Two Landmarks, Two Heroes

The experts have weighed in, the dust has settled, and we can all now celebrate the proof of the Poincaré Conjecture. And if you also raised a cheer for the proof of Thurston's Geometrization Conjecture, not many would try to shush you.

More than three years were needed for the experts to work carefully through the papers of Grigory Perelman. During this time the conviction gradually mounted that his work does indeed validate the vision of Richard Hamilton for attacking the conjectures. The climax came at the International Congress of Mathematicians in Madrid in August 2006, when experts stated publicly that Perelman had proved Poincaré (a bit more caution was expressed about Geometrization, but no doubts were voiced).

The saga of Perelman and his work catapulted mathematics into headlines in a way not seen since Andrew Wiles's proof of Fermat's Last Theorem in the mid-1990s. As with Fermat, the intense media blitz was followed by a period of silence during which books were written. Two have appeared recently: *The Poincaré Conjecture: In Search of the Shape of the Universe*, by Donal O'Shea (Walker, March 2007), and *Poincaré's Prize: The Hundred-Year Quest to Solve One of Math's Greatest Problems*, by George Szpiro (Dutton, June 2007).

Fermat's Last Theorem and the Poincaré Conjecture are similar in that they both inspired a great deal of mathematical development. Many special cases were established: for example, FLT was proved for large classes of exponents, and the Poincaré Conjecture was established for many special types of manifolds. Despite these similarities, as mathematical statements, FLT and Poincaré are quite different. FLT is a somewhat isolated statement that does not by itself have important implications that could be explored. By contrast, the Poincaré Conjecture captures a deep truth about the nature of three-dimensional shapes.

The importance of Poincaré was reinforced when it was shown to be a particular case of Thurston's sweeping Geometrization Conjecture, which has enormous implications in topology and geometry. For years many papers have appeared with careful caveats about how far the results could be pushed given the current status of the Geometrization Conjecture. Probably few papers have begun, "Assuming Fermat's Last Theorem, we prove…" Much more comparable to Geometrization is the deep conjecture that connected FLT to modern number theory, namely, the Taniyama-Shimura Conjecture, the centerpiece of Wiles's proof of FLT.

Another big difference between FLT and Poincaré is their fame outside of mathematics. How many among the general public had heard of the Poincaré Conjecture before Perelman's story became front-page news? I am guessing few. Being highlighted by the Clay Mathematics Institute as one of its seven Millennium Prize Problems raised the conjecture's profile a bit. But the Poincaré Conjecture is not a statement that is easily explained to those without mathematical background. Fermat's Last Theorem, of course, can be understood by schoolchildren and thus attracted the efforts of legions of amateur mathematicians—and continues to attract them, despite Wiles's proof. And who can blame them? Who can be 100 percent certain that there is not a really simple proof that has yet to be discovered?

One interesting similarity between the two results is that Perelman and Wiles both worked in isolation. Wiles confided in his Princeton colleague Nicholas Katz; it seems likely Perelman confided in no one at all. But the unveiling of their respective results could not have been more different. Wiles revealed his proof before a cheering crowd of his number theory colleagues, who had gathered at the Newton Institute in Cambridge, one of the world's major mathematics centers. By contrast, Perelman, having become increasingly isolated from the mathematical community, posted his preprints on the arXiv, let a few selected people know they were there, and then waited for the world to respond. Wiles attended the Berlin ICM in 1998 and collected his "special award" from the International Mathematical Union to thunderous applause from the audience gathered at the opening ceremonies. Perelman not only chose to skip the Madrid Congress in 2006, where he was to be awarded the Fields Medal, but he refused to accept the honor altogether.

These two landmark results are very different, but they both show how mathematics proceeds: It starts with a tantalizing question or a flash of insight compelling enough to spark the search for the *why*. This basic human desire for understanding is one reason Wiles and Perelman became heroes to the general public—and to many mathematicians as well.

—Allyn Jackson