A Certain Ambiguity

Reviewed by Danny Calegari

A Certain Ambiguity (A Mathematical Novel)
Gaurav Suri and Hartosh Singh Bal
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In "A negative review of negative reviews" ([6]), Doron Zeilberger contends that "(a)nyone who wastes his time writing a review of a book that he or she dislikes, is a frustrated mathematician, who has an axe to grind, and just enjoys being mean."

Writing a novel, especially a novel with serious philosophical aims, is not easy, and Suri and Bal have made a serious attempt. They have chosen an ambitious and significant theme—the story of the gradual discovery and development of nonstandard axioms for geometry (hyperbolic geometry) and for set theory (undecidability of the Continuum Hypothesis), and the implications for epistemology. This theme reveals itself through the story of a young man called Ravi, who by chance takes a liberal arts mathematics undergraduate course at Stanford. The topic of the course is infinity, and Ravi's intuitions and naive notions are sharpened by an exposure to ideas such as cardinality, power sets, and so forth. At the same time, Ravi stumbles on evidence of a surprising episode in the life of his deceased grandfather Vijay and, by hunting through newspaper clippings and court transcripts, comes to learn about a parallel formative experience his grandfather underwent at a young age. Ravi's gradual discovery of the subtle and indirect nature of mathematical and historical reality is interspersed with (fabricated) "diary entries" by such historical figures as Pythagoras, Bolyai, and Cantor.

I find the premise of this book fascinating and am excited by the idea of a simultaneous examination of these issues from historical, scientific, and psychological viewpoints. I therefore regret (at the risk of arousing Zeilberger's spleen) that, despite finding some things to like here, I cannot recommend this book on literary, philosophical, or mathematical grounds (however, read on). The result is more of a novelty than a novel, and the authors' ambitions seem more grandiose than grand. Structurally, the book is sound, and even innovative. Numerous historical and fictional strands are played out, and the literary device of switching back and forth between narrative, newspaper excerpt, transcripted dialogue, diary entry, and so forth does a good job of organizing and integrating these strands.

The main disappointment is the disparity between the potential sophistication and depth of the subject matter, and the unremarkable depths to which Suri and Bal plumb it. Easiest to criticize (and least serious, in my opinion) is the cursory investigation of the mathematics. We are led through the usual pedestrian examples of sets that are in bijection to proper subsets (the real line and an open interval, the positive integers and the squares), Cantor's diagonal argument that for any set S the power set $\mathcal{P}(S)$ has greater cardinality than S, and so on. Irritatingly, there are some minor goofs even in the exposition of such basic material. In order to disprove the existence of a bijection of a particular set $A = \{c, *, a, ?, \#, q, t, \dots\}$ with $\mathcal{P}(A)$, the authors instead start to give a partial

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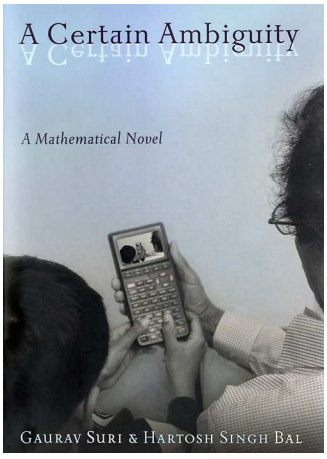
bijection of A with $\mathcal{P}(\mathcal{P}(A))$

$$c \longleftrightarrow \{?, a\}, \{q\}$$
$$* \longleftrightarrow \{c, a\}, \{\#, ?, *\}, \ldots$$
$$a \longleftrightarrow \{\}$$

and so on. Actually, I assume this is what they are doing; the notation makes it unclear exactly what set the range is supposed to be. However, in the next paragraph, the confusion is compounded by the assertion that "the element c is mapped to the subsets $\{?, a\}, \{q\}$, and so c does not occur in the set it is mapped into" (p. 171). Set or sets? If the authors cannot get basic things right, at least they can eulogize them: "(f)rom my current perch in retrospective adulthood I consider Cantor's proof about power sets to be one of the flagship creations of the human race. We have thought nothing more elegant or powerful, only different" (p. 170). If "perch" doesn't make you wince, the contrast between this encomium and the subsequent muddle will. There is inconsistent notation (the power set of A is $P[A]$ at one point, and $[A]$ later on, and then finally $P(A)$) and confusing exposition of elementary theorems in plane geometry. And for the reader who already has some familiarity with mathematics, there is boredom. Mathematically speaking, there is nothing here that is not already in every other popular or informal account of non-Euclidean geometry or the Continuum Hypothesis. All the high points are hit, like a tour bus doing Europe in seven days. Tangible examples of negative curvature in nature (lettuce leaves, seaweed) or sewing (pleats) are missing here; instead we read a newspaper account of an expedition to measure the effect of gravity on light rays. At one point, Nico the course instructor declares "I've read some things about this but I don't understand it fully, so I'm not going to talk about it" (p. 222). One gets the impression that the authors have taken a different position. There is a lot of waffle about the implications of the work of Gödel and Cohen, but no discussion of what they actually did. Mathematics in this book is a strut on which a range of philosophical opinions are propped up. Perhaps in reaction to this, I was provoked into reading something more substantial about the mathematics of the Continuum Problem. To the nonspecialist like myself, I can recommend *Set Theory and the Continuum Problem* by Smullyan and Fitting ([3]), or Woodin's pair of articles which appeared in the *Notices* ([5]).



