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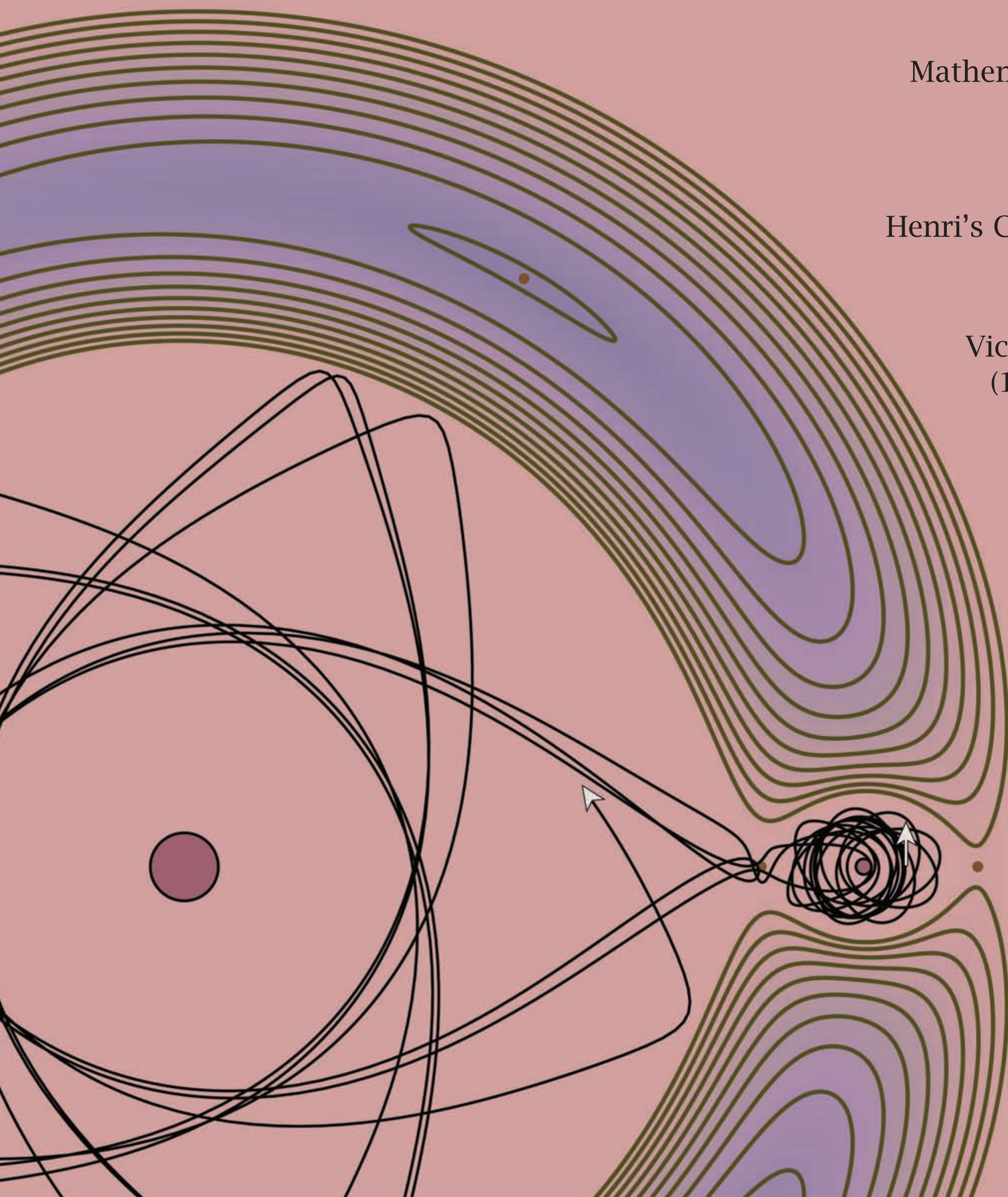

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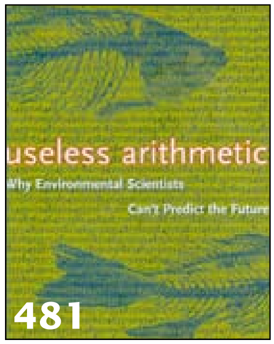

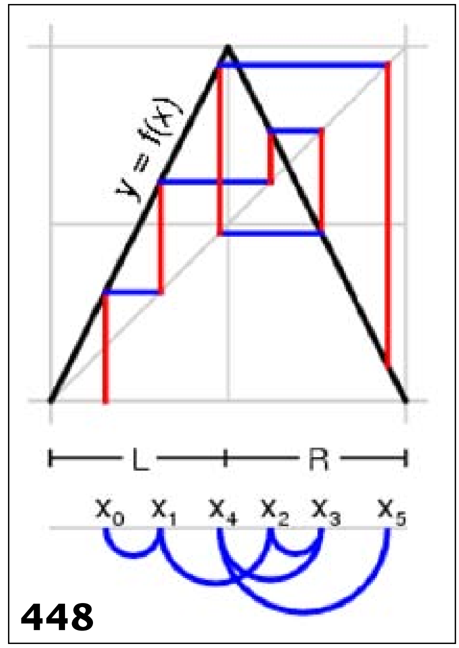

Features

Notices
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Departments

From the AMS Secretary

DARPA and Hilbert

In August last year, a document called *23 Mathematical Challenges* appeared on the website of the Defense Advanced Research Projects Agency. DARPA, which operates under the U.S. Department of Defense, funds high-risk research and development projects that could eventually lead to technology of use to the military. The document consists of one- or two-sentence summaries of 23 problems that fall within the field of mathematics or that could require mathematics for their solution. On the final page, Benjamin Mann, a mathematician and DARPA program manager, states that he is "fully responsible for the selection and statement of these challenges" and thanks a few other mathematicians for their help.

The background color of the document suggests parchment, and the whole appearance conveys an aura of the classic and the hallowed. Given that there are exactly 23 problems listed, the document seems intended to evoke David Hilbert's famous list of 23 problems presented at the International Congress of Mathematicians in Paris in 1900.

But DARPA is offering something Hilbert wasn't: money. On the same webpage there is a link to a description of a new program called *DARPA Mathematical Challenges*, which will support work on the 23 problems. This is not DARPA's first foray into funding pure mathematics. Over the past few years, for example, it has funded work on the geometric Langlands program.

Exactly how much money DARPA will devote to its *Challenges* is not specified in the program announcement. According to reports in the *Notices* by AMS Washington Office director Samuel M. Rankin III, over the past several years the DARPA mathematics budget has hovered around US$16 million per year. According to a report in the August 2007 *Notices*, the requested budget for the current fiscal year of 2008 is 50% higher than for 2007, up from an estimated US$18.0 million to a requested US$27 million.

Issuing a list of problems to be solved is quite an unusual way for a government agency to organize a proposal solicitation in mathematics. Some reactions to it can be found on Internet blogs, such as the n-Category Café (`http://golem.ph.utexas.edu/category/`). There, in addition to simple perplexity, one finds a range of reactions. For example, one contributor complained that the problems are "poorly phrased or extremely speculative"; another, noting the limited funding for mathematics research, called the program "a serious attempt" to widen funding possibilities.

The new DARPA program makes me wonder whether the U.S. mathematical community is becoming more receptive to military funding than it was, say, twenty years ago. Back in the mid-1980s, a group of mathematics organizations, including the AMS, began to collaborate on strategies for increasing government funding for research. This effort triggered a searching debate within the Society about whether it is ethical for mathematicians to take research grants from the military. One of the main advocates against military funding was Fields Medalist William Thurston, who during the 1980s served as an AMS vice president. Panel discussions were held at the Joint Meetings, and the *Notices* established a special section to air debate on the topic.

The climax was a referendum put before the AMS membership in January 1988. One of the motions in the referendum stated:

> The AMS is concerned about the large proportion of military funding of mathematics research. There is a tendency to distribute this support through narrowly focused (mission-oriented) programs and to circumvent peer review procedures. This situation may skew and ultimately injure mathematics in the United States. Therefore those representing the AMS are requested to direct their efforts towards increasing the fraction of non-military funding for mathematics research, as well as towards increasing total research support.

With a turnout of voters much larger than for any other AMS election or ballot issue, the referendum passed by a wide margin. (The full text of all five motions in the referendum appeared in the November 1987 *Notices*, page 1014.)

The AMS membership was not unanimously against military funding for mathematics, however. Many thought the Society had no business telling mathematicians who they should and should not take funding from. I remember a heated Council debate in which William Browder, then AMS president, expressed his strong opposition to blanket condemnations of research grants from the military. He likened such condemnations to a "witch hunt" against those who opted to take such grants. (Browder, whose father Earl was persecuted because of his leadership of the American Communist Party, is not one to use the term "witch hunt" lightly.)

Will the AMS Council or other venues within the mathematical community renew debate over military funding for mathematics? A mathematical generation later, it seems unlikely. But that doesn't mean the question raised in the 1980s—do mathematicians have a responsibility to try to influence how their work is used?—has lost validity.

—Allyn Jackson

[1] *Available at* `http://www.darpa.mil/dso/personnel/mann.htm`*; the PDF file is 1.7MB.*

Value of a Game

The assignment of values to objects such as outcomes and coalitions, i.e., the construction of value functions, is a fundamental concept of game theory. Value (or utility, or preference) is not a physical property of the objects being valued, that is, value is a subjective (or psychological, or personal) property. Therefore, the definition of value requires specifying both what is being valued and whose values are being measured.

Game theory's characteristic function assigns values to coalitions so that what is being valued by this function is clear but von Neumann and Morgenstern do not specify whose values are being measured in the construction of this function. Since it is not possible to construct a value (or utility) scale of an unspecified person or a group of persons, game theory's characteristic function is not well-defined. Likewise, all game theory solution concepts that do not specify whose values are being measured are ill-defined.

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Kaplansky's Lecture Notes

Kaplansky's works and influence were aptly presented in "Irving Kaplansky (1917–2006)", *Notices* of the American Math. Soc. **54** (2007), 1477–1493. It seems strange that three lecture notes of Kaplansky were not mentioned in this article. Nor were they included in the bibliography contained in *Selected Papers and Other Writings of Irving Kaplansky*, Springer-Verlag, 1995. These lecture notes belong to the *Lecture Notes in Mathematics* series published by the department of mathematics, the University of Chicago. They are

Topics in commutative ring theory, 1974;

Bialgebras, 1975;

Hilbert's problems (preliminary edition), 1977.

The first two lecture notes were reviewed in *Math Reviews*. However, the third one has not been reviewed anywhere; only three chapters of it were translated into Swedish and Norwegian.

When I was a graduate student studying commutative algebra at Chicago during the 1970s, I was stumped by the definition of multiplicities of local rings defined purely algebraically through Hilbert-Samuel polynomials. Kaplansky gave me a copy of the chapter on the 15th problem of his lecture notes on "Hilbert's problem". It provided an excellent lesson of learning mathematics. A panorama of this famous and important problem was exhibited. Many names, e.g., Hilbert, van der Waerden, Weil, Serre, etc., appeared in this article and they became my heroes henceforth. It was really an effective way of teaching and a blessing to a naive graduate student.

After thirty years, although there have been many new publications on Hilbert's problems (e.g., Felix Browder's *Proceedings of Symposia in Pure Mathematics*, vol. 28, and B. H. Yandell's *The Honor Class: Hilbert's Problems and Their Solvers*), I still believe that Kaplansky's preliminary edition of Hilbert's problems should stand in the bookshelf of every graduate student's desk. Together with the expository books about Riemann and Poincaré (is there any such book?), Kaplansky's lecture notes will tell you what is good mathematics. It confirms Kaplansky's motto "Spend some time every day learning something new that is disjoint from the problem on which you are currently working. And read the masters."

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Octonion Algebras and Cohomology Classes

In the *Notices* November 2007 issue, p. 1297, the theorem on the first line is not correct (same for the last two lines of p. 1296). The author claims that the octonion algebras over a field F (of characteristic not 2) are in 1-1 correspondence with the elements of the cohomology group $H^3(F, Z/2Z)$. No. These algebras correspond to the elements of $H^3(F, Z/2Z)$ which are "decomposable" (or "symbols"), i.e., which are cup products of 3 elements of $H^1(F, Z/2Z)$. In a few simple cases, such as $F = \mathbb{Q}$, every element is decomposable. Not so in general: it is usually a difficult problem to decide when this happens. See for instance my Bourbaki seminar report no. 783 (1994).

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Non-English Names of Prominent Mathematicians

There are some names which are internationally well-known and pronounced in the same manner all over the world. Such are the names of prominent musicians, artists and politicians like Mozart, Gandhi, Gauguin. It is important also for the mathematical community to treat its prominent representatives with due respect and pronounce their names in a uniform manner. This will enhance their international recognition and standing. The natural choice is their original phonetics. Thus Euler should be pronounced ['Oy-lehr] (first syllable stressed) and Cauchy should be [Ko:'shi] (second syllable stressed). It is painful to hear the names of Weierstrass, Lie, Hurwitz, Poincaré, Dirichlet, Plancherel (and many others) pronounced sometimes in a strange, unrecognizable manner. The Voice of America (http://names.voa.gov) has developed a pronunciation guide for prominent foreign politicians. Following the spirit of that guide, I have written a short list of some European (non-English) mathematicians published under the Pronunciation guide at http://www2.onu.edu/%7Emcaragiu1/bonus_files.html. Any suggestions and corrections are very welcome.

I think the AMS could do the same as the VOA and create an online pronunciation guide for the names of prominent mathematicians.

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Mathematics and Voting

Donald G. Saari

The 2008 annual *Math Awareness Month* theme features the unusual combination of "Mathematics and Voting". The importance of voting is obvious; indeed, with the United States election season hard upon us, discussions about voting are seemingly nonstop. More generally, on almost any day of any year the news media reports on some consequential election going on somewhere in the world. But what does mathematics have to do with any of this?

Actually, a lot. In fact, mathematics has been central to this area since 1770, when the mathematician Jean Charles de Borda challenged whether the French Academy's elections for new members accurately reflected the views of the voters: Borda blamed their voting rule, the plurality vote. It is this concern, whether an election rule can faithfully produce outcomes capturing "the will of the voters", that requires serious mathematical attention. Consider the consequences; as it is well understood, electing the "wrong leader" can cause serious, lasting problems. But while the mathematical study of this topic is more than a couple of centuries old, there remain more mysteries than answers, and the mathematical development of this area is in its early stages.

To illustrate what can happen, suppose the voter preferences among candidates $\{A, B, C\}$ in an election for the next chair of your department are

Number	Ranking	Number	Ranking
3	$A \succ B \succ C$	2	$B \succ C \succ A$
2	$A \succ C \succ B$	4	$C \succ B \succ A$

If your department uses the plurality "vote-for-one" rule, A wins with the $A \succ C \succ B$ ranking. But with a vote-for-two rule, the ranking *reverses* to $B \succ C \succ A$ so formerly last-place B now wins. By using the *Borda Count* (introduced in 1770 by Borda), where a ballot is tallied by assigning a first and second positioned candidate, respectively, two and one points, C wins with the $C \succ B \succ A$ ranking. As *any* candidate can win with an "appropriate" voting rule, not all three outcomes register the will of the voters. Who should be the winner?

Accompanying the disturbing reality that *election outcomes can more accurately reflect the choice of an election rule than the voters' wishes* are the mathematical issues of understanding why this is so, determining whether "paradoxical outcomes" are unlikely anomalies or reasonably prevalent behavior, developing appropriate mathematical structures to permit a systematic rather than an ad hoc analysis, and identifying whether any election rule reliably produces outcomes that, arguably, represent the views of the voters. Can the reader, for instance, determine what mathematical structures force the radically different conclusions for the above example?

In this article, I answer some of these questions while indicating mathematical structures currently being developed to analyze voting systems. More generally, I treat the "mathematics of voting" as a prototype to identify mathematical concerns that are associated with aggregation rules. For

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instance, I will indicate why orbits of symmetry groups and even notions from chaotic dynamics play roles in understanding voting problems: expect similar mathematical approaches to identify which non-parametric statistical procedures best represent the data and whether Adam Smith's Invisible Hand story from economics captures the wishes of consumers.

Before seeking solutions, we need to appreciate what kinds of problems can arise. Thus the first part of this article illustrates certain ways in which election outcomes can widely differ. After understanding what can go wrong, I will indicate a way to analyze these issues.

How Bad Can It Get?

With the above 11-voter example, each candidate can win with some *positional voting rule*: this is a voting rule where an n-candidate ballot is tallied by assigning a specified weight w_j to the candidate in the j^{th} *position,* $j = 1, \dots, n$. (The obvious constraints are that $w_j \geq w_{j+1}$ for all j and the weights are not all equal.) The choice matters; e.g., by using different weights, this introductory example has *seven* different election rankings; four of them are strict (without ties).

While it has been known since Borda that the choice of a voting rule matters, to approach this area systematically, we need results demonstrating how serious the problem can be. To do so, treat the weights as components of a *voting vector* $\mathbf{w} = (w_1, w_2, \dots, w_n)$, where $w_n = 0$. A normalized version (where $w_1 = 1$) of any voting vector is a convex combination of the "vote-for-k" vectors $(1, 0, \dots, 0), \dots, (1, 1, \dots, 1, 0)$. Using this convexity with the geometry of $\mathbb{R}^{n-1}$ and the linearity of the tallying process, it is not overly difficult to prove the following disturbing conclusion.

Theorem 1. *(Saari* [6]*). For $n \geq 2$ candidates and any k satisfying $1 \leq k \leq (n-1)(n-1)!$, there is a profile (i.e., a list of each voter's complete, transitive ranking of the candidates) where, with different choices of the positional voting rule, precisely k different, strict election rankings arise. (No profile generates more than $(n-1)(n-1)!$ strict rankings.) Indeed, for $n \geq 4$, a profile can be found where each candidate is ranked in first, second, …, last place with different positional election rules.*

With three candidates, then, a profile can generate four different strict election rankings. But with $n = 10$, which is about the number of candidates starting in a U.S. presidential election, a single profile could have over three million different election rankings where each candidate could be the "winner" with some rules, and the "loser" with others. The 2002 French presidential election started with 16 candidates; here a single profile could admit well over 19 *trillion* different positional election rankings. Somewhat surprisingly but because of the astronomical values that can arise with combinatorics, there are reasons to believe [10] that examples exhibiting trillions of outcomes can be created with, say, no more than 30 or 50 voters. The point to be made for this article is that if changing rules generates trillions of different rankings, the mathematical challenge is to determine which positional voting rules, if any, can be trusted to get the job done accurately.

As the "mathematics of voting" is a prototype, expect the Theorem 1 behavior to accompany other aggregation rules. As an illustration, if political parties A and B have, respectively, 49 and 51 of the senate seats and 217 and 218 of the congressional seats, one might mistakenly accept that they have essentially equal power. But as B enjoys a majority in both houses, it has complete power; i.e, simple counts are misleading; a more accurate measure is a group's ability to effect change. "Power indices" were developed in game theory to measure these differences in the ability of competing groups, or even individuals, to determine outcomes; these tools have been cited in Supreme Court decisions and used to analyze European Union voting rules. While the best known ones are the Shapley and Banzhef values, many others have been introduced. Because of their importance and alerted by what can happen with voting rules, one should worry whether the choice of an index matters. It does; by using Theorem 1 Lauruelle and Merlin [2] extended its conclusion to power indices; i.e., with the same data or game theoretic structure, it is possible to have up to $(n-1)(n-1)!$ different rankings of the n parties by using different power indices. Independently and using different techniques, Saari and Sieberg [11] proved similar results.

What about the nagging concerns whether these voting examples are cooked up anomalies or reflect what must be expected? For decades, researchers such as Fishburn, Gehrlein, Riker, and others have estimated the likelihoods of various voting paradoxes with consistently discouraging conclusions. These results were hampered by the complexity of the associated computations, which forced using unrealistic assumptions such as that each profile is equally likely. The next result, which is based on a six-dimensional central limit theorem and where the computational difficulties are handled by borrowing ideas from Schläfli [13], provides more realistic estimates for the issues described here while underscoring the severity of these problems. Incidentally, the reason a zero probability is associated with even k values is that here one of the tie rankings is easily broken.

Theorem 2. *(Saari and Tataru* [12]*). For three candidates, assume there are n voters where as $n \to \infty$ the distribution of voter choices is asymptotically independent with an asymptotic common*

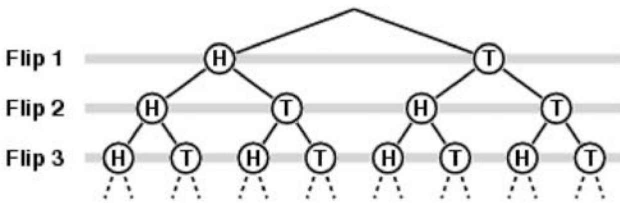


Figure 1. Randomness of flipping a coin.

finite variance and the asymptotic mean has an equal distribution of voters of each type. The limiting probability as $n \to \infty$ that a profile permits precisely k different outcomes as the positional rule choice varies is zero if k is an even integer and

k	*Probability*	k	*Probability*
1	0 : 31	3	0 : 44
5	0 : 19	7	0 : 06

With a surprisingly high probability of 0.69, then, the choice of the positional voting rule matters for a close three-candidate election! Different rules can have different outcomes. Indeed, it is easy to find many actual elections where it appears that the outcomes reflect the voting rule rather than the voters' intent.

The mathematical approach Tataru and I developed for this theorem has been applied by others, including various combinations of Merlin, Tataru, Valognes, Gehrlein, and Lepelley, to obtain related three-candidate conclusions. While a definitive answer for four or more candidates has yet to be investigated, it is clear that the severity of the problem escalates with the number of candidates. For instance, with more than five candidates in a closely contested election, there are reasons to believe that, with probability close to certainty, the election ranking changes with positional methods.

Chaotic Effects

To illustrate what else can go wrong, suppose during hiring season your department will make an offer to one of the four candidates $\{A, B, C, D\}$. Preferences within the department are

(1)

Number	Ranking	Number	Ranking
3	$A \succ C \succ D \succ B$	2	$C \succ B \succ D \succ A$
6	$A \succ D \succ C \succ B$	5	$C \succ D \succ B \succ A$
3	$B \succ C \succ D \succ A$	2	$D \succ B \succ C \succ A$
5	$B \succ D \succ C \succ A$	4	$D \succ C \succ B \succ A$

which define the plurality ranking of $A \succ B \succ C \succ D$. Just before A is contacted to offer her a position, C calls to drop out because she just accepted a position elsewhere.

With C dropping out, should the decision be re-evaluated? With C's low ranking, I doubt whether any group would hold another election. They should; two of C's previous voters would now vote for B and five for D, creating the conflicting $D \succ B \succ A$ outcome! Indeed, with this example, if *any* one or two of the candidates drops out, the plurality ranking reverses; it is compatible with the opposite $D \succ C \succ B \succ A$. So, is A or D the "true choice of these voters?"

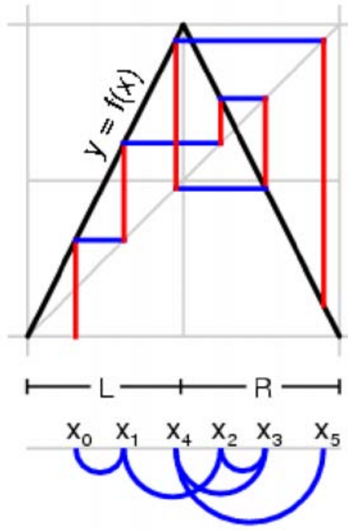


Figure 2. Iterations of the function $f(x)$ simulate the random behavior of the previous figure. In this case the initial value x_0 produces $LLRRLR\dots$.

The mathematical issue prompted by this example is clear. How bad can it get? What combinations of rankings for the different subsets of candidates can be actual election rankings? Combinatoric complexities discourage exploring this issue by creating examples. But as my research includes analyzing the dynamics of Newtonian N-body systems, a natural way to address this potentially chaotic state of affairs is to borrow notions from "chaos".

Start with a highly random situation; flipping a penny. The first flip outcome is either Heads or Tails. In either case, the second flip could be H or T. A tree, such as Figure 1, indicates all possibilities, while the mechanism of random behavior ensures that any branch of this tree, any listing of H's and T's, can arise. A sense of "chaotic dynamics" is created for a deterministic system if its orbits mimic the random behavior depicted by this tree by permitting all of the same possibilities. This can happen; e.g., a way to describe the orbits of the iterative system

$$x_{n+1} = f(x_n),$$

where the graph of f on the unit interval is given in Figure 2, is with this tree of random coin-flipping behavior. On the tree, replace H with L for "Left", and T with R for "Right". For any sequence of L's and R's, there exists an initial point so that for each k, the k^{th} iterate is in the indicated region. Because each tree branch represents an admissible orbit, this deterministic system captures the randomness of Figure 1. A power of this approach is that actual initial points for the deterministic system are not found; only their existence is verified.

To explore how to use a similar approach to determine whether a "chaotic state of affairs" occurs with voting rules, create a "tree of possible election outcomes". With three candidates $\{A, B, C\}$, start with the pair $\{A, B\}$ and draw three legs representing the three possible $A \succ B, A \sim B, B \succ A$ rankings. Pair $\{B, C\}$ has a similar three-legged tree; append a copy to each of the three possible $\{A, B\}$ legs, and then do the same for $\{A, C\}$. This leads to a tree with 27 branches; what remains are the 13 rankings for the triplet $\{A, B, C\}$. Attaching this 13-legged object to each of the 27 branches creates a tree with 351 legs that lists all possible combinations of rankings over all subsets; denote this three-candidate tree by $\mathcal{T}^3$. Similarly, $\mathcal{T}^4$ represents the four-candidate tree that has over 1.4 billion legs. Just as with deterministic dynamics, a way to determine whether a sense of randomness can be associated with voting outcomes is to determine which branches, which combinations of rankings over the different subsets of candidates, can be realized as election outcomes with a profile.

Theorem 3. *(Saari [7, 8]). For any three-candidate positional voting rule where $w_1 \neq 2w_2$, anything is possible; for any branch on the $\mathcal{T}^3$ tree, there exists a profile where the sincere election outcome of each subset is as specified. Only the Borda Count, where $w_1 = 2w_2$, does not admit all $\mathcal{T}^3$ branches; e.g., any branch on this tree where a candidate is ranked last in all pairwise rankings and ranked first in the ranking of all three cannot be a Borda outcome.*

As true with chaotic deterministic dynamics, but with the sole exception of the Borda Count, Theorem 3 indicates the "chaotic" sense allowed by election outcomes in that they permit anything to happen; e.g., a non-Borda positional election winner could even be the loser in "head-to-head" majority vote comparison with all other candidates. (Below I show how to create such examples.) Notice that knowing which $\mathcal{T}^3$ branches are non-admissible identifies properties for the positional voting rule. For instance, some of the branches that are not permitted by Borda outcomes prove that it is the only positional rule where its outcomes are related to how the same voters majority-rank the candidates in pairs.

With $n \geq 3$ candidates, we need terminology to represent which voting rules are used to tally elections for different subsets of candidates. Let $\mathbf{W}^n$ list the voting vectors assigned to tally the different subsets of candidates; after removing obvious redundancies, these vectors include an open set in an appropriate Euclidean space. So, if the plurality vote always is used, $\mathbf{W}^n$ is a concatenated list of plurality vectors. Let $\mathbf{B}^n$ represent where all elections are tallied with the Borda Count; i.e., the voting vector for each subset of candidates is such that the differences between successive weights is the same. Adopting terminology from dynamics, a list of rankings created by a profile, where the election for each subset is tallied as required by $\mathbf{W}^n$, is called a $\mathbf{W}^n$ *word*. For instance, the Equation 1 plurality word consists of all rankings described above. The $\mathbf{W}^n$ *dictionary*, denoted by $\mathcal{D}^n(\mathbf{W}^n) \subset \mathcal{T}^n$, is the set of all possible $\mathbf{W}^n$ words. We now can state what happens in general.

Theorem 4. *(Saari [7]). For $n \geq 3$, there exists a proper, lower-dimensional algebraic variety $\mathcal{V}^n$ such that if $\mathbf{W}^n \notin \mathcal{V}^n$, then*

$$\mathcal{D}(\mathbf{W}^n) = \mathcal{T}^n. \tag{2}$$

However, $\mathbf{B}^n \in \mathcal{V}^n$, and for all $\mathbf{W}^n$ where at least one voting vector is not a Borda vector,

$$\mathcal{D}(\mathbf{B}^n) \subsetneq \mathcal{D}(\mathbf{W}^n). \tag{3}$$

Oh my; this result, which proves that the Equation 1 example is tame compared to what else can happen, explains my lost faith in the plurality vote. This is because all of the "vote-for-k" rules normally used in departmental and societal elections, including the plurality vote, are *not* in the algebraic variety $\mathcal{V}^n$. Consequently (from Equation 2), any perverse listing of election rankings, even if the ranking for each of the $2^n - (n+1)$ different subsets of candidates is selected in a random manner, can actually occur! The good news is that the rare $\mathbf{W}^n$ systems belonging to the algebraic variety $\mathcal{V}^n$ are spared certain paradoxes involving weird conflicting rankings for different subsets of candidates; the Borda Count always belongs to $\mathcal{V}^n$. In fact, while the Borda Count admits some inconsistent behavior (I indicate how to find all examples), Equation 3 proves that any list of questionable Borda election outcomes must also occur with any other collection of voting vectors.

At this point, please forgive me for tossing in a "gee-whiz" comparison to demonstrate the benefit derived by using the Borda Count over, say, the plurality vote. The plurality dictionary for seven candidates is identified with $\mathcal{T}^7$, so we could compare the cardinality of $|\mathcal{T}^7|$ with $|\mathcal{D}(\mathbf{B}^7)|$. While the difference $|\mathcal{T}^7| - |\mathcal{D}(\mathbf{B}^7)| > 10^6$ sounds impressive, by asserting that the plurality vote admits a million more paradoxical settings, a more impressive comparison is that $|\mathcal{T}^7|/|\mathcal{D}(\mathbf{B}^7)|$ exceeds a billion times the number of droplets of water in all oceans in the world. Thus Borda provides a shockingly higher level of consistency; the choice of a voting system matters! Incidentally, I have characterized the algebraic variety $\mathcal{V}^n$;

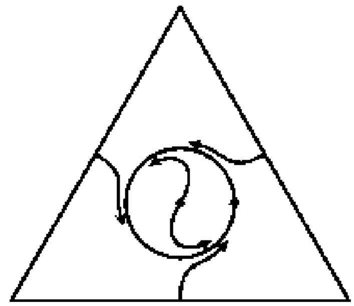


Figure 3. Scarf's example contradicting the "Invisible Hand Theory" .

e.g., in terms of voting outcomes, different $\mathcal{V}^n$ branches categorize those $\mathbf{W}^n$ that have similar, new kinds of election inconsistencies.

Before explaining why these different election outcomes arise, it is worth indicating how these Theorem 4 results extend to other disciplines. In her thesis and a *JASA* article [1], Deanna Haunsperger used Theorem 4 to prove a similar result for non-parametric statistics. Namely, she replaced profiles with data sets, and positional rules with non-parametric statistical rules. As above, a "statistical dictionary" collects all lists of rankings, over different subsets of alternatives that come from some data set by using a specified collection of statistical procedures. In proving that Theorem 4 extends to non-parametric statistics, Haunsperger showed that the Kruskal-Wallis rule assumes the role of the Borda Count. Thus, she proved that while the Kruskal-Wallis rule admits many new kinds of paradoxical behavior, when judged against all other choices, the Kruskal-Wallis rule is by far the most consistent!

The "algebraic variety" conclusion accurately suggests that the analysis involved using different symmetry groups. Just by changing the groups, Theorem 4 extends to areas such as probability where the algebraic variety corresponds to various independence conditions. An interesting extension, where an algebraic variety result has yet to be identified, is the Adam Smith "Invisible Hand" story. To introduce the ideas for a "pure exchange" model, prices for n commodities can be normalized to the "price sphere", which is the positive orthant of S^{n-1}. The aggregate excess demand function, which is the difference between the total demand and supply of each commodity at given prices, defines a continuous tangent vector field. Smith's story requires the vector field to have an attracting equilibrium. Contradicting this story was a stunning example created by H. Scarf where, instead, the only price equilibrium is a repeller forcing prices away from the equilibrium! This was followed by a Theorem 1 type result first recognized by H. Sonnenschein, advanced by R. Mantel, and posed in its current form by G. Debreu showing that beyond Scarf's example, "anything can happen." First, bound all prices away from zero by any specified $\epsilon > 0$. They showed it is possible to select *any* continuous tangent vector field to this price sphere, and then an initial endowment of goods and a nice utility function for each customer can be found so that, with the ϵ restriction, the aggregate excess demand function agrees with the selected vector field.

For a result parallel to Theorem 4, I extended this conclusion to all possible subsets of two or more commodities. Namely, with n commodities and $a \geq n$ agents, and any $\epsilon > 0$, select any continuous tangent vector field for the appropriate sphere for each subset of two or more commodities. There exists a utility function and initial endowment for each agent so that, with each subset of commodities, the aggregate excess demand function agrees with the selected vector field within the ϵ restriction. In other words, with the same economic agents, the economics associated with different subsets of commodities need not have anything to do with each other; chaos reigns. From a mathematical perspective, the proof of this theorem involved creating appropriate continuous foliations, with appropriate convexity properties, to represent the level sets of the individual utility functions. My main point is that we must expect Theorem 4 type of results with aggregations.

Symmetry Structures

The next step is to develop appropriate mathematical tools to analyze and explain all of these voting problems. The complexity is a dimensional issue; the $n!$ dimension of the domain, which is the space of preferences for n alternatives, quickly exceeds the dimension of the range, which is the space of election outcomes. As we know, a larger domain allows more kinds of outcomes: they become the voting paradoxes. So, a way toward creating a systematic analysis of voting issues is to develop appropriate structures to better understand the domain.

Clues about what should be done are abundant throughout mathematics; e.g., in Galois theory, we are interested in elements that are fixed by automorphisms from particular subgroups. The approach adopted here is to find appropriate configurations of profiles that force a neutral, tied outcome for certain classes of voting rules, but non-tied outcomes for others. As an illustration, the orbit of the Klein four-group leads to the configuration

$$\begin{aligned} (4) \qquad & A \succ B \succ C \succ D, \quad D \succ B \succ C \succ A, \\ & B \succ A \succ D \succ C, \quad C \succ D \succ A \succ B. \end{aligned}$$

Each candidate is in first, second, third, and fourth place precisely once, so all positional outcomes

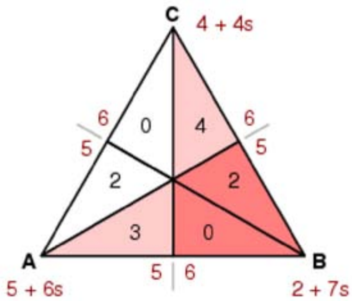


Figure 4. Geometric tally. The score $2 + 7s$ for B counts the chambers nearest to B as well as the second-nearest chamber. Similarly for other vertices.

are ties. Moreover, each ranking is accompanied by its reversal, so all pairwise rankings are ties. But the situation changes with triplets; e.g., dropping D results in

$$A \succ B \succ C, \quad B \succ C \succ A, \quad B \succ A \succ C, \quad C \succ A \succ B, \tag{5}$$

where of all positional outcomes, only the Borda Count defines a complete tie so only it maintains consistency with the rankings of the other sets of candidates. As adding configurations of the Equation 4 type to a profile can change the rankings of triplets, but affects nothing else, it is clear that these configurations play a fundamental role in explaining Theorem 4 features where positional rankings of triplets differ from those of other subsets of candidates; it also helps explain why the Borda Count admits significantly fewer paradoxical outcomes than other positional rules.

To indicate how appropriate profile configurations are found, I'll outline the three-candidate structure. To do so, let me introduce a geometric way I developed to simplify the tallying of three-candidate ballots. As in Figure 4, assign each candidate to a vertex of an equilateral triangle. Divide the triangle into regions based on the distance to each vertex where "closer is better;" e.g., as all points on the vertical line are equal distance to vertices A and B, this line corresponds to a tied $A \sim B$ ranking. Points to the left and right represent, respectively, $A \succ B$ and $B \succ A$. The resulting 13 regions represent the 13 transitive rankings; the regions on lines correspond to rankings with ties.

In each open *ranking region*, place the number of the voters with this preference. Figure 4, for instance, has the introductory example data. To use the geometry to tally ballots, notice that all votes with $A \succ B$ are to the left of the vertical line. So, to compute the $\{A, B\}$ outcome, simply add the numbers on each side of the vertical line obtaining the $B \succ A$ outcome by $6 : 5$. The pairwise outcomes for each other pair are similarly computed and listed by the appropriate edge.

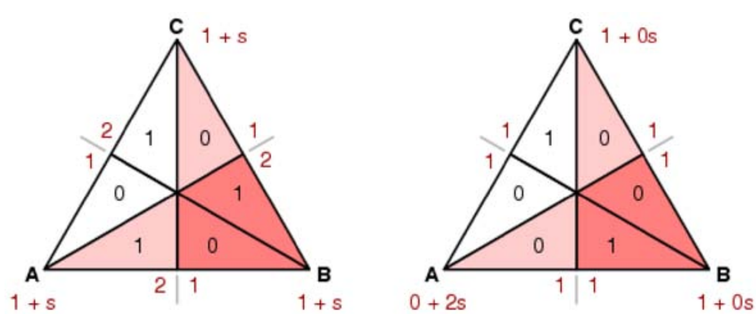
To compute all positional outcomes, normalize each $(w_1, w_2, 0)$ vector into a $\mathbf{w}_s = (1, s, 0)$ form, $s \in [0, 1]$, by dividing by w_1. This means, for instance, that the Borda $(2, 1, 0)$ vector is normalized to $(1, \frac{1}{2}, 0)$. The $\mathbf{w}_s$ tally for candidate C is

> [number of voters with C top-ranked] + s[number of voters with C second-ranked].

Using the geometry, this is the sum of numbers in the two regions with C as a vertex plus s times the sum of numbers in the two adjacent regions, or $[4 + 0] + s[2 + 2]$. The $\mathbf{w}_s$ tallies for all three candidates are similarly computed and listed near the appropriate vertex on the triangle. In this simple manner, all positional and pairwise outcomes are easily computed.

The geometry of tallying suggests we should examine the triangle's symmetry structure. A first choice is the *kernel* where each ranking is supported by the same number of voters; here all positional and pairwise outcomes are ties. The next obvious choice is the 120° symmetries, or a Z_3 orbit defining, say, $A \succ B \succ C, B \succ C \succ A, C \succ A \succ B$. The tallies in Figure 5a show that this configuration never affects positional rules, as all outcomes are ties, but it does influence the pairs as they define a cycle. Thus, this kind of configuration in a profile causes the majority vote pairwise outcomes to differ from positional outcomes. A final symmetry is 180°, or a Z_2 reversal such as $C \succ A \succ B$, $B \succ A \succ C$. As Figure 5b shows, this configuration never affects pairs as they are all ties, but it affects all non-Borda positional outcomes, $s \neq \frac{1}{2}$, as they are not ties. Consequently, reversal configurations in a profile force differences among all non-Borda positional outcomes and differences from pairwise outcomes.

While a bit more work is required to convert all of this into a working tool for social scientists, such as creating a coordinate system for profile space [10], this structure explains all possible positional and pairwise three-candidate election inconsistencies. For instance, in the first paragraphs, I asked



Figures 5a (left) and 5b. Symmetry structures.

if the reader could explain why the introductory example allowed conflicting positional outcomes. As just indicated, the answer is that all possible differences among positional outcomes are caused by reversal configuration components in a profile. Indeed, to create the introductory example, I started with $C \succ B \succ A$ and added two reversal components of $A \succ C \succ B, B \succ C \succ A$ and three of $A \succ B \succ C$, $C \succ B \succ A$ to force the positional differences. While these terms influence the positional outcomes, they do not affect Borda or pairwise rankings, which remain $C \succ B \succ A$. If you wish to enhance the example with a cyclic pairwise outcome, just add an appropriate multiple of either the $A \succ B \succ C, B \succ C \succ A, C \succ A \succ B$ or the $A \succ C \succ B, C \succ B \succ A, B \succ A \succ C$ configuration; this term never affects positional rankings but it can change pairwise rankings. Notice, only the Borda Count is not affected by reversal and cyclic profile configurations; this turns out to be an explanation for the many favorable properties of the Borda Count, and it explains why it is immune to many of the voting paradoxes.

The general n-candidate case is analyzed in a similar manner. Here, to use the geometry of tallying, the equilateral triangle is replaced with an equilateral n-simplex. Thus the symmetries of this simplex capture the structures of voting. A lesson learned from algebraic topology is how permutations in the interior of a simplex have an interesting effect on the faces; some of this is captured here by wreath products of permutation groups. Thus, a way to identify which profile configurations explain voting inconsistencies is to determine which orbits of subgroups create configurations that force completely tied outcomes for some subsets of candidates but not for others.

For a flavor of what happens in general, it turns out for n candidates that Z_n orbits of the

$$(6)\quad A_1 \succ A_2 \succ \cdots \succ A_n, A_2 \succ \cdots \succ A_n \succ A_1, \ldots, A_n \succ A_1 \succ \cdots \succ A_{n-1}$$

type explain all possible problems, inconsistencies, etc., of majority votes over pairs. Adding such a configuration to any profile does not affect n-candidate positional rankings, but it can change majority vote rankings with pairs. Such orbits, however, affect *all positional outcomes* for smaller subsets of candidates. (To see why, consider the $n = 4$ case and compute what happens when a candidate is dropped.)

Incidentally, the Equation 6 type of profile configurations turns out to be the sole cause of any inconsistent rankings for the Borda Count. As such, a strong case can be made that the Borda ranking over all candidates most accurately reflects the views of the voters. A stronger case comes from noting that certain profile configurations that are orbits of symmetry groups should result in ties, but only the Borda Count always respects this.

Other Rules and Extensions

As this kind of symmetry structure answers questions about voting rules, it also answers questions about other aggregation methods. For instance, Anna Bargagliotti and I are developing related symmetry arguments to explain mysteries about non-parametric statistical rules. This structure also answers other voting mysteries such as Arrow's Theorem and Sen's Theorem, which have disturbing conclusions asserting that it is impossible for any voting rule to do what appears to be obviously possible to do. But by examining these results in the light of the above symmetry structures, it turns out [9, 10] that these seminal theorems occur because assumptions requiring the decision rule to emphasize pairs negate the crucial assumption that voters have transitive preferences. By understanding the mathematical source of the problem, benign resolutions are immediate.

There are all sorts of other voting concerns; e.g., voters might be strategic, or there might be too many candidates to realistically expect voters to rank them. But once we understand the mathematical structure of voting rules, all such issues can be addressed. A similar comment applies to wide classes of other voting rules; e.g., both the AMS and the Mathematical Association of America use something called "Approval Voting" (AV) where a voter votes "approval" for as many candidates as he or she wishes. Stated in another manner, a voter ranks the candidates and selects which "vote-for-k" rule to tally the preferences. From Theorem 4, where it is shown how the "vote-for-k" rules cause so many problems, and the fact that the dimension of the associated domain increases significantly, it is easy to show that AV introduces many new and troubling problems. (With the introductory example, for instance, all 13 ways to rank the candidates are admissible AV outcomes.)

Perhaps an appropriate concluding comment is to recall how for a couple of millennia, mathematics and the physical sciences have enjoyed a symbiotic relationship where advances in one area motivated advances in the other. There is a new opportunity; within the last couple of decades, the social and behavioral sciences have become mathematically more sophisticated, which suggests that a similar, mutually advantageous relationship can be developed. Beyond voting, areas that appear ripe for mathematical analysis include behavioral sciences such as psychology and social sciences such as economics and political science. Several mathematicians are creating such connections with voting. What I find particularly attractive about these areas is that their issues differ from what is found in the physical sciences, so new kinds of mathematics often is needed. In other words, the underlying mathematical structures needed to convert the somewhat ad-hoc mathematical analysis into a systematic approach awaits some mathematician to develop them. I invite more mathematicians to examine these fascinating topics.

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Henri's Crystal Ball

Philip J. Davis and David Mumford

On April 10, 1908, at the general session of the Fourth International Congress of Mathematicians held in Rome, Gaston Darboux presented a talk by Henri Poincaré (1854–1912) entitled *The Future of Mathematics* (Poincaré was unfortunately unable to deliver the talk himself). The original can be found on the web at: http://gallica.bnf.fr/ark:/12148/bpt6k17083c/f934n10.capture.

It is now a full century since this date, and it is of some interest (and amusement) to see how its contents shape up in view of the historically turbulent years and the tremendously productive mathematical ones that followed. Poincaré's talk has long been available in an English translation. This translation can now be downloaded either in pdf or html form: http://www-history.mcs.st-andrews.ac.uk/Extras/Poincare_Future.html or http://portail.mathdoc.fr/BIBLIOS/PDF/Poincare.pdf. The remarks in this paper are based on the latter translation.

The talk is divided into two parts; the first presents generalities. In the second part, numerous specific problems are mentioned in ten different areas, in which the current status of the problems is described and suggestions are made that research would be welcome along certain follow-up lines. The language of the first part is vivid and clear. This is not the case with the second part, but what impresses there is the wide range of mathematical material that Poincaré had at his command.

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Poincaré's talk should be compared and contrasted with the earlier talk by David Hilbert (1862–1943) at the Second International Congress of Mathematics held in Paris in 1900. Hilbert specified twenty-three problems that he said were important and open for solution. History has given the accolade and notoriety to Hilbert's problems, whereas Poincaré, who did not list specific problems, has attracted not nearly as much publicity and éclat within the mathematical world.

