the best by readers tend to relate different areas of mathematics to each other.

By design the *Notices* is partly magazine and partly journal, and authors' expository styles should take this into account. For example, many readers want to understand the mathematics articles without undue effort and without consulting other sources.

Mathematics feature articles in the *Notices* are normally six to nine pages, sometimes a little longer. Shorter articles are more likely to be read fully than are longer articles. The first page is 400 or 500 words, and subsequent pages are about 800 words. From this one should subtract an allowance for figures, photos, and other illustrations, and an appropriate allowance for any displayed equations and any bibliography.

Form of articles. Except with very short articles, authors are encouraged to use section headings and subsection headings to help orient readers. Normally there is no section heading at the beginning of an article. Despite the encouraged use of internal headings, the assigning of numbers to sections and subsections is not permitted in any article.

The bibliography should be kept short. In the case of mathematics articles, bibliographies are normally limited to about ten items and should consist primarily of entries like books in which one may do further reading. To help readers who might want lists of recent literature, an author might include a small number of recent publications with good bibliographies.

Editing process. Most articles that are destined to be accepted undergo an intensive editing process. The purposes of this process are to ensure that the target audience is as large as practicable, that the content of the article is clear and unambiguous, and that the article is relatively easy to read. Usually it is the members of the editorial board who are involved in this process. Sometimes outside referees are consulted.

Preparation of articles for submission. The preferred form for submitted articles is as electronic files. Authors who cannot send articles electronically may send the articles by fax or by postal mail.

Articles with a significant number of mathematical symbols are best prepared in TEX, LATEX, or AMS-TEX. There are no special style files for the *Notices*, because TEX code gets converted to something else during the production process. Since the *Notices* is set in narrow columns, keeping displayed formulas relatively short helps to minimize adjustments during the production process; avoiding nonstandard supplementary files and complex sequences of TEX definitions also helps. For the handling of figures and other illustrations, please consult the editor.

Articles without a significant number of mathematical symbols may be prepared as text files or in Microsoft Word. In the case of files prepared in Microsoft Word, it is advisable to send both the file and a fax of a printout.

Instructions for Authors of "WHAT IS…?" Columns

The purpose of the "WHAT IS…?" column is to provide brief, nontechnical descriptions of mathematical objects in use in current research. The target audience for the columns is first-year graduate students.

Each "WHAT IS…?" column provides an expository description of a single mathematical object being used in contemporary research. Thus "WHAT IS M-Theory?" would be too broad, but "WHAT IS a Brane?" would be appropriate; ideally, "WHAT IS a Brane?" would give a flavor of what M-theory is.

The writing should be nontechnical and informal. The level should be a little higher than the level of popular articles about mathematical developments one finds in magazines like *Science* that are aimed at a general audience.

There is a strict limit of two *Notices* pages (1,400 words with no picture, or 1,200 words with one picture). A list of "Further Reading" should contain no more than three references.

Inquiries and comments about the "WHAT IS…?" column are welcome and may be sent to notices-whatis@ ams.org.

Upcoming Deadlines

June 30, 2007: Nominations for 2007 Fermat Prize. See http://www.math.ups-tlse.fr/Fermat/.

July 1, 2007: Nominations for 2008 Dannie Heineman Prize for Mathematical Physics. See "Mathematics Opportunities" in this issue.

July 19, 2007: Proposals for NSF CAREER program. See "Mathematics Opportunities" in this issue.

July 31, 2007: Nominations for ICTP Ramanujan Prize. See "Mathematics Opportunities" in this issue.

August 15, 2007: Call for Nominations for SASTRA Ramanujan Prize. See "Mathematics Opportunities" in this issue.

September 15, 2007: Nominations for Sloan Research Fellowships. See "Mathematics Opportunities" in this issue.

October 1, 2007: Applications for AWM Travel Grants. See http://www.awm-math.org/travelgrants.html; telephone 703-934-0163; email: awm@math.umd.edu; or contact Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030.

October 5, 2007: Full proposals for NSF IGERT competition. See http://www.nsf.gov/pubs/2007/nsf07540/nsf07540.htm.

October 15, 2007: Preferred deadline for January entrance in junior-year program at the Smith College Center for Women in Mathematics.

See http://www.math.smith.edu/center.

October 17, 2007: Full proposals for NSF Computational Science Training for Undergraduates in the Mathematical Sciences (CSUMS). See "Mathematics Opportunities" in this issue.

October 17, 2007: Proposals for NSF Postdoctoral Research Fellowships. See http://www.nsf.gov/funding/pgm_summ.jsp?pims_id=5301&org=DMS.

January 5, 2008: Applications for IMA postdoctoral and New Directions program. See http://www/ima.umn.edu.

New Journals for 2005

Below is a list of mathematical journals appearing for the first time in 2005, as compiled by *Mathematical Reviews*. This list, as well as the listings for new journals for other years, can be found on the Web at http://www.ams.org/mathweb/mi-newjs.html.

ACM Transactions on Algorithms. 1549-6325, Association for Computing Machinery, US$225/4 Issues/yr.

Boundary Value Problems, 1687-2762, Hindawi Publishing, US$214.50/3 Issues/yr.

Global Journal of Pure and Applied Mathematics, 0973-1768, Research India Publications, US$150/3 Issues/yr.

IMRP, International Mathematics Research Papers, 1687-3017, Hindawi Publishing, US$214.50. Frequency irregular.

IMRS, International Mathematics Research Surveys, 1687-1308, Hindawi Publishing, US$195/yr. Frequency unknown.

International Journal of Evolution Equations, 1549-2907, Nova Science Publishers, US$195/4 Issues/yr.

International Journal of Number Theory, 1793-0421, World Scientific Publishing, US$224/4 Issues/yr.

Journal of Applied Mathematics and Decision Sciences, [CONTINUED JOURNAL] 1173-9126, Hindawi Publishing, US$324.50/yr. Frequency unknown.

Journal of Computational Mathematics and Optimization, 0972-9372, SAS International Publications, Euro 150/3 Issues/yr.

Journal Électronique d'Histoire des Probabililtés et de la Statistique, 1173-0074, Séminaire d'Histoire du Calcul des Probabilités de la Statistique, Free of charge/2 Issues/yr. Electronic.

Journal of Industrial and Management Optimization, 1547-5816, American Institute of Mathematical Sciences, US$450/4 Issues/yr.

Journal of Mathematics and Statistics, 1549-3644, Science Publications, US$500/4 Issues/yr.

New Mathematics and Natural Computation, 1793-0057, World Scientific Publishing, US$301/3 Issues/yr.

Notas de Matemática, Universidad de Los Andes, Price not listed, 2 Issues/yr.

Pacific Journal of Optimization, 1348-9151, Yokohama Publishers, US$150/3 Issues/yr.

Pure and Applied Mathematics Quarterly, 1558-8599, International Press, US$330/4 Issues/yr.

Sarajevo Journal of Mathematics, [CONTINUED JOURNAL] 1840-0655, Academy of Sciences and Arts of Bosnia and Herzegovina, US$60/2 Issues/yr.

SIGMA, Symmetry, Integrability and Geometry, 1815-0659, National Academy of Sciences of Ukraine, Free of charge/Frequency unknown, Electronic.

New Journals for 2006

Communications in Applied Mathematics and Computational Science Mathematical Sciences Publishers. Price to be announced.

International Electronic Journal of Algebra, 1306-6048, Hacettepe University, Ankara, Turkey, Price unstated, Electronic, Announced for 2007.

International Journal of Mathematics and Computer Science, 1814-0424 (Paper);1814-0432 (Electronic), Lebanese University, Hadath, Lebanon, US$300/yr.

JNAIAM, Journal of Numerical Analysis, Industrial and Applied Mathematics, ISSN: 1790-8140 EISSN: 1790-8159, Promacon, Athens, Greece.

Journal of Generalized Lie Theory and Applications, [see 2007 listing which supersedes old 2006 listing as from Department of Mathematics, Tallinn University of Technology in Estonia and Centre for Mathematical Sciences, Lund University in Sweden].

Online Journal of Analytic Combinatorics, University of Missouri at Columbia, Free of charge, Electronic.

Surveys in Mathematics and its Applications, 1842-6298, MODOP Project of West University of Timisoara, University of Bucharest, University Constantin Brâncuşi of Târgu-Jiu, Free of charge, Electronic.

Book List

The Book List highlights books that have mathematical themes and are aimed at a broad audience potentially including mathematicians, students, and the general public. When a book has been reviewed in the Notices, *a reference is given to the review. Generally the list will contain only books published within the last two years, though exceptions may be made in cases where current events (e.g., the death of a prominent mathematician, coverage of a certain piece of mathematics in the news) warrant drawing readers' attention to older books. Suggestions for books to include on the list may be sent to* notices-booklist@ams.org.

*Added to "Book List" since the list's last appearance.

An Abundance of Katherines, by John Green. Dutton Juvenile Books, September 2006. ISBN 0-525-47688-1.

Analysis and Probability: Wavelets, Signals, Fractals, by Palle E. T. Jorgensen. Springer, September 2006. ISBN 0-387-29519-4.

**Ants, Bikes, and Clocks: Problem Solving for Undergraduates*, by William Briggs. Society for Industrial and Applied Mathematics, 2005. ISBN 0-89871-574-1.

**The Archimedes Codex*, by Reviel Netz and William Noel. Weidenfeld and Nicolson, May 2007. ISBN-13: 978-0-29764-547-4.

**The Art of Mathematics: Coffee Time in Memphis*, by Béla Bollobás. Cambridge University Press, September 2006. ISBN-13978-0-52169-395-0.

Arthur Cayley: Mathematician Laureate of the Victorian Age, by Tony Crilly. Johns Hopkins University Press, December 2005. ISBN 0-801-88011-4.

The Artist and the Mathematician: The Story of Nicolas Bourbaki, the Genius Mathematician Who Never Existed, by Amir D. Aczel. Thunder's Mouth Press, August 2006. ISBN 1-560-25931-0.

A Beautiful Math: John Nash, Game Theory, and the Modern Quest for a Code of Nature, by Tom Siegfried. Joseph Henry Press, October 2006. ISBN 0-309-10192-1.

The Best of All Possible Worlds: Mathematics and Destiny, by Ivar Ekeland. University of Chicago Press, October 2006. ISBN-13: 978-0-226-19994-8.

Bourbaki, a Secret Society of Mathematicians, by Maurice Mashaal. AMS, June 2006. ISBN 0-8218-3967-5.

The Cat in Numberland, by Ivar Ekeland. Cricket Books, April 2006. ISBN-13: 978-0-812-62744-2.

**Chases and Escapes: The Mathematics of Pursuit and Evasion*, by Paul J. Nahin. Princeton University Press, May 2007. ISBN-13: 978-0-69112-514-5.

Descartes: A Biography, by Desmond Clarke. Cambridge University Press, March 2006. ISBN 0-521-82301-3.

Descartes: The Life and Times of a Genius, by A. C. Grayling. Walker & Company, November 2006. ISBN 0-8027-1501-X.

Einstein's Heroes: Imagining the World through the Language of Mathematics, by Robyn Arianrhod. Oxford University Press, July 2006. ISBN-13: 978-0-195-30890-7.

The Essential Turing, edited by B. Jack Copeland. Oxford University Press, September 2004. ISBN 0-198-25080-0. (Reviewed November 2006.)

Euclid in the Rainforest: Discovering Universal Truths in Logic and Math, by Joseph Mazur. Pi Press, October 2004. ISBN 0-131-47994-6.

Evolutionary Dynamics: Exploring the Equations of Life, by Martin Nowak. Belknap Press, September 2006. ISBN 0-674-02338-2.

The Fabulous Fibonacci Numbers, by Alfred S. Posamentier and Ingmar Lehmann. Prometheus Books, February 2007. ISBN 1-591-02475-7.

Fearless Symmetry: Exposing the Hidden Patterns of Numbers, by Avner Ash and Robert Gross. Princeton University Press, May 2006. ISBN 0-691-12492-2. (Reviewed January 2007.)

**Fly Me to the Moon: An Insider's Guide to the New Science of Space Travel*, by Edward Belbruno. Princeton University Press, January 2007. ISBN-13: 978-0-691-12822-1.

From Cosmos to Chaos: The Science of Unpredictability, by Peter Coles. Oxford University Press, August 2006. ISBN 0-198-56762-6.

From Zero to Infinity: What Makes Numbers Interesting, by Constance Reid. Fiftieth anniversary edition, A K Peters, February 2006. ISBN 1-568-81273-6. (Reviewed February 2007.)

Gödel's Theorem: An Incomplete Guide to Its Use and Abuse, by Torkel Franzen. A K Peters, May 2005. ISBN 1-568-81238-8. (Reviewed March 2007.)

Great Feuds in Mathematics: Ten of the Liveliest Disputes Ever, by Hal Hellman. Wiley, September 2006. ISBN 0-471-64877-9.

How Mathematics Happened, by Peter S. Rudman. Prometheus Books, October 2006. ISBN 1-591-02477-3.

How to Cut a Cake: And Other Mathematical Conundrums, by Ian Stewart. Oxford University Press, November 2006. ISBN 0-199-20590-6.

**I Am a Strange Loop*, by Douglas R. Hofstadter. Basic Books, March 2007. ISBN-13: 978-0-46503-078-1.

John von Neumann: Selected Letters, edited by Miklós Rédei. AMS, November 2005. ISBN 0-8218-3776-1. (Reviewed in this issue.)

**Karl Pearson: The Scientific Life in a Statistical Age*, by Theodore M. Porter. Princeton University Press, (new edition) December 2005. ISBN-13: 978-0-69112-635-7.

King of Infinite Space: Donald Coxeter, the Man Who Saved Geometry, by Siobhan Roberts. Walker & Company, September 2006. ISBN 0-802-71499-4.

Leonhard Euler, by Emil A. Fellmann. Birkhäuser, 2007. ISBN-13: 978-3-7643-7538-6.

Leonhard Euler, a Man to Be Reckoned With, by Andreas K. Heyne and Alice K. Heyne. Birkhäuser, 2007. ISBN-13: 978-3-7643-8332-9.

Letters to a Young Mathematician, by Ian Stewart. Perseus Books, April 2006. ISBN-13: 978-0-465-08231-5. (Reviewed May 2007.)

A Madman Dreams of Turing Machines, by Janna Levin. Knopf, August 2006. ISBN 1-400-04030-2.

The Man Who Knew Too Much: Alan Turing and the Invention of the Computer, by David Leavitt. Great Discoveries series, W. W. Norton, December 2005. ISBN 0-393-05236-2. (Reviewed November 2006.)

Mathematical Illustrations: A Manual of Geometry and PostScript, by Bill Casselman. Cambridge University Press, December 2004. ISBN 0-521-54788-1. (Reviewed January 2007.)

Mathematics and Common Sense: A Case of Creative Tension, by Philip J. Davis. A K Peters, October 2006. ISBN 1-568-81270-1.

Measuring the World, by Daniel Kehlmann. Pantheon, November 2006. ISBN 0-375-42446-6.

More Mathematical Astronomy Morsels, by Jean Meeus. Willmann-Bell, 2002. ISBN 0-943396-743.

**The Motion Paradox: The 2,500-Year Old Puzzle Behind All the Mysteries of Time and Space*, by Joseph Mazur. Dutton Adult, April 2007. ISBN-13: 978-0-52594-992-3.

Musimathics: The Mathematical Foundations of Music, Volume 1, by Gareth Loy. MIT Press, September 2006. ISBN 0-262-12282-0.

Negative Math: How Mathematics Rules Can Be Positively Bent, by Alberto A. Martinez. Princeton University Press, November 2005. ISBN-13: 978-0-691-12309-7.

**Nonplussed!: Mathematical Proof of Implausible Ideas*, by Julian Havil. Princeton University Press, May 2007. ISBN-13: 978-0-691-12056-0.

Not Even Wrong: The Failure of String Theory and the Continuing Challenge to Unify the Laws of Physics, by Peter Woit. Jonathan Cape, April 2006. ISBN 0-224-07605-1.

Once upon Einstein, by Thibault D'Amour. A K Peters, March 2006. ISBN 1-568-81289-2.

**Out of the Labyrinth: Setting Mathematics Free*, by Robert Kaplan and Ellen Kaplan. Oxford University Press, January 2007. ISBN-13: 978-0-19514-744-5.

The Pea and the Sun: A Mathematical Paradox, by Leonard M. Wapner. A K Peters, April 2005. ISBN 1-568-81213-2. (Reviewed October 2006.)

Piano Hinged Dissections: Time to Fold!, by Greg Frederickson. A K Peters, October 2006. ISBN 1-568-81299-X.

Piero della Francesca: A Mathematician's Art, by J. V. Field. Yale University Press, August 2005. ISBN 0-300-10342-5. (Reviewed March 2007.)

**The Poincaré Conjecture: In Search of the Shape of the Universe*, by Donal O'Shea. Walker, March 2007. ISBN-13: 978-08027-1532-6.

Prince of Mathematics: Carl Friedrich Gauss, by M. B. W. Tent. A K Peters, January 2006. ISBN 1-568-81261-2.

Project Origami: Activities for Exploring Mathematics, by Thomas Hull. A K Peters, March 2006. ISBN-10 1-568-81258-2. (Reviewed May 2007.)

Pursuit of Genius: Flexner, Einstein, and the Early Faculty at the Institute for Advanced Study, by Steve Batterson. A K Peters, June 2006. ISBN 1-568-81259-0.

**Pythagoras: His Life, Teaching and Influence*, by Christoph Riedweg. Translated by Steven Rendall. Cornell University Press, March 2005. ISBN-13: 978-0-80144-240-7.

**Pythagoras: The Mathemagician*, by Karim El-koussa. Cloonfad Press, September 2005. ISBN-13: 978-0-97694-042-5.

Reality Conditions: Short Mathematical Fiction, by Alex Kasman. Mathematical Association of America, May 2005. ISBN 0-88385-552-6. (Reviewed August 2006.)

The Secret Life of Numbers: 50 Easy Pieces on How Mathematicians Work and Think, by George G. Szpiro. Joseph Henry Press, March 2006. ISBN 0-309-09658-8.

Shadows of Reality: The Fourth Dimension in Relativity, Cubism, and Modern Thought, by Tony Robbin. Yale University Press, March 2006. ISBN 0-300-11039-1. (Reviewed April 2007.)

The Shoelace Book: A Mathematical Guide to the Best (and Worst) Ways to Lace Your Shoes, by Burkard Polster. AMS, June 2006. ISBN 0-8218-3933-0. (Reviewed December 2006.)

**The Square Root of 2: A Dialogue Concerning a Number and a Sequence*, by David Flannery. Springer, December 2005. ISBN-13: 978-0-38720-220-4.

Stalking the Riemann Hypothesis: The Quest to Find the Hidden Law of Prime Numbers, by Dan Rockmore. Pantheon, April 2005. ISBN 0-375-42136-X. (Reviewed September 2006.)

Superior Beings: If They Exist, How Would We Know?: Game-Theoretic Implications of Omnipotence, Omniscience, Immortality, and Incomprehensibility, by Steven Brams. Springer, second edition, November 2007. ISBN-13: 978-0-387-48065-7.

Symmetry and the Monster: The Story of One of the Greatest Quests of Mathematics, by Mark Ronan. Oxford University Press, May 2006. ISBN 0-192-80722-6. (Reviewed February 2007.)

The Three Body Problem, by Catherine Shaw. Allison and Busby, March 2005. ISBN 0-749-08347-6. (Reviewed October 2006.)

The Triumph of Numbers: How Counting Shaped Modern Life, by I. B. Cohen. W. W. Norton, July 2006. ISBN-13: 978-0-393-32870-7.

The Trouble with Physics: The Rise of String Theory, the Fall of a Science, and What Comes Next, by Lee Smolin. Joseph Henry Press, October 2006. ISBN 0-309-10192-1.

Unknown Quantity: A Real and Imaginary History of Algebra, by John Derbyshire. Joseph Henry Press, May 2006. ISBN 0-309-09657-X.

Useless Arithmetic: Why Environmental Scientists Can't Predict the Future, by Orrin Pilkey and Linda Pilkey-Jarvis. Columbia University Press, February 2007. ISBN 0-231-13212-3.

**Why Beauty Is Truth: The Story of Symmetry*, by Ian Stewart. Perseus Books Group, April 2007. ISBN-13: 978-0-46508-236-0.

Yearning for the Impossible: The Surprising Truths of Mathematics, by John Stillwell. A K Peters, May 2006. ISBN 1-568-81254-X. (Reviewed in this issue.)

REFERENCE & BOOK LIST

AMERICAN MATHEMATICAL SOCIETY

MATHEMATICAL REVIEWS ASSOCIATE EDITOR

Applications and recommendations are invited for a full-time position as an Associate Editor of Mathematical Reviews (MR), to commence as soon as possible after June 1, 2008, and no later than January 1, 2009.

The Mathematical Reviews division of the American Mathematical Society (AMS) is located in Ann Arbor, Michigan, not far from the campus of the University of Michigan. The editors are employees of the AMS; they also enjoy many privileges at the University. At present, the AMS employs over seventy people including fifteen mathematical editors at Mathematical Reviews. MR's mission is to develop and maintain the AMS databases covering the published mathematical literature. The chief responsibility is the development and maintenance of the MR Database, from which all MR-related products are produced: MathSciNet, the journals *Mathematical Reviews* and *Current Mathematical Publications*, and MathSciDisc.

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The twelve-month salary will be commensurate with the experience the applicant brings to the position. Interested applicants are invited to write (or telephone) for further information.

Applications (including curriculum vitae; bibliography; and the names, addresses, phone numbers, and email addresses of at least three references) and recommendations should be sent to

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Mathematics Calendar

The most comprehensive and up-to-date Mathematics Calendar information is available on the AMS website at `http://www.ams.org/mathcal/`.

June 2007

2007 **International Conference on Learning**, Johannesburg, South Africa. (Apr. 2007, p. 556)

* 1–3 **The 16th International Workshop on Matrices and Statistics**, University of Windsor, Windsor, Ontario, Canada.

Purpose: To stimulate research, in an informal setting, and to foster the interaction of researchers in the interface between matrix theory and statistics. The workshop will include both invited and contributed talks, and a special session with talks and posters by graduate students is planned. A special issue of the journal Linear Algebra and its Applications will be published devoted to selected papers presented at the conference. Please note that the International Organizing Committee includes George P. H. Styan, Chair (McGill), Simo Puntanen, Vice-chair (Finland), S. Ejaz Ahmed, LOC chair (Windsor), Jeffrey J. Hunter (New Zealand), Augustyn Markiewicz (Poland), Gatz Trenkler (Germany), Dietrich von Rosen (Sweden), Hans Joachim Werner (Germany).

Information: For information about abstract submission, or to participate as a speaker, please email Dr. Styan at email: `styan@math.mcgill.ca`; `http://www.uwindsor.ca/iwms`; email: `iwms@uwindsor.ca`.

1–5 **Sampling Theory and Applications 07 (SAMPTA07)**, Aristotle Univ. of Thessaloniki, Thessaloniki, Greece. (Dec. 2006, p. 1378)

1–7 **International Conference—Leonhard Euler and Modern Combinatorics: Applications to Logic, Representation Theory, Mathematical Physics**, Euler IMI, St. Petersburg, Russia. (Mar. 2007, p. 435)

2 **Developments in Algebraic Geometry**, Brown University, Providence, Rhode Island. (Apr. 2007, p. 556)

2–9 **Symmetry and Perturbation Theory**, Otranto (Lecce), Italy. (Feb. 2007, p. 305)

* 4–5 **Colloquium António Aniceto Monteiro on the centenary of his birth**, Museu de Ciência da Universidade de Lisboa, Lisboa, Portugal.

Description: On the centenary of the birth of the Portuguese mathematician António Aniceto Monteiro, the Portuguese Mathematical Society (SPM) and †the Museum of Science of the University of Lisbon jointly organize a colloquium in his honour. This colloquium is mostly devoted to the historical aspects of the life and scientific work of A. A. Monteiro, as well as their impact on the development of science in Portugal, Brazil and Argentina. In addition, the colloquium comprises a number of invited talks on the scientific aspects and recent developments in research topics closely related†to the work of A.A. Monteiro.

Speakers: Kátia Amaral Monteiro (Universidade de Évora, Portugal), Gabriela Bordalo (Universidade de Lisboa, Portugal), Oswaldo Chateaubriand (Universidade Católica do Rio de Janeiro, Brazil), Roberto Cignoli (Universidad de Buenos Aires, Argentina), Augusto Fitas (Universidade de Évora, Portugal), Ernesto Garcia Camarero (Universidad Complutense de Madrid, Spain), Laura Lopes (Brazil), Luiz Monteiro (Universidad Nacional del Sur, Bahía Blanca, Argentina), Daniele Mundici (Università di Firenze, Italy), Eduardo Ortiz (Imperial College, London, United Kingdom), António Passos Videira (Universidade do Estado do Rio de Janeiro, Brazil), Margarita Ramalho (Universidade de Lisboa, Portugal), Jorge Rezende (Universidade de Lisboa, Portugal), Fernando Rosas (Universidade Nova de Lisboa, Portugal), Edgardo Stacco (Universidad Nacional del Sur, Bahía Blanca, Argentina), Júlia Vaz de Carvalho (Universidade

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This section contains announcements of meetings and conferences of interest to some segment of the mathematical public, including ad hoc, local, or regional meetings, and meetings and symposia devoted to specialized topics, as well as announcements of regularly scheduled meetings of national or international mathematical organizations. A complete list of meetings of the Society can be found on the last page of each issue.

An announcement will be published in the *Notices* if it contains a call for papers and specifies the place, date, subject (when applicable), and the speakers; a second announcement will be published only if there are changes or necessary additional information. Once an announcement has appeared, the event will be briefly noted in every third issue until it has been held and a reference will be given in parentheses to the month, year, and page of the issue in which the complete information appeared. Asterisks (*) mark those announcements containing new or revised information.

In general, announcements of meetings and conferences held in North America carry only the date, title of meeting, place of meeting, names of speakers (or sometimes a general statement on the program), deadlines for abstracts or contributed papers, and source of further information. Meetings held outside the North American area may carry more detailed information. In any case, if there is any application deadline with respect to participation in the meeting, this fact should be noted. All communications on meetings and conferences in the mathematical sciences should be sent to the Editor of the *Notices* in care of the American Mathematical Society in Providence or electronically to `notices@ams.org` or `mathcal@ams.org`.

In order to allow participants to arrange their travel plans, organizers of meetings are urged to submit information for these listings early enough to allow them to appear in more than one issue of the *Notices* prior to the meeting in question. To achieve this, listings should be received in Providence **eight months** prior to the scheduled date of the meeting.

The complete listing of the Mathematics Calendar will be published only in the September issue of the *Notices*. The March, June/July, and December issues will include, along with new announcements, references to any previously announced meetings and conferences occurring within the twelve-month period following the month of those issues. New information about meetings and conferences that will occur later than the twelve-month period will be announced once in full and will not be repeated until the date of the conference or meeting falls within the twelve-month period.

The Mathematics Calendar, as well as Meetings and Conferences of the AMS, is now available electronically through the AMS website on the World Wide Web. To access the AMS website, use the URL: `http://www.ams.org/`.

Nova de Lisboa, Portugal).
Organizer: Sociedade Portuguesa de Matemática, Museu de Ciência da Universidade de Lisboa.
Deadline for Registration: May 31, 2007 (April 30, 2007 at a lower rate).
Information: http://aam.cii.fc.ul.pt/.

* 4–7 **DIMACS Workshop on Markov Chain Monte Carlo: Synthesizing Theory and Practice**, DIMACS Center, CoRE Bldg, Rutgers University, Piscataway, New Jersey.
Short Description: In the past two decades researchers in discrete mathematics and the theory of computing have made enormous theoretical strides in sampling via Markov chains, sometimes known as MCMC (Markov Chain Monte Carlo). We plan to begin with a one-day tutorial on MCMC and on sampling issues in general, understandable to graduate students and a general math/statistics/computer science audience. The tutorial will be about half devoted to mathematical results on MCMC and half to MCMC in statistics.
Organizers: Jim Fill, Johns Hopkins University, jimfill@jhu.edu; Jim Hobert, University of Florida, jhobert@stat.ufl.edu; Antonietta Mira, University of Insubria (Italy), amira@eco.unisubria.it; Luke Tierney, University of Iowa, luke@stat.uiowa.edu; Peter Winkler, Dartmouth College, peter.winkler@dartmouth.edu.
Local Arrangements: Linda Casals, Workshop Coordinator, DIMACS Center, lindac@dimacs.rutgers.edu. DIMACS Publicity Coordinator, workshop@dimacs.rutgers.edu; 732-445-5928; http://dimacs.rutgers.edu/Workshops/MonteCarlo.

