AMS Short Course

Applications of Knot Theory

San Diego, California, January 4–5, 2008

Organized by
Dorothy Buck, Imperial College of London
Erica Flapan, Pomona College

Over the past twenty years, knot theory has rekindled its historic ties with biology, chemistry, and physics. While the original motiviation for understanding and classifying knots—Lord Kelvin's correlation of chemical elements with particular knotted configurations in the "ether"—proved erroneous, mathematicians continued to develop the theory of knots, and until the 1980s this remained a primarily pure field of mathematics.

At this time, chemists (most notably Ned Seeman and Kurt Mislow), biologists (most notably Nick Cozzarelli and Andrzej Stasiak), and physicists began searching for more sophisticated descriptions of the entanglements of natural phenomena—from strings to small organic compounds to, most famously, DNA.

Since their discovery in the late 1960s, DNA knots and links have been implicated in a number of cellular processes. In particular, they have been found during replication and recombination and as the products of protein actions, notably with topoisomerases, recombinases, and transposases. The variety of DNA knots and links observed made biologically separating and distinguishing these molecules a critical issue. While DNA knots and links can be visualized via electron microscopy, this process can be both difficult and time-consuming. So topological methods of characterizing and predicting their behavior can be helpful.

Topological techniques (notably the node number for knots, the Jones polynomial for catenanes/links, and the work of Schubert for 4-plats) played a significant role in characterizing knotted and linked DNA that arises from the biochemical process of site-specific recombination. Additionally, building on the experimental work of Wasserman and Cozzarelli as well as Conway's theory of tangles, Ernst and Sumners developed a tangle model of recombination to make predictions—later experimentally verified—about how a particular protein interacts with DNA. Modified versions of the tangle model have since been used to determine various features of protein-DNA interactions for a number of specific proteins.

Similarly, in chemistry, Pasteur began the study of molecular chirality. Since two enantiomers of the same drug can interact with a host's metabolism very differently, pharmaceutical companies are particularly interested in topological stereochemistry. Thus, knot theory techniques have been used to understand whether—and if so how intrinsically—a number of synthetic compounds are chiral.

In addition to the examples described above, there are many other deep interactions of knot theory with biology, chemistry and physics. While this Short Course can not cover all aspects of applied knot theory, the organizers' goal is to provide the participants with an appetizer—both as a small taste and to stimulate the (mathematical) appetite.

It is planned that lecture notes will be available to those who register for this course. Advance registration fees are: member of the AMS—US$94; nonmember—US$125; student, unemployed, emeritus—US$42. On-site fees are: member of the AMS—US$125; nonmember—US$155; student, unemployed, emeritus—US$63. Registration and housing information can be found in this issue of the *Notices*; see the section "Registering in Advance and Hotel Accommodations" in the announcement for the meetings in San Diego. The registration form is at the back of this issue.

Format of the Short Course:

This AMS Short Course will introduce knots, and some of their recent applications in molecular biology, chemistry, and physics.

No prior knowledge of knots, biology, or physics is assumed. In particular, the first day of the Short Course will include introductory lectures by Colin Adams on knot theory, Dorothy Buck on DNA and knots, and Erica

Flapan on topological stereochemistry. The second day will include lectures on particular aspects of these subjects: Lou Kauffman on applications of knot theory to physics; Ned Seeman, who uses topology for DNA nanotechnology; and Jon Simon on the statistical and energetic properties of knots and their relation to molecular biology.

Speakers will highlight both their own motivation and projects, as well as describing new avenues for interested researchers (and their students) to explore.

The Short Course will conclude with a panel discussion of the putative trajectories of these applications of knot theory, and summarize the major open problems and challenges.

In addition to the formal activities led by the speakers and organizers, the organizers will ensure that the participants themselves have adequate time to discuss topics of mutual interest—during the panel discussion, during smaller group discussions at the end of the first day, and at a dinner organized for the first evening.

Introduction to Knot Theory

Colin Adams, Williams College

Abstract: This talk will be an introduction to the mathematical theory of knots, including Reidemeister moves, surfaces, types of knots, and various invariants associated to knots. We will also touch on the stick number for knots and its implications for chemistry.

