2008 Steele Prizes

The 2008 Leroy P. Steele Prizes were awarded at the 114th Annual Meeting of the AMS in San Diego in January 2008.

The Steele Prizes were established in 1970 in honor of George David Birkhoff, William Fogg Osgood, and William Caspar Graustein. Osgood was president of the AMS during 1905–1906, and Birkhoff served in that capacity during 1925–1926. The prizes are endowed under the terms of a bequest from Leroy P. Steele. Up to three prizes are awarded each year in the following categories: (1) Lifetime Achievement: for the cumulative influence of the total mathematical work of the recipient, high level of research over a period of time, particular influence on the development of a field, and influence on mathematics through Ph.D. students; (2) Mathematical Exposition: for a book or substantial survey or expository research paper; (3) Seminal Contribution to Research: for a paper, whether recent or not, that has proved to be of fundamental or lasting importance in its field or a model of important research. Each Steele Prize carries a cash award of US$5,000.

The Steele Prizes are awarded by the AMS Council acting on the recommendation of a selection committee. For the 2008 prizes the members of the selection committee were: Rodrigo Bañuelos, Enrico Bombieri, Russel Caflisch, Lawrence C. Evans, Lisa C. Jeffrey, Nicholas M. Katz, Julius L. Shaneson, Richard P. Stanley, and David A. Vogan (chair).

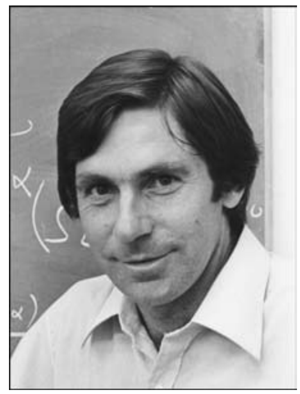
The list of previous recipients of the Steele Prize may be found on the AMS website at `http://www.ams.org/prizes-awards`.

The 2008 Steele Prizes were awarded to Neil Trudinger for Mathematical Exposition, to Endre Szemerédi for a Seminal Contribution to Research, and to George Lusztig for Lifetime Achievement. The text that follows presents, for each awardee, the selection committee's citation, a brief biographical sketch, and the awardee's response upon receiving the prize.

Mathematical Exposition: Neil Trudinger

Citation

The Leroy P. Steele Prize for Mathematical Exposition is awarded to Neil Trudinger in recognition of his book *Elliptic Partial Differential Equations of Second Order*, written with the late David Gilbarg.



Neil Trudinger

The global theory of nonlinear partial differential equations was mostly restricted to PDE involving two variables until the late 1950s, when fundamental estimates of DeGiorgi and Nash for second-order elliptic (and parabolic) equations finally broke open such PDE in more variables. The subject thereupon exploded beyond all expectations, and nowadays the analysis of even extremely degenerate and highly nonlinear second-order elliptic PDE in many variables is fairly routine, if very technical in detail.

Neil Trudinger, starting with the original 1977 edition of his book with Gilbarg, has recorded the progress of the field. He has reworked the breakthroughs, many due to him, recasting these technical estimates into understandable form within the fixed notation and framework of this highly cited book in its various domestic and foreign editions. His service has been invaluable. Having this foundational reference has made it possible for young researchers to enter the field, which would otherwise have been impenetrable. Here they can read in full detail all about Schauder estimates, Sobolev spaces, boundary estimates, Harnack inequalities, a priori derivative bounds, and much, much more.

Good mathematical exposition is always difficult, but it is especially so for technical estimates. The heights to which the research community has pushed the analysis of nonlinear second-order elliptic PDE is amazing, but the fundamental inequalities are mostly without any good heuristic interpretations. Hard analysis is both hard and hard to explain: Neil Trudinger's concise, elegant exposition in this outstanding book is magnificent.

Biographical Sketch

Neil S. Trudinger was born in Ballarat, Australia, in 1942. After schooling and undergraduate education at the University of New England in Australia, he completed his Ph.D. at Stanford University in 1966.

Following appointments at the Courant Institute (1966–67); University of Pisa, Italy (1967); Macquarie University, Australia (1968–70); University of Queensland, Australia (1970–73); University of Minnesota (1970–71); and Stanford University (1971), he took up a chair of mathematics at the Australian National University in 1973, where he has been since. During this period he has also held numerous visiting positions at universities in Asia, Europe, and the United States, as well as a professorship at Northwestern University from 1989 to 1993. Among various administrative positions at the Australian National University, he was head of the Department of Pure Mathematics from 1973 to 1980, director of the Commonwealth Special Research Centre for Mathematical Analysis from 1982 to 1990, and dean of the School of Mathematical Sciences from 1992 to 2000.