Poincaré's Generalities

First: a brief summary of the first part of Poincaré's paper. He proposes that some people considered mathematics in 1908 to be rich in ideas, having developed in "every sense". But then he says, if this was absolutely true, "Our riches would become an encumbrance" and produce an incomprehensible increase in knowledge. One answer to the plethora of material is professional specialization. But this may be a "vexatious obstacle to the progress of our science". Instead, he affirms that we must fight specialization by seeking unifying ideas:

> If a new result has value it is when, by binding together long known elements, until now scattered and appearing unrelated to each other, it suddenly brings order where there reigned apparent disorder.

Poincaré is fond of the Viennese physicist/philosopher Ernst Mach (1838–1916). "The role of science," Poincaré quotes Mach as saying, is "the production of economy of thought, just as a machine produces economy of labor." Poincaré carries this over to mathematics, citing both formulas and unifying theories.

He further asserts that the aesthetic element is often bound up with an achieved economy of thought as well as labor. The aesthetic element in methods and results is thus of great importance. It is not pure "dilettantism" because it brings "a comprehension at the same time of the whole and the parts". He asserts that long calculations alone cannot reveal the general structure of the originating problem:

> When a somewhat long calculation has led us to a simple and striking result we are not fully satisfied until we have shown that we could have foreseen…its most characteristic details. … how vain would be any attempt to replace by any mechanical process the free initiative of the mathematician.

Poincaré appreciates the rigor that the preceding fifty years had brought to mathematics but is wary of making a fetish out of it:

> In mathematics rigor is not everything, but without it there would be nothing…But is it necessary to repeat every time this discussion?…I fear that in this lengthening of our demonstrations they will lose that appearance of harmony.

The linguistic element—i.e., the creation of new terms—is also of great importance. An older example is the word "convergence", but he gives as newer examples "group", "invariant", "isomorphic", and "transformation".

> One of those marks by which we recognize the pregnancy of a result is in that it permits a happy innovation in our language. The mere fact is oftentimes without interest; it has been noted many times, but it is of no service to science. It becomes of value only on that day when some happily advised thinker perceives a relationship which he indicates and symbolizes by a word.

Poincaré acknowledges "the study of postulates, of unusual geometries, of functions having unusual values" as showing us "the workings of the human mind… when freed from the tyranny of the external world." But he is not impressed: "It is to the opposite side—the side of nature—against which we must direct the main corps of our army."

He imagines a physicist or engineer coming to a mathematician with a problem. Sometimes but not often the solution can be expressed explicitly in terms of known functions. But there is likely to be a power series solution: still, does it converge fast enough to be useful? His engineer has a time constraint and cares little for what "the engineer of the twenty-second century" can do (is he imagining a future of ultra-fast computers?). But for the mathematician, the conclusion is that "there are no longer some problems which are solved and others which are not" because one has qualitative approaches as well as quantitative, computationally useful as well as computationally useless power series in addition to traditional solutions. Poincaré promotes the qualitative as opposed to the quantitative when the latter is not immediately or easily forthcoming.

Henri Poincaré

Poincaré's overall conclusion is that the best that one can do in predicting the future of mathematics is to start with the present and give heed to these rubrics: take the various general lines along which progress has been accomplished, extrapolate these by generalization, abstraction, analogies, etc. But we should expect the greatest advances when two branches of mathematics find a "similarity of their forms despite the dissimilarity of material", where "each takes profit from the other."

A Few Comments

The comments that follow draw, of course, on our knowledge of post-Poincaré developments in mathematics.

Predicting the future

Is the possession of a crystal ball a specific skill possessed by some and not others, and do the people who are considered the most brilliant and prominent in a field have a better crystal ball? Perhaps their prominence and their lines of thought in part shape the future. Can we do no better than, as Poincaré suggests, to extrapolate from the present by calling for intensifications, generalizations, analogies, abstractions, etc., of what is already around? What about the genuinely new? Historians always find seeds of such developments in the past, but these are post-hoc judgments.

Freeman Dyson analyzed how well one can predict the future and came to the conclusion that, in science, unexpected technological breakthroughs were often the events that led to the discovery of wholly new and unexpected phenomena and thus to new theories. In mathematics it is harder to separate technology and theory, but we shall find much of Dyson's caution reflected in mathematical developments that Poincaré did not predict when we take up specific fields below.

Increase in the corpus of mathematics and specialization

Poincaré's remarks on the increase of the mathematical corpus, of the subsequent specialization and the limitations it causes, are certainly valid. Specialization and specialized vocabulary have exploded and create huge barriers to sharing ideas both within mathematics and to neighboring fields. When Alexander Ostrowski (1893–1986) came up for his doctoral examination in 1920 (under the supervision of Hilbert and Landau) he once confided to one of us—perhaps in jest—that he was the last student in mathematics who would ever be expected to answer questions in any part whatever of mathematics. Poincaré himself has been called "the last of the universalists".

At least the number of Ph.D.'s in mathematics who have been able to find gainful employment has increased. Specialization has led to a vast increase in the number of journals, societies, meetings, papers (the last can be accurately tracked in the exponential growth of *Math Reviews* and *Zentralblatt*).

Rigor

As Poincaré feared, there has occurred a widespread tendency towards more and more lengthy rigorous expositions and a resulting increase in the difficulty of finding the essential ideas often buried in a mathematician's papers. Sometimes an increase in abstraction as well as the explosion of new technical terms has compounded the bad effects of meticulous rigor (although they are not the same). All of these make papers in any but your own narrow field harder to read.

But the question may be asked, insofar as absolute rigor is an unattainable ideal, how is it to be attained and how much rigor suffices? An extreme, not envisioned in Poincaré's day, is computer verification of proofs carried out within a precisely defined set of predicate calculus axioms, as in the work of R. S. Moore and J. S. Boyer. The history of mathematical proof shows that great mathematicians of the past were not hung up on rigor, that the standards of rigor have waned and waxed. "Sufficient unto the day is the rigor thereof." However, the twentieth century did bring some debacles: many papers in algebraic geometry in the period 1920–1950 did contain "proofs" which were wrong or uncorrectable.

Computation

Poincaré seems rather hard on what might be called "naked" computation, denying it a role in the discovery process. Of course, the electronic digital computer was not around in Poincaré's day, and for him computation meant hand calculation, algebraic as well as numerical. We believe computation does have an important role. It has had differing impacts on different areas of mathematics, but few areas have not felt its impact. We will discuss several examples below.

In every generation, some mathematicians have used calculation extensively and some not. Gauss is a clear example that some of the most brilliant mathematicians have loved to calculate. With contemporary computers, those who do like to calculate have the power of a race car at their disposal compared with Gauss's horse and buggy.

One can cite the existence of journals devoted to "experimental mathematics". Virtually all the papers published in them draw sustenance from computer results. In this connection, it is appropriate and revealing to quote from the announced philosophy of the *Journal of Experimental Mathematics*:

> Experiment has always been, and increasingly is, an important method of mathematical discovery. (Gauss declared that his way of arriving at mathematical truths was "through systematic experimentation".) Yet this tends to be concealed by the tradition of presenting only elegant, well-rounded, and rigorous results. While we value the theorem-proof method of exposition, and do not depart from the established view that a result can only become part of mathematical knowledge once it is supported by a logical proof, we consider it anomalous that an important component of the process of mathematical creation is hidden from public discussion. It is to our loss that most of us in the mathematical community are almost always unaware of how new results have been discovered. It is especially deplorable that this knowledge is not made part of the training of graduate students, who are left to find their own way through the wilderness.

While we agree largely with the sentiments expressed here, we disagree with the above stated "established view" that all mathematical knowledge rests on logical proof. We would assert that in various ways "mathematical knowledge" goes beyond that which is supported by logical proof.

The quantitative vs. the qualitative

Poincaré, of course, was one of the discoverers of the whole field of topology, and this is the prime area where qualitative approaches to geometry superseded quantitative ones. But in many other fields of pure and applied mathematics these two approaches still vie for dominance.

In analysis, Poincaré uses the term "quantitative" to indicate not just isolated numbers but the whole theory of particular special functions from which specific numbers relevant to a theory or experiment have been derived and which then allow automatic application to other parallel theories.

From Poincaré's own work to the present day, the replacement of the "quantitative" by the "qualitative" has played a great role in the theory of differential equations. It was challenged by an old comment of the Nobelist in physics, Ernest Rutherford, that "the qualitative is naught but poor quantitative." What might have been in Rutherford's mind was set out for us recently by a physicist friend:

> Suppose, for example, someone has a theory of capillary attraction. Try it: water rises in the tube. Is the theory correct? Unless measurement agrees with a quantitative prediction you can't possibly know; a qualitative experiment would be a waste of time and money. Of course, it can happen that a false theory gives a correct answer but it happens rarely; also no experiment is foolproof, but everybody knows there is room for coincidence and error. Science does not deal with facts but with probable facts. These would be a logician's nightmare but they are part of a scientist's everyday thinking.

The so-called "catastrophe theory" of Thom, Zeeman, Arnold, and others is a prime example of a qualitative theory whose validity many have questioned. Its mathematical elegance is obvious but since it avoids ever committing to specific models and differential equations, its applicability is uncertain. Self-similar "fractal" models are another example of a theory in this gray area. There is extensive numerical evidence in extremely diverse fields for self-similarity over several orders of magnitude but there are relatively few physical models that demonstrably exhibit this.

In many areas of applied mathematics, models are proposed for some aspect of highly complex systems that cannot be modeled in their entirety. These models are qualitatively reasonable but, in order to argue for their validity, they are fleshed out in quantitative guise in order to make "predictions" about experimental results and/or computational simulations. This is sometimes a dubious procedure that can be summarized by the skeptical sentiment "every model is doomed to succeed." However, there is much that can be said pro qualitative analysis, and there is an ongoing and red-hot debate between the qualitative and the quantitative that will most certainly be argued beyond our lifetimes.

Aesthetics as an element of discovery and presentation

Poincaré's sentiments, when he asserts that for mathematicians elegance means a quality of a proof that makes the whole comprehensible, have been echoed by Gian-Carlo Rota. "A proof is beautiful," Rota wrote, "when it gives away the secret of the theorem, when it leads us to perceive the actual and not the logical inevitability of the statement that is proved." Aesthetics are certainly an important part of mathematics, one that has attracted much comment and speculation. But it should not be overstressed:

> I once heard [Paul] Dirac (1902–1984, British physicist) say in a lecture, which largely consisted of students, that students of physics shouldn't worry too much about what the equations of physics mean, but only about the beauty of the equations. The faculty members present groaned at the prospect of all our students setting out to imitate Dirac.
>
> —Steven Weinberg
> *Towards the Final Laws of Physics*

The aesthetic and the useful should not be confused. Thus, there are very many computer programs useful in promoting mathematical discovery and scientific computation that are hardly aesthetic by the criterion of simplicity or any other criterion such as that proposed by Gian-Carlo Rota. The proofs of the four-color theorem and of Kepler's conjecture, which rely heavily on computation, attest to their usefulness.

The linguistic element

Poincaré's comment on the role of the linguistic element in mathematics is both sharp and prophetic. It has been explored only in recent decades and deserves more elaboration and attention. The famous linguist, Benjamin Whorf, proposed that the structure of your language affects, in fact constrains, your understanding of a situation, the way you think about it. In a recent letter to us, semioticist and mathematician Kay O'Halloran put Poincaré's perception in current semiotic terminology (or jargon):

> The "word" gives rise to existence![1] The relationship symbolized by the word undergoes processes of co-contextualisation and re-contextualisation to enter

[1] *This sounds almost theological. Cf. John: 1:1 "In the beginning was the word." But see: Kay L. O'Halloran,* Mathematical Discourse: Language, Symbolisms and Visual Images*, Continuum, London & New York, 2005.*

> into other relationships: a never-ending ongoing phenomenon.

On the other hand, the development of twentieth century mathematics has seen the explosion of specialized vocabularies in each sub-sub-area of mathematics. Who is conversant with all the concepts of Woodin cardinals, Lie superalgebras, algebraic stacks, perverse sheaves, Weyl tensor, Thom spectra, Besov spaces, semi-martingales, chromatic index, and trapdoor functions, each basic in its own field? It is not that these concepts are minor—they are each part of the standard vocabulary in their area. But sadly, they are a huge impediment to Poincaré's dream that "interlockings" between diverse fields will drive the deepest future discoveries.

Poincaré's specific predictions

In the second half of his talk, Poincaré takes up each of the areas of mathematics and makes specific comments. Poincaré, in contrast to Hilbert, does not pinpoint the problems to be worked on; he merely points in a general way to certain "subareas" and issues in each field, sometimes with frustratingly vague phrases. Ideally, the second part of the talk should be responded to by experts in the various fields or sub-fields—thus confirming Poincaré's concern about the specialization and the fragmentation of mathematics. Nonetheless, we will do our best to say something about what he had right and what he missed! In some instances, what followed after 1908 cannot be adequately described except at the monograph level.

In each section, the heading is Poincaré's, and, in parentheses, we put in some cases the more standard contemporary name. A word of caution: much of what he says is pretty vague, and one needs (or at least we need) to interpret his text and guess what he is suggesting.

Arithmetic (number theory)

In this field, Poincaré is quite successful in predicting the twentieth century developments. His first point seems to us to foreshadow clearly the work of André Weil creating characteristic p algebraic geometry alongside traditional algebraic geometry over the complex numbers:

> The first example which comes to mind is the theory of congruences where we find a perfect parallelism with that of algebraic equations. And we will certainly complete this parallelism which must exist between the theory of algebraic curves and that of congruences of two variables, for instance. And when the problems relative to congruences of several variables are solved we shall have taken the first step toward the solution of many of the questions of indeterminate analysis.

By "congruences",[2] he clearly means polynomial equations mod p. Although "indeterminate analysis" is pretty vague, a sympathetic interpretation would be that he is asking for connections between solutions of polynomials in many variables mod p and their solutions over the integers, Diophantine equations. And this is what Weil's conjectures made precise and Dwork, Grothendieck, and Deligne proved. He pursues the analogy between number theory and algebraic geometry in the next paragraph:

> Another example where the analogy has not always been seen at first sight is given to us by the theory of corpora and ideals. For a counterpart let us consider the curves traced upon a surface; to the existing numbers correspond the complete intersections, to the ideals the incomplete intersections, and to the prime ideals the indecomposable curves; the various classes of ideals thus have their analogs.

Here he seems to be talking about the theory of divisors on varieties, the "Picard" group or ideal class group and the analogy again between the number-theoretic situation and the algebro-geometric situation. In the twentieth century, class field theory in the number-theoretic case, and the theory of generalized Jacobians and Picard varieties in the algebro-geometric case, have developed this analogy. But note that Hilbert's famous *Zahlbericht* had appeared when this lecture was given and contained clearer leads to later developments.

His next topic is the theory of quadratic forms for which he says "(It) was one of the first to take shape … when the arithmeticians introduced unity through the consideration of groups of linear transformations." He suggests that further groups may yield more fruit, and he brings up discontinuous groups and Minkowski's *Geometrie der Zahlen*. Although this is a bit of a leap, one might say that his ideas are leading to the theory of semi-simple algebraic groups and their discrete subgroups. This has been one of the major themes of work in the twentieth century.

Finally, there is a paragraph about prime numbers, where, he says,

> I believe I have a glimpse of the wished for unity…All leads back without doubt to the study of a family of transcendental functions which, through the study of their singular points and the application of the method of M. Darboux, will permit the calculation asymptotically of certain functions of very great numbers.

[2] *The translator mistakenly wrote "congruents" for Poincaré's word "congruences".*

In this rather mysterious passage, it is possible to guess that he is foreshadowing the tremendously successful use of L-functions in number theory. If so, he has touched on all the major themes of twentieth century number theory.

Algebra

In Section II (Algebra), Poincaré focuses narrowly on polynomial equations. He starts by saying "the most important [subject here] is that of groups…," obviously meaning Galois groups, but he will treat groups in a separate section. He discusses instead "the question of the calculation of the numerical value of roots and the discussion of the number of real roots."

Concerning the numerical calculation of roots of polynomials, it is hard to decipher Poincaré's specific remarks but there has certainly been much work and much success both experimental and theoretical—all stimulated by the appearance of increasingly powerful digital computers. Indeed, the whole of numerical analysis, relatively stagnant, burst forth in the digital age like the desert cactus that blooms when it suddenly rains. Now, there is hardly a package for scientific computation that does not have a reasonably high precision polynomial root finder. Many diverse attacks on the problem have been made, each with its pluses and minuses.

An allied problem, perhaps of more applied significance than "mere" root finding, is that of the numerical calculation of the eigenvalues of a square matrix. The roots of a polynomial are the eigenvalues of its companion matrix. The QR algorithm gives a reliable method for eigenvalue calculation. So a method of choice, valid for polynomials of degree, say, less than several hundred, first inaugurated by Cleve Moler of (Matlab fame) and later provided a substantial theoretical underpinning by Edelman and Murakami, also Trefethen, is to go that route. The companion matrix must first be "balanced" by a standard similarity transformation to reduce the condition of the matrix. For polynomials of enormously high degree, arising in special problems, effective special algorithms have been devised. Future work on root finding will very likely be stimulated by improvements in digital computers combined with demands from scientific/technological applications.

Poincaré goes on to talk about invariants of homogeneous polynomials, i.e., functions of their coefficients invariant by linear substitutions, and mentions Gordon and Hilbert's work here. He then writes "If we have an infinity of whole polynomials, depending algebraically on a finite number among them, can we always deduce them from a finite number among them by addition and multiplication?" This would seem to be Hilbert's 14th problem. It was disproved by Nagata in 1959 for the ring of invariants of a representation of a power of the additive group. Both Hilbert and Poincaré seem to have been overly optimistic about finiteness results for rings of polynomials. He ends this section by proposing that questions about algebra should be done over rings of polynomials with integer or other coefficients but not pursuing this.

Differential equations (dynamical systems)

Poincaré starts off with a very astute proposal: we need a group of transformations that will group dynamical systems into classes that are easier to describe. He proposes the analogy with using birational transformations to classify algebraic curves. One can read this as foreshadowing Smale's idea of using the full group of homeomorphisms to classify dynamical systems, more precisely defining two systems to be topologically equivalent if there is a homeomorphism taking the orbits of one system to the orbits of the other. As an example, Poincaré raises the question of counting the number of limit cycles of two-dimensional dynamical systems.

Curiously, Poincaré does not talk about the complexities of dynamical systems that he had encountered in his work on the three-body problem theory. The modern theory of dynamical systems has been dominated by the struggle to find a satisfactory theory for such chaotic systems, the split between the relatively simple hyperbolic systems and those with strange attractors. Simple chaotic systems, such as the famous Lorenz system modeling convection cells in the atmosphere, were found to be ubiquitous in three or more dimensions.

Instead Poincaré mentions holomorphic vector fields in the plane and asks when they have integrals and what you can say about the functions that uniformize their orbits. One can imagine links with the discovery and exploration in the century to follow of the many unexpected completely integrable dynamical systems such as KdV, the Toda lattice, etc.

Equations with partial derivatives (linear PDEs)

Poincaré reviews what was then recent work of Fredholm on integral equations and clearly envisions the idea that linear PDEs are going to require an understanding of infinite-dimensional space and the extension of linear algebra to these spaces. He describes the analogy he sees between Hill's work on infinite determinants and Fredholm's theory, the analogy between an infinite-dimensional space of sequences and an infinite-dimensional space of functions. At the end, he acknowledges that "Thanks to M. Hilbert, who has been doubly an initiator, we are already on that path." That path is unifying these "two methods" and applying it to problems such as the Dirichlet problem.

As it turned out, linear PDEs were essentially mastered using function space techniques, distributions, and Fourier analysis a little after the middle of the twentieth century. Then the cutting edge turned to nonlinear PDEs which remain an area full of mysteries. Poincaré says nothing about, for example, the Euler and Navier-Stokes fluid flow equations.

The Abelian functions

This very short section is remarkably specific. The question Poincaré raises is

> What is the relationship of the Abelian functions begot by the integrals relative to an algebraic curve to the general Abelian functions and how shall we classify the latter?

The question in the last part of this quote has proven to be by far the more important one. It leads directly to the construction of Siegel's modular variety, the moduli space that indeed classifies what are now called principally polarized Abelian varieties. These spaces are the simplest arithmetic quotients of Hermitian symmetric spaces and all such modular varieties and the more general arithmetic quotients are a key component of the theories linking number theory, algebraic geometry and representations of Lie groups (especially the "Langlands conjecture"). Poincaré was certainly on the right track in raising this classification question.

The first part of his question is a much more special one, although it has been studied by quite a few mathematicians. In modern algebro-geometric language, it asks what is special about the Jacobian varieties of curves in the bigger set of all Abelian varieties? Finding ways to characterize Jacobians is now called the "Schottky problem". There are remarkably very many quite different ideas for solving this problem whose interrelations are still not completely clear: a review up to 1996 is in an appendix to one of the second author's books.[3]

The theory of functions (complex variables)

In another short section, Poincaré's main concern is the theory of analytic functions of several variables as opposed to one:

> The analogy with the functions of a single variable gives a valuable but insufficient guide; there is an essential difference between the two classes of functions (one and more than one variable) and every time a generalization is attempted by passing from one to the other, an unexpected obstacle has been encountered…

Thus:

> Why is a conformal representation more often impossible in the domain of four dimensions and what shall we substitute for it? Does not the true generalization of functions of one variable come in the harmonic functions of four variables …In what sense may we say that the transcendental functions of two variables are to transcendental functions of one variable as (algebraic or) rational functions of two variables are to (algebraic or) rational functions of one variable?

Poincaré certainly hit on a ripe area here. The work of William Fogg Osgood on functions of several complex variables that date shortly after 1908 can be found in his influential *Lehrbuch der Funktionentheorie*. The field opened up wide in the first half of the twentieth century, and we have the later theories and books of e.g., Behnke and Thullen, Bochner and Martin, Bergman, Kodaira and Spencer, Hörmander, Remmert, Krantz, Scheidemann. It gave birth to topics such as pseudo-convexity, Stein manifolds, and sheaf theory. The link with the theory of algebraic varieties of dimension two or more has been extremely fruitful, and the algebraic and transcendental theory have intertwined continuously. The ghost of Poincaré is very pleased.

Though Poincaré failed to mention analytic functions of one complex variable, this field also flourished in the years following 1908. It also has a rich history. The Riemann mapping theorem for simply connected regions was worked on by Osgood, Carathéodory, and Perron. The Bieberbach conjecture and the extensive theory of conformal mappings, Nevanlinna theory and the work of Ahlfors on meromorphic curves, Teichmüller theory and its connection to three-manifolds via Thurston theory all drove the field far.

The theory of groups

In a third short section, Poincaré states he will talk only about Lie groups and Galois groups, thus ignoring both the growing general theory of finite groups and the discontinuous Kleinian groups on which he had worked himself. He recalls how Lie groups have been tamed by the use of Lie algebras (which he describes as a "special symbolism upon which you will excuse me for not dwelling"). He says justly that "The study of the groups of Galois is much less advanced" and hopes that, as in the links between number theory and algebraic geometry, links can be made between Lie theory and Galois theory. The search for a better understanding of Galois groups has proven to be very difficult and continues to this day.

Geometry

Poincaré first asks if geometry is nothing more than "the facts of algebra and analytical geometry expressed in another language?" No, he says, "Common geometry has a great advantage in that the senses may come to the help of our reason and aid it in finding what path to follow." But "our senses fail us when we try to escape from the

[3] *D. Mumford,* The Red Book of Varieties and Schemes, *2nd edition, Springer Lecture Notes 1358.*

classical three dimensions." One should not forget that it was during Poincaré's lifetime that mathematicians had come to accept higher-dimensional spaces as a matter of course:

> We have nowadays become so familiar with this notion of more than three dimensions that we may speak of it even in the university without arousing astonishment.

He states most eloquently that geometric intuition is more robust than one might expect and can be useful in higher dimensions:

> It guides us into that space which is too vast for us and which we may not see; it does this by ever bringing to mind the relationship of the latter space to our ordinary, visible space, which without doubt is only a very imperfect image, but which nevertheless is an image.

He then introduces Analysis Situs (topology) as a creation of Riemann and states that its importance is very great, that it is leading the way into higher dimensions and, indeed, must be studied in all dimensions. Of course, Poincaré is now usually considered as its creator. He is certainly on the money in foreseeing the central role topology will play in the twentieth century, creating key elements of our vocabulary (such as homology and homotopy groups) and giving us some intuition about higher-dimensional space.

It is interesting that he makes no speculations about how geometry in higher dimensions will differ from what we know in three dimensions. There had been one hint at his time: Schläfli's classification of regular polytopes showed that in dimension 5 or more, life got simpler. It is interesting that this is exactly what happened with the higher-dimensional versions of Poincaré's conjectured characterization of spheres: Stallings and Smale showed that this was true in dimension 5 or more because, in some sense, life was simpler due to there being more "elbow room". This phenomenon of things stabilizing as dimensions get higher has occurred over and over in many fields.

Poincaré goes on to doff his hat towards both algebraic geometry and differential geometry, "a vast field from which to reap a harvest". This is certainly right but he has nothing specific to say about them—a bit sad considering their great flowering in the twentieth century.

Cantorism (set theory and foundations)

One senses in this section considerable ambivalence of Poincaré towards Cantor's ideas. While acknowledging that "(His) services to science we all know," he ends the paragraph by saying "(with this theory) we can promise ourselves the joy of the physician called in to follow a beautiful pathological case!" It seems that uppermost in his mind in this short paragraph are the paradoxes that arise in this field, the apparent contradictions which "would have overwhelmed Zeno…with joy."

As we know, from our vantage point, it was Gödel's ingenious use of exactly these paradoxes that led to the deepest result in the foundations of mathematics, to Gödel's magnificent incompleteness theorem, whose philosophical significance continues to reverberate. Starting from Russell and Whitehead (1910–1913), the foundations of mathematics, the search for universal axioms for integers, real numbers, and set theory, developed into a field of its own. But Gödel showed that any finite set of axioms could not be a complete foundation for mathematics, and attempts to found mathematics on sets are not now universally accepted. Some of these skeptics (admittedly a minority of mathematicians) assert that mathematics can't have "ultimate" foundation stones, and, in any case, it doesn't need them.

In the century following 1908, brilliant mathematicians have created a large corpus of material that goes under the rubric of logic, sets, and foundations. To name but a few: Zermelo, Fraenkel, Ramsey, Łukasiewicz, Post, von Neumann, Bernays, Gödel, Turing, Cohen, Martin Davis, Henkin, Feferman, Chaitin. Scanning a recent text on mathematical logic yields postulate systems such as PA (Peano arithmetic), ZF (Zermelo (1908), Fraenkel: (1891–1965), ZFC (ZF + the axiom of choice), ZFL (ZF + constructibility). It yields such topics as decidability, consistency, forcing, generalized continuum hypotheses, non-standard analysis, hyper-hyper inaccessible cardinals, alternate logics.

But, on the other hand, there is also a widespread feeling among working mathematicians that measurable cardinals and the like, that is to say, present day set theory, are indeed some kind of "pathological case" as Poincaré put it, ideas that can give the uninitiated existential angst. So Poincaré perhaps caught the future mainstream reaction to this area as well as pinpointing its arguably most significant idea.

The research of postulates (axiomatic analysis)

This short section of Poincaré's article may be differentiated from the previous one by saying that under "Cantorism", he was thinking of the theoretical side of the logical analysis of the foundations of mathematics, while in this section, he was thinking of the applied side. If he was skeptical of Cantor, he is even more so of the usefulness of axiomatic analysis:

> We are trying to enumerate the axioms and postulates, <u>more or less deceiving</u>, which serve as the foundation stones of our various mathematical theories. M. Hilbert has obtained the most brilliant results. It seems now that this domain must be very limited and there will not be any more to be done when this

> inventory is finished, and that will be very soon.

Consideration of the underlined phrase might very well suggest that Poincaré believed such an inventory or enumeration of postulates was unnecessary or misleading. Poincaré was really wrong in this instance: Hilbert's initiative not only at listing all necessary postulates to complete Euclid, but at constructing alternate geometric universes where all but one axiom held, has had a major influence on twentieth century mathematics.

In the 1920s, the German school of "modern algebra", with its completely general rings, its abstract ideal theory, and Noether's spectacular generalizations of Hilbert's results, seems to us the spiritual descendent of Hilbert's inventory. Rings were now divorced from specific examples such as rings of algebraic integers, polynomials, or matrices and instead were considered as having a vast array of possible incarnations, cases where some standard axioms held and others did not. Likewise, the abstract theory of topological spaces and of Banach spaces developed by the Polish school followed the same path: look at all combinations of postulates and see what spaces they deliver.

This point of view was thoroughly absorbed in the culture of twentieth century mathematics and was clearly enunciated in Bourbaki's monumental treatise. It is now taken for granted as the "obvious" way to do things in the pure math community, the way to find the best abstract setting for every argument, the most general form for every theorem. But it seems that Poincaré missed it, that it was definitely not his cup of tea.

We summarize our thoughts in the presumptuous table below. It is interesting that in many areas he saw the possibility of links between fields and, perhaps because it was less exciting, deemphasized the deepening of existing fields. His negative feelings about the "Research of Postulates" seems to lie behind his missing the explosion of work in the first half of the twentieth century setting almost every area of mathematics, but especially algebra, in its most general abstract form and investigating all mathematical objects that this led to (e.g., all finite simple groups).

SCORECARD

FORESEEN	MISSED
Importance of linking number theory and algebraic geometry	Theory of general commutative, noncommutative rings
Importance of analytic methods in number theory, L-functions	
Topological equivalence of dynamical systems	Deeper theory of chaotic dynamical systems
Importance of function spaces and their linear algebra	Small successes, challenges of nonlinear PDEs
Differences of several complex variables from one, links of complex analytic geometry with algebraic geometry	Deeper theory of one complex variable (e.g., Teichmüller, Bieberbach)
Importance of Lie and Galois groups	Theory of general finite groups
Topology as the key to higher dimensions	Rich diversity of dimensions 3 and 4 and 7 (exotic spheres)
	Gödel and deep significance of the paradoxes
	Axiomatic treatment of every field (eventually: categories)
	Explosion of computational methods, computational experiments, numerical analysis
	Development of probability theory, stochastic differential equations, information theory

Victor L. Klee 1925–2007

Peter Gritzmann and Bernd Sturmfels

Victor L. Klee passed away on August 17, 2007, in Lakewood, Ohio. Born in San Francisco in 1925, he received his Ph.D. in mathematics from the University of Virginia in 1949. In 1953 he moved to the University of Washington in Seattle, where he was a faculty member for 54 years. Klee specialized in convex sets, functional analysis, analysis of algorithms, optimization, and combinatorics, writing more than 240 research papers. He received many honors, including a Guggenheim Fellowship; the Ford Award (1972), the Allendoerfer Award (1980 and 1999), and the Award for Distinguished Service (1977) from the Mathematical Association of America; and the Humboldt Research Award (1980); as well as honorary doctorates from Pomona College (1965) and the Universities of Liège (1984) and Trier (1995). For collaborations with the first listed editor he received the Max Planck Research Award (1992). Klee served as president of the Mathematical Association of America from 1971 to 1973, was a fellow of the American Academy of Arts and Sciences, and was a fellow of the American Association for the Advancement of Science.

In 1990, in honor of Klee's 65th birthday and the broad range of his mathematical interests, the two of us (long-time co-worker and former Ph.D. student, respectively) edited the volume *Applied Geometry and Discrete Mathematics*, which was published by the American Mathematical Society.

For this obituary, we invited a group of former colleagues and mentees to contribute short pieces on Klee's mathematical life. This resulted in ten individual spotlights, followed by some personal remarks by the editors. The emphasis lies on Klee's work in the more recent decades of his rich scientific life, and hence they focus on finite-dimensional convexity, discrete mathematics, and optimization. His bibliography, however, makes it clear that by the late 1960s he already had more than a career's worth of papers in continuous and infinite dimensional convexity.

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Louis J. Billera
Richard P. Stanley

Algebraic Combinatorics and the g-Theorem

Victor Klee was a pioneer in two closely related aspects of convex polytopes that have subsequently played an important role in algebraic combinatorics, namely, f-vectors and shellings. The f-vector of a polytope (or of more general geometric complexes) encodes the number of faces of each dimension [33].

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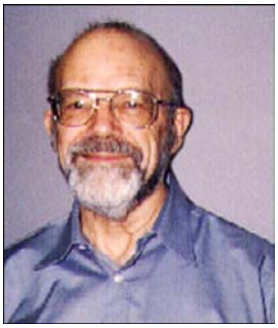

Victor L. Klee

Klee had the key insight of proving results concerning f-vectors in their greatest possible generality. Thus he proved the Dehn-Sommerville equation for Eulerian manifolds (a vast generalization of polytopes), generalized the Upper Bound Conjecture for simplicial polytopes to triangulations of spheres, and proved a special case for Eulerian manifolds [29]. He also proved the Lower Bound Conjecture for polytopes in the general setting of pseudomanifolds [34].

Klee was the first person to deal with shellings of simplicial complexes in a systematic way [11]. His papers in this area paved the way for the use of shellings as a major tool in proving combinatorial and topological properties of wide classes of complexes. His work on shellings and f-vectors had a big influence on our own research and led to some of our best papers.

Much of Klee's interest in polytopes related to questions originally arising in optimization theory. In the early 1960s he began to write and lecture on f-vectors and diameters of polytopes. Interest in both of these topics was spurred by their relevance to computational techniques for linear programming problems. The d-step conjecture poses a linear bound on the diameter of the graph of a polytope in terms of its dimension and number of facets. (In spite of all the progress on the combinatorics of polytopes since that time, this question remains unsettled.) Klee's paper [32] was seminal to part of the proof of the g-theorem, which characterizes f-vectors of simplicial polytopes [3, 47]. It suggested a way to construct extremal examples of polytopes by placing points over cyclic polytopes and led to the essential geometric step in the construction of simplicial polytopes having predetermined f-vectors.

Richard A. Brualdi

Sign Patterns of Matrices

The idea that the signs (some or all) of the solution vector of some linear systems of equations could be determined knowing only the signs of the system parameters originated in the economics literature [46] in 1947. The subject, including the consideration of other matrix properties, e.g., stability, that could be determined solely on the basis of the signs of entries, caught the attention of a few economists, mathematicians, and computer scientists (including Klee) in the 1960s and 1970s. But it was the 1984 paper by Klee et al. [36] that was a catalyst for its further substantial development and its further extension to many matrix properties. The authors of the book [9] were enticed into the subject by this paper.

The study of sign solvability can be broken down into the study of L-matrices (linear independence of the rows can be determined solely from the sign pattern) and S-matrices (the sign pattern implies that the null space is spanned by one positive vector). In [36] it was shown that recognizing L-matrices is NP-complete, even when the matrix is "almost square". The recognition problem for square L-matrices (also called sign-nonsingular matrices), which can be formulated as a pure graph-theoretic problem, was later shown to be of polynomial complexity [45].

In the 1980s, Klee, with various coauthors, continued his work on the class of S-matrices, including recursive structure and recognition algorithms. He also investigated linear systems of differential equations from the sign pattern point of view, constructing and classifying such systems [5]. In what I believe to be his last paper in this area, Klee investigated in 2000 the idea of conditional sign solvability where the sign pattern determines the sign pattern of a solution *when a solution exists*. Klee's papers in this area were full of original ideas and clever combinatorial, geometrical, and analytic arguments. He, more than anyone, is responsible for the explosion of interest in sign patterns in the last twenty years.

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Jacob E. Goodman
Richard Pollack

Geometric Transversal Theory

Vic Klee's interest in the combinatorial and topological properties of convex sets, manifested in his early papers [26, 27, 28], led him to help found the subfield of discrete geometry that has recently been recognized with its own AMS classification, geometric transversal theory. After Helly's theorem was published [22, 44], there were some scattered papers written by Santaló, Vincensini, Horn, and others; but it was not until after Vic published [27] his first paper on transversals, that new people, such as Grünbaum, Hadwiger, and

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Danzer, began publishing results relating to common transversals of families of convex sets. In the past few decades the subject has blossomed, and many long-outstanding problems have been resolved and new problems have taken their place. A major inspiration to the field was the very comprehensive survey [12] that Vic wrote with Danzer and Grünbaum in 1963, which summarized virtually all the work that had been done in the field up to that time. Some recent surveys of GTT can be found in [13, 14, 48, 49]. Vic's interest in geometric transversal theory resurfaced in recent years, as evidenced in the papers [37, 42].

In addition to his foundational work in a subject close to our hearts, we can attest to Vic's personal generosity and his encouragement. His interests, perhaps more than anyone's, spanned the complete scope of the journal *Discrete & Computational Geometry* in its early years, on whose editorial board he served from its beginning in 1986. He once commented that DCG was his favorite journal, the one whose new issues he looked forward to reading the most. That offhand remark has meant a great deal to us over the years and continues to inspire us as editors.

Peter Gritzmann
Bernd Sturmfels

From the Klee-Minty Cube to Computational Convexity

Vic Klee has always applied his strong geometric insight to problems in mathematical programming. Arguably his most famous contribution to this field was his paper with Minty [38] on the worst-case behavior of the simplex algorithm. Since the simplex method worked so well in practice, there was a long-standing conjecture that the number of required arithmetic operations (in particular, the number of pivots) is bounded by some polynomial in the dimension n and the number m of inequalities. However, [38] showed that the worst-case behavior of Dantzig's pivot rule is exponentially bad. The offending polytope is combinatorially equivalent to an n-cube; in particular, it is defined by $2n$ linear inequalities in n variables. For a bad choice of starting vertex, the resulting path to the maximizer involves all 2^n vertices of P. Similarly bad behavior was later established by other authors for other pivot rules. However, it is still unknown whether there exists a pivot rule under which the worst-case behavior of the simplex method is polynomially bounded, though certain pivot rules have been shown to have good average-case behavior. The Klee-Minty example was the starting point for the quest for less combinatorial paradigms leading to polynomial-time algorithms for linear programming like the ellipsoid and the interior point methods; see [18].

In retrospect, his work on the simplex method is at the heart of the more recent field of *computational convexity*, the name having first appeared in print in 1988 in [15]. The subject of computational convexity draws its methods from discrete mathematics and convex geometry, and many of its problems from operations research, computer science, and other applied areas. In essence, it is the study of the computational and algorithmic aspects of convex bodies in normed vector spaces of finite but generally not restricted dimension, especially polytopes, with a view to applying the knowledge gained to bodies that arise in a wide range of disciplines in the mathematical sciences.

Basic and typical problems deal with the efficient computation or approximation of geometric functionals such as the volume or the diameter of a polytope, or with the algorithmic reconstruction of a polytope from data concerning it, or with algorithmic versions of geometric theorems; see [19, 20] for surveys. One emphasis in Vic's work on computational convexity was the computation of radii of convex polytopes and more general bodies, leading to far reaching theoretical and algorithmic results that have turned out to be of great relevance in applications ranging from data analysis to medical surgery planning; see [6, 7, 16, 17].

Branko Grünbaum

Convexity

Convexity is a topic that has been studied since the late nineteenth century. A "final report" of sorts was the survey *Theorie der konvexen Körper* by Bonnesen and Fenchel, published in 1934. A new direction in convexity research arose mid-century, combining aspects of convexity and discrete (or combinatorial) mathematics. This was a topic to which Vic was attracted all his life; in later years he also dealt with computational aspects of convexity.

In June 1961 Vic organized the first ever symposium entitled *Convexity*. This served, in many ways, as a starting point of widespread interest in questions of combinatorial convexity. One of the papers included in the proceedings of that symposium was the 80-page survey "Helly's Theorem and its Relatives" [12]. Its genesis occurred through Vic's diplomacy: Ludwig Danzer, I, and Vic were all three interested in writing a survey about Helly's theorem. As the organizer of the symposium and the editor of the proceedings (not

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to mention his senior status relative to Danzer and myself) Vic could have decided that only his paper was to be published. Instead, he proposed to have a joint paper coauthored by all three of us (there were very few three-authors papers at that time). Although Danzer and I did contribute to it, the overwhelming majority of the work on the survey paper was done by Vic. The paper was immediately hailed as a landmark; even more than forty years after it was written, this survey is the most quoted paper of Vic's: According to the MR Citation Database it was referenced in 72 works reviewed since 2000.

Another notable activity of Vic's was the Unsolved Problems column in the *American Mathematical Monthly*, which Vic started in 1969 and to which he contributed many items. This was an outgrowth of an earlier endeavor: Vic compiled in the early 1960s a collection of unsolved problems, meant to be part of a joint effort with Paul Erdős, Laszlo Fejes Tóth, and Hugo Hadwiger; however, this collaboration never materialized.

The level of Vic's activity during the 1960s can also be appreciated by recalling that it was during this time that he wrote his well-known papers on convex polytopes, which ushered in the still-continuing flourishing of that field. Indirectly, these papers are responsible for my book *Convex Polytopes*: In 1963 I conducted a seminar on convex polytopes at the Hebrew University, based on preprints of Vic's papers. The students had difficulties understanding the material, so I started writing explanatory notes; ultimately, these notes became the book [21].

Robert Jamison

The Shift from Continuous to Discrete

Klee's early work was largely in the area of the topology of normed spaces and the geometry of convex bodies. But right from the start there was an indication of the flexibility of his interest. Among his earliest papers are several on the Euler totient [25]. When I arrived in Seattle to study with Vic in 1970, a major shift from the continuous and infinite to the discrete and finite was taking place. It is only fair to say that Vic did not drop one subject for another, rather he expanded his field of interest.

One of Vic's major discrete papers, co-authored with George Minty [38], showed the simplex algorithm could be exponentially bad. He also did work on the greedy algorithm in infinite matroids [31]. In addition to his own work, Klee was a popularizer of ideas and problems. In public lectures he liked to link Hamiltonian cycles with "life on Mars", the idea being that a Hamiltonian cycle in a chemical structure would make the encoding and transmission of its structure easier. Vic was also fond of talking about the Lekkerkerker-Boland characterization of interval graphs. In a research problem in the *American Mathematical Monthly*, he promoted research on a variant, the circular arc graphs, a class that is now widely studied and known to have many interesting properties.

In addition to being an outstanding mathematician, Vic was an outstanding person. Once he agreed to hear a graduate student present a "proof" of Fermat's Last Theorem. I asked Vic why he was willing to invest his time in a project that almost certainly would end in failure. He said, "There was a small chance he was right. Then he would need someone to vouch for him." Vic was generous with his time, with his encouragement, and with his friendship.

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Peter Kleinschmidt

The d-Step Conjecture

The d-step conjecture, first formulated by Warren Hirsch in 1957 and published in 1963 in George Dantzig's classical book on linear programming, arose from an attempt to understand the computational complexity of edge-following algorithms for linear programming as exemplified by the simplex algorithm. It can be stated in terms of diameters of graphs of convex polytopes, in terms of the existence of nonrevisiting paths in such graphs, in terms of an exchange process for simplicial bases of a vector space, and in terms of matrix pivot operations. This variety of equivalent formulations of the conjecture—largely due to Klee himself and reported in Klee-Walkup [40], Klee-Kleinschmidt [35] and Grünbaum [21]—made it a typical field of his research areas: geometry and combinatorics of polytopes, linear programming, and complexity theory. Warren Hirsch died about a month prior to Victor on July 9, 2007. He spent most of his career at NYU were he worked mainly as a probabilist and statistician. He is best known for his work in mathematical biology, particularly on the transmission of parasitic diseases, but several of his earlier papers concerned optimization.

The conjecture states that the maximum diameter of (the graphs of) d-polytopes with $2d$ facets is d. It is equivalent—though not necessarily on a dimension-for-dimension basis—to the Hirsch conjecture, which states that $\Delta(d, n)$, the maximum diameter of the graph of a d-polytope with

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n facets, is not greater than $n - d$. Klee believed the conjecture to be false. However, the fact that it obviously reflects the worst-case behavior of a best edge-following LP-algorithm makes the study of the behavior of $\Delta(d, n)$ very important. The various edge-following LP algorithms, apart from numerical and implementational issues, differ principally in the pivot rule by which the sequence of edges is chosen. A pivot rule that generates a polynomial (in n and d) number of edges would imply a polynomial bound for $\Delta(d, n)$. However, to this day no such rule is known. The famous Klee-Minty example [38] shows that the original rule of Dantzig that maximizes the gradient in the space of nonbasic variables is exponential. For the most common pivot rules exponential behavior was proved subsequently by various authors. The best currently known general upper bound for $\Delta(d, n)$ is quasi-polynomial (Kalai-Kleitman [24]):

$$\Delta(d, n) \leq 2n^{log(d)+1}.$$

This result contains the unbounded case. Results of Barnette [2] and Larman [41] provide an upper bound that is linear in the number of facets but exponential in the dimension. For the important class of 0-1 polytopes Naddef [43] has proved the validity of the Hirsch conjecture. For other classes of polyhedra arising in applications it is also known to be correct. For a survey see [35]. Slightly stronger versions of the Hirsch conjecture have been proved to be false (e.g., the case of unbounded polyhedra and the monotone version). Holt-Klee [23] showed that $n - d$ is the best possible lower bound for $\Delta(n, d)$ in the bounded case for $n > d \geq 8$.

Jim Lawrence

Unfinished Business

Not long ago I received email from Vic in which he mentioned health problems and described actions that he had taken relating to the end of his career; but he said that he was so encumbered by unfinished business that his wife Jodey had made a plaque for him reading "I was put on this earth to accomplish certain things. Right now, I'm so far behind that I'm sure I'll never die." Perhaps one of those pieces of unfinished business related to some hand-written, mimeographed notes from a class he taught when I was a graduate student. They seem to be part of Chapter 2 of a book in preparation, on the subject of convexity. (Some years later, I overheard his answer when asked why he hadn't finished that book; after hesitating a moment, he said, "I think it's because I don't know enough about the subject yet." This engendered much laughter, although he seemed serious.)

Items in *Mathematical Reviews* concerning Vic's papers often use modifiers such as "commendably clear", "precise", "elegant", occasionally preceded by the phrase "as always". I'd like to add two others: "fun" and "exciting". Chapter 2 certainly deserved all of these. It was an energetic account of such topics as Helly's Theorem, Radon's Theorem, and the notion of a positive basis.