4–8 **Arithmetic Harmonic Analysis on Character and Quiver Varieties**, AIM Research Conference Center, Palo Alto, California. (Jun/Jul. 2006, p. 714)

4–9 **Algebraic Geometry in Higher Dimensions**, G. Hotel Bellavista, Levico Terme (Trento), Italy. (Apr. 2007, p. 557)

4–15 **Moduli spaces of Riemann surfaces and related topics**, Centre de Recherches Mathematiques, Montreal, Canada. (Jan. 2007, p. 63)

7–9 **International Conference—Mathematical Hydrodynamics: Euler Equations and Related Topics**, Euler IMI, St. Petersburg, Russia. (Mar. 2007, p. 439)

7–9 **Ordinal and Symbolic Data Analysis (OSDA) 2007**, Ghent University, Ghent, Belgium. (Jan. 2007, p. 64)

7–10 **74th Workshop on General Algebra**, Tampere University of Technology, Tampere, Finland. (Apr. 2007, p. 557)

8–13 **The Ninth International Conference on Geometry, Integrability and Quantization**, Sts. Constantine and Elena resort, Varna, Bulgaria. (Feb. 2007, p. 305)

8–14 **33rd International Conference: Applications of Mathematics in Engineering and Economics**, Sozopol, Bulgaria. (Mar. 2007, p. 435)

10–12 **Leonhard Euler Festival**, Euler IMI, St. Petersburg, Russia. (Mar. 2007, p. 435)

10–12 **Special Congress on the occasion of the 300th anniversary of Leonhard Euler's birth**, St. Petersburg, Russia. (Feb. 2007, p. 305)

10–15 **IPM Logic Conference 2007**, School of Mathematics, Institute for Studies in Theoretical Physics and Mathematics (IPM), Tehran, Iran. (May 2007, p. 661)

11–12 **SPEED: Software Performance Enhancement for Encryption and Decryption**, Amsterdam, the Netherlands. (Apr. 2007, p. 557)

11–15 **An Algebraic Geometry Conference**, IHP, Paris, France. (Sept. 2006, p. 960)

11–15 **Barcelona Conference on C*- Algebras and Their Invariants**, Centre de Recerca Matemàtica, Barcelona, Spain. (Jan. 2007, p. 64)

11–15 **Conference on Complex Function Theory and Geometry**, Warsaw, Poland. (Mar. 2007, p. 435)

11–16 **QTRF4 – Quantum Theory: Reconsiderations and Foundations 4**, International Centre for Mathematical Modelling in Physics, Engineering and Cognitive Sciences at Vaxjo University, Sweden. (May 2007, p. 661)

* 11–22 **Sixth Working Days of Introduction of Dynamical Systems and PDE's: Sixth *JORNADES D'INTRODUCCIO ALS SISTEMES DINAMICS I A LES EDP'S (JISD2007)**, Barcelona, Spain.
Organizers: This year the Jornades are organized jointly with the Summer School NEEDS 2007 (Nonlinear evolution equations and dynamical systems): http://www.needs-conferences.net/2007/, and with GLOBAL School on PDEs: layers and dislocations: http://www.ma1.upc.edu/recerca/seminaris/JISD2007/indexpdes07.html.
Description: The JISD2007 courses belong to the Master in Applied Mathematics, inside the Graduate studies at UPC, and are organized by Xavier Cabre, Amadeu Delshams, and Tere M. Seara. The Summer School NEEDS 2007 courses are organized by David Gomez-Ullate, Andrew Hone, Sara Lombardo and Joaquim Puig.
Courses: 1ST WEEK (June 11-15): FME, UPC, Barcelona. Dario Bambusi (Univ. Milano): Normal Form for Hamiltonian PDEs; Vadim Kaloshin (Penn State Univ. and Univ. Maryland): Instabilities in the 3 Body Problem; Rafael de la Llave (Univ. Texas at Austin): Introduction to the Properties of Extended Systems. Summer School NEEDS (June 16-17), L'Ametlla de Mar, Tarragona. Alastair Rucklidge (Leeds University), An Introduction to Pattern Formation; Carles Simo (Univ. Barcelona), Properties of Low Dimensional Dynamical Systems in the Largem; Paolo M. Santini (Univ. Roma "La Sapienza"), The Transition from Regular to Irregular Motion: As Travel on Riemann Surfaces; Steven H. Strogatz (Cornell University), Synchronization and Networks. 2ND WEEK: (June 18-22), FME, UPC, Barcelona. Changfeng Gui (Univ. Connecticut), Multi Peak Solutions and Solutions of Semilinear Equations in Phase Transitions and a Conjecture of De Giorgi; Regis Monneau (CERMICS-ENPC, France), Introduction to Dislocations Dynamics.
Deadline: To apply for financial support: April 30, 2007.
Information: http://www.ma1.upc.edu/recerca/seminaris/JISD2007/indexjisd2007.html.

11–22 **Workshop: Real, Tropical, and Complex Enumerative Geometry**, Centre de recherches mathématiques, Montréal, Québec, Canada. (Feb. 2007, p. 305)

11–July 6 **Clay Mathematics Institute 2007 Summer School on "Homogeneous flows, moduli spaces, and arithmetic"**, Centro di Ricerca Matematica Ennio De Giorgi, Pisa, Italy. (Feb. 2007, p. 305)

12–14 **Conference on Ordered Statistical Data and Inequalities: Theory & Applications**, The University of Jordan, Amman, Jordan. (Dec. 2006, p. 1378)

* 13–16 **Recent Advances in Differential Geometry: International Conference in honour of Prof. O. Kowalski**, University of Lecce, Italy.
Second Announcement: A celebration of the scientific activity of Prof. Kowalski (Prague), the conference will be the opportunity for the presentation of recent results in the framework of Differential Geometry.
Information: In the future, further news and information will be available at the Conference website: http://www.diffgeo.unile.it. Contact: diffgeo@unile.it.

13–19 **International Conference on Arithmetic Geometry**, Euler IMI, St. Petersburg, Russia. (Mar. 2007, p. 435)

14–16 **Model Theory and Algebra**, Department of Mathematics and Computer Science, Camerino, Italy. (Mar. 2007, p. 435)

14–16 **The XVIth International Colloquium on Integrable Systems and Quantum symmetries**, Czech Technical University, Prague, Czech Republic. (Nov. 2006, p. 1253)

15–17 **Algebraic Methods in Functional Analysis**, Chalmers University of Technology and Gothenburg University, Gothenburg, Sweden. (May 2007, p. 662)

15–17 **International Conference on Rings and Things: Dedicated to Carl Faith on his 80th birthday and Barbara Osofsky on her 70th**, Ohio University, Zanesville, Ohio. (Mar. 2007, p. 436)

15–24 **NEEDS 2007 (Nonlinear Evolution Equations and Dynamical Systems)**, L'Ametlla de Mar, Spain. (Feb. 2007, p. 305)

16–17 **NEEDS 2007 School**, L'Ametlla de Mar, Spain. (Feb. 2007, p. 305)

16–19 **XVI Congreso Nacional de Matematicas**, Medellin, Colombia. (May 2007, p. 662)

* 17–22 **2007 USENIX Annual Technical Conference (USENIX '07)**, Hyatt Regency Santa Clara, 5101 Great America Parkway, Santa Clara, California.
Description: USENIX '07 will feature an extensive training program, covering crucial topics and led by highly respected instructors. Technical sessions, featuring the Refereed Papers Track, Invited Talks, and a Poster Session, plus BoFs and more! Join the community of programmers, developers, and systems professionals in sharing solutions and fresh ideas.
Information: email: conference@usenix.org.

17–23 **International Conference: Skorokhod Space. 50 Years On**, Institute of Mathematics of the National Academy of Sciences of Ukraine, Kyiv, Ukraine. (Feb. 2007, p. 305)

17–23 **Trends in Harmonic Analysis**, Strobl, Salzburg, Austria. (Feb. 2007, p. 305)

18–21 **Summer School in Model Theory**, Department of Mathematics and Computer Science, Camerino, Italy. (Mar. 2007, p. 436)

* 18–22 **2007 International Conference on Geometric Analysis**, Department of Mathematics, National Taiwan University, Taipei, Taiwan.
Speakers: H. D. Cao (Lehigh University, USA), Jianguo Cao (University of Notre Dame, USA), Der-Chen Chang (Georgetown University, USA), Richard Hamilton (Columbia University, USA), (To be confirmed), Xiaojun Huang (Rutgers University, USA), Shanyu Ji (University of Houston, USA), Peter Li (University of California, Irvine, USA), John Loftin (Rutgers University, USA), Lei Ni (University of California, San Diego, USA), D. H. Phong (Columbia University, USA), L-F. Tam (Chinese University, Hong Kong), Burkhard Wilking (Fachbereich Mathematik und Informatik, Germany), Jon Wolfson (Michigan State University, East Lansing, USA), Sumio Yamada (Tohoku University, Japan), Stephen Yau (University of Illinois at Chicago, USA).
Information: http://www.tims.ntu.edu.tw/GA2007/. For any questions related to the conference, please email ga2007@math.ntu.edu.tw.

18–22 **Cherednik Algebras**, University of Edinburgh, Edinburgh, Scotland. (May 2007, p. 662)

18–23 **Combinatorics and Optimization 40th Anniversary Conference**, University of Waterloo, Waterloo, Ontario, Canada. (Sept. 2006, p. 960)

18–23 **Computability in Europe 2007 (CiE 2007)**, Siena, Italy. (Feb. 2007, p. 305)

18–23 **Russian-German Geometry Meeting dedicated to the 95th anniversary of A. D. Alexandrov**, Euler IMI, St. Petersburg, Russia. (Mar. 2007, p. 436)

18–24 **Algebraic Topology: Old and New (M. M. Postnikov Memorial Conference)**, Stefan Banach International Mathematical Center (Bedlewo, Poland). (Jan. 2007, p. 64)

18–29 **Flow in Porous Media with Emphasis on Modeling Oil Reservoirs**, University of Wyoming, Laramie, Wyoming. (Jan. 2007, p. 64)

18–29 **Hamiltonian Dynamical Systems and Applications Systèmes Dynamiques Hamiltoniens et Applications**, Université de Montréal, Montréal, Québec, Canada. (Feb. 2007, p. 306)

19–22 **Computational Algebraic Geometry**, Oakland University, Rochester, Michigan. (Feb. 2007, p. 306)

19–22 **Thirteenth Annual Conference of African American researchers in the mathematical sciences (CAARMS13)**, Northeastern University and University of Massachusetts, Boston, Massachusetts. (May 2007, p. 662)

* 20–22 **International Conference "in Nonsmooth and Variational Analysis in Sciences and Engineering" (NVA 2007)**, Université de Limoges, France.
Topics: Convex analysis and optimization, Nonsmooth analysis, Variational analysis, Nonsmooth dynamical systems, Complementarity problems, Applications in mechanics and in electronics.
Invited Speakers: H. Attouch (Université Montpellier II, France), Gerald A. Beer (California State Universilty, Los Angeles, USA), Maïtine Bergounioux (Université d'Orléans, France), Jonathan M. Borwein (Dalhousie University, Canada), J.-Fréderic Bonnans (Inria, France), Guy Bouchitté (Université de Toulon et du Var, France), Bernard Brogliato (INRIA Rhône-Alpes, France), Patrick-Louis Combettes (Université Paris 6, France), Darinka Dentcheva (Stevens Institute of Technology, USA), Daniel Goeleven (Université de La Réunion, France), Masao Fukushima (Kyoto University, Japan), Marco Lopez (Universidad de Alicante, Spain), Roberto Lucchetti (Politecnico di Milano, Italy), Antonino Maugeri (Università di Catania, Italy), Boris Mordukhovich (Wayne State University, USA), Jean-Paul Penot (Université de Pau, France), Julian Revalski (Bulgarian Academy of Sciences, Bulgary), Stephen Simons (University of California, Santa Barbara, USA), Jie Sun (National University of Singapore, Singapore), Lionel Thibault (Université de Montpellier II, France).
Important dates: Inscription: April 30, 2007. Deadline of submission (pdf abstract 1-2 pages): April 30, 2007. Notification of acceptance: May 15, 2007. Final program: May 31, 2007.
Information: http://www.unilim.fr/nva2007.

20–23 **International Conference "Trends and Challenges in Applied Mathematics" (ICTCAM 2007)**, Technical University of Civil Engineering, Bucharest, Romania. (Feb. 2007, p. 306)

* 20–23 **Mathematics of Social Justice Course Development Workshop**, Middlebury College, Middlebury, Vermont.
Program: Workshop participants will work collaboratively on modules for a variety of mathematics and statistics courses, exploring connections between quantitative literacy and social justice issues such as apportionment, income distribution, and the environment. Working groups will also consider models for service-learning projects in mathematics courses. This meeting will build on the work begun at Lafayette College in May of 2006; as with that workshop, some specific topics will be determined by the participants.
Information: http://community.middlebury.edu/~bremser/MSJ2Announce.htm.

21–26 **Conference on Riemannian Geometry and Applications: Dedicated to the memory of Radu Rosca**, Brasov, Romania. (Mar. 2007, p. 436)

24–26 **Mathematical Modelling in Sport**, Salford Quays Conference Centre, Salford, Manchester, UK. (Apr. 2007, p. 557)

24–29 **Fifth School on Analysis and Geometry in Metric Spaces**, Grand Hotel Bellavista, Levico Terme (Trento), Italy. (Apr. 2007, p. 557)

24–30 **Lyapunov Memorial Conference: International Conference on the occasion of the 150th Birthday of Aleksandr Lyapunov**, Karazin Kharkiv National University and Verkin Institute for Low Temperature Physics, Kharkiv, Ukraine. (Sept. 2006, p. 960)

24–30 **Seventh International Conference "Symmetry in Nonlinear Mathematical Physics"**, Institute of Mathematics, Kiev, Ukraine. (Sept. 2006, p. 960)

24–July 1 **45th International Symposium on Functional Equations**, Bielsko-Biala, Poland. (Dec. 2006, p. 1379)

25–28 **The 2007 World Congress in Computer Science, Computer Engineering, and Applied Computing: WORLDCOMP'07**, Las Vegas, Nevada. (May 2007, p. 662)

25–29 **Braids**, Inst. Math. Sciences, National Univ., Singapore, Singapore. (Mar. 2007, p. 436)

25–29 **Conference on Enumeration and Probabilistic Methods in Combinatorics**, Centre de Recerca Matemàtica, Barcelona, Spain. (Jan. 2007, p. 64)

25–29 **First International Summer School on Geometry, Mechanics and Control**, Castro Urdiales, Cantabria, Spain. (Apr. 2007, p. 557)

25–29 **Number Theory and Computability**, ICMS, Edinburgh, Scotland. (May 2007, p. 662)

25–29 **Summer School on Algebra and Combinatorics**, CELC, University of Lisbon, Portugal. (May 2007, p. 662)

* 25–29 **International Workshop on Zeta Functions in Algebra and Geometry**, Escuela de Informatica de Segovia, Segovia, Spain.
Aims and Scope: The conference will focus on current advances and perspectives related with the following topics: (1) arithmetic and geometric aspects of local, topological, and motivic zeta functions, (2) Poincare Series of Valuations, (3) Zeta functions in algebra, (4) prehomogeneous vector spaces and zeta functions, (5) Orbital Integrals, representation theory and motivic integration, (6) Kapranov zeta function.
Scientific Committee: A. Campillo (Spain), M. Du Satoy (U.K.), S. M. Gusein-Zade (Russia), F. Loeser (France), I. Luengo (Spain), Y. Manin (Germany), W. Veys (Belgium), F. Sato (Japan).
Invited Speakers: David Bourqui, Pierrette Cassou-Noguès, Raf Cluckers, Félix Delgado, Christopher Deninger, Wolfgang Ebeling, Sabir Gusein-Zade, Fritz Grunewald, Andrei Jaikin, Franziska Heinloth, Mikhail Kapranov, Michael Larsen, Ben Lichtin, Johannes Nicaise, Alexei Pantchichkine, Fumihiro Sato, Hiroshi Saito, Dirk Segers, Duco van Straten, Yuri Tschinkel, Christopher Voll.
Information: `http://euclid.barry.edu/~zuniga/Workshop.htm`.

25–29 **Workshop on "Modeling, Analysis and Simulation of Multiscale Nonlinear Systems" in cooperation with SIAM Activity Group on Geosciences**, Oregon State University, Corvallis, Oregon. (Mar. 2007, p. 436)

25–30 **16th Summer St. Petersburg Meeting on Mathematical Analysis**, Euler IMI, St. Petersburg, Russia. (Mar. 2007, p. 457)

25–30 **ERLOGOL-2007: Intermediate problems of model theory and universal algebra**, State Technical University–Math Institute, Novosibirsk, Novosibirsk-Altai/Russia. (Oct. 2006, p. 1091)

25–30 **International Conference "Algebraic Analysis and Around": in honor of Masaki Kashiwara's 60th birthday**, Kyoto University, Kyoto, Japan. (Jan. 2007, p. 64)

* 26–28 **Workshop on Nonparametric Inference—WNI2008**, Coimbra, Portugal.
Information: `http://www.mat.uc.pt/~wni2008`.

26–29 **ALM (Adults learning mathematics) 14th International Conference**, Limerick University, Limerick, Republic of Ireland. (Feb. 2007, p. 306)

26–29 **Nonlinear Modeling and Control, An International Seminar**, Nayanova University, Samara, Russia. (Feb. 2007, p. 306)

26–30 **ICDS International Conference on Dynamical Systems 2007**, Abant Ýzzet Baysal University, Bolu, Turkey. (May 2007, p. 662)

28–30 **Fourth European PKI Workshop: Theory and Practice**, University of Illes Balears, Mallorca, Spain. (Mar. 2007, p. 437)

28–30 **Third Statistical Days at the University of Luxembourg**, University of Luxembourg, Luxembourg, Luxembourg. (Apr. 2007, p. 557)

28–July 4 **6th Congress of Romanian Mathematicians**, Faculty of Mathematics and Computer Science, University of Bucharest, Bucharest, Romania. (Sept. 2006, p. 960)

29–July 1 **SIAM Conference on Control and Its Applications (CT07)**, Hyatt Regency San Francisco Airport, San Francisco, California. (Feb. 2007, p. 306)

30–July 1 **Advanced Numerical Techniques in Applied Dynamical Systems**, Centre de recherches mathématiques, Université de Montréal, Montréal, Canada. (May 2007, p. 662)

30–July 7 **14th International Conference on Waves and Stability in Continuous Media (WASCOM07)**, Hotel Village Baia Samuele, Scicli, Ragusa, Italy. (Mar. 2007, p. 437)

July 2007

1–4 **Fifth Mathematics & Design International Conference (M&D-2007)**, Universidade Regional de Blumenau, Blumenau, SC - Brazil. (Mar. 2007, p. 437)

1–7 **The VI International Algebraic Conference in Ukraine, dedicated to the 100th anniversary of D. K. Faddeev**, Kamenets-Podol'sky State University, Kamenets-Podol'sky, Ukraine. (Dec. 2006, p. 1379)

1–13 **Cohomology of groups and Algebraic K-theory**, Center of Math Sciences, Zhejiang University, China. (Dec. 2006, p. 1379)

2–4 **The 2007 International Conference of Applied and Engineering Mathematics**, Imperial College London, London, U.K. (Nov. 2006, p. 1253)

* 2–4 **Workshop: Hyperbolic Volume 2007**, University of Fribourg, Fribourg, Switzerland.
Purpose: To learn about the diverse new developments around hyperbolic volume.
Sponsors: University of Fribourg, Swiss National Science Foundation, IIIcycle romand des mathématiques, Stiftung zur Förderung der Mathematischen Wissenschaften in der Schweiz.
Organizers: Ruth Kellerhals (Fribourg) and Robert Meyerhoff (Boston).
Information: `ruth.kellerhals@unifr.ch`; `http://commonweb.unifr.ch/math/events/WorkshopHypVol07/welcome.html`.

2–6 **9th Conference on Orthogonal Polynomials Special Functions and Applications (OPSFA9)**, International Center for Mathematical Meetings, Marseille, France. (May 2007, p. 662)

2–6 **19th International Conference on Formal Power Series and Algebraic Combinatorics**, Nankai University, Tianjin, China. (Feb. 2007, p. 306)

2–6 **25th Journees Arithmetiques**, University of Edinburgh, Scotland, UK. (May 2006, p. 612)

2–6 **Advanced Algorithms and Numerical Software for the Bifurcation Analysis of Dynamical Systems**, Centre de recherches mathématiques, Université de Montréal, Montréal, Canada. (May 2007, p. 662)

2–6 **Des équations aux dérivées partielles au calcul scientifique: Congrès en l'honneur de Luc Tartar à l'occasion de son soixantième anniversaire**, Carré des Sciences, Ancienne Ecole Polytechnique, Paris, France. (Jan. 2007, p. 64)

2–6 **Design Theory of Alex Rosa, a meeting in celebration of Alex Rosa's 70th Birthday**, Bratislava, Slovakia. (Sept. 2006, p. 961)

* 2–6 **French-Israeli-Russian Workshop "Global Fields: Algebra, Geometry, Arithmetic, Asymptotics"**, Independent University of Moscow, Moscow, Russia.

Program: The workshop will cover all aspects of number fields and function fields, including algebraic groups over such fields. It is conceived for bringing together graduate students, postdocs, and more experienced researchers working in the area.

Organizers: Boris Kunyavskii (Bar-Ilan University), kunyav@macs.biu.ac.il; Gilles Lachaud (IML), lachaud@iml.univ-mrs.fr; Michael Tsfasman (Poncelet lab and IITP RAS), tsfasman@iml.univ-mrs.fr; Serge Vladuts (IML and IITP RAS), vladut@iml.univ-mrs.fr; Alexey Zykin (Independent University of Moscow, Steklov Institute, and IML), zykin@iml.univ-mrs.fr.

Visas: If you need a Russian visa, you are requested to contact Liza Kryukova at poncelet@mccme.ru as soon as possible.

Information: email: kunyav@macs.biu.ac.il.

* 2–6 **Representation Theory, System of Differential Equations and their Related Topics**, Hokkaido University, Sapporo, Japan.

Organizers: Keiji Matsumoto (Hokkaido University), Hiroyuki Ochiai (Nagoya University), Toshio Oshima (University of Tokyo), Mutsumi Saito (Hokkaido University), Hiroaki Terao (Hokkaido University), Hiroshi Yamashita (Hokkaido University).

Information: http://coe.math.sci.hokudai.ac.jp/sympo/070702/;; cri@math.sci.hokudai.ac.jp. This conference is supported by The Pacific Rim Mathematical Association (PRIMA) http://www.primath.org/.

2–7 **International Conference: Modular Forms and Moduli Spaces**, Euler IMI, St. Petersburg, Russia. (Mar. 2007, p. 437)

2–13 **Geometry and Lie Theory**, Australian National University, Canberra, and University of Sydney, Australia. (Feb. 2007, p. 306)

2–20 **Geometric and Topological Methods for Quantum Field Theory**, Villa de Leyva, Colombia. (Apr. 2007, p. 558)

3–6 **18th International Workshop on Operator Theory and its Applications (IWOTA-2007)**, North-West University, Potchefstroom, South Africa. (Dec. 2006, p. 1379)

3–7 **Symposium on the differential geometry of submanifolds**, University of Valenciennes, Valenciennes, France. (Mar. 2007, p. 437)

4–7 **APFA 6: Applications of Physics in Financial Analysis Conference**, ISCTE Business School, Lisbon, Portugal. (Feb. 2007, p. 307)

4–8 **International Conference on Nonlinear Operators, Differential Equations and Applications (ICNODEA 2007)**, Babes-Bolyai University, Cluj-Napoca, Romania. (Oct. 2006, p. 1091)

5–8 **XVIth Oporto Meeting on Geometry, Topology and Physics**, University of the Algarve, Faro, Portugal. (Apr. 2007, p. 558)

* 6–11 **The Second International Congress of Algebra and Combinatorics: Dedicated to the 70th birthday of Leonid A. Bokut**, Beijing University of Aseonautics and Aeronautics, Beijing, China.

Topics: Group Theory, Ring Theory, Hopf Algebras and Lie Algebras, Graph Theory, Codes and Designs, Combinatorics Methods.

Scientific Committee: Will be chaired by Professor Efim Zelmanov (USA): Zhe-Xian Wan, (China), T. Arad (Israel), E. Bannai (Japan), M. Broue (France), K. Denecke (Germany), M. M. Deza (France), R. Gonchigdorzh (Mongolia), R. Graham (USA), M. Ito (Japan), K. Ueno (Japan), T. Y. Lam (USA), V. Latyshev (Russia), L. Marki (Hungary), A. V. Mikhalev (Russia), H. C. Myung (South Korea), M. Jambu (France), C. Procesi (Italy), C. Ringel (Germany), K Saito (Japan), I. Shestakov (Brazil), L. Shevrin (Russia), L. Small (USA), B. Wegner (Germany), V. A. Artamonov (Moscow), E. C. Tan (Singapore).

Information: Efim Zelmanov: email: ezelmanov@ucsd.edu.

* 8–11 **Invited Session: "The Educational Method of Management on Conditions of Extreme Situations" along with the workshop International Conference on Knowledge Generation, Communication and Management: KGCM 2007**, Orlando, Florida.

Deadlines: Papers/Abstract Submissions and Invited Sessions Proposal: April 12th. Authors Notification: May 3rd. Camera-ready, full papers: May 24th.

Session organizer: All abstracts and papers should be sent to and more information can be obtained by contacting: Professor Boli Yarkulov, profyarkulov@yahoo.com. phone: +998662291481; http://iiis-cyber.org/kgcm2007/Invitedsession/InvitedSessionPre.asp?vc=18.

* 12–15 **The Second International Congress of Algebra and Combinatorics: Dedicated to the 70th birthday of Leonid A. Bokut**, Xi'an University of Architecture and Technology, Xi'an, China.

Topics: Semigroups and Semi-rings, Words and Languages, Applications of Algebra and Combinatorics in Theorectical Computer Science.

Scientific Committee: Will be chaired by Professor Efim Zelmanov (USA): Zhe-Xian Wan, (China), T. Arad (Israel), E. Bannai (Japan), M. Broue (France), K. Denecke (Germany), M. M. Deza (France), R. Gonchigdorzh (Mongolia), R. Graham (USA), M. Ito (Japan), K. Ueno (Japan), T. Y. Lam (USA), V. Latyshev (Russia), L. Marki (Hungary), A. V. Mikhalev (Russia), H. C. Myung (South Korea), M. Jambu (France), C. Procesi (Italy), C. Ringel (Germany), K Saito (Japan), I. Shestakov (Brazil), L. Shevrin (Russia), L. Small (USA), B. Wegner (Germany), V. A. Artamonov (Moscow), E. C. Tan (Singapore).

Information: Efim Zelmanov: email: ezelmanov@ucsd.edu.

6–12 **Sixth Summer School on Potential Theory and Applications**, Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Sofia, Bulgaria. (Feb. 2007, p. 307)

8–12 **2007 von Neumann Symposium**, Snowbird, Utah. (Dec. 2006, p. 1379)

8–12 **International Conference on Analytical Methods of Celestial Mechanics**, Euler IMI, St. Petersburg, Russia. (Mar. 2007, p. 438)

8–21 **37th Probability Summer School**, Saint-Flour, France. (May 2007, p. 663)

9–12 (REVISED DEADLINE) **International Conference on Artificial Intelligence and Pattern Recognition**, Orlando, Florida. (Aug. 2006, p. 824)

9–12 **International Conference on Enterprise Information Systems and Web Technologies**, Orlando, Florida. (Aug. 2006, p. 824)

9–12 **International Conference on High Performance Computing, Networking and Communication Systems**, Orlando, Florida. (Aug. 2006, p. 824)

9–12 **International Conference on Software Engineering Theory and Practice (SETP-07)**, Orlando, Florida. (Aug. 2006, p. 824)

9–13 **Conference on Applied Mathematics and Scientific Computing**, Brijuni Island, Croatia. (Dec. 2006, p. 1379)

9–13 **Dynamics Days Europe 2007**, Loughborough University, United Kingdom. (Jan. 2006, p. 70)

* 9–13 **First French-Spanish Congress of Mathematics**, Universidad de Zaragoza, Spain.

Plenary Speakers: Enrique Artal, Yves Benoist, Xavier Cabré, Michel Ledoux, François Loeser, Consuelo Martínez, Carlos Parés, Benoit Perthame, Olivier Pironneau, Ana Vargas.

Organizer: "Real Sociedad Matemática Española" (RSME), the "Société Mathématique de France" (SMF), and the "Sociedad Española de Matemática Aplicada" (SEMA).

Information: Besides the plenary lectures, there will be special sessions covering a broad range of mathematics, together with some parallel activities (expositions, round tables, ...). More information at the web: http://www.unizar.es/ICHFM07.

* 9–13 **Further Developments in Quantitative Finance**, International Centre for Mathematical Sciences, ICMS, Edinburgh, Scotland, United Kingdom.
Information: http://www.icms.org.uk/workshops/quantfin2.

9–13 **PRACQSYS 2007: The Principles and Applications of Control in Quantum Systems**, Sydney, Australia. (May 2007, p. 663)

9–27 **Graduate Summer School: Probabilistic Models of Cognition: The Mathematics of Mind**, UCLA, Los Angeles, California. (May 2007, p. 663)

9–August 3 **Program on Computational Methods in Biomolecular Structures and Interaction Networks**, Institute for Mathematical Sciences, National University of Singapore, Singapore. (Mar. 2007, p. 438)

12–14 **Sixth International Conference on Lattice Path Combinatorics and Applications**, East Tennessee State University, Johnson City, Tennessee. (Oct. 2006, p. 1091)

13–19 **1007 ASL European Summer Meeting (Logic Colloquium '07)**, Wroclaw, Poland. (Jan. 2007, p. 64)

16–19 **Communicating Mathematics**, Duluth, Minnesota. (Apr. 2007, p. 558)

* 16–20 **Conference on Algorithmic Complexity and Universal Algebra**, Bolyai Institute, University of Szeged, Szeged, Hungary.
Program: Many real-life and combinatorial problems can be naturally expressed as a constraint satisfaction problem (CSP), making it a universal, easy to understand declarative programming language. The two key open problems are to characterize those classes of CSPs that are solvable in polynomial time, and to prove the dichotomy conjecture stating that every CSP class is either in P or NP-complete. The focus of this conference is to explore and disseminate the interplay between universal algebra and the CSP that has already led to a number of interesting (and deep) results, and to study novel properties of algebras, related to or implied by CSP conjectures, but of independent interest.
Speakers: Victor Dalmau, Marcin Kozik, Ralph McKenzie, Jaroslav Nesetril, Vera Vertesi.
Deadlines: Early registration till June 12, 2007.
Information: http://www.math.u-szeged.hu/confer/algebra/.

16–20 **International Congress on Industrial and Applied Mathematics, ICIAM07**, ETH and University Zurich, Zurich, Switzerland. (Dec. 2006, p. 1379)

16–20 **Optimal Transportation, and Applications to Geophysics and Geometry**, University of Edinburgh, Edinburgh, Scotland. (May 2007, p. 663)

16–22 **The 8th International Conference on Fixed Point Theory and Its Applications**, Department of Mathematics, Faculty of Science, Chiang Mai University, Chiang Mai, Thailand. (Sept. 2006, p. 961)

17–30 **The Second International Conference on Optimization and Optimal Control**, National University of Mongolia, Ulaanbaatar, Mongolia. (Dec. 2006, p. 1380)

18–August 3 **X Diffiety School—School in Geometry of Partial Differential Equations**, Santo Stefano del Sole (AV), Italy. (May 2007, p. 663)

19–22 **Applications of Computer Algebra (ACA 2007)**, Oakland University, Rochester, Michigan. (Apr. 2007, p. 558)

20–22 **Ross Program Fiftieth Anniversary Reunion-Conference**, Ohio State University, Columbus, Ohio. (Mar. 2007, p. 438)

21–23 **5th International Summer School and Workshop on Pattern Recognition: Call for Participation and Papers**, Plymouth, United Kingdom. (Apr. 2007, p. 558)

22–25 **OPTIMIZATION 2007**, Faculty of Economics, University of Porto, Porto, Portugal. (Dec. 2006, p. 1380)

22–27 **CIM/UC Summer School: Topics in Nonlinear PDEs**, Centro Internacional de Matemática (CIM), Coimbra, Portugal. (Nov. 2006, p. 1253)

22–28 **Topological Theory of Fixed and Periodic Points (TTFPP 2007)**, Conference Center of the Mathematical Institute of the Polish Academy of Sciences, Bedlewo near Poznan, Poland. (Feb. 2007, p. 307)

22–August 4 **Biomedical Modeling and Cardiovascular-Respiratory Control: Theory and Practice**, Schloss Seggau, Leibnitz, (near Graz) Austria. (Apr. 2007, p. 558)

23–27 **The Twelfth International Conference on Difference Equations and Apllications (ICDEA07)**, The Technical University of Lisbon, Portugal. (Mar. 2007, p. 438)

23–27 **VII Americas School in Differential Equations**, University of Cartagena, Cartagena, Colombia (South America). (Feb. 2007, p. 307)

23–27 **XVII Latinamerican Colloquium of Algebra**, Antioquia University, Medellin, Antioquia/Colombia. (May 2007, p. 663)

23–August 31 **Cemracs'07: Summer Mathematical Research Center on Scientific Computing and Its Applications**, CIRM, Luminy, France. (May 2007, p. 663)

23–December 21 **Strong Fields, Integrability and Strings**, Isaac Newton Institute for Mathematical Sciences, Cambridge, UK. (Nov. 2005, p. 1264)

24–27 (REVISED) **3rd International workshop "Reliable Methods of Mathematical Modelling" (RMMM 2007)**, Euler IMI, Saint Petersburg, Russia. (Mar. 2007, p. 438)

25–27 **Symbolic-Numeric Computation 2007 (SNC 2007)**, London, Canada. (May 2007, p. 664)

* 27–28 **Parallel Symbolic Computation 2007 (PASCO 2007)**, London, Canada.
Program: The program of PASCO 2007 will include invited presentations, contributed research papers and posters. Specific topics include, but are not limited to: Parallel computer algebra, Design of high performance software tools and interfaces for exact and approximate procedures, Design and analysis of parallel algorithms for algebraic computations, Parallel computing for number theory, geometry, automatic theorem proving, combinatorial and discrete methods, Distributed data-structures for matrices and polynomials, Implementations of solvers on multi-cores, SMPs, clusters, supercomputers and grids, Interactive parallel symbolic computation, Volunteer computing for symbolic problems, Applications of parallel symbolic computation.
Invited Speakers: The conference is pleased to announce the following invited speakers: Michael A. Bauer, Univ. of Western Ontario; Matteo Frigo, Cilk Arts; Thierry Gautier, INRIA; Anthony D. Kennedy, Edinburg Univ. (Jointly with SNC); Katherine Yelick, Univ. of California at Berkeley.
Information: http://www.orcca.on.ca/conferences/pasco2007.