Colin Adams is the the Francis Christopher Oakley Third Century Professor of Mathematics at Williams College. He authored the now-standard undergraduate knot theory text, "The Knot Book", and is renowned for his witty and deceptively sophisticated introductory geometry and topology talks. His own research focuses on hyperbolic knots and 3-manifolds, and he has involved numerous undergraduates in annual summer research projects at Williams. He is a recipient of the Deborah and Franklin Tepper Haimo Distinguished Teaching Award from the MAA, a Polya Lecturer for the MAA, and a Sigma Xi Distinguished Lecturer.

Introduction to Topological Chirality

Erica Flapan, Pomona College

Abstract: Symmetry plays an important role in predicting the behavior of molecules. A particular type of symmetry that is chemically important is mirror image symmetry. A molecule is said to be chiral if it cannot change into its mirror image. In this talk we will explain why chirality is important; discuss the differences between chemical, geometric, topological, and intrinsic chirality; and introduce various techniques to show that a molecule is topologically chiral.

Erica Flapan is the Lingurn H. Burkhead Professor of Mathematics at Pomona College. Her research is in 3-dimensional topology and applications of topology to chemistry. Her book "When Topology Meets Chemistry", was jointly published by the Mathematical Association of America and Cambridge University Press. From 2000 to 2004, she was the principle investigator of an NSF-CCLI grant entitled "Enhancing the mathematical understanding of students in chemistry". As part of this grant, she developed a course entitled "Problem Solving in the Sciences", to help students with weak math skills succeed in general chemistry. Together with an organic chemist, she also developed an interdisciplinary upper division course on Symmetry and Chirality.

Introduction to Knots and DNA

Dorothy Buck, Imperial College London

Abstract: This talk will introduce DNA, and explain why knot theorists are interested in this molecule. We will explore the topological techniques used to understand both DNA itself and how it interacts with proteins in the cell. As an extended example, we will give an overview of the tangle model and its variations to understand the molecular process of site-specific recombination. We will also discuss mathematicians' contributions to several open questions involving DNA, including how a protein unknots DNA effectively and how complicated linked DNA is copied accurately.

Dorothy Buck is a mathematical biologist at Imperial College London in the Department of Mathematics and Centre for Bioinformatics. She specializes in 3-manifold topology and its applications to mathematical biology. Her training is in both mathematics and microbiology—she spent six years, both at University of Texas-Austin and Johns Hopkins Medical School, working in molecular biology labs. Before joining the faculty at Imperial, she was an NSF Postdoctoral Fellow with Craig Benham at the University of California Davis Genome Center, and an assistant professor in the Applied Mathematics Department at Brown University.

Knots and Physics

Lou Kauffman, University of Illinois–Chicago

Abstract: Knots are mathematical abstractions of the topological properties of rope in physical space. As such, there are immediate relationships of knots with the physics of ropes, weaves, long-chain molecules, and other knotting phenomena in Nature. There are also beautiful and surprising relationships of knot theory with the structures and methods of statistical mechanics and quantum theory. This talk will survey some of the speaker's favorite interactions between knots and physics.

Louis Kauffman is professor of mathematics at University of Illinois–Chicago. He authored the interdisciplinary text "Knots and Physics". He discovered the bracket polynomial state model for the Jones polnomial and the first direct relationship between statistical mechanics models and knot invariants. As a topologist, he is omnivorous, working in knot theory and its relationships with statistical mechanics, quantum theory, algebra, combinatorics, and more recently, biology. He is editor of the *Journal of Knot Theory and its Ramifications.*

Single-Stranded DNA Topology

Ned Seeman, New York University

Abstract: The double helical nature of the DNA molecule has a wide variety of topological implications. Most biologists are familiar with the notion that circular DNA molecules are catenanes/links, so that the strands are linked about once every 10 nucleotides. Consequently, biological systems contain topoisomerases which change the linking topology of the molecule, thereby solving a variety of problems in the metabolism of the genetic material. Today, the realm of DNA extends beyond its biological role as a molecule with an unbranched helix axis. Branched DNA molecules exist as intermediates in genetic recombination, but for twenty-five years synthetic branched DNA molecules have been built for a variety of purposes that are important for nanotechnology and for molecular computation. The ability to assemble branched DNA backbones has enabled the deliberate construction of single-stranded knots, polyhedral catenanes and Borromean rings. New branched DNA motifs have been derived by using techniques from knot theory. Branched DNA molecules have enabled the deliberate construction of periodic and aperiodic DNA crystals. The applications of these systems include analysis of biological systems, nanoelectronics and nanorobotics.