Vic greatly liked David Barnette's work in settling what (before Barnette) was called the "lower bound conjecture" for convex polytopes [1], and his paper [34] related to that work provides another example in which a complicated topic is treated with utter simplicity. In this paper, by using the method of Barnette in a graph-theoretical setting, he obtained a generalization for connected pseudomanifolds. The mathematics was quite technically involved, but the paper was written in such a way that one could read it in an almost leisurely manner, and it imparted the feeling that he had actually had a lot of fun in writing it! (I once mentioned to him that I liked the paper. He minimized his contribution, saying that he was just making use of David's methods.)

I suppose that he didn't finish his book on convexity. We would certainly have liked to see the rest of it.

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Joseph Zaks

Shapes of the Future

I first met Victor Klee when I came to the University of Washington as a graduate student in 1966; I was fascinated by his talks in the Geometry Seminar and later on by his Open Problems section in the *Monthly*. In the early 1970s I had enjoyed his two films "Shapes of the Future". (See also [30].) I worked on many problems that Vic raised in his papers and films, in particular on the illumination of planar polygonal simply-connected regions. I gave an example of a non-spherical, non-convex body in 3-space that has constant HA-Measurements, and I have shown, with the aid of a computer (and my son Ayal) that there exist no nine neighborly tetrahedra in 3-space—these are two of the many open problems that Vic mentioned in his articles. I frequently use Vic's part in his book [39] with S. Wagon, and quite often I refer to Chapter 8, concerning the colorings of the rational spaces $\mathbf{Q}^d$; this is related to the Beckman-Quarles Theorem, concerning one-distance preserving mappings from $\mathbf{Q}^d$ to itself.

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I have enjoyed Vic's company during my recent visits to UW, and during many conferences all over. I last saw Vic when he came to my talk at UW in October 2006. In a small tribute to Branko and to Vic, I had organized "The Klee-Grünbaum Festival of Geometry" in Ein Gev, Israel, in 2000, in which we celebrated the 70th birthday of Branko and the 75th birthday of Vic. My wife Sara and I had the pleasure of having Jodey and Vic spend an overnight at our house in Israel, a few days before the festival. Vic was a great teacher, a devoted inspirer, and an extremely warm colleague. He will be missed by all of his students, colleagues, and friends.

Günter M. Ziegler

Generosity

Vic Klee was a wonderful poser of problems, who guided *others* to great success: For example, problems asked by Klee and Erdős in the early 1960s (before I was born) led to the 1962 Danzer–Grünbaum paper about point sets without obtuse angles—which suddenly in 2006 was the key to breakthroughs in Barvinok–Novik's work on centrally symmetric polytopes. But I also remember him asking about cube tilings in Oberwolfach, which stimulated Lagarias and Shor in their stunning disproof of Keller's cube tiling conjecture in 1992.

From our work on the second edition of Branko Grünbaum's *Convex Polytopes* [21], I remember most vividly Vic's generosity: Indeed, the first edition from 1967 appeared "with the cooperation of Victor Klee, Micha Perles, and Geoffrey C. Shephard"; Vic wrote two influential chapters for the book; the re-edition of course would have never happened without his sense of duty, taking responsibility for the project. He provided a wealth of ideas, references, and suggestions from his decades of work on convexity and polytopes, which reflected richly his stunning influence on work of others; this wealth shows in the final product. In the end, the book won the 2005 Steele Prize. I hope that our lasting image of Vic Klee reflects some of the splendor and success that he helped others achieve.

Editors' Epilogue

Looking at the photographs we have of Vic and his family, we know that Vic will live on in our personal memories. He remains with us through all the things we learned from him and admired in him, through his strong direct influence on our paths of life, but also through his modesty, his warmth, and his humor that enriched us, and the thousand little things that left their traces.

Of course, Vic found immortal results in many different branches of mathematics and neighboring fields. What was even more important to us was his wonderful personality. Given his scientific achievements, his numerous awards, and his exceptional standing in the scientific community, it is by no means self-evident how modest, friendly, open-minded, and encouraging he was, how fun to be with, great to talk to, hike with, or play table tennis or billiards with.

Vic always created an atmosphere that made it easy for others to grow despite their own imperfection. He always encouraged others to explain their ideas, no matter how vague they were. It has been incredibly wonderful to be with him and to enjoy mathematics together. **Thank you, Vic!**

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WHAT IS...

Property A?

Piotr Nowak and Guoliang Yu

Amenability of metric spaces usually manifests itself in the existence of large sets with relatively small boundary. Think of Euclidean n-space. Here the volume of the ball of radius r is proportional to r^n while the area of its boundary, the sphere of radius r, is proportional to r^{n-1}. Thus the area of the boundary is roughly $1/r$ of the volume of the ball, and the ratio tends to 0 as r increases. Such amenability properties are very useful in many contexts. Unfortunately, it usually does not take much to prevent them from being satisfied. A simple example of a space that is not amenable in this sense is the hyperbolic plane $\mathbb{H}^2$. Property A is a weak amenability-type condition that is much less restrictive than the one described above and is satisfied by many known metric spaces. It was introduced in [3] and turns out to be of great importance in many areas of mathematics. We start with the definition.

Definition ([3]). *A discrete metric space X has Property A if for every $\varepsilon > 0$ and every $R > 0$ there is a family $\{A_x\}_{x\in X}$ of finite subsets of $X \times \mathbb{N}$ and a number $S > 0$ such that*

(1) $\dfrac{\#(A_x \Delta A_y)}{\#(A_x \cap A_y)} < \varepsilon$ *whenever* $d(x,y) \le R$,

(2) $A_x \subseteq B(x,S) \times \mathbb{N}$ *for every* $x \in X$.

Here $A_x \,\Delta\, A_y = (A_x \setminus A_y) \cup (A_y \setminus A_x)$ denotes the symmetric difference. In simple words, these conditions mean that when points x and y are at most R apart, then the sets A_x and A_y are almost equal, but if the distance between x and y is at least $2S$, then A_x and A_y are disjoint. Property A is a large-scale geometric property, meaning that it is preserved by quasi-isometries—the copy of $\mathbb{N}$ appearing in the definition of the sets A_x is exactly to guarantee this invariance. We can thus say that a locally compact metric space has Property A if it is quasi-isometric to a discrete metric space with Property A.

Let us look at some examples. Observe that any finite metric space X has Property A trivially: simply take $A_x = X \times \{1\}$ for all $x \in X$, and the ratio in the above definition vanishes. Next in line is the set of integers $\mathbb{Z}$, with the usual Euclidean metric. To show Property A for $\mathbb{Z}$, given R and ε, choose an integer $r \ge 2R\varepsilon^{-1}$ and define A_x to be the ball of radius r centered at x for all $x \in \mathbb{Z}$.

The most interesting case is when X in the definition above is a finitely generated group G, equipped with a metric in the following way. Having fixed a finite set of generators, we declare the distance between two elements $g, h \in G$ to be the smallest number of generators and their inverses necessary to write $g^{-1}h$ as a word. Such a metric is called the *word length metric* on G. For example, if we consider $\mathbb{Z}$ as an additive group generated by a single element $\{1\}$, the word length metric is simply the usual metric on $\mathbb{Z}$ induced by absolute value.

In this setting the resemblance of Property A to amenability can be easily seen through Følner's criterion. A group G is called amenable if for every $\varepsilon > 0$ and every $R > 0$ there exists a finite set $F \subseteq G$ such that

$$\frac{\#(F \,\Delta\, gF)}{\#F} < \varepsilon,$$

whenever the distance of g to the identity element is less than R. Above gF is the translation of F by element g, and the sets F are called Følner sets.

It follows from the definition that amenable groups satisfy Property A: given $\varepsilon > 0$ and a corresponding Følner set F, one simply takes $A_g = gF$, and it can be easily verified that the sets A_g defined this way satisfy the required condition. In fact, close examination of definitions reveals that for groups, Property A is "non-equivariant" amenability—simply imagine what "equivariant Property A" would mean to end up exactly with amenability, modulo some simple calculations.

But Property A covers a much wider class of groups than that of amenable groups. Typical examples of non-amenable groups are the free groups on $n \ge 2$ generators $\mathbb{F}_n$, but as it turns out, free groups satisfy Property A. To give the proof recall that the Cayley graph of a free group is a tree. In this tree fix a geodesic ray γ that originates at the identity, and given $n \in \mathbb{N}$ for every $x \in \mathbb{F}_2$ define the set A_x to be the the unique geodesic segment of length $2n$ from x in the direction of the ray γ. It is not hard

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to check that for any ε there is an $n \in \mathbb{N}$ such that the required conditions are satisfied. This method of proof can be generalized to show that hyperbolic groups and the hyperbolic plane $\mathbb{H}^2$ mentioned earlier have Property A.

Another natural example is furnished by discrete linear groups, i.e., subgroups of the group $GL_n(F)$ of invertible $n \times n$ matrices over a field F. The fact that they satisfy Property A was proved by Guentner, Higson and Weinberger. The list of classes of groups for which Property A has been verified also includes one-relator groups, Coxeter groups, groups acting on finite dimensional CAT(0) cube complexes, and many more.

The original motivation for introducing Property A, see [3], was that it is a sufficient condition to coarsely embed a group into a Hilbert space. A coarse embedding is a natural notion of inclusion in large-scale geometry. We recall the definition.

Definition (Gromov)**.** *A function $f : X \to Y$ between metric spaces is a coarse embedding if*

$$\varphi_-(d_X(x_1, x_2)) \leq d_Y(f(x_1), f(x_2)) \leq \varphi_+(d_X(x_1, x_2))$$

for all $x_1, x_2 \in X$, where $\varphi_-, \varphi_+ : [0, \infty) \to [0, \infty)$ are nondecreasing functions and $\lim_{t\to\infty} \varphi_-(t) = \infty$.

In [3] the Novikov conjecture was proved under the assumption of coarse embeddability into Hilbert spaces. The Novikov conjecture is a rigidity statement about high-dimensional, compact, smooth manifolds. A compact manifold is said to be aspherical if its universal cover is contractible (a typical example is a torus T^n with $\mathbb{R}^n$ as its universal cover). The Borel conjecture claims that every aspherical manifold M is rigid in the sense that if another compact manifold N is homotopy equivalent to M, then N is actually homemorphic to M. In the case of aspherical manifolds, the Novikov conjecture is an infinitesimal version of the Borel conjecture. It asserts that the rational Pontryagin classes, i.e., certain characteristic classes associated to the tangent bundle of the manifold, are homotopy invariants. For more general manifolds the Novikov conjecture claims that higher signatures are homotopy invariants.

Thus the results of [3] yield the following implication:

if G has Property A then the Novikov conjecture is true for all closed manifolds with fundamental group G.

This theorem sparked significant interest in the notion of Property A, and as a result Higson and Roe characterized Property A in terms of existence of a topologically amenable action on some compact space. The notion of amenability for group actions was introduced by Zimmer in ergodic theory and later adapted to the topological setting by Anantharaman-Delaroche and Renault.

Another connection arose in the theory of C^*-algebras. The work of Guentner and Kaminker and subsequently Ozawa showed that, for groups, Property A is equivalent to exactness of $C^*_r(G)$, the reduced C^*-algebra of G. Exactness of C^*-algebras was introduced by Kirchberg, and a long-standing problem was whether $C^*_r(G)$ is exact for every G.

These results immediately prompted the question of whether there exist metric spaces and, more importantly, finitely generated groups, that do not have Property A. However this question turned out to be quite difficult, and still only a handful of examples is known. One of them is due to Gromov, who observed that a metric space constructed out of a sequence of expanders does not admit a coarse embedding into a Hilbert space and therefore cannot satisfy Property A. We refer to [2] for a description of expanders. For finitely generated groups the problem of finding examples that would not satisfy Property A is much harder. A natural idea is to try to find a group that would metrically contain, in its Cayley graph, a sequence of expanders. Such a group could not satisfy Property A for the same reason given earlier. Gromov implemented this idea in the realm of his random groups, but a search for more examples is under way. If found, such groups might shed some light on many problems in geometric group theory as well as for example index theory, where groups without Property A served as counterexamples to some versions of the Baum-Connes conjecture.

Another major question is to what degree Property A captures coarse embeddability into a Hilbert space. In particular, it was not known whether the two notions coincide. An example of a metric space distinguishing the two properties was given in [1] and can be described as follows. Take discrete cubes $\{0,1\}^n$, $n = 1, 2, \ldots$. We define two points in a cube to be at distance k if they differ in exactly k coordinates. It is not hard to see that the disjoint union of these cubes (with the distance between cubes defined appropriately) embeds coarsely into a Hilbert space. However using the fact that the cube $\{0,1\}^n$ has a structure of an amenable group—namely that of $\mathbb{Z}_2^n$—it was proved that such a disjoint union of cubes does not have Property A.

It would be extremely interesting to find more examples of finitely generated groups that do not satisfy Property A, especially ones that *do* embed coarsely into a Hilbert space. Such groups would have to exhibit completely new geometric phenomena. At present these issues are far from being understood.

Further Reading

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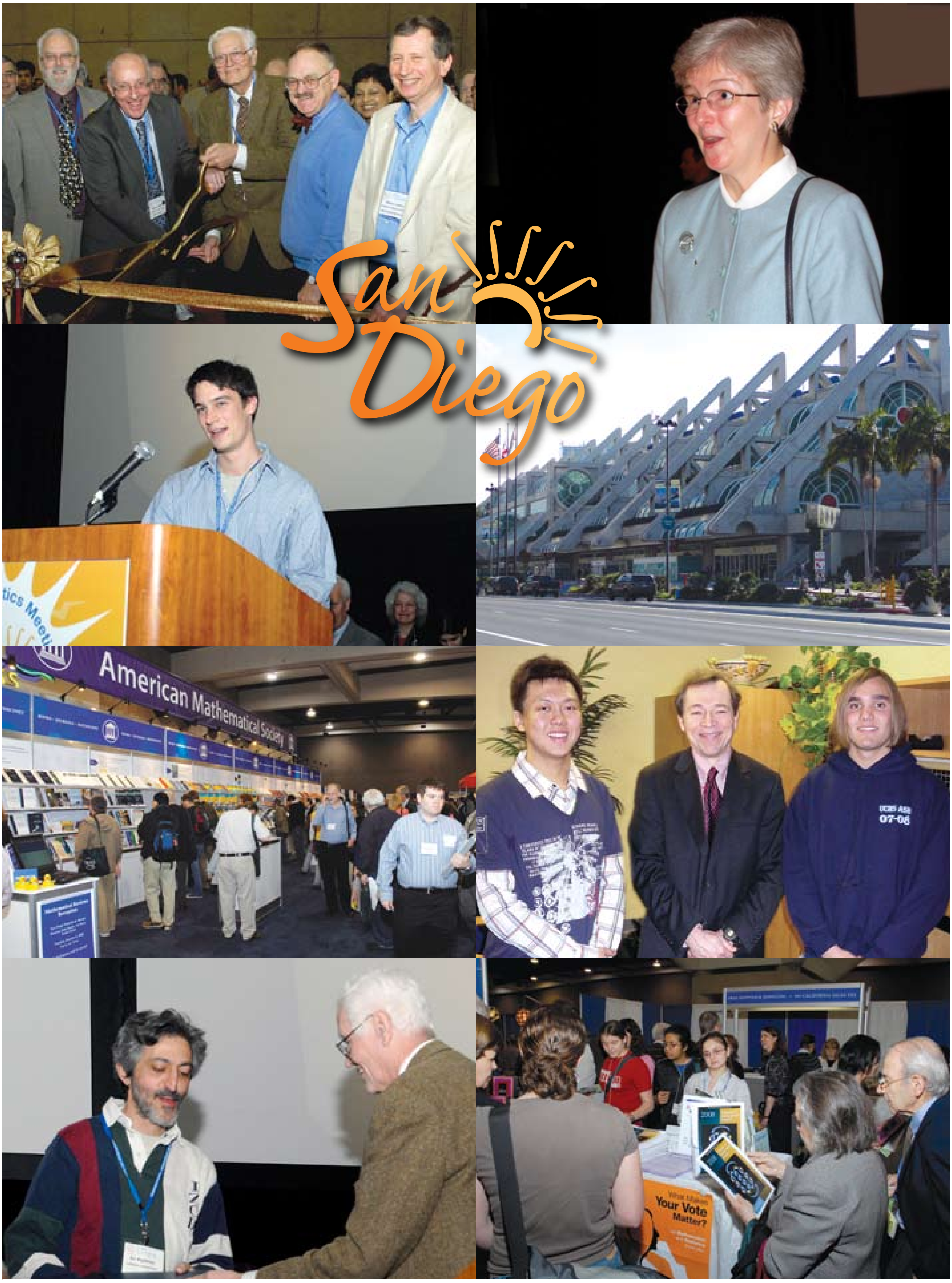
See photo key on page 510.



Book Review

Fly Me to the Moon

Reviewed by Shane Ross

Fly Me to the Moon
Edward Belbruno
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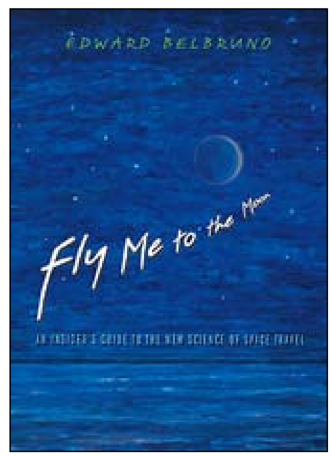
There is a revolution afoot in how space travel is done. Forget the big rocket burns of the past, which sent spacecraft hurtling away from Earth only to need more large rocket burns to slow them down and enter orbit at their destination, be it the Moon, Mars, or beyond. Think instead of comets, wise old travelers of the solar system, which follow the complex interplay of gravitational forces to move from one place to another, sometimes finding themselves in a temporary orbit around a planet. While we can't (yet) influence the motion of comets, we can learn lessons from them as to how we can control spacecraft, directing them in such a way that they travel vast distances and get captured around planets and moons using practically no fuel.

Edward Belbruno's *Fly Me to the Moon* provides an insider's look at this revolution, what the subtitle calls "the new science of space travel". The book intermingles popular-level explanations of some new and fascinating concepts regarding gravity, chaos, and spacecraft trajectory design with the interesting personal story of Belbruno's moments of discovery and struggle. The book provides excellent, readily accessible, and profusely illustrated examples of the uses of chaotic dynamics in celestial mechanics. And Belbruno's story provides insight into how new ideas gain acceptance within the scientific and engineering communities,



sometimes at great personal cost to the proponents.

The setting of celestial mechanics provides a seemingly timeless context that conjures up images of famous regularity, the clockwork cosmos of circles within circles. But that's only when one considers the two-body problem of a spacecraft and only one massive body at a time. Belbruno infuses the element of chaos by adding one more massive body to the mix and considering the three-body problem, for example a spacecraft moving in the Earth-Moon system. By "chaos", Belbruno is referring to sensitive dependence on initial conditions. The motion of an object is considered chaotic "if a tiny change in the motion at some moment results in a large change [in] the motion of the object and a substantially different trajectory."

To help us visualize the chaotic motion of the third (and smallest) body in a three-body system, Belbruno introduces the concept of a "weak stability boundary" around one of the larger, massive bodies. Take the Moon, for example. The weak stability boundary of the Moon is a set of locations where the gravitational attraction of the Moon is almost balanced by the attraction of the Earth. This boundary is a separatrix in the phase space, a critical surface where a

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spacecraft is particularly sensitive to perturbations. A slight jostle one way or the other and the spacecraft heads off to very different fates, either falling toward the Moon or hurtling away from it. On the boundary itself, however, a spacecraft is weakly captured in a kind of celestial limbo. But with a small rocket maneuver, a weakly captured spacecraft can be placed on a stable orbit around the Moon. If trajectories can be found from Earth to the Moon's weak stability boundary, these routes could potentially be very useful in practical applications as a cheaper (if not the cheapest!) way to the Moon.

Throughout the book, Belbruno leads us through the professional and mental trajectory he took to arrive at the weak stability boundary, its uses, and its myriad insights. Through personal accounts, a picture is painted of a man uniquely suited to tackling an interesting problem. Leaving an assistant professorship post in a mathematics department, in 1985 Belbruno headed to NASA's Jet Propulsion Laboratory in California, in search of "some new ideas, often not easy in an academic setting in which one has to pay attention to what is acceptable or trendy at the time." This gives us a glimpse of what motivated Belbruno, but more generally what motivates all good and lasting mathematics research: a dedication to the discovery of timeless truth, independent of the momentary and ever-changing winds of fad and research dollars.

In a new position as a trajectory mission analyst, Belbruno found himself wondering if it were possible to find a path through space that, after an initial boost near Earth, could get a spacecraft captured by the Moon using no fuel (sometimes called a "ballistic capture"). This would be analogous to standing on a beach in Florida and throwing a bottle into the ocean, into just the right current and at just the right time, so the bottle washes ashore in France. The ballistic capture concept was a far cry from the trajectories used by missions like Apollo, which were moving so fast relative to the Moon when they approached it that another large expenditure of fuel was necessary to get into lunar orbit. If a ballistic capture path could be found, it could dramatically reduce the cost of a mission.

After asking around and searching the literature, Belbruno found that in the 1960s Charles Conley conjectured that ballistic capture trajectories from the Earth to the Moon might exist, but Conley never found them. Belbruno had a hunch they did indeed exist and was determined to find them. Much of the book discusses both the theory and the personal story of how he did find them.

The first success came in 1986. Using his concept of the weak stability boundary, Belbruno found a low-fuel trajectory using ballistic (or weak) capture for the Lunar Get Away Special (LGAS) mission. The LGAS mission went through planning stages but never flew. However, the European Space Agency's SMART 1 mission to the Moon, which launched in 2003 and arrived at the Moon in 2004, was inspired by the LGAS design—a vindication of that initial discovery.

The most spectacular success of Belbruno's weak stability approach was in the rescue of a Japanese mission to the Moon. The mission originally had two spacecraft, MUSES-A and MUSES-B; B was to go into orbit around the Moon, with A remaining in Earth orbit as a communications relay. But B failed, and A did not have sufficient fuel to make the journey to the Moon by the conventional route planned for B. However, by utilizing the ballistic capture concept, Belbruno and colleague James Miller found a way to get A to the Moon that fit within the fuel budget. MUSES-A (renamed Hiten) left Earth orbit in April 1991 and reached the Moon that October, making it the first ballistic capture trajectory to actually fly. As a result, Japan became the third nation to send a spacecraft to the Moon.

As Belbruno shares, his ballistic capture approach was not appreciated by many of his colleagues at the time. The trajectory flight times were long, and the idea of using chaos to design a trajectory conjured images of unpredictability that were not consistent with the way trajectories had previously been designed. The idea of using "chaos to guide a spacecraft to the Moon and have it achieve orbit with no fuel" was "the shattering of a paradigm". And paradigms, bulwarked by the human tendency to depend on the tried and true, do not like being shattered.

The book begins by considering spacecraft trajectories, but goes on to describe an entire zoo of strange orbital behaviors and their significance for natural objects. For example, while studying chaotic motion of small objects, or rocks, in the Earth-Moon system, Belbruno found interesting cases of intermittent behavior wherein the rock would perform peculiar resonance transitions. The rock would start out in an orbit around the Earth that was beyond the Moon's, but in resonance with the Moon's orbit. After passing closely by the Moon, the rock would then get into an Earth orbit interior to the Moon's, and also in resonance. This kind of resonance transition had not been observed before, and Belbruno linked it with weak stability boundaries. Interestingly, Belbruno relays how the same behavior has been seen among comets under the sway of Jupiter's weak stability boundary. During encounters with Jupiter, some comets loosely orbit the planet, becoming temporarily captured moons. Analogous behavior has been seen in Kuiper belt objects, large comet-like objects that interact strongly with Neptune. Also intriguing is the possibility of near-Earth objects that are caught in a chaotic tangle related to the Earth's weak stability boundary. Belbruno speculates that perhaps the large impactor that collided with Earth to form the

Moon was in this precarious dynamical situation some four billion years ago.

The principles of the weak stability boundary are not limited in scale, being a general feature of the three- (or more) body problem. Near the end of the book, Belbruno discusses recent work on comets moving between the stars. These interstellar wanderers get temporarily captured at a star before moving on to the next. These are of course slow processes. Nevertheless hitchhiking the galaxy from star system to star system may in fact take place, albeit on very large time scales. Belbruno points out the implication for life-bearing material hitchhiking on interstellar comets, which may collide with a planet while temporarily captured around a star. This mechanism may provide "a key to the origin of life within our own solar system."

Fly Me to the Moon provides a fast, very readable account of new developments in chaotic celestial mechanics, especially low-fuel space travel, at a level appropriate for a general audience. By the end, nonmathematicians will have gained some intuition about one of the hallmarks of chaos, sensitive dependence on initial conditions, and how chaos can be harnessed to good purpose. All readers will walk away thinking differently about the cosmos. Far from being a clockwork, it will seem more dynamic, more turbulent, and full of diverse possibilities.

Is Mathematics Misapplied to the Environment?

Review by Christopher K. R. T. Jones

Useless Arithmetic: Why Environmental Scientists Can't Predict the Future
Orrin H. Pilkey and Linda Pilkey-Jarvis
Columbia University Press, 2006
US$29.95, 248 pages, ISBN 978-0231132121

This book is about disastrous decisions made on critical environmental issues. The authors document multiple cases ranging from nuclear waste disposal to plant invasion. Between the initial acceptance of a particular environmental issue and the eventual decision for action (or inaction), they show that something all too often goes very wrong. The cases they cite are unarguable, and they are right, at least about the outcome, in each instance: extremely bad decisions have been made that have led to situations that should outrage all of us. These have included the almost complete depletion of cod stocks on George's Bank and the engineering of beaches that has had the reverse effect of what was purportedly intended, the saving of those beaches.

Having been living recently in North Carolina, I am aware of the role of one of the authors, Orrin Pilkey, in combating beach development. He has stood up to the rampant and unthinking commercial exploitation of the coastline and exposed the obfuscation of developers and politicians as well as the miscalculations of coastal engineers. He is a hero of the environmental movement in North Carolina and deserves a hearing, even with a book having a title as provocative as this one.

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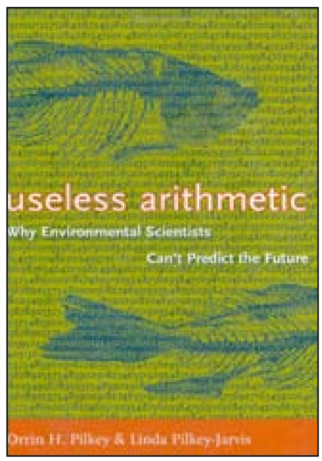

One of the most compelling examples is the authors' analysis of pit lake contamination. This is in particular reference to the Berkeley pit in Montana. The problem is to predict the environmental impact of open-pit mining many years down the road when the various contaminating by-products of the mining operation have been allowed to seep away from the original pit and potentially contaminate groundwater. The various parties, either governmental or with direct business interests, have used models to make predictions. In spite of the underlying physics and chemistry of the pit lake pollution being only crudely estimated and modeled, the resulting predictions are presented with an unwarranted level of specificity and certainty. The authors claim that the predictions have not been borne out by later facts. Although they, unfortunately, do not make a detailed comparison, they do make a strong case that the original modeling is misguided and that little later verification and assessment was carried out despite many key decisions being based on the earlier predictions.

I was convinced by their arguments that something goes awry in the process of environmental decision-making and that absurdly optimistic predictions, based on the use of mathematical models, are involved. The important question is what exactly does go wrong and how can it be fixed? The answer offered by the authors is unequivocal: the real culprit is the very use of mathematical models in environmental science, and the fix is to rid the subject of such models altogether.

This is a fairly extreme position. Models, after all, are neutral in themselves, and if they are at the root of the problem then it is surely because they are being misused. Such misuse can often be traced to people seeking to bolster their self-interest by appealing to supposedly objective results. The authors make this point themselves, but they also hammer on the inappropriateness of mathematical models and advocate their near total banishment from environmental sciences. Although I ultimately disagree with their position, it is worthwhile to give it full consideration as there are serious issues raised in the book and important messages for mathematics as a discipline.

The authors make a distinction between quantitative and qualitative models. In their lexicon, quantitative models are designed to give numbers representing a specific prediction, while qualitative models are used for understanding underlying physical processes. This distinction is based on the uses of the models rather than any of their intrinsic properties and is, in my opinion, misleading.

A clearer view of modeling is as a continuum, with one end being computational models, which involve intensive use of numerical solutions of some complex system of equations, and, at the other end, models that are more like rules-of-thumb. The authors use their distinction inappropriately at times as they classify almost anything they disagree with as quantitative. For example, they devote a considerable portion of the book to the discussion of beach erosion in which the Bruun Rule, which describes the amount of horizontal retreat of the shore-face as a response to sea-level rise, plays a starring role. In my view, this rule is far on the "rule-of-thumb" end of the modeling spectrum, and yet they group it with quantitative modeling. It should surely be viewed as qualitative modeling because it makes a broad brush stroke of the physics. They disagree with its use in practice and even its applicability, but that would make it bad qualitative—not quantitative—modeling.

A more straightforward distinction to make would be between the uses of models for prediction as opposed to understanding. Models all along the spectrum may be used for either objective. It is the use of models for prediction that the authors question. I will argue below that to understand the limits to the predictive capacity of models, we must view them as working in concert with data. But, first, I will look in more detail at their arguments.

Models of environmental situations involve the setting of a large number of parameters. The proper values of these parameters are largely unknown and are often estimated in a subjective process with settings that are based, at best, on convenience or, at worst, with the intention of producing a particular outcome. Moreover, the complexity of the models means that many subprocesses are either included only very approximately or omitted altogether. The result is a crude model whose predictive output may have little to do with the true outcome. This inadequacy of the models is compounded by their intrinsic limits to predictability due to internal dynamics; for instance they may well be chaotic.

Nevertheless, the models are used extensively by (coastal) engineers, government planners, and others for precise and specific predictions. Given this state of affairs, it is not surprising that the predictions have often proved to be inaccurate and have led to disastrous decisions. A modest and reasonable response at this point would be to propose caution and advocate the restricted use of models and then only with considerable validation and verification. This, however, is not the authors' approach, as they take the extreme position of advocating the wholesale rejection of mathematical models in environmental sciences. Their case for such a drastic proposal takes us into the psycho-sociological realm.

The authors have no fond feelings for the politicians, planners, and developers who ultimately make the decisions about policies affecting the environment. My guess is that they would agree these decision-makers would use other convenient justifications for their decisions were the output of models not available or not well regarded. Their point, however, is that mathematical models carry an inherent susceptibility to manipulation in the hands of not-so-well-intentioned.

They use terms like "priesthood" for the practitioners of mathematical modeling and quote a number of times a colleague whom they report as having said, "I stress that the problem was not mathematics per se but the place of idolatry we have given it. And it is idolatry" [1]. Perhaps we should be relieved that mathematics itself is freed of blame in their view, but, as an applied mathematician, these statements give me considerable cause for concern.

Their point is that mathematical modeling is being applied in a domain to which it is not relevant, namely studies of the environment, and yet is held in such awe by the public that even ludicrous decisions can hold sway if backed by mathematical predictions. They seem to be suggesting that a form of "math anxiety" is at play here, and I cannot help feeling that they are betraying a certain degree of such a condition themselves. We are all very aware in the mathematics community of the reaction of the public to our erudite subject, and it is sometimes all too easy to hide behind its mystique. It is, however, in our interest to demystify mathematics and its uses as much as possible. If they are correct that bad decisions are being made in our name, then that should certainly concern us. But is this an argument for the complete rejection of mathematical models in environmental sciences?

A closer look at their scientific case for the inappropriateness of mathematical models to the environment is warranted. They argue that this use of mathematical models is extending mathematics beyond its proper home into a setting that is just too complex to model. Its success in concrete physical situations is well-known; they call this the land of bricks and mortar. In a recent review of this book in *Nature* by Roger Pielke Jr. [2], the reviewer points out that mathematics has met with extraordinary success in two areas of great relevance to flying in an airplane, which is what he was doing while writing the review, namely weather prediction and the successful flight of the plane itself. He writes that these uses of mathematical modeling are very different from those discussed in the book. Unfortunately, he does not fully explore what makes them different. The prediction of weather, even on short time scales, involves an extremely complex system that suffers from all the issues present in the kind of environmental systems addressed in the book: unknown parameters, unresolved processes, and underlying chaotic dynamics. Airplane flight is arguably more straightforward to model, but is still a highly complex system given that the ambient air is undergoing all the dynamic effects that make weather prediction so complex.

What then is the key difference? I would contend that it is not the physical basis of these models that makes them so different but rather the data that are being incorporated into the predictive process. The assimilation of data into the process of forecasting weather is critical in its success. We know that weather models will fail to predict accurately after not too many hours without the incorporation of available data. The data serves to correct the inaccuracies due to the inadequacies of the model as well as its intrinsic instabilities. The mechanics of compromising between model output and data is emerging as both an important and fascinating area of science: data assimilation. It involves a blend of statistical and dynamical thinking that in itself offers many mathematical challenges; see [3].

The flying of an airplane is a similar story. There is continual incorporation of information on the response of the plane to its ambient environment, and the plane is then controlled to keep its flight on track. It is an extraordinary feat of engineering that flights run so smoothly given the complexity of the physics. In both of these cases it is then the mutual support that data and model give each other that leads to successful predictions. Neither can live without the other: models will go wrong fairly quickly without corrections in light of data, and the data are insufficient to provide a description from which a prediction might be extrapolated without the use of a model to fill in away from the data points.

A serious weakness of this book is the lack of discussion of data: how data can and indeed must be incorporated into models. There is, of course, a vicious circle here in that the topic is prediction and we do not have data from future events. The point is that once we realize the significance of data, we can start to circumscribe the valid use of models in making predictions. It is determined by not going too far from where data are available. In the case of weather prediction, it means forecasting a few days out and understanding that longer time predictions are to be taken much more lightly.

The authors argue that mathematical models are being taken out of their domain of applicability by moving from the safe ground of physically based modeling, such as for bridges and buildings, to complex situations like those that occur in the environment. I would argue that they are correct in saying that models are being taken to a place they do not belong, but it is not the complexity of the situation that is the problem, but its disconnection from data. Once the point is understood properly that data and models go hand-in-hand, a new focus can be adopted that places data assimilation in a central position.

Reading this book, one realizes that the scientific research community has a very different view of mathematical models from those who typically put them into use, and the authors point this out repeatedly. The scientific community sees models as a testing ground for ideas. New phenomena can be discovered in models and relationships between physical effects exposed and understood; in other words, hypotheses about cause and effect can be tested.

One of my favorite examples is the work of Hodgkin and Huxley on nerve impulses [4]. They formulated a model for the propagation of a voltage action potential along a nerve axon. It involved all kinds of approximations concerning the chemical concentration differences across the membrane. The biochemical processes are not even physically modeled, only their effect on the membrane is accounted for. Nevertheless, they showed, using a primitive computational device which was little more than an adding machine, that this modeling of the mechanisms for chemical passage through the membrane leads to a propagating wave (the nerve impulse) in the equation for the voltage. They thus showed that the postulated chemical concentration differences and their changes in response to electrical excitation could explain why nerve axons support propagating impulses. This groundbreaking piece of work won them the Nobel Prize and would not have been possible without the computations they performed on the mathematical model. This example is not of environmental modeling, but neither is it from the land of bricks-and-mortar.

Another variant of the use of mathematical models is to test "what-if scenarios". Information is fed into a model that reflects a particular set of choices or decisions. The output of computational runs of the model can then give tremendous insight into the potential consequences of the original choices. The importance of this use of modeling is emphasized by Naomi Oreskes in a series of very insightful and interesting articles about prediction and models [5].

A fascinating example of this type, which is in the context of an environmental issue, was pointed out to me by Margaret Beck [6]. Loggerhead sea turtles are a species that has been in danger of extinction. Crouse et al. [7] tested various conservation management strategies in a population dynamics model. They concluded that the management practices of the time, with their focus on eggs on nesting beaches, were not the most effective as this is the least responsive life stage. Based on runs of the model, they proposed specific protection efforts for juvenile loggerheads, for instance using turtle excluder devices that prevent turtles from getting caught in nets. There is evidence that this approach has been successful; see the recent articles [8].

These examples are typical of the great successes of mathematics applied through models. Different physical effects are shown to be connected through their being modeled and the resulting equations solved computationally. The suggestions of a replacement for mathematical models by the authors are rather vague. Their recommendation appears to amount to putting trust in the environmental experts who understand the underlying physical processes. Without belittling their own expertise and contributions, I would suggest that depending solely on expert advice would constitute a system more vulnerable to abuse than one based on mathematical models.

It is almost certain that mathematical models are here to stay. Moreover, we should be happy about this as they can be used to expose an enormous amount about the underlying physical mechanisms. It is critical to understand cause and effect in environmental situations and to be able to test the possible outcomes of different decision strategies. Mathematical models play the key role in this enterprise, and they allow us to cover cases that are far more complex than can be handled by well-informed expertise. We do need to be circumspect, however, about very specific predictions and reserve our faith in such predictions for cases where reasonable data have been available and assimilated into the model. The issue of delineating the validity of predictions and their presentation is fascinating and critically important. This is essentially a mathematical subject but also has sociological and philosophical dimensions. It has received much recent attention because of its significance in addressing climate change; see [9] for an interesting and current discussion.

It should be of concern to us as a community that mathematical models have been abused in environmental engineering as described in this book. But this is reason to get rid of the abuse, not the models.

References

[1] The authors attribute this to Jim O'Malley, who is a representative of the fishing industry and is referred to extensively in Chapter 1. This quote is on p. xiii in the Preface, see also p. 156.

[2] Roger Pielke Jr., When the numbers don't add up, *Nature* **447** (2007), 35–6.

[3] The recent special issue of *Physica D* gives a good overview of the state-of-the-art in data assimilation: Data Assimilation, (Kayo Ide and Christopher K. R. T. Jones, eds.), *Physica D* **230** (2007). This volume is based on a special semester of activity held at SAMSI (Statistical and Applied Mathematical Sciences Institute), an NSF funded mathematics institute in Research Triangle Park, North Carolina.

[4] A. Hodgkin and A. Huxley, A quantitative description of membrane current and its application to conduction and excitation in nerve, *J. Physio.* **117** (1952), 500–544.

[5] N. Oreskes, Why believe a computer? Models, measures, and meaning in the natural world, *The Earth Around Us: Maintaining a Livable Planet*, (Jill S. Schneiderman, ed.), W. H. Freeman and Co., San Francisco, 2000, pp. 70–82; The role of quantitative models in science, *Models in Ecosystem Science*, (Charles D. Canham, Jonathan J. Cole, and William K. Lauenroth, eds.), Princeton University Press, Princeton, 2003, pp. 13–31.

[6] Margaret Beck is currently an NSF Mathematical Sciences Postdoctoral Fellow, and she dates her interest in applied mathematics to the time she read this paper as an undergraduate.

[7] Deborah T. Crouse, Larry B. Crowder, and Hal Caswell, A stage-based population model for loggerhead sea turtles and implications for conservation, *Ecology* **68** (1987), 1412–1423; and Rebecca L. Lewison, Sloan A. Freeman, and Larry B. Crowder, Quantifying the effects of fisheries on threatened species: the impact of pelagic longlines on loggerhead and leatherback sea turtles, *Ecology Letters* **7** (2004), 221–231.

[8] See: `http://www.ens-newswire.com/ens/dec2002/2002-12-18-06.asp`; `http://www.dukeresearch.duke.edu/database/pagemaker.cgi?992632903`; `http://www.commondreams.org/news2004/1129-06.htm`; `http://moray.ml.duke.edu/faculty/crowder/`.

[9] D. A. Stainforth, M. R. Allen, E. R. Tredger, and L. A. Smith, Confidence, uncertainty and decision-support relevance in climate predictions, *Phil. Trans. Roy. Soc. A* **365** (2007), 2145–2161.

Mathematics
Awareness Month

April 2008

Math and Voting is the theme for Mathematics Awareness Month, held each year in April. Its goal is to increase public understanding of and appreciation for mathematics. Visit the Mathematics Awareness Month web site at ***www.mathaware.org*** for details.

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2008 Steele Prizes

The 2008 Leroy P. Steele Prizes were awarded at the 114th Annual Meeting of the AMS in San Diego in January 2008.

The Steele Prizes were established in 1970 in honor of George David Birkhoff, William Fogg Osgood, and William Caspar Graustein. Osgood was president of the AMS during 1905–1906, and Birkhoff served in that capacity during 1925–1926. The prizes are endowed under the terms of a bequest from Leroy P. Steele. Up to three prizes are awarded each year in the following categories: (1) Lifetime Achievement: for the cumulative influence of the total mathematical work of the recipient, high level of research over a period of time, particular influence on the development of a field, and influence on mathematics through Ph.D. students; (2) Mathematical Exposition: for a book or substantial survey or expository research paper; (3) Seminal Contribution to Research: for a paper, whether recent or not, that has proved to be of fundamental or lasting importance in its field or a model of important research. Each Steele Prize carries a cash award of US$5,000.

The Steele Prizes are awarded by the AMS Council acting on the recommendation of a selection committee. For the 2008 prizes the members of the selection committee were: Rodrigo Bañuelos, Enrico Bombieri, Russel Caflisch, Lawrence C. Evans, Lisa C. Jeffrey, Nicholas M. Katz, Julius L. Shaneson, Richard P. Stanley, and David A. Vogan (chair).

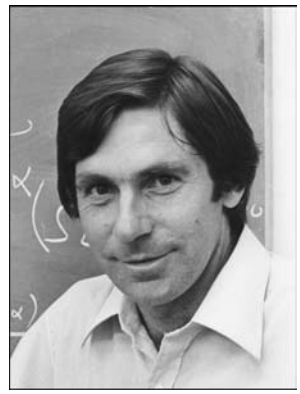
The list of previous recipients of the Steele Prize may be found on the AMS website at `http://www.ams.org/prizes-awards`.

The 2008 Steele Prizes were awarded to Neil Trudinger for Mathematical Exposition, to Endre Szemerédi for a Seminal Contribution to Research, and to George Lusztig for Lifetime Achievement. The text that follows presents, for each awardee, the selection committee's citation, a brief biographical sketch, and the awardee's response upon receiving the prize.

Mathematical Exposition: Neil Trudinger

Citation

The Leroy P. Steele Prize for Mathematical Exposition is awarded to Neil Trudinger in recognition of his book *Elliptic Partial Differential Equations of Second Order*, written with the late David Gilbarg.



Neil Trudinger

The global theory of nonlinear partial differential equations was mostly restricted to PDE involving two variables until the late 1950s, when fundamental estimates of DeGiorgi and Nash for second-order elliptic (and parabolic) equations finally broke open such PDE in more variables. The subject thereupon exploded beyond all expectations, and nowadays the analysis of even extremely degenerate and highly nonlinear second-order elliptic PDE in many variables is fairly routine, if very technical in detail.

Neil Trudinger, starting with the original 1977 edition of his book with Gilbarg, has recorded the progress of the field. He has reworked the breakthroughs, many due to him, recasting these technical estimates into understandable form within the fixed notation and framework of this highly cited book in its various domestic and foreign editions. His service has been invaluable. Having this foundational reference has made it possible for young researchers to enter the field, which would otherwise have been impenetrable. Here they can read in full detail all about Schauder estimates, Sobolev spaces, boundary estimates, Harnack inequalities, a priori derivative bounds, and much, much more.

Good mathematical exposition is always difficult, but it is especially so for technical estimates. The heights to which the research community has pushed the analysis of nonlinear second-order elliptic PDE is amazing, but the fundamental inequalities are mostly without any good heuristic interpretations. Hard analysis is both hard and hard to explain: Neil Trudinger's concise, elegant exposition in this outstanding book is magnificent.

Biographical Sketch

Neil S. Trudinger was born in Ballarat, Australia, in 1942. After schooling and undergraduate education at the University of New England in Australia, he completed his Ph.D. at Stanford University in 1966.

Following appointments at the Courant Institute (1966–67); University of Pisa, Italy (1967); Macquarie University, Australia (1968–70); University of Queensland, Australia (1970–73); University of Minnesota (1970–71); and Stanford University (1971), he took up a chair of mathematics at the Australian National University in 1973, where he has been since. During this period he has also held numerous visiting positions at universities in Asia, Europe, and the United States, as well as a professorship at Northwestern University from 1989 to 1993. Among various administrative positions at the Australian National University, he was head of the Department of Pure Mathematics from 1973 to 1980, director of the Commonwealth Special Research Centre for Mathematical Analysis from 1982 to 1990, and dean of the School of Mathematical Sciences from 1992 to 2000.

Neil Trudinger is a fellow of the Australian Academy of Science and a fellow of the Royal Society of London. He was also chief judge in the Singapore National Science Talent Search in 2002. His research contributions, while largely focused on nonlinear elliptic partial differential equations, have also spread into functional analysis, geometry, computational mathematics, and, more recently, optimal transportation.

Response

I am very honoured and pleased to receive the Steele Prize for Mathematical Exposition. I could never have imagined forty years ago when my book with David Gilbarg on elliptic partial differential equations was first published that it would get such recognition. The book was originally conceived by us after I had prepared lecture notes for the spring quarter of the graduate PDE course at Stanford in 1971. My topics were Sobolev spaces and their application to linear elliptic PDE, and we decided to start by blending these with earlier notes of Dave on the Schauder theory. Six years later and after a lot of hard work, including long and painful negotiations over language, the first edition appeared. We were extremely fortunate to have incredible assistance. First was the impeccable typing of Anna Zalucki in Canberra and Isolde Field at Stanford. Isolde had already typed my Ph.D. thesis at Stanford several years earlier, and Dave had been my supervisor, so the Stanford team was ready to roll from the outset. In Australia I had an amazing research assistant, Andrew Geue, who checked every bibliographical reference against its original publication so that titles and page numbers were always correct. We also got plenty of encouragement and support from many colleagues over the succeeding years to whom I am very grateful, as well as to those old friends Catriona Byrne and Joachim Heinze at Springer in Heidelberg.