29–August 4 **Reconnect Conference 2007**, DyDAn Center, Rutgers University, Piscataway, New Jersey. (May 2007, p. 664)

30–August 3 **L-functions and modular forms**, AIM Research Conference Center, Palo Alto, California. (Dec. 2006, p. 1380)

30–August 3 **The 15th International Conference on Finite or Infinite Dimensional Complex Analysis and Applications**, Media Center and Library (Sugimoto Campus), Osaka City University, Osaka, Japan. (Mar. 2007, p. 439)

30–August 4 **On Certain L-functions**, Purdue University, West Lafayette, Indiana. (Apr. 2007, p. 558)

31–August 3 **First Joint International Meeting between the AMS and the Polish Mathematical Society**, Warsaw, Poland. (Jun/Jul. 2006, p. 714)

August 2007

5–11 **Equadiff 2007**, Vienna University of Technology, Vienna, Austria. (Apr. 2007, p. 559)

6–8 **Joint SOCR CAUSEway Continuing Education Workshop 2007**, UCAL, Los Angeles, California. (Oct. 2006, p. 1091)

6–10 **Enhancing the problem authoring capabilities of WeBWorK**, AIM Research Conference Center, Palo Alto, California. (Apr. 2007, p. 559)

6–10 **Mathematical Virology**, Edinburgh, Scotland. (May 2007, p. 664)

6–10 **Security '07: 16th USENIX Security Symposium**, Sheraton Boston Hotel, 39 Dalton Street, Boston, Mass. (Dec. 2006, p. 1380)

6–17 **19th European Summer School in Logic, Language and Information**, Dublin, Ireland. (Mar. 2007, p. 440)

* 9–12 **Satellite Summer School on Lévy Processes: Theory and Applications**, Sandbjerg Manor, Soenderborg, Denmark.
Information: This Summer school will comprise four minicourses on different areas related to Lévy process theory and its applications. The intended audience are Ph.D. and postdoctoral students, or researchers who want to get an introduction to the theory of Levy processes and an overview of applications in different fields.
Minicourses: The minicourses will be devoted to the following topics: An introduction to the theory of Lévy processes by Andreas Kyprianou; Lévy processes in finance by Jan Kallsen; Fractal and sample path properties of Lévy processes by Davar Khosnevisan; Lévy processes in telecommunications by Sid Resnick.

12–15 **The 13th International Conference of Knowledge Discovery and Data Mining: ACM/SIGKDD**, San Jose, California. (Mar. 2007, p. 440)

12–16 **Summer Symposium in Real Analysis XXXI**, Trinity College, Oxford, United Kingdom. (Apr. 2007, p. 559)

12–17 **Geometry and Algebra of PDEs**, University of Tromso, Norway. (May 2007, p. 664)

* 13–17 **5th International Conference on Lévy Processes: Theory and Applications**, University of Copenhagen, Denmark.
Conference: As in the previous meetings, the 2007 meeting will schedule review papers and original research on all aspects of Lévy process theory and its applications. It is the aim of the conference to bring together a wide range of researchers, practitioners, and graduate students whose work is related to Lévy processes and infinitely divisible distributions in a wide sense.
Topics of Interest: Structural results for Lévy processes: distribution and path properties; Lévy trees, superprocesses and branching theory; Fractal processes and fractal phenomena; Stable and infinitely divisible processes and distributions; Applications in finance, physics, biosciences and telecommunications; Lévy processes on abstract structures; Statistical, numerical and simulation aspects of Lévy processes; Lévy and stable random fields.
Organizers in Copenhagen: Thomas Mikosch, Michael Sorensen, Niels Richard Hansen, Anders Tolver Jensen.
Information: `http://www.math.ku.dk/conf/levy2007/levy.html`.

13–17 **Generic Case Complexity**, AIM Research Conference Center, Palo Alto, California. (Feb. 2007, p. 307)

13–17 **Groups, rings, Lie and Hopf algebras. II**, Bonne Bay Marine Station of Memorial University of Newfoundland, Norris Point, NL, Canada, A0K 3V0. (May 2007, p. 664)

* 13–24 **Fields Institute, University of Ottawa Summer School in Operator Algebras**, University of Ottawa, Ottawa, Ontario, Canada.
Program: The Summer School is organized in conjunction with the Fields Institute Thematic Program on Operator Algebras (July-December, 2007) and will consist of short introductory courses in various areas of operator algebra theory that are of major current interest, mainly targeted at graduate students and postdoctoral fellows.
Course lecturers: Dietmar Bisch (Vanderbilt University), Matthias Neufang (Carleton University), Narutaka Ozawa (University of Tokyo), N. Christopher Phillips (University of Oregon), Jean Renault (Université d'Orléans), Stefaan Vaes (K.U. Leuven).
Financial support: Available for graduate students and postdoctoral fellows. Application deadline: April 30.
Local organizers: Thierry Giordano (main organizer), Benoît Collins, David Handelman, Vladimir Pestov (all University of Ottawa).
Information: `http://www.fields.utoronto.ca/programs/scientific/07-08/opalg_school/`.

15–18 **International Conference on Integral Geometry, Harmonic Analysis and Representation Theory in honor of Sigurdur Helgason on the occasion of his 80th birthday**, University of Iceland, Reykjavik, Iceland. (May 2007, p. 664)

19–25 **Loops'07**, Charles University, Prague, Czech Republic. (Apr. 2007, p. 559)

19–25 **The Eighth International Workshop on Differential Geometry and its Applications**, Cluj-Napoca, Romania. (Feb. 2007, p. 307)

* 20–24 **Analysis and Singularities: International conference dedicated to the 70th anniversary of V. I. Arnold**, Steklov Mathematical Institute, Moscow, Russia.
Conference Topics: Algebraic Geometry, Singularity Theory, Differential Equations, Mechanics, Hydrodynamics, Number Theory.
Invited speakers: A. Agrachev (Trieste, Italy), J. Palis (IMPA, Brasil), A. Varchenko (Chapel-Hill, USA), O. Viro (Uppsala, Sweden), B. Khesin (Toronto, Canada), E. Shustin (Tel-Aviv, Israel), P. Biran (Tel-Aviv, Israel), S. Novikov (Mariland, USA and Moscow, Russia), Ya. Sinai (Princeton, USA), V. Goryunov (Liverpool, UK), S. Kuksin (Edinburgh, UK), D. McDuff (Stony Brook, USA), A. Vershik (St.Petersburg, Russia), V. Kharlamov (Strasburg, France), D. Siersma (Utrecht, Holland), J. Steenbrink (Nijmegen, Holland).
Deadline for registration: April 15, 2007.
Information: `http://arnold-70.mi.ras.ru/index_e.html`. Mail: "Analysis and Singularities", Steklov Mathematical Institute, Gubkina 8, Moscow, 119991, Russia, fax: +7 (495) 135 05 55; Email: (with mark "ANALYSIS and SINGULARITIES") `arnold-70@mi.ras.ru`.

20–24 **Fourier analytic methods in convex geometry**, AIM Research Conference Center, Palo Alto, California. (Apr. 2007, p. 559)

20–24 **Geometric Aspects of Analysis and Mechanics: A Conference in Honor of the 65th Birthday of Hans Duistermaat**, Utrecht University, Utrecht, The Netherlands. (Dec. 2006, p. 1380)

* 20–24 **Lars Ahlfors Centennial Celebration**, Helsinki, Finland.
Program: Plenary and invited lectures.
Topics: Hyperbolic geometry and Kleinian groups, harmonic analysis and PDE's, geometric measure theory, geometric function theory and complex dynamics, analysis on metric spaces.
Plenary Speakers: Lennart Carleson, Alice Chang, Jeff Cheeger, Guy David, Juha Heinonen, Bruce Kleiner, Olli Lehto, Curt McMullen, Jill Pipher, David Preiss, Yum-Tong Siu, Dennis Sullivan, Xavier Tolsa, Gunther Uhlmann, Wendelin Werner.
Invited Speakers: Luigi Ambrosio, Zoltán Balogh, Rodrigo Bañuelos, Walter Bergweiler, Mario Bonk, Danny Calegari, Kevin Costello, Marianna Csörnyei, Benson Farb, Ursula Hamenstädt, Håkan Hedenmalm, Aimo Hinkkanen, Tadeusz Iwaniec, Peter Jones, Maarit Järvenpää, Bernd Kirchheim, Leonid Kovalev, Bryna Kra, John Lewis, Vladimir Markovic, Gaven Martin, Jani Onninen, Stefanie Petermichl, Lassi Päivärinta, Eero Saksman, Stanislav Smirnov, Mikhail Sodin,

Teruhiko Soma, Tatiana Toro, Joan Verdera, Alexander Volberg, Jang-Mei Wu, Xiao Zhong, Michel Zinsmeister.
Information: http://mathstat.helsinki.fi/ahlfors100/.

24–26 **32nd Sapporo Symposium on Partial Differential Equations**, Department of Mathematics, Hokkaido University, Sapporo, Japan. (Mar. 2007, p. 439)

26–September 1 **Conference "Algebras, Representations and Applications" Lie and Jordan Algebras, their Representations and Applications, III: In Honour of Ivan Shestakov's 60th Birthday**, Maresias Beach Hotel, Maresias, São Paulo, Brazil. (Mar. 2007, p. 439)

27–29 **Automata 2007: 13th International Workshop on Cellular Automata**, The Fields Institute for Research in Mathematical Science, Toronto, Canada. (May 2007, p. 664)

27–29 **International Conference on Biomathematics 2007**, ITB, Bandung, Indonesia. (Feb. 2007, p. 307)

27–31 **LMS workshop on motives, quadratic forms and algebraic groups**, Queen's University, Belfast, United Kingdom. (May 2007, p. 665)

28–30 **2nd Regional Conference on Ecological and Environmental Modelling (ECOMOD 2007)**, Gurney Hotel, Penang, Malaysia. (Apr. 2007, p. 559)

September 2007

* 3–6 **38th Annual Iranian Mathematics Conference**, Zanjan University, Zanjan, Iran.
Scientific Committee: F. Mirzapour (The head of conference), f.mirza@mail.znu.ac.ir; M. T. Dastjerdi (The head of scientific committee of conference), tdast@yahoo.com; E. S. Mahmoodian, emahmood@sharif.edu; M. Adib, madib@mail.znu.ac.ir; M. Ariannejad, m_ariannejad@yahoo.com; M. Emami, emami@mail.znu.ac.ir; J. Malaki, j.malaki@mail.znu.ac.ir; H. Mohebi, hmohbei@mail.uk.ac.ir; A. Rasooli, rasooli@yahoo.com; Sh.Rezapour, sh.rezapour@azaruni.ac.ir; S. Salehipourmehr, saeed@iasbs.ac.ir; S. Varsaie, varsaie@iasbs.ac.ir; R. Zarenahandi, rashidzn@iasbs.ir.
Information: Contact: 6-Kilometer of Tabriz Road, Zanjan, IRAN, Zanjan University Po Box: 313; Tel: (+98) 0241 5152681; Fax: (+98) 0241 5152514; aimc38@znu.ac.ir; http://aimc38.znu.ac.

* 3–7 **Algebraic and Arithmetic Structures of Moduli Spaces**, Hokkaido University, Sapporo, Japan.
Organizers: Iku Nakamura (Hokkaido University) and Lin Weng (Kyushu University).
Fee: No registration fee is required.
Information and Contact: http://coe.math.sci.hokudai.ac.jp/sympo/moduli2007/; cri@math.sci.hokudai.ac.jp.

3–7 **Conference in Numerical Analysis 2007 (NumAn 2007): Recent Approaches To Numerical Analysis: Theory, Methods and Applications**, Kalamata, Greece. (May 2007, p. 665)

3–7 **The Riemann-Hilbert Problem and Toeplitz Operators**, Heriot Watt University, Edinburgh, Scotland. (May 2007, p. 665)

3–14 **Epidemiology & Control of Infectious Diseases: Introduction to mathematical models of global and emerging infections**, Imperial College, London, England. (Apr. 2007, p. 559)

3–December 21 **Phylogenetics**, Isaac Newton Institute for Mathematical Sciences, Cambridge, UK. (Nov. 2006, p. 1264)

4–6 **International Conference on Mathematical Biology 2007 (ICMB07)**, Universiti Putra Malaysia, Serdang, Malaysia. (Nov. 2006, p. 1253)

4–8 **Potential Theory and Stochastics**, Albac, Romania. (May 2007, p. 665)

* 7–9 **Geometry, Rigidity, and Group Actions: A Conference in Honor of Robert J. Zimmer's 60th Birthday**, University of Chicago, Illinois.
Speakers: David Fisher (Indiana University), Alex Furman (University of Illinois, Chicago), Anatole Katok (Pennsylvania State University), Alexander Lubotzky (Hebrew University, Israel), Gregory Margulis (Yale University), Amos Nevo (Technion, Israel), Leonid Polterovich (Tel Aviv University, Israel), Sorin Popa (University of California, Los Angeles), Yehuda Shalom (Tel Aviv University, Israel), and Shmuel Weinberger (University of Chicago).
Schedule: The conference will begin Friday morning, and end before noon on Sunday. Talks will be 45 minutes in length, and limited in number, to allow time for informal discussions. On Sunday afternoon, after the conference, there will be a workshop for young researchers to give a short talk on their work.
Organizers: Benson Farb (University of Chicago), David Fisher (Indiana University), Dave Witte Morris (University of Lethbridge), and Ralf Spatzier (University of Michigan).
Information: http://www.math.uchicago.edu/zimmer60/.

7–11 **KIAS Conference on Geometric Analysis**, KIAS, Seoul, Korea. (Apr. 2007, p. 559)

7–13 **9th International Conference of The Mathematics Education into the 21st Century Project**, Charlotte, North Carolina. (Apr. 2007, p. 498)

9–10 **SHARCS'07: Special-purpose Hardware for Attacking Cryptographic Systems**, Vienna Marriott Hotel, Vienna, Austria. (Mar. 2007, p. 439)

10–11 **Journées Jean-Yves Girard, Conference in Honour of his 60th Birthday**, Institut Henri Poincaré, Paris, France. (May 2007, p. 665)

* 10–14 **1st IMACS International Conference on Computational Biomechanics and Biology ICCBB 2007**, University of West Bohemia, Pilsen, Czech Republic.
Scientific topics: The conference, under the auspices of IMACS, will be devoted to certain problems in biomechanical, biological, medical modelling, and mathematical methods for their solutions. The conference features a number of invited scientific sessions in the following areas: Soft tissue and muscular mechanics, Bone and dental mechanics, Cardiovascular mechanics, Micro-circular and respiratory systems, Cellular and molecular mechanics, Tissue engineering, biomaterials, Biotransport and multi-field problems, Organ biomechanics and fluid-structure interaction, Impact and injury biomechanics, sport biomechanics, Orthopedics and implant modelling, Muscle-skeletal systems and performance, Computational biomechanics and large simulations, Imaging and computer assisted surgery, Biological, biomechanical and medical modelling.
Information: http://www.iccbb.zcu.cz.

10–14 **5th Symposium on Nonlinear Analysis**, Nicolaus Copernicus University, Torun, Poland. (Apr. 2007, p. 559)

* 10–14 **11th Workshop on Well-Posedness of Optimization Problems and Related Topics**, University of Alicante, Alicante, Spain.
Description: The Workshop started in 1987 in Milan, Italy, as a small meeting between Bulgarian and Italian groups working on the subject: Well-Posedness of Optimization Problems and Related Topics. A Summer School on "Stability and Well-Posedness in Convex Optimization" will be held parallel to the workshop.
Topics: Well-posedness and stability of optimization models and problems in calculus of variations, optimal control and mathematical programming; Hadamard, Tykhonov and similar type of concepts of well-posedness; topological aspects of well-posedness, with applications to special classes of optimization problems; game theory and equilibrium; variational principles; well-posedness concepts for vector optimization problems; regularization techniques for ill-posed problems; stability in stochastic optimization; applications to the performance analysis of numerical optimization methods and their stable behavior under perturbations; critical point theory.

Deadline: For abstracts: April 16, 2007.
Information: http://www.eio.ua.es/congreso/index.html.

10–14 **High-order methods for computational wave propagation and scattering**, AIM Research Conference Center, Palo Alto, California. (Aug. 2006, p. 824)

10–15 **10th Quantum Mathematics International Conference: QMath10**, Moeciu, Romania. (Apr. 2007, p. 560)

10–15 **International Conference on Nonlinear Partial Differential Equations dedicated to the memory of Igor V. Skrypnik**, NPDE2007, Yalta, Crimea, Ukraine. (Feb. 2007, p. 307)

* 10–15 **School (and Workshop) on the Geometry of Special Varieties**, Fondazione Bruno Kessler-IRST, Povo, Trento, Italy.
Information: The School is mainly aimed at Ph.D. students and young researchers in Algebraic Geometry, introducing the participants to research, beginning from a basic level with a view towards the applications and to the most recent results. A tentative program is as follows: Secant and defective varieties; Classification of varieties with extremal tangential properties; Degeneration of varieties (expecially of toric varieties); Degeneration of maps (expecially of projections); Applications to the study of secant varieties; Toric degeneration; Tropical geometry. The Workshop is intended to discuss the state of the art.
Lecturers: C. Ciliberto (University of Roma II) and F. Russo (Universidade Federal de Pernambuco). Speakers in the workshop: M. Andreatta (University Trento), L. Badescu (University Genova), L. Chiantini (University Siena), K. Ranestad (University Oslo).
Registration Deadline: June 30, 2007 (for researchers asking a financial grant) and July 20, 2007 (for other participants).
Information: http://www.science.unitn.it/cirm/; A. Micheletti, Secretary of CIRM, Fondazione Bruno Kessler, Via Sommarive 14, I-38050 Povo (Trento), Italy. email: michelet@science.unitn.it, Tel. +39-0461-881628, Fax +39-0461-810629.

10–December 14 **Mathematics of Knowledge and Search Engines (Long Program)**, UCLA, Los Angeles, California. (May 2007, p. 665)

11–14 **Fall 2007 Workshop for Young Researchers in Mathematical Biology (WYRMB)**, Columbus, Ohio. (May 2007, p. 665)

11–15 **CSL07: 16th EACSL Annual Conference on Computer Science and Logic**, Lausanne, Switzerland. (Feb. 2007, p. 308)

13–14 **IMA Tutorial: Mathematics of Nucleic Acids**, University of Minnesota, Minneapolis, Minnesota. (Dec. 2006, p. 1380)

* 16–19 **Mathematical Neuroscience**, Centre de recherches mathématiques, Université de Montréal, Montréal, Canada.
Goal: The goal of this workshop is to provide an overview of the current state of research in mathematical approaches to neuroscience. This workshop will not be preceded by a minicourse. Graduate students and early career scientists seeking further preparation before attending this workshop might consider the computational Neuroscience Summer School in Ottawa in June 2007.
Organizers: S. Coombes (Nottingham), A. Longtin (Ottawa), J. Rubin (Pittsburgh).
Information: http://www.crm.math.ca/Dynamics2007/.

16–20 **International Conference of Numerical Analysis and Applied Mathematics 2007 (ICNAAM 2007)**, Corfu, Greece. (Mar. 2007, p. 440)

17–20 **Fourth International Workshop on Meshfree Methods for Partial Differential Equations**, Universitaet Bonn, Bonn, Germany. (Mar. 2007, p. 440)

17–21 **13th Czech-French-German Conference on Optimization**, University of Heidelberg, Heidelberg, Germany. (Feb. 2007, p. 308)

17–21 **IMA Workshop: Mathematics and Biology of Nucleic Acids**, University of Minnesota, Minneapolis, Minnesota. (Dec. 2006, p. 1380)

17–21 **Manifolds with nonnegative sectional curvature**, American Institute of Mathematics, Palo Alto, California. (May 2007, p. 665)

17–21 **SECURECOMM 2007: Third International Conference on Security and Privacy for Communication Networks**, Nice, France. (May 2007, p. 665)

* 17–21 **Summer School on "New Trends and Directions in Harmonic Analysis, Approximation Theory, and Image Analysis"**, Inzell, Germany.
Aims and Scope: The workshop is intended to bring together leading international scientists and young researchers from different facets of harmonic analysis, approximation theory and signal and image analysis to present their latest cutting-edge research and to establish new and exciting directions for future investigations.
Organizers: Brigitte Forster-Heinlein, Peter Massopust, and Rupert Lasser Centre of Mathematical Sciences, Technische Universität München, Garching, Germany, and Institute for Biomathematics and Biometry, GSF National Research Centre for Environment and Health, Neuherberg, Germany. The Summer School partially financed by means of the Marie Curie Excellence Team MAMEBIA funded by the European Commission.
Invited Plenary Speakers: John Benedetto (University of Maryland), Ole Christensen (Technical University of Denmark), Karlheinz Gröchenig (University of Vienna), Michael Unser (École Polytechnique Fédérale de Lausanne), Guido Weiss (Washington University, St. Louis).
Information: http://www.mamebia.de/inzell.

20–22 **Finsler Geometry (Mathematics and Physics)**, Institut de Recherche Mathématique Avancée, Strasbourg, France. (May 2007, p. 666)

* 22–23 **Minicourse on Quantitative Biology**, Centre de recherches mathématiques, Université de Montréal, Montréal, Canada.
Description: Biology is becoming a quantitative science. For example, new technologies allow the motion of individual proteins to be visualized in single cells and the expression levels of all genes in a genome to be measured simultaneously. Nevertheless, a traditional biology training does not give the mathematical tools required to analyse such data. Across the life sciences, there is a need for researchers who are familiar with both mathematics and biology. This minicourse will introduce the fundamentals of quantitative biology. Its goal will be to give sufficient background to enable mathematics students to start reading the scientific literature and to enable biology students to critique the assumptions behind some mathematical techniques. Students will be taught by experts in quantitative biology, all of whom have themselves moved their research from the physical to the biological sciences.
Organizers: P. S. Swain (McGill), B. P. Ingalls (Waterloo), M. C. Mackey (McGill).
Information: http://www.crm.math.ca/Dynamics2007/.

* 22–24 **Geometrization of Probability: A Fields Institute/ University of Ottawa workshop**, University of Ottawa, Ottawa, Ontario, Canada.
Program: The workshop, mostly aimed at graduate students and recent Ph.D. recepients, will feature minicourses, one-hour invited lectures, and possibly time for short communications and a discussion of open problems.
Topics: Recent decades have seen a remarkable degree of interaction between probability theory and high-dimensional convexity theory. The framework of the subject involves very high dimensional spaces (normed spaces, convex bodies) and accompanying asymptotic phenomena as the dimension goes to infinity. Concepts, results and approaches of a new theory developed in the framework of geometrization of probability will benefit many mathematicians using in one form or another probabilistic and functional-analytic methods in their work. The lecturers at the workshop are today's foremost experts in the area.

Minicourse lecturers: Shiri Artstein-Avidan (Tel-Aviv University), Robert McCann (University of Toronto), Boaz Klartag (IAS and Clay Institute).
Invited Speakers: Alexander Litvak (University of Alberta), Sasha Sodin (Tel-Aviv University), Stanislaw Szarek (Case Western Reserve University), Nicole Tomczak-Jaegermann (University of Alberta), Elisabeth Werner (Case Western Reserve University).
Financial support: Limited financial support will be available for Ph.D. students and postdoctoral fellows, cf. the web page for details.
Information: `http://www.fields.utoronto.ca/programs/scientific/07-08/geometrization/index.html`.

* 23–26 **Applied Statistics 2007**, Ribno (Bled), Slovenia.
Program: The conference will provide an opportunity for researchers in statistics, data analysts, and other professionals from various statistical and related fields to come together, present their research, and learn from each other. A three day program consists of invited paper presentations, contributed paper sections from diverse topics, and finishes with a workshop. Cross-discipline and applied paper submissions are especially welcome.
Plenary speakers: Alan Agresti, University of Florida, USA; Ingram Olkin, Stanford University, USA; Rolf Steyer, Friedrich-Schiller-University Jena, Germany.
Organizers: Statistical Society of Slovenia.
Deadlines: Abstract Submission: July 1, 2007. Registration: August 15, 2007.
Information: `info.AS@nib.si`; `http://ablejec.nib.si/AS2007/`.

23–28 **14th Workshop on Stochastic Geometry, Stereology and Image Analysis**, Friedrich-Schiller-University Jena, Department of Stochastics, Jena, Germany. (Feb. 2007, p. 308)

* 24–27 **International Conference on Applications in Nonlinear Dynamics**, Poipu Beach, Koloa, Kauai, Hawaii.
Information: All information about conference can be found at `http://www.icand2007.org`.

* 24–28 **Towards Relative Symplectic Field Theory**, CUNY Graduate Center, New York, New York.
Description: The goal of this workshop, sponsored by AIM, NSF, CUNY Graduate Center and Stanford Mathematical Research Center (MRC), is to understand the structure of relative Symplectic Field Theory (SFT), discuss and reconcile different versions of its algebraic formalism, and to work towards building rigorous foundations of the theory. There will also be explored applications of relative SFT to symplectic and contact topology, as well as low-dimensional topology.
Organizers: Kai Cieliebak, Tobias Ekholm, Yakov Eliashberg, Kenji Fukaya, Dennis Sullivan, and Michael Sullivan.
Deadline: May 21, 2007.
Information: `http://aimath.org/ARCC/workshops/relsymplectic.html`.

* 24–28 **Workshop on Deconstructing Biochemical Networks**, Centre de recherches mathématiques, Université de Montréal, Montréal, Canada.
Organizers: P. S. Swain (McGill), B. P. Ingalls (Waterloo), M. C. Mackey (McGill).
Description: New technologies have led to a huge increase in molecular level data in biological systems. As well as fueling the transition of molecular biology to a quantitative science, this data is also revealing new complexity in biochemical networks. This workshop will bring together both modellers and experimentalists, and will focus on network "design".
Information: `http://www.crm.math.ca/Dynamics2007/`.

24–29 **18th Congress of Unione Matematica Italiana**, Bari, Italy. (May 2007, p. 666)

24–29 **International Algebraic Conference dedicated to the 100th anniversary of D. K. Faddeev**, Euler International Mathematical Institute, St. Petersburg, Russia. (Apr. 2007, p. 560)

25–28 **The 2nd International Conference on Nonlinear Dynamics: KhPI 2007 in honor of Alexander Lyapunov 150th Anniversary**, National Technical University, Kharkov Polytechnical Institute, Kharkov, Ukraine. (Feb. 2007, p. 308)

October 2007

* 1–5 **Dynamic Searches and Knowledge Building**, UCLA, Los Angeles, California.
Topics: User-tailored search, Textual entailment, Knowledge Discovery and Data Mining, Analysis and organization of search results, Measures of document or semantic similarity, Multi-media data mining and semantic annotation, including images, video and audio, Applications integrating search and knowledge (e.g. biomedicine, customer relationship management).
Speakers: Will be announced soon on: `http://www.ipam.ucla.edu/programs/sews1/`.
Organizing Committee: Karin Verspoor, Chair (Los Alamos National Laboratory, CCS-3), Jennifer Chu-Carroll (IBM Watson Research Center), Ronald Coifman (Yale University), Carey Priebe (Johns Hopkins University, Center for Imaging Science).
Application/Registration: An application/registration form is available at: `http://www.ipam.ucla.edu/programs/sews1/`. The application part is for people requesting financial support to attend the workshop. If you don't intend to do this, you may simply register. We urge you to apply as early as possible. Applications received by August 20, 2007 will receive fullest consideration. Successful applicants will be notified as soon as funding decisions are made.
Funding: To support the attendance of recent Ph.D.'s, graduate students, and researchers in the early stages of their career; however, mathematicians and scientists at all levels who are interested in this area are encouraged to apply for funding. Encouraging the careers of women and minority mathematicians and scientists is an important component of IPAM's mission and we welcome their applications.

5–6 **AMS Central Section Meeting**, DePaul University, Chicago, Illinois. (Dec. 2006, p. 1380)

6–7 **AMS Eastern Section Meeting**, Rutger University-New Brunswick, Busch Campus, New Brunswick, New Jersey. (Dec. 2006, p. 1380)

8–12 **Dichotomy Amenable/Nonamenable in Combinatorial Group Theory**, AIM Research Conference Center, Palo Alto, California. (Jun/Jul. 2006, p. 714)

9–11 **SIAM Conference on Mathematics for Industry: Challenges and Frontiers (MI07)**, Hyatt Regency Philadelphia, Philadelphia, Pennsylvania. (Mar. 2007, p. 440)

* 11–12 **Tenth New Mexico Analysis Seminar**, University of New Mexico, Albuquerque, New Mexico.
Keynote Speakers: Andrea Nahmod, University of Massachussets, Amherst. Minicourse on "Bilinear Operators in Analysis and PDEs", and Rodrigo Banuelos, Purdue University Minicourse on "Martingales and Fourier multipliers. What's new in this old marriage?".
Conference Information and Registration: This seminar is organized by analysis aficionados at New Mexico State University and The University of New Mexico. This year the conference is scheduled in the Fall prior to an AMS meeting that will be held in Albuquerque on Oct 13–14, 2007. The goal is to provide an opportunity for scientific exchange and cooperation among broadly defined analysts. This year the centerpieces of the seminar will be two minicourses given by the keynote speakers.