Ned Seeman is professor of chemistry at New York University. He founded the field of single-stranded Nucleic Acid Topology. Among other work, his lab has characterized the interactions of synthetic DNA knots with topoisomerases, developed a general algorithm for the construction of any DNA knot, synthesized a DNA molecule that can be built to yield four different topological species, and discovered an RNA topoisomerase. For his innovation, he was awarded the Feynman Prize in Nanotechnology, the Emerging Technology Award from *Discover Magazine*, and elected Fellow of the Royal Society of Chemistry. He is the founding president of the International Society for Nanoscale Science, Computation and Engineering.

Long Tangled Filaments

Jon Simon, University of Iowa

Abstract: We are interested in filaments, from rope and string and hair to DNA and proteins, anything that might be understood as one-dimensional strands wiggling and tangling in three-dimensional space.

If the filaments are short, we can try to describe the exact geometric shape and understand how the shape relates to physical behavior. If the filaments are somewhat long and flexible, then topological knot type can be very useful, as evidenced by the success of topological methods for studying the actions of DNA enzymes. But if the filaments are very long (think of a complicated 3-dimensional scribble) or somehow random (think of a lot of complicated 3-dimensional scribbles) then it may be impractical to try describe the exact shapes or even knot types. We need to develop a vocabulary of ideas and models that describe physically important geometric/topological properties of long tangled things.

In this talk, we will consider ideas, experiments, and theorems dealing with packing, curvature, tangling, and knotting of individual complicated filaments as well as statistical ensembles. We will explore some of the work that has been done, some open research problems, and some topics that seem well-suited for undergraduate research activities.

Jon Simon is professor of mathematics at the University of Iowa. He, along with the chemist Kurt Mislow, pioneered the rigorous application of knot theory to chemistry, in particular by determining chirality of synthetic compounds. He codeveloped the idea of Möbius energy of thick knots. His current research also includes particular knotting and tangling of filaments; "energy" of knots; and applications to molecular biology, e.g., knotted DNA loops.

References

[1] *The Knot Book*, by Colin Adams, American Mathematical Society (2004).

[2] *Knots and Physics*, by L. Kauffman, World Scientific, Third Edition (2001).

[3] `http://www.math.uic/kauffman/Alex.pdf` (Alexander's original paper on the Alexander polynomial).

[4] `http://www.math.uic/kauffman/QuickTrip.pdf` (Fox's Quick Trip through Knot Theory).

[5] arXiv Math 0410329 (Kauffman's paper on Knot Diagrammatics).

[6] *DNA Topology*, by A. Maxwell and A. Bates, Cambridge University Press, Second Edition (2005).

[7] *When Topology Meets Chemistry: A Topological Look at Molecular Chirality*, by E. Flapan, Cambridge University Press and Mathematical Association of America (2000).

[8] Thickness of knots, by R. Litherland, J. Simon, E. Rawdon, and O. Durumeric, *Topology and its Applications* **91** (1999), 233–244.

[9] Physical models for exploring DNA topology, by N. C. Seeman, *Journal of Biomolecular Structure and Dynamics* **5** (1988), 997–1004.

[10] DNA nicks and nodes and nanotechnology, by N. C. Seeman, *NanoLetters* **1** (2001), 22–26.

[11] Models of entanglement, by G. Buck and J. Simon, Knot Theory for Scientific Objects, *OCAMI Studies* **1** (2007), 51–74.

[12] Total curvature and packing of knots, by G. Buck and J. Simon, *Topology and its applications* **154** (2007), 192–204.

[13] Thickness and crossing number of knots, by G. Buck and J. Simon, *Topology and its Applications* **91** (1999), 245–257.