My own passage into mathematical exposition was rather severe, akin to learning to swim by being thrown in a deep ocean. My first postdoctoral position in 1966 was a Courant Instructorship, and I was assigned an advanced topics course in PDE for the full year. Armed with books by Bers, John, and Schechter on partial differential equations; Morrey on multiple integrals in the calculus of variations; Friedman on parabolic partial differential equations; as well as works of Ladyzhenskaya and Ural'tseva, Moser, Serrin, and Stampacchia from my graduate days, I struggled to teach a full-year course on elliptic and parabolic equations to students who all looked older than my meagre twenty-four years. But this torture had its rewards. I presented a then recent and now famous paper by John and Nirenberg on BMO as it was needed for the Moser Harnack inequality. Subsequently, I found that it could be bypassed for the Harnack inequality through a simpler argument, a byproduct of which was an exponential-type imbedding result, later sharpened by Moser and now well known as the Moser-Trudinger inequality. At the same time, my quest to understand loss of compactness in Sobolev imbeddings led to the Yamabe "problem". But most of all I was extremely well equipped when I started work on the book a few years later.

I conclude on a sad note. Both David Gilbarg and Isolde Field passed away in recent years. This honour is for you, Dave and Isolde!

Seminal Contribution to Research: Endre Szemerédi

Citation

The Steele Prize in 2008 for a Seminal Contribution to Mathematical Research is awarded to Endre Szemerédi for the paper "On sets of integers containing no k elements in arithmetic progression", *Acta Arithmetica* XXVII (1975), 199–245.

A famous result of arithmetic combinatorics due to van der Waerden in 1927 proving an earlier conjecture of Baudet states that if we partition the natural integers into finitely many subsets, then one of these subsets contains arithmetic progressions of arbitrary length. In its finite version, because of the inevitable use of a multiple induction argument, it leads to incredibly large bounds for the size of a set of consecutive integers such that for every k-partition of it there is always a subset containing an arithmetic progression of k terms. In 1936 Erdős and Turán proposed, as a natural extension of van der Waerden's theorem, the conjecture that any infinite set of integers of positive density contained arbitrarily long arithmetic progressions; this may be viewed as a discrete analog of the classical theorem of Lebesgue that almost every point of a set of positive measure of real numbers has density 1. This conjecture quickly became one of the major open questions in Ramsey theory.

Endre Szemerédi

The first nontrivial result about the Erdős-Turán conjecture was obtained by K. F. Roth in 1953 using harmonic analysis, proving it for progressions of length 3, but his method did not extend to length 4 in any obvious way. In 1969 Szemerédi proved the Erdős-Turán conjecture for length 4 using a difficult combinatorial method. Finally, the Erdős-Turán conjecture was settled in the affirmative by Szemerédi in his landmark 1975 paper.

The solution is a true masterpiece of combinatorics, containing new ideas and tools whose impact go well beyond helping to solve a specific hard problem. One of these new tools, his by now famous Regularity Lemma, has become a foundation of modern combinatorics. Its statement of striking simplicity asserts roughly that any sufficiently large dense graph can be approximated by a union of a bounded number of very regular subgraphs of almost equal size, looking in pairs like very regular bipartite graphs; the upper and lower bounds for the number of subgraphs are determined only by the desired quality of approximation and are independent of the size of the graph. In essence, every large dense graph is well approximated by a controlled bounded union of quasirandom bipartite graphs of almost equal size. This is a very surprising result, far from intuitive. The proof is short but very subtle, leading to bounds for the number of components larger than any tower of exponentials. The subtlety of the statement has been confirmed by recent work by Gowers, showing that these gigantic bounds are indeed necessary for the validity of the Regularity Lemma in all cases.

The impact in combinatorics of the Regularity Lemma and of the numerous variants that followed it is due to the fact that there are many techniques available for studying random graphs and, via the Regularity Lemma, they can be transferred to the study of completely arbitrary graphs. It is fair to say that the Regularity Lemma has transformed the focus of graph theory from the study of special graphs and of extremal problems to the study of general graphs and random graphs. Beyond combinatorics it has found applications in number theory and in computer science, in particular in complexity theory.

However, the impact of Szemerédi's paper goes beyond this. The solution of the Erdős-Turán conjecture stimulated other mathematicians to find other lines of attack. In 1977 Furstenberg found a new proof of Szemerédi's theorem using deep methods of ergodic theory, together with a correspondence principle showing the equivalence of Szemerédi's theorem with his new ergodic theorem. Furstenberg's new method could then be used to attack multidimensional versions of the theorem as well as nonlinear versions. In 2001 Gowers obtained a new proof of Szemerédi's theorem, based on his novel idea of a Fourier analysis with nonlinear phases. More recently, Green and Tao were able to replace the positive density condition in Szemerédi's theorem by other arithmetical conditions, which allowed them, using again a suitable transference principle, to prove the same result for any sequence of primes of relative positive density, thereby solving another famous conjecture of Erdős considered inaccessible by standard methods of analytic number theory.

Recent work by many authors strongly indicates that these different approaches to Szemerédi's theorem are all interrelated. There is no doubt that Szemerédi's landmark paper is the source of these beautiful developments in mathematics.

Biographical Sketch

Endre Szemerédi was born in Budapest in 1940. He finished university in Budapest, at ELTE University. He received his Ph.D. at the Moscow State University. He has been a member of the Renyi Institute of the Hungarian Academy of Sciences since 1970. Currently he is a professor in the Department of Computer Sciences, Rutgers University. He is a member of the Hungarian Academy of Sciences. In 1976 he received the Pólya Prize.

Response

I am really grateful to the AMS, to the Steele Prize Committee, and to those people who recommended me. This prize is a great honor.

Here is what actually sparked my work on $R_4(n)$. Assuming that it was a well-known fact that dense sets of integers have arithmetic progressions of length four, I proudly showed Paul Erdős a proof that no positive fraction of elements in a long arithmetic progression could be squares. Erdős pointed out a flaw in the argument, namely that $R_4(n)$ was actually an open problem and that the rest of my proof was in fact already known to Euler. So now I really had to work on $R_4(n)$. Once $R_4(n)$ was settled, so was the original problem about squares. Later, Bombieri, Granville, and Pintz greatly improved my result. Luckily for me this occurred several years after $R_4(n)$; otherwise I would never have worked on it.

It is my opinion (and maybe only mine) that the Regularity Lemma was born after the $R_k(n)$ result, though certainly inspired by ideas from that paper. It is necessary to acknowledge Andras Hajnal for the $R_k(n)$ paper and Vasek Chvatal for the Regularity Lemma paper. These friends literally wrote every word of the papers based on my explanations. I also want to express my gratitude to Paul

Erdős and to K. F. Roth for their encouragement to persevere with $R_k(n)$.

This award could not have occurred were it not for the fundamental work of other mathematicians who developed the field of additive combinatorics and established its relations with many other areas. Without them my theorem is only a fairly strong result, but no "seminal contribution to research". I acknowledge my debt to them. Finally, I want to thank my wife, Anna, for all her patience, good humor, and support.

Lifetime Achievement: George Lusztig

Citation

The work of George Lusztig has entirely reshaped representation theory and in the process changed much of mathematics.

Here is how representation theory looked before Lusztig entered the field in 1973. A central goal of the subject is to describe the irreducible representations of a group. The case of reductive groups over locally compact fields is classically one of the most difficult and important parts. There were three more or less separate subjects, corresponding to groups over $\mathbb{R}$ (Lie groups), $\mathbb{Q}_p$ (p-adic groups), and finite fields (finite Chevalley groups).

Lusztig's first great contribution was to the representation theory of groups over finite fields. In a 1974 book he showed how to construct "standard" representations—the building blocks of the theory—in the case of general linear groups. Then, working with Deligne, he defined standard representations for all finite Chevalley groups. This was mathematics that had been studied for nearly a hundred years; Lusztig and Deligne did more in one paper than everything that had gone before.

With the standard representations in hand (in the finite field case), Lusztig turned to describing irreducible representations. The first step is simply to get a list of irreducible representations. This he did almost immediately for the "classical groups", like the orthogonal groups over a finite field. The general case required deep new ideas about connections among three topics: irreducible representations of reductive groups, the representations of the Weyl group, and the geometry of the unipotent cone. Although some key results were contributed by other (great!) mathematicians like T. Springer, the deepest new ideas about these connections came from Lusztig, sometimes in work with Kazhdan.

Lusztig's results allowed him to translate the problem of describing irreducible representations of a finite Chevalley group into a problem about the Weyl group. This allowed results about the symmetric group (like the Robinson-Schensted algorithm and the character theory of Frobenius and Schur) to be translated into descriptions of the irreducible representations of finite classical groups. For the exceptional groups, Lusztig was asking an entirely new family of questions about the Weyl groups, and considerable insight was needed to arrive at complete answers, but eventually he did so.

George Lusztig

Lusztig's new questions about Weyl groups originate in his 1979 paper with Kazhdan. The little that was known about irreducible representations first becomes badly behaved in some very specific examples in $SL(4, \mathbb{C})$. Kazhdan and Lusztig noticed that their new questions about Weyl groups first had nontrivial answers in exactly these same examples (for the symmetric group on four letters). In an incredible leap of imagination, they conjectured a complete and detailed description of singular irreducible representations (for reductive groups over the complex numbers) in terms of their new ideas about Weyl groups. This (in its earliest incarnation) is the Kazhdan-Lusztig conjecture. The first half of the proof was given by Kazhdan and Lusztig themselves, and the second half by Beilinson-Bernstein and Brylinski-Kashiwara independently.

The structure of the proof is now a paradigm for representation theory: use combinatorics on a Weyl group to calculate some geometric invariants, relate the geometry to representation theory, and draw conclusions about irreducible representations. Lusztig has used this paradigm in an unbelievably wide variety of settings. One striking case is that of groups over p-adic fields. In that setting Langlands formulated a conjectural parametrization of irreducible representations around 1970. Deligne refined this conjecture substantially, and many more mathematicians have worked on it. Lusztig (jointly with Kazhdan) showed how to prove the Deligne-Langlands conjecture in an enormous family of new cases. This work has given new direction to the representation theory of p-adic groups.

There is much more to say: about Lusztig's work on quantum groups, on modular representation theory, and on affine Hecke algebras, for instance. His work has touched widely separated parts of mathematics, reshaping them and knitting them together. He has built new bridges to combinatorics and algebraic geometry, solving classical problems in those disciplines and creating exciting new ones. This is a remarkable career and as exciting to watch today as it was at the beginning more than thirty years ago.

Biographical Sketch

George Lusztig was born in Timisoara, Romania, in 1946. After graduating from the University of Bucharest in 1968, he was an assistant at the University of Timisoara and then a member of the Institute for Advanced Study in Princeton, where he studied with Michael Atiyah. During his second year at IAS he was also a graduate student at Princeton University and received a Ph.D. degree (1971) for work on Novikov's higher signature and families of elliptic operators. He then moved to the University of Warwick, U.K., becoming a professor in 1974. For the last thirty years he has been a professor at the Massachusetts Institute of Technology. He has been a frequent visitor to the IHÉS (Institut des Hautes Études Scientifiques) and spent the academic year 1985–86 at the University of Rome. Lusztig received the Berwick Prize (London Mathematical Society, 1977), the Cole Prize in Algebra (American Mathematical Society, 1985), and the Brouwer Medal (Dutch Mathematical Society, 1999). He is a fellow of the Royal Society of London, a fellow of the American Academy of Arts and Sciences, and a member of the National Academy of Sciences.

Response

When writing a response it is very difficult to say something that has not been said before. Therefore, I thought that I might give some quotes from responses of previous Steele Prize recipients which very accurately describe my sentiments.

"What a pleasant surprise!" (Y. Katznelson, 2002). "I feel honored and pleased to receive the Steele prize—with a small nuance, that it is awarded for work done up to now" (D. Sullivan, 2006). "I always thought this prize was for an old person, certainly someone older than I, and so it was a surprise to me, if a pleasant one, to learn that I was chosen a recipient" (G. Shimura, 1996). "But if ideas tumble out in such a profusion, then why aren't they here now when I need them to write this little acceptance?" (J. H. Conway, 2000).

Now, I thank the Steele Prize Committee for selecting me for this prize. It is an unexpected honor, and I am delighted to accept it. I am indebted to my teachers, collaborators, colleagues at MIT, and students for their encouragement and inspiration over the years.

Around the time of my Ph.D., I switched from being a topologist with a strong interest in Lie theory to being a representation theorist with a strong interest in topology. (The switch happened with some coaching by Michael Atiyah and later by Roger Carter.) After that most of my research was concerned with the study of representations of Chevalley groups over a finite field or used the experience I gained from groups over a finite field to explore neighboring areas such as p-adic groups (which can be viewed as groups over a finite field that are infinite dimensional) or quantum groups (which can be viewed as analogues of the Iwahori-Hecke algebras, familiar from the finite group case).

Here are three topics from my research which I am particularly fond of:

(i) the classification of complex irreducible representations of a finite Chevalley group;

(ii) the theory of character sheaves, which helps in computing the irreducible characters in (i);

(iii) the theory of canonical bases arising from quantum groups, which unexpectedly provides a very rigid structure with coefficients in the natural numbers for several of the known objects in Lie theory.

I would like to make some comments on the period in which I focused on topic (i) above, from late 1975 (when my paper with Deligne (DL) was just completed) to the spring of 1978. In the first few months of that period I worked on the "Coxeter paper" (CP), in which I studied in detail the cohomology with compact support of the variety attached in (DL) to a Coxeter element in the Weyl group. Luckily, in this case the eigenvalues of Frobenius could be explicitly computed, and the eigenspaces provided a complete decomposition into irreducible representations, giving several new key examples of cuspidal representations. Then during the next year I found the classification and degrees of the irreducible representations of classical groups over a finite field using an extension of the method of (DL). After this (in 1977), as I wrote the notes for my lectures in the CBMS Regional Conference Series, No. 39, I found the classification and degrees of the irreducible unipotent representations of the finite exceptional groups of type other than E_8, based on (DL) and (CP). Towards the end of 1977 I discovered the nonabelian Fourier transform attached to any finite group H (which in the case where H is abelian reduces to the ordinary Fourier transform for functions on H times its dual). This new Fourier transform allowed me to find (in the spring of 1978) the classification and degrees of the irreducible unipotent representations for E_8. The same (or somewhat easier) methods can be used to obtain the classification and degrees of nonunipotent irreducible representations of finite exceptional groups. Thus, contrary to what the citation says, the classification of irreducible representations of finite exceptional groups does not depend on the "geometry of the unipotent cone" or on my work with Kazhdan done in 1979 (KL). On the other hand, the latter (KL) did play a role in my work (1981, 1982) on computing the values of irreducible characters on semisimple elements, and the former played a role in my work (1983–1986) on character sheaves. Moreover, the use of (KL) simplifies some of the arguments in the classification, as I showed in my 1984 book.

2008 Conant Prize

The 2008 Levi L. Conant Prize was awarded at the 114th Annual Meeting of the AMS in San Diego in January 2008.

The Conant Prize is awarded annually to recognize an outstanding expository paper published in either the *Notices of the AMS* or the *Bulletin of the AMS* in the preceding five years. Established in 2001, the prize honors the memory of Levi L. Conant (1857–1916), who was a mathematician at Worcester Polytechnic University. The prize carries a cash award of US$1,000.

The Conant Prize is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2008 prize the members of the selection committee were: Noam D. Elkies, Stephen J. Greenfield, and Carl R. Riehm (chair).

Previous recipients of the Conant Prize are: Carl Pomerance (2001), Elliott Lieb and Jakob Yngvason (2002), Nicholas Katz and Peter Sarnak (2003), Noam D. Elkies (2004), Allen Knutson and Terence Tao (2005), Ronald M. Solomon (2006), and Jeffrey Weeks (2007).

The 2008 Conant Prize was awarded to J. Brian Conrey; and to Shlomo Hoory, Nathan Linial, and Avi Wigderson. The text that follows presents the committee's citations, brief biographical sketches of the authors, and their responses upon receiving the prize.

J. Brian Conrey

Citation

"The Riemann Hypothesis", *Notices*, March 2003, pages 341–353.

The Riemann Hypothesis (RH) has a strong claim to being the outstanding open problem in mathematics. Much has been written about RH, but rarely with anything like the scope that Conrey covers in but a dozen *Notices* pages, outlining the mathematical context that justifies the importance of RH, key moments in the problem's 140-plus-year history, known partial results and blind alleys, various threads of numerical and theoretical evidence, and suggestive connections with disparate branches of mathematics and theoretical physics. The mathematical exposition is enhanced by the judicious use of anecdotes illustrating the human drama of the quest for a proof and of figures that help the reader visualize the zeta function as a function of a complex variable and the key connections between the distribution of prime numbers, the distribution of the zeros of the Riemann zeta function, and conjecturally also the distribution of the eigenvalues of random Hermitian operators.

Conrey remarks on one of those fascinating connections (Gauss's class number problem and a "conspiracy of L-functions") that "we seem to be players in the middle of a mystery novel." The same can be said of the status of the Riemann Hypothesis itself. Conrey has given a masterly and lucid introduction to the plot thus far, to the detectives who brought us to this point, and to what may be called the main suspects: the mathematical structures that might be expected to figure in the eventual resolution of this central mystery of modern mathematics.

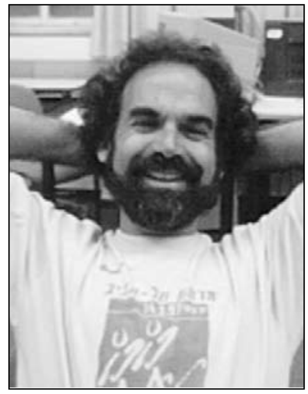
Biographical Sketch

J. Brian Conrey is the founding executive director of the American Institute of Mathematics (AIM). In this position, he oversees AIM's operations and helps to initiate programs that further AIM's goal of solving problems through focused collaborative efforts.

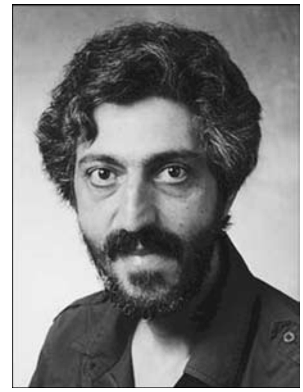
Conrey received his Bachelor of Science from Santa Clara University in 1976 and his doctorate from the University of Michigan in 1980. He conducted his postdoctoral studies at the University of Illinois, 1980–1982, and the Institute for Advanced Study in Princeton, 1982–1983. He was awarded an Alfred P. Sloan Fellowship in 1986.

J. Brian Conrey

Shlomo Hoory



Nathan Linial



Avi Wigderson

Conrey was a mathematics professor at Oklahoma State University, serving as head of the department from 1991 to 1997. He joined AIM in 1997. In 2005 he also became a professor at the University of Bristol.

Conrey's mathematical specialty is number theory, and he has a particular interest in the nearly 150-year-old Riemann Hypothesis. He has published more than fifty research papers and serves as an editor of the *Journal of Number Theory*.

Conrey has also helped launch several outreach programs for students interested in mathematics, including the San Jose Math Circle, MathCounts, and the Math MardiGras in Morgan Hill, and has been involved in several Research Experiences for Undergraduates programs working with undergraduates doing research. Conrey lives in San Martin, California, with his wife, Jan. They have three children: Brianna, Jennifer, and Rick.

Response

I am honored to receive the Levi Conant Prize for my article on the Riemann Hypothesis. I really enjoyed working on it and found the endeavor to be interesting and instructive. I hope that the article will play some small role in the eventual solution of this beautiful problem, perhaps by inspiring a young mathematician to think about it.

I would like to thank the people who helped me with the writing: Harold Boas, Brianna Conrey, David Farmer, Roger Heath-Brown, and K. Soundararajan, and with the graphics: Sandra Frost, Andrew Odlyzko, Mike Rubinstein, and Nina Snaith.

Shlomo Hoory, Nathan Linial, and Avi Wigderson

Citation

Expander graphs are (finite) graphs that are both sparse and highly connected: a sequence of graphs G_i of increasing size is a family of expander graphs if there is an $\epsilon > 0$ such that for each i and each subset S of G_i, the number of edges from S to its complement is at least $\epsilon|G_i|$. Since their introduction thirty years ago, the study of these graphs has blossomed into a substantial area of research with many branches. One direction involves understanding the relationship of graph expansion to other graph invariants, most notably the second largest eigenvalue of its adjacency matrix. Identifying classes of expanders and proving that they are indeed expanders involves a variety of techniques from harmonic analysis, group representation theory, graph theory, and information theory. Expanders have found a variety of applications within the theory of computing and other fields, from direct application to interconnection networks, to more surprising applications such as the problem of understanding the relative power of deterministic and randomized computation, the construction of computationally efficient error-correcting codes, and the construction of finite metric spaces that cannot be well approximated in Euclidean space. These applications confirm that computer science is an area with problems, techniques, and results that engage mathematicians in many fields.

This very readable article, "Expander graphs and their applications", which appeared in *Bull. Amer. Math. Soc. (N.S.)* **43** (2006), 439–561, provides a thorough overview of these and other developments. It is readily accessible for self-study by experienced graduate students and, with appropriate guidance, could even be appropriate for an advanced undergraduate seminar.

Biographical Sketch: Shlomo Hoory

Shlomo Hoory received his Ph.D. in computer science in 2002 under Nathan Linial at the Hebrew University of Jerusalem. His postdoctoral work was done at the University of Toronto and at the University of British Columbia. Currently he is working at the IBM Haifa research labs in the Constraint Satisfaction and Machine Learning group.

Response: Shlomo Hoory

It is a great honor for me to receive the Conant Prize for my joint paper with Nati Linial and Avi Wigderson. I would like to thank Nati and Avi for the pleasure of being a teacher assistant in their course on expander graphs at the Hebrew University and later for their help and encouragement while I taught the course at the University of

Toronto. Special thanks are due to the students of the course who wrote the scribe notes that formed the foundation for our paper and to Mark Goresky, who convinced us to make the effort and turn the notes into a full-scale review of the subject. Mark Goresky also assisted us throughout the writing process. I see great potential in the field of expander graphs for advancing areas in mathematics, computer science, and engineering. I hope that our expository paper will make the subject accessible to a wide audience.

Biographical Sketch: Nathan Linial

Nathan (Nati) Linial was born in Haifa, Israel, in 1953. He received his undergraduate education in mathematics at the Technion. His Ph.D. thesis in graph theory was written under Micha Perles at the Hebrew University of Jerusalem in 1978. Following a postdoctoral period at the University of California, Los Angeles, he returned to the Hebrew University to become a professor of computer science, a position he has held ever since. His main research interests include the mathematical foundations of computer science and combinatorics. He is particularly fascinated by the interaction between geometry and combinatorics. In addition, he is interested in mathematical problems that are motivated by other scientific disciplines, such as bioinformatics.

Response: Nathan Linial

I was first exposed to graph theory in a class for mathematically oriented high school kids. As my mathematical horizons expanded, I came to like the connections between combinatorics and other parts of mathematics. There are few places where these connections shine as brightly as in the study of expander graphs. I believe that the full potential impact of combinatorics on the rest of mathematics is only starting to reveal itself and the study of expander graphs can give us some idea of the true power of these connections.

Biographical Sketch: Avi Wigderson

Avi Wigderson is a professor at the School of Mathematics, Institute for Advanced Study (IAS), Princeton. He obtained his B.Sc. in computer science from the Technion in 1980 and his Ph.D. from Princeton in 1983. He was a member of the faculty at the Hebrew University in Jerusalem from 1986 to 2003. He joined the permanent faculty of the IAS in 1999. His research interests lie principally in complexity theory, algorithms, randomness, and cryptography. His awards include the Nevanlinna Prize (1994).

Response: Avi Wigderson

I am honored to receive the Conant Prize for my joint paper with Shlomo Hoory and Nati Linial. Many thanks are in order. First and foremost, to Nati and Shlomo for the pleasure of teaching together (at the Hebrew University) the course which resulted in this manuscript and for the big effort of writing it. Thanks to the many students of this course whose scribe notes formed the foundation of that paper. Special thanks to Mark Goresky, who convinced us to write it and whose enthusiasm and meticulous reading of earlier drafts helped get us through the process. Thanks to the many others who read and corrected earlier versions. And finally, thanks to the many colleagues and collaborators from whom I learned so much in the wonderful world of expander graphs.

2008 Morgan Prize

Nathan Kaplan

The 2008 AMS-MAA-SIAM Frank and Brennie Morgan Prize for Outstanding Research in Mathematics by an Undergraduate Student was awarded at the Joint Mathematics Meetings in San Diego in January 2008.

The Morgan Prize is awarded annually for outstanding research in mathematics by an undergraduate student (or students having submitted joint work). Students in Canada, Mexico, or the United States or its possessions are eligible for consideration for the prize. Established in 1995, the prize was endowed by Mrs. Frank (Brennie) Morgan of Allentown, Pennsylvania, and carries the name of her late husband. The prize is given jointly by the AMS, the Mathematical Association of America (MAA), and the Society for Industrial and Applied Mathematics (SIAM) and carries a cash award of US$1,000.

Recipients of the Morgan Prize are chosen by a joint AMS-MAA-SIAM selection committee. For the 2008 prize, the members of the selection committee were Kelly J. Black, James H. Curry, Karen E. Smith, Kannan Soundararajan, Judy L. Walker, and Paul Zorn (chair).

Previous recipients of the Morgan Prize are Kannan Soundararajan (1995), Manjul Bhargava (1996), Jade Vinson (1997), Daniel Biss (1998), Sean McLaughlin (1999), Jacob Lurie (2000), Ciprian Manolescu (2001), Joshua Greene (2002), Melanie Wood (2003), Reid Barton (2005), Jacob Fox (2006), and Daniel Kane (2007).

The 2008 Morgan Prize was awarded to NATHAN KAPLAN. The text that follows presents the selection committee's citation, a brief biographical sketch, and the awardee's response upon receiving the prize.

Citation

Nathan Kaplan has been named the recipient of the 2008 Morgan Prize for Outstanding Research by an Undergraduate. He graduated in 2007 from Princeton University with high honors. He also received the mathematics department's Peter Greenberg Prize, which honors outstanding mathematical accomplishments.

This award is based principally on four impressive papers in algebraic number theory, two of them individual and two with other authors. (Coauthors of the joint papers were careful to highlight Kaplan's substantial contributions.) At least three of these papers have been accepted for publication in such venues as the *Journal of Number Theory*, the *Journal of Algebra and Its Applications*, and *Acta Arithmetica*. Concerning Nathan's paper "Flat cyclotomic polynomials of order three", the *Journal of Number Theory* wrote that the work "contains…rather definitive results substantially advancing our understanding of cyclotomic polynomials of order three." Another recommender observed that this and related work of Kaplan demonstrates "remarkable creativity [and] technical facility…[and] will provide researchers new tools."

Kaplan participated in three summer REU [Research Experiences for Undergraduates] programs (at Trinity University, Williams College, and the University of Minnesota-Duluth) during his undergraduate career and produced publishable, professional-level work at all three. One of his supervisors described him as the most outstanding undergraduate with whom he had worked. Another supervisor described Kaplan as an extraordinary student—brilliant, friendly, outgoing, polite, and fun to work with. All of his recommenders, and this committee, fully expect Kaplan to become a very successful research mathematician.

Biographical Sketch

Nathan Kaplan was raised in Brooklyn, New York, and began taking mathematics classes at Columbia University while in high school. He graduated in June 2007 with a degree in mathematics from Princeton University and is currently at Cambridge

University doing Part III of the Mathematical Tripos.

His first research experience was in the summer of 2004 at the Trinity University REU program studying numerical monoids with Scott Chapman. The following summer he worked in the algebraic number theory group at the Williams College SMALL program under the direction of Allison Pacelli. In 2006 Kaplan attended Joe Gallian's REU at the University of Minnesota-Duluth and studied cyclotomic polynomials. This past summer he returned to the Trinity REU and worked as a graduate assistant. He also participated in independent research at Princeton with Ramin Takloo-Bighash, who has advised him since his first week on campus.

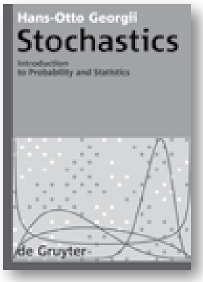
Next fall he will begin the mathematics Ph.D. program at Harvard University on a National Science Foundation Graduate Fellowship. He plans to study algebraic number theory. He is enthusiastic about teaching and has been active in tutoring since high school. Outside of math he is a dedicated New York Mets fan, enjoys theater and film, and once bowled a 162.

Response

I am very honored to be selected for the 2008 Morgan Prize. I would like to thank Mrs. Frank Morgan for endowing the award and the AMS, MAA, and SIAM for sponsoring it. I am very grateful to all of my advisers who have taught me what research is all about: Ramin Takloo-Bighash and Manjul Bhargava at Princeton, Scott Chapman at Trinity University, Allison Pacelli at Williams, and Joe Gallian at the University of Minnesota-Duluth. I owe a lot of thanks to the other students I worked with at summer REU programs and also to the students in my problem set groups at Princeton for helping me get the most out of my academic experiences. I would also like to thank my friends in Princeton, NYC, and elsewhere for giving me something to do when I needed a mathematical break. Most importantly, I must thank my parents for their love and support and for giving me so many opportunities to succeed.

DE GRUYTER

Just published



Hans-Otto Georgii

■ Stochastics

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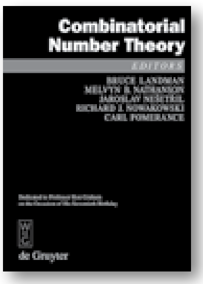

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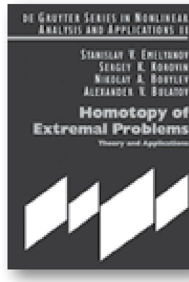

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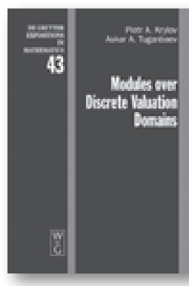

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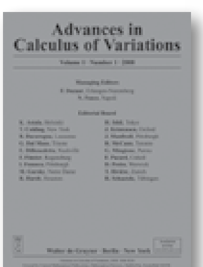

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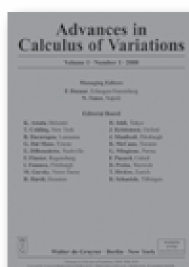

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2008 Cole Prize in Number Theory

The 2008 Frank Nelson Cole Prize in Number Theory was awarded at the 114th Annual Meeting of the AMS in San Diego in January 2008.

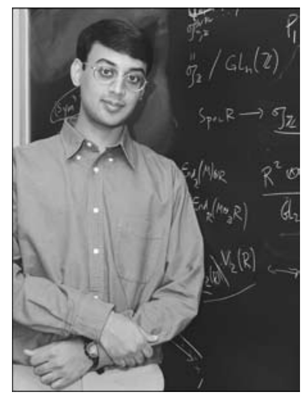
The Cole Prize in Number Theory is awarded every three years for a notable research memoir in number theory that has appeared during the previous five years. The awarding of this prize alternates with the awarding of the Cole Prize in Algebra, also given every three years. These prizes were established in 1928 to honor Frank Nelson Cole (1861–1926) on the occasion of his retirement as secretary of the AMS after twenty-five years of service. He also served as editor-in-chief of the *Bulletin* for twenty-one years. The endowment was made by Cole, contributions from Society members, and his son, Charles A. Cole. The Cole Prize carries a cash award of US$5,000.

The Cole Prize in Number Theory is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2008 prize the members of the selection committee were: Nicholas M. Katz, Kenneth A. Ribet (chair), and Alice Silverberg.

Previous recipients of the Cole Prize in Number Theory are: H. S. Vandiver (1931), Claude Chevalley (1941), H. B. Mann (1946), Paul Erdős (1951), John T. Tate (1956), Kenkichi Iwasawa (1962), Bernard M. Dwork (1962), James B. Ax and Simon B. Kochen (1967), Wolfgang M. Schmidt (1972), Goro Shimura (1977), Robert P. Langlands (1982), Barry Mazur (1982), Dorian M. Goldfeld (1987), Benedict H. Gross and Don B. Zagier (1987), Karl Rubin (1992), Paul Vojta (1992), Andrew J. Wiles (1997), Henryk Iwaniec (2002), Richard Taylor (2002), and Peter Sarnak (2005).

The 2008 Cole Prize in Number Theory was awarded to MANJUL BHARGAVA. The text that follows presents the selection committee's citation, a brief biographical sketch, and the awardee's response upon receiving the prize.

Citation



Manjul Bhargava

Professor Manjul Bhargava of Princeton University is cited for his revolutionary work on higher composition laws. His series of articles on this subject introduced completely new and unexpected ideas into a subject that began with work of Carl Friedrich Gauss in 1801.

At that time, Gauss anticipated the modern theory of abelian groups by constructing a law of composition on the set of equivalence classes of binary quadratic forms of given discriminant. By the end of the nineteenth century, the fundamental concept of an abstract group allowed one to view the system of equivalence classes of binary quadratic forms of given discriminant as the "ideal class group" of a quadratic field. Once this recasting of Gauss's work became established, Gauss's "law of composition" became something of a curiosity—evidence of how a brilliant mathematician can uncover a fundamental phenomenon even without the right tools to think about it.

Bhargava's original and surprising contribution is the discovery of laws of composition on forms of higher degree. His techniques and insights into this question are dazzling; even in the case considered by Gauss, they lead to a new and clearer presentation of that theory. If Bhargava had stopped with this discovery, his work would already be quite remarkable. But Bhargava has gone on to use his composition laws to solve a new case of one of the fundamental questions of number theory, that of asymptotic enumeration of number fields of given degree as the discriminant grows. The question is trivial for degree 1, and the quadratic case was solved by Gauss's work. Davenport and Heilbronn treated the cubic case in 1971. Bhargava used his new composition laws to solve the degree 4 case, brilliantly overcoming very serious analytic problems that had completely blocked all previous work on the problem.

Biographical Sketch

Manjul Bhargava was born in Hamilton, Ontario, Canada, but spent most of his early years in Long Island, New York. He received his A.B. in mathematics summa cum laude from Harvard University in 1996 and his Ph.D. from Princeton University in 2001. After holding visiting positions at the Mathematical Sciences Research Institute in Berkeley, the Institute for Advanced Study in Princeton, and Harvard University, he joined the faculty at Princeton University as professor of mathematics in 2003. He was also named the Clay Mathematics Institute's first Five-Year Long-Term Prize Fellow in 2001. An accomplished tabla player whose research interests span number theory, combinatorics, and representation theory, Bhargava has received numerous awards and honors, including the Hoopes Prize for Excellence in Scholarly Work and Research from Harvard University (1996), the AMS-MAA-SIAM Frank and Brennie Morgan Prize for Outstanding Undergraduate Research in Mathematics (1997), the MAA Merten M. Hasse Prize for Exposition (2003), the Packard Foundation Fellowship in Science and Engineering (2004), the Clay Research Award (2005), the SASTRA Ramanujan Prize (2005), and the Blumenthal Award for the Advancement of Research in Pure Mathematics (2005). He has been a three-time recipient of the Derek Bok Award for Excellence in Teaching and was named one of *Popular Science* magazine's "Brilliant 10" in 2002. Bhargava was an invited speaker at the International Congress of Mathematicians in Madrid in 2006 and has given numerous other invited addresses, colloquia, seminars, and public lectures at colleges and universities across North America and Europe.

Response

I am very grateful and honored to be the recipient of the 2008 Cole Prize. During the past few years I have had the good fortune of interacting with many wonderful mathematicians (both faculty and students) whose friendship and wisdom have been a constant source of inspiration for me. I would like to thank them all. In particular, I wish to express my deep gratitude to my graduate school teachers, Andrew Wiles, Peter Sarnak, and John Conway; and my undergraduate teachers and mentors, Dick Gross, Barry Mazur, Persi Diaconis, Joe Gallian, and Dave Cargo, from whom I have learned (and continue to learn!) so much and by whom I have been constantly inspired. I am also extremely grateful to Hendrik Lenstra and Don Zagier for their kindness and generosity and for always being available to discuss interesting mathematics!

I thank the Department of Mathematics at Princeton University for providing me with a wonderful work environment and the Clay Mathematics Institute and the Packard Foundation for funding my work.

The papers cited above build on ideas that go way back, starting with the mathematical works of Brahmagupta, Gauss, Dirichlet, Eisenstein, and Dedekind and leading up to the works of modern mathematicians such as Delone-Faddeev, Davenport-Heilbronn, Sato-Kimura, Wright-Yukie, and Gan-Gross-Savin. I gratefully acknowledge my indebtedness to all these mathematicians!

Perhaps I should also take this opportunity to thank here Erno Rubik for making his cube!

Finally, I thank my family for all their love and support.

2008 Bôcher Prize

The 2008 Maxime Bôcher Memorial Prize was awarded at the 114th Annual Meeting of the AMS in San Diego in January 2008.

Established in 1923, the prize honors the memory of Maxime Bôcher (1867–1918), who was the Society's second Colloquium Lecturer in 1896 and who served as AMS president during 1909–1910. Bôcher was also one of the founding editors of *Transactions of the AMS*. The original endowment was contributed by members of the Society. The prize is awarded for a notable paper in analysis published during the preceding six years. To be eligible, the author should be a member of the AMS or the paper should have been published in a recognized North American journal. The prize is given every three years and carries a cash award of US$5,000.

The Bôcher Prize is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2008 prize the members of the selection committee were: Peter S. Constantin, Tai-Ping Liu (chair), and Elias M. Stein.

Previous recipients of the Bôcher Prize are: G. D. Birkhoff (1923), E. T. Bell (1924), Solomon Lefschetz (1924), J. W. Alexander (1928), Marston Morse (1933), Norbert Wiener (1933), John von Neumann (1938), Jesse Douglas (1943), A. C. Schaeffer and D. C. Spencer (1948), Norman Levinson (1953), Louis Nirenberg (1959), Paul J. Cohen (1964), I. M. Singer (1969), Donald S. Ornstein (1974), Alberto P. Calderón (1979), Luis A. Caffarelli (1984), Richard B. Melrose (1984), Richard M. Schoen (1989), Leon Simon (1994), Demetrios Christodoulou (1999), Sergiu Klainerman (1999), Thomas Wolff (1999), Daniel Tataru (2002), Terence Tao (2002), Fanghua Lin (2002), and Frank Merle (2005).

The 2008 Bôcher Prize was awarded to Alberto Bressan, Charles Fefferman, and Carlos Kenig. The text that follows presents, for each awardee, the selection committee's citation, a brief biographical sketch, and the awardee's response upon receiving the prize.

Alberto Bressan

Citation

Alberto Bressan of Penn State University is awarded the Bôcher Prize for his fundamental works on hyperbolic conservation laws. Professor Bressan has made important contributions to the well-posedness theory; the results have been summarized in his monograph *Hyperbolic Systems of Conservation Laws. The One-Dimensional Cauchy Problem* (Oxford Lecture Series in Mathematics and Its Applications, 20, Oxford University Press, Oxford, 2000, xii + 250 pp.). Another landmark achievement is the work on zero dissipation limit (with Stefano Bianchini), "Vanishing viscosity solutions of nonlinear hyperbolic systems", *Ann. of Math. (2)* **161** (2005), no. 1, 223–342.

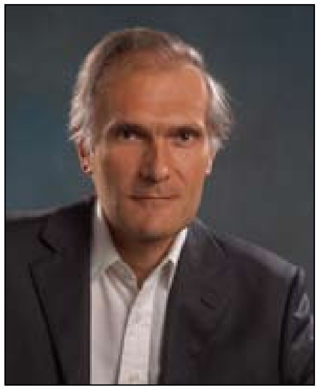
Biographical Sketch

Alberto Bressan was born in Venice, Italy. He completed his undergraduate studies at the University of Padova, Italy, and received a Ph.D. from the University of Colorado, Boulder, in 1982. He has held faculty positions at the University of Colorado and at the International School for Advanced Studies in Trieste, Italy. Presently he holds the Eberly Chair Professor of Mathematics at Pennsylvania State University. His scientific interests lie in the areas of differential inclusions, control theory, differential games, partial differential equations, and hyperbolic systems of conservation laws. He gave a plenary lecture at the International Congress of Mathematicians, Beijing, 2002. In 2006 he received the A. Feltrinelli Prize for Mathematics, Mechanics, and Applications from the Accademia Nazionale dei Lincei in Rome. Besides mathematics he enjoys playing piano and flute. He lives in State College, Pennsylvania, with his wife, Wen Shen, and two daughters, Luisa Mei and Maria Lan.

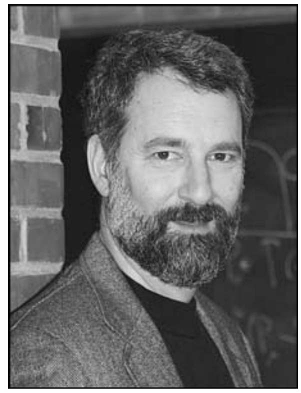
Response

It is a great honor for me to receive this prize. It was also a pleasant surprise to discover that my name is now listed among the 1,631 direct descendents of Maxime Bôcher listed in the Math Genealogy Project.

When I first became interested in hyperbolic conservation laws in the 1980s, my main training had been in other fields: parabolic equations, differential inclusions, and control theory. But as a fresh Ph.D. recipient, I was intrigued by the fact that something apparently so basic as the well-posedness of the equations for gas dynamics could have remained an open problem for so many years.



Alberto Bressan



Charles Fefferman

Carlos Kenig

The key estimates needed to establish continuous dependence of solutions were something I could figure out fairly quickly. However, it took me nearly ten years to fix details and achieve a rigorous proof in some significant cases. When I attended my first hyperbolic meeting in Stony Brook in 1994, I was still an outsider. Within the research community on hyperbolic problems I found very friendly and encouraging people. One can now say that the well-posedness for hyperbolic conservation laws in one space dimension has really been a cooperative accomplishment. In particular, the ideas contributed by Tai Ping Liu and Tong Yang have been instrumental in creating the polished theory we now have.

Understanding vanishing viscosity approximations was a second major challenge. This was achieved in 2001 in joint work with Stefano Bianchini at the International School for Advanced Studies in Trieste. Bianchini was the kind of student that you can call yourself fortunate if you find one in a lifetime. He took up my research program and contributed a new and fundamental idea: using the center manifold theorem to decompose a solution as local superposition of traveling waves. He also found the energy and determination to push his way through an incredible amount of computational details, eventually completing the proof.

In the end, all this is far beyond anything I could have hoped for when I first started reading about conservation laws and the Glimm scheme in Joel Smoller's book. I am delighted to receive this prize, and I thank the American Mathematical Society for the award.

Charles Fefferman

Citation

Charles Fefferman of Princeton University is awarded the Bôcher Prize for his many fundamental contributions to different areas of analysis, including his recent work on the Whitney extension problem. His important work in this area is contained in his papers "A sharp form of Whitney's extension theorem", *Annals of Math.* **161** (2005), 509–577, and "Whitney's extension problem for C^m", *Annals of Math.* **164** (2006), 313–359.

Biographical Sketch

Charles Fefferman was born in Washington, D.C., in 1949. He received his B.S. at the University of Maryland in 1966 and his Ph.D. at Princeton in 1969 under E. M. Stein. He taught at Princeton from 1969 to 1970, at the University of Chicago from 1970 to 1974, and again at Princeton since 1974. Fefferman has worked in classical Fourier analysis, partial differential equations, several complex variables, conformal geometry, quantum mechanics, fluid mechanics, and computational geometry. His honors include the Salem Prize, the Waterman Award, the Fields Medal, the Bergman Prize, and several honorary doctorates. He has served as chairman of the Princeton mathematics department and currently chairs the board of trustees of the Mathematical Sciences Research Institute in Berkeley. He is a member of the American Academy of Arts and Sciences, the National Academy of Sciences, and the American Philosophical Society.

Response

I am grateful for my selection for the Bôcher Prize and for the recognition of my work on Whitney's problem. That question and its close relatives have fascinated me for years. In solving them, I've had crucial help in the form of beautiful, highly original ideas due to several people. Let me mention especially G. Glaeser, who invented a key geometric construction; E. Bierstone, P. Milman, and W. Pawłucki, who discovered a general form of Glaeser's construction; and Y. Brudnyi and P. Shvartsman, who conjectured a basic finiteness principle and proved it in the first hard case.

It has been a joy to collaborate with Bo'az Klartag on the effective finite version of Whitney's problem, which I hope will one day connect to applied problems. Bo'az's brilliant ideas (he insists they are obvious) have gotten us out of many an impasse.

Most of all, I am grateful that I can share the pleasure of this occasion with my wife, Julie.

Carlos Kenig

Citation

Carlos Kenig of the University of Chicago is awarded the Bôcher Prize for his important contributions to harmonic analysis, partial differential equations, and in particular to nonlinear dispersive PDE. Kenig's work has been influential in the analysis of well-posedness under minimal regularity assumptions for physical equations. Examples of this work include his seminal paper with G. Ponce and L. Vega, "Well-posedness and scattering results for generalized Korteweg-de Vries equations via the contraction principle", *Comm. Pure Appl. Math.* **46** (1993), 527–620; his remarkable work with A. Ionescu, "Global well-posedness of the Benjamin-Ono equation in low regularity spaces", *J. Amer. Math. Soc.* **20** (2007), 3, 753–798; and his outstanding work with F. Merle, "Global well-posedness, scattering and blow-up for the energy critical focusing nonlinear wave equation", to appear, *Acta Math.*

Biographical Sketch

Carlos E. Kenig was born on November 25, 1953, in Buenos Aires, Argentina, where he received his early education. He obtained his Ph.D. at the University of Chicago in 1978 under the direction of Alberto Calderón. From 1978 to 1980 he was an instructor at Princeton University, after which he held positions at the University of Minnesota, becoming professor in 1983. In 1985 he returned to the University of Chicago, where he now is the Louis Block Distinguished Service Professor.