There is time allocated for shorter contributed talks, most of them will be presented in special sessions during the AMS meeting. If you would like to attend and give a talk, please contact one of the organizers by June 26, 2007. Doctoral students and recent Ph.D.s are specially encouraged to apply.

The registration form can be found at http://www.math.unm.edu/conferences/10thAnalysis/registration.html.

The seminar is being sponsored by NSF. We will provide travel stipends for qualified graduate students. We intend to pay, at least partially, shared accommodations for all participants, and if there are funds left, we will reimburse some travel expenses to those participants who have no other sources of funding (priority given to speakers and junior participants).

Information about the conference will be posted at: http://www.math.unm.edu/conferences/10thAnalysis/.

Organizers: Cristina Pereyra (crisp@math.unm.edu), Tiziana Giorgi (tgiorgi@nmsu.edu), Joseph Lakey (jlakey@nmsu.edu), Adam Sikora (asikora@nmsu.edu), Robert Smits (rsmits@nmsu.edu).

13–14 **AMS Western Section Meeting**, University of New Mexico, Albuquerque, New Mexico. (Jun/Jul. 2006, p. 714)

* 19–20 **27th Southeast Atlantic Regional Conference on Differential Equations (SEARCDE)**, Murray State University, Murray, Kentucky.

Organizers: Maeve L. McCarthy (maeve.mccarthy@murraystate.edu) and K. Renee Fister.

Speakers: Jeff Borggaard, Virginia Tech; Gerda deVries, University of Alberta; Barbara L. Keyfitz, Fields Institute and University of Houston.

Deadlines: October 1, 2007.

Information: http://campus.murraystate.edu/searcde/.

* 21–27 **The ADONET-CIRM School on Graphs and Algorithms**, Grand Hotel Bellavista, Levico Terme, Trento, Italy.

Minicourses: The school offers six minicourses (four lectures each) on state-of-the art topics on graphs and algorithms and related subjects. The minicourses are given by: Maria Chudnowsky (Columbia University): "Forbidding induced subgraphs", Jim Geelen (University of Waterloo): "Graph minors: Structure and algorithms", Michel Goemans (Massuchusetts Institute of Technology): "Uncrossing techniques", Bertrand Guenin (University of Waterloo): "Flows in graphs and matroids", Satoru Iwata (University of Tokyo): "Submodular function", Bill Jackson (University of London): "Rigidity of graphs". Institutional lectures will be given by Carsten Thomassen (Technical University of Denmark) and Wolfgang Mader (University of Hannover).

Scientific Organizers: Michele Conforti (University of Padova) and Bert Gerards (CWI, Amsterdam and Technische Universiteit Eindhoven).

Information: http://www.science.unitn.it/cirm/ADONETCIRM07.html; registration deadline is July 31, 2007. A. Micheletti, CIRM, Fondazione Bruno Kessler, 38050 Povo (Trento), Italy. email: michelet science.unitn.it, Tel. +39-0461-881628, Fax +39-0461-810629.

* 23–25 **Petra International Conference on Mathematics**, Al-Hussein Bin Talla Unversity, Ma'an, Jordan.

Information: http://www.ahu.edu.jo/picom2007/.

24–26 **2007 International Conference in Modeling Health Advances**, Clark Kerr Campus, UC Berkeley, California. (Mar. 2007, p. 440)

24–27 **Integers Conference 2007**, University of West Georgia, Carrollton, Georgia. (Apr. 2007, p. 560)

29–November 2 **IMA Workshop: RNA in Biology, Bioengineering and Nanotechnology**, University of Minnesota, Minneapolis, Minnesota. (Dec. 2006, p. 1381)

November 2007

* 1–5 **Joint AARMS-CRM Workshop on Recent Advances in Functional and Delay Differential Equations**, Dalhousie University, Halifax, Canada.

Organizers: J. Appleby (Dublin City), H. Brunner (Memorial), A. R. Humphries (McGill), D.E. Pelinovsky (McMaster).

Local Organizers: P. Keast (Dalhousie), P. Muir (St Mary's).

Description: Delay differential equations arise in many applications, and in the case of constant delays solutions give rise to semi-flows on function spaces. This workshop will provide a wide perspective on current research and open problems, covering theory, applications and numerical analysis of these equations.

Concentrated Topics: Dissipative Advanced Retarded Equations, Hamiltonian Advanced Retarded Equations Numerical DDEs (Chinese & Italian schools, numerics also in other concentrations), Applications in Mathematical Biology (Mathematical Physiology and Pop Dynamics), Volterra and Integral Equations, State Dependent Delays.

Information: http://www.crm.math.ca/Dynamics2007/.

1–December 31 **Program on Bose-Einstein Condensation and Quantized Vortices in Superfluidity and Superconductivity**, Institute for Mathematical Sciences, Singapore, Singapore. (May 2007, p. 666)

* 2–3 **Seventh Annual Prairie Analysis Seminar**, Kansas State University, Manhattan, Kansas.

Speakers: Luis Caffarelli, University of Texas, Antoine Mellet, University of British Columbia, and Daniel Phillips, Purdue University. There will be sufficient time for contributed talks; mathematicians early in their careers are especially encouraged to contribute a 20-minute talk.

Organizers: Marianne Korten, Diego Maldonado, Charles Moore, Virginia Naibo, Kansas State University and Estela Gavosto, Rodolfo Torres, University of Kansas.

Information: For information and registration see http://www.math.ksu.edu/pas/2007/. Support for travel may be possible.

3–4 **AMS Southeastern Section Meeting**, Middle Tennessee State University, Murfreesboro, Tennessee. (Jun/Jul. 2006, p. 714)

5–9 **Algorithmic Convex Geometry**, AIM Research Conference Center, Palo Alto, California. (Mar. 2007, p. 440)

* 11–16 **21st Large Installation System Administration Conference (LISA '07)**, Hyatt Regency Dallas, 300 Reunion Boulevard, Dallas, Texas 75207.

Description: The most in-depth, real-world system administration training available. For twenty years, the annual LISA conference has been the foremost worldwide gathering for everyone interested in the technical and administrative issues of running a large computing facility. Administrators of all specialties and levels of expertise meet at LISA to exchange ideas, sharpen old skills, learn new techniques, debate current issues, and meet colleagues and friends.

Information: http://www.usenix.org/events/lisa07/.

* 14–16 **Workshop on Dynamical Systems and Continuum Physics**, Centre de recherches mathématiques, Université de Montréal, Montréal, Québec, Canada.

Organizer: L. Tuckerman (PMMH-ESPCI, France).

Description: Dynamical systems theory provides astonishingly faithful representations of systems from continuum physics. This workshop will focus on recent research in laminar and turbulent hydrodynamics and in some more discrete systems, notably granular media and foams.

Information: http://www.crm.math.ca/Dynamics2007/.

* 28–29 **International Conference on Mathematical Sciences (ICMS'07)**, Equatorial Hotel, Bangi-Putrajaya, Malaysia.

Description: "Integrating Mathematical Sciences within Society".

Focus: Theory and applications of mathematical sciences in engineering, medical, biotechnology, insurance, finance and other related areas.

Important Dates: Abstract deadline: July 1, 2007. Conference Registration: August 1, 2007 (early birds). Full paper deadline: Sept. 30, 2007.

Organizers: Universiti Kebangsaan Malaysia & Malaysian Mathematical Sciences Society.
Information: email: seminarppsm@lycos.com; http://pkukmweb.ukm.my/~ppsmfst/icoms/.

December 2007

* 3–7 **Rigidity and polyhedral combinatorics**, American Institute of Mathematics, Palo Alto, California.
Organizers: Robert Connelly, Ezra Miller, and Igor Pak.
Description: This workshop, sponsored by AIM and the NSF, will be devoted to polyhedral objects in Euclidean spaces. Specifically, which metric or combinatorial properties must change or remain constant under certain classes of deformations, such as bending, folding, stretching, or flexing?
Deadline: July 15, 2007.
Information: http://aimath.org/ARCC/workshops/polyhedralcomb.html.

7–11 **Fourth Pacific Rim Conference on Mathematics: Celebrating the Tenth Anniversary of the Liu Bie Ju Centre for Mathematical Sciences**, City University of Hong Kong, Hong Kong. (Jan. 2007, p. 64)

10–12 **SIAM Conference on Analysis of Partial Differential Equations (PD07)**, Hilton Phoenix East/Mesa, Mesa, Arizona. (Apr. 2007, p. 560)

10–14 **Triangulations, Heegaard splittings and hyperbolic geometry**, American Institute of Mathematics, Palo Alto, California. (Apr. 2007, p. 560)

* 11–14 **Workshop on Chaos and Ergodicity of Realistic Hamiltonian Systems**, Centre de recherches mathématiques, Université de Montréal, Montréal, Québec, Canada.
Organizers: H. Broer (Groningen), P. Tupper (McGill).
Topics: Is mathematical ergodicity too strong a property for realistic systems? What is the relation between chaos and ergodicity for Hamiltonian systems? What about stronger properties, such as mixing, and CLT? What are the limits imposed by KAM? What is the relevance to molecular dynamics and fluid mechanics? What are the prospects for the Arnold-Avez Conjecture: For rather general Hamiltonian systems show that areas of positive measure exist with positive Lyapunov exponent.
Information: email: paradis@crm.umontreal.ca; http://www.crm.math.ca/Dynamics2007/.

12–15 **First Joint International Meeting between the AMS and the New Zealand Mathematical Society (NZMS)**, Wellington, New Zealand. (Jun/Jul. 2006, p. 714)

* 15–17 **First Announcement and Call for Papers: The Eighth Asian Symposium on Computer Mathematics (ASCM 2007)**, National University of Singapore, Singapore.
Description: The Asian Symposia on Computer Mathematics (ASCM) are a series of conferences which offer a forum for participants to present original research, to learn of research progress and new developments, and to exchange ideas and views on doing mathematics using computers. ASCM 2007 will consist of invited talks, regular sessions of contributed papers, and software demonstrations.
Call for Papers: Research papers on all aspects of the interaction between computers and mathematics are solicited for the symposium. Papers should be written in English, in single column, not exceeding 15 pages, and the main text font not smaller than 10 pt. Authors are expected to submit their papers electronically (in postscript, or pdf format).
Information: http://www.comp.nus.edu.sg/~ascm2007/. See submission instructions from the conference http://www.comp.nus.edu.sg/~ascm2007/A_Instructions.htm.

* 15–17 **International Symposium on Recent Advances in Mathematics & Its Applications (ISRAMA 2007)**, Calcutta, India.
Topics: Algebra, Discrete Mathematics & Theoretical Computer Science, Analysis & Topology and their Applications, Geometry and its Applications, Dynamical Systems, Chaos and Fractals, Continuum Mechanics, Plasma Physics, Control Theory and Optimization Theory, Bio-mechanics and Bioinformatics, Applications of Mathematics to Environmental Problems, History and Philosophy of Physical Science, Quantum Information Theory, Relativity and its Applications.
Information: Professor M. R. Adhikari, Secretary, Calcutta Mathematical Society, AE-374, Sector-1, Salt Lake City, India; email: cms@cal2.vsnl.net.in or cms_mra@yahoo.co.in.

16–20 **The Twelfth Asian Technology Conference in Mathematics (ATCM2007)**, Taipei, Taiwan. (Mar. 2007, p. 441)

* 17–19 **The 3rd Indian International Conference on Artificial Intelligence (IICAI-07)**, Pune, India.
Description: IICAI-07 is one of the major AI events in the world. This conference focuses on all areas of AI and related fields. We invite paper submissions. Please visit on the conference website for more details.
Information: Bhanu Prasad, IICAI-07 Chair, Department of Computer and Information Sciences, Florida A & M University, Tallahassee, FL 32307; email: bhanupvsr@gmail.com; tel: 850-412-7350; http://www.iiconference.org.

17–22 **Transformation Groups 2007**, Independent University of Moscow, Moscow, Russia. (Apr. 2007, p. 560)

* 30–January 4 **Conference on Representations of Algebras, Groups and Semigroups**, Bar-Ilan University, Ramat-Gan, Israel and the Netanya Academic College, Netanya, Israel.
Information: email: margolis@math.biu.ac.il; http://www.math.biu.ac.il/~margolis.

January 2008

* 6–9 **Joint Mathematics Meetings**, San Diego, California.
Information: http://www.ams.org/amsmtgs/national.html.

7–June 27 **Statistical Theory and Methods for Complex, High-Dimensional Data**, Isaac Newton Institute for Mathematical Sciences, Cambridge, United Kingdom. (Jun/Jul. 2006, p. 714)

10–11 **IMA Tutorial: Mathematics of Proteins**, University of Minnesota, Minneapolis, Minnesota. (Dec. 2006, p. 1381)

14–18 **IMA Workshop: Protein Folding**, University of Minnesota, Minneapolis, Minnesota. (Dec. 2006, p. 1381)

14–18 **The uniform boundedness conjecture in arithmetic dynamics**, American Institute of Mathematics, Palo Alto, California. (May 2007, p. 666)

14–July 4 **Combinatorics and Statistical Mechanics**, Isaac Newton Institute for Mathematical Sciences, Cambridge, United Kingdom. (Dec. 2006, p. 1381)

* 20–22 **ACM-SIAM Symposium on Discrete Algorithms (SODA08)**, Holiday Inn Golden Gateway, San Francisco, California.
Information: This symposium focuses on research topics related to efficient algorithms and data structures for discrete problems. In addition to the design of such methods and structures, the scope also includes their use, performance analysis, and the mathematical problems related to their development or limitations. Performance analyses may be analytical or experimental and may address worst-case or expected-case performance. Studies can be theoretical or based on data sets that have arisen in practice and may address methodological issues involved in performance analysis.
Submission Deadline: July 6, 2007.

March 2008

* 2–7 **IX International Conference "Approximation and Optimiza-**

tion in the Caribbean", Sunrise Beach Hotel, San Andres Island, Colombia.
Information: For more information about this event please visit the website http://matematicas.univalle.edu.co/~appopt2008/.

3–7 **IMA Workshop: Organization of Biological Networks**, University of Minnesota, Minneapolis, Minnesota. (Dec. 2006, p. 1381)

* 10–June 13 **Optimal Transport (Long Program)**, UCLA, Los Angeles, California.
Aim: The aim of the workshop is to put together physicists, biologists, mathematicians working on the optimization of transportation networks.
Organizing Committee: Andrea Bertozzi (UCLA, Mathematics), Yann Brenier (Université de Nice Sophia Antipolis), Wilfrid Gangbo (Georgia Institute of Technology), Peter Markowich (Universität Wien, Institute of Mathematics), Jean-Michel Morel (École Normale Supérieure de Cachan, CMLA).
Funding: We have funding especially to support the attendance of recent Ph.D.'s, graduate students, and researchers in the early stages of their career; however, mathematicians and scientists at all levels who are interested in this area are encouraged to apply for funding. Encouraging the careers of women and minority mathematicians and scientists is an important component of IPAM's mission and we welcome their applications.
Information: Please apply online to request financial support to attend and participate for extended periods up to the entire length of the program. The application and more information is available at http://www.ipam.ucla.edu/programs/ot2008/.

* 15–16 **AMS Eastern Section Meeting**, Courant Institute of New York University, New York, New York.
Information: http://www.ams.org/amsmtgs/sectional.html.

* 17–21 **Nonlinear PDEs of mixed type arising in mechanics and geometry**, American Institute of Mathematics, Palo Alto, California.
Organizers: Gui-Qiang Chen, Tai-Ping Liu, Richard Schoen, and Marshall Slemrod.
Description: This workshop, sponsored by AIM and the NSF, will be devoted to the study of nonlinear partial differential equations of mixed hyperbolic and elliptic type arising in conservation laws, continuum mechanics, differential geometry, relativity, and string theory.
Deadline: November 1, 2007.
Information: See http://aimath.org/ARCC/workshops/pdesofmixedtype.html.

* 19–21 **The IAENG International Conference on Operations Research 2008**, Regal Kowloon Hotel, Kowloon, Hong Kong.
Topics: Management Science, Managerial economics, Systems thinking and analysis, Optimization, Integer programming, Linear programming, Nonlinear programming, Assignment problem, Transportation network design, Simulation, Statistical Analysis, Stochastics Modelling, Reliability and maintenance, Queueing theory, Game theory, Graph theory, OR algorithms and software developments, OR applications and case studies.
Information:http://www.iaeng.org/IMECS2008/ICOR2008.html.

* 28–30 **AMS Southeastern Section Meeting**, Louisiana State University, Baton Rouge, Louisiana.
Information: http://www.ams.org/amsmtgs/sectional.html.

* 31–April 4 **Applications of universal algebra and logic to the constraint satisfaction problem**, American Institute of Mathematics, Palo Alto, California.
Description: This workshop, sponsored by AIM and the NSF, will be devoted to advancing the understanding of the computational complexity of the constraint satisfaction problem using methods and techniques from universal algebra and logic. The intended participants are researchers in computational complexity, universal algebra, and logic. By bringing together researchers from these three areas, progress could be made towards resolving long-standing open problems about the complexity of constraint satisfaction.
Organizers: Anuj Dawar, Phokion Kolaitis, Benoit Larose, and Matt Valeriote.
Deadline: November 15, 2007.
Information: See http://aimath.org/ARCC/workshops/constraintsatis.html.

April 2008

* 4–6 **AMS Central Section Meeting**, Indiana University, Bloomington, Indiana.
Information: http://www.ams.org/amsmtgs/sectional.html.

17–18 **IMA Workshop: Network Dynamics and Cell Physiology**, University of Minnesota, Minneapolis, Minnesota. (Dec. 2006, p. 1381)

21–25 **IMA Workshop: Design Principles in Biological Systems**, University of Minnesota, Minneapolis, Minnesota. (Dec. 2006, p. 1381)

May 2008

* 3–4 **AMS Western Section Meeting**, Claremont McKenna College, Claremont, California.
Information: http://www.ams.org/amsmtgs/sectional.html.

* 11–16 **CHT-08: Advances in Computational Heat Transfer**, Kenzi Farah Hotel, Marrakech, Morocco.
Goal: Of the symposium is to provide a forum for the exposure and exchange of ideas, methods and results in computational heat transfer. Papers on all aspects of computational heat transfer - both fundamental and applied - will be welcome.
Organizers: International Centre for Heat and Mass Transfer, the CFD Research Laboratory of the University of NSW, Australia and the LEEVAM Research Laboratory of the University of Cergy-Pontoise, France with the participation of the Heat Transfer Division of ASME International and the assistance of the University of Cadi Ayyad, Marrakech and the University of Limoges, France .
Co-chairs: Graham de Vahl Davis and Eddie Leonardi, CFD Research Laboratory, School of Mech. & Manuf. Engineering, The University of NSW, Sydney, NSW, Australia 2052.
Deadlines: One page abstract: October 1, 2007. Full text of paper: December 15, 2007.
Information: email: cht08@cfd.mech.unsw.edu.au; http://cht08.mech.unsw.edu.au/.

* 12–16 **Ferroelectric Phenomena in Soft Matter Systems**, American Institute of Mathematics, Palo Alto, California.
Organizers: Maria-Carme Calderer and Jie Shen.
Description: This workshop, sponsored by AIM and the NSF, will be devoted to the modeling, simulation and analysis of the ferroelectric phenomena in soft matter systems such as liquid crystals, elastomers and gels. The ferroelectric phenomena of these materials, such as its interaction with electric fields in switching processes, introduce new challenging problems that can only be properly addressed with a combined effort of theoretical and experimental physicists, and analytical and computational mathematicians.
Deadline: January 10, 2008.
Details: http://aimath.org/ARCC/workshops/ferroelectric.html..

* 15–17 **Twelfth International Conference Devoted to the Memory of Academician Mykhailo Kravchuk (Krawtchouk) (1892–1942)**, Kyiv, Ukraine.
Topics: (1) Differential and integral equations, its applications; (2) Algebra, geometry, mathematical and numerical analysis; (3) Theory of probability and mathematical statistics; (4) History, methods of teaching of mathematics.

Organizers: National Tech. Univ. of Ukraine (KPI), Inst. of Math. of NASU, National Shevchenko Univ., National Drahomanov Pedagogical Univ.
Deadlines: Abstracts of one page by March 1, 2008.
Languages: English, Ukraine, Russian.
Information: Ukraine, 03056, Kyiv-56, Peremohy Ave. 37, National Technical University of Ukraine (KPI), Phys.-Math. Departments, Corpus 7, Room 437, M. Kravchuk Conference, N. Virchenko; tel. (380) 44 454-97-40; email: kravchukconf.@yandex.ru.

18–21 **The 7th AIMS International Conference on Dynamical Systems, Differential Equations and Applications**, University of Texas at Arlington, Arlington, Texas. (May 2007, p. 666)

26–30 **IMA Workshop: Quantitative Approaches to Cell Motility and Chemotaxis**, University of Minnesota, Minneapolis, Minnesota. (Dec. 2006, p. 1381)

The following new announcements will not be repeated until the criteria in the next to the last paragraph at the bottom of the first page of this section are met.

June 2008

* 4–7 **First Joint International Meeting with the Sociedade Brasileira de Matematica**, Instituto Nacional de Matematica Pura e Aplicada (IMPA), Rio de Janeiro, Brazil.
Information: http://www.ams.org/amsmtgs/internmtgs.html.

* 9–13 **12th International Conference on Hyperbolic Problems: Theory, Numerics, Applications**, University of Maryland, College Park, Maryland.
Important dates: Full consideration will be given to abstracts which are submitted before February 29, 2008. Notification of acceptance of papers: March 28, 2008. Registration fee at a reduced rate: April 11, 2008.
Information: http://hyp2008.umd.edu.

* 9–19 **Advances in Set-Theoretic Topology: Conference in Honour of Tsugunori Nogura on his 60th Birthday**, Centre for Scientific Culture "Ettore Majorana", Erice, Sicily, Italy.
Organizers: Szymon Dolecki, Yasunao Hattori, Dmitri Shakhmatov, Gino Tironi.
Topics: Convergence properties and convergence structures; Dimension theory and related fields; General topology and its applications in other areas of mathematics; Hyperspaces, set-valued mapping and their selections; Set theoretic methods in mathematics; Set theory; Topological algebra (topological groups, functions spaces, etc.).
Information: http://www.math.sci.ehime-u.ac.jp/erice/; email: erice@dmitri.math.sci.ehime-u.ac.jp.

* 17–20 **4th Croatian Mathematical Congress**, Department of Mathematics, University of Osijek, Osijek, Croatia.
Description: The Congress will have an international component, and it is open to all areas of mathematics.
Program: Includes Plenary Lectures, Croatian Mathematical Society award lecture, parallel sessions and posters. The parallel sessions include invited lectures and contributed talks selected by the Scientific Committee.
Organizers: Department of Mathematics, University of Osijek, Trg Ljudevita Gaja 6, HR-31 000 Osijek, Croatia. Croatian Mathematical Society, Bijenicka 30, HR-10 000 Zagreb, Croatia . Osijek Mathematical Society, Trg Ljudevita Gaja 6, HR-31 000 Osijek, Croatia .
Deadlines: March 15, 2008: For individual abstract submission. May 1, 2008: For notification of acceptance. May 15, 2008: For registration fee payment.
Information: Up-to-date information about the Congress will be available at the website http://www.mathos.hr/congress2008.

* 23–27 **Hermitian Symmetric Spaces, Jordan Algebras and Related Problems**, CIRMLuminy, Marseille, France.
Information: This international conference is in honor of Prof. Jean-Louis Clerc; see http://www.cirm.univ-mrs.fr/liste_rencontre/Rencontres2008/Koufany08/Koufany08.html or http://hssja08.iecn.u-nancy.fr/.

* 25–27 **ICNPAA 2008: Mathematical Problems in Engineering, Aerospace and Sciences [Theory, Methods (includes Experimental, Computational) and Applications]**, University of Genoa, Italy.
Sponsors: IFNA, IFIP, AIAA University of Genoa, Italy
Deadlines: Organizing Special Session (To send the title of the session, name of the organizers): November 30, 2007. Final deadline for abstracts of the talks: February 15, 2008.
Scope: Includes mathematical problems in all areas of Engineering, Aerospace and sciences.
Organizers: Seenith Sivasundaram, USA; Marcello Sanguineti, Italy.
Information: Contact: ICNPAA 2008, 104 Snow Goose Court, Daytona Beach, Florida 32119; email: Seenithi@aol.com, seenithi@gmail.com; http://www.icnpaa.com.

* 29–July 4 **IWASAWA 2008**, Kloster Irsee, Germany.
Information: Conference website: http://www.irsee2008.de.

October 2008

* 4–5 **AMS Western Section Meeting**, University of British Columbia and the Pacific Institute of Mathematical Sciences, Vancouver, Canada.
Information: http://www.ams.org/amsmtgs/sectional.html.

* 11–12 **AMS Eastern Section Meeting**, Wesleyan University, Middletown, Connecticut.
Information: http://www.ams.org/amsmtgs/sectional.html.

* 17–19 **AMS Central Section Meeting**, Western Michigan University, Kalamazoo, Michigan.
Information: http://www.ams.org/amsmtgs/sectional.html.

* 24–26 **AMS Southeastern Section Meeting**, University of Alabama, Huntsville, Alambama.
Information: http://www.ams.org/amsmtgs/sectional.html.

December 2008

* 17–21 **First Joint International Meeting with the Shanghai Mathematical Society**, Shanghai, China.
Information: http://www.ams.org/amsmtgs/internmtgs.html.

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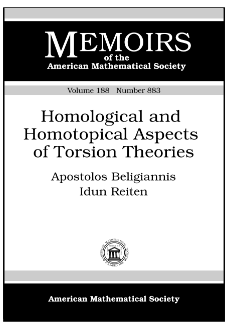
Algebra and Algebraic Geometry



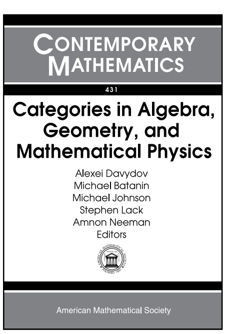
Homological and Homotopical Aspects of Torsion Theories

Apostolos Beligiannis, *University of Ioannina, Greece*, and **Idun Reiten**, *Norwegian University of Science and Technology, Trondheim, Norway*

Contents: Introduction; Torsion pairs in abelian and triangulated categories; Torsion pairs in pretriangulated categories; Compactly generated torsion pairs in triangulated categories; Hereditary torsion pairs in triangulated categories; Torsion pairs in stable categories; Triangulated torsion(-free) classes in stable categories; Gorenstein categories and (co)torsion pairs; Torsion pairs and closed model structures; (Co)torsion pairs and generalized Tate-Vogel cohomology; Nakayama categories and Cohen-Macaulay cohomology; Bibliography; Index.

Memoirs of the American Mathematical Society, Volume 188, Number 883

July 2007, 207 pages, Softcover, ISBN: 978-0-8218-3996-6, 2000 *Mathematics Subject Classification:* 18E40, 18E30, 18E35, 18G55, 18G60; 16G10, 18E30, 18E10, 20C05, 20J05, 55U35, **Individual member US$43**, List US$72, Institutional member US$58, Order code MEMO/188/883



Categories in Algebra, Geometry, and Mathematical Physics

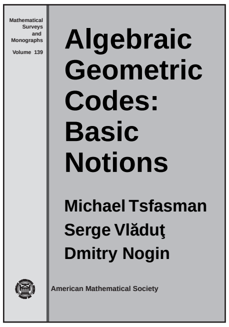
Alexei Davydov, **Michael Batanin**, and **Michael Johnson**, *Macquarie University, Sydney, Australia*, **Stephen Lack**, *University of Western Sydney, Penrith South, Australia*, and **Amnon Neeman**, *Australian National University, Canberra, Australia*, Editors

Category theory has become the universal language of modern mathematics. This book is a collection of articles applying methods of category theory to the areas of algebra, geometry, and mathematical physics. Among others, this book contains articles on higher categories and their applications and on homotopy theoretic methods. The reader can learn about the exciting new interactions of category theory with very traditional mathematical disciplines.

Contents: **G. M. Kelly**, The beginnings of category theory in Australia; **J. C. Baez** and **U. Schreiber**, Higher gauge theory; **C. Berger** and **I. Moerdijk**, Resolution of coloured operads and rectification of homotopy algebras; **J. Bergner**, Simplicial monoids and Segal categories; **F. Borceux** and **D. Bourn**, Split extension classifier and centrality; **D. Bourn**, Moore normalization and Dold-Kan theorem for semi-abelian categories; **M. Bunge** and **J. Funk**, An intrinsic characterization of branched coverings; **E. Cheng** and **N. Gurski**, The periodic table of n-categories for low dimensions I: Degenerate categories and degenerate bicategories; **B. Chorny**, Abstract cellularization as a cellularization with respect to a set of objects; **D.-C. Cisinski**, Batanin higher groupoids and homotopy types; **B. Day** and **R. Street**, Centres of monoidal categories of functors; **J. Fuchs**, **I. Runkel**, and **C. Schweigert**, Ribbon categories and (unoriented) CFT: Frobenius algebras, automorphisms, reversions; **I. Runkel**, **J. Fjelstad**, **J. Fuchs**, and **C. Schweigert**, Topological and conformal field theory as Frobenis algebras; **G. Janelidze** and **W. Tholen**, Characterization of torsion theories in general categories; **A. Joyal** and **J. Kock**, Weak units and homotopy 3-types; **A. Joyal** and **M. Tierney**, Quasi-categories vs Segal spaces; **J.-L. Loday**, Parking functions and triangulation of the associahedron; **G. Maltsiniotis**, La K-théorie d'un dérivateur triangulé; **B. Keller**, Appendice: Le dérivateur triangulé associé à une catégorie exacte; **T. Porter**, Formal homotopy quantum field theories, II: Simplicial formal maps; **J. Power**, Three dimensional monad theory; **R. Steiner**, Orientals; **D. Verity**, Weak complicial sets II Nerves of complcial gray-categories.

Contemporary Mathematics, Volume 431

July 2007, 467 pages, Softcover, ISBN: 978-0-8218-3970-6, LC 2007060669, 2000 *Mathematics Subject Classification:* 18-XX, 55Uxx, 81Txx, 19Dxx, **AMS members US$103**, List US$129, Order code CONM/431



Algebraic Geometric Codes: Basic Notions

Michael Tsfasman, *French-Russian Poncelet Laboratory (CNRS and Ind. Univ. Moscow), Russia, and Institute for Information Transmission Problems, Moscow, Russia*, **Serge Vlăduţ**, *Institut de Mathématiques de Luminy, France, and Institute for Information Transmission Problems, Moscow, Russia*, and **Dmitry Nogin**, *Institute for Information Transmission Problems, Moscow, Russia*

The book is devoted to the theory of algebraic geometric codes, a subject formed on the border of several domains of mathematics. On one side there are such classical areas as algebraic geometry and number theory; on the other, information transmission theory, combinatorics, finite geometries, dense packings, etc.

