About the Cover

Chaos in the weak stability boundary

This month's cover was suggested by Shane Ross' review of Edward Belbruno's recent book *Fly Me to the Moon*. It displays, in a rotating coordinate system, one of Hill's regions for the restricted three-body problem modeling the Earth-Moon system, as well as a sample low-energy spacecraft trajectory starting near the Moon. What it illustrates is that certain orbits, starting near the region Belbruno calls the *weak stability boundary*, behave chaotically. Very roughly, this is because they pass through the narrow neck of Hill's surface surrounding the libration point between Earth and Moon. The global extension of that point's local invariant manifolds possesses the familiar structure of the homoclinic tangle. The sensitive behavior associated with chaos makes energy-efficient space travel feasible.

Belbruno first formulated the concept of WSB in 1986, and proposed spacecraft trajectories associated with it to offer low-energy lunar transfer. At the time his ideas were met with much skepticism, but in time they have become part of mainstream technology. Belbruno's intuitive notion of the WSB as a region lying on the boundary of stable trajectory behavior has become more and more precise as time goes on, but it is still not possible to find a completely satisfactory account in the literature, and indeed there remain many mysteries in the subject. That Belbruno's own formulation is not totally accurate is demonstrated in the preprint "A note on weak stability boundaries" by F. Garcìa and G. Gòmez, *Celestial Mechanics and Dynamical Astronomy*, February 2007. These authors show that the WSB is more complicated than Belbruno imagined, but also present striking evidence that his intuition relating the WSB to chaotic dynamics on the invariant manifolds of dynamical system theory is valid. The book *Dynamical Systems, the Three-Body Problem, and Space Mission Design* by W. S. Koon et al. presents a slightly different, and in many aspects clearer, way of looking at the same problem of planning space voyages. Ross is one of its authors, and it is available from his website at Virginia Tech (`http://www.esm.vt.edu/~sdross/`).

—Bill Casselman, Graphics Editor
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