Kenig has been a recipient of Sloan and Guggenheim Fellowships. In 1984 he was awarded the Salem Prize. He was an invited speaker at the International Congress of Mathematicians in Berkeley (1986) and in Beijing (2002). Since 2002 he has been a fellow of the American Academy of Arts and Sciences.

Kenig's current research interests include boundary value problems under minimal regularity conditions, degenerate diffusions, free boundary problems, inverse problems, and nonlinear dispersive equations.

Response

It is a great honor to be a corecipient of this year's Bôcher Memorial Prize. I am grateful to the American Mathematical Society and to the selection committee for their recognition of my research. I would like to thank my family—my wife, Sarah, and my daughters, Lucy and Anna—for their love and support throughout the years. I would also like to thank my teachers, my many collaborators, and my students, all of whom have shared many insights with me. I am especially indebted to my long-time collaborators Gustavo Ponce and Luis Vega for more than twenty years (and still counting) of joint work, friendship, and shared fun.

There are many people who have influenced my mathematical career to whom I owe thanks, beginning with Alberto Calderón, my advisor, and Antoni Zygmund (both now deceased), who introduced me as a graduate student to the Calderón-Zygmund school of analysis. Eli Stein was my postdoctoral mentor, and I have greatly profited from many mathematical discussions with him and from his continued support and encouragement. The late Gene Fabes introduced me to research in partial differential equations; he was my mentor, collaborator, and dear friend. I continue to miss him. I am also particularly indebted to David Jerison and to the late Björn Dahlberg for their influence on me early on in my career. The three papers cited by the selection committee are joint works. I am very thankful to Gustavo Ponce, Luis Vega, Alex Ionescu, and Frank Merle, my coauthors in the cited papers, for their fundamental contributions to these joint works, without which these projects could not have been carried out. Finally, I would like to thank the University of Chicago, my home institution for more than twenty years, for providing me with the excellent working conditions in which my research is carried out.

The use of harmonic analysis techniques in the study of nonlinear dispersive equations was pioneered in works of I. Sigal, R. Strichartz, J. Ginibre-G. Velo, and T. Kato. In the late 1980s in joint work with Ponce and Vega, we introduced the use of the machinery of modern harmonic analysis for the study of nonlinear dispersive equations with derivatives in the nonlinearity. We showed for the first time that the initial value problem for the generalized Korteweg-de Vries equation with data in Sobolev spaces can be solved by the contraction mapping principle. In doing so, we obtained results that (for many powers in the nonlinearity) turned out to give the minimal regularity assumptions on the data for which this can be done. This was not the case with our first results for the quadratic nonlinearity in the KdV equation. Here, fundamental work of J. Bourgain (1993) expanded the functional framework for the use of the contraction mapping principle in this setting. This eventually led, in joint work with Ponce and Vega (1996), to the minimal regularity result for this case too. The resulting body of techniques (with refinements and extensions by many authors) has proved extremely powerful in many problems and settings and has attracted the attention of a large community of researchers.

In recent years I have been interested in some natural equations for which there is an exact balance between the smoothing properties of the linear part and the strength of the nonlinearity, which precludes the direct application of the techniques described before. The Benjamin-Ono equation is one such model. For this equation, examples of Molinet-Saut-Tzvetkov (2001) show that it is not possible to use the contraction mapping principle on any Sobolev space. After an important contribution by Tao (2004), who introduced a gauge transform into the problem (with a further extension by Burq-Planchon (2005) simultaneous to our work), Ionescu and I were able to obtain the conjectured global well-posedness for data of finite mass. This was achieved by combining the gauge transform of Tao with some new function spaces inspired by earlier work of Tataru in the wave map problem. These new functional structures have since proved useful for Schrödinger maps in joint works with Ionescu and with Bejenaru and Ionescu.

Lately there has been considerable interest in the study (for nonlinear dispersive and wave equations) of the long-time behavior of solutions. Issues like blow-up, global existence, and scattering have come to the forefront, especially in critical problems. The case of the energy critical, defocusing nonlinear wave equation was studied in pioneering works of many researchers in the 1980s and 1990s. (For instance M. Struwe (radial case), M. Grillakis (general case), J. Shatah-M. Struwe, H. Bahouri-J. Shatah, H. Bahouri-P. Gerard, and others.) These works show that for general data in the energy space we have global existence and scattering. Corresponding results for the energy critical, defocusing nonlinear Schrödinger equations were obtained in groundbreaking works of Bourgain (radial case, 1998), Colliander-Keel-Staffilani-Takaoka-Tao (general three-dimensional case), with higher-dimensional extensions due to Ryckman-Visan and to Visan (2005). For the corresponding focusing problems, say in the case of the wave equation, H. Levine (1974) had shown that blow-up in finite time can occur. Moreover, there is a stationary solution W (which solves the corresponding elliptic problem and plays an important role in the Yamabe problem). For this solution, scattering obviously does not occur. In a series of joint works with Merle, partly inspired by the elliptic case and also by works of Merle and Martel-Merle in mass critical problems, we have developed an approach to critical dispersive problems that applies to defocusing and for the first time also to focusing problems. The approach goes through a concentrated compactness procedure that reduces matters to a rigidity theorem. For instance, for the case of the energy critical focusing nonlinear wave equation, we show that the energy of W is a threshold. For data of energy smaller than that of W, if the critical Sobolev norm is smaller than the one of W, we have global existence and scattering; while if it is bigger, there is finite time blow-up.

There are many natural directions for future research in the areas just described. I look forward to continued research in them. I thank the selection committee once more for honoring these lines of research.

2008 Doob Prize

The 2008 Joseph Doob Prize was awarded at the 114th Annual Meeting of the AMS in San Diego in January 2008.

This prize was established by the AMS in 2003 and endowed in 2005 by Paul and Virginia Halmos in honor of Joseph L. Doob (1910–2004). Paul Halmos (1916–2006) was Doob's first Ph.D student. Doob received his Ph.D. from Harvard in 1932 and three years later joined the faculty at the University of Illinois, where he remained until his retirement in 1978. He worked in probability theory and measure theory, served as AMS president in 1963–1964, and received the Steele Prize in 1984. The Doob Prize recognizes a single, relatively recent, outstanding research book that makes a seminal contribution to the research literature, reflects the highest standards of research exposition, and promises to have a deep and long-term impact in its area. The book must have been published within the six calendar years preceding the year in which it is nominated. Books may be nominated by members of the Society, by members of the selection committee, by members of AMS editorial committees, or by publishers. The prize of US$5,000 is given every three years.

The Doob Prize is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2008 prize, the members of the selection committee were Andrew Granville, Robin Hartshorne, Steven G. Krantz, Dale P. Rolfsen (chair), and Bhama Srinivasan.

The previous recipient of the Doob Prize is William P. Thurston.

The 2008 Doob Prize was awarded to Enrico Bombieri and Walter Gubler. The text that follows presents the selection committee's citation and, for each awardee, a brief biographical sketch and the awardee's response upon receiving the prize.

Citation

Heights in Diophantine Geometry, by Enrico Bombieri and Walter Gubler (Cambridge University Press, 2006).

The book is a research monograph on all aspects of Diophantine geometry, both from the perspective of arithmetic geometry and of transcendental number theory. The key emphasis is on the (delicate) theory of heights, which is developed with extraordinary precision and elegance.

The choice of subjects is broad and gives the sense of a powerful body of ideas. The great results of arithmetic geometry, the theorems of Mordell-Weil, Roth, Siegel, and Faltings, are all proved with a consistent, remarkably accessible point of view. The book also develops the extraordinary work of Zhang and others on the Bogomolov conjecture, puts forward an elegant approach to Hilbert irreducibility, and includes a detailed discussion of the Nevanlinna-Vojta theory. There is a lovely exposition of the important theory of unit equations and a most brilliant discussion of the Subspace Theorem of Schmidt and Schlickewei, as well as the possibilities afforded by the abc-conjecture and further developments along these lines.

Enrico Bombieri

Walter Gubler

The book is self-contained, yet surprisingly accessible given the depth of the material. Links between classical Diophantine arithmetic and modern arithmetic geometry are emphasized throughout the text in an appealing way. There are well-constructed appendices on key technical issues such as basic algebraic geometry, algebraic ramification theory, and the geometry of numbers (a subject which is going through a revival at the moment).

One gets the sense that every lemma, every theorem, every remark has been carefully considered, and every proof has been thought through in every detail. There are well-chosen illuminating examples throughout every chapter. The book is a masterpiece in terms of its original approach, its unrivaled comprehensiveness, and the sheer elegance of the exposition. There can be no doubt that this book will become the basis for the future development of this central subject of modern mathematics.

Biographical Sketch: Enrico Bombieri

Enrico Bombieri was born in Milan, Italy, in 1940. He started studying mathematics, and in particular number theory, at an early age with Giovanni Ricci. He graduated from the University of Milan in 1963 and became assistant professor there immediately after. He spent the next year in Cambridge, England, working with Davenport and Swinnerton-Dyer, studying geometry over finite fields and the distribution of prime numbers. He became a full professor in 1965, with his first appointment at the University of Cagliari and in 1966 at the University of Pisa. In 1975 he moved to the Scuola Normale Superiore in Pisa and in 1977 joined the School of Mathematics of the Institute for Advanced Study in Princeton as a full professor. He became a U.S. citizen in 1994.

He was elected a member of the U.S. National Academy of Sciences in 1965 and of the Accademia Nazionale dei Lincei, Italy, in 1976; fellow of the American Academy of Arts and Sciences in 1979; foreign member of the Institut de France, Académie des Sciences in 1984; foreign member of the Royal Swedish Academy of Sciences in 1982; honorary member of the London Mathematical Society in 1977; Chevalier de l'Ordre des Palmes Académiques, France, in 1993; Doctor Honoris Causa, University of Pisa, in 2001; and Cavaliere di Gran Croce al Merito della Repubblica, Italy, in 2002. He received the Fields Medal at the International Congress of Mathematicians in 1974 in Vancouver, the Feltrinelli Prize in 1976, and the Balzan Prize in 1980. His first studies in number theory were with Giovanni Ricci and Davenport and in algebraic geometry with Swinnerton-Dyer and Aldo Andreotti. During his tenure in Pisa he was initiated into the theory of partial differential equations and minimal surfaces by Guido Stampacchia and Ennio De Giorgi.

His main interests in number theory are prime number theory, zeta functions, Diophantine geometry, and Diophantine approximation; in analysis, complex function theory in one and several variables, minimal surfaces, and geometric measure theory; in algebraic geometry, geometry over finite fields, arithmetic geometry, and classification problems.

He is the author of two short monographs, a comprehensive monograph (with Walter Gubler) on the theory of heights in Diophantine geometry, and over 160 research papers published in leading mathematical journals. After mathematics his main activities are painting and drawing.

Response: Enrico Bombieri

It is indeed a great surprise for me, and certainly a great honor, to receive the Doob Prize for my book with Walter Gubler on the theory of heights in Diophantine geometry. The origin of this book goes back to 1992 after I found a simplification of Vojta's landmark new proof of the Mordell conjecture. I had been invited to give a series of lectures to graduate students and young researchers in Pisa, and I thought it appropriate to give a short course on Diophantine geometry, culminating with the proof of the Mordell conjecture. This course was well received, so when a little later I was asked by Wüstholz to give a *Nachdiplom* course to students at the ETH in Zürich, we quickly agreed that the same topic would be fine. There was a little condition, namely, to develop all the material from scratch. Walter Gubler, who was then just finishing his Ph.D. thesis with Professor Wüstholz, was given the job of taking notes in the best old-fashioned European style.

To my great surprise, Walter's notes were absolutely superb: well organized, clearly written, amplified in places, and correcting the inaccuracies and mistakes I had made during my lectures. They formed an excellent basis for an introductory course, so it was decided to expand them to book form. Walter collaborated enthusiastically in the writing, and after a short while when the rough notes expanded well beyond the initial text in order to include more and more foundational material as well as complements to the main theory, he became a coauthor. The unifying theme would be the theory of heights and its application to Diophantine geometry on commutative groups.

Without Walter, this book could not have been written.

It was a long task to write up and organize the material, and in the meantime the subject itself kept growing and we had to play a catch-up game. So it took almost twelve years to write and revise the book. It was not the first one on the subject, and there were already several other excellent monographs where one could learn the subject. So why one more book? For me, writing this book was like preparing carefully a series of lectures to bright students, and I received a lot of satisfaction doing it. Now it is time for it to go out and establish its little place in the mathematical world, with the hope that it will be well received and prove itself to be useful to young mathematicians entering the beautiful subject of Diophantine geometry and arithmetic geometry.

Biographical Sketch: Walter Gubler

Walter Gubler was born October 30, 1965, in Olten, Switzerland. He received his diploma in mathematics at the Eidgenössisches Technische Hochschule Zürich in 1989. At the same place, he earned his Ph.D. in 1992 under Gisbert Wüstholz. For his thesis, *Heights of subvarieties*, he won the silver medal of the ETH. From 1992 to 1993 he visited the Institute for Advanced Study in Princeton. Then he held postdoc positions at the ETH Zürich and at the Humboldt University in Berlin. In 2003 Walter Gubler received the *venia legendi* at the ETH for his habilitation thesis. From 2003 to 2007 he was a lecturer at the University of Dortmund. Currently he is BMS substitute professor at the Humboldt University in Berlin.

Response: Walter Gubler

It is an honour for me to receive the Doob Prize 2008 together with my coauthor, Enrico Bombieri. Our book project started with a lecture by Enrico at the ETH Zürich. I had not anticipated that we would have to invest more than ten years of hard work to finish this book. On the one hand, new results came from research, and on the other hand, a lot of efforts were necessary to make the book self-contained. From my point of view, the time was well invested, as I learned so much about the subject and it was great fun to work with Enrico. I wish to thank him for giving me the opportunity to collaborate. I am very gratified to receive this prize for all the effort. Thank you.

2008 Eisenbud Prize

The 2008 Leonard Eisenbud Prize for Mathematics and Physics was awarded at the 114th Annual Meeting of the AMS in San Diego in January 2008.

The Eisenbud Prize was established in 2006 in memory of the mathematical physicist Leonard Eisenbud (1913–2004) by his son and daughter-in-law, David and Monika Eisenbud. Leonard Eisenbud, who was a student of Eugene Wigner, was particularly known for the book *Nuclear Structure* (1958), which he coauthored with Wigner. A friend of Paul Erdős, he once threatened to write a dictionary of "English to Erdős and Erdős to English". He was one of the founders of the physics department at the State University of New York, Stony Brook, where he taught from 1957 until his retirement in 1983. His son David was president of the American Mathematical Society in 2003–2004. The Eisenbud Prize for Mathematics and Physics honors a work or group of works that brings the two fields closer together. Thus, for example, the prize might be given for a contribution to mathematics inspired by modern developments in physics or for the development of a physical theory exploiting modern mathematics in a novel way. The US$5,000 prize will be awarded every three years for a work published in the preceding six years. This is the first time the prize has been awarded.

The Eisenbud Prize is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2008 prize, the members of the selection committee were Joel L. Lebowitz, David R. Morrison (chair), and Edward Witten.

The 2008 Eisenbud Prize was awarded to Hirosi Ooguri, Andrew Strominger, and Cumrun Vafa. The text that follows presents the selection committee's citation, brief biographical sketches of the awardees, and their responses upon receiving the prize.

Citation

The Eisenbud Prize for 2008 is awarded to Hirosi Ooguri, Andrew Strominger, and Cumrun Vafa for their paper "Black hole attractors and the topological string" (*Physical Review D (3)* **70** (2004), 106007). This paper contains a beautiful and highly unexpected proposal: that the counting of black hole states, in certain string theories obtained by compactification on a Calabi-Yau manifold X, can be expressed in terms of the topological string partition function of X (i.e., in terms of the so-called Gromov-Witten invariants of X). The proposal explains some mysterious earlier results to the effect that certain scattering amplitudes in physical string theory can be expressed in terms of the topological string; the authors here argue that these amplitudes control the counting of microscopic states of certain electrically and magnetically charged black holes. Black holes and enumerative invariants such as Gromov-Witten invariants are both intensively studied but had not been significantly related to each other prior to this work.

Biographical Sketch: Hirosi Ooguri

Hirosi Ooguri was born on March 13, 1962, in Japan. He attended Gifu High School, whose notable alumni include Teiji Takagi, who developed class field theory. Ooguri received a B.A. in 1984 and an M.S. in 1986 from Kyoto University.

In 1986 Ooguri became an assistant professor at the University of Tokyo. After a year at the Institute for Advanced Study in Princeton, he moved to the University of Chicago as an assistant professor in 1989. In the same year he was awarded an Sc.D. from the University of Tokyo. A year later he returned to Japan as an associate professor at the Research Institute for Mathematical Sciences in Kyoto University. In 1994 he became a professor at the University of California at Berkeley and was appointed a faculty senior scientist at the Lawrence Berkeley National Laboratory in 1996. Since 2000 he has been at Caltech, where he is now Fred Kavli Professor of Theoretical Physics.

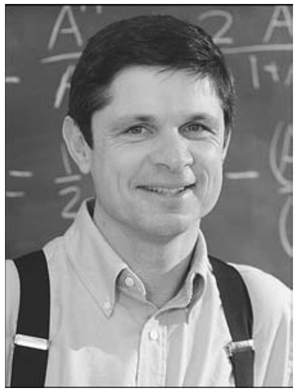
In 2007 Ooguri and his friends in Japan proposed establishing the Institute for the Physics and Mathematics of the Universe at the University of Tokyo. The proposal was approved with funding for two hundred staff scientists and visitors for the next ten years. Ooguri will continue to keep his intellectual base at Caltech, but he will spend a few months a year in Tokyo as a principal investigator at the new institute to lead activities at the interface of mathematics and physics.

Response: Hirosi Ooguri

I am deeply honored to share the Leonard Eisenbud Prize for Mathematics and Physics with such outstanding physicists as Andy Strominger and Cumrun Vafa.

In an early stage of my career I had the good fortune to work with Tohru Eguchi in Tokyo and to experience the power of quantum field theory in revealing new connections between different areas of mathematics. I have collaborated with Cumrun Vafa for over eighteen years on various aspects of gauge theory and string theory, including $N = 2$

Hirosi Ooguri



Andrew Strominger

Cumrun Vafa

string theory, topological string theory, gauge theories on D-branes, and their geometric engineering. Our collaborations have almost always aimed to discover hidden geometric structures in physical problems and to exploit them to develop new theoretical tools. Cumrun brims over with ideas that he has generously shared with me and many others. I thank him for the collaboration and friendship. I have always admired Andy Strominger for his creative insights, and I am happy to have had the chance to collaborate with both Andy and Cumrun in the academic year of 2003–2004, which led to the paper cited above. In this work we formulated a conjecture that relates two different concepts: topological string theory, which computes the Gromov-Witten invariants, and the counting of quantum states of black holes, which has to do with topological invariants of gauge theories in various dimensions. I would like to make a brief comment on each of them.

Topological string theory was introduced by Edward Witten. The construction of mirror pairs of Calabi-Yau manifolds by Brian Greene and Ronen Plesser and their application to the computation of the genus-zero Gromov-Witten invariants by Philip Candelas, Xenia De La Ossa, Paul Green, and Linda Parkes sparked interest in the mathematics community. I spent the academic year of 1991–1992 at Harvard University and collaborated with Michael Bershadsky, Sergio Cecotti, and Cumrun Vafa to generalize their results to higher genus. We found that the higher genus topological string partition functions can be used to compute certain scattering amplitudes in superstring theory compactified on a Calabi-Yau manifold. It took another twelve years to find the compelling question in physics, i.e., the counting of quantum states of black holes, to which these amplitudes give an answer. We also derived the holomorphic anomaly equations for the topological string partition functions and developed a method to solve them recursively in the genus. In this work we made several mathematical conjectures. Recently, the conjecture on the genus-one Gromov-Witten invariants for a quintic threefold was proven by Aleksey Zinger, and the conjecture on the so-called BCOV torsion for the mirror of the quintic was proven by Hao Fang, Zhiqin Lu, and Ken-ichi Yoshikawa. The conjectures for genus greater than one remain open.

The black hole entropy formula was proposed by Jacob Bekenstein and Stephen Hawking based on a remarkable mathematical analogy between thermodynamics and black hole mechanics and on the semiclassical theory of black hole radiance. It was expected that if there is a theory that successfully unifies quantum mechanics and general relativity, in such a theory the Bekenstein-Hawking formula can be derived as the statistical entropy of quantum states of black holes. Thanks to the D-brane construction by Joseph Polchinski for a certain class of black holes in string theory, it has become possible to count quantum states by evaluating topological invariants of gauge theory on D-branes, such as the Euler characteristic of instanton moduli space. The counting was carried out by Strominger and Vafa in 1996, and they found a perfect agreement with the Bekenstein-Hawking formula in the limit of large black holes, for which the approximation used by Bekenstein and Hawking becomes precise. Our paper cited above showed that this approximation can be significantly improved by using topological string theory. I was surprised and delighted to find the application of topological string theory to the counting of quantum states of black holes. This reaffirmed my belief that exact results in quantum field theory and string theory have enduring value and unintended applications.

When I was a high school student, physics was my least favorite subject until I learned calculus. Clearly, physicists need mathematics to formulate fundamental laws of nature. In return, physicists' search for fundamental laws has inspired many important developments in mathematics. In the past couple of decades interactions of mathematicians and physicists have been particularly intense and productive in the area involving quantum field theory and string theory. Since neither of them has a proper definition, mathematicians often view them as black boxes from which interesting conjectures materialize. I think that collaborations of mathematicians and physicists can be elevated to an even higher level if these physical theories are placed on more solid mathematical foundations.

I would like to thank Andy Strominger and Cumrun Vafa for the wonderful collaboration. Topological string theory has been developed by many people. In particular, I would like to acknowledge the influence of the earlier work by Gabriel Lopes Cardoso, Bernard de Wit, and Thomas Mohaupt.

I would like to thank the American Mathematical Society and the Eisenbud Prize Committee for recognizing the progress in this line of research. I am grateful to my teachers, collaborators, and friends for helping me make contributions to this area. Finally, I would like to thank my wife, Kyoko, for her love and support and my daughter, Tomoko, for adding extra dimensions to my life.

Biographical Sketch: Andrew Strominger

Andrew Strominger, the son of biochemist Jack Strominger, is an American theoretical physicist whose research centers around string theory. He is currently a professor at Harvard University, cofounder of the Center for the Fundamental Laws of Nature at Harvard, and a senior fellow at the Society of Fellows. He received his undergraduate degree from Harvard University in 1977 and his Ph.D. from the Massachusetts Institute of Technology in 1982 under the supervision of Roman Jackiw. His wide and varied contributions to physics include:

- a paper with Cumrun Vafa that explains the microscopic origin of the black hole entropy, originally calculated thermodynamically by Stephen Hawking and Jacob Bekenstein from string theory;
- a paper with Philip Candelas, Gary Horowitz, and Edward Witten about the relevance of Calabi-Yau manifolds for obtaining the Standard Model from string theory;
- other articles discussing the dS/CFT correspondence (a variation of AdS/CFT correspondence), S-branes (a variation of D-branes), and OM-theory (with Shiraz Minwalla and Nathan Seiberg);
- research on massless black holes in the form of wrapped D3-branes that regulate the physics of a conifold and allow topology change interpretation of mirror symmetry as a special case of T-duality (with Eric Zaslow and Shing-Tung Yau).

The fundamental laws of nature as we currently understand them are both incomplete and contradictory. Unsolved problems concerning these laws include the incompatibility of quantum mechanics and Einstein's theory of gravity, the origin of the universe, and the origin of the masses of the elementary particles. Strominger's research has concerned various aspects of these problems. The emergence of string theory as the most promising approach to these problems began with Strominger's 1985 codiscovery of so-called Calabi-Yau compactifications. This construction demonstrated that string theory not only reconciles quantum mechanics and gravity but can also contain within it electrons, protons, photons, and all the other observed particles and forces and hence is a viable candidate for a complete unified theory of nature. In 1991 Strominger codiscovered the brane solutions of string theory, which have played a crucial role in unraveling the beautiful mathematical structure and duality symmetries of the theory. The branes were eventually used by Strominger and collaborators to give a microscopic explanation of how black holes are able to store information, finally resolving a deep paradox uncovered by Hawking and Bekenstein a quarter century earlier. He and coworkers also used the branes to derive new relations in algebraic geometry, equating the moduli space of a brane in a Calabi-Yau space to the mirror Calabi-Yau. Preliminary attempts have been made to apply these insights to cosmology. Current research continues attempts to better understand the fundamental laws of nature.

Response: Andrew Strominger

I am greatly honored to receive, along with my collaborators Cumrun Vafa and Hirosi Ooguri, the first Leonard Eisenbud Prize of the American Mathematical Society for our work demonstrating a connection between Gromov-Witten invariants and microstate degeneracies of black hole attractors. Our success in discovering this connection relied on the uncanny ability of physical reasoning to lead to insights into pure mathematics.

Biographical Sketch: Cumrun Vafa

Cumrun Vafa is a Donner Professor of Science at Harvard University, where he teaches and does research on theoretical physics.

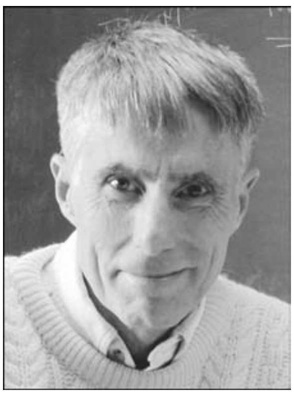
Vafa was born in Tehran, Iran, in 1960 and came to the U.S. for continuation of his education in 1977. He earned his B.S. in mathematics and physics from the Massachusetts Institute of Technology in 1981. He went on to earn his Ph.D. in physics from Princeton University in 1985 under the supervision of Edward Witten. He came to Harvard University in 1985 as a junior fellow of the Harvard Society of Fellows and has been on the Harvard faculty since 1988. He is married to Afarin Sadr, and they are the proud parents of three sons: Farzan, Keyon, and Neekon.

Response: Cumrun Vafa

It is a great pleasure to receive the 2008 Leonard Eisenbud Prize, together with my collaborators. I view this not only as an acknowledgment of a single paper but also as an appreciation of the work of so many physicists and mathematicians that led to this work. With the intrinsic beauty of the connection between mathematics and physics and with so many talented researchers, I hope to witness the continuing development of this remarkable area of science.

I am greatly indebted for the support I have received from my family and my parents, as well as my teachers over the years.

2008 Award for Distinguished Public Service



Herbert Clemens

The 2008 Award for Distinguished Public Service was presented at the 114th Annual Meeting of the AMS in San Diego in January 2008.

The Award for Distinguished Public Service is presented every two years to a research mathematician who has made a distinguished contribution to the mathematics profession during the preceding five years. The purpose of the award is to encourage and recognize those individuals who contribute their time to public service activities in support of mathematics. The award carries a cash prize of US$4,000.

The Award for Distinguished Public Service is made by the AMS Council acting on the recommendation of a selection committee. For the 2008 award the members of the selection committee were: William J. Lewis, Carolyn R. Mahoney, Paul J. Sally (chair), Richard A. Tapia, and Margaret H. Wright.

Previous recipients of the award are: Kenneth M. Hoffman (1990), Harvey B. Keynes (1992), I. M. Singer (1993), D. J. Lewis (1995), Kenneth C. Millett (1998), Paul J. Sally Jr. (2000), Margaret H. Wright (2002), Richard A. Tapia (2004), and Roger Howe (2006).

The 2008 Award for Distinguished Public Service was presented to Herbert Clemens. The text that follows presents the selection committee's citation, a brief biographical sketch, and the recipient's response upon receiving the award.

Citation

The American Mathematical Society's Distinguished Public Service Award for 2008 is awarded to Herbert Clemens for his superb research in complex algebraic geometry, for his continuing efforts in education, beginning with his days at Columbia University and his work with teachers in Chile to his teaching and collaborating with teachers in the Salt Lake City public schools and his influence in mathematics education at the national level; and, in addition, for his seminal role in the founding and continuation of the Park City/IAS Mathematics Institute.

Biographical Sketch

Herbert Clemens earned his Ph.D. in 1966 from the University of California, Berkeley, under the direction of Phillip A. Griffiths. He has taught at Columbia University, the University of Utah, and the Ohio State University, where he has been on the faculty since 2002. He has served as director of the NSF Regional Geometry Institute, Park City, UT, and chair of the Steering Committee for the IAS Park City Mathematics Institute. He was an invited speaker at the International Congress of Mathematicians in 1974 and in 1986. His academic honors include a Silver Medal from the Italian Mathematical Society and a Laurea de honoris causa from the Universita di Torino, among others. His research area is complex geometry.

Response

I feel very honored to receive the 2008 Award for Distinguished Public Service from the American Mathematical Society and regret that I am unable to be present in person to receive the award. I accept the award in the name of the hundreds of AMS members engaged in professional outreach, which, though often viewed to lie at the margins of our calling as mathematicians, is vital to the long-term sustainability of our discipline, especially as pertains to the discipline's continued support by society at large.

More particularly and currently, I accept this award as recognizing the more than forty AMS members offering their services for university lecturing in Cambodia, as recognizing an equivalent number working with African mathematicians through the International Mathematical Union, and finally as recognizing the countless AMS members currently working in cooperation with the education community to improve pre-university mathematics education in our country's schools.

Mathematics People

Awards Presented at 2007 ICCM

At each International Congress of Chinese Mathematicians (ICCM), the winners of several prestigious awards are announced during the opening ceremony. These awards include the Morningside Medal of Mathematics, the Chern Prize in Mathematics, and the ICCM International Cooperation Award. The Fourth ICCM was held in Hangzhou, China, December 17–22, 2007. At ICCM 2007, two new prizes were introduced: the New World Mathematics Awards and the S. T. Yau Mathematics Awards.

The Morningside Medal of Mathematics is awarded to exceptional mathematicians of Chinese descent under the age of forty-five for their seminal achievements in mathematics and applied mathematics. The winners of the Morningside Medal of Mathematics are traditionally announced at the ICCM. The inaugural medals were presented in 1998. Each Morningside Medalist receives a certificate and medal, as well as a cash award of US$25,000 for a gold medal or US$10,000 for a silver medal.

The 2007 Morningside Gold Medal of Mathematics is awarded to Jianqing Fan, Princeton University, and Xujia Wang, Australian National University. The 2007 Morningside Silver Medal of Mathematics is awarded to Chiu-Chu Liu, Northwestern University and Columbia University; Lizhen Ji, University of Michigan and Zhejiang University; Shi Jin, University of Wisconsin at Madison; Chiun-Chuan Chen, Taiwan University; and Ye Tian, Morningside Center of Mathematics at the Chinese Academy of Sciences.

The Chern Prize in Mathematics was established in 2001 in honor of Shing-Shen Chern, one of the greatest geometers and Chinese mathematicians of the twentieth century. The Chern Prize is presented every three years at the ICCM to mathematicians of Chinese descent who have made exceptional contributions to mathematical research or to public service activities in support of mathematics. The 2007 Chern Prize is awarded to Shiu-Yuen Cheng, Hong Kong University of Science and Technology, and Mu-Tao Wang, Columbia University.

The ICCM International Cooperation Award is presented to an individual who has promoted the development of mathematics in China, Hong Kong, and Taiwan through collaboration, teaching, and support of Chinese mathematicians. The inaugural award was presented in 2004. The 2007 ICCM International Cooperation Award is awarded to Stanley Osher, University of California at Los Angeles.

Supported by the New World Development Company Ltd., the New World Mathematics Awards recognize outstanding doctoral, master's, and undergraduate theses written by mathematicians of Chinese descent who have graduated from universities and institutes in the past three years. The purpose is to provide encouragement to talented Chinese mathematicians and to promote creativity and innovation in mathematics. Six Ph.D. Thesis Awards, five Master Thesis Awards, and ten Bachelor Thesis Awards were presented at the 2007 ICCM.

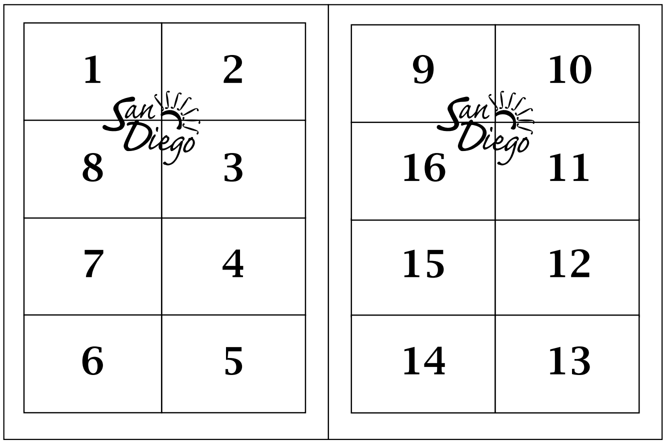
Coorganized by the International Congress of Chinese Mathematicians and the Taikang Life Insurance Company Ltd., the S. T. Yau Mathematics Awards recognize excellence in mathematics research projects among high school students of Chinese descent throughout the world. The goal is to identify gifted mathematicians at a young age and foster their interest in this field of study. A press briefing marking the establishment of the awards was held in December 2007. The first awards will be presented in October 2008 in Beijing.

—From an ICCM announcement

Deloro and Moczydlowski Awarded ASL Sacks Prize

Adrien Deloro of Rutgers University and Wojciech Moczydlowski of Cornell University have been awarded the 2007 Sacks Prizes of the Association for Symbolic Logic

2008 San Diego Joint Mathematics Meetings Photo Key



Page 476 (left), clockwise from Number 1. Page 477 (right), clockwise from Number 9:

1. Welcome!
2. David Eisenbud (right) presenting the 2008 Eisenbud Prize to Cumrun Vafa.
3. 3-D sculptures in the mathematical art exhibit.
4. In the Networking Center.
5. Message Board area in the San Diego Convention Center.
6. AMS Colloquium speaker Wendelin Werner.
7. Entrance to the Employment Center.
8. AMS-MAA Joint Invited Address speaker Terence Tao.
9. Opening ceremony ribbon cutting for the JMM Exhibits area (left to right: AMS executive director John Ewing, MAA president Joseph Gallian, AMS president James Glimm, MAA associate secretary James Tattersall, AMS associate secretary Michel Lapidus).
10. MAA Invited Address speaker Karen Parshall.
11. San Diego Convention Center.
12. Who Wants to Be a Mathematician game host Mike Breen and San Diego game winners Hansen Han (left) and Ben Wendel (right).
13. AMS Membership Booth.
14. Avi Wigderson (left) receives Conant Prize from AMS president James Glimm.
15. AMS Booth in the Exhibits area.
16. Morgan Prize winner Nathan Kaplan.

(ASL). The prize is awarded to the most outstanding doctoral dissertation or dissertations in mathematical logic.

Deloro received his Ph.D. in 2007 from the Université Paris 7. The prize citation notes that his thesis “deals with the Cherlin-Zilber conjecture, according to which every simple group of finite Morley rank is isomorphic to an algebraic group over an algebraically closed field. In particular, the thesis removes the assumption that there are no bad fields from the classification of minimal counterexamples.” Moczydlowski received his Ph.D. in 2007 from Cornell University. According to the prize citation, his thesis “contains groundbreaking results on constructive set theory and its relation to type theory” and proves weak normalization for the intuitionistic Zermelo-Fraenkel set theory with replacement rather than collection.

The Sacks Prize was established in honor of Gerald Sacks for his unique contribution to mathematical logic. It consists of a cash award and five years’ free membership in the ASL.

—From an ASL announcement

AWM Essay Contest Winners Announced

The Association for Women in Mathematics (AWM) has announced the winners of its 2007 essay contest, “Biographies of Contemporary Women in Mathematics”.

The grand prize was awarded to Leena Shah, Hartland Middle School at Ore Creek, Brighton, Michigan, for her essay “The Creation of a Female Mathematician: Ms. Melanie Wood”. Shah’s essay won first place in the Grade 6–8 category. As the grand prize winner, this essay will be published in the *AWM Newsletter*.

In the College category, first place went to Sarah Budrus, Hollins University, Huntington, West Virginia, for “Dr. Marjorie Senechal: What do Silk, Crystals, Culture, and History Have in Common?” In the Grades 9–12 category, first place went to Elizabeth Faiella, homeschooled in Northwood, New Hampshire, for “Dr. Rita Hibschweiler: Exploring the Pure Beauty of Mathematics”, while the honorable mention went to Haley Kossek, Elk Rapids High School, Williamsburg, Michigan, for “Mrs. Ann Weber: Hard Work Pays Off”. An honorable mention in the Grade 6–8 category went to Helen A. Rawlins, Brier Terrace Middle School, Bothell, Washington, for “Dr. Eve Riskin: Engineer, Professor, Role Model”.

—From an AWM announcement

Correction

Because of incorrect information supplied to the *Notices*, the list of Doctoral Degrees Conferred that appeared in the February 2008 issue contained a misspelling of the name of John Kittrell, who received a doctorate from the University of California, Los Angeles.

—Allyn Jackson

Mathematics Opportunities

DMS Workforce Program in the Mathematical Sciences

The Division of Mathematical Sciences (DMS) of the National Science Foundation (NSF) welcomes proposals for the Workforce Program in the Mathematical Sciences. The long-range goal of the program is increasing the number of well-prepared U.S. citizens, nationals, and permanent residents who successfully pursue careers in the mathematical sciences and in other NSF-supported disciplines. Of primary interest are activities centered on education that broaden participation in the mathematical sciences through research involvement for trainees at the undergraduate-through-postdoctoral educational levels. The program is particularly interested in activities that improve recruitment and retention, educational breadth, and professional development.

The submission period for unsolicited proposals is **May 15–June 15, 2008**. For more information and a list of cognizant program directors, see the website `http://www.nsf.gov/funding/pgm_summ.jsp?pims_id=503233`.

—From a DMS announcement

Project NExT: New Experiences in Teaching

Project NExT (New Experiences in Teaching) is a professional development program for new and recent Ph.D.'s in the mathematical sciences (including pure and applied mathematics, statistics, operations research, and mathematics education). It addresses all aspects of an academic career: improving the teaching and learning of mathematics, engaging in research and scholarship, and participating in professional activities. It also provides the participants with a network of peers and mentors as they assume these responsibilities. In 2008 about seventy faculty members from colleges and universities throughout the country will be selected to participate in a workshop preceding the Mathematical Association of America (MAA) summer meeting, in activities during the summer MAA meetings and the Joint Mathematics Meetings in January, and in an electronic discussion network. Faculty for whom the 2008–2009 academic year will be the first or second year of full-time teaching (post-Ph.D.) at the college or university level are invited to apply to become Project NExT Fellows.

The application deadline is **April 18, 2008**. For more information see the Project NExT website, `http://archives.math.utk.edu/projnext/`. Project NExT is a program of the MAA. It receives major funding from the ExxonMobil Foundation, with additional funding from the Dolciani-Halloran Foundation, the American Mathematical Society, the Educational Advancement Foundation, the American Institute of Mathematics, the American Statistical Association, the National Council of Teachers of Mathematics, Texas Instruments, the Association of Mathematics Teacher Educators, the Association for Symbolic Logic, the W. H. Freeman Publishing Company, Maplesoft, John Wiley & Sons, MAA Sections, and the Greater MAA Fund.

—From a Project NExT announcement

AP Calculus Readers Sought

The Educational Testing Service and the College Board invite interested college faculty to apply to be readers for the Advanced Placement Calculus Exam. Each June, high school AP teachers and college faculty members from around the world gather in the United States for the annual AP Reading. There they evaluate and score the free-response sections of the AP Exams. AP Exam readers are led by a chief reader, a college professor who has the responsibility of ensuring that students receive grades that accurately reflect college-level achievement. Readers find the experience an intensive collegial exchange in which they can receive professional support and training.

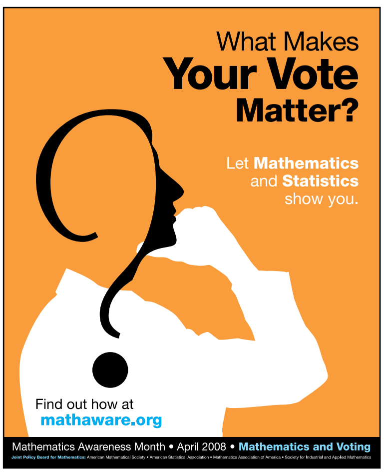
To learn more about this opportunity or to apply for a position as a reader, see the website `http://apcentral.collegeboard.com/apc/public/homepage/4137.html`; email: `apreader@ets.org`; telephone: 609-406-5384.

—Michael Boardman, Pacific University

Inside the AMS

From the AMS Public Awareness Office

• **Mathematics Awareness Month, April 2008.** Mathematics Awareness Month, held each year in April, is sponsored by the Joint Policy Board for Mathematics—a collaborative effort of the AMS, the American Statistical Association (ASA), the Mathematical Association of America (MAA), and the Society for Industrial and Applied Mathematics (SIAM)—to increase public understanding of and appreciation for mathematics. This year's theme, *Mathematics and Voting*, has been developed and publicized by the American Statistical Association, and the AMS Public Awareness Office served in an advisory role to support this outreach program. Activities for Mathematics Awareness Month have been organized on local, state, and regional levels by college and university departments, institutional public information offices, student groups, and related associations and interest groups. Download the theme poster, read essays, and view activities at `http://www.mathaware.org/mam/08/`.



• **This Mathematical Month: Monthly postings of vignettes on people, publications, and mathematics to inform and entertain.** April: Oscar Zariski was born, world-famous mathematician and Fields Medalist Enrico Bombieri played an April Fool's Day joke, Robert P. Langlands of the Institute for Advanced Study in Princeton received the first NAS Award in Mathematics from the National Academy of Sciences, and more. Read about these anecdotes and more events in April and link to vignettes from other months at `http://www.ams.org/ams/thismathmonth-apr.html`.

• **Headlines & Deadlines for Students.** Students and advisors subscribe to this service, which provides email notification of news and upcoming deadlines. Recent news includes the winner of the AMS-MAA-SIAM Frank and Brennie Morgan Prize for Outstanding Research in Mathematics by an Undergraduate Student, Nathan Kaplan (Princeton University); and spring deadlines for stipends and fellowships. The postings and sign-up pages are at `http://www.ams.org/news-for-students/`.

• **Reviews.** The Reviews webpage, part of the monthly *Math in the Media* online magazine, contains links to reviews of books, plays, films, and television shows that are related to mathematics (but are not aimed solely at the professional mathematician). The Reviews page is updated on an ongoing basis. Links are given to reviews posted on the Web, and each citation includes review title, reviewer, and source. In some cases the source requires a subscription, a password, or a search of the archives to access the review; in other cases the review may be available only in print directly from the source. The Reviews page is a great resource for general readers, math clubs, librarians, and mathematicians who want to keep abreast of what's new in mathematical fiction, biographies, histories, movies, plays, and television. Bookmark `http://www.ams.org/ams/reviews.html`.

• **Feature Column.** Recent columns include "Pulling Digits out of Pi", by David Austin; and "Urban Geometry", by Joe Malkevitch, at `http://www.ams.org/featurecolumn/`.

—Annette Emerson and Mike Breen
AMS Public Awareness Officers
`paoffice@ams.org`

Deaths of AMS Members

Eleazer Bromberg, from North Wales, PA, died on March 28, 2006. Born on October 7, 1913, he was a member of the Society for 57 years.

Ralph Byers, University of Kansas, Lawrence, died on December 15, 2007. He was a member of the Society for 30 years.

Ralph E. DeMarr, professor emeritus, University of New Mexico, died in October 2007. Born on January 17, 1930, he was a member of the Society for 47 years.

Karl-Heinz Diener, professor, University of Cologne, died on September 18, 2007. Born on January 8, 1930, he was a member of the Society for 42 years.

Henry R. Dowson, professor, University of Glasgow, Scotland, died on January 28, 2008. Born on March 2, 1939, he was a member of the Society for 41 years.

Roy Dubisch, professor emeritus, University of Washington, Seattle, died on January 20, 2008. Born on February 5, 1917, he was a member of the Society for 60 years.

Eugene Georg D'yakonov, professor, Moscow State University, died on November 8, 2006. Born on July 2, 1935, he was a member of the Society for 13 years.

Richard E. Ewing, professor, Texas A&M University, College Station, died on December 5, 2007. Born on November 24, 1946, he was a member of the Society for 33 years.

Sr. Claude Marie Faust, from San Antonio, TX, died on July 3, 2007. Born on November 18, 1917, she was a member of the Society for 45 years.

Richard F. Gabriel, professor emeritus, Seton Hall University, died on October 2, 2007. Born on November 30, 1920, he was a member of the Society for 58 years.

Robert S. Johnson, retired, from Cinnaminson, NJ, died on January 1, 2008. Born in November 1928, he was a member of the Society for 52 years.

Sam Karlin, professor, Stanford University, died on December 18, 2007. Born on June 8, 1924, he was a member of the Society for 21 years.

Herbert B. Keller, professor, California Institute of Technology, died on January 26, 2008. Born on June 19, 1925, he was a member of the Society for 47 years.

Henry E. Kyburg Jr., professor, University of Rochester, died on October 30, 2007. Born on October 9, 1928, he was a member of the Society for 48 years.

Jean E. Lebel, retired, University of Toronto, died on December 14, 2007. Born on March 21, 1922, he was a member of the Society for 56 years.

William J. LeVeque, retired AMS executive director, died on December 1, 2007. Born on August 9, 1923, he was a member of the Society for 63 years.

Stuart P. Lloyd, retired, Bell Telephone Laboratories, Murray Hill, died on October 20, 2007. Born on March 23, 1923, he was a member of the Society for 51 years.