The authors give a unique perspective on the subject. Whereas most books on coding theory build up coding theory from within, starting from elementary concepts and almost always finishing without reaching a certain depth, this book constantly looks for interpretations that connect coding theory to algebraic geometry and number theory.

There are no prerequisites other than a standard algebra graduate course. The first two chapters of the book can serve as an introduction to coding theory and algebraic geometry respectively. Special attention is given to the geometry of curves over finite fields in the third chapter. Finally, in the last chapter the authors explain relations between all of these: the theory of algebraic geometric codes.

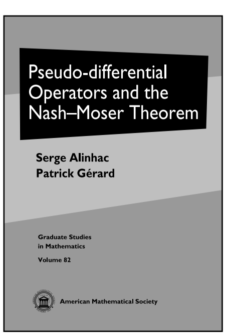
This item will also be of interest to those working in applications.

Contents: Codes; Curves; Curves over finite fields; Algebraic geometry codes; Summary of results and tables; Bibliography; List of names; Index.

Mathematical Surveys and Monographs, Volume 139

July 2007, 338 pages, Hardcover, ISBN: 978-0-8218-4306-2, LC 2007061731, 2000 *Mathematics Subject Classification:* 14Hxx, 94Bxx, 14G15, 11R58; 11T23, 11T71, **AMS members US$71**, List US$89, Order code SURV/139

Analysis



Pseudo-differential Operators and the Nash–Moser Theorem

Serge Alinhac and **Patrick Gérard**, *Université Paris-Sud, Orsay, France*
Translated by Stephen S. Wilson

This book presents two essential and apparently unrelated subjects. The first, microlocal analysis and the theory of pseudo-differential operators, is a basic tool in the study of partial differential equations and in analysis on manifolds. The second, the Nash–Moser theorem, continues to be fundamentally important in geometry, dynamical systems, and nonlinear PDE.

Each of the subjects, which are of interest in their own right as well as for applications, can be learned separately. But the book shows the deep connections between the two themes, particularly in the middle part, which is devoted to Littlewood–Paley theory, dyadic analysis, and the paradifferential calculus and its application to interpolation inequalities.

An important feature is the elementary and self-contained character of the text, to which many exercises and an introductory Chapter 0 with basic material have been added. This makes the book readable by graduate students or researchers from one subject who are interested in becoming familiar with the other. It can also be used as a textbook for a graduate course on nonlinear PDE or geometry.

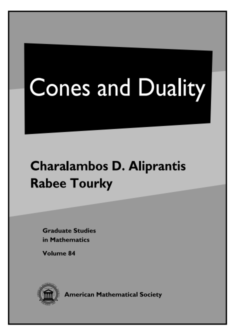
This item will also be of interest to those working in geometry and topology.

Contents: General introduction; Notation and review of distribution theory; Pseudo-differential operators; Nonlinear dyadic analysis, microlocal analysis, energy estimates; Implicit function theorems; Bibliography; Main notation introduced; Index.

Graduate Studies in Mathematics, Volume 82

May 2007, 168 pages, Hardcover, ISBN: 978-0-8218-3454-1, LC 2006047985, 2000 *Mathematics Subject Classification:* 35-02; 35Sxx, 47G30, 47N20, **AMS members US$31**, List US$39, Order code GSM/82

COURSE ADOPTION



Cones and Duality

Charalambos D. Aliprantis, *Purdue University, West Lafayette, IN*, and **Rabee Tourky**, *The University of Queensland, Brisbane, Queensland, Australia*

Ordered vector spaces and cones made their debut in mathematics at the beginning of the twentieth century. They were developed in parallel (but from a different perspective) with functional analysis and operator theory. Before the 1950s, ordered vector spaces appeared in the literature in a fragmented way. Their systematic study began around the world after 1950 mainly through the efforts of the Russian, Japanese, German, and Dutch schools.

Since cones are being employed to solve optimization problems, the theory of ordered vector spaces is an indispensable tool for solving a variety of applied problems appearing in several diverse areas, such as engineering, econometrics, and the social sciences. For this reason this theory plays a prominent role not only in functional analysis but also in a wide range of applications.

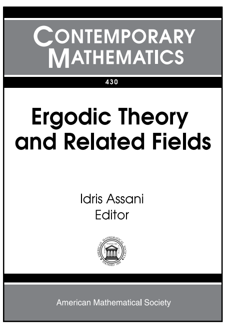
This is a book about a modern perspective on cones and ordered vector spaces. It includes material that has not been presented earlier in a monograph or a textbook. With many exercises of varying degrees of difficulty, the book is suitable for graduate courses.

Most of the new topics currently discussed in the book have their origins in problems from economics and finance. Therefore, the book will be valuable to any researcher and graduate student who works in mathematics, engineering, economics, finance, and any other field that uses optimization techniques.

Contents: Cones; Cones in topological vector spaces; Yudin and pull-back cones; Krein operators; $\mathcal{K}$-lattices; The order extension of L'; Piecewise affine functions; Appendix: Linear topologies; Bibliography; Index.

Graduate Studies in Mathematics, Volume 84

July 2007, approximately 288 pages, Hardcover, ISBN: 978-0-8218-4146-4, LC 2007060758, 2000 *Mathematics Subject Classification:* 46A40, 46B40, 47B60, 47B65; 06F30, 28A33, 91B28, 91B99, **AMS members US$44**, List US$55, Order code GSM/84



Ergodic Theory and Related Fields

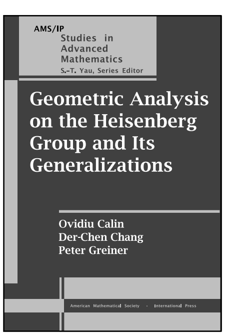
Idris Assani, *University of North Carolina, Chapel Hill, NC*, Editor

The book contains papers by participants of the Chapel Hill Ergodic Theory Workshops organized in February 2004, 2005, and 2006. Topics covered by these papers illustrate the interaction between ergodic theory and related fields such as harmonic analysis, number theory, and probability theory.

Contents: I. Assani, Averages along cubes for not necessarily commuting m.p.t.; **I. Assani** and **M. Lin**, On the one-sided ergodic Hilbert transform; **Z. Buczolich** and **R. D. Mauldin**, Concepts behind divergent ergodic averages along the squares; **L. A. Bunimovich** and **A. Yurchenko**, Deterministic walk in Markov environments with constant rigidity; **G. Cohen**, On random Fourier-Stieltjes transforms; **J.-P. Conze** and **A. Raugi**, Limit theorems for sequential expanding dynamical systems on [0,1]; **M. K. Roychowdhury**, m_n-odometer and the binary odometer are finitarily orbit equivalent; **I. Assani**, Some open problems.

Contemporary Mathematics, Volume 430

May 2007, 145 pages, Softcover, ISBN: 978-0-8218-3869-3, LC 2007060037, 2000 *Mathematics Subject Classification:* 28D05, 37A05, 37A20, 37A50, 47A35, 47A16, 60F15, 82C20, 60G50, **AMS members US$39**, List US$49, Order code CONM/430



Geometric Analysis on the Heisenberg Group and Its Generalizations

Ovidiu Calin, *Eastern Michigan University, Ypsilanti, MI*, **Der-Chen Chang**, *Georgetown University, Washington, DC*, and **Peter Greiner**, *University of Toronto, ON, Canada*

The theory of subRiemannian manifolds is closely related to Hamiltonian mechanics. In this book, the authors examine the properties and applications of subRiemannian manifolds that automatically satisfy the Heisenberg principle, which may be useful in quantum mechanics. In particular, the behavior of geodesics in this setting plays an important role in finding heat kernels and propagators for Schrödinger's equation. One of the novelties of this book is the introduction of techniques from complex Hamiltonian mechanics.

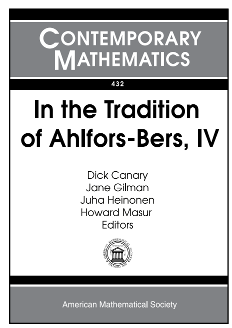
This item will also be of interest to those working in geometry and topology.

Titles in this series are co-published with International Press, Cambridge, MA.

Contents: Geometric mechanics on the Heisenberg group; Geometric analysis of step 4 case; The geometric analysis of step $2(k+1)$ case; Geometry on higher dimensional Heisenberg groups; Complex Hamiltonian mechanics; Quantum mechanics on the Heisenberg group; Bibliography; Index.

AMS/IP Studies in Advanced Mathematics, Volume 40

June 2007, 244 pages, Hardcover, ISBN: 978-0-8218-4319-2, LC 2007060760, 2000 *Mathematics Subject Classification:* 53C17, 53C22, 35H20; 46E25, 20C20, **AMS members US$47**, List US$59, Order code AMSIP/40



In the Tradition of Ahlfors-Bers, IV

Dick Canary, *University of Michigan, Ann Arbor, MI*, **Jane Gilman**, *Yale University, New Haven, CT*, **Juha Heinonen**, *University of Michigan, Ann Arbor, MI*, and **Howard Masur**, *University of Illinois at Chicago, IL*, Editors

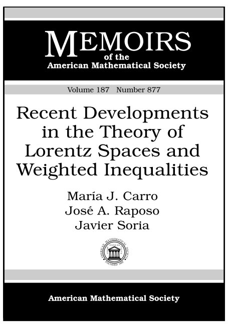
The Ahlfors–Bers Colloquia commemorate the mathematical legacy of Lars Ahlfors and Lipman Bers. The core of this legacy lies in the fields of geometric function theory, Teichmüller theory, hyperbolic manifolds, and partial differential equations. However, the work of Ahlfors and Bers has impacted and created interactions with many other fields, such as algebraic geometry, mathematical physics, dynamics, geometric group theory, number theory, and topology. The triannual Ahlford–Bers Colloquia serve as a venue to disseminate the relevant work to the wider mathematical community and bring the key participants together to ponder future directions in the field.

The present volume includes a wide range of articles in the fields central to this legacy. The majority of articles present new results, but there are expository articles as well.

Contents: J. W. Anderson, **J. Aramoyona**, and **K. J. Shackleton**, Uniformly exponential growth and mapping class groups of surfaces; **C. J. Bishop**, An A_1 weight not comparable with any quasiconformal Jacobian; **M. Duchin**, Curvature, stretchiness, and dynamics; **C. J. Earle**, Some special loci in the Siegel space of genus two; **E. Fujikawa**, Another approach to the automorphism theorem for Teichmüller spaces; **W. M. Goldman** and **R. A. Wentworth**, Energy of twisted harmonic maps of Riemann surfaces; **P. Hästö**, **Z. Ibragimov**, **D. Minda**, **S. Ponnusamy**, and **S. Sahoo**, Isometries of some hyperbolic-type path metrics, and the hyperbolic medial axis; **J. Hu**, From left earthquakes to right; **C. M. Judge**, Small eigenvalues and maximal laminations on complete surfaces of negative curvature; **L. Keen** and **N. Lakic**, A generalized hyperbolic metric for plane domains; **R. P. Kent, IV** and **C. J. Leininger**, Subgroups of mapping class groups from the geometrical viewpoint; **Y.-H. Kim**, Determinants of Laplacians, quasifuchsian spaces, and holomorphic extensions of Laplacians; **L. V. Kovalev** and **J. T. Tyson**, Hyperbolic and quasisymmetric structure of hyperspace; **K. Matsuzaki**, A classification of the modular transformations of infinite dimensional Teichmüller spaces; **M. Mirzakhani**, Random hyperbolic surfaces and measured laminations; **S. Mitra**, Extensions of holomorphic motions to quasiconformal motions; **R. Schul**, Analyst's traveling salesman theorems. A survey; **R. A. Wentworth**, Energy of harmonic maps and Gardiner's formula.

Contemporary Mathematics, Volume 432

July 2007, 229 pages, Softcover, ISBN: 978-0-8218-4227-0, 2000 *Mathematics Subject Classification:* 14H15, 20H10, 28A75, 30F40, 30C62, 32G15, 54E40, 57M50, 54F99, **AMS members US$63**, List US$79, Order code CONM/432



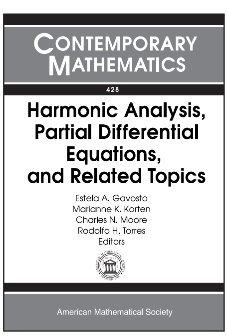
Recent Developments in the Theory of Lorentz Spaces and Weighted Inequalities

María J. Carro, *University of Barcelona, Spain*, **José A. Raposo**, and **Javier Soria**, *University of Barcelona, Spain*

Contents: Introduction; Boundedness of operators on characteristic functions and the Hardy operator; Lorentz spaces; The Hardy-Littlewood maximal operator in weighted Lorentz spaces; Bibliography; Index.

Memoirs of the American Mathematical Society, Volume 187, Number 877

March 2007, 128 pages, Softcover, ISBN: 978-0-8218-4237-9, LC 2007060666, 2000 *Mathematics Subject Classification:* 42B25; 26D10, 46E30, 47B38, 47G10, **Individual member US$37**, List US$62, Institutional member US$50, Order code MEMO/187/877



Harmonic Analysis, Partial Differential Equations, and Related Topics

Estela A. Gavosto, *University of Kansas, Lawrence, KS*, **Marianne K. Korten** and **Charles N. Moore**, *Kansas State University, Manhattan, KS*, and **Rodolfo H. Torres**, *University of Kansas, Lawrence, KS*, Editors

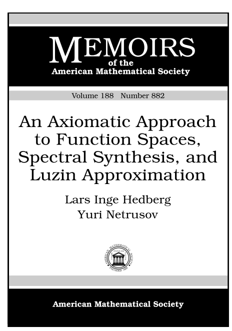
This collection of contributed articles comprises the scientific program of the fifth annual Prairie Analysis Seminar. All articles represent important current advances in the areas of partial differential equations, harmonic analysis, and Fourier analysis. A range of interrelated topics is presented, with articles concerning Painlevé removability, pseudodifferential operators, A_p weights, nonlinear Schrödinger equations, singular integrals, the wave equation, the Benjamin–Ono equation, quasi-geostrophic equations, quasiconformal mappings, integral inclusions, Bellman function methods, weighted gradient estimates, Hankel operators, and dynamic optimization problems.

Most importantly, the articles illustrate the fruitful interaction between harmonic analysis, Fourier analysis, and partial differential equations, and illustrate the successful application of techniques and ideas from each of these areas to the others.

Contents: **K. Astala**, **A. Clop**, **J. Mateu**, **J. Orobitg**, and **I. Uriarte-Tuero**, Improved Painlevé removability for bounded planar quasiregular mappings; **A. Bényi** and **K. A. Okoudjou**, Time-frequency estimates for pseudodifferential operators; **R. Berndt**, Extrapolation of operators defined on domains and boundary respecting A_p weights; **I. A. Blank**, **M. K. Korten**, and **C. N. Moore**, Existence, uniqueness and regularity of the free boundary in the Hele-Shaw problem with a degenerate phrase; **J. Colliander**, On blowup solutions of NLS with low regularity initial data; **L. Grafakos**, **P. Honzík**, and **D. Ryabogin**, Are L^2-bounded homogeneous singular integrals necessarily L^p-bounded?; **A. Grinshpan**, A note on Chebyshev polynomials and finite difference wave equation; **A. D. Ionescu** and **C. E. Kenig**, Complex-valued solutions of the Benjamin–Ono equation; **N. Ju**, The 2D quasi-geostrophic equations in the Sobolev space; **L. V. Kovalev** and **D. Maldonado**, Convex functions and quasiconformal mappings; **P. S. Macansantos**, Volterra integral inclusions: Existence of solutions; **L. Slavin** and **A. Volberg**, Bellman function and the H^1–BMO duality; **C. Sweezy**, Weights and Hölder norms for solutions to a second order elliptic Dirichlet problem on nonsmooth domains; **E. Terwilleger**, Hankel operators and VMO on the bi-disc; **L. Wang**, Necessary conditions for optimization problems governed by differential algebraic inclusions.

Contemporary Mathematics, Volume 428

May 2007, 173 pages, Softcover, ISBN: 978-0-8218-4093-1, LC 2006053098, 2000 *Mathematics Subject Classification:* 30-06, 33-06, 35-06, 42-06, 45-06, 46-06, 47-06, 49-06, 76-06, **AMS members US$47**, List US$59, Order code CONM/428



An Axiomatic Approach to Function Spaces, Spectral Synthesis, and Luzin Approximation

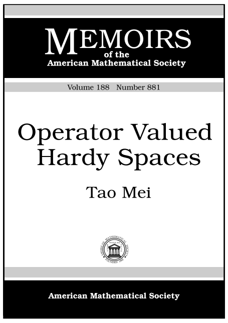
Lars Inge Hedberg, *Linköping University, Sweden*, and **Yuri Netrusov**, *University of Bristol, UK*

Contents: Introduction. Notation; A class of function spaces; Differentiability and spectral synthesis; Luzin type theorems; Appendix. Whitney's approximation theorem in $L_p(\mathbf{R}^N)$, $p > 0$; Bibliography.

Memoirs of the American Mathematical Society, Volume 188, Number 882

July 2007, 97 pages, Softcover, ISBN: 978-0-8218-3983-6, 2000 *Mathematics Subject Classification:* 46E35; 26B35, 31B15, 31C15,

31C45, 41A17, **Individual member US$36**, List US$60, Institutional member US$48, Order code MEMO/188/882



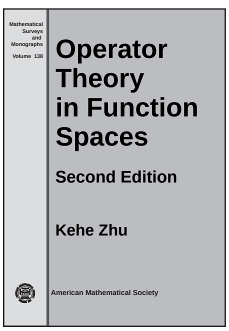
Operator Valued Hardy Spaces

Tao Mei, *Texas A&M University, College Station, TX*

Contents: Introduction; Preliminaries; The Duality between $\mathcal{H}^1$ and **BMO**; The maximal inequality; The duality between $\mathcal{H}^p$ and BMO^q, $1 < p < 2$; Reduction of BMO to dyadic BMO; Interpolation; Bibliography.

Memoirs of the American Mathematical Society, Volume 188, Number 881

July 2007, 64 pages, Softcover, ISBN: 978-0-8218-3980-5, LC 2007060755, 2000 *Mathematics Subject Classification:* 46L52, 32C05, **Individual member US$31**, List US$52, Institutional member US$42, Order code MEMO/188/881



Operator Theory in Function Spaces

Second Edition

Kehe Zhu, *State University of New York at Albany, NY*

This book covers Toeplitz operators, Hankel operators, and composition operators on both the Bergman space and the Hardy space. The setting is the unit disk and the main emphasis is on size estimates of these operators: boundedness, compactness, and membership in the Schatten classes.

Most results concern the relationship between operator-theoretic properties of these operators and function-theoretic properties of the inducing symbols. Thus a good portion of the book is devoted to the study of analytic function spaces such as the Bloch space, Besov spaces, and BMOA, whose elements are to be used as symbols to induce the operators we study.

The book is intended for both research mathematicians and graduate students in complex analysis and operator theory. The prerequisites are minimal; a graduate course in each of real analysis, complex analysis, and functional analysis should sufficiently prepare the reader for the book. Exercises and bibliographical notes are provided at the end of each chapter. These notes will point the reader to additional results and problems.

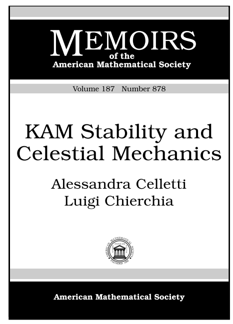
Kehe Zhu is a professor of mathematics at the State University of New York at Albany. His previous books include *Theory of Bergman Spaces* (Springer, 2000, with H. Hedenmalm and B. Korenblum) and *Spaces of Holomorphic Functions in the Unit Ball* (Springer, 2005). His current research interests are holomorphic function spaces and operators acting on them.

Contents: Bounded linear operators; Interpolation of Banach spaces; Integral operators on L^p spaces; Bergman spaces; Bloch and Besov spaces; The Berezin transform; Toeplitz operators on the Bergman space; Hankel operators on the Bergman space; Hardy spaces and BMO; Hankel operators on the Hardy space; Composition operators; Bibliography; Index.

Mathematical Surveys and Monographs, Volume 138

June 2007, 348 pages, Hardcover, ISBN: 978-0-8218-3965-2, LC 2007060704, 2000 *Mathematics Subject Classification:* 47-02, 30-02, 46-02, 32-02, **AMS members US$71**, List US$89, Order code SURV/138

Differential Equations



KAM Stability and Celestial Mechanics

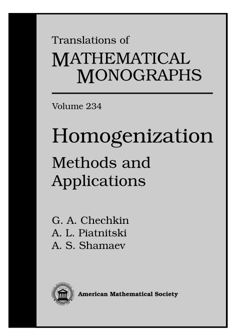
Alessandra Celletti, *Universitá di Roma Tor Vergata, Rome, Italy*, and **Luigi Chierchia**, *Universitá "Roma Tre", Rome, Italy*

Contents: Introduction; Iso-energetic KAM theory; The restricted, circular, planar three-body problem; KAM stability of the Sun-Jupiter-Victoria problem; Appendix A. The ellipse; Appendix B. Diophantine estimates; Appendix C. Interval arithmetic; Appendix D. A guide to the computer programs; Bibliography.

Memoirs of the American Mathematical Society, Volume 187, Number 878

March 2007, 134 pages, Softcover, ISBN: 978-0-8218-4169-3, LC 2007060667, 2000 *Mathematics Subject Classification:* 70F07, 70-04; 70H08, 37J40, **Individual member US$38**, List US$64, Institutional member US$51, Order code MEMO/187/878

COURSE ADOPTION



Homogenization

Methods and Applications

G. A. Chechkin, *Moscow State University, Russia, and Narvik University College, Norway*, **A. L. Piatnitski**, *Lebedev Physical Institute, Moscow, Russia, and Narvik University College, Norway*, and **A. S. Shamaev**, *Institute for Problems in Mechanics, Moscow, Russia, and Moscow State University, Russia*

Homogenization is a collection of powerful techniques in partial differential equations that are used to study differential operators with rapidly oscillating coefficients, boundary value problems with rapidly varying boundary conditions, equations in perforated domains, equations with random coefficients, and other objects of theoretical and practical interest.

The book focuses on various aspects of homogenization theory and related topics. It comprises classical results and methods of homogenization theory, as well as modern subjects and techniques developed in the last decade. Special attention is paid to averaging of random parabolic equations with lower order terms, to homogenization of singular structures and measures, and to problems with rapidly alternating boundary conditions.

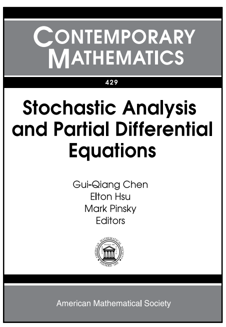
The book contains many exercises, which help the reader to better understand the material presented. All the main results are illustrated with a large number of examples, ranging from very simple to rather advanced.

This item will also be of interest to those working in applications.

Contents: Related topics; Homogenization methods; Applications of homogenization methods; Bibliography; Index.

Translations of Mathematical Monographs, Volume 234

September 2007, approximately 256 pages, Hardcover, ISBN: 978-0-8218-3873-0, LC 2007060746, 2000 *Mathematics Subject Classification:* 35Bxx, **AMS members US$71**, List US$89, Order code MMONO/234



Stochastic Analysis and Partial Differential Equations

Gui-Qiang Chen, **Elton Hsu**, and **Mark Pinsky**, *Northwestern University, Evanston, IL*, Editors

This book is a collection of original research papers and expository articles from the scientific program of the 2004–05 Emphasis Year on Stochastic Analysis and Partial Differential Equations at Northwestern University. Many well-known mathematicians attended the events and submitted their contributions for this volume.

Topics from stochastic analysis discussed in this volume include stochastic analysis of turbulence, Markov processes, microscopic lattice dynamics, microscopic interacting particle systems, and stochastic analysis on manifolds. Topics from partial differential equations include kinetic equations, hyperbolic conservation laws, Navier–Stokes equations, and Hamilton–Jacobi equations. A variety of methods, such as numerical analysis, homogenization, measure-theoretical analysis, entropy analysis, weak convergence analysis, Fourier analysis, and Itô's calculus, are further developed and applied. All these topics are naturally interrelated and represent a cross-section of the most significant recent advances and current trends and directions in stochastic analysis and partial differential equations.

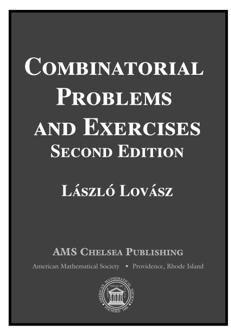
This volume is suitable for researchers and graduate students interested in stochastic analysis, partial differential equations, and related analysis and applications.

Contents: **A. Biryuk**, **W. Craig**, and **S. Ibrahim**, Construction of suitable weak solutions of the Navier-Stokes equations; **I. H. Biswas**, **E. R. Jakobsen**, and **K. H. Karlsen**, Error estimates for finite difference-quadrature schemes for a class of nonlocal Bellman equations with variable diffusion; **W. Bo**, **B. Cheng**, **J. Du**, **B. Fix**, **E. George**, **J. Glimm**, **J. W. Grove**, **X. Jia**, **H. Jin**, **H. Lee**, **Y. Li**, **X. Li**, **X. Liu**, **D. H. Sharp**, **L. Wu**, and **Y. Yu**, Recent progress in the stochastic analysis of turbulent mixing; **V. Calvez**, **B. Perthame**, and **M. S. tabar**, Modified Keller-Segel system and critical mass for the log interaction kernel; **G.-Q. Chen**, **N. Even**, and **C. Klingenberg**, Entropy solutions to conservation laws with discontinuous fluxes via microscopic interacting particle systems; **Z.-Q. Chen** and **R. Song**, Spectral properties of subordinate processes in domains; **P. Constantin**, Smoluchowski Navier-Stokes systems; **S. Fang**, Recent developments in stochastic differential equations; **E. P. Hsu**, Heat equations on manifolds and Bismut's formula; **N. Ikeda** and **Y. Ogura**, On a class of one-dimensional Markov processes with continuous paths; **M. A. Katsoulakis**, **A. J. Majda**, and **A. Sopasakis**, Prototype hybrid couplings of macroscopic deterministic models and microscopic stochastic lattice dynamics; **E. Kosygina**, Homogenization of stochastic Hamilton-Jacobi equations: Brief review of methods and applications; **P. Michel**, General relative entropy in a nonlinear McKendrick model; **M. A. Pinsky**, Pointwise Fourier inversion in analysis and geometry; **S. Taniguchi**, Stochastic analysis and the KdV equation; **K. Trivisa**, On binary fluid mixtures.

Contemporary Mathematics, Volume 429

May 2007, 278 pages, Softcover, ISBN: 978-0-8218-4059-7, 2000 *Mathematics Subject Classification:* 35-06, 60-06, 82-06, 60H15, 76F55, 76M35, 58J65, 65C20, 35F20, 35Q30, **AMS members US$63**, List US$79, Order code CONM/429

Discrete Mathematics and Combinatorics



Combinatorial Problems and Exercises

Second Edition

László Lovász, *Eötvös Loránd University, Budapest, Hungary*

The main purpose of this book is to provide help in learning existing techniques in combinatorics. The most effective way of learning such techniques is to solve exercises and problems. This book presents all the material in the form of problems and series of problems (apart from some general comments at the beginning of each chapter). In the second part, a hint is given for each exercise, which contains the main idea necessary for the solution, but allows the reader to practice the techniques by completing the proof. In the third part, a full solution is provided for each problem.

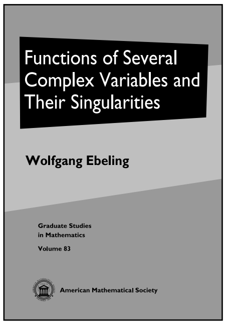
This book will be useful to those students who intend to start research in graph theory, combinatorics or their applications, and for those researchers who feel that combinatorial techniques might help them with their work in other branches of mathematics, computer science, management science, electrical engineering and so on. For background, only the elements of linear algebra, group theory, probability and calculus are needed.

Contents: Problems; Hints; Solutions; Dictionary of the combinatorial phrases and concepts used; Notation; Index of the abbreviations of textbooks and monographs; Subject index; Author index; Errata.

AMS Chelsea Publishing

July 2007, 639 pages, Hardcover, ISBN: 978-0-8218-4262-1, LC 2007060765, 2000 *Mathematics Subject Classification:* 05-01, **AMS members US$62**, List US$69, Order code CHEL/361.H

Geometry and Topology



Functions of Several Complex Variables and Their Singularities

Wolfgang Ebeling, *Leibniz Universität Hannover, Germany*
Translated by Philip G. Spain

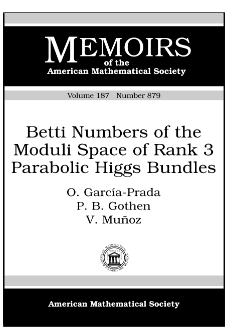
The book provides an introduction to the theory of functions of several complex variables and their singularities, with special emphasis on topological aspects. The topics include Riemann surfaces, holomorphic functions of several variables, classification and deformation of singularities, fundamentals of differential topology, and the topology of singularities. The aim of the book is to guide the reader from the fundamentals to more advanced topics of recent research. All the necessary prerequisites are specified and carefully explained. The general theory is illustrated by various examples and applications.

This item will also be of interest to those working in analysis.

Contents: Riemann surfaces; Holomorphic functions of several variables; Isolated singularities of holomorphic functions; Fundamentals of differential topology; Topology of singularities; Bibliography; Index.

Graduate Studies in Mathematics, Volume 83

June 2007, 312 pages, Hardcover, ISBN: 978-0-8218-3319-3, LC 2007060745, 2000 *Mathematics Subject Classification:* 32-01; 32S10, 32S55, 58K40, 58K60, **AMS members US$47**, List US$59, Order code GSM/83



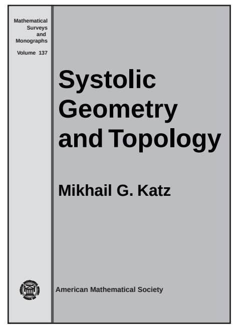
Betti Numbers of the Moduli Space of Rank 3 Parabolic Higgs Bundles

O. García-Prada, *Consejo Superior de Investigaciones Científicas, Madrid, Spain*, **P. B. Gothen**, *Universidade do Porto, Portugal*, and **V. Muñoz**, *Consejo Superior de Investigaciones Científicas, Madrid, Spain*

Contents: Introduction; Parabolic Higgs bundles; Morse theory on the moduli space; Parabolic triples; Critical values and flips; Parabolic triples with $r_1 = 2$ and $r_2 = 1$; Critical submanifolds of type (1, 1, 1); Critical submanifolds of type (1, 2); Critical submanifolds of type (2, 1); Betti numbers of the moduli space of rank three parabolic bundles; Betti numbers of the moduli space of rank three parabolic Higgs bundles; The fixed determinant case; Bibliography.