The philosophical ambitions of the novel are more off-putting. In a broad sense, the novel preaches a kind of relativism in which truth is provisional and relative to a set of axioms that are accepted on some unknown basis. Further, there is an important distinction between the consistency of a theory and the question of whether that theory accurately models some phenomenon under discussion. Fine. A page and a half is devoted to a token comparison of several schools of thought on the interpretation of mathematical knowledge, e.g., Platonism, formalism, constructivism, quasi-empiricism, but the real target is elsewhere. The subplot revolving around Vijay is a sustained effort to "equate ... faith in God with a mathematician's belief in absolute mathematics" (p. 258). In this subplot, opposite (but apparently symmetric) points of view are taken up by Vijay and by Judge John Taylor. The independence of the parallel postulate is taken to undermine Vijay's initial position that the Christian belief in the authority of the Bible is illogical and has no place in America, the "land of 'rationality and objectivity'" (p. 51). Leaving aside the question of whether a published professional number theorist working in 1919 could be unaware of the existence of a consistent theory of non-Euclidean geometry, there is no man in this argument who is not a straw man. I was irritated on behalf of mathematicians and theists alike by their banal and predictable interchanges. Ultimately, Vijay's position is undermined when he discovers that the "self-evident" parallel postulate may not correctly describe nature after all. Judge Taylor undergoes a similar examination of his faith. "That night I let myself see the world as an atheist must: a desolate planet occupied by people who had abandoned themselves to amoral meaninglessness" he writes (pp. 248-249). As for the axiomatic method, we get the following platitude: "(a)s long as (a man) is true to some core beliefs, he can't go too far wrong. Which starting point is true is not something we humans can make much progress on" (p. 255). Bleagh. The best one can say about this bit of homespun wisdom is that it has the virtue of being hopelessly naive about something important. From the Habermas-Lyotard debate (see [1] for an introduction) to the Sokal hoax ([4]), to recent atheist manifestos on the bestseller lists (e.g., [2]) the question of foundations for intellectual thought and especially for intellectual debate has never been more critical or urgent. Never mind.

Turning to the literary dimensions of the work, I am compelled to say that the prose is usually workmanlike, and the main characters' attitudes are generally sophomoric. But at least in this domain there are parts of the novel worth enjoying. The description of Ravi's emotional state as he lights his grandfather's funeral pyre is delicate and moving. And the story of his ambivalent quest to join the firm of Goldman-Sachs is compellingly factual (and, given the background of one of

the authors, one suspects factually correct) and concrete. The character of Nico is sympathetic and admirable, and mainly by a process of tactful omission ("from the lower pitch of his voice I got that he didn't really want to debate his beliefs with me" (p. 157)) is made to seem wise and generous. The narrative is well-plotted, dynamic, and compelling, and rarely drags. There is even a latent love interest. I found it easy to start reading this book, and to keep on reading. While some of the fictitious "diary entries" are somewhat unconventional (e.g., Cantor's journal entry in which he records his wife saying "You are a huggable bear, Georg" (p. 105)), one can read these in the playful spirit in which they are offered without taking offense. The book plausibly succeeds somewhere at the interface of entertainment and journalism. Readers with no prior interest in mathematics or philosophy may find these aspects of the book at their level and be drawn in by the entertainment the book provides to a fascinating subject whose rewards and subtleties the book points to even if it does not illustrate them.

References

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[4] A. SOKAL and P. BRICMONT, *Fashionable Nonsense: Postmodern Intellectuals' Abuse of Science*, Picador, 1999.

[5] H. WOODIN, The continuum hypothesis, Parts I and II, *Notices AMS* **48**, no. 6, pp. 567–576 and no. 7, pp. 681–690.

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