Ralph Mansfield, retired, from Santa Rosa, CA, died on December 17, 2007. Born on August 21, 1912, he was a member of the Society for 69 years.

Robert B. Reisel, professor emeritus, Loyola University of Chicago, died on November 16, 2007. Born on April 27, 1925, he was a member of the Society for 56 years.

Alex Rosenberg, professor emeritus, University of California Santa Barbara, died on October 27, 2007. Born on December 5, 1926, he was a member of the Society for 57 years.

Robert J. Rubin, retired from the National Institute of Standards and Technology, died on January 18, 2008. Born on August 17, 1926, he was a member of the Society for 23 years.

Richard C. Sacksteder, professor emeritus, City University of New York, died on November 4, 2007. Born on February 11, 1928, he was a member of the Society for 50 years.

William K. Smith, retired, from Northumberland, PA, died on October 9, 2007. Born on March 8, 1920, he was a member of the Society for 58 years.

Masaru Takeuchi, professor emeritus from Kawanishi, Japan, died on January 8, 2001. Born on January 12, 1932, he was a member of the Society for 37 years.

Henry S. Tropp, retired, from Humboldt State University, died on March 24, 2007. Born on July 15, 1927, he was a member of the Society for 39 years.

Andrew H. Wallace, professor emeritus, University of Pennsylvania, died on January 18, 2008. Born on June 14, 1926, he was a member of the Society for 57 years.

Izaak Wirszup, retired professor from the University of Chicago, died on January 30, 2008. Born on January 5, 1915, he was a member of the Society for 51 years.

Reference and Book List

*The **Reference** section of the* Notices *is intended to provide the reader with frequently sought information in an easily accessible manner. New information is printed as it becomes available and is referenced after the first printing. As soon as information is updated or otherwise changed, it will be noted in this section.*

Contacting the *Notices*

The preferred method for contacting the *Notices* is electronic mail. The editor is the person to whom to send articles and letters for consideration. Articles include feature articles, memorial articles, communications, opinion pieces, and book reviews. The editor is also the person to whom to send news of unusual interest about other people's mathematics research.

The managing editor is the person to whom to send items for "Mathematics People", "Mathematics Opportunities", "For Your Information", "Reference and Book List", and "Mathematics Calendar". Requests for permissions, as well as all other inquiries, go to the managing editor.

The electronic-mail addresses are `notices@math.ou.edu` in the case of the editor and `notices@ams.org` in the case of the managing editor. The fax numbers are 405-325-7484 for the editor and 401-331-3842 for the managing editor. Postal addresses may be found in the masthead.

Upcoming Deadlines

March 31, 2008: Nominations for prizes of the Academy of Sciences for the Developing World (TWAS). See `http://www.twas.org/`.

March 31, 2008: Nominations for 2008 Prize for Achievement in Information-Based Complexity. Contact Joseph Traub at `traub@cs.columbia.edu`.

April 15, 2008: Applications for Math in Moscow for fall 2008. See `http://www.mccme.ru/mathinmoscow`, or write to: Math in Moscow, P.O. Box 524, Wynnewood, PA 19096; fax +7095-291-65-01; email: `mim@mccme.ru`; or contact Math in

Where to Find It

A brief index to information that appears in this and previous issues of the *Notices*.

Moscow Program, Membership and Programs Department, American Mathematical Society, 201 Charles Street, Providence RI 02904-2294; email: student-serv@ams.org.

April 18, 2008: Applications for Project NExT: New Experiences in Teaching. See "Mathematics Opportunities" in this issue.

May 1, 2008: Applications for AWM Travel Grants. See http://www.awm-math.org/travelgrants.html; telephone 703-934-0163; email: awm@awm-math.edu; or contact Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030.

May 1, 2008: Applications for May review for the National Academies Postdoctoral and Senior Research Associateship Programs. See http://www7.nationalacademies.org/rap/index.html or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email: rap@nas.edu.

May 15-June 15, 2008: Proposals for DMS Workforce Program in the Mathematical Sciences. See "Mathematics Opportunities" in this issue.

June 1, 2008: Applications for Christine Mirzayan Science and Technology Policy Graduate Fellowship Fall Program. See http://www7.nationalacademies.org/policyfellows; or contact The National Academies Christine Mirzayan Science and Technology Policy Graduate Fellowship Program, 500 Fifth Street, NW, Room 508, Washington, DC 20001; telephone: 202-334-2455; fax: 202-334-1667; email: policyfellows@nas.edu.

June 10, 2008: Proposals for Enhancing the Mathematical Sciences Workforce in the Twenty-First Century. See http://www.nsf.gov/publications/pub_summ.jsp?ods_key=nsf05595.

August 1, 2008: Applications for August review for the National Academies Postdoctoral and Senior Research Associateship Programs. See http://www7.nationalacademies.org/rap/index.html or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email: rap@nas.edu.

August 18, 2008: Applications for NSF Research Experiences for Undergraduates (REU) program sites. See http://www.nsf.gov/publications/pub_summ.jsp?ods_key=nsf07569.

October 1, 2008: Applications for AWM Travel Grants. See http://www.awm-math.org/travelgrants.html; telephone: 703-934-0163; email: awm@awm-math.edu; or contact Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030.

November 1, 2008: Applications for November review for the National Academies Postdoctoral and Senior Research Associateship Programs. See http://www7.nationalacademies.org/rap/index.html or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email: rap@nas.edu.

Mathematical Sciences Education Board, National Research Council

Jan de Lange, Freudenthal Institute, The Netherlands

Keisha M. Ferguson, Pattengill Elementary School, Ann Arbor, MI

Louis Gomez, Northwestern University

Javier Gonzalez, Pioneer High School, Whittier, CA

Sharon Griffin, Clark University

Phillip A. Griffiths (chair), Institute for Advanced Study

Arthur Jaffe, Harvard University

Jeremy Kilpatrick, University of Georgia

Julie Legler, St. Olaf College

W. James Lewis, University of Nebraska, Lincoln

Kevin F. Miller, University of Michigan, Ann Arbor

Marge Petit (vice chair), Consultant, Fayston, VT

Donald Saari, University of California, Irvine

Nancy J. Sattler, Terra State Community College, Freemont, OH

Richard J. Schaar, Texas Instruments

Frank Wang, Oklahoma School of Science and Mathematics

MSEB Staff

David R. Mandel, Director

The contact information is: Mathematical Sciences Education Board, National Academy of Sciences, 500 Fifth Street, NW, 11th Floor, Washington, DC 20001; telephone 202-334-2353; fax 202-344-2210; email: mseb@nas.edu; World Wide Web http://www7.nationalacademies.org/MSEB/About%20MSEB.html.

Book List

The Book List highlights books that have mathematical themes and are aimed at a broad audience potentially including mathematicians, students, and the general public. When a book has been reviewed in the Notices, *a reference is given to the review. Generally the list will contain only books published within the last two years, though exceptions may be made in cases where current events (e.g., the death of a prominent mathematician, coverage of a certain piece of mathematics in the news) warrant drawing readers' attention to older books. Suggestions for books to include on the list may be sent to* notices-booklist@ams.org.

*Added to "Book List" since the list's last appearance.

Alfred Tarski: Life and Logic, by Anita Burdman Feferman and Solomon Feferman. Cambridge University Press, October 2004. ISBN 0-521-80240-7. (Reviewed September 2007.)

**Amongst Mathematicians: Teaching and Learning Mathematics at University Level*, by Elena Nardi. Springer, November 2007. ISBN: 978-0-387-37141-2.

Ants, Bikes, and Clocks: Problem Solving for Undergraduates, by William Briggs. Society for Industrial and Applied Mathematics, 2005. ISBN 0-89871-574-1.

The Archimedes Codex, by Reviel Netz and William Noel. Weidenfeld and Nicolson, May 2007. ISBN-13: 978-0-29764-547-4.

The Art of Mathematics: Coffee Time in Memphis, by Béla Bollobás. Cambridge University Press, September 2006. ISBN-13: 978-0-52169-395-0.

The Artist and the Mathematician: The Story of Nicolas Bourbaki, the Genius Mathematician Who Never Existed, by Amir D. Aczel. Thunder's Mouth Press, August 2006. ISBN 1-560-25931-0. (Reviewed October 2007.)

Benjamin Franklin's Numbers: An Unsung Mathematical Odyssey, by Paul C. Pasles. Princeton University Press, October 2007. ISBN-13: 978-0-69112-956-3.

Bourbaki, a Secret Society of Mathematicians, by Maurice Mashaal. AMS, June 2006. ISBN 0-8218-3967-5. (Reviewed October 2007.)

The Calculus Wars: Newton, Leibniz, and the Greatest Mathematical Clash of All Time, by Jason Socrates Bardi. Thunder's Mouth Press, April 2007. ISBN-13: 978-1-56025-992-3.

The Cat in Numberland, by Ivar Ekeland. Cricket Books, April 2006. ISBN-13: 978-0-8126-2744-2.

A Certain Ambiguity: A Mathematical Novel, by Gaurav Suri and Hartosh Singh Bal. Princeton University Press, June 2007. ISBN-13: 978-0-691-12709-5. (Reviewed February 2008.)

Chases and Escapes: The Mathematics of Pursuit and Evasion, by Paul J. Nahin. Princeton University Press, May 2007. ISBN-13: 978-0-69112-514-5.

Descartes: A Biography, by Desmond Clarke. Cambridge University Press, March 2006. ISBN 0-521-82301-3. (Reviewed January 2008.)

Does Measurement Measure Up?: How Numbers Reveal and Conceal the Truth, by John Henshaw. Johns Hopkins University Press, March 2006. ISBN-13: 978-08018-8375-0.

Ernst Zermelo: An Approach to His Life and Work, by Heinz-Dieter Ebbinghaus. Springer, April 2007. ISBN-13: 978-3-540-49551-2.

Flatland—The Movie: A Journey of Many Dimensions. Flatworld Productions, 2007. Special Educator Edition DVD, http://store.flatlandthemovie.com. (Reviewed November 2007.)

Fly Me to the Moon: An Insider's Guide to the New Science of Space Travel, by Edward Belbruno. Princeton University Press, January 2007. ISBN-13: 978-0-691-12822-1. (Reviewed in this issue.)

The Great π/e Debate: Which Is the Better Number?, DVD by Colin Adams and Thomas Garrity. Mathematical Association of America, 2007. ISBN 0-88385-900-9.

A History of Abstract Algebra, by Israel Kleiner. Birkhäuser, October 2007. ISBN-13: 978-0-8176-4684-4.

How Mathematicians Think: Using Ambiguity, Contradiction, and Paradox to Create Mathematics, by William Byers. Princeton University Press, May 2007. ISBN-13: 978-0-6911-2738-5. (Reviewed December 2007.)

I Am a Strange Loop, by Douglas R. Hofstadter. Basic Books, March 2007. ISBN-13: 978-0-46503-078-1. (Reviewed August 2007.)

The Indian Clerk, by David Leavitt. Bloomsbury USA, September 2007. ISBN-13: 978-15969-1040-9.

An Introduction to Gödel's Theorems, by Peter Smith. Cambridge University Press, August 2007. ISBN-13: 978-0-52167-453-9.

John von Neumann: Selected Letters, edited by Miklós Rédei. AMS, November 2005. ISBN 0-8218-3776-1. (Reviewed June/July 2007.)

Karl Pearson: The Scientific Life in a Statistical Age, by Theodore M. Porter. Princeton University Press, (new edition) December 2005. ISBN-13: 978-0-69112-635-7. (Reviewed December 2007.)

The Legacy of Mario Pieri in Geometry and Arithmetic, by Elena Anne Marchisotto and James T. Smith. Birkhäuser, May 2007. ISBN-13: 978-0-8176-3210-6.

Leonhard Euler, by Emil A. Fellmann. Birkhäuser, 2007. ISBN-13: 978-3-7643-7538-6.

Leonhard Euler, a Man to Be Reckoned With, by Andreas K. Heyne and Alice K. Heyne. Birkhäuser, 2007. ISBN-13: 978-3-7643-8332-9. (Reviewed March 2008.)

Letters to a Young Mathematician, by Ian Stewart. Perseus Books, April 2006. ISBN-13: 978-0-465-08231-5. (Reviewed May 2007.)

The Math behind the Music, by Leon Harkleroad. Cambridge University Press, August 2006. ISBN-13: 978-0-521-00935-5.

Math Doesn't Suck: How to Survive Middle-School Math without Losing Your Mind or Breaking a Nail, by Danica McKellar. Hudson Street Press, August 2007. ISBN-13: 978-1-5946-3039-2.

Mathematical Mind-Benders, by Peter Winkler. A K Peters, August 2007. ISBN-13: 978-15688-1336-3.

The Mathematician's Brain, by David Ruelle. Princeton University Press, July 2007. ISBN-13: 978-0-691-12982-2.

Mathematics at Berkeley: A History, by Calvin C. Moore. A K Peters, February 2007. ISBN-13: 978-15688-130-28.

The Millennium Prize Problems, edited by James Carlson, Arthur Jaffe, and Andrew Wiles. AMS, June 2006. ISBN-13: 978-0-8218-3679-8.

The Mind of the Mathematician, by Michael Fitzgerald and Ioan James. Johns Hopkins University Press, May 2007. ISBN-13: 978-0-8018-8587-7.

More Mathematical Astronomy Morsels, by Jean Meeus. Willmann-Bell, 2002. ISBN 0-943396-743.

More Sex Is Safer Sex: The Unconventional Wisdom of Economics, by Steven E. Landsburg. Free Press, April 2007. ISBN-13: 978-1-416-53221-7.

The Motion Paradox: The 2,500-Year Old Puzzle behind All the Mysteries of Time and Space, by Joseph Mazur. Dutton Adult, April 2007. ISBN-13: 978-0-52594-992-3.

Mr. Hopkins' Men: Cambridge Reform and British Mathematics in the 19th Century, by A. D. D. Craki. Springer, July 2007. ISBN-13: 978-1-8462-8790-9.

Music and Probability, by David Temperley. MIT Press, January 2007. ISBN-13: 978-0-262-20166-7.

Music: A Mathematical Offering, by David J. Benson. Cambridge University Press, December 2006. ISBN-13: 978-0-521-61999-8.

Negative Math: How Mathematics Rules Can Be Positively Bent, by Alberto A. Martinez. Princeton University Press, November 2005. ISBN-13: 978-0-691-12309-7.

New Theories of Everything, by John D. Barrow. Oxford University Press, July 2007. ISBN-13: 978-0-192-80721-2.

Nonplussed! Mathematical Proof of Implausible Ideas, by Julian Havil. Princeton University Press, May 2007. ISBN-13: 978-0-691-12056-0.

The Numbers behind NUMB3RS: Solving Crime with Mathematics, by Keith Devlin and Gary Lorden. Plume, August 2007. ISBN-13: 978-0-4522-8857-7.

Out of the Labyrinth: Setting Mathematics Free, by Robert Kaplan and Ellen Kaplan. Oxford University Press, January 2007. ISBN-13: 978-0-19514-744-5.

Perfect Figures: The Lore of Numbers and How We Learned to Count, by Bunny Crumpacker. Thomas Dunne Books, August 2007. ISBN-13: 978-03123-6005-4.

The Poincaré Conjecture: In Search of the Shape of the Universe, by Donal O'Shea. Walker, March 2007. ISBN-13: 978-0-8027-1532-6. (Reviewed January 2008.)

Poincaré's Prize: The Hundred-Year Quest to Solve One of Math's Greatest Puzzles, by George Szpiro. Dutton Adult, June 2007. ISBN-13: 978-0-525-95024-0. (Reviewed January 2008.)

The Probability of God: A Simple Calculation That Proves the Ultimate Truth, by Stephen D. Unwin. Three Rivers Press (October 26, 2004). ISBN-13: 978-14000-5478-7. (Reviewed February 2008.)

Project Origami: Activities for Exploring Mathematics, by Thomas Hull. A K Peters, March 2006. ISBN 1-568-81258-2. (Reviewed May 2007.)

Pythagoras: His Life, Teaching and Influence, by Christoph Riedweg. Translated by Steven Rendall. Cornell University Press, March 2005. ISBN-13: 978-0-80144-240-7.

Pythagoras: The Mathemagician, by Karim El-koussa. Cloonfad Press, September 2005. ISBN-13: 978-0-97694-042-5.

The Pythagorean Theorem: A 4000-Year History, by Eli Maor. Princeton University Press, May 2007. ISBN-13: 978-0-69112-526-8.

Solving Mathematical Problems: A Personal Perspective, by Terence Tao. Oxford University Press, September 2006. ISBN-13: 978-0-199-20560-8.

The Square Root of 2: A Dialogue Concerning a Number and a Sequence, by David Flannery. Springer, December 2005. ISBN-13: 978-0-38720-220-4.

Superior Beings: If They Exist, How Would We Know? Game-Theoretic Implications of Omnipotence, Omniscience, Immortality, and Incomprehensibility, by Steven Brams. Springer, second edition, November 2007. ISBN-13: 978-0-387-48065-7. (Reviewed February 2008.)

Thinking about Gödel and Turing: Essays on Complexity, 1970–2007, by Gregory J. Chaitin. World Scientific, August 2007. ISBN-13: 978-9-8127-0895-3.

The Triumph of Numbers: How Counting Shaped Modern Life, by I. B. Cohen. W. W. Norton, July 2006. ISBN-13: 978-0-393-32870-7. (Reviewed December 2007.)

The Trouble with Physics: The Rise of String Theory, the Fall of a Science, and What Comes Next, by Lee Smolin. Joseph Henry Press, October 2006. ISBN 0-309-10192-1. (Reviewed September 2007.)

Useless Arithmetic: Why Environmental Scientists Can't Predict the Future, by Orrin Pilkey and Linda Pilkey-Jarvis. Columbia University Press, February 2007. ISBN 0-231-13212-3. (Reviewed in this issue.)

The Volterra Chronicles: The Life and Times of an Extraordinary Mathematician, by Judith R. Goodstein. AMS, February 2007. ISBN-13: 978-0-821-83969-0. (Reviewed March 2008.)

Why Beauty Is Truth: The Story of Symmetry, by Ian Stewart. Perseus Books Group, April 2007. ISBN-13: 978-0-46508-236-0.

Yearning for the Impossible: The Surprising Truths of Mathematics, by John Stillwell. A K Peters, May 2006. ISBN 1-568-81254-X. (Reviewed June/July 2007.)

You Failed Your Math Test, Comrade Einstein: Adventures and Misadventures of Young Mathematicians, or Test Your Skills in Almost Recreational Mathematics, edited by M. Shifman. World Scientific, June 2005. ISBN-13: 978-9-8125-6279-1.

REFERENCE & BOOK LIST

Call for Nominations

2009

Frank and Brennie Morgan AMS-MAA-SIAM Prize

for Outstanding Research in Mathematics by an Undergraduate Student

The prize is awarded each year to an undergraduate student (or students having submitted joint work) for outstanding research in mathematics. Any student who is an undergraduate in a college or university in the United States or its possessions, or Canada or Mexico, is eligible to be considered for this prize.

The prize recipient's research need not be confined to a single paper; it may be contained in several papers. However, the paper (or papers) to be considered for the prize must be submitted while the student is an undergraduate; they cannot be submitted after the student's graduation. The research paper (or papers) may be submitted for consideration by the student or a nominator. All submissions for the prize must include at least one letter of support from a person, usually a faculty member, familiar with the student's research. Publication of research is not required.

The recipients of the prize are to be selected by a standing joint committee of the AMS, MAA, and SIAM. The decisions of this committee are final. The 2009 prize will be awarded for papers submitted for consideration no later than June 30, 2008, by (or on behalf of) students who were undergraduates in December 2007.

Questions may be directed:

Dr. Martha J. Siegel, MAA Secretary
Mathematics Department
Stephens Hall 302
Towson University
8000 York Road
Towson, MD 21252-0001
telephone: 410-704-2980
e-mail: siegel@towson.edu

Nominations and submissions should be sent to:

Morgan Prize Committee
c/o Robert J. Daverman, Secretary
American Mathematical Society
312D Ayres Hall
University of Tennessee
Knoxville, TN 37996-1330

THE MATHEMATICAL ASSOCIATION OF AMERICA

siam

Call for NOMINATIONS

The selection committees for these prizes request nominations for consideration for the 2009 awards, which will be presented at the Joint Mathematics Meetings in Washington, DC, in January 2009. Information about these prizes may be found in the November 2007 issue of the *Notices*, pp. 1372–1390, and at http://www.ams.org/prizes-awards.

RUTH LYTTLE SATTER PRIZE. The Ruth Lyttle Satter Prize is presented every two years in recognition of an outstanding contribution to mathematics research by a woman in the previous six years.

ALBERT LEON WHITEMAN MEMORIAL PRIZE. The Albert Leon Whiteman Memorial Prize is awarded every four years, for notable exposition on the history of mathematics. The ideas expressed and understandings embodied in that exposition should reflect exceptional mathematical scholarship.

Nominations for all prizes should be submitted to the secretary, Robert J. Daverman, American Mathematical Society, 312D Ayres Hall, University of Tennessee, Knoxville, TN 37996-1330. Include a short description of the work that is the basis of the nomination, with complete bibliographic citations when appropriate. A brief curriculum vitae should be included for the nominee. The nominations will be forwarded by the secretary to the appropriate prize selection commttee, which, as in the past, will make final decisions on the awarding of these prizes.

Deadline for nominations is June 30, 2008.

AMS
AMERICAN MATHEMATICAL SOCIETY

Mathematics Calendar

Please submit conference information for the Mathematics Calendar through the Mathematics Calendar submission form at `http://www.ams.org/cgi-bin/mathcal-submit.pl`.
The most comprehensive and up-to-date Mathematics Calendar information is available on the AMS website at `http://www.ams.org/mathcal/`.

April 2008

* 2–4 **Ischia Group Theory 2008 (3rd International Conference on Group Theory)**, Ischia (NA - Italy).
Information: `http://www.dmi.unisa.it/ischia2008/`.

* 7–11 **Topics in PDE's and applications 2008**, University of Granada, Granada, Spain.
Courses: Luigi Ambrosio (Scuola Normale, Pisa), Fokker-Planck equations, Wasserstein gradient flows and Markov semigroups; Luis Caffarelli (Austin), Nonlinear problems involving integral diffusions; François Golse (École Polytechnique), Recent results on the periodic Lorenz gas; Pierre-Louis Lions (Collège de France, Paris), On mean field games; Horng-Tzer Yau (Harvard), Dynamics of Bose Gas and Gross-Pitaevskii equation.
Conferences: Yan Guo (Brown), Decay and continuity of the Boltzmann equation in bounded domains; Pierre-Emmanuel Jabin (Nice), Derivation of Vlasov equations from systems of interacting particles with singular forces; Frank Merle (Cergy-Pontoise), On interaction in the non integrable case for Generalized KdV.
Information: `http://www.ugr.es/~kinetic/PDE/`.

* 28–May 4 **Geometry and Group theory**, Mathematics Instiute, University of Strasbourg, France.
Description: Mini-courses and talks on group representations, asymptotic geometry, hyperbolic geometry, symmetric spaces, applications to number theory. Mini-courses by Marc Burger, Viktor Schroeder, Norbert A'Campo, Gabriele Link and Françoise Dal'Bo. Financial support for graduate students and new PhD's.
Information:`http://www-irma.u-strasbg.fr/article593.html`.

May 2008

* 1–3 **Milestones in Computer Algebra (MICA 2008): A Conference in Honour of Keith Geddes' 60th Birthday**, Stonehaven Bay, Trinidad and Tobago.
Description: Keith Geddes' research has spanned the areas of numerical approximation, algebraic algorithms for symbolic computation, hybrid symbolic-numeric computation and the design and implementation of computer algebra systems. He is perhaps best known as co-founder of the Maple computer algebra system. Through his teaching, research and software, the work of Keith Geddes has touched millions of individuals. In honour of Keith Geddes' 60th birthday, the 40th year of his scientific career and the 20th anniversary of the founding of Maplesoft, a conference will be held May 1–3, 2008 near Scarborough, Tobago.
Information: `http://www.orcca.on.ca/conferences/mica2008`; Stephen.Watt@uwo.ca—.

* 12–16 **Quasiconformal Mappings and Analysis on Metric Spaces. A conference in memory of Juha Heinonen**, University of Michigan, Ann Arbor, Michigan.
Description: Juha Heinonen was a leading figure in geometric function theory and contributed greatly to the theory of quasiconformal mappings and analysis on metric spaces. The conference will be dedicated to his legacy and new developments in the area.
Organizers: M. Bonk, T. Kilpeläinen, P. Koskela, S. Rohde, E.

This section contains announcements of meetings and conferences of interest to some segment of the mathematical public, including ad hoc, local, or regional meetings, and meetings and symposia devoted to specialized topics, as well as announcements of regularly scheduled meetings of national or international mathematical organizations. A complete list of meetings of the Society can be found on the last page of each issue.
An announcement will be published in the *Notices* if it contains a call for papers and specifies the place, date, subject (when applicable), and the speakers; a second announcement will be published only if there are changes or necessary additional information. Once an announcement has appeared, the event will be briefly noted in every third issue until it has been held and a reference will be given in parentheses to the month, year, and page of the issue in which the complete information appeared. Asterisks (*) mark those announcements containing new or revised information.
In general, announcements of meetings and conferences held in North America carry only the date, title of meeting, place of meeting, names of speakers (or sometimes a general statement on the program), deadlines for abstracts or contributed papers, and source of further information. Meetings held outside the North American area may carry more detailed information. In any case, if there is any application deadline with respect to participation in the meeting, this fact should be noted. All communications on meetings and conferences in the mathematical sciences should be sent to the Editor of the *Notices* in care of the American Mathematical Society in Providence or electronically to `notices@ams.org` or `mathcal@ams.org`.
In order to allow participants to arrange their travel plans, organizers of meetings are urged to submit information for these listings early enough to allow them to appear in more than one issue of the *Notices* prior to the meeting in question. To achieve this, listings should be received in Providence **eight months** prior to the scheduled date of the meeting.
The complete listing of the Mathematics Calendar will be published only in the September issue of the *Notices*. The March, June/July, and December issues will include, along with new announcements, references to any previously announced meetings and conferences occurring within the twelve-month period following the month of those issues. New information about meetings and conferences that will occur later than the twelve-month period will be announced once in full and will not be repeated until the date of the conference or meeting falls within the twelve-month period.
The Mathematics Calendar, as well as Meetings and Conferences of the AMS, is now available electronically through the AMS website on the World Wide Web. To access the AMS website, use the URL: `http://www.ams.org/`.

Saksman.
Information: http://www.math.lsa.umich.edu/conferences/heinonen/.

*14-16 **DIMACS/DyDAn Workshop on Internet Tomography**, DIMACS/DyDAn Center, CoRE Building, Rutgers University, Piscataway, New Jersey.
Short Description: A large-scale network such as the Internet is typically controlled by multiple entities, and it is not possible to directly obtain information about the internal characteristics. Desired internal characteristics include network structure and topology, loss rate and delay distribution, and the origin-to-destination traffic matrix, which are important for dynamic routing, optimized service provision, service level verification, and detection of anomalous or malicious behavior. Internet tomography is the study of methods for inferring unknown internal characteristics of large-scale networks on the basis of peripheral information that can be easily obtained. Tomography is made difficult by the heterogeneity and the largely unregulated structure of the Internet. Furthermore, one cannot rely on the cooperation of individual servers and routers.
This workshop will address open research areas including: algorithms for placement of network monitors; space-efficient and time-efficient on-line algorithms to process even high volumes of traffic; machine- learning and statistical algorithms to analyze traffic patterns and detect anomalous behavior; algorithms for computing traffic flows and using results to better distribute network capacity and resources.
Organizers: Jasleen Kaur, University of North Carolina, jasleen@cs.unc.edu; Don Towsley, University of Massachusetts, towsley@cs.umass.edu; Walter Willinger, AT&T Labs-Research, walter@research.att.com.
Local Arrangements: Workshop Coordinator, DIMACS Center, workshop@dimacs.rutgers.edu, 732-445-5928; http://dimacs.rutgers.edu/Workshops/Tomography/.
Information: Presented under the auspices of the http://dimacs.rutgers.edu/SpecialYears/2007_AFI/. Special Focus on Algorithmic Foundations of the Internet and the http://www.dydan.rutgers.edu/; Center for Dynamic Data Analysis (DyDAn).

*22-24 **Interactions between Classical and Numerical Algebraic Geometry**, University of Notre Dame, Notre Dame, Indiana.
Description: The chief objective of this conference is to bring together researchers from classical and numerical algebraic geometry to spark joint efforts between these two communities. The intersection of these fields is ripe for growth fueled by joint work. Furthermore, this conference coincides with the 60th birthday of Andrew Sommese, Duncan Chair of Mathematics at the University of Notre Dame and one of the pioneers of the field. A secondary objective of this conference is to honor his achievements.
Speakers include: E. Allgower (Colorado State), M. Beltrametti (Genova), M. de Cataldo (SUNY Stony Brook), L. Goettsche (ICTP Trieste), T.Y. Li (Michigan State), C. Peterson (Colorado State), F.O. Schreyer (Saarbrücken), B. Shiffman (Johns Hopkins), J. Verschelde (Univ. of Illinois at Chicago), C. Wampler (General Motors), and J. Wisniewski (Warsaw). Grad students and postdocs, especially those from groups underrepresented in mathematics, are encouraged to attend and to apply for potential funding.
Information: http://www.depaul.edu/~gbesana/interactions2008;email: dabates@ima.umn.edu.

June 2008

*1-7 **ULTRAMATH 2008 Applications of Ultrafilters and Ultraproducts in Mathematics**, Pisa, Italy.
Description: The international Congress ULTRAMATH aims to present recent results in the whole spectrum of mathematics which are grounded on the use of ultrafilters and ultraproducts. Its main goals: Disseminate information about the various techniques related to the use of ultrafilters and ultraproducts, and their potential to attack open problems. Bring together researchers with different backgrounds, and encourage their collaborations and interactions, especially on topics connecting different areas of mathematics.
Topics: The covered topics include (but are not limited to): Additive/combinatorial number theory; combinatorics, Ramsey theory, ergodic theory; algebra; geometry; general topology; measure theory; metric spaces; functional analysis; nonstandard methods; set theory and foundations.
Information: http://www.dm.unipi.it/~ultramath.

*2-6 **International conference on linear preservers and operator theory**, Faculté des Sciences, Université Moulay Ismaïl, Meknès, Morocco.
Description: The main themes of international conference on linear preservers and operator theory include linear preserver, local spectral theory, commutators, derivations and elementary operators.
Information: http://maths.fs-umi.ac.ma/congresmath/; email: bourhim@mat.ulaval.ca.

*2-13 **Motives, Quantum Field Theory, and Pseudodifferential Operators**, Boston University, Boston, Massachusetts.
Description: This ten-day conference will explore the rapidly developing relationships among motives, perturbative quantum field theory, and pseudodifferential operators. The first week of the conference will have introductory lectures on each of the three fields, and the second week will have introductory lectures on Hopf algebras and QFT. Both weeks will also have advanced lectures on research topics. Limited financial aid is available for graduate students, postdocs, and faculty with Ph.D.s awarded after 2005. Women and underrepresented minorities are encouraged to apply.
Information: http://math.bu.edu/qftconference.

*5-6 **Mini-Workshop: Entropies and Optimal Transport in Quantum Mechanics**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California.
Overview: In many important applications (such as ultra integrated solid state devices, or other applications in nanoscience) so called open quantum systems occur, which model the interaction of the particles not only with each other (electrostatically) but also with their environment. Typical questions that occur are existence of steady states, convergence to equilibria, and speed of convergence, which in some cases can be dealt with by exploiting the mathematical properties of the quantum entropy. Another important topic of the meeting is going to be the application of optimal mass transportation techniques to linear and nonlinear Schrödinger equations.
Organizer: Peter Markowich.
Application Registration: A registration form is available at http://www.ipam.ucla.edu/programs/otws5/. There is no registration fee for this two-day workshop.
Information: http://www.ipam.ucla.edu/programs/otws5/.

*9-14 **II International Summer School on Geometry, Mechanics and Control**, La Palma (Canary Islands), Spain.
Description: The school is oriented to young researchers, Ph.D. and postdoctoral students in mathematics, physics and engineering, in particular those interested in focusing their research on geometric reduction in mechanics and its applications and on stochastic mechanics. It is intended to present an updated view of the knowledge on some basic problems in these topics and bring to the participants open problems in the research area (in particular, related with applications) by specialized courses imparted by the best international scientists on the respective topics. This activity is a part of the Intensive Research Program "Geometric Mechanics and Control Theory" which will be held in 2008 in Spain.
Information: http://webpages.ull.es/users/gmcnet/—Summer-School08/home1.htm.

*11-15 **3rd Conference on Analysis and Probability on Fractals**, Cornell University, Ithaca, New York.

Description: The purpose of this conference is to bring together mathematicians who are already working in the area of analysis and probability on fractals, and also graduate students and researchers from related areas who would like to learn more about it. Toward this end, the conference will include a mini-course intended to bring neophytes up to speed on the subject, starting with basic definitions and examples, and reaching to the frontiers of research.

Information: http://www.math.cornell.edu/event/conf/fractals3/index.php.

* 13–16 **8Th International Confernce of Computational and Mathematical Methods in Science and Engeneering CMMSE-2008**, Hotel Melia Galua, Manga del Mar Menor, Murcia, Spain.

Description: The 2008 Conference on Mathematical Methods in Science and Engineering Conference (CMMSE-2008), is the eighth of this conference series. CMMSE2008 aims to be a unifying, cross-cutting, interdisciplinary gathering, where specialists can have exposure to diverse fields and participate in special sessions different from but still close to their own interests.

Talks: Standard presentation: 30 minutes. Plenary presentations: 60 minutes.

Important Dates: Deadline for submission of extended Abstracts (3–4 pages): May 1, 2008. Notification of the acceptance/rejection of submitted contributions: May 7, 2008. Deadline for Special Issues submission: September 10, 2008.

Previous Conference: CMMSE-2007 Chicago.

Information: http://www.uwm.edu/Dept/CIM/CMMSE/.

Information: http://www.usal.es/~CMMSE; email: jvigo@usal.es.

* 15–24 **CIMPA Summer School Nonlinear Analysis and Geometric PDE**, The Conference Centre of Yerevan State University, Tsaghkadzor, Armenia.

Description: CIMPA and the Institute of Mathematics of Armenian National Academy of Sciences, in association with IMPRS Mathmatics in the Sciences (Leipzig), Yerevan State University and the Royal Institute of Technology (Stockholm) are organizing a Summer School on Nonlinear Analysis and Geometric PDE.

Speakers: Luis A. Caffarelli (University of Texas at Austin, USA), The porous media equation and geometric methods in non linear PDE; Henri Berestycki (EHESS, Paris, France), Reaction-diffusion equations and propagation phenomena; Yann Brenier (CNRS, Univ. de Nice, France), Optimal transport, convection and magnetic relaxation; Stephan Luckhaus (Universität Leipzig, Germany), Minimalsurfaces, free boundary problems and geometric measure theory; Nina Uraltseva (St. Petersburg State University, Russia), Regularity of free boundaries in Obstacle-type problems.

Language: English.

Information: http://www.imprs-mis.mpg.de/CIMPA/.

* 16–20 **15th Conference of the International Linear Algebra Society**, Westin Resort and Spa, Cancun, Mexico.

Description: The conference deals with all themes related to linear algebra; theory, applications, numerical aspects, and education. The program includes 14 plenary talks, 8 mini-symposia, and contributed talks. The conference proceedings will appear as a special issue of linear algebra and its applications.

Information: http://star.izt.uam.mx/ILAS08.

* 16–20 **15th Conference of the International Linear Algebra Society**, Westin Resort and Spa, Cancun, Mexico.

Description: The conference deals with all themes related to Linear Algebra; theory, applications, numerical aspects, and education. The program includes 14 plenary talks, 8 mini-symposia, and contributed talks. The conference proceedings will appear as a special issue of Linear Algebra and its Applications.

Information: http://star.izt.uam.mx/ILAS08; email: star.izt.uam.mx/ILAS08.

* 23–27 **First Iberoamerican Meeting on Geometry, Mechanics, and Control**, Santiago de Compostela, Spain.

Description: The goal of this event is bringing together specialists and students working on Geometric Mechanics and Control Theory from Spain, Portugal and Latin America, in order to create new links and reinforce existing ones.

Information: http://www.gmcnetwork.org/eia08.

* 25–28 **VII Iberoamerican Conference on Topology and its Applications**, Valencia, Spain.

Topics: Set-Theoretical Topology, General Topology and Function Spaces, Topological Algebra and Topological Dynamics, Asymmetric Topology and Applied Topology, Topology for Functional Analysis.

Plenary Speakers: Gerald Beer, Fernando Hernández-Hernández, Jorge Galindo.

Invited Speakers: Antonio Fernández Carrión, Michael Hrúák, Miguel Ángel Sánchez-Granero, Ángel Tamariz-Mascarua, José Valero.

Information: http://cita.webs.upv.es.

* 29–July 4 **The 8th Brauer Group Meeting, in honor of Adrian Wadsworth's 60th Birthday**, Colorado State University's Pingree Park Campus and Conference Center, Pingree Park, Colorado.

Description: Following the tradition of previous entries in the series, this is an informal conference on the Brauer group and related topics such as linear algebraic groups and valuation theory. The aim of the conference is both to give researchers in the field the opportunity to meet and exchange ideas, as well as give graduate students and postdocs the chance to get acquainted with the senior researchers in the area.

Information: http://www.math.rice.edu/~klm1/pingree.html.

* 30–July 4 **The European Consortium of Mathematics in Industry**, University College, London, United Kingdom.

Description: The biennial conference of the European Consortium for Mathematics in Industry which will be held in central London in 2008.

Plenary talks: The plenary talks will cover a wide range of applied mathematical topics and there will be a strong industrial presence particularly from the financial district in the City of London.

Themes: Highlighted themes of the meeting are: Socio-economic interactions, medicine, sport and leisure, uncertainty and risk, optimisation and control as well as more traditional industrial sectors.

Information: http://www.ecmi2008.org/.

July 2008

* 7–10 **The 2008 International Conference on Artificial Intelligence and Pattern Recognition (AIPR-08)**, Orlando, Florida.

Description: We invite draft paper submissions.

Deadline: The deadline for submissions is approaching very soon.

Information: The conference will be held at the same time and place where several other major events are taking place. The website contains more details: http://www.promoteresearch.org/; email: jeedward@gmail.com.

* 16–18 **Numerical Mathematics for Engineering Problems (NMEP)**, São Paulo, Brazil.

Topics (not limited to): Novel mesh/grid generation techniques for engineering problems; Efficient numerical methods for discretizing multiphysics continuum equations; New industrial applications of numerical mathematics. Numerical methods and algorithms with industrial applications; Mathematical/numerical formulation of engineering problems; Ideal algorithms for solving algebraic equations associated with multiphysics continuum equations; Effective software designs for multiphysics processes; Simulation of important multiphysics processes. For example, deposition of CO2; Application of mathematics for modeling engineering problems; Modelling multiphase flow in process equipments such as pipelines;

Applications of the state of art simulators (such as Fluent, CFX, STAR-CD) for simulating important engineering processes.
Information: Workshop site: http://ans.hsh.no/home/skk/Workshop/workshop.html.Conference site:http://www.icmc.usp.br/\%7Ecse08/; email: sanjaykhattri@gmail.com.

* 25–31 **15th International Mathematics Competition for University Students**, Blagoevgrad, Bulgaria.
Description: Every participating university is invited to send several students and one teacher. Individual students are welcome. The competition is planned for students completing their first, second, third or fourth year of university education and will consist of 2 Sessions of 5 hours each. Problems will be from the fields of Algebra, Analysis (Real and Complex) and Combinatorics.
Language: The working language will be English.
Information: Although this is an individual event, the universities traditionally divide their participants into groups of four each. The number of students in the teams is, however, not fixed. The professor who accompanies the students is expected to be a member of the jury. Participants are invited to confirm their intention to participate, either by on-line registration or by e-mail, by the end of May 2008, providing the following information: University, City, Country; Leader of the team (name, e-mail address); Students (number); Mailing address; e-mail address; Fax. Visas: Participants from some countries will need a visa to enter Bulgaria. Please, contact your travel agent or the Bulgarian Consulate in your country for details. If necessary, the organizers will post formal invitations for participation in the competition. Further information: http://www.imc-math.org.uk/.

* 28–August 1 **Generalizing theta correspondences**, American Institute of Mathematics, Palo Alto, California.
Description: This workshop, sponsored by AIM and the NSF, concerns extensions and applications of the method of theta correspondence, including a discussion of outstanding problems and future directions.
Information: http://aimath.org/ARCC/workshops/thetacorr.html.

August 2008

* 18–22 **Frames for the finite world: Sampling, coding and quantization**, American Institute of Mathematics, Palo Alto, California.
Description: This workshop, sponsored by AIM and the NSF, will be devoted to frame theory in finite dimensions with an emphasis on compressive sensing and quantization theory for frames.
Information:http://aimath.org/ARCC/workshops/frames.html.

* 21–23 **20th International Congress of Jangjeon Mathematical Society**, Uludag University, Bursa, Turkey.
Description: The proposed conference aims to bring together all the researchers working in various fields of Mathematics, Mathematical Physics and related areas such as Analysis, Non-linear Analysis, Number Theory, p-adic Analysis, Special Functions, q-Analysis, Mathematical Physics and their applications.
Information: All the details of the conference can be followed at the web site http://icjms20.uludag.edu.tr/. Please contact cangul@uludag.edu.tr, hozden@uludag.edu.tr or inam@uludag.edu.tr for further requests. The full-text papers will be published at ASCM (Advanced Studies in Contemporary Mathematics) or PJMS (Proceedings of Jangjeon Mathematical Society) after the refereeing process.

* 22–29 **Function Spaces, Differential Operators and Nonlinear Analysis**, University of Helsinki, Helsinki, Finland.
Description: As a continuation of the previous conferences held in Czech Republic, Finland and Germany, we are now organizing the 70th International Conference on function spaces, differential operators and nonlinear analysis. The conference celebrates twenty years of FSDONA conferences and will be organized between August 25–29, 2008 in Helsinki, Finland. The conference is preceded by a workshop, August 22–24, aimed especially at younger researchers.
Information: http://mathstat.helsinki.fi/fsdona; email: petteri.harjulehto@helsinki.fi.

* 28–September 2 **12th Serbian Mathematical Congress**, Faculty of Science and Mathematics, University of Novi Sad, Serbia.
Description: The 12th Serbian Mathematical Congress is organized by the Department of Mathematics and Informatics (Faculty of Science and Mathematics, University of Novi Sad, Serbia) and the Mathematical Society of Serbia. The meeting has an international character and is open for all fields of mathematics.
Information: http://www.im.ns.ac.yu/events/2008/smkongres/.

September 2008

* 8–12 **International Workshop on Orthogonal Polynomials and Approximation Theory 2008: In honor of Professor Guillermo López Lagomasino on his 60th Anniversary**, Universidad Carlos III de Madrid, Madrid, Spain.
Description: It is well known the increasing attention paid in the last decades to the theory of Orthogonal Polynomials. Numerous applications of these mathematical objects in different areas of mathematics like numerical integration, spectral methods, interpolation, combinatorics, mathematical physics, quantum physics, and approximation theory among others have been particularly relevant. The workshop will take place at Escuela Politécnica Superior of the Universidad Carlos III de Madrid, Leganés (Madrid).
Topics: Approximation theory, numerical analysis, in particular quadrature formulas, orthogonal polynomials and special functions, analytic properties and applications, integrable systems.
Information: email: hpijeira@math.uc3m.es; http://www.uc3m.es/iwopa08.

* 19–26 **Harmonic Analysis and Approximations, IV (International Conference)**, Tsaghkadzor, Armenia.
Description: The Conference continues the series of International Conferences organized in Armenia (HAA I - Nor Amberd, 1998, HAA II - Nor Amberd, 2001, HAA III - Tsaghkadzor, 2005) which were attended by 177 participants from 19 countries. This conference is dedicated to the 80th anniversary of academician Alexandr Talalian. The program of the conference will consist of invited 50-minute plenary lectures and contributed 20-minutes talks.
Speakers: The following mathematicians have agreed to give a plenary lecture at the conference: Sergey Konyagin (Russia), Thomas Korner (UK), Michael Lacey (USA), Konstantin Oskolkov (USA), Allan Pinkus (Israel), Vilmos Totik (USA), Przemyslaw Wojtaszczyk (Poland).
Information: http://math.sci.am/conference/sept2008/conf.html.

* 27–29 **Discrete Analysis and Applications (Walsh-Fourier Series, Symbolic Sequences-Complexity and Cryptography)**, Department of Informatics, Aristotle University of Thessaloniki, Thessaloniki, Greece.
Description: The aim of this workshop is to focus on both theoretical–applied results and new potentialities of applications of Discrete Analysis. Special attention is given to Walsh-Fourier analysis, symbolic sequences analysis, complexity, non-linearity and cryptography.
Information: http://web.auth.gr/DiscreteAnal08/.

* 29–October 3 **13th GAMM–IMACS International Symposium on Scientific Computing, Computer Arithmetic, and Verified Numerical Computations SCAN'2008**, El Paso, Texas.
Description: The conference continues the series of international SCAN symposia held under the joint sponsorship of GAMM (International Association of Applied Mathematics and Mechanics) and IMACS (International Association for Mathematics and Computers

in Simulation). These symposia have covered the numerical and algorithmic aspects of scientific computing, with a strong emphasis on verification of computed results, as well as on arithmetic, programming, and algorithmic tools for this purpose. Their objectives have been both to propagate current applications and research and to promote a greater understanding and increased awareness of the subject matters. SCAN 2008 strives to become a forum for the researchers of various fields in numerical verification to discuss many existing verification tools and approaches.

Information: http://www.cs.utep.edu/interval-comp/scan08.html; email: vladik@utep.edu.