Memoirs of the American Mathematical Society, Volume 187, Number 879

March 2007, 80 pages, Softcover, ISBN: 978-0-8218-3972-0, LC 2007060665, 2000 *Mathematics Subject Classification:* 14D20, 14H60, **Individual member US$34**, List US$57, Institutional member US$46, Order code MEMO/187/879



Systolic Geometry and Topology

Mikhail G. Katz, *Bar Ilan University, Ramat Gan, Israel*

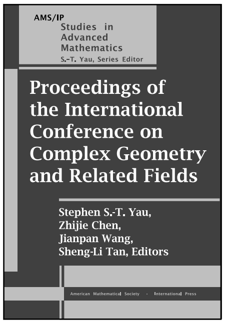
The systole of a compact metric space X is a metric invariant of X, defined as the least length of a noncontractible loop in X. When X is a graph, the invariant is usually referred to as the girth, ever since the 1947 article by W. Tutte. The first nontrivial results for systoles of surfaces are the two classical inequalities of C. Loewner and P. Pu, relying on integral-geometric identities, in the case of the two-dimensional torus and real projective plane, respectively. Currently, systolic geometry is a rapidly developing field, which studies systolic invariants in their relation to other geometric invariants of a manifold.

This book presents the systolic geometry of manifolds and polyhedra, starting with the two classical inequalities, and then proceeding to recent results, including a proof of M. Gromov's filling area conjecture in a hyperelliptic setting. It then presents Gromov's inequalities and their generalisations, as well as asymptotic phenomena for systoles of surfaces of large genus, revealing a link both to ergodic theory and to properties of congruence subgroups of arithmetic groups. The author includes results on the systolic manifestations of Massey products, as well as of the classical Lusternik-Schnirelmann category.

Contents: *Systolic geometry in dimension 2:* Geometry and topology of systoles; Historical remarks; The *theorema egregium* of Gauss; Global geometry of surfaces; Inequalities of Loewner and Pu; Systolic applications of integral geometry; A primer on surfaces; Filling area theorem for hyperelliptic surfaces; Hyperelliptic surfaces are Loewner; An optimal inequality for CAT(0) metrics; Volume entropy and asymptotic upper bounds; *Systolic geometry and topology in n dimensions:* Systoles and their category; Gromov's optimal stable systolic inequality for $\mathbb{CP}^n$; Systolic inequalities dependent on Massey products; Cup products and stable systoles; Dual-critical lattices and systoles; Generalized degree and Loewner-type inequalities; Higher inequalities of Loewner-Gromov type; Systolic inequalities for L^p norms; Four-manifold systole asymptotics; Period map image density (by Jake Solomon); Open problems; Bibliography; Index.

Mathematical Surveys and Monographs, Volume 137

March 2007, 222 pages, Hardcover, ISBN: 978-0-8218-4177-8, LC 2007060668, 2000 *Mathematics Subject Classification:* 53C23; 11R52, 16K20, 17B25, 28D20, 30F10, 37C35, 52C07, 53C20, 55M30, 57M27, 55R37, 57N65, **AMS members US$55**, List US$69, Order code SURV/137



Proceedings of the International Conference on Complex Geometry and Related Fields

Stephen S.-T. Yau, *East China Normal University, Shanghai, People's Republic of China, and University of Illinois at Chicago, IL*, and **Zhijie Chen**, **Jianpan Wang**, and **Sheng-Li Tan**, *East China Normal University, Shanghai, People's Republic of China*, Editors

In commemoration and celebration of the tenth anniversary of the Institute of Mathematics at East China Normal University, an International Conference on complex geometry and related fields recently convened. This collection presents some of the conference highlights, dealing with various and significant topics of differential and algebraic geometry, while exploring their connections to number theory and mathematical physics.

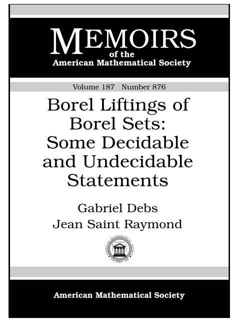
Titles in this series are co-published with International Press, Cambridge, MA.

Contents: **N. Bergeron**, **Y. Gao**, and **N. Hu**, Representations of two-parameter quantum orthogonal and symplectic groups; **R. W. Brockett**, Optimal control of the Liouville equation; **F. Catanese** and **P. Frediani**, Real structures on torus bundles and their deformations; **Z. Chen**, **R. Du**, **S.-L. Tan**, and **F. Yu**, Cubic equations of rational triple points of dimension two; **J. Du** and **H. Rui**, Multi-parameter cells of finite Coxeter groups; **X. Huang** and **S. Ji**, On some rigidity problems in Cauchy-Riemann analysis; **L. Jia**, **H. S. Luk**, and **S. S.-T. Yau**, From CR geometry to algebraic geometry and combinatorial geometry; **D. Jiang**, Periods of automorphic forms; **H.-J. Lai** and **Y. Shao**, Some problems related to hamiltonian line graphs; **S.-Y. Li**, Composition operators and isometries on holomorphic function spaces over domains in $\mathbb{C}^n$; **K. Liu**, Localization and string duality; **Z. Long**, **X. Wang**, and **Y. Wang**, Polyhedral and geometric convergence of Kleinian groups; **N. Mok**, Rigidity problems on compact quotients of bounded symmetric domains; **M. Ru**, The second main theorem with hypersurfaces over function fields; **J.-Y. Shi**, Presentations for finite complex reflection groups; **B. Shu**, Representations of finite Lie algebras and geometry of reductive Lie algebras; **S.-T. Yau**, Perspectives on geometric analysis; **D.-Q. Zhang**, Automorphisms of K3 surfaces; **L. Zhao**, **X. Zhou**, and **Q. Li**, Vector bundles on certain surfaces without divisors.

AMS/IP Studies in Advanced Mathematics, Volume 39

May 2007, 402 pages, Softcover, ISBN: 978-0-8218-3949-2, LC 2007060748, 2000 *Mathematics Subject Classification:* 32-XX, 14-XX, 53-XX, 17-XX, 20-XX, 11-XX, 05Cxx, 93-XX, 30Fxx, **AMS members US$63**, List US$79, Order code AMSIP/39

Logic and Foundations



Borel Liftings of Borel Sets: Some Decidable and Undecidable Statements

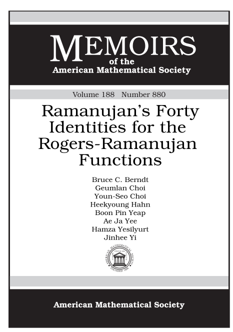
Gabriel Debs and **Jean Saint Raymond**, *Institut de Mathématiques de Jussieu, Paris, France*

Contents: Introduction; A tree representation for Borel sets; A double-tree representation for Borel sets; Two applications of the tree representation; Borel liftings of Borel sets; More consequences and reverse results; Proof of the main result; Bibliography; Index.

Memoirs of the American Mathematical Society, Volume 187, Number 876

March 2007, 118 pages, Softcover, ISBN: 978-0-8218-3971-3, LC 2007060664, 2000 *Mathematics Subject Classification:* 03E15; 03E45, 54H05, **Individual member US$37**, List US$62, Institutional member US$50, Order code MEMO/187/876

Number Theory



Ramanujan's Forty Identities for the Rogers-Ramanujan Functions

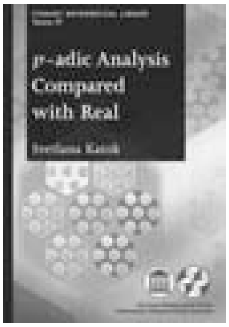
Bruce C. Berndt and **Geumlan Choi**, *University of Illinois, Urbana, IL*, **Youn-Seo Choi**, *Institute for Advanced Study, Seoul, Republic of Korea*, **Heekyoung Hahn**, *University of Rochester, NY*, **Boon Pin Yeap**, *University of Illinois, Urbana, IL*, **Ae Ja Yee**, *Pennsylvania State University, College Park, PA*, **Hamza Yesilyurt**, *University of Florida, Gainesville, FL*, and **Jinhee Yi**, *Korea Science Academy, Pusan, Republic of Korea*

Contents: Introduction; Definitions and preliminary results; The forty identities; The principal ideas behind the proofs; Proofs of 35 of the 40 entries; Asymptotic "proofs" of Entries 3.28 (second part), 3.29, 3.30, 3.31, and 3.35; New identities for $G(q)$ and $H(q)$ and final remarks; Bibliography.

Memoirs of the American Mathematical Society, Volume 188, Number 880

July 2007, 96 pages, Softcover, ISBN: 978-0-8218-3973-7, LC 2007060759, 2000 *Mathematics Subject Classification:* 11P82, 11F27, 33D15, **Individual member US$36**, List US$60, Institutional member US$48, Order code MEMO/188/880

COURSE ADOPTION



p-adic Analysis Compared with Real

Svetlana Katok, *Pennsylvania State University, University Park, PA*

The book gives an introduction to p-adic numbers from the point of view of number theory, topology, and analysis. Compared to other books on the subject, its novelty is both a particularly balanced approach to these three points of view and an emphasis on topics accessible to undergraduates. In addition, several topics from real analysis and elementary topology which are not usually covered in undergraduate courses (totally disconnected spaces and Cantor sets, points of discontinuity of maps and the Baire Category Theorem, surjectivity of isometries of compact metric spaces) are also included in the book. They will enhance the reader's understanding of real analysis and intertwine the real and p-adic contexts of the book.

The book is based on an advanced undergraduate course given by the author. The choice of the topic was motivated by the internal beauty of the subject of p-adic analysis, an unusual one in the undergraduate curriculum, and abundant opportunities to compare it with its much more familiar real counterpart. The book includes a large number of exercises. Answers, hints, and solutions for most of them appear at the end of the book. Well written, with obvious care for the reader, the book can be successfully used in a topic course or for self-study.

This item will also be of interest to those working in analysis.

Contents: Arithmetic of the p-adic numbers; The topology of $\mathbb{Q}_p$ vs. the topology of $\mathbb{R}$; Elementary analysis in $\mathbb{Q}_p$; p-adic functions; Answers, hints, and solutions for selected exercises; Bibliography; Index.

Student Mathematical Library, Volume 37

April 2007, 152 pages, Softcover, ISBN: 978-0-8218-4220-1, LC 2006047983, 2000 *Mathematics Subject Classification:* 11-01, 26E30, 12Jxx, **AMS members US$23**, List US$29, Order code STML/37

New AMS-Distributed Publications

Algebra and Algebraic Geometry

Propriétés de Lefschetz Automorphes pour les Groupes Unitaires et Orthogonaux

Nicolas Bergeron, *Unité Mixte de Recherche 8553 du CNRS, Paris, France*

This mémoire deals with a number of results and conjectures relating the cohomology of an arithmetic manifold with the cohomology of a totally geodesic submanifold. This echoes the Lefschetz Theorem, which states that appropriate cohomology groups of a generic hyperplane section of a projective manifold inject into the corresponding cohomology groups of that manifold. It may sound quite surprising that such an analogy still exists in the case of real arithmetic manifolds.

This item will also be of interest to those working in geometry and topology.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Contents: Introduction; Représentations cohomologiques; Théorème 1.3 et cohomologie L^2; Isolation des représentations cohomologiques; Restriction des représentations cohomologiques; Géométrie de l'espace symétrique associé au groupe $O(p,q)$; Calcul de la cohomologie L^2; Démonstration des principaux résultats; Généralisations et perspectives; Bibliographie.

Mémoires de la Société Mathématique de France, Number 106

January 2007, 125 pages, Softcover, ISBN: 978-2-85629-219-8, 2000 *Mathematics Subject Classification:* 11F75, 22E47, 22E55, 11G18, 14G35, 58J50, **Individual member US$34**, List US$38, Order code SMFMEM/106

Groups de Galois Arithmétiques et Différentiels

Daniel Bertrand, *Université Pierre et Marie Curie, Paris, France*, and **Pierre Dèbes**, *Université Lille 1, Villeneuve, France*, Editors

On March 8–13, 2004, a meeting was organized at the Luminy CIRM (France) on arithmetic and differential Galois groups, reflecting the growing interactions between the two theories. The present volume contains the proceedings of this conference. It covers the following themes: moduli spaces (of curves, of coverings, of connexions), including the recent developments on modular towers; the arithmetic of coverings and of differential equations (fields of definition, descent theory); fundamental groups; the inverse problems and methods of deformation; and the algorithmic aspects of the theories, with explicit computations or realizations of Galois groups.

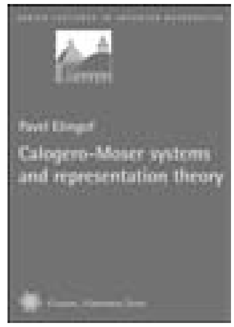
This item will also be of interest to those working in differential equations and number theory.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Contents: **M. Berkenbosch**, Algorithms and moduli spaces for differential equations; **M. Berkenbosch** and **M. van der Put**, Families of linear differential equations on the projective line; **P. Boalch**, Brief introduction to Painlevé VI; **A. Buium**, Correspondences, Fermat quotients, and uniformization; **J.-M. Couveignes**, Jacobiens, jacobiennes et stabilité numérique; **P. Dèbes**, An introduction to the modular tower program; **M. Dettweiler** and **S. Wewers**, Variation of parabolic cohomology and Poincaré duality; **M. D. Fried**, The main conjecture of modular towers and its higher rank generalization; **R. Liţcanu** and **L. Zapponi**, Properties of Lamé operators with finite monodromy; **S. Malek**, On the Riemann-Hilbert problem and stable vector bundles on the Riemann sphere; **B. H. Matzat**, Integral p-adic differential modules; **F. Pop**, Galois theory of Zariski prime divisors; **M. Romagny** and **S. Wewers**, Hurwitz spaces; **D. Semmen**, The group theory behind modular towers; **C. Simpson**, Formalized proof, computation, and the construction problem in algebraic geometry; Annexe. Liste des participants.

Séminaires et Congrès, Number 13

February 2007, 391 pages, Softcover, ISBN: 978-2-85629-222-8, 2000 *Mathematics Subject Classification:* 03B35, 11F11, 11F25, 11F30, 11F32, 11Gxx, 11G18, 11R58, 11Y16, 11Y35, 12Exx, 12E30, 12Fxx, 12F10, 12F12, 12Gxx, 12G99, 12H05, 12H25, 12Jxx, 13Nxx, 13N05, 13N10, 14-04, 14Dxx, 14D22, 14F05, 14G05, 14G32, 14G35, 14H05, 14H10, 14H30, 18A25, 20B05, 20C05, 20C20, 20C25, 20D25, 20E18, 34M55, 20F34, 20F69, 20Gxx, 20G25, 20J05, 20J06, 32J25, 32S40, 33C05, 34-XX, 34A20, 35C10, 35C20, 65E05, 65Y20, 68Q15, 53Cxx, **Individual member US$99**, List US$110, Order code SECO/13



Calogero–Moser Systems and Representation Theory

Pavel Etingof, *Massachusetts Institute of Technology, Cambridge, MA*

Calogero–Moser systems, which were originally discovered by specialists in integrable systems, are currently at the crossroads of many areas of mathematics and within the scope of interests of many mathematicians. More specifically, these systems and their generalizations turned out to have intrinsic connections with such fields as algebraic geometry (Hilbert schemes of surfaces), representation theory (double affine Hecke algebras, Lie groups, quantum groups), deformation theory (symplectic reflection algebras), homological algebra (Koszul algebras), Poisson geometry, etc.

The goal of the present lecture notes is to give an introduction to the theory of Calogero–Moser systems, highlighting their interplay with these fields. Since these lectures are designed for non-experts, the author gives short introductions to each of the subjects involved and provides a number of exercises.

This item will also be of interest to those working in mathematical physics.

A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.

Contents: Introduction; Poisson manifolds and Hamiltonian reduction; Classical mechanics and integrable systems; Deformation theory; Moment maps, Hamiltonian reduction and the Levasseur–Stafford theorem; Quantum mechanics, quantum integrable systems and the Calogero–Moser system; Calogero–Moser systems associated to finite Coxeter groups; The rational Cherednik algebra; Symplectic reflection algebras; Deformation-theoretic interpretation of symplectic reflection algebras; The center of the symplectic reflection algebra; Representation theory of rational Cherednik algebras; Bibliography; Index.

Zurich Lectures in Advanced Mathematics

March 2007, 102 pages, Softcover, ISBN: 978-3-03719-034-0, 2000 *Mathematics Subject Classification:* 16G30, 70H06, 16S80, 14A22, **AMS members US$27**, List US$34, Order code EMSZLEC/4

Kobayashi–Hitchin Correspondence for Tame Harmonic Bundles and an Application

Takuro Mochizuki, *Kyoto University, Japan*

The author establishes the correspondence between tame harmonic bundles and μ_L-polystable parabolic Higgs bundles with trivial characteristic numbers. He also shows the Bogomolov–Gieseker type inequality for μ_L-stable parabolic Higgs bundles.

The author shows that any local system on a smooth quasiprojective variety can be deformed to a variation of polarized Hodge structure. He then concludes that some kind of discrete groups cannot be a split quotient of the fundamental group of a smooth quasiprojective variety.

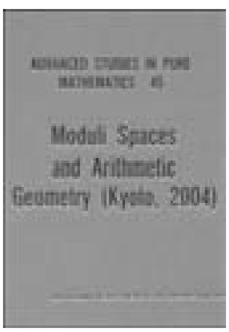
This item will also be of interest to those working in geometry and topology.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Contents: Introduction; Preliminary; Parabolic Higgs bundle and regular filtered Higgs bundle; An ordinary metric for a parabolic Higgs bundle; Parabolic Higgs bundle associated to tame harmonic bundle; Preliminary correspondence and Bogomolov–Gieseker inequality; Construction of a frame; Some convergence results; Existence of adapted pluri–harmonic metric; Torus action and the deformation of representations; G-harmonic bundle; Bibliography.

Astérisque, Number 309

December 2006, 117 pages, Softcover, ISBN: 978-2-85629-226-6, 2000 *Mathematics Subject Classification:* 14J60, 53C07, **Individual member US$35**, List US$39, Order code AST/309



Moduli Spaces and Arithmetic Geometry (Kyoto, 2004)

Shigeru Mukai, *Kyoto University, Japan*, **Yoichi Miyaoka**, *University of Tokyo, Japan*, **Shigefumi Mori** and **Atsushi Moriwaki**, *Kyoto University, Japan*, and **Iku Nakamura**, *Hokkaido University, Sapporo, Japan*, Editors

Since its birth, algebraic geometry has been closely related to and deeply motivated by number theory. The modern study of moduli spaces and arithmetic geometry demonstrates that these two areas have many important techniques and ideas in common. With this close relation in mind, the RIMS conference "Moduli Spaces and Arithmetic Geometry" was held at Kyoto University during September 8–15, 2004 as the 13th International Research Institute of the Mathematical Society of Japan.

This volume is the outcome of this conference and consists of thirteen papers by invited speakers, including C. Soulé, A. Beauville and C. Faber, and other participants. All papers, with two exceptions by C. Voisin and Yoshinori Namikawa, treat moduli problem and/or arithmetic geometry. Algebraic curves, Abelian varieties, algebraic vector bundles, connections and D-modules are the subjects of those moduli papers. Arakelov geometry and rigid geometry are studied in arithmetic papers. In the two exceptions, integral Hodge classes on Calabi–Yau threefolds and symplectic resolutions of nilpotent orbits are studied.

This item will also be of interest to those working in number theory.

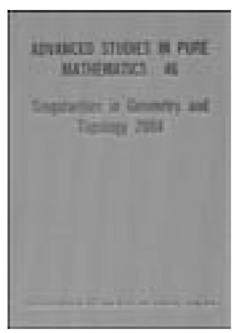
Published for the Mathematical Society of Japan by Kinokuniya, Tokyo, and distributed worldwide, except in Japan, by the AMS.

Contents: K. Yoshioka, Moduli spaces of twisted sheaves on a projective variety; **D. Huybrechts** and **P. Stellari**, Appendix. Proof of Caldararu's conjecture; **C. Voisin**, On integral Hodge classes on uniruled or Calabi–Yau threefolds; **Y. Namikawa**, Birational geometry of symplectic resolutions of nilpotent orbits; **T. Abe**, The moduli stack of rank-two Gieseker bundles with fixed determinant on a nodal curve; **A. Beauville**, Vector bundles on curves and theta functions; **A. Moriwaki**, On the finiteness of abelian varieties with bounded modular height; **N. Nitsure**, Moduli of regular holonomic $\mathcal{D}_X$-modules with natural parabolic stability; **I. Nakamura** and **K. Sugawara**, The cohomology groups of stable quasi-abelian schemes and degenerations associated with the E_8-lattice; **C. Soulé**, Semi-stable extensions on arithmetic surfaces; **C. Consani** and **C. Faber**, On the cusp form motives in genus 1 and level 1; **S. Mukai**, Polarized K3 surfaces of genus thirteen; **K. Fujiwara** and **F. Kato**, Rigid geometry and applications; **M. Inaba**, **K. Iwasaki**, and **M. Saito**, Moduli of stable parabolic connections, Riemann–Hilbert correspondence and geometry of Painlevé equation of type VI, part II.

Advanced Studies in Pure Mathematics, Volume 45

January 2007, 432 pages, Hardcover, ISBN: 978-4-931469-38-9, 2000 *Mathematics Subject Classification:* 14-06; 11-06, 32-06, **AMS members US$82**, List US$102, Order code ASPM/45

Geometry and Topology



Singularities in Geometry and Topology 2004

Jean-Paul Brasselet, *CNRS, Marseille, France*, and **Tatsuo Suwa**, *Niigata University, Japan*, Editors

This volume constitutes the proceedings of the Third Franco–Japanese Symposium on Singularities in Geometry and Topology, held in Sapporo in September 2004. It contains not only research papers on the most advanced topics in the field but also some survey articles, which give a broad scope of some areas of the subject.

Published for the Mathematical Society of Japan by Kinokuniya, Tokyo, and distributed worldwide, except in Japan, by the AMS.

Contents: V. Blanlœil and **O. Saeki**, Cobordism of fibered knots and related topics; **J.-P. Brasselet**, **J. Seade**, and **T. Suwa**, Proportionality of indices of 1-forms on singular varieties; **M. Hanamura**, Motivic sheaves and intersection cohomology; **E. Hironaka**, On hyperbolic perturbations of algebraic links and small Mahler measure; **V. P. Kostov**, Stably hyperbolic polynomials; **Y. Nakamura** and **S. Tajima**, On weighted-degrees for algebraic local cohomologies associated with semiquasihomogeneous singularities; **P. Popescu-Pampu**, The geometry of continued fractions and the topology of surface singularities; **G. Rond**, Exemples de fonctions de Artin de germes d'espaces analytiques; **K. Takeuchi**, Perverse sheaves and Milnor fibers over singular varieties; **S. Tanabé**, On Horn–Kapranov uniformisation of the discriminantal loci; **M. Tibuar**, Duality of Euler data for affine varieties; **M. Vaquié**, Algebre graduée associée à une valuation de $K[x]$; **M. Watari**, Plane curve singularities whose Milnor and Tjurina numbers differ by three; **S. Yokura**, Characteristic classes of (pro)algebraic varieties.

Advanced Studies in Pure Mathematics, Volume 46

February 2007, 329 pages, Hardcover, ISBN: 978-4-931469-39-6, 2000 *Mathematics Subject Classification:* 12D10, 13A18, 14B05, 14C05, 14C17, 14C22, 14D05, 14E15, 14E20, 14Gxx, 32A27, 32B20, 32S05, 32S10, 32S15, 32S25, 32S45, 32S60, 53D35, 58K05, **AMS members US$82**, List US$102, Order code ASPM/46

Les Préfaisceaux Comme Modèles des Types d'Homotopie

Denis-Charles Cisinski, *University of Paris XIII, Villetaneuse, France*

Grothendieck introduced in *Pursuing Stacks* the notion of *test categories*. These, by definition, are small categories on which presheaves of sets are models for homotopy types of CW-complexes. A well-known example of this is the category of simplices. (The corresponding presheaves are then simplicial sets.) Furthermore, Grothendieck defined the notion of *basic localizer*, which gives an axiomatic approach to the homotopy theory of small categories and gives a natural setting to extend the notion of test category with respect to some localizations of the homotopy category of CW-complexes. This text is the sequel to Grothendieck's homotopy theory. The author proves in particular two conjectures made by Grothendieck: any category of presheaves on a test category is canonically endowed with a Quillen closed model category structure, and the smallest basic localizer defines the homotopy theory of CW-complexes.

The author shows how a local version of the theory allows consideration in a unified setting of the equivariant homotopy theory as well. The realization of this program goes through the construction and the study of model category structures on any category of presheaves on an abstract small category, as well as the study of the homotopy theory of small categories following and completing the contributions of Quillen, Thomason and Grothendieck.

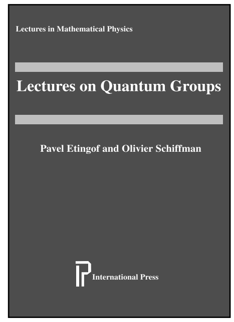
This item will also be of interest to those working in algebra and algebraic geometry.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Contents: *Partie I. Algèbre homotopique des préfaisceaux:* Constructions de catégories de modèles; Yoga simplicial; Densité homotopique; *Partie II. À la poursuite des modèles:* Correspondances fondamentales; Factorisations de foncteurs; Changements de base homotopiques; Représentations et structures fibrées; *Partie III. Zoologies:* Exemples de catégories test locales; Exemples de localisateurs fondamentaux; Bibliographie; Index; Index des notations.

Astérisque, Number 308

January 2007, 392 pages, Softcover, ISBN: 978-2-85629-225-9, 2000 *Mathematics Subject Classification:* 55-02, 18F20, 18G50, 18G55, 18G30, 54B30, 54B40, 55P57, 55P60, 55P92, 55U35, 55U40, **Individual member US$102**, List US$113, Order code AST/308



Lectures on Quantum Groups

Second Edition

Pavel Etingof, *Massachusetts Institute of Technology, Cambridge, MA*, and **Olivier Schiffmann**, *Ecole Normale Superieur, Paris, France*

Quantum groups, an exciting new area of mathematics which originated from mathematical physics, has developed greatly over the last 15 years. This book arose from a graduate course on quantum groups given by P. Etingof at Harvard. The purpose of this book is to give an elementary introduction to the theory of quantum groups. It is written for a general mathematical audience and assumes only a knowledge of basic algebra and geometry.

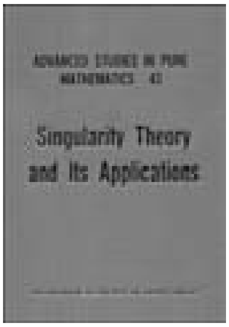
Two unusual features of this book, compared to other textbooks, are the extensive use of pictorial language for writing and checking algebraic relations, and over 55 problems and exercises with solutions.

A publication of International Press. Distributed worldwide by the American Mathematical Society.

Contents: Poisson algebras and quantization; Poisson–Lie groups; Coboundary Lie bialgebras; Drinfeld's double construction; Belavin–Drinfeld classification (I); Infinite dimensional Lie bialgebras; Belavin–Drinfeld classification (II); Hopf algebras; Quantized universal enveloping algebras; Formal groups and h-formal groups; Infinite dimensional quantum groups; The quantum double; Tensor categories and quasi-Hopf algebras; Braided tensor categories; KZ equations and the Drinfeld category; Quasi-Hopf quantized enveloping algebras; Lie associators; Fiber functors and Tannaka–Krein duality; Quantization of finite dimensional Lie bialgebras; Universal constructions; Universal quantization; Dequantization and the equivalence theorem; Appendix; Solutions to problems and exercises.

International Press

June 2002, 250 pages, Hardcover, ISBN: 978-1-57146-094-3, 2000 *Mathematics Subject Classification:* 35J60, 53C21, 53C44, 53C50, 58J05, **AMS members US$38**, List US$48, Order code INPR/33.R



Singularity Theory and Its Applications

Shyuichi Izumiya and **Goo Ishikawa**, *Hokkaido University, Sapporo, Japan*, **Hiroo Tokunaga**, *Tokyo Metropolitan University, Japan*, **Ichiro Shimada**, *Hokkaido University, Sapporo, Japan*, and **Takasi Sano**, *Hokkaigakuen University, Kitami, Japan*, Editors

This volume is the proceedings of the 12th International Research Institute of the Mathematical Society of Japan, held during 2003. The papers cover several important topics in singularity theory, including survey on motivic integrations, Thom polynomials, complex analytic singularity theory, generic differential geometry, etc.

This item will also be of interest to those working in algebra and algebraic geometry.

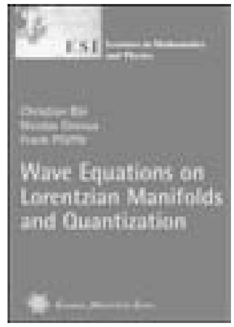
Published for the Mathematical Society of Japan by Kinokuniya, Tokyo, and distributed worldwide, except in Japan, by the AMS.

Contents: **E. A. Bartolo**, **J. C. Ruber**, **J. I. Cogolludo-Agustĭn**, and **M. A. Marco-Buzunăriz**, Invariants of combinatorial line arrangements and Rybnikov's example; **A. Davydov**, On time averaged optimization of dynamic inequalities on a circle; **L. M. Féher** and **R. Rimányi**, Thom polynomial computing strategies. A survey; **V. Goryunov** and **S. H. Man**, The complex crystallographic groups and symmetries of J_{10}; **C. Hertling**, tt^* geometry and mixed Hodge structures; **M. Kazarian**, Thom polynomials; **K. Kurdyka** and **A. Parusiński**, Quasi-convex decomposition in o-minimal structures. Application to the gradient conjecture; **A. Libgober**, Homotopy groups of complements to ample divisors; **D. Matei**, Massey products of complex hypersurface complements; **V. S. Matveev**, On degree of mobility for complete metrics; **P. Mormul**, Valuations and moduli of Goursat distributions; **C. Murolo** and **D. Trotman**, Semidifférentiabilité et version lisse de la conjecture de fibration de Whitney; **J. J. Nuño-Ballesteros**, Submanifolds with a nondegenerate parallel normal vector field in euclidean spaces; **O. M. Abderrahmane**, Weighted homogeneous polynomials and blow-analytic equivalence; **A. Parusiński**, Characteristic classes of singular varieties; **G. M. Polotovskiy**, On the classification of 7th degree real decomposable curves; **M. J. Saia** and **L. M. Soares**, $\mathcal{A}$-topological triviality of map germs and Newton filtrations; **V. D. Sedyh**, On the topology of symmetry sets of smooth submanifolds in $\mathbb{R}^k$; **M. A. S. Ruas** and **J. N. Tomazella**, An infinitesimal criterion for topological triviality of families of sections of analytic varieties; **J. Sotomayor** and **R. Garcia**, Lines of principal curvature near singular end points of surfaces in $\mathbb{R}^3$; **D. Trotman** and **L. Wilson**, r does not imply n or (npf) for definable sets in non polynomially bounded o-minimal structures; **M. Vaquié**, Valuations and local uniformization; **W. Veys**, Arc spaces, motivic integration and stringy invariants; **Y. Yamada**, Finite Dehn surgery along A'Campo's divide knots.