October 2008

* 9–11 **Algebra, Geometry and Mathematical Physics, Baltic-Nordic Workshop**, University of Tartu, Tartu, Estonia.

Description: The aim of this conference is to strengthen interaction between algebra, differential geometry and theoretical physics. The scope of conference includes the following topics: noncommutative geometry, operad and group theoretical methods, generalizations of Lie algebras, non-associative systems, quantum groups, Hopf algebras, categorical physics, integrable systems.

Information: email: viktor.abramov@ut.ee; http://www.agmf.astralgo.eu/tartu08/.

* 22–23 **The Second Conference on Mathematical Sciences (CMS'2008)**, Department of Mathematics. Faculty of Science and Information Technology, Zarqa Private University, Zarqa 13110, Jordan.

Scope: Pure Mathematics, Applied Mathematics and Statistics and its Techniques.

Languages: Arabic or English.

Deadlines: For Abstract Submission: March 30th, 2008. Full Paper Submission: May 30th, 2008. Notification of Acceptance: July 31st, 2008.

Information: There is no registration fee. http://www.zpu.edu.jo/cms/cms.htm.

* 22–24 **International Conference in Modeling Health Advances 2008**, San Francisco, California.

Description: To tackle these illnesses, the cooperation of modelers, mathematicians, statisticians, computer scientists, and others, and of researchers from the medical community is absolutely essential. Modeling is important because it gives important insight into the method of treatment. In the case of HIV/AIDS, for example, mathematical modeling indicated that a combination of both protease inhibitors and reverse transcriptase inhibitors would be far more effective than any one of these two drugs. The purpose of this conference is to bring all the people working in the area of epidemiology under one roof and encourage mutual interaction.

Information:http://www.iaeng.org/WCECS2008/ICMHA2008.html.

March 2009

* 9–June 12 **Quantum and Kinetic Transport: Analysis, Computations, and New Applications**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California.

Overview: This long program will focus on the mathematical analysis, computational challenges and new applications of quantum and kinetic transport theory. Besides applied mathematicians, we will invite researchers in science and engineering, representing academic, national lab and industrial research.

Organizing Committee: Irene Gamba and Shi Jin (chairs), Eric Carlen, Pierre Degond, Frank Graziani, Karl Kempf, Dave Levermore, Peter Markowich, Stanley Osher, Christian Ringhofer, Marshall Slemrod.

Funding: We have funding especially to support the attendance of recent Ph.D.s, graduate students and researchers in the early stages of their career. Encouraging the careers of women and minority mathematicians and scientists is an important component of IPAM's mission and we welcome their applications.

Application: Please apply online to request financial support to attend and participate for extended periods up to the entire length of the program. The application is available online.

Information: http://www.ipam.ucla.edu/programs/kt2009/.

New Publications Offered by the AMS

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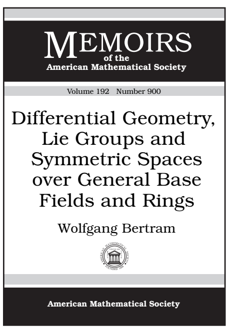
Algebra and Algebraic Geometry



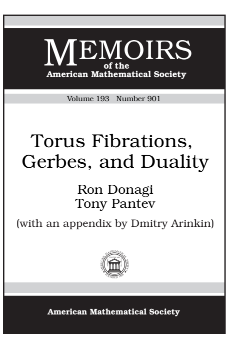
Differential Geometry, Lie Groups and Symmetric Spaces over General Base Fields and Rings

Wolfgang Bertram, *Université Henri Poincaré (Nancy I), Vandœuvre-lés-Nancy, France*

Contents: Introduction; Basic notions; Interpretation of tangent objects via scalar extensions; Second order differential geometry; Third and higher order differential geometry; Lie theory; Diffeomorphism groups and the exponential jet; Appendix L. Limitations; Appendix G. Generalizations; Appendix: Multilinear geometry; References.

Memoirs of the American Mathematical Society, Volume 192, Number 900

March 2008, 202 pages, Softcover, ISBN: 978-0-8218-4091-7, LC 2007060585, 2000 *Mathematics Subject Classification:* 22E65, 53B05, 53C35, 58A05, 58A20, 58A32; 22E20, 14L05, 15A69, 51K10, 58B20, **Individual member US$45**, List US$75, Institutional member US$60, Order code MEMO/192/900



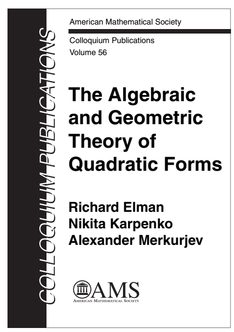
Torus Fibrations, Gerbes, and Duality

Ron Donagi and **Tony Pantev**, *University of Pennsylvania, Philadelphia, PA*

Contents: Introduction; The Brauer group and the Tate-Shafarevich group; Smooth genus one fibrations; Surfaces; Modified T-duality and the SYZ conjecture; Appendix A. Duality for representations of 1-motives, by Dmitry Arinkin; Bibliography.

Memoirs of the American Mathematical Society, Volume 193, Number 901

March 2008, 90 pages, Softcover, ISBN: 978-0-8218-4092-4, LC 2008060002, 2000 *Mathematics Subject Classification:* 14A20, 14J32; 14A22, 14J27, **Individual member US$37**, List US$62, Institutional member US$50, Order code MEMO/193/901



The Algebraic and Geometric Theory of Quadratic Forms

Richard Elman, *University of California, Los Angeles, CA*, **Nikita Karpenko**, *Université Pierre et Marie Curie - Paris 6, France*, and **Alexander Merkurjev**, *University of California, Los Angeles, CA*

This book is a comprehensive study of the algebraic theory of quadratic forms, from classical theory to recent developments, including results and proofs that have never been published. The book is written from the viewpoint of algebraic geometry and includes the theory of quadratic forms over fields of characteristic two, with proofs that are characteristic independent whenever possible. For some results both classical and geometric proofs are given.

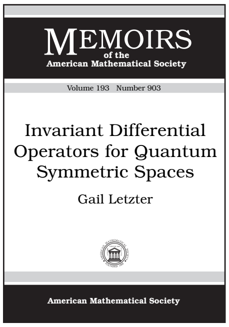
Part I includes classical algebraic theory of quadratic and bilinear forms and answers many questions that have been raised in the early stages of the development of the theory. Assuming only a basic course in algebraic geometry, Part II presents the necessary additional topics from algebraic geometry including the theory of Chow groups, Chow motives, and Steenrod operations. These topics are used in Part III to develop a modern geometric theory of quadratic forms.

Contents: *Classical theory of symmetric bilinear forms and quadratic forms:* Bilinear forms; Quadratic forms; Forms over rational function fields; Function fields of quadrics; Bilinear and quadratic forms and algebraic extensions; u-invariants; Applications of the Milnor conjecture; On the norm residue homomorphism of degree two; *Algebraic cycles:* Homology and cohomology; Chow groups; Steenrod operations; Category of Chow motives; *Quadratic forms and algebraic cycles:* Cycles on powers of quadrics; The Izhboldin dimension; Application of

Steenrod operations; The variety of maximal totally isotropic subspaces; Motives of quadrics; Appendices; Bibliography; Notation; Terminology.

Colloquium Publications, Volume 56

April 2008, approximately 431 pages, Hardcover, ISBN: 978-0-8218-4329-1, 2000 *Mathematics Subject Classification:* 11E04, 11E81, 14C15, 14C25, 14C17, **AMS members US$63**, List US$79, Order code COLL/56



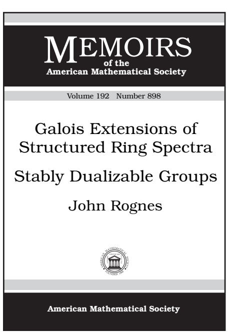
Invariant Differential Operators for Quantum Symmetric Spaces

Gail Letzter, *Viriginia Polytechnic Institute and State University, Blacksburg, VA*

Contents: Introduction; Background and notation; A comparison of two root systems; Twisted Weyl group actions; The Harish-Chandra map; Quantum radial components; The image of the center; Finding invariant elements; Symmetric pairs related to type AII; Four exceptional cases; Appendix: Commonly used notation; Bibliography.

Memoirs of the American Mathematical Society, Volume 193, Number 903

March 2008, 90 pages, Softcover, ISBN: 978-0-8218-4131-0, LC 2008060004, 2000 *Mathematics Subject Classification:* 17B37, **Individual member US$37**, List US$62, Institutional member US$50, Order code MEMO/193/903



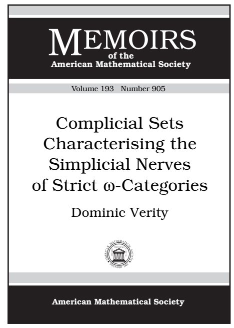
Galois Extensions of Structured Ring Spectra/Stably Dualizable Groups

John Rognes, *University of Oslo, Norway*

Contents: *Galois Extensions of Structured Ring Spectra:* Abstract; Introduction; Galois extensions in algebra; Closed categories of structured module spectra; Galois extensions in topology; Examples of Galois extensions; Dualizability and alternate characterizations; Galois theory I; Pro-Galois extensions and the Amitsur complex; Separable and étale extensions; Mapping spaces of commutative S-algebras; Galois theory II; Hopf–Galois extensions in topology; References; *Stably Dualizable Groups:* Abstract; Introduction; The dualizing spectrum; Duality theory; Computations; Norm and transfer maps; References; Index.

Memoirs of the American Mathematical Society, Volume 192, Number 898

March 2008, 137 pages, Softcover, ISBN: 978-0-8218-4076-4, LC 2007060583, 2000 *Mathematics Subject Classification:* 55M05, 55P35, 57T05, **Individual member US$40**, List US$67, Institutional member US$54, Order code MEMO/192/898



Complicial Sets Characterising the Simplicial Nerves of Strict ω-Categories

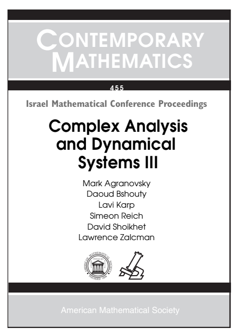
Dominic Verity, *Macquarie University, Sydney, Australia*

Contents: Simplicial operators and simplicial sets; A little categorical background; Double categories, 2-categories and n-categories; An introduction to the decalage construction; Stratifications and filterings of simplicial sets; Pre-complicial sets; Complicial sets; The path category construction; Complicial decalage constructions; Street's ω-categorical nerve construction; Bibliography.

Memoirs of the American Mathematical Society, Volume 193, Number 905

March 2008, 184 pages, Softcover, ISBN: 978-0-8218-4142-6, LC 2008060006, 2000 *Mathematics Subject Classification:* 18D05, 55U10; 18D15, 18D20, 18D35, 18F99, 18G30, **Individual member US$44**, List US$74, Institutional member US$59, Order code MEMO/193/905

Analysis



Complex Analysis and Dynamical Systems III

A Conference in Honor of the Retirement of Dov Aharonov, Lev Aizenberg, Samuel Krushkal, and Uri Srebro

Mark Agranovsky, *Bar-Ilan University, Ramat-Gan, Israel*, **Daoud Bshouty**, *Technion, Haifa, Israel*, **Lavi Karp**, *ORT Braude College, Karmiel, Israel*, **Simeon Reich**, *Technion, Haifa, Israel*, **David Shoikhet**, *ORT Braude College, Karmiel, Israel*, and **Lawrence Zalcman**, *Bar-Ilan University, Ramat-Gan, Israel*, Editors

The papers in this volume cover a wide variety of topics in the geometric theory of functions of one and several complex variables, including univalent functions, conformal and quasiconformal mappings, minimal surfaces, and dynamics in infinite-dimensional spaces. In addition, there are several articles dealing with various aspects of approximation theory and partial differential equations. Taken together, the articles collected here provide the reader with a panorama of activity in complex analysis, drawn by a number of leading figures in the field.

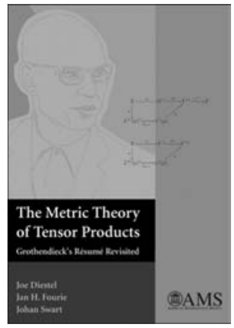
This book is co-published with Bar-Ilan University (Ramat-Gan, Israel).

Contents: **L. Aizenberg** and **N. Tarkhanov**, Stable expansions in homogeneous polynomials; **S. Aizicovici**, **S. Reich**, and

A. J. Zaslavski, Dynamics of approximate solutions to a class of evolution equations in Banach spaces; **K. Astala**, **T. Iwaniec**, **G. Martin**, and **J. Onninen**, Schottky's theorem on conformal mappings between annuli: A play of derivatives and integrals; **D. Bshouty** and **A. Lyzzaik**, Univalent convex functions in the positive direction of the real axis; **M. Budzyńska** and **T. Kuczumow**, The common fixed point set of commuting nonexpansive mappings in infinite products of unit balls; **M. Elin**, **D. Shoikhet**, and **L. Zalcman**, Controlled approximation for some classes of holomorphic functions; **M. I. Ganzburg** and **D. S. Lubinsky**, Best approximating entire functions to $|x|^\alpha$ in L_2; **V. M. Gichev**, Orbits of tori extended by finite groups and their polynomial hulls: The case of connected complex orbits; **D. H. Hamilton**, QC Riemann mapping theorem in space; **G. Harutyunyan** and **B.-W. Schulze**, Boundary value problems in weighted edge spaces; **H. T. Kaptanoğlu** and **A. E. Üreyen**, Analytic properties of Besov spaces via Bergman projections; **B. A. Kats**, The Cauchy integral over non-rectifiable paths; **O. Kounchev** and **H. Render**, Holomorphic continuation via Laplace-Fourier series; **Y. Krasnov**, Analytic functions in algebras; **S. L. Krushkal**, Rational approximation of holomorphic functions and geometry of Grunsky inequalities; **R. Kühnau**, Quadratic forms in geometric function theory, quasiconformal extensions, Fredholm eigenvalues; **A. Kytmanov**, Elimination methods of unknowns from nonlinear systems; **P. Liczberski** and **V. V. Starkov**, On locally biholomorphic finitely valent mappings from multiply connected to simply connected domains; **Y. Lutsky** and **V. S. Rabinovich**, Parabolic pseudodifferential operators in exponential weighted spaces; **O. Makhmudov**, **I. Niyozov**, and **N. Tarkhanov**, The Cauchy problem of couple-stress elasticity; **S. Myslivets**, On the zeta-function of a nonlinear system; **E. Reich**, Some questions of uniqueness for extremal quasiconformal mappings; **E. Saucan**, Remarks on the existence of quasimeromorphic mappings; **A. Shlapunov**, On the Cauchy problem for the Cauchy-Riemann operator in Sobolev spaces; **V. Strauss**, On spectral functions for commutative J-self-adjoint operator families of the D_κ^+-class; **A. Ukhlov** and **S. K. Vodopyanov**, Mappings associated with weighted Sobolev spaces; **E. Vesentini**, Inner maps and Banach algebras; **A. Vidras**, Reconstructing holomorphic functions in a domain from their values on a part of its boundary; **A. Weitsman**, A note on the parabolicity of minimal graphs; **K. Włodarczyk**, **D. Klim**, and **R. Kowalczyk**, Localization of fixed points and zeros for holomorphic maps in locally convex spaces and nonexpansive maps in J*-algebras; **V. Zahariuta**, On harmonic polynomial interpolation.

Contemporary Mathematics, Volume 455

May 2008, approximately 455 pages, Softcover, ISBN: 978-0-8218-4150-1, 2000 *Mathematics Subject Classification:* 16W30, 81R50, 18D10, 17B37, **AMS members US$103**, List US$129, Order code CONM/455



The Metric Theory of Tensor Products

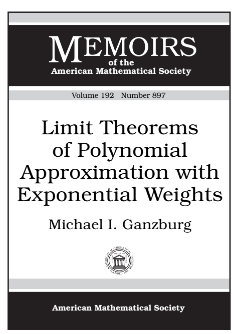
Grothendieck's Résumé Revisited

Joe Diestel, *Kent State University, OH*, **Jan H. Fourie**, *North-West University, Potchefstroom, South Africa*, and **Johan Swart**, *University of Pretoria, South Africa*

Grothendieck's Resumé is a landmark in functional analysis. Despite having appeared more than a half century ago, its techniques and results are still not widely known nor appreciated. This is due, no doubt, to the fact that Grothendieck included practically no proofs, and the presentation is based on the theory of the very abstract notion of tensor products. This book aims at providing the details of Grothendieck's constructions and laying bare how the important classes of operators are a consequence of the abstract operations on tensor norms. Particular attention is paid to how the classical Banach spaces ($C(K)$'s, Hilbert spaces, and the spaces of integrable functions) fit naturally within the mosaic that Grothendieck constructed.

Contents: Basics on tensor norms; The role of $C(K)$-spaces and L^1-spaces; ⊗-norms related to Hilbert space; The fundamental theorem and its consequences; Glossary of terms; The problems of the Résumé; The Blaschke selection principle and compact convex sets in finite dimensional Banach spaces; A short introduction to Banach lattices; Stonean spaces and injectivity; Epilogue; Bibliography; Author index; Index of notation; Index.

June 2008, approximately 287 pages, Hardcover, ISBN: 978-0-8218-4440-3, LC 2007062042, 2000 *Mathematics Subject Classification:* 46-02, 46B03, 46B04, 46B28, 47L20, **AMS members US$63**, List US$79, Order code MBK/52



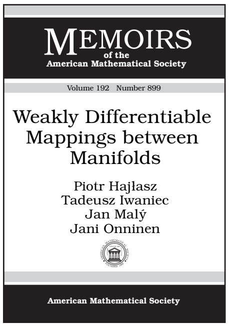
Limit Theorems of Polynomial Approximation with Exponential Weights

Michael I. Ganzburg, *Hampton University, VA*

Contents: Introduction; Statement of main results; Properties of harmonic functions; Polynomial inequalities with exponential weights; Entire functions of exponential type and their approximation properties; Polynomial interpolation and approximation of entire functions of exponential type; Proofs of the limit theorems; Applications; Multidimensional limit theorems of polynomial approximation with exponential weights; Examples; Appendix A. Negativity of a kernel; Bibliography; Index.

Memoirs of the American Mathematical Society, Volume 192, Number 897

March 2008, 161 pages, Softcover, ISBN: 978-0-8218-4063-4, LC 2007060582, 2000 *Mathematics Subject Classification:* 30D15, 41A30; 31A05, 41A17, **Individual member US$41**, List US$69, Institutional member US$55, Order code MEMO/192/897



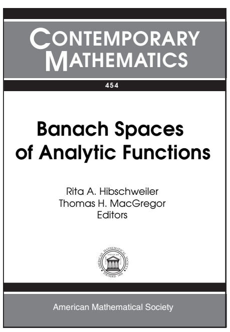
Weakly Differentiable Mappings between Manifolds

Piotr Hajłasz, *University of Pittsburgh, PA*, **Tadeusz Iwaniec**, *Syracuse University, NY*, **Jan Malý**, *Charles University, Prague, Czech Republic, and J. E. Purkyně University, Ústí nad Labem, Czech Republic*, and **Jani Onninen**, *Syracuse University, NY*

Contents: Introduction; Preliminaries concerning manifolds; Examples; Some classes of functions; Smooth approximation; $\mathcal{L}^1$-Estimates of the Jacobian; $\mathcal{H}^1$-Estimates; Degree theory; Mappings of finite distortion; Bibliography.

Memoirs of the American Mathematical Society, Volume 192, Number 899

March 2008, 72 pages, Softcover, ISBN: 978-0-8218-4079-5, LC 2007060584, 2000 *Mathematics Subject Classification:* 58D15; 46E35, **Individual member US$35**, List US$59, Institutional member US$47, Order code MEMO/192/899



Banach Spaces of Analytic Functions

Rita A. Hibschweiler, *University of New Hampshire, Durham, NH*, and **Thomas H. MacGregor**, *State University of New York at Albany, NY*, Editors

This volume is focused on Banach spaces of functions analytic in the open unit disc, such as the classical Hardy and Bergman spaces, and weighted versions of these spaces. Other spaces under consideration here include the Bloch space, the families of Cauchy transforms and fractional Cauchy transforms, BMO, VMO, and the Fock space. Some of the work deals with questions about functions in several complex variables.

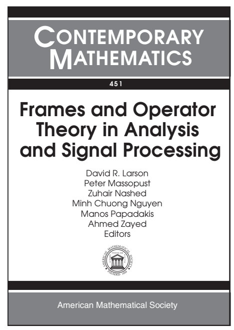
Multiplication operators, composition operators and weighted composition operators form a central topic of the volume. This topic has been an extensive area of research for the past twenty years. This volume includes results characterizing bounded, compact and isometric composition operators in various settings.

Graduate students who are interested in analysis will find an overview of current work in the field. Specialists will find interesting questions and new methods, as well as familiar ideas (such as composition operators) seen in new settings or in more general form. Mathematicians with an interest in modern analysis will gain insight into the interplay between function theory and operator theory which is central to this work.

Contents: **J. A. Cima**, A note on the life of Alec Matheson and some of his work; **J. A. Cima**, On the inverse of an analytic mapping; **J. Cohen** and **F. Colonna**, Isometric composition operators on the Bloch space in the polydisk; **O. Eroshkin**, Pluripolarity of manifolds; **R. Fournier** and **L. Salinas**, On a question of Brézis and Korevaar concerning a class of square-summable sequences; **Z. Guadarrama** and **D. Khavinson**, Approximating $\overline{z}$ in Hardy and Bergman norms; **D. Hadwin** and **E. Nordgren**, A geneal view of multipliers and composition operators II; **D. Hadwin** and **H. Yousefi**, A general view of BMO and VMO; **R. A. Hibschweiler**, Order bounded weighted composition operators; **T. H. MacGregor**, Fractional Cauchy transforms and composition; **A. L. Matheson**, Continuous functions in star-invariant subspaces; **W. T. Ross**, Indestructible Blaschke products; **J. Tung**, On Taylor coefficients and multipliers in Fock spaces.

Contemporary Mathematics, Volume 454

April 2008, 147 pages, Softcover, ISBN: 978-0-8218-4268-3, LC 2007060849, 2000 *Mathematics Subject Classification:* 30D15, 30D45, 30D50, 30D55, 30E20, 30H05, 32A18, 46E15, 47B33, **AMS members US$47**, List US$59, Order code CONM/454



Frames and Operator Theory in Analysis and Signal Processing

David R. Larson, *Texas A&M University, College Station, TX*, **Peter Massopust**, *Technische Universität München, Munich, Germany*, **Zuhair Nashed**, *University of Central Florida, Orlando, FL*, **Minh Chuong Nguyen**, *Vietnamese Academy of Science and Technology, Hanoi, Vietnam*, **Manos Papadakis**, *University of Houston, TX*, and **Ahmed Zayed**, *DePaul University, Chicago, IL*, Editors

This volume contains articles based on talks presented at the Special Session *Frames and Operator Theory in Analysis and Signal Processing*, held in San Antonio, Texas, in January of 2006.

Recently, the field of frames has undergone tremendous advancement. Most of the work in this field is focused on the design and construction of more versatile frames and frames tailored towards specific applications, e.g., finite dimensional uniform frames for cellular communication. In addition, frames are now becoming a hot topic in mathematical research as a part of many engineering applications, e.g., matching pursuits and greedy algorithms for image and signal processing. Topics covered in this book include:

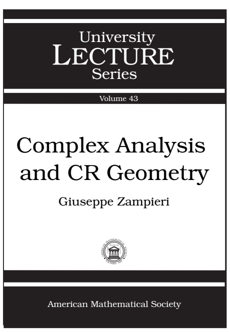
- Application of several branches of analysis (e.g., PDEs; Fourier, wavelet, and harmonic analysis; transform techniques; data representations) to industrial and engineering problems, specifically image and signal processing.
- Theoretical and applied aspects of frames and wavelets.
- Pure aspects of operator theory emphasizing the connections to applied mathematics, frames, and signal processing.

This volume will be equally attractive to pure mathematicians working on the foundations of frame and operator theory and their interconnections as it will to applied mathematicians investigating applications and to physicists and engineers employing these designs. It thus may appeal to a wide target group of researchers and may serve as a catalyst for cross-fertilization of several important areas of mathematics and the applied sciences.

Contents: **N. D. Atreas** and **C. Karanikas**, Haar-type orthonormal systems, data presentation as Riesz products and a recognition on symbolic sequences; **P. G. Casazza** and **N. Leonhard**, Classes of finite equal norm Parseval frames; **N. M. Chuong** and **N. V. Co**, *P*-adic pseudodifferential operators and wavelets; **E. Cordero** and **L. Rodino**, Short-time Fourier transform analysis of localization operators; **J. D'Andrea**, **K. D. Merrill**, and **J. Packer**, Fractal wavelets of Dutkay-Jorgensen type for the Sierpinski gasket space; **L. Dettori** and **A. I. Zayed**, Texture identification of tissues using directional wavelet, ridgelet and curvelet transforms; **M. Dobrescu** and **G. Ólafsson**, Coxeter groups, wavelets, multiresolution and sampling; **C. Heil** and **D. Larson**, Operator theory and modulation spaces; **P. E. T. Jorgensen**, Frame analysis and approximation in reproducing kernel hilbert spaces; **V. Kaftal**, **D. Larson**, and **S. Zhang**, Operator-valued frames on C*-modules; **D. R. Larson** and **P. Massopust**, Coxeter groups and wavelet sets; **C. Pöschl** and **O. Scherzer**, Characterization of minimizers of convex regularization functionals; **G. G. Walter** and **T. Soleski**, Error estimates for the PSWF method in MRI; **H. Šikić**, **D. Speegle**, and **G. Weiss**, Structure of the set of Dyadic PFW's.

Contemporary Mathematics, Volume 451

March 2008, 291 pages, Softcover, ISBN: 978-0-8218-4144-0, LC 2007060586, 2000 *Mathematics Subject Classification:* 11S80, 20F55, 35S99, 41A30, 42C15, 42C40, 47N40, 47S10, 94A12, **AMS members US$71**, List US$89, Order code CONM/451



Complex Analysis and CR Geometry

Giuseppe Zampieri, *University of Padova, Italy*

Cauchy-Riemann (CR) geometry is the study of manifolds equipped with a system of CR-type equations. Compared to the early days when the purpose of CR geometry was to supply tools for the analysis of the existence and regularity of solutions to the $\bar{\partial}$-Neumann problem, it has rapidly acquired a life of its own and has became an important topic in differential geometry and the study of non-linear partial differential equations. A full understanding of modern CR geometry requires knowledge of various topics such as real/complex differential and symplectic geometry, foliation theory, the geometric theory of PDE's, and microlocal analysis. Nowadays, the subject of CR geometry is very rich in results, and the amount of material required to reach competence is daunting to graduate students who wish to learn it.

However, the present book does not aim at introducing all the topics of current interest in CR geometry. Instead, an attempt is made to be friendly to the novice by moving, in a fairly relaxed way, from the elements of the theory of holomorphic functions in several complex variables to advanced topics such as extendability of CR functions, analytic discs, their infinitesimal deformations, and their lifts to the cotangent space. The choice of topics provides a good balance between a first exposure to CR geometry and subjects representing current research. Even a seasoned mathematician who wants to contribute to the subject of CR analysis and geometry will find the choice of topics attractive.

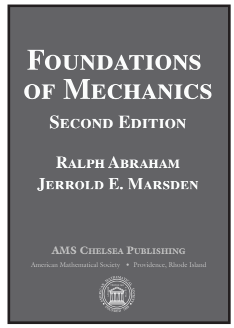
Contents: Several complex variables; Real structures; Real/complex structures; Bibliography; Subject index; Symbols index.

University Lecture Series, Volume 43

May 2008, approximately 204 pages, Softcover, ISBN: 978-0-8218-4442-7, LC 2007060580, 2000 *Mathematics Subject Classification:* 32-02, 32Qxx, 32Vxx, **AMS members US$36**, List US$45, Order code ULECT/43

Differential Equations

COURSE ADOPTION



Foundations of Mechanics

Second Edition

Ralph Abraham, *University of California, Santa Cruz, CA*, and **Jerrold E. Marsden**, *California Institute of Technology, Pasadena, CA*

Undoubtedly [the book] will be for years the standard reference on symplectic geometry, analytical mechanics and symplectic methods in mathematical physics.

—***Zentralblatt für Mathematik***

For many years, this book has been viewed as a classic treatment of geometric mechanics. It is known for its broad exposition of the subject, with many features that cannot be found elsewhere. The book is recommended as a textbook and as a basic reference work for the foundations of differentiable and Hamiltonian dynamics.

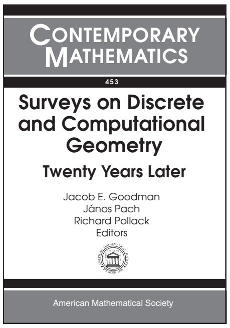
This item will also be of interest to those working in mathematical physics.

Contents: *Preliminaries:* Differential theory; Calculus on manifolds; *Analytical dynamics:* Hamiltonian and Lagrangian systems; Hamiltonian systems with symmetry; Hamilton-Jacobi theory and mathematical physics; *An outline of qualitative dynamics:* Topological dynamics; Differentiable dynamics; Hamiltonian dynamics; *Celestial mechanics:* The two-body problem; The three-body problem; The general theory of dynamical systems and classical mechanics by A. N. Kolmogorov; Bibliography; Index; Glossary of symbols; Errata.

AMS Chelsea Publishing

June 2008, 806 pages, Hardcover, ISBN: 978-0-8218-4438-0, LC 2008005206, 2000 *Mathematics Subject Classification:* 70-01; 37Jxx, 70Hxx, **AMS members US$89**, List US$99, Order code CHEL/364.H

Discrete Mathematics and Combinatorics



Surveys on Discrete and Computational Geometry

Twenty Years Later

Jacob E. Goodman and **János Pach**, *City College, CUNY, New York, NY*, and **Richard Pollack**, *Courant Institute, New York, NY*, Editors

This volume contains nineteen survey papers describing the state of current research in discrete and computational geometry as well as a set of open problems presented at the 2006 AMS-IMS-SIAM Summer Research Conference "Discrete and Computational Geometry—Twenty Years Later", held in Snowbird, Utah, in June 2006. Topics surveyed include metric graph theory, lattice polytopes, the combinatorial complexity of unions of geometric objects, line and pseudoline arrangements, algorithmic semialgebraic geometry, persistent homology, unfolding polyhedra, pseudo-triangulations, nonlinear computational geometry, k-sets, and the computational complexity of convex bodies.

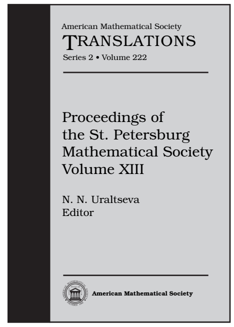
Discrete and computational geometry originated as a discipline in the mid-1980s when mathematicians in the well-established field of discrete geometry and computer scientists in the (then) nascent field of computational geometry began working together on problems of common interest. The combined field has experienced a huge growth in the past twenty years, which the present volume attests to.

Contents: B. Grünbaum, Musings on discrete geometry and "20 years of discrete & computational geometry"; **P. K. Agarwal**, **J. Pach**, and **M. Sharir**, State of the union (of geometric objects); **H.-J. Bandelt** and **V. Chepoi**, Metric graph theory and geometry: A survey; **I. Bárány**, Extremal problems for convex lattice polytopes: a survey; **N. Bartholdi**, **J. Blanc**, and **S. Loisel**, On simple arrangements of lines and pseudo-lines in $\mathbb{P}^2$ and $\mathbb{R}^2$ with the maximum number of triangles; **A. Barvinok** and **E. Veomett**, The computational complexity of convex bodies; **S. Basu**, Algorithmic semi-algebraic geometry and topology—recent progress and open problems; **R. Connelly**, Expansive motions; **E. D. Demaine**, **B. Gassend**, **J. O'Rourke**, and **G. T. Toussaint**, All polygons flip finitely…right?; **H. Edelsbrunner** and **J. Harer**, Persistent homology—a survey; **A. F. Holmsen**, Recent progress on line transversals to families of translated ovals; **G. Nivasch**, An improved, simple construction of many halving edges; **J. O'Rourke**, Unfolding orthogonal polyhedra; **R. Radoičić** and **G. Tóth**, The discharging method in combinatorial geometry and the Pach-Sharir conjecture; **G. Rote**, **F. Santos**, and **I. Streinu**, Pseudo-triangulations—a survey; **F. Sottile** and **T. Theobald**, Line problems in nonlinear computational geometry; **P. Valtr**, On empty hexagons; **U. Wagner**, k-sets and k-facets; **X. Wei** and **R. Ding**, An Erdős-Szekeres type problem for interior points; **C. Zong**, The kissing number, blocking number and covering number of a convex body; **J. Pach**, Open problems.

Contemporary Mathematics, Volume 453

March 2008, 556 pages, Softcover, ISBN: 978-0-8218-4239-3, LC 2007060848, 2000 *Mathematics Subject Classification:* 01A65, 05Cxx, 14Pxx, 51M20, 52Axx, 52Bxx, 52Cxx, 55-04, 68U05, **AMS members US$103**, List US$129, Order code CONM/453

General and Interdisciplinary



Proceedings of the St. Petersburg Mathematical Society, Volume XIII

N. N. Uraltseva, *St. Petersburg State University, Russia*, Editor

This volume contains articles on analysis, theory of algebraic groups, partial differential equations, pseudodifferential operators, wavelets, and other areas of mathematics.

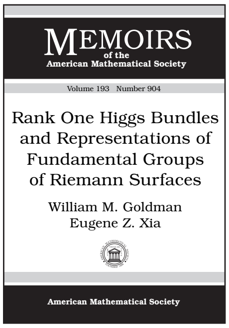
This book is suitable for a broad group of graduate students and researchers interested in the topics presented here.

Contents: Y. A. Alkhutov and **A. N. Gordeev**, L_p-estimates for solutions to second order parabolic equations; **Y. K. Dem'yanovich**, Wavelet decompositions on nonuniform grids; **A. I. Karol'**, Newton polyhedra and estimates for differential operators; **N. G. Kuznetsov**, Nodal lines and uniqueness of solutions to linear water-wave problems; **B. E. Kunyavskii** and **B. Z. Moroz**, On integral models of algebraic tori and affine toric varieties; **A. I. Lipovetskii**, A geometrical approach to computation of the optimal solution to the rectangle packing problem; **S. Matyukevich** and **P. Neittaanmäki**, Nonstationary Maxwell system with nonhomogeneous boundary conditions in domains with conical points; **J. Mashreghi** and **E. Fricain**, Exceptional sets for the derivatives of Blaschke products; **S. A. Nazarov** and **I. L. Sofronov**, Asymptotics of solutions and artificial boundary conditions in the transmission problem with a layer-like inhomogeneity.

American Mathematical Society Translations—Series 2, Volume 222

May 2008, approximately 238 pages, Hardcover, ISBN: 978-0-8218-4461-8, 2000 *Mathematics Subject Classification:* 11-06, **AMS members US$87**, List US$109, Order code TRANS2/222

Geometry and Topology



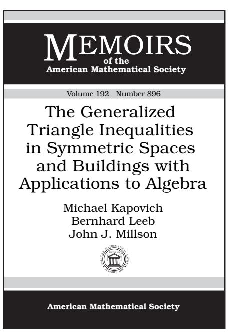
Rank One Higgs Bundles and Representations of Fundamental Groups of Riemann Surfaces

William M. Goldman, *University of Maryland, College Park, MD*, and **Eugene Z. Xia**, *National Cheng Kung University, Taiwan, Republic of China*

Contents: Introduction; Equivalences of deformation theories; The Betti and de Rham deformation theories and their moduli spaces; The Dolbeault groupoid; Equivalence of de Rham and Dolbeault groupoids; Hyperkähler geometry on the moduli space; The twistor space; The moduli space and the Riemann period matrix; Bibliography.

Memoirs of the American Mathematical Society, Volume 193, Number 904

March 2008, 69 pages, Softcover, ISBN: 978-0-8218-4136-5, LC 2008060005, 2000 *Mathematics Subject Classification:* 14H40, 30F30, 57M05, 53C26, **Individual member US$35**, List US$59, Institutional member US$47, Order code MEMO/193/904



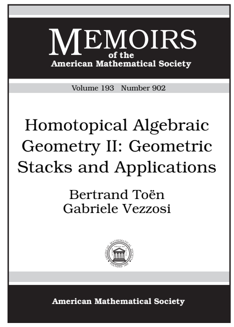
The Generalized Triangle Inequalities in Symmetric Spaces and Buildings with Applications to Algebra

Michael Kapovich, *University of California, Davis, CA*, **Bernhard Leeb**, *Universität München, Munich, Germany*, and **John J. Millson**, *University of Maryland, College Park, MD*

Contents: Introduction; Roots and Coxeter groups; The first three algebra problems and the parameter spaces Σ for $K\backslash\overline{G}/K$; The existence of polygonal linkages and solutions to the algebra problems; Weighted configurations, stability and the relation to polygons; Polygons in Euclidean buildings and the generalized invariant factor problem; The existence of fixed vertices in buildings and computation of the saturation factors for reductive groups; The comparison of problems **Q3** and **Q4**; Appendix A. Decomposition of tensor products; Appendix. Bibliography.

Memoirs of the American Mathematical Society, Volume 192, Number 896

March 2008, 83 pages, Softcover, ISBN: 978-0-8218-4054-2, LC 2007060581, 2000 *Mathematics Subject Classification:* 22E46, 20G15; 14L24, 20E42, **Individual member US$35**, List US$59, Institutional member US$47, Order code MEMO/192/896



Homotopical Algebraic Geometry II: Geometric Stacks and Applications

Bertrand Toën, *Université Paul Sabatier, Toulouse, France*, and **Gabriele Vezzosi**, *Università di Firenze, Italy*

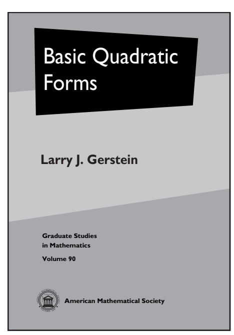
Contents: Introduction; *Part 1. General theory of geometric stacks:* Introduction to Part 1; Homotopical algebraic context; Preliminaries on linear and commutative algebra in an HA context; Geometric stacks: Basic theory; Geometric stacks: Infinitesimal theory; *Part 2. Applications:* Introduction to Part 2; Geometric n-stacks in algebraic geometry (after C. Simpson); Derived algebraic geometry; Complicial algebraic geometry; Brave new algebraic geometry; Appendix A. Classifying spaces of model categories; Appendix B. Strictification; Appendix C. Representability criterion (after J. Lurie); Bibliography; Index.

Memoirs of the American Mathematical Society, Volume 193, Number 902

March 2008, 224 pages, Softcover, ISBN: 978-0-8218-4099-3, LC 2008060003, 2000 *Mathematics Subject Classification:* 14A20, 18G55, 18F10, 55U40, 55P42, 55P43, 18F20, 18D10, 18E30, 18G35, 18G30, 13D10, 55N34, **Individual member US$47**, List US$79, Institutional member US$63, Order code MEMO/193/902

Number Theory

COURSE ADOPTION



Basic Quadratic Forms

Larry J. Gerstein, *University of California, Santa Barbara, CA*

The arithmetic theory of quadratic forms is a rich branch of number theory that has had important applications to several areas of pure mathematics—particularly group theory and topology—as well as to cryptography and coding theory. This book is a self-contained introduction to quadratic forms that is based on graduate courses the author has taught many times. It leads the reader from foundation material up to topics of current research interest—with special attention to the theory over the integers and over polynomial rings in one variable over a field—and requires only a basic background in linear and abstract algebra as a prerequisite. Whenever possible, concrete constructions are chosen over more abstract arguments. The book includes many exercises and explicit examples, and it is appropriate as a textbook for graduate courses or for independent study. To facilitate further study, a guide to the extensive literature on quadratic forms is provided.

Contents: A brief classical introduction; Quadratic spaces and lattices; Valuations, local fields, and p-adic numbers; Quadratic spaces over $\mathbb{Q}_p$; Quadratic spaces over $\mathbb{Q}$; Lattices over principal ideal domains; Initial integral results; Local classification of lattices; The local-global approach to lattices; Lattices over $\mathbb{F}_q[x]$; Applications to cryptography; Further reading; Bibliography; Index.

Graduate Studies in Mathematics, Volume 90

May 2008, approximately 260 pages, Hardcover, ISBN: 978-0-8218-4465-6, LC 2007062041, 2000 *Mathematics Subject Classification:* 11Exx, 12Exx, 15-XX, **AMS members US$44**, List US$55, Order code GSM/90

New AMS-Distributed Publications

Algebra and Algebraic Geometry

Lectures on Algebraic Geometry I

Günter Harder, *Max Planck Institute for Mathematics, Bonn, Germany*

This book—the first of two volumes—is an introduction to modern algebraic geometry. In the first volume the methods of homological algebra, theory of sheaves, and sheaf cohomology are developed. These methods are indispensable for modern algebraic geometry, but they are also fundamental for other branches of mathematics and of great interest in their own right.

In the last chapter of volume I these concepts are applied to the theory of compact Riemann surfaces. In this chapter the author makes clear how influential—particularly on many of the modern methods—the ideas of Abel, Riemann and Jacobi were.

A publication of Vieweg Verlag. The AMS is exclusive distributor in North America. Vieweg Verlag Publications are available worldwide from the AMS outside of Germany, Switzerland, Austria, and Japan.

Contents: Categories, products, projective and inductive limits; Basic concepts of homological algebra; Presheaves and sheaves; Cohomology of sheaves; Compact Riemann surfaces and abelian varieties.

Vieweg Aspects of Mathematics, Volume 35

December 2007, 300 pages, Hardcover, ISBN: 978-3-528-03136-7, 2000 *Mathematics Subject Classification:* 14-XX, **AMS members US$60**, List US$67, Order code VWAM/35

Analysis

Traces in Number Theory, Geometry and Quantum Fields

Sergio Albeverio, *Universitát Bonn, Germany*, **Matilde Marcolli**, *Max Planck Institute for Mathematics, Bonn, Germany*, **Sylvie Paycha**, *Université Blaise Pascal, Aubiere, France, and* **Jorge Plazas**, *Institut des Hautes Etudes Scientifiques, Bures-sur-Yvette, France*

This book collects recent interactions between the fields of global analysis, noncommutative geometry, number theory and dynamical systems, organized around the common theme of traces, residues and zeta functions. It consists of a series of articles, contributed by specialists in the various disciplines, aimed at stressing interactions and different perspectives.

This item will also be of interest to those working in number theory.

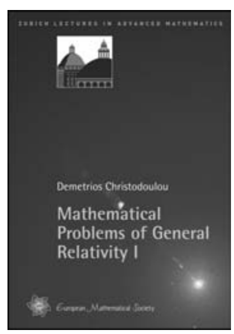
A publication of Vieweg Verlag. The AMS is exclusive distributor in North America. Vieweg Verlag Publications are available worldwide from the AMS outside of Germany, Switzerland, Austria, and Japan.

Contents: Number theory, dynamical systems, noncommutative geometry, differential geometry and quantum field theory are five areas of mathematics represented in this volume, which presents an overview of different ongoing research directions around the main theme of traces and zeta functions.

Vieweg Aspects of Mathematics, Volume 38

December 2007, 223 pages, Hardcover, ISBN: 978-3-8348-0371-9, 2000 *Mathematics Subject Classification:* 58J42, 58J52, 11M36, 81S40, **AMS members US$60**, List US$67, Order code VWAM/38

Mathematical Physics



Mathematical Problems of General Relativity I

Demetrios Christodoulou, *ETH-Zurich, Switzerland*

General relativity is a theory proposed by Einstein in 1915 as a unified theory of space, time and gravitation. It is based on and extends Newton's theory of gravitation as well as Newton's equations of motion. It is thus fundamentally rooted in classical mechanics. The theory can be seen as a development of Riemannian geometry, itself an extension of Gauss' intrinsic theory of curved surfaces in Euclidean space. The domain of application of the theory is astronomical systems.

One of the mathematical methods analyzed and exploited in the present volume is an extension of Noether's fundamental principle connecting symmetries to conserved quantities. This is involved at a most elementary level in the very definition of the notion of hyperbolicity for an Euler-Lagrange system of partial differential equations. Another method, the study and systematic use of foliations by characteristic (null) hypersurfaces, is in the spirit of Roger Penrose's approach in his incompleteness theorem. The methods have applications beyond general relativity to problems in fluid mechanics and, more generally, to the mechanics and electrodynamics of continuous media.

The book is intended for advanced students and researchers seeking an introduction to the methods and applications of general relativity.

This item will also be of interest to those working in differential equations.

A publication of the European Mathemati cal Society (EMS). Distributed within the Americas by the American Mathematical Society.

Contents: Introduction; The laws of general relativity; Asymptotic flatness at spacelike infinity and conserved quantities; The global stability of Minkowski spacetime; Bibliography; Index.

Zurich Lectures in Advanced Mathematics

February 2008, 157 pages, Softcover, ISBN: 978-3-03719-005-0, 2000 *Mathematics Subject Classification:* 83C05, 35L70, 35Q75, 53C50, 58J45, 83C40, 83C45, **AMS members US$26**, List US$32, Order code EMSZLEC/6

Classified Advertisements

Positions available, items for sale, services available, and more

WEST VIRGINIA

WEST VIRGINIA STATE UNIVERSITY MATHEMATICS DEPARTMENT ASSISTANT PROFESSOR

The Department of Mathematics at West Virginia State University announces a tenure-track position at the rank of assistant professor starting in August of 2008. A Ph.D. in one of the mathematical sciences or near completion of the doctorate is required. Ability and/or interest in teaching computer science is a plus. Send letter of application, resume, statement of teaching philosophy, statement of research interests, copies of transcripts (originals required upon acceptance of offer), and three letters of recommendation to: Dr. Katherine Harper, Dean, College of Natural Sciences and Mathematics, West Virginia State University, P. O. Box 1000, Institute, WV 25112-1000. Review of applications will begin in January, 2008, and continue until the position is filled. Incomplete applications will not be considered. The Mathematics Department at WVSU offers Bachelor of Science Degrees in Mathematics and Computer Science. Please see the WVSU homepage at `http://www.wvstateu.edu` for more information. West Virginia State University is a historically black land-grant institution, which has evolved into a fully accessible, racially diverse, multi-generational masters-granting university. West Virginia State University is the largest institution of higher education in the Charleston metropolitan area, serving approximately 5,000 students.