Advanced Studies in Pure Mathematics, Volume 43

February 2007, 583 pages, Hardcover, ISBN: 978-4-931469-32-7, 2000 *Mathematics Subject Classification:* 58K99; 32S99, **AMS members US$90**, List US$113, Order code ASPM/43

Mathematical Physics



Wave Equations on Lorentzian Manifolds and Quantization

Christian Bär, **Nicolas Ginoux**, and **Frank Pfäffle**, *University of Potsdam, Germany*

This book provides a detailed introduction to linear wave equations on Lorentzian manifolds (for vector-bundle valued fields). After a collection of preliminary material in the first chapter, one finds in the second chapter the construction of local fundamental solutions together with their Hadamard expansion. The third chapter establishes the existence and uniqueness of global fundamental solutions on globally hyperbolic spacetimes and discusses Green's operators and well-posedness of the Cauchy problem. The last chapter is devoted to field quantization in the sense of algebraic quantum field theory. The necessary basics on C^*-algebras and CCR-representations are developed in full detail.

The text provides a self-contained introduction to these topics addressed to graduate students in mathematics and physics. At the same time, it is intended as a reference for researchers in global analysis, general relativity, and quantum field theory.

This item will also be of interest to those working in differential equations and geometry and topology.

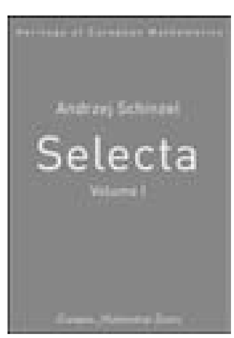
A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.

Contents: Preliminaries; The local theory; The global theory; Quantization; Appendix. Background material; Bibliography; Figures; Symbols; Index.

ESI Lectures in Mathematics and Physics

March 2007, 202 pages, Softcover, ISBN: 978-3-03719-037-1, 2000 *Mathematics Subject Classification:* 58J45, 81T20; 35L05, 35L15, 53C50, 81T05, **AMS members US$35**, List US$44, Order code EMSESILEC/3

Number Theory



Andrzej Schinzel, Selecta

Andrzej Schnizel, *Polish Academy of Sciences, Warsaw, Poland*

Andrzej Schinzel, born in 1937, is a leading number theorist whose work has had a lasting impact on modern mathematics. He is the author of over 200 research articles in various branches of arithmetics, including elementary, analytic, and algebraic number theory. He has also been, for nearly 40 years, the editor of *Acta Arithmetica*, the first international journal devoted exclusively to number theory.

Selecta, a two-volume set, contains Schinzel's most important articles published between 1955 and 2006. The arrangement is by topic, with each major category introduced by an expert's comment. Many of the hundred selected papers deal with arithmetical and algebraic properties of polynomials in one or several variables, but there are also articles on Euler's totient function, the favorite subject of Schinzel's early research, on prime numbers (including the famous paper with Sierpiński on the Hypothesis "H"), algebraic number theory, diophantine equations, analytical number theory and geometry of numbers. *Selecta* concludes with some papers from outside number theory, as well as a list of unsolved problems and unproved conjectures, taken from the work of Schinzel.

A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.

Contents: Volume 1: Diophantine equations and integral forms; Continued fractions; Algebraic number theory; Polynomials in one variable; Polynomials in several variables; Hilbert's irreducibility theorem; **Volume 2:** Arithmetic functions; Divisibility and congruences; Primitive divisors; Prime numbers; Analytic number theory; Geometry of numbers; Other papers; Unsolved problems and unproved conjectures; Publication list of Andrzej Schinzel.

Heritage of European Mathematics

March 2007, 1417 pages, Hardcover, ISBN: 978-3-03719-038-8, 2000 *Mathematics Subject Classification:* 11-XX, 12-XX, **AMS members US$174**, List US$218, Order code EMSHEM/1

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Please send curriculum vitae, description of research interests, and at least three letters of recommendation by July 2, 2007.

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Yale University
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Applications are invited for visiting and permanent faculty positions at all levels. Applicants should have a Ph.D. and an excellent academic record. Outstanding candidates in all areas of Mathematical Sciences are encouraged to apply. Current departmental interests include Algebra, Algebraic Geometry, Algebraic Topology, Combinatorics, Differential Geometry, Functional Analysis, Harmonic Analysis, Number Theory, Numerical Analysis, Partial Differential Equations, Probability and Statistics. The Department of Mathematics and IIT Bombay offer an environment conducive to research. Teaching duties are about 5 hours a week and consist of at most two courses per semester at the undergraduate (B.Tech.), postgraduate (M.Sc.), or doctoral (Ph.D.) levels. A substantial "seed grant" of up to Rs. 5,00,000 is available for each new faculty member. In addition, the institute periodically funds participation in international conferences. It is also possible to raise grant money for research through various government and industry sources. Faculty enjoy several personal benefits including on-campus housing with free high-speed LAN connection, free or subsidized medical care, and easy access to schools on campus for children. Further information is available at: `http://www.math.iitb.ac.in/`. Applications including a curriculum vitae, a

Suggested uses for classified advertising are positions available, books or lecture notes for sale, books being sought, exchange or rental of houses, and typing services.

The 2007 rate is $110 per inch or fraction thereof on a single column (one-inch minimum), calculated from top of headline. Any fractional text of 1/2 inch or more will be charged at the next inch rate. No discounts for multiple ads or the same ad in consecutive issues. For an additional $10 charge, announcements can be placed anonymously. Correspondence will be forwarded.

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There are no member discounts for classified ads. Dictation over the telephone will not be accepted for classified ads.

Upcoming deadlines for classified advertising are as follows: August 2007 issue–May 29, 2007; September 2007 issue–June 28, 2007; October 2007 issue–July 26, 2007; November 2007 issue–August 28, 2007; December 2007 issue–October 1, 2007; January 2008 issue–October 26, 2007.

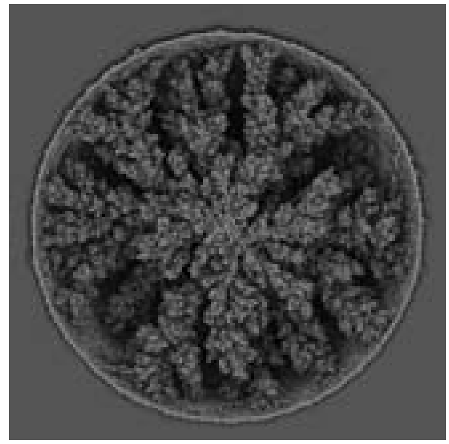
U.S. laws prohibit discrimination in employment on the basis of color, age, sex, race, religion, or national origin. "Positions Available" advertisements from institutions outside the U.S. cannot be published unless they are accompanied by a statement that the institution does not discriminate on these grounds whether or not it is subject to U.S. laws. Details and specific wording may be found on page 1373 (vol. 44).

Situations wanted advertisements from involuntarily unemployed mathematicians are accepted under certain conditions for free publication. Call toll-free 800-321-4AMS (321-4267) in the U.S. and Canada or 401-455-4084 worldwide for further information.

Submission: Promotions Department, AMS, P.O. Box 6248, Providence, Rhode Island 02940; or via fax: 401-331-3842; or send email to `classads@ams.org`. AMS location for express delivery packages is 201 Charles Street, Providence, Rhode Island 20904. Advertisers will be billed upon publication.

About the Cover

Aggregation 22



This month's cover is from the collection of images http://www.bridgesmathart.org/art-exhibits/bridges06/lomas.html by Andy Lomas, part of the "Bridges London 2006" conference covered by Mike Field elsewhere in this issue. Andy studied mathematics as an undergraduate at Trinity College Cambridge, but is currently head of computer graphics at the English company Framestore CFC, Europe's largest digital effects and animation studio. This company produced, among other things, the title sequence for *Casino Royale*.

Andy tells us, "Mathematics and visual imagery have played an important role throughout my life. From an early age I have also had deep interests in photography and filmmaking. Digital art, computer graphics, and animation have provided a very natural and fulfilling way of bringing these two paths together. Examples where I have used mathematics in my professional work include fluid effects for the 'pool of tears' sequence in a TV series *Alice in Wonderland* and an illumination model for rain that allowed us to emulate the effects of rain being illuminated from different angles, to integrate with live action in *The Matrix: Revolutions*."

As to how the image was constructed, he wrote:

> The structures in the *Aggregation* series are produced by variations on Diffusion Limited Aggregation (DLA), a stochastic model for fractal growth originally proposed by T. L. Witten and L. M. Sander (*Physical Review Letters* **47**).
>
> DLA starts with a seed particle, then deposits new particles onto the structure by creating new 'walker' particles and allowing them to move randomly by Brownian motion until they reach the aggregated cluster where they are deposited. Iterating this process over many particles, extremely complex fractal structures called 'Brownian Trees' are formed. Computer implementations of DLA commonly restrict the particles to a lattice grid, but my implementation is lattice-free and calculates the intersections of the walker particles with the clustered form analytically. I have used a piece-wise linear approximation to Brownian motion.
>
> One thing that I find particularly intriguing about *Aggregation 22* is that there appear to be distinctly different structures in the form: an interior shape and an outer shell…although there is no point in the simulation where the processes were altered to produce these two structures.
>
> The simulation stage to create the structure of *Aggregation 22* took 182 hours on a 3.2 GHz Pentium 4 processor. Rendering the image at a resolution of 8,192 by 8,192 pixels took 28 hours on the same machine.

Andy's home page is at www.andylomas.com.

—Bill Casselman, Graphics Editor
(notices-covers@ams.org)

Meetings & Conferences of the AMS

IMPORTANT INFORMATION REGARDING MEETINGS PROGRAMS: AMS Sectional Meeting programs do not appear in the print version of the *Notices*. However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on the AMS website. See `http://www.ams.org/meetings/`. Final programs for Sectional Meetings will be archived on the AMS website accessible from the stated URL and in an electronic issue of the *Notices* as noted below for each meeting.

Warsaw, Poland

University of Warsaw

July 31 – August 3, 2007

Tuesday – Friday

Meeting #1029

First Joint International Meeting between the AMS and the Polish Mathematical Society
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: May 2007
Program first available on AMS website: Not applicable
Program issue of electronic *Notices*: Not applicable
Issue of *Abstracts*: Not applicable

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/internmtgs.html`.

Invited Addresses

Henryk Iwaniec, Rutgers University, *Golden nuggets of sieve methods*.

Tomasz J. Luczak, Adam Mickiewicz University, *Title to be announced*.

Tomasz Mrowka, Massachusetts Institute of Technology, *Reflections on homological invariants for knots*.

Ludomir Newelski, University of Wroclaw, *Topological dynamics and model theory*.

Madhu Sudan, Massachusetts Institute of Technology, *List decoding: A survey*.

Anna Zdunik, Warsaw University, *Title to be announced*.

Special Sessions

Arithmetic Algebraic Geometry, **Grzegorz Banaszak**, Adam Mickiewicz University, **Eric Friedlander**, Northwestern University, **Wojciech Gajda**, Adam Mickiewicz University, **Piotr Krason**, Szczecin University, and **Wieslawa Nizio**.

Complex Analysis, **Zeljko Cuckovic**, University of Toledo, **Zbigniew Blocki**, Jagiellonian University, and **Marek Ptak**, University of Agriculture.

Complex Dynamics, **Robert Devaney**, Boston University, **Jane N. Hawkins**, University of North Carolina, and **Janina Kotus**, Warsaw University of Technology.

Complexity of Multivariate Problems, **Joseph F. Traub**, Columbia University, **Grzegoorz W. Wasilkowski**, University of Kentucky, and **Henryk Wozniakowski**, Columbia University.

Control and Optimization of Non-linear PDE Systems, **Irena Lasiecka**, University of Virginia, and **Jan Sokolowski**, Systems Research institute.

Dynamical Systems, **Steven Hurder**, University of Illinois at Chicago, **Michal Misiurewicz**, Indiana University-Purdue University Indianapolis, and **Pawel Walczak**, University of Lodz.

Dynamics, Control and Optimization of Finite Dimensional Systems: Theory and Applications to Biomedicine, **Urszula Forys**, Warsaw University, **Urszula Ledzewicz**, Southern Illinois University, and **Heinz Schaettler**, Washington University.

Ergodic Theory and Topological Dynamics, **Dan Rudolph**, Colorado State University, and **Mariusz Lemanczyk**, Nicholas Copernicus University.

Extremal and Probabilistic Combinatorics, **Joel Spencer**, New York University-Courant Institute, and **Michal Karonski** and **Andrzej Rucinski**, Adam Mickiewicz University.

Function Spaces, Theory of Operators and Geometry of Banach Spaces, **Henryk Hudzik**, Adam Mickiewicz University, **Anna Kaminska**, University of Memphis, and **Mieczyslaw Mastylo**.

Geometric Applications of Homotopy Theory, **Yuli B. Rudyak**, University of Florida, **Boguslaw Hajduk**, Warsaw University, **Jaroslaw Kedra**, University of Aberdeen, and **Aleksy Tralle**, The College of Economics & Comp. Science.

Geometric Function Theory, **Michael Dorff**, Brigham Young University, **Piotr Liczberski**, University of Lodz, **Maria Nowak**, Biblioteka Instytutu Matematyki, and **Ted Suffridge**, University of Kentucky.

Geometric Group Theory, **Mladen Bestvina**, University of Utah, **Tadeusz Januszkiewicz**, Ohio State University, and **Jacek Swiatkowski**, University of Wroclaw.

Geometric Topology, **Jerzy Dydak**, University of Tennessee, **Slawomir Nowak**, and **Stanislaw Spiez**, University of Warsaw.

Homotopy Methods in Algebra and Topology, **Wojciech Chacholski**, KTH, Stockholm, **Jan Spalinski**, Politechnika Warszawska, and **Michele Intermont**, Kalamazoo College.

Invariants of Links and 3-manifolds, **Mieczyslaw Dabkowski**, University of Texas at Dallas, **Jozef H. Przytycki**, George Washington University, **Adam S. Siroka**, State University of New York at Buffalo, and **Pawel Traczyk**, Warsaw University.

Issues in Reforming Mathematics Education, **Jeremy Kilpatrick**, University of Georgia, and **Zbigniew Semadeni**, University of Warsaw.

Mathematics of Large Quantum Systems, **Michael Loss**, Georgia Institute of Technology, **Jan Philip Solovej**, University of Copenhagen, and **Jan Derezinski**, University of Warsaw.

Noncommutative Geometry and Quantum Groups, **Paul Baum**, Pennsylvania State University, and **Ulrich Kraehmer** and **Tomasz Maszczyk**.

Partial Differential Equations of Evolution Type, **Susan J. Friedlander**, University of Illinois at Chicago, and **Grzegorz A. Karch**, University of Wroclaw.

Quantum Information Theory, **Robert Alicki**, University of Gdansk, and **Mary Beth Ruskai**, Tufts University.

Topological Fixed Point Theory and Related Topics, **Jerzy Jezierski**, University of Agriculture, **Wojciech Kryszewski**, Nicholas Copernicus University, and **Peter Wong**, Bates College.

Topology of Manifolds and Transformation Groups, **Slawomir Kwasik,** Tulane University, **Krzysztof Pawalowski,** Adam Mickiewicz University, and **Dariusz Wilczynski,** Utah State University.

Chicago, Illinois

DePaul University

October 5–6, 2007

Friday – Saturday

Meeting #1030

Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: August 2007
Program first available on AMS website: August 16, 2007
Program issue of electronic *Notices*: October 2007
Issue of *Abstracts*: Volume 28, Issue 3

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: June 19, 2007
For abstracts: August 7, 2007

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Martin Golubitsky, University of Houston, *Title to be announced.*

Matthew J. Gursky, University of Notre Dame, *Title to be announced.*

Alex Iosevich, University of Missouri, *Title to be announced.*

David E. Radford, University of Illinois at Chicago, *Title to be announced.*

Special Sessions

Algebraic Coding Theory (in Honor of Harold N. Ward's Retirement) (Code: SS 19A), **Jay A. Wood**, Western Michigan University.

Algebraic Combinatorics: Association Schemes and Related Topics (Code: SS 1A), **Sung Y. Song**, Iowa State University, and **Paul Terwilliger**, University of Wisconsin.

Algebraic Geometry (Code: SS 5A), **Lawrence Man Hou Ein** and **Anatoly S. Libgober**, University of Illinois at Chicago.

Algorithmic Probability and Combinatorics (Code: SS 22A), **Manuel Lladser**, University of Colorado, and **Robert S. Maier**, University of Arizona.

Analysis and CR geometry (Code: SS 12A), **Song-Ying Li**, University of California Irvine, and **Stephen S-T. Yau**, University of Illinois at Chicago.

Applied Harmonic Analysis (Code: SS 13A), **Jonathan Cohen** and **Ahmed I. Zayed**, DePaul University.

Automorphic Forms: Representation Theory of p-adic and Adelic Groups (Code: SS 8A), **Mahdi Asgari** and **Anantharam Raghuram**, Oklahoma State University.

Differential Geometry and its Applications (Code: SS 17A), **Jianguo Cao**, University of Notre Dame.

Ergodic Theory and Symbolic Dynamical Systems (Code: SS 7A), **Ayse A. Sahin** and **Ilie D. Ugarcovici**, DePaul University.

Extremal and Probabilistic Combinatorics (Code: SS 3A), **Jozsef Balogh**, University of Illinois at Urbana-Champaign, and **Dhruv Mubayi**, University of Illinois at Chicago.

Free Resolutions (Code: SS 21A), **Noam Horwitz** and **Irena Peeva**, Cornell University.

Geometric Combinatorics (Code: SS 15A), **Caroline J. Klivans**, University of Chicago, and **Kathryn Nyman**, Loyola University Chicago.

Graph Theory (Code: SS 20A), **Hemanshu Kaul** and **Michael J. Pelsmajer**, Illinois Institute of Technology.

Hopf Algebras and Related Areas (Code: SS 2A), **Yevgenia Kashina** and **Leonid Krop**, DePaul University, **M. Susan Montgomery**, University of Southern California, and **David E. Radford**, University of Illinois at Chicago.

Mathematical Modeling and Numerical Methods (Code: SS 16A), **Atife Caglar**, University of Wisconsin-Green Bay.

Model Theory of Non-elementary Classes (Code: SS 23A), **John T. Baldwin**, University of Illinois at Chicago, **David W. Kueker**, University of Maryland, and **Rami Grossberg**, Carnegie Mellon University.

Nonlinear Conservation Laws and Related Problems (Code: SS 11A), **Cleopatra Christoforou** and **Gui-Qiang Chen**, Northwestern University.

Numerical and Symbolic Techniques in Algebraic Geometry and Its Applications (Code: SS 18A), **GianMario Besana**, DePaul University, **Jan Verschelde**, University of Illinois at Chicago, and **Zhonggang Zeng**, Northeastern Illinois University.

Sequence Spaces and Transformations (Code: SS 10A), **Constantine Georgakis**, DePaul University, and **Martin Buntinas**, Loyola University of Chicago.

Singular Integrals and Related Problems (Code: SS 14A), **Laura De Carli**, Florida International University, and **A. M. Stokolos**, DePaul University.

Smooth Dynamical Systems (Code: SS 6A), **Marian Gidea**, Northeastern Illinois University, and **Ilie D. Ugarcovici**, DePaul University.

The Euler and Navier-Stokes Equations (Code: SS 4A), **Alexey Cheskidov**, University of Michigan, and **Susan J. Friedlander** and **Roman Shvydkoy**, University of Illinois at Chicago.

Wave Propagation from Mathematical and Numerical Viewpoints (Code: SS 9A), **Gabriel Koch**, University of Chicago, **Catalin Constantin Turc**, Caltech and University of North Carolina at Charlotte, and **Nicolae Tarfulea**, Purdue University Calumet.

New Brunswick, New Jersey

Rutgers University-New Brunswick, College Avenue Campus

October 6–7, 2007

Saturday – Sunday

Meeting #1031

Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: August 2007
Program first available on AMS website: August 16, 2007
Program issue of electronic *Notices*: October 2007
Issue of *Abstracts*: Volume 28, Issue 3

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: June 19, 2007
For abstracts: August 7, 2007

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Satyan L. Devadoss, Williams College, *Title to be announced.*

Tara S. Holm, University of Connecticut, *Title to be announced.*

Sir Roger Penrose, University of Oxford, *Title to be announced* (Einstein Public Lecture in Mathematics).

Scott Sheffield, Courant Institute and Institute for Advanced Study, *Title to be announced.*

Mu-Tao Wang, Columbia University, *Title to be announced.*

Special Sessions

Commutative Algebra (Code: SS 4A), **Jooyoun Hong**, University of California Riverside, and **Wolmer V. Vasconcelos**, Rutgers University.

Geometric Analysis of Complex Laplacians (Code: SS 8A), **Siqi Fu**, Rutgers University, Camden, **Xiaojun Huang**, Rutgers University, New Brunswick, and **Howard J. Jacobowitz**, Rutgers University, Camden.

Invariants of Lie Group Actions and Their Quotients (Code: SS 9A), **Tara S. Holm**, Cornell University, and **Rebecca F. Goldin**, George Mason University.

Mathematical and Physical Problems in the Foundations of Quantum Mechanics (in honor of Shelly Goldstein's 60th birthday) (Code: SS 3A), **Roderich Tumulka** and **Detlef Dürr**, München University, and **Nino Zanghi**, University of Genova.

Noncommutative Geometry and Arithmetic Geometry (Code: SS 10A), **Caterina Consani**, Johns Hopkins University, and **Li Guo**, Rutgers University.

Partial Differential Equations in Mathematical Physics (in honor of Shelly Goldstein's 60th birthday) (Code: SS 2A), **Sagun Chanillo**, **Michael K.-H. Kiessling**, and **Avy Soffer**, Rutgers University.

Partial Differential Equations of Mathematical Physics, I (dedicated to the memory of Tom Branson) (Code: SS 7A), **Sagun Chanillo**, **Michael K.-H. Kiessling**, and **Avy Soffer**, Rutgers University.

Probability and Combinatorics (Code: SS 1A), **Jeffry N. Kahn** and **Van Ha Vu**, Rutgers University.

Set Theory of the Continuum (Code: SS 5A), **Simon R. Thomas**, Rutgers University.

Toric Varieties (Code: SS 6A), **Milena S. Hering**, Institute for Mathematics and Its Applications, and **Diane Maclagan**, Rutgers University.

Albuquerque, New Mexico

University of New Mexico

October 13–14, 2007

Saturday – Sunday

Meeting #1032

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: August 2007
Program first available on AMS website: August 30, 2007
Program issue of electronic *Notices*: October 2007
Issue of *Abstracts*: Volume 28, Issue 4

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: June 26, 2007
For abstracts: August 21, 2007

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Emmanuel Candes, California Institute of Technology, *Title to be announced*.

Alexander Polischuk, University of Oregon, *Title to be announced*.

Eric Raines, University of California Davis, *Title to be announced*.

William E. Stein, University of California San Diego, *SAGE: Software for Algebra and Geometry Experimentation*.

Special Sessions

Affine Algebraic Geometry (Code: SS 2A), **David Robert Finston**, New Mexico State University.

Arithmetic and Algebraic Geometry (Code: SS 10A), **Alexandru Buium** and **Michael J. Nakamaye**, University of New Mexico.

Computational Applications of Algebraic Topology (Code: SS 6A), **Ross Staffeldt**, New Mexico State University.

Computational Methods in Harmonic Analysis and Signal Processing (Code: SS 1A), **Emmanuel Candes**, California Institute of Technology, and **Joseph D. Lakey**, New Mexico State University.

Financial Mathematics: The Mathematics of Financial Markets and Structures (Code: SS 12A), **Cristina Mariani** and **Kenneth Martin**, New Mexico State University.

Geometric Structures on Manifolds (Code: SS 11A), **Charles Boyer** and **Krzysztof Galicki**, University of New Mexico.

Harmonic Analysis Applied to Partial Differential Equations (Code: SS 7A), **Justin Homer**, University of California Berkeley, **Changxing Miao**, Institute of Applied Physics and Computational Mathematics, and **Jiaong Wu**, Oklahoma State University.

Harmonic Analysis and Operator Theory (Code: SS 9A), **Maria C. Pereyra** and **Wilfredo O. Urbina**, University of New Mexico.

Mathematical and Computational Aspects of Compressible Flow Problems (Code: SS 8A), **Jens Lorenz** and **Thomas M. Hagstrom**, University of New Mexico.

Methods of Heterogeneous Data Analysis (Code: SS 14A), **Hanna Ewa Makaruk**, Los Alamos National Laboratory, and **Nikita A. Sakhanenko**, University of New Mexico.

Nonlinear Waves in Optics, Hydrodynamics and Plasmas (Code: SS 13A), **Alejandro Aceves** and **Pavel Lushnikov**, University of New Mexico.

Recent Developments in 2-D Turbulence (Code: SS 3A), **Michael S. Jolly**, Indiana University, and **Greg Eyink**, Johns Hopkins University.

Topics in Mathematical Physics (Code: SS 4A), **Rafal Komendarczyk**, University of Pennsylvania, and **Robert Michal Owczarek**, Los Alamos National Laboratory.

Variational Problems in Condensed Matter (Code: SS 5A), **Lia Bronsard**, McMaster University, and **Tiziana Giorgi**, New Mexico State University.

Murfreesboro, Tennessee

Middle Tennessee State University

November 3–4, 2007

Saturday – Sunday

Meeting #1033

Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: September 2007
Program first available on AMS website: September 20, 2007
Program issue of electronic *Notices*: November 2007
Issue of *Abstracts*: Volume 28, Issue 4

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: July 17, 2007
For abstracts: September 11, 2007

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Sergey Gavrilets, University of Tennessee, *Mathematical models of speciation*.

Daniel K. Nakano, University of Georgia, *Bridging algebra and geometry via cohomology*.

Carla D. Savage, North Carolina State University, *The mathematics of lecture hall partitions.*

Sergei Tabachnikov, Pennsylvania State University, *Ubiquitous billiards.*

Special Sessions

Advances in Algorithmic Methods for Algebraic Structures (Code: SS 3A), **James B. Hart**, Middle Tennessee State University.

Applied Partial Differential Equations (Code: SS 4A), **Yuri A. Melnikov**, Middle Tennessee State University, and **Alain J. Kassab**, University of Central Florida.

Billiards and Related Topics (Code: SS 6A), **Sergei Tabachnikov**, Pennsylvania State University, and **Richard Schwartz**, Brown University.

Combinatorial Enumeration, Optimization, Geometry, and Statistics (Code: SS 13A), **Nicholas A. Loehr**, College of William and Mary, **Gabor Pataki**, University of North Carolina, Chapel Hill, **Margaret A. Readdy**, University of Kentucky and M.I.T., **Carla D. Savage**, North Carolina State University, and **Ruriko Yoshida**, University of Kentucky.

Combinatorial Methods in Continuum Theory (dedicated to Jo Heath, Auburn University, on the occasion of her retirement) (Code: SS 8A), **Judy A. Kennedy**, University of Delaware and Lamar University, **Krystyna M. Kuperberg**, Auburn University, and **Van C. Nall**, University of Richmond.

Differential Equations and Dynamical Systems (Code: SS 1A), **Wenzhang Huang** and **Jia Li**, University of Alabama, Huntsville, and **Zachariah Sinkala**, Middle Tennessee State University.

Financial Mathematics (Code: SS 16A), **Abdul Khaliq**, Middle Tennessee State University.

Graph Theory (Code: SS 2A), **Rong Luo**, **Don Nelson**, **Chris Stephens**, and **Xiaoya Zha**, Middle Tennessee State University.

Lie and Representation Theory (Code: SS 11A), **Terrell L. Hodge**, University of Virginia and Western Michigan University, **Daniel K. Nakano**, University of Georgia, and **Brian J. Parshall**, University of Virginia.

Mathematical Modeling in Biological Systems (Code: SS 9A), **Terrence J. Quinn**, Middle Tennessee State University.

Mathematical Tools for Survival Analysis and Medical Data Analysis (Code: SS 7A), **Curtis Church**, Middle Tennessee State University, **Chang Yu**, Vanderbilt University, and **Ping Zhang**, Middle Tennessee State University.

Nonlinear Partial Differential Equations and Applications (Code: SS 14A), **Emmanuele DiBenedetto**, **Mikhail Perepelitsa**, and **Gieri Simonett**, Vanderbilt University.

Physical Knots and Links (Code: SS 10A), **Yuanan Diao**, University of North Carolina at Charlotte, and **Claus Ernst**, Western Kentucky University.

Recent Advances in Algebraic Topology (Code: SS 12A), **Mark W. Johnson**, Pennsylvania State University, Altoona, and **Donald Yau**, The Ohio State University at Newark.

Splines and Wavelets with Applications (Code: SS 5A), **Don Hong**, Middle Tennessee State University, and **Qingtang Jiang**, University of Missouri-St. Louis.

Using National Assessment of Educational Progress (NAEP) Data to Enhance Assessment and Inform Instruction (Code: SS 15A), **Michaele F. Chappell**, Middle Tennessee State University, and **Judith H. Hector**, Walters State Community College.

Wellington, New Zealand

Victoria University of Wellington

December 12–15, 2007

Wednesday – Saturday

Meeting #1034

First Joint International Meeting between the AMS and the New Zealand Mathematical Society (NZMS).
Associate secretary: Matthew Miller
Announcement issue of *Notices*: June 2007
Program first available on AMS website: Not applicable
Program issue of electronic *Notices*: Not applicable
Issue of *Abstracts*: Not applicable

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: September 28, 2007
For abstracts: October 31, 2007

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/internmtgs.html.

AMS Invited Addresses

Marston Conder, University of Auckland, *Chirality.*

Rodney G. Downey, Victoria University of Wellington, *Practical FPT and foundations of kernelization.*

Michael H. Freedman, Microsoft Research/University of California Santa Barbara, *Physically motivated questions in topology: manifold pairings.*

Gaven J. Martin, Massey University, *Curvature and dynamics.*

Assaf Naor, Microsoft Research/Courant Institute, *Title to be announced.*

Theodore A. Slaman, University of California Berkeley, *Title to be announced.*

Matthew J. Visser, Victoria University of Wellington, *Title to be announced.*

AMS Special Sessions

Computability Theory, **Rodney G. Downey** and **Noam Greenberg**, Victoria University of Wellington.

Dynamical Systems and Ergodic Theory, **Arno Berger**, University of Canterbury, **Rua Murray**, University of Waikato, and **Matthew J. Nicol**, University of Houston.

Geometric Numerical Integration, **Laurent O. Jay**, The University of Iowa, and **Robert McLachlan**, Massey University.

Group Theory, Actions, and Computation, **Marston Conder**, University of Auckland, and **Russell Blyth**, Saint Louis University.