WVSU and WVSCTC are Equal Opportunity, Affirmative Action Institutions that do not discriminate. For full disclaimer see `http://mail.wvstateu.edu/disclaimer.html`.

000027

CHILE

PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE Departamento de Matemáticas

The Department of Mathematics invites applications for two tenure-track positions at the assistant professor level beginning either March or August 2009. Applicants should have a Ph.D. in mathematics, proven research potential either in pure or applied mathematics, and a strong commitment to teaching and research. The regular teaching load for assistant professors consists of three one-semester courses per year, reduced to two during the first two years. The annual salary will be US$36,000. Please send a letter indicating your main research interests, potential collaborators in our department (`http://www.mat.puc.cl`), detailed curriculum vitae, and three letters of recommendation to:

Director
Departamento de Matemáticas
Pontificia Universidad Católica de Chile
Av. Vicuña Mackenna 4860
Santiago, Chile;
fax: (56-2) 552-5916;
email: `mchuaqui@mat.puc.cl`

For full consideration, complete application materials must arrive by May 31, 2008.

000022

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Looking for professional help to solve (for compensation) a mathematical problem (eventually to be included in scientific paper for publication). The problem resembles tracking a very small number of decaying radioactive particles having a lifetime that is a continuous function of time. An exact mathematical description (assumingly involving path-integral formalism) and a method of precise statistical computation (possibly requiring Monte Carlo methods) is needed. Please email William Atkins at `waarc@grics.net`.

000026

Suggested uses for classified advertising are positions available, books or lecture notes for sale, books being sought, exchange or rental of houses, and typing services.

The 2007 rate is $110 per inch or fraction thereof on a single column (one-inch minimum), calculated from top of headline. Any fractional text of 1/2 inch or more will be charged at the next inch rate. No discounts for multiple ads or the same ad in consecutive issues. For an additional $10 charge, announcements can be placed anonymously. Correspondence will be forwarded.

Advertisements in the "Positions Available" classified section will be set with a minimum one-line headline, consisting of the institution name above body copy, unless additional headline copy is specified by the advertiser. Headlines will be centered in boldface at no extra charge. Ads will appear in the language in which they are submitted.

There are no member discounts for classified ads. Dictation over the telephone will not be accepted for classified ads.

Upcoming deadlines for classified advertising are as follows: May 2008–February 28, 2008; June/July 2008 issue–April 28, 2008; August 2008 issue–May 28, 2008; September 2008 issue–June 30, 2008; October 2008 issue–July 29, 2008; November 2008 issue–August 28, 2008.

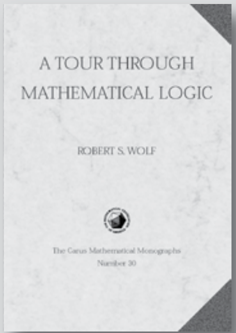
U.S. laws prohibit discrimination in employment on the basis of color, age, sex, race, religion, or national origin. "Positions Available" advertisements from institutions outside the U.S. cannot be published unless they are accompanied by a statement that the institution does not discriminate on these grounds whether or not it is subject to U.S. laws. Details and specific wording may be found on page 1373 (vol. 44).

Situations wanted advertisements from involuntarily unemployed mathematicians are accepted under certain conditions for free publication. Call toll-free 800-321-4AMS (321-4267) in the U.S. and Canada or 401-455-4084 worldwide for further information.

Submission: Promotions Department, AMS, P.O. Box 6248, Providence, Rhode Island 02940; or via fax: 401-331-3842; or send email to `classads@ams.org`. AMS location for express delivery packages is 201 Charles Street, Providence, Rhode Island 20904. Advertisers will be billed upon publication.

From the **Mathematical Association of America**®

A Tour through Mathematical Logic



Catalog Code: CAM-30/NT
408 pp., Hardbound, 2005
SBN 978-0-88385-036-7
List: $59.95
MAA Member: $47.95

SERIES: Carus Mathematical Monographs
Robert S. Wolf

"This is a remarkable book, both in terms of the considerable material that it covers and the "user friendliness" with which it is presented.... The author ought to be commended for having written such a clear and carefully crafted introduction to the whole field of mathematical logic."

—Yehuda Rav, *Mathematical Reviews*

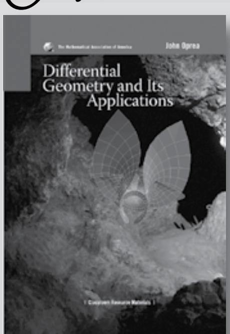
"This is a book I would have cherished when I was beginning research in mathematical logic. It surveys the background knowledge that every researcher in this field should have.... This is a very welcome book. Its enthusiasm and semiformality make it a novel introduction to mathematical logic."

—Wilfrid Hodges, SIAM

This addition to the Carus Mathematical Monograph Series provides a tour through the main branches of the foundations of mathematics. It contains chapters covering elementary logic, basic set theory, recursion theory, Gödel's (and others') incompleteness theorems, model theory, and independence results in set theory, nonstandard analysis, and constructive mathematics. The book could serve as the basis for a course at the undergraduate or graduate level in the foundations of mathematics.

New

Differential Geometry and Its Applications | Second Edition



Catalog Code: DGA/NT
510 pp., Hardbound, 2007
SBN 978-0-88385-748-9
List: $69.96
MAA Member: $47.00

SERIES: MAA Textbooks
John Oprea

Can be used as a textbook for a graduate or undergraduate course in Differential Geometry or the Calculus of Variations.

Differential geometry has a long, wonderful history, and it has found new relevance in areas ranging from machinery design to the classification of four-manifolds to the creation of theories of nature's fundamental forces to the study of DNA. This book studies the differential geometry of surfaces with the goal of helping students make the transition from the compartmentalized courses of a standard university curriculum to a type of mathematics that is a unified whole, mixing together geometry, calculus, linear algebra, differential equations, complex variables, the calculus of variations, and various notions from the sciences.

Differential Geometry and Its Applications is not just for mathematics majors, but encompasses techniques and ideas relevant to students in engineering and the sciences. Into the mix of these ideas now comes the opportunity to visualize concepts through the use of computer algebra systems such as *Maple*.

Table of Contents:

To order, **call 1-800-331-1622 or go online to www.maa.org.**

Meetings & Conferences of the AMS

IMPORTANT INFORMATION REGARDING MEETINGS PROGRAMS: AMS Sectional Meeting programs do not appear in the print version of the *Notices.* However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on the AMS website. See `http://www.ams.org/meetings/`. Final programs for Sectional Meetings will be archived on the AMS website accessible from the stated URL and in an electronic issue of the *Notices* as noted below for each meeting.

Baton Rouge, Louisiana

Louisiana State University, Baton Rouge

March 28–30, 2008

Friday – Sunday

Meeting #1037

Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: February 2008
Program first available on AMS website: February 14, 2008
Program issue of electronic *Notices*: March 2008
Issue of *Abstracts*: Volume 29, Issue 2

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Soren Galatius, Stanford University, *Stable homology of automorphism groups of free groups*.

Maria Chudnovsky, Department of IEOR, Columbia University, *Even pairs in perfect graphs*.

Zhongwei Shen, University of Kentucky, *The celebrated Calderón-Zygmund lemma revisited*.

Mark Shimozono, Virginia Polytechnic Institute & State University, *Schubert calculus for the affine Grassmannian*.

Special Sessions

Actions of Quantum Algebras, **Lars Kadison**, University of Pennsylvania, and **Alexander Stolin**, University of Gothenburg/Chalmers University of Technology.

Algebraic Geometry of Matrices and Determinants, **Zachariah C. Teitler**, Texas A&M University, and **Kent M. Neuerburg**, Southeastern Louisiana University.

Arrangements and Related Topics, **Daniel C. Cohen**, Louisiana State University.

Current Challenges in Financial Mathematics, **Arkadev Chatterjea**, Kenan-Flagler Business School, The University of North Carolina at Chapel Hill, and **Ambar Sengupta**, Louisiana State University.

Elementary Mathematics from an Advanced Perspective, **James J. Madden**, Louisiana State University, and **Kristin L. Umland**, University of New Mexico.

Gauge Theory in Smooth and Symplectic Topology, **Scott J. Baldridge** and **Brendan E. Owens**, Louisiana State University.

Geometric Group Theory, **Noel Brady**, University of Oklahoma, **Tara E. Brendle**, Louisiana State University, and **Pallavi Dani**, University of Oklahoma.

Geometric and Combinatorial Representation Theory, **Pramod N. Achar** and **Daniel S. Sage**, Louisiana State University.

Harmonic Analysis and Partial Differential Equations in Real and Complex Domains, **Loredana Lanzani**, University of Arkansas, and **Zhongwei Shen**, University of Kentucky.

Knot and 3-Manifold Invariants, **Oliver T. Dasbach** and **Patrick M. Gilmer**, Louisiana State University.

Lie Groups and Holomorphic Function Spaces: Analysis, Geometry, and Mathematical Physics, **Brian C. Hall**, University of Notre Dame, and **Jeffrey J. Mitchell**, Robert Morris University.

Lie Groups, Lie Algebras, and Their Representations, **Mark C. Davidson**, Louisiana State University, and **Ronald Stanke**, Baylor University.

Mathematical Modeling in Biology, **Hongyu He**, Louisiana State University, **Sergei S. Pilyugin**, University of Florida, and **Jianjun Tian**, College of William and Mary.

Matroid Theory, **Bogdan S. Oporowski** and **James G. Oxley**, Louisiana State University.

Nonlinear Evolution Equations of Mathematical Physics, **Jerry L. Bona**, University of Illinois at Chicago, and **Michael M. Tom**, Louisiana State University.

Number Theory and Applications in Other Fields, **Jorge Morales**, Louisiana State University, **Robert Osburn**, University College Dublin, and **Robert V. Perlis** and **Helena Verrill**, Louisiana State University.

Radon Transforms, Tomography, and Related Geometric Analysis, **Fulton B. Gonzalez**, Tufts University, **Isaac Pesenson**, Temple University, **Todd Quinto**, Tufts University, and **Boris S. Rubin**, Louisiana State University.

Recent Advances in Knot Theory: Quandle Theory and Categorified Knot Invariants, **Sam Nelson**, Pomona College, and **Alissa S. Crans**, Loyola Marymount University.

Recent Trends in Partial Differential Equations, **Wai Yuen Chan**, Southeast Missouri State University.

Structural Graph Theory, **Maria Chudnovsky**, Columbia University.

Wavelets, Frames, and Multi-Scale Constructions, **Palle E. T. Jorgensen**, University of Iowa, **David R. Larson**, Texas A&M University, **Gestur Olafsson**, Louisiana State University, and **Darrin Speegle**, Saint Louis University.

White Noise Distribution Theory and Orthogonal Polynomials, **Jeremy J. Becnel**, Stephen F. Austin State University, and **Aurel I. Stan**, The Ohio State University at Marion.

Bloomington, Indiana

Indiana University

April 5–6, 2008

Saturday – Sunday

Meeting #1038

Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: February 2008
Program first available on AMS website: February 21, 2008
Program issue of electronic *Notices*: April 2008
Issue of *Abstracts*: Volume 29, Issue 3

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Shi Jin, University of Wisconsin, *Hamiltonian systems and Liouville equations with discontinuous Hamiltonians: Computataional methods for high frequency waves in heterogeneous media*.

Michael J. Larsen, Indiana University, *Solving equations in finite groups*.

Mircea Mustata, University of Michigan, *Invariants of singularities in algebraic geometry*.

Margaret H. Wright, Courant Institute, NYU, *Nonderivative optimization: Mathematics, heuristics, or hack?*

Special Sessions

Algebraic Aspects of Coding Theory, **Heide Gluesing-Luerssen**, University of Kentucky, and **Roxana Smarandache**, San Diego State University.

Algebraic K-theory and Nil groups in Algebra and Topology, **James F. Davis**, Indiana University, and **Christian Haesemeyer**, University of Illinois at Chicago.

Applications of Ring Spectra, **Randy McCarthy**, University of Illinois at Urbana-Champaign, and **Ayelet Lindenstrauss**, Indiana University.

Birational Algebraic Geometry, **Mircea I. Mustata**, University of Michigan, and **Mihnea Popa**, University of Illinois at Chicago.

Combinatorial Representation Theory, Topological Combinatorics, and Interactions Between Them, **Patricia Hersh**, Indiana University, **Cristian P. Lenart**, State University of New York at Albany, and **Michelle Wachs**, University of Miami.

Combinatorial and Geometric Aspects of Commutative Algebra, **Juan Migliore**, University of Notre Dame, and **Uwe Nagel**, University of Kentucky.

D-modules, **Mathias Schulze**, Oklahoma State University, and **Hans Ulrich Walther**, Purdue University.

Discrete Structures in Conformal Dynamics and Geometry, **Kevin M. Pilgrim**, Indiana University, and **William J. Floyd**, Virginia Polytech Institute & State University.

Financial Mathematics, **Victor Goodman**, Indiana University, and **Kiseop Lee**, University of Louisville.

Finite Element Methods and Applications, **Nicolae Tarfulea**, Purdue University Calumet, and **Sheng Zhang**, Wayne State University.

Geometry and Dynamics, **Chris Connell**, **David M. Fisher**, and **Marlies Gerber**, Indiana University.

Graph Theory, **Jozsef Balogh**, University of Illinois at Urbana-Champaign, **Hemanshu Kaul**, Illinois Institute of Technology, and **Tao Jiang**, Miami University.

Harmonic Analysis Methods in Mathematical Fluid Dynamics, **Zoran Grujic** and **Irina Mitrea**, University of Virginia.

Harmonic Analysis and Related Topics, **Ciprian Demeter**, Institute for Advance Study, and **Nets Katz**, Indiana University.

Hyperbolic and Kinetic Equations, **Shi Jin** and **Marshall Slemrod**, University of Wisconsin.

Mathematical Modeling of Cell Motility: From Molecular Events to Mechanical Movement, **Anastasios Matzavinos**, Ohio State University, and **Nicoleta Eugenia Tarfulea**, Purdue University Calumet.

Minimal and CMC Surfaces, **Bruce Michael Solomon** and **Matthias Weber**, Indiana University, and **Adam Weyhaupt**, Southern Illinois University.

Operator Algebras and Applications, **Hari Bercovici**, Indiana University, **Marius Dadarlat**, Purdue University, and **Mihai Popa**, Indiana University.

Probability and Spatial Systems, **Russell D. Lyons**, Indiana University, and **Alexander Holroyd**, University of British Columbia.

Recent Advances in Classical and Geophysical Fluid, **Roger Temam** and **Shouhong Wang**, Indiana University.

Some Mathematical Problems in Biology, from Macromolecules to Ecosystems, **Santiago David Schnell** and **Roger Temam**, Indiana University.

Weak Dependence in Probability and Statistics, **Richard C. Bradley** and **Lahn T. Tran**, Indiana University.

Claremont, California

Claremont McKenna College

May 3–4, 2008
Saturday – Sunday

Meeting #1039
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: March 2008
Program first available on AMS website: March 20, 2008
Program issue of electronic *Notices*: May 2008
Issue of *Abstracts*: Volume 29, Issue 3

Deadlines
For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Michael Bennett, University of British Columbia, *Title to be announced.*

Chandrashekhar Khare, University of Utah, *Title to be announced.*

Huaxin Lin, University of Oregon, *Title to be announced.*

Anne Schilling, University of California Davis, *Title to be announced.*

Special Sessions

Algebraic Combinatorics, **Anne Schilling**, University of California Davis, and **Michael Zabrocki**, York University.

Applications of Delay-Differential Equations to Models of Disease, **Ami Radunskaya**, Pomona College.

C-algebras, subfactors and free probability*, **Huaxin Lin**, University of Oregon, and **Feng Xu**, University of California Riverside.

Combinatorics of Partially Ordered Sets, **Timothy M. Hsu**, San Jose State University, **Mark J. Logan**, University of Minnesota-Morris, and **Shahriar Shahriari**, Pomona College.

Diophantine Problems and Discrete Geometry, **Matthias Beck**, San Francisco State University, and **Lenny Fukshansky**, Texas A&M University.

Dynamical Systems and Differential Equations, **Adolfo Rumbos**, Pomona College, **Mario Martelli**, Claremont McKenna College, and **Alfonso Castro**, Harvey Mudd College.

Hopf Algebras and Quantum Groups, **Gizem Karaali**, Pomona College, **M. Susan Montgomery**, University of Southern California, and **Serban Raianu**, California State University Dominguez Hills.

Knot Theory and the Topology of 3-manifolds, **Sam Nelson** and **Erica Flapan**, Pomona College, **David Bachman** and **Jim Hoste**, Pitzer College, and **Patrick Shanahan**, Loyola Marymount University.

Operators, Functions and Linear Spaces, **Asuman G. Aksoy** and **Michael Davlin O'Neill**, Claremont McKenna College, **Stephan R. Garcia**, Pomona College, and **Winston C. Ou**, Scripps College.

Recent Developments in Riemannian and Kaehlerian Geometry, **Hao Fang**, University of Iowa, **Zhiqin Lu**, University of California, Irvine, **Dragos-Bogdan Suceava**, California State University Fullerton, and **Mihaela B. Vajiac**, Chapman University.

Rio de Janeiro, Brazil

Instituto Nacional de Matemática Pura e Aplicada (IMPA)

June 4–7, 2008
Wednesday – Saturday

Meeting #1040
First Joint International Meeting between the AMS and the Sociedade Brasileira de Matemática (SBM).
Associate secretary: Lesley M. Sibner

Announcement issue of *Notices*: February 2008
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/internmtgs.html.

AMS Invited Addresses

Ruy Exel, Universidade Federal de Santa Catarina, *Noncommutative dynamics*.

Velimir Jurdjevic, University of Toronto, *Integrable Hamiltonian systems on symmetric spaces*.

Andre Nachbin, IMPA, *Wave dynamics: Asymptotics with differential operators and solutions*.

Richard M. Schoen, Stanford University, *Riemannian manifolds of positive curvature*.

Ivan P. Shestakov, University of Sao Paulo, *Automorphisms of free algebras*.

Amie Wilkinson, Northwestern University, *Partially hyperbolic dynamics*.

AMS Special Sessions

Some sessions are cosponsored with other organizations. These are noted within the parenthesis at the end of each listing, where applicable.

Commutative Algebra and Algebraic Geometry, **Izzet Coskun**, University of Illinois at Chicago, and **Israel Vainsencher**, UFMG (AMS-SBM).

Complexity, **Gregorio Malajovich**, Universidade Federal do Rio de Janeiro, and **J. Maurice Rojas**, Texas A&M University (AMS-SBM).

Control and Related Topics, **Jair Koiller**, FGV, **Velimir Jurdgevic**, University of Toronto, and **Fernando Lizarralde**, COPPE/UFRJ (AMS-SBM).

Extremal and Probablistic Combinatorics, **Bela Bollobas**, The University of Memphis, and **Yoshiharu Kohayakawa**, University of Sao Paulo (AMS-SBM).

Geometry, Representation Theory, and Mathematical Physics, **Henrique Bursztyn**, IMPA, **Anthony Licata**, Stanford University, and **Alistair Savage**, University of Ottawa (AMS-SBM).

Group Theory, **Rostislav I. Grigorchuk**, **Volodymyr Nekrashevych**, and **Zoran Sunic**, Texas A&M University, and **Said N. Sidki** and **Pavel Zalesskii**, University of Brasilia (AMS-SBM).

History and Philosophy of Mathematics, **Sergio Nobre**, Universidade Estadual Paulista-Rio Claro, and **James J. Tattersall**, Providence College (AMS-SBM).

Lie and Jordan Algebras and Their Applications, **Ivan K. Dimitrov**, Queen's University, **Vyacheslav Futorny**, University of Sao Paulo, and **Vera Serganova**, University of California Berkeley (AMS-SBM).

Low Dimensional Dynamics, **Andre de Carvalho**, University of Sao Paulo, and **Misha Lyubich** and **Marco Martens**, SUNY at Stony Brook (AMS-SBM).

Low Dimensional Topology, **Louis H. Kauffman**, University of Illinois at Chicago, and **Pedro M. Lopes**, Instituto Superior Tecnico, Technical University of Lisbon (AMS-SBM).

Mathematical Fluid Dynamics, **Susan J. Friedlander**, University of Southern California, **Milton Lopes Filho** and **Helena Nussenzveig Lopes**, University of Campinus, and **Maria Elena Schonbek**, University of California Santa Cruz (AMS-SBM).

Mathematical Methods in Biology, **Julie C. Mitchell**, University of Wisconsin-Madison, and **Max O. Souza**, Universidade Federal Fluminense (AMS-SBM).

Mathematical Methods in Image Processing, **Stacey Levine**, Duquesne University, and **Celia A. Zorzo Barcelos**, Federal University of Uberlândia (AMS-SBM).

Partial Differential Equations, Harmonic Analysis, and Related Questions, **Haroldo R. Clark**, Universidade Federal Fluminense, **Michael Stessin**, University at Albany, and **Geraldo Soares de Souza**, Auburn University (AMS-SBM).

Several Complex Variables and Partial Differential Equations, **Shiferaw Berhanu**, Temple University, and **Jorge Hounie**, Federal University of San Carlos (AMS-SBM).

Vancouver, Canada

University of British Columbia and the Pacific Institute of Mathematical Sciences (PIMS)

October 4–5, 2008

Saturday – Sunday

Meeting #1041

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: August 2008
Program first available on AMS website: August 21, 2008
Program issue of electronic *Notices*: October 2008
Issue of *Abstracts*: Volume 29, Issue 4

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: June 17, 2008
For abstracts: August 12, 2008

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Freeman Dyson, Institute for Advanced Study, *Birds and Frogs* (Einstein Public Lecture in Mathematics).

Richard Kenyon, Brown University, *Title to be announced.*

Alexander S. Kleshchev, University of Oregon, *Title to be announced.*

Mark Lewis, University of Alberta, *Title to be announced.*

Audrey A. Terras, University of California San Diego, *Title to be announced.*

Special Sessions

Algebraic Topology and Related Topics (Code: SS 3A), **Alejandro Adem**, University of British Columbia, and **Stephen Ames Mitchell**, University of Washington.

Combinatorial Representation Theory (Code: SS 1A), **Sara C. Billey**, University of Washington, **Alexander S. Kleshchev**, University of Oregon, and **Stephanie Jane Van Willigenburg**, University of British Columbia.

Hilbert Functions and Free Resolutions (Code: SS 4A), **Susan Cooper**, California Polytechnic State University, **Christopher A. Francisco**, Oklahoma State University, and **Benjamin P. Richert**, California Polytechnic State University.

History and Philosophy of Mathematics (Code: SS 11A), **Shawnee L. McMurran**, California State University San Bernardino, and **James J. Tattersall**, Providence College.

Moduli Spaces and Singularity Theory (Code: SS 8A), **James B. Carrell**, **Patrick Brosnan**, and **Kalle Karu**, University of British Columbia.

Noncommutative Algebra and Geometry (Code: SS 6A), **Jason Bell**, Simon Fraser University, and **James Zhang**, University of Washington.

Nonlinear Functional Analysis, Approximations and Applications (Code: SS 10A), **S. P. Singh** and **Mahi Singh**, University of Western Ontario, and **Pranesh Kumar**, University of Northern British Columbia.

Probability and Statistical Mechanics (Code: SS 7A), **David Brydges**, University of British Columbia, and **Richard Kenyon**, Brown University.

Special Functions and Orthogonal Polynomials (Code: SS 2A), **Mizanur Rahman**, Carleton University, and **Diego Dominici**, State University of New York New Paltz.

Wavelets, Fractals, Tilings and Spectral Measures (Code: SS 5A), **Dorin Ervin Dutkay**, University of Central Florida, **Palle E. T. Jorgensen**, University of Iowa, and **Ozgur Yilmaz**, University of British Columbia.

West End Number Theory (Code: SS 9A), **Nils Bruin**, Simon Fraser University, **Matilde N. Lalin**, University of Alberta, and **Greg Martin**, University of British Columbia.

Middletown, Connecticut

Wesleyan University

October 11–12, 2008

Saturday – Sunday

Meeting #1042

Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: August 2008
Program first available on AMS website: August 28, 2008
Program issue of electronic *Notices*: October 2008
Issue of *Abstracts*: Volume 29, Issue 4

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: June 24, 2008
For abstracts: August 19, 2008

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Pekka Koskela, University of Jyväskylän, *Title to be announced.*

Monika Ludwig, Polytechnic University New York, *Title to be announced.*

Duong Hong Phong, Columbia University, *Title to be announced.*

Thomas W. Scanlon, University of California, Berkeley, *Title to be announced.*

Special Sessions

Algebraic Geometry (Code: SS 1A), **Eyal Markman** and **Jenia Tevelev**, University of Massachusetts, Amherst.

Complex Geometry and Partial Differential Equations. (Code: SS 3A), **Jacob Sturm**, Rutgers University.

Geometric Function Theory and Geometry (Code: SS 5A), **Petra Bonfert-Taylor**, Wesleyan University, **Katsuhiko Matsuzaki**, Okayama University, and **Edward C. Taylor**, Wesleyan University.

Geometric Group Theory and Topology (Code: SS 6A), **Matthew Horak**,
University of Wisconsin-Stout, **Melanie Stein**, Trinity College, and **Jennifer Taback**, Bowdoin College.

Number Theory (Code: SS 4A), **Wai Kiu Chan** and **David Pollack**, Wesleyan University.

Riemannian and Lorentzian Geometries (Code: SS 2A), **Ramesh Sharma**, University of New Haven, and **Philippe Rukimbira**, Florida International University.

Kalamazoo, Michigan

Western Michigan University

October 17–19, 2008

Friday – Sunday

Meeting #1043

Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: August 2008
Program first available on AMS website: September 4, 2008
Program issue of electronic *Notices*: October 2008
Issue of *Abstracts*: Volume 29, Issue 4

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: July 1, 2008
For abstracts: August 26, 2008

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

M. Carme Calderer, University of Minnesota, *Title to be announced.*

Alexandru Ionescu, University of Wisconsin, *Title to be announced.*

Boris S. Mordukhovich, Wayne State University, *Title to be announced.*

David Nadler, Northwestern University, *Title to be announced.*

Special Sessions

Affine Algebraic Geometry (Code: SS 9A), **Shreeram Abhyankar**, Purdue University, **Anthony J. Crachiola**, Saginaw Valley State University, and **Leonid G. Makar-Limanov**, Wayne State University.

Computation in Modular Representation Theory and Cohomology (Code: SS 2A), **Christopher P. Bendel**, University of Wisconsin-Stout, **Terrell L. Hodge**, Western Michigan University, **Brian J. Parshall**, University of Virginia, and **Cornelius Pillen**, University of South Alabama.

Graph Labeling, Graph Coloring, and Topological Graph Theory (Code: SS 5A), **Arthur T. White**, Western Michigan University, and **David L. Craft**, Muskingum College.

Homotopy Theory (Code: SS 8A), **Michele Intermont**, Kalamazoo College, and **John R. Martino** and **Jeffrey A. Strom**, Western Michigan University.

Mathematical Finance (Code: SS 3A), **Qiji J. Zhu**, Western Michigan University, and **George Yin**, Wayne State University.

Mathematical Knowledge for Teaching (Code: SS 7A), **Kate Kline** and **Christine Browning**, Western Michigan University.

Nonlinear Analysis and Applications (Code: SS 1A), **S. P. Singh**, University of Western Ontario, and **Bruce B. Watson**, Memorial University.

Optimization/Midwest Optimization Seminar (Code: SS 6A), **Jay S. Treiman** and **Yuri Ledyaev**, Western Michigan University, and **Ilya Shvartsman**, Penn State Harrisburg.

Representations of Real and P-adic Lie Groups (Code: SS 10A), **Alessandra Pantano**, University of California Irvine, **Annegret Paul**, Western Michigan University, and **Susana Alicia Salamanca-Riba**, New Mexico State University.

Variational Analysis and its Applications (Code: SS 4A), **Yuri Ledyaev** and **Jay S. Treiman**, Western Michigan University, **Ilya Shvartsman**, Penn State Harrisburg, and **Qiji J. Zhu**, Western Michigan University.

Huntsville, Alabama

University of Alabama, Huntsville

October 24–26, 2008

Friday – Sunday

Meeting #1044

Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: August 2008
Program first available on AMS website: September 11, 2008
Program issue of electronic *Notices*: October 2008
Issue of *Abstracts*: Volume 29, Issue 4

Deadlines

For organizers: March 24, 2008
For consideration of contributed papers in Special Sessions: July 8, 2008
For abstracts: September 2, 2008

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Mark Behrens, Massachusetts Institute of Technology, *Title to be announced.*

Anthony Michael Bloch, University of Michigan, Ann Arbor, *Title to be announced.*

Roberto Camassa, University of North Carolina, Chapel Hill, *Title to be announced.*

Mark V. Sapir, Vanderbilt University, *Title to be announced.*

Special Sessions

Dynamics and Applications of Differential Equations (Code: SS 3A), **Wenzhang Huang** and **Shangbing Ai**, University of Alabama in Huntsville, and **Weishi Liu**, University of Kansas.

Graph Decompositions (Code: SS 2A), **Robert A. Beeler** and **Robert B. Gardner**, East Tennessee State University.

Mathematical Biology: Modeling, Analysis, and Simulations (Code: SS 1A), **Jia Li**, University of Alabama in Huntsville, **Azmy S. Ackleh**, University of Louisiana at Lafayette, and **Maia Martcheva**, University of Florida.

Shanghai, People's Republic of China

Fudan University

December 17–21, 2008

Wednesday – Sunday

Meeting #1045

First Joint International Meeting Between the AMS and the Shanghai Mathematical Society
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: June 2008
Program first available on AMS website: Not applicable
Program issue of electronic *Notices*: Not applicable
Issue of *Abstracts*: Not applicable

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/internmtgs.html.

Invited Addresses

L. Craig Evans, University of California Berkeley, *Title to be announced.*

Zhi-Ming Ma, Chinese Academy of Sciences, *Title to be announced.*

Richard Schoen, Stanford University, *Title to be announced.*

Richard Taylor, Harvard University, *Title to be announced.*

Xiaoping Yuan, Fudan University, *Title to be announced.*

Weiping Zhang, Chern Institute, *Title to be announced.*

Special Sessions

Biomathematics: Newly Developed Applied Mathematics and New Mathematics Arising from Biosciences, **Banghe Li**, Chinese Academy of Sciences, **Reinhard C. Laubenbacher**, Virginia Bioinformatics Institute, and **Jianju Tian**, College of William and Mary.

Combinatorics and Discrete Dynamical Systems, **Reinhard C. Laubenbacher**, Virginia Bioinformatics Institute, **Klaus Sutner**, Carnegie Mellon University, and **Yaokun Wu**, Shanghai Jiao Tong University.

Differential Geometry and Its Applications, **Jianguo Cao**, University of Notre Dame, and **Yu Xin Dong**, Fudan University.

Dynamical Systems Arising in Ecology and Biology, **Qishao Lu**, Beijing University of Aeronautics & Astronautics, and **Zhaosheng Feng**, University of Texas-Pan American.

Harmonic Analysis and Partial Differential Equations with Applications, **Yong Ding**, Beijing Normal University, and **Guo-Zhen Lu**, Wayne State University.

Integrable System and Its Applications, **En-Gui Fan**, Fudan University, **Sen-Yue Lou**, Shanghai Jiao Tong University and Ningbo University, and **Zhi-Jun Qiao**, University of Texas-Pan American.

Integral and Convex Geometric Analysis, **Deane Yang**, Polytechnic University, and **Jiazu Zhou**, Southwest University.

Nonlinear Systems of Conservation Laws and Related Topics, **Gui-Qiang Chen**, Northwestern University, and **Shuxing Chen** and **Yi Zhou**, Fudan University.

Quantum Algebras and Related Topics, **Naihuan N. Jing**, North Carolina State University, **Quanshui Wu**, Fudan University, and **James J. Zhang**, University of Washington.

Recent Developments in Nonlinear Dispersive Wave Theory, **Jerry Bona**, University of Illinois at Chicago, **Bo Ling Guo**, Institute of Applied Physics and Computational Mathematics, **Shu Ming Sun**, Virginia Polytech Institute and State University, and **Bingyu Zhang**, University of Cincinnati.

Several Topics in Banach Space Theory, **Gerard J. Buskes** and **Qingying Bu**, University of Mississippi, and **Lixin Cheng**, Xiamen University.

Stochastic Analysis and Its Application, **Jiangang Ying**, Fudan University, and **Zhenqing Chen**, University of Washington.

Washington, District of Columbia

Marriott Wardman Park Hotel and Omni Shoreham Hotel

January 5–8, 2009

Monday – Thursday

Meeting #1046

Joint Mathematics Meetings, including the 115th Annual Meeting of the AMS, 92nd Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Bernard Russo
Announcement issue of *Notices*: October 2008

Program first available on AMS website: November 1, 2008
Program issue of electronic *Notices*: January 2009
Issue of *Abstracts*: Volume 30, Issue 1

Deadlines

For organizers: April 1, 2008
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Urbana, Illinois

University of Illinois at Urbana-Champaign

March 27–29, 2009

Friday – Sunday
Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: August 29, 2008
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Special Sessions

Geometric Group Theory (Code: SS 2A), **Sergei V. Ivanov**, **Ilya Kapovich**, **Igor Mineyev**, and **Paul E. Schupp**, University of Illinois at Urbana-Champaign.

q-Series and Partitions (Code: SS 1A), **Bruce Berndt**, University of Illinois at Urbana-Champaign, and **Ae Ja Yee**, Pennsylvania State University.

Raleigh, North Carolina

North Carolina State University

April 4–5, 2009

Saturday – Sunday
Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 4, 2008
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

San Francisco, California

San Francisco State University

April 25–26, 2009

Saturday – Sunday
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 25, 2008
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Worcester, Massachusetts

Worcester Polytechnic Institute

April 25–26, 2009

Saturday – Sunday
Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 25, 2008
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Waco, Texas

Baylor University

October 16–18, 2009

Friday – Sunday
Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 17, 2009
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Special Sessions

Dynamic Equations on Time Scales: Analysis and Applications (Code: SS 1A), **John M. Davis**, **Ian A. Gravagne**, and **Robert J. Marks**, Baylor University.

Boca Raton, Florida

Florida Atlantic University

October 30 – November 1, 2009

Friday – Sunday
Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 30, 2009
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Riverside, California

University of California

November 7–8, 2009

Saturday – Sunday
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 6, 2009
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

San Francisco, California

Moscone Center West and the San Francisco Marriott

January 6–9, 2010

Wednesday – Saturday
Joint Mathematics Meetings, including the 116th Annual Meeting of the AMS, 93rd Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society of Industrial and Applied Mathematics (SIAM).
Associate secretary: Matthew Miller
Announcement issue of *Notices*: October 2009
Program first available on AMS website: November 1, 2009
Program issue of electronic *Notices*: January 2010
Issue of *Abstracts*: Volume 31, Issue 1

Deadlines

For organizers: April 1, 2009
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Lexington, Kentucky

University of Kentucky

March 27–28, 2010

Saturday – Sunday

Southeastern Section

Associate secretary: Matthew Miller

Announcement issue of *Notices*: To be announced

Program first available on AMS website: To be announced

Program issue of electronic *Notices*: To be announced

Issue of *Abstracts*: To be announced

Deadlines

For organizers: August 28, 2009

For consideration of contributed papers in Special Sessions: To be announced

For abstracts: To be announced

St. Paul, Minnesota

Macalester College

April 10–11, 2010

Saturday – Sunday

Central Section

Associate secretary: Susan J. Friedlander

Announcement issue of *Notices*: To be announced

Program first available on AMS website: To be announced

Program issue of electronic *Notices*: To be announced

Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 10, 2009

For consideration of contributed papers in Special Sessions: To be announced

For abstracts: To be announced

New Orleans, Louisiana

New Orleans Marriott and Sheraton New Orleans Hotel

January 5–8, 2011

Wednesday – Saturday

Joint Mathematics Meetings, including the 117th Annual Meeting of the AMS, 94th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Susan J. Friedlander

Announcement issue of *Notices*: October 2010

Program first available on AMS website: November 1, 2010

Program issue of electronic *Notices*: January 2011

Issue of *Abstracts*: Volume 32, Issue 1

Deadlines

For organizers: April 1, 2010

For consideration of contributed papers in Special Sessions: To be announced

For abstracts: To be announced

Boston, Massachusetts

John B. Hynes Veterans Memorial Convention Center, Boston Marriott Hotel, and Boston Sheraton Hotel

January 4–7, 2012

Wednesday – Saturday

Joint Mathematics Meetings, including the 118th Annual Meeting of the AMS, 95th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Michel L. Lapidus

Announcement issue of *Notices*: October 2011

Program first available on AMS website: November 1, 2011

Program issue of electronic *Notices*: January 2012

Issue of *Abstracts*: Volume 33, Issue 1

Deadlines

For organizers: April 1, 2011

For consideration of contributed papers in Special Sessions: To be announced

For abstracts: To be announced

San Diego, California

San Diego Convention Center and San Diego Marriott Hotel and Marina

January 9–12, 2013

Wednesday – Saturday

Joint Mathematics Meetings, including the 119th Annual Meeting of the AMS, 96th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the

National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 1, 2012
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Baltimore, Maryland

Baltimore Convention Center

January 15–18, 2014

Wednesday – Saturday
Joint Mathematics Meetings, including the 120th Anual Meeting of the AMS, 97th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Matthew Miller
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 1, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

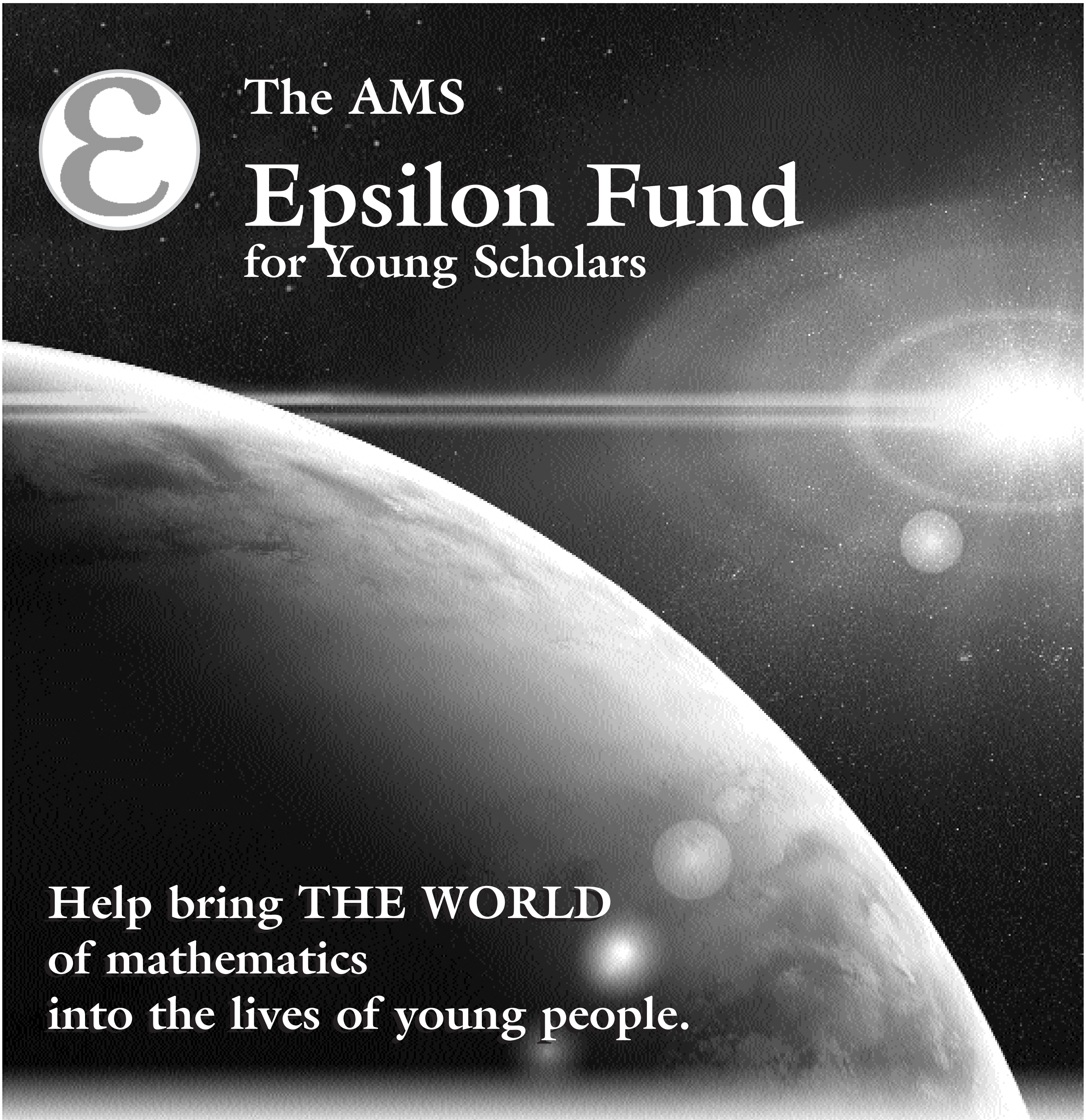
About the Cover

Chaos in the weak stability boundary

This month's cover was suggested by Shane Ross' review of Edward Belbruno's recent book *Fly Me to the Moon*. It displays, in a rotating coordinate system, one of Hill's regions for the restricted three-body problem modeling the Earth-Moon system, as well as a sample low-energy spacecraft trajectory starting near the Moon. What it illustrates is that certain orbits, starting near the region Belbruno calls the *weak stability boundary*, behave chaotically. Very roughly, this is because they pass through the narrow neck of Hill's surface surrounding the libration point between Earth and Moon. The global extension of that point's local invariant manifolds possesses the familiar structure of the homoclinic tangle. The sensitive behavior associated with chaos makes energy-efficient space travel feasible.

Belbruno first formulated the concept of WSB in 1986, and proposed spacecraft trajectories associated with it to offer low-energy lunar transfer. At the time his ideas were met with much skepticism, but in time they have become part of mainstream technology. Belbruno's intuitive notion of the WSB as a region lying on the boundary of stable trajectory behavior has become more and more precise as time goes on, but it is still not possible to find a completely satisfactory account in the literature, and indeed there remain many mysteries in the subject. That Belbruno's own formulation is not totally accurate is demonstrated in the preprint "A note on weak stability boundaries" by F. Garcìa and G. Gòmez, *Celestial Mechanics and Dynamical Astronomy*, February 2007. These authors show that the WSB is more complicated than Belbruno imagined, but also present striking evidence that his intuition relating the WSB to chaotic dynamics on the invariant manifolds of dynamical system theory is valid. The book *Dynamical Systems, the Three-Body Problem, and Space Mission Design* by W. S. Koon et al. presents a slightly different, and in many aspects clearer, way of looking at the same problem of planning space voyages. Ross is one of its authors, and it is available from his website at Virginia Tech (`http://www.esm.vt.edu/~sdross/`).

—Bill Casselman, Graphics Editor
(`notices-covers@ams.org`)



Whether they become scientists, engineers, or entrepreneurs, young people with mathematical talent need to be nurtured. Income from this fund supports the Young Scholars Program, which provides grants to summer programs for talented high school students.

Please give generously.

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(worldwide)
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09/04

Meetings and Conferences of the AMS

Associate Secretaries of the AMS

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Central Section: Susan J. Friedlander, Department of Mathematics, University of Illinois at Chicago, 851 S. Morgan (M/C 249), Chicago, IL 60607-7045; e-mail: susan@math.nwu.edu; telephone: 312-996-3041.

Eastern Section: Lesley M. Sibner (until January 31, 2009), Department of Mathematics, Polytechnic University, Brooklyn, NY 11201-2990; e-mail: lsibner@duke.poly.edu; telephone: 718-260-3505. **Steven Weintraub** (after January 31, 2009), Department of Mathematics, Lehigh University, Bethlehem, PA 18105-3174; e-mail: steve.weintraub@lehigh.edu; telephone: 610-758-3717.

Southeastern Section: Matthew Miller, Department of Mathematics, University of South Carolina, Columbia, SC 29208-0001, e-mail: miller@math.sc.edu; telephone: 803-777-3690.

The Meetings and Conferences section of the *Notices* gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited in the table of contents on this page for more detailed information on each event. Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. **Information in this issue may be dated. Up-to-date meeting and conference information can be found at** www.ams.org/meetings/.

Meetings:

Important Information Regarding AMS Meetings

Potential organizers, speakers, and hosts should refer to page 95 in the January 2008 issue of the *Notices* for general information regarding participation in AMS meetings and conferences.

Abstracts

Speakers should submit abstracts on the easy-to-use interactive Web form. No knowledge of LaTeX is necessary to submit an electronic form, although those who use LaTeX may submit abstracts with such coding, and all math displays and similarily coded material (such as accent marks in text) must be typeset in LaTeX. Visit http://www.ams.org/cgi-bin/abstracts/abstract.pl. Questions about abstracts may be sent to abs-info@ams.org. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

Conferences: (see http://www.ams.org/meetings/ for the most up-to-date information on these conferences.)

Co-sponsored conferences:

June 30–July 3, 2008: Tenth Conference on p-adic and Non-Archimedean Analysis, Michigan State University. See http://bt.pa.msu.edu/NA08/ for more information.


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