History and Philosophy of Mathematics, **James J. Tattersall**, Providence College, **Ken Pledger**, Victoria University of Wellington, and **Clemency Williams**, University of Canterbury.

Hopf Algebras and Quantum Groups, **M. Susan Montgomery**, University of Southern California, and **Yinhuo Zhang**, Victoria University of Wellington.

Infinite-Dimensional Groups and Their Actions, **Christopher Atkin**, Victoria University of Wellington, **Greg Hjorth**, University of California Los Angeles/University of Melbourne, **Alica Miller**, University of Louisville, and **Vladimir Pestov**, University of Ottawa.

Integrability of Continuous and Discrete Evolution Systems, **Mark Hickman**, University of Canterbury, and **Willy A. Hereman**, Colorado School of Mines.

Mathematical Models in Biomedicine, **Ami Radunskaya**, Pomona College, **James Sneyd**, University of Auckland, **Urszula Ledzewicz**, University of Southern Illinois at Edwardsville, and **Heinz Schaettler**, Washington University.

Matroids, Graphs, and Complexity, **Dillon Mayhew**, Victoria University of Wellington, and **James G. Oxley**, Louisiana State University.

New Trends in Spectral Analysis and Partial Differential Equations, **Boris P. Belinskiy**, University of Tennessee, Chattanooga, **Anjan Biswas**, Delaware State University, and **Boris Pavlov**, University of Auckland.

Quantum Topology, **David B. Gauld**, University of Auckland, and **Scott E. Morrison**, University of California Berkeley.

Special Functions and Orthogonal Polynomials, **Shaun Cooper**, Massey University, **Diego Dominici**, SUNY New Paltz, and **Sven Ole Warnaar**, University of Melbourne.

Water-Wave Scattering Focusing on Wave-Ice Interactions, **Michael H. Meylan**, Massey University, and **Malte Peter**, University of Bremen.

Conference Website and Contact Information

This announcement was composed with information taken from the website maintained by the local organizers at `http://www.mcs.vuw.ac.nz/%7Emathmeet/amsnzms2007/index.shtml`. Please note that when dialing the telephone numbers listed below from the U.S., one should dial 011 to connect to the international network, then 64 (country code for New Zealand), then 4 (Wellington city code).

Abstract Submission

Talks are invited in any area of the mathematical sciences, however acceptance is at the discretion of the organizers. If your proposed talk falls under the heading of one of the special sessions please contact one of the session organizers before submitting an abstract, as Special Sessions have limited time slots. Talks not in Special Sessions will be timetabled into general contribution sessions. Talks should be of 20 minutes duration, unless an invitation to give a longer plenary or keynote talk has been given.

Titles and abstracts must be submitted to Atlas Conference at `http://atlas-conferences.com/cgi-bin/abstract/submit/catm-01`. **The submission deadline for all abstracts is October 31**.

Accommodations

Weir House (student hostel), Victoria University of Wellington, Gladstone Terrace, off Salamanca Road, Kelburn. There are a large number of single rooms and a smaller number of double rooms and self-contained flats which are about a five-minute walk from the university. Rooms are available from the 11th till the 16th of December (six nights). It may be possible to extend this at either end by special request. The single and double room rate (bed and breakfast) is NZ$40 per person per night. The rate for the flats is NZ$90 per night. Some amenities include TV rooms, snooker/pool room, pianos, kitchenettes, and video/DVD player. Nearby there are tennis and squash courts, croquet green, Kelburn Park, Anderson Park, and the cable car to the city centre. Bookings can be made when you register for the conference.

Quest Atrium, 154 The Terrace, Wellington City, New Zealand. Studios and one- and two-bedroom apartments are available at this property, about a 10-minute walk from the university and close to the center of the city. Reservation of an apartment can be made by emailing Ginny Nikorima-Robinson, `ginny.nikorima@mcs.vuw.ac.nz`. Rates are studio/NZ$143–152, one bedroom/NZ$163, two bedroom/NZ$196 per night. Payment is made directly to Quest Atrium. This property is situated on one of Wellington's most prestigious central business district locations. The apartments are within easy access of Lambton Quay, Parliament, boutique shopping, cafes, restaurants, and theaters. Within walking distance are the waterfront, Te Papa Museum, Victoria University, and the Botanical Gardens. Apartment features include a fully equipped kitchen, laundry, lounge with TV, video and stereo, tiled balcony, ironing board and iron. High speed wireless broadband Internet is available for an extra charge.

Other hotels recommended by local organizers are listed below; participants should make their own arrangements directly. The AMS is not responsible for rate changes or for the quality of the accommodations. When making a reservation, participants should state that they are with the AMS-NZMS Meeting at Victoria University of Wellington. **Cancellation and early checkout penalties vary with each hotel; be sure to check the policy when you make your reservations**.

Bolton Hotel, Corner Bolton & Mowbray Streets, Wellington, New Zealand, Reservations (NZ only) 0800-99-66-22, international +64-4-472-9966, Fax +64-4-472-9955, `info@boltonhotel.co.nz` or `http://www.boltonhotel.co.nz/rooms.html`. Rooms feature high-speed Internet (including WiFi), a personal bar, and air conditioning. Most enjoy views of the adjacent park or downtown Wellington. Suites have separate bedroom, dining and living areas, laundry facilities, and kitchen with refrigerator/freezer, microwave, stovetop, oven, and dishwasher. Many package rates are available.

James Cook Hotel Grand Chancellor Wellington, 147 The Terrace, Wellington, New Zealand, +64-4-499-9500, fax +64-4-499-9800 or reservations@jamescookhotel.co.nz, Standard rooms at NZ$185 have one double bed and one single bed, couch, mini bar, high speed Internet access, desk, air conditioning, and tea/coffee making facilities, with harbor or city views. Suites are available.

Duxton Hotel Wellington, 170 Wakefield St., Wellington, New Zealand, +64-4-473-3900, fax +64-4-473-3929 or res@wellington.duxton.co.nz. Rooms range from NZ$195 to $400 and are equipped with multi-channel television, desk, guest Internet access, coffeemakers, personal bar, marble bathroom. Most rooms have a spectacular view over Wellington Harbour.

Mercure Hotel Wellington, 345 The Terrace, Wellington, New Zealand, +64-4-385-9829, fax +64-4-385-2119 or H1991-RE01@accor.com or http://www.mercure.co.nz. Studio rooms range from NZ$99-$195, and feature desktop modem jack, personal voicemail and opening windows. Facilities include free guest parking, indoor heated pool, Sky (cable) TV, gym, sauna, bar/restaurant. Close to the university.

Novotel Capital Wellington, 133-137 The Terrace, Wellington, New Zealand, +64-4-918-1900, Fax +64-4-918-1901 or H3276-RE01@accor.com. See http://www.novotel.co.nz. In-house restaurant and bar, gymnasium, valet parking, Internet access, refrigerator, twin beds. Rooms range from NZ$223 to $421.

Capital View Motor Inn, CNR Webb & Thompson St., Wellington, New Zealand, toll free (NZ only) 0800-43-85-05, phone/fax +64-4-385-0515 or accommodation@capitalview.co.nz. See http://www.capitalview.co.nz. Studio and one-bedroom apartments with fully equpped kitchens with microwave. One or two persons/NZ$95-$180, extra person/NZ$20.

For backpacking accommodations and hostels visit http://www.backpack.co.nz. Additional bed and breakfast options can be found at http://www.bedandbreakfast.collection.co.nz.

Conference Location and Preliminary Program

All scientific sessions and events will take place on the Kelburn campus of the Victoria University of Wellington in New Zealand's capital city, from Wednesday through Saturday, December 12-15, 2007. A preliminary program sketch may be found at http://www.mcs.vuw.ac.nz/~mathmeet/amsnzms2007/timetable.

Local Information/Tourism

New Zealand offers a huge selection of high-quality tourist attractions and activities for a vacation while in New Zealand. A list of some of these attractions can be found at http://www.tourism.net.nz/attractions.

The electrical standard in New Zealand is 230 volts AC at 50 Hz. Power sockets use a three-pin plug and are compatible with Australian power sockets. Please check before you plug in any appliance or equipment that you bring with you that it can operate safely with 230VAC/50Hz power.

Exchange rates at the time this announcement went to press were US$1= $1.37 NZ. See http://www.oanda.com/convert/classic for up-to-date rates.

The Victoria University of Wellington School of Mathematics, Statistics and Computer Science webpage is at http://www.mcs.vuw.ac.nz.

Registration and Conference Dinner

On-line registration is available at https://www.mcs.vuw.ac.nz/cgi-bin/events/amsnzms2007-registration. **The deadline for registration is October 31, 2007**. Registration includes morning and afternoon refreshments. Regular registration is NZ$140 and student registration is NZ$40. Payment must be made by VISA or MasterCard or a NZ check or bank draft. The conference dinner will be held at the Skyline, a wonderful restaurant with spectacular harbor, hilltop, and cityscape views, on Friday evening, December 14. The cost per person is NZ$60 (NZ$40 for students). See http://www.theskyline.co.nz.

Student Travel Grants

The organizers are pleased to offer limited travel grants available to New Zealand and Pacific students wishing to attend the meeting. Please send a brief letter of application which includes: (1) your name, (2) your home institution, (3) your thesis supervisor's name and email address, (4) details of your travel and an estimate of the expenses you will incur. Send to Dr. Peter Donelan, School of Mathematics, Statistics and Computer Science, Victoria University of Wellington, PO Box 600, Wellington 6140, New Zealand.

Applications should be made by September 30. Decisions on grants will be made shortly after that.

Travel

U.S. nationals do not require a visa to enter New Zealand for a period of less than three months. You may obtain up-to-date information on immigration and visas at http://www.immigration.govt.nz/migrant/stream/visit/.

Participants should plan to fly to Wellington International Airport (WLG) but, unless traveling via Australia, will probably arrive in New Zealand at either Auckland (AKL) or Christchurch (CHC) International Airports and transfer there to a domestic flight. At Auckland a free shuttle is available between the international and domestic terminals or transferring passengers can take the short 10-minute walk, which is clearly marked. Christchurch and Wellington have single terminals catering to international and domestic flights. Alternatively you may want to travel the scenic routes to Wellington by car, bus, train, and/or ferry. In that case you should allow two days for comfort.

At Wellington Airport, taxi stands are located directly out of the ground floor entrance of the terminal. A taxi from the airport to the city will cost between NZ$25-NZ$35, and will take around 20 minutes, though it may take longer during Wellington's rush hour (between 4:30 p.m. and 5:30 p.m.). A number of companies run door-to-door shuttle services from the airport to Wellington city. Shuttles are located on the ground floor level at the southern end of the terminal. The airport-to-city trip costs around NZ$15, although couples or groups may qualify for discounts. The trip to the city should take around 30 minutes, depending on where passengers are dropped off. The Airport Flyer is a bus service between the airport, Wellington City, and the Hutt Valley. The bus stop is on the ground floor level at the southern end of the terminal. Buses depart approximately every 30 minutes, but hourly in the early morning and at night during weekends. The fare from the airport to Wellington city is NZ$4.50 for adults and NZ$2 for children. The Stagecoach Flyer makes a number of stops including all bus stops through the city center. The trip to the city takes about 45 minutes.

See `www.tourism.net.nz` and `www.wellingtonnz.com` for more information. Cars can be hired for around NZ$50-80 per day, depending on vehicle and length of hire.

Weather

December is early summer in New Zealand, and the weather is generally mild and sunny but can be damp. See `www.wellingtonnz.com/AboutWellington/Weather.htm`.

San Diego, California

San Diego Convention Center

January 6–9, 2008

Sunday – Wednesday

Meeting #1035

Joint Mathematics Meetings, including the 114th Annual Meeting of the AMS, 91st Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: October 2007
Program first available on AMS website: November 1, 2007
Program issue of electronic *Notices*: January 2008
Issue of *Abstracts*: Volume 29, Issue 1

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: July 26, 2007
For abstracts: September 20, 2007

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/2109_intro.html`.

AMS-MAA Invited Addresses

Terence Chi-Shen Tao, UCLA, *Structure and randomness in the prime numbers.*

AMS Invited Addresses

James G. Arthur, University of Toronto, *Title to be announced* (AMS Retiring Presidential Address).

Constantine M. Dafermos, Brown University, *Title to be announced.*

Wen-Ching Winnie Li, National Tsing Hua University and Pennsylvania State University, *Title to be announced.*

Donald G. Saari, University of California Irvine, *Title to be announced.*

Peter Teichner, Universityof California Berkeley, *Title to be announced.*

Wendelin Werner, University of Paris-Sud, *Title to be announced* (Colloquium Lectures).

Avi Wigderson, Hebrew University of Jerusalem, *Title to be announced* (AMS Josiah Willard Gibbs Lecture).

New York, New York

Courant Institute of New York University

March 15–16, 2008

Saturday – Sunday

Meeting #1036

Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: January 2008
Program first available on AMS website: January 31, 2008
Program issue of electronic *Notices*: March 2008
Issue of *Abstracts*: Volume 29, Issue 2

Deadlines

For organizers: August 15, 2007
For consideration of contributed papers in Special Sessions: November 27, 2007
For abstracts: January 22, 2008

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Special Sessions

L-Functions and Automorphic Forms (Code: SS 1A), **Alina Bucur**, Institute for Advanced Study, **Ashay Venkatesh**, Courant Institute of Mathematical Sciences, **Stephen D. Miller**, Rutgers University, and **Steven J. Miller**, Brown University.

Baton Rouge, Louisiana

Louisiana State University, Baton Rouge

March 28–30, 2008
Friday – Sunday

Meeting #1037
Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: February 2008
Program first available on AMS website: February 14, 2008
Program issue of electronic *Notices*: March 2008
Issue of *Abstracts*: Volume 29, Issue 2

Deadlines
For organizers: August 28, 2007
For consideration of contributed papers in Special Sessions: December 11, 2007
For abstracts: February 5, 2008

Bloomington, Indiana

Indiana University

April 4–6, 2008
Friday – Sunday

Meeting #1038
Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: February 2008
Program first available on AMS website: February 21, 2008
Program issue of electronic *Notices*: April 2008
Issue of *Abstracts*: Volume 29, Issue 3

Deadlines
For organizers: September 4, 2007
For consideration of contributed papers in Special Sessions: December 18, 2007
For abstracts: February 12, 2008

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Shi Jin, University of Wisconsin, *Title to be announced.*

Michael J. Larsen, Indiana University, *Title to be announced.*

Mircea Mustata, University of Michigan, *Title to be announced.*

Margaret H. Wright, New York University-Courant Institute, *Title to be announced.*

Special Sessions

Birational Algebraic Geometry (Code: SS 3A), **Mircea I. Mustata**, University of Michigan, and **Mihnea Popa**, University of Chicago.

Combinatorial and Geometric Aspects of Commutative Algebra (Code: SS 1A), **Juan Migliore**, University of Notre Dame, and **Uwe Nagel**, University of Kentucky.

Hyperbolic and Kinetic Equations (Code: SS 2A), **Shi Jin**, University of Wisconsin, and **Marshall Slemrod**, University of Wisconsin.

Weak Dependence in Probability and Statistics (Code: SS 4A), **Richard C. Bradley**, University of Indiana, and **Lahn T. Tran**, University of Indiana.

Claremont, California

Claremont McKenna College

May 3–4, 2008
Saturday – Sunday

Meeting #1039
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: March 2008
Program first available on AMS website: March 20, 2008
Program issue of electronic *Notices*: May 2008
Issue of *Abstracts*: Volume 29, Issue 3

Deadlines
For organizers: October 4, 2007
For consideration of contributed papers in Special Sessions: January 15, 2008
For abstracts: March 11, 2008

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Michael Bennett, University of British Columbia, *Title to be announced.*

Chandrashekhar Khare, University of Utah, *Title to be announced.*

Huaxin Lin, University of Oregon, *Title to be announced.*

Anne Schilling, University of California Davis, *Title to be announced.*

Special Sessions

Diophantine Problems and Discrete Geometry (Code: SS 3A), **Matthias Beck**, San Francisco State University, and **Lenny Fukshansky**, Texas A&M University.

Dynamical Systems and Differential Equations (Code: SS 1A), **Adolfo Rumbos**, Pomona College, **Mario Martelli**, Claremont McKenna College, and **Alfonso Castro**, Harvey Mudd College.

Operators, Functions and Linear Spaces (Code: SS 2A), **Asuman G. Aksoy**, Claremont McKenna College, **Stephan R. Garcia**, Pomona College, **Michael Davlin O'Neill**, Claremont McKenna College, and **Winston C. Ou**, Scripps College.

Rio de Janeiro, Brazil

Instituto Nacional de Matemática Pura e Aplicada (IMPA)

June 4–7, 2008

Wednesday – Saturday

Meeting #1040

First Joint International Meeting between the AMS and the Sociedade Brasileira de Matemática.
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: February 2008
Program first available on AMS website: Not applicable
Program issue of electronic *Notices*: Not applicable
Issue of *Abstracts*: Not applicable

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Vancouver, Canada

University of British Columbia and the Pacific Institute of Mathematical Sciences (PIMS)

October 4–5, 2008

Saturday – Sunday

Meeting #1041

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: August 2008
Program first available on AMS website: August 21, 2008
Program issue of electronic *Notices*: October 2008
Issue of *Abstracts*: Volume 29, Issue 4

Deadlines

For organizers: March 9, 2008
For consideration of contributed papers in Special Sessions: June 17, 2008
For abstracts: August 12, 2008

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Richard Kenyon, University of British Columbia, *Title to be announced.*

Alexander S. Kleshchev, University of Oregon, *Title to be announced.*

Mark Lewis, University of Alberta, *Title to be announced.*

Audrey A. Terras, University of California San Diego, *Title to be announced.*

Middletown, Connecticut

Wesleyan University

October 11–12, 2008

Saturday – Sunday

Meeting #1042

Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: August 2008
Program first available on AMS website: August 28, 2008
Program issue of electronic *Notices*: October 2008
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 11, 2008
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: Volume 29, Issue 4

Kalamazoo, Michigan

Western Michigan University

October 17–19, 2008

Friday – Sunday

Meeting #1043

Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: August 2008
Program first available on AMS website: September 4, 2008
Program issue of electronic *Notices*: October 2008
Issue of *Abstracts*: Volume 29, Issue 4

Deadlines

For organizers: March 17, 2008
For consideration of contributed papers in Special Sessions: July 1, 2008

For abstracts: August 26, 2008

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

M. Carme Calderer, University of Minnesota, *Title to be announced.*

Alexandru Ionescu, University of Wisconsin, *Title to be announced.*

David Nadler, Princeton University, *Title to be announced.*

Huntsville, Alabama

University of Alabama, Huntsville

October 24–26, 2008

Friday – Sunday

Meeting #1044

Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: August 2008
Program first available on AMS website: September 11, 2008
Program issue of electronic *Notices*: October 2008
Issue of *Abstracts*: Volume 29, Issue 4

Deadlines

For organizers: March 24, 2008
For consideration of contributed papers in Special Sessions: July 8, 2008
For abstracts: September 2, 2008

Shanghai, People's Republic of China

Fudan University

December 17–21, 2008

Wednesday – Sunday

Meeting #1045

First Joint International Meeting between the AMS and the Shanghai Mathematical Society
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: June/July 2008
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/internmtgs.html.

Invited Addresses

L. Craig Evans, University of California Berkeley, *Title to be announced.*

Zhi-Ming Ma, Chinese Academy of Sciences, *Title to be announced.*

Richard Schoen, Stanford University, *Title to be announced.*

Richard Taylor, Harvard University, *Title to be announced.*

Xiaoping Yuan, Fudan University, *Title to be announced.*

Weiping Zhang, Chern Institute, *Title to be announced.*

Washington, District of Columbia

Marriott Wardman Park Hotel and Omni Shoreham Hotel

January 7–10, 2009

Wednesday – Saturday

Joint Mathematics Meetings, including the 115th Annual Meeting of the AMS, 92nd Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: October 2008
Program first available on AMS website: November 1, 2008
Program issue of electronic *Notices*: January 2009
Issue of *Abstracts*: Volume 30, Issue 1

Deadlines

For organizers: April 1, 2008
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Urbana, Illinois

University of Illinois at Urbana-Champaign

March 27–29, 2009
Friday – Sunday
Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines
For organizers: August 29, 2008
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Raleigh, North Carolina

North Carolina State University

April 4–5, 2009
Saturday – Sunday
Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines
For organizers: September 4, 2008
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

San Francisco, California

San Francisco State University

April 25–26, 2009
Saturday – Sunday
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines
For organizers: September 25, 2008
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Boca Raton, Florida

Florida Atlantic University

October 30 – November 1, 2009
Friday – Sunday
Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines
For organizers: March 30, 2009
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Riverside, California

University of California Riverside

November 7–8, 2009
Saturday – Sunday
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines
For organizers: Expired
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

San Francisco, California

Moscone Center West and the San Francisco Marriott

January 6–9, 2010

Wednesday – Saturday

Joint Mathematics Meetings, including the 116th Annual Meeting of the AMS, 93rd Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society of Industrial and Applied Mathematics (SIAM).

Associate secretary: Matthew Miller
Announcement issue of *Notices*: October 2009
Program first available on AMS website: November 1, 2009
Program issue of electronic *Notices*: January 2010
Issue of *Abstracts*: Volume 31, Issue 1

Deadlines

For organizers: April 1, 2009
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Lexington, Kentucky

University of Kentucky

March 27–28, 2010

Saturday – Sunday

Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: August 28, 2009
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

New Orleans, Louisiana

New Orleans Marriott and Sheraton New Orleans Hotel

January 5–8, 2011, *Wednesday – Saturday*

Joint Mathematics Meetings, including the 117th Annual Meeting of the AMS, 94th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: October 2010
Program first available on AMS website: November 1, 2010
Program issue of electronic *Notices*: January 2011
Issue of *Abstracts*: Volume 32, Issue 1

Deadlines

For organizers: April 1, 2010
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Boston, Massachusetts

John B. Hynes Veterans Memorial Convention Center, Boston Marriott Hotel, and Boston Sheraton Hotel

January 4–7, 2012,*Wednesday – Saturday*

Joint Mathematics Meetings, including the 118th Annual Meeting of the AMS, 95th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: October 2011
Program first available on AMS website: November 1, 2011
Program issue of electronic *Notices*: January 2012
Issue of *Abstracts*: Volume 33, Issue 1

Deadlines

For organizers: April 1, 2011

For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

San Diego, California

San Diego Convention Center and San Diego Marriott Hotel and Marina

January 9–12, 2013, *Wednesday – Saturday*

Joint Mathematics Meetings, including the 119th Annual Meeting of the AMS, 96th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 1, 2012
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Baltimore, Maryland

Baltimore Convention Center

January 15–18, 2014, *Wednesday – Saturday*

Joint Mathematics Meetings, including the 120th Anual Meeting of the AMS, 97th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Matthew Miller
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 1, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Meetings and Conferences of the AMS

Associate Secretaries of the AMS

Western Section: Michel L. Lapidus, Department of Mathematics, University of California, Sproul Hall, Riverside, CA 92521-0135; e-mail: lapidus@math.ucr.edu; telephone: 951-827-5910.

Central Section: **Susan-J. Friedlander**, Department of Mathematics, University of Illinois at Chicago, 851 S. Morgan (M/C 249), Chicago, IL 60607-7045; e-mail: susan@math.nwu.edu; telephone: 312-996-3041.

Eastern Section: **Lesley-M. Sibner**, Department of Mathematics, Polytechnic University, Brooklyn, NY 11201-2990; e-mail: lsibner@duke.poly.edu; telephone: 718-260-3505.

Southeastern Section: **Matthew Miller,** Department of Mathematics, University of South Carolina, Columbia, SC 29208-0001, e-mail: miller@math.sc.edu; telephone: 803-777-3690.

The Meetings and Conferences section of the *Notices* gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited in the table of contents on this page for more detailed information on each event. Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. **Information in this issue may be dated. Up-to-date meeting and conference information can be found at** www.ams.org/meetings/.

Meetings:

2007

July 31–August 3	Warsaw, Poland	p. 801
October 5–6	Chicago, Illinois	p. 802
October 6–7	New Brunswick, New Jersey	p. 803
October 13–14	Albuquerque, New Mexico	p. 804
November 3–4	Murfreesboro, Tennessee	p. 804
December 12–15	Wellington, New Zealand	p. 805

2008

January 6–9	San Diego, California Annual Meeting	p. 808
March 22–23	New York, New York	p. 808
March 28–30	Baton Rouge, Louisiana	p. 809
April 4–6	Bloomington, Indiana	p. 809
May 3–4	Claremont, California	p. 809
June 4–7	Rio de Janeiro, Brazil	p. 810
October 4–5	Vancouver, Canada	p. 810
October 11–12	Middletown, Connecticut	p. 810
October 17–19	Kalamazoo, Michigan	p. 810
October 24–26	Huntsville, Alabama	p. 811
December 17–21	Shanghai, People's Republic of China	p. 811

2009

January 7–10	Washington, DC Annual Meeting	p. 811
March 27–29	Urbana, Illinois	p. 812
April 4–5	Raleigh, North Carolina	p. 812
April 25–26	San Francisco, California	p. 812
Oct. 30–Nov. 1	Boca Raton, Florida	p. 812
Nov. 7–8	Riverside, California	p. 812

2010

January 6–9	San Francisco, California Annual Meeting	p. 813
March 27–29	Lexington, Kentucky	p. 813

2011

January 5–8	New Orleans, Louisiana Annual Meeting	p. 813

2012

January 4–7	Boston, Massachusetts Annual Meeting	p. 813

2013

January 9–12	San Diego, California Annual Meeting	p. 814

2014

January 15–18	Baltimore, Maryland Annual Meeting	p. 814

Important Information Regarding AMS Meetings

Potential organizers, speakers, and hosts should refer to page 78 in the the January 2007 issue of the *Notices* for general information regarding participation in AMS meetings and conferences.

Abstracts

Speakers should submit abstracts on the easy-to-use interactive Web form. No knowledge of LaTeX is necessary to submit an electronic form, although those who use LaTeX may submit abstracts with such coding, and all math displays and similarily coded material (such as accent marks in text) must be typeset in LaTeX. Visit http://www.ams.org/cgi-bin/abstracts/abstract.pl. Questions about abstracts may be sent to abs-info@ams.org. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

Conferences: (see http://www.ams.org/meetings/ for the most up-to-date information on these conferences.)

June 16–July 6, 2007: Joint Summer Research Conferences, Snowbird, Utah.

July 8–July 12, 2007: von Neumann Symposium on Sparse Representation and High-Dimensional Geometry, Snowbird, Utah.

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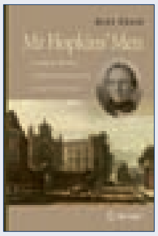
New and Noteworthy from Springer



Mr Hopkins' Men:

Cambridge Reform and British Mathematics in the 19th Century

A. D. D. Craik, University of St. Andrews, UK

This wide-ranging book tells the story of the remarkable William Hopkins and his top "wranglers", many of whom went on to have illustrious careers as bishops, judges, politicians, scientists or educators. It draws on first-hand accounts of life at Cambridge in the 19th century to give the reader a glimpse inside its colleges, and charts the evolution of the curriculum and the slow reforms that led to Cambridge's dominance of British higher education. As a special highlight, Hopkins' own collection of portraits of his top students are published - in full colour - for the first time.

2007. Approx. 432 p. 78 illus., 48 in colour Hardcover
ISBN 978-1-84628-790-9 ► **$129.00**

Modern Methods in the Calculus of Variations

L^p Spaces

I. Fonseca, **G. Leoni**, both at Carnegie Mellon University, Pennsylvania

This is the first of two books on methods and techniques in the calculus of variations. Contemporary arguments are used throughout the text to streamline and present in a unified way classical results, and to provide novel contributions at the forefront of the theory.
This book addresses fundamental questions related to lower semicontinuity and relaxation of functionals within the unconstrained setting, mainly in L^p spaces. It prepares the ground for the second volume where the variational treatment of functionals involving fields and their derivatives will be undertaken within the framework of Sobolev spaces.

2007. Approx. 385 p. (Springer Monographs in Mathematics) Hardcover
ISBN 978-0-387-35784-3 ► **approx. $59.95**

More Math Into LaTeX, 4th Edition

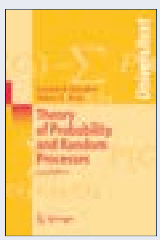
4th Edition

G. Grätzer, University of Manitoba, Winnipeg, MB, Canada

For nearly two decades **Math into Latex** has been the standard introductory text and complete reference for scientists in all disciplines and engineers. In this fourth edition, the reader is provided with important updates on articles and books; an important new topic is also discussed: how to create transparencies (computer projections) for classrooms and for professional meetings.

There are several Latex guides, but this one wins hands down for the elegance of its approach and breadth of coverage.
► **Amazon.com, Best of 2000, Editor's Choice**

2007. approx. 656 p. 80 illus. Softcover
ISBN 978-0-387-32289-6 ► **approx. $49.95**



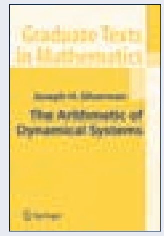
2nd Edition

Theory of Probability and Random Processes

L. B. Koralov, **Y. G. Sinai**, both at Princeton University, New Jersey

A one-year course in probability theory and the theory of random processes, taught at Princeton University to undergraduate and graduate students, forms the core of this book. It provides a comprehensive and self-contained exposition of classical probability theory and the theory of random processes. The book includes detailed discussion of Lebesgue integration, Markov chains, random walks, laws of large numbers, limit theorems, and their relation to Renormalization Group theory. It also includes the theory of stationary random processes, martingales, generalized random processes, and Brownian motion.

2nd ed. 2007. Approx. 400 p. (Universitext) Softcover
ISBN 978-3-540-25484-3 ► **$44.95**



The Arithmetic of Dynamical Systems

J. Silverman, Brown University, Rhode Island

This book provides an introduction to the relatively new discipline of arithmetic dynamics. A principal theme of arithmetic dynamics is that many of the fundamental problems in the theory of Diophantine equations have dynamical analogs. This graduate-level text provides an entry for students into an active field of research and serves as a standard reference for researchers.

2007. Approx. 520 p. (Graduate Texts in Mathematics, Volume 241) Hardcover
ISBN 978-0-387-69903-5 ► **$49.95**

Now in Softcover

Applied Linear Algebra and Matrix Analysis

T. S. Shores, University of Nebraska

2007. 388 p. 27 illus. (Undergraduate Texts in Mathematics) Softcover
ISBN 978-0-387-33195-9 ► **$39.95**

Additive Number Theory

Density Theorems and the Growth of Sumsets

M. B. Nathanson, City University of New York

Additive Number Theory presents material that deals with a central problem in additive number theory--the growth of sumsets. Ideas and techniques from many parts of mathematics including number theory, combinatorics, commutative algebra, ultrafilters and logic, and nonstandard analysis, are used to prove theorems in the subject. The book is self-contained, and provides short introductions to the various techniques that are not standard in this field.

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