Neil Trudinger is a fellow of the Australian Academy of Science and a fellow of the Royal Society of London. He was also chief judge in the Singapore National Science Talent Search in 2002. His research contributions, while largely focused on nonlinear elliptic partial differential equations, have also spread into functional analysis, geometry, computational mathematics, and, more recently, optimal transportation.

Response

I am very honoured and pleased to receive the Steele Prize for Mathematical Exposition. I could never have imagined forty years ago when my book with David Gilbarg on elliptic partial differential equations was first published that it would get such recognition. The book was originally conceived by us after I had prepared lecture notes for the spring quarter of the graduate PDE course at Stanford in 1971. My topics were Sobolev spaces and their application to linear elliptic PDE, and we decided to start by blending these with earlier notes of Dave on the Schauder theory. Six years later and after a lot of hard work, including long and painful negotiations over language, the first edition appeared. We were extremely fortunate to have incredible assistance. First was the impeccable typing of Anna Zalucki in Canberra and Isolde Field at Stanford. Isolde had already typed my Ph.D. thesis at Stanford several years earlier, and Dave had been my supervisor, so the Stanford team was ready to roll from the outset. In Australia I had an amazing research assistant, Andrew Geue, who checked every bibliographical reference against its original publication so that titles and page numbers were always correct. We also got plenty of encouragement and support from many colleagues over the succeeding years to whom I am very grateful, as well as to those old friends Catriona Byrne and Joachim Heinze at Springer in Heidelberg.

My own passage into mathematical exposition was rather severe, akin to learning to swim by being thrown in a deep ocean. My first postdoctoral position in 1966 was a Courant Instructorship, and I was assigned an advanced topics course in PDE for the full year. Armed with books by Bers, John, and Schechter on partial differential equations; Morrey on multiple integrals in the calculus of variations; Friedman on parabolic partial differential equations; as well as works of Ladyzhenskaya and Ural'tseva, Moser, Serrin, and Stampacchia from my graduate days, I struggled to teach a full-year course on elliptic and parabolic equations to students who all looked older than my meagre twenty-four years. But this torture had its rewards. I presented a then recent and now famous paper by John and Nirenberg on BMO as it was needed for the Moser Harnack inequality. Subsequently, I found that it could be bypassed for the Harnack inequality through a simpler argument, a byproduct of which was an exponential-type imbedding result, later sharpened by Moser and now well known as the Moser-Trudinger inequality. At the same time, my quest to understand loss of compactness in Sobolev imbeddings led to the Yamabe "problem". But most of all I was extremely well equipped when I started work on the book a few years later.

I conclude on a sad note. Both David Gilbarg and Isolde Field passed away in recent years. This honour is for you, Dave and Isolde!

Seminal Contribution to Research: Endre Szemerédi

Citation

The Steele Prize in 2008 for a Seminal Contribution to Mathematical Research is awarded to Endre Szemerédi for the paper "On sets of integers containing no k elements in arithmetic progression", *Acta Arithmetica* XXVII (1975), 199–245.

A famous result of arithmetic combinatorics due to van der Waerden in 1927 proving an earlier conjecture of Baudet states that if we partition the natural integers into finitely many subsets, then one of these subsets contains arithmetic progressions of arbitrary length. In its finite version, because of the inevitable use of a multiple induction argument, it leads to incredibly large bounds for the size of a set of consecutive integers such that for every k-partition of it there is always a subset containing an arithmetic progression of k terms. In 1936 Erdős and Turán proposed, as a natural extension of van der Waerden's theorem, the conjecture that any infinite set of integers of positive density contained arbitrarily long arithmetic progressions; this may be viewed as a discrete analog of the classical theorem of Lebesgue that almost every point of a set of positive measure of real numbers has density 1. This conjecture quickly became one of the major open questions in Ramsey theory.

Endre Szemerédi

The first nontrivial result about the Erdős-Turán conjecture was obtained by K. F. Roth in 1953 using harmonic analysis, proving it for progressions of length 3, but his method did not extend to length 4 in any obvious way. In 1969 Szemerédi proved the Erdős-Turán conjecture for length 4 using a difficult combinatorial method. Finally, the Erdős-Turán conjecture was settled in the affirmative by Szemerédi in his landmark 1975 paper.

The solution is a true masterpiece of combinatorics, containing new ideas and tools whose impact go well beyond helping to solve a specific hard problem. One of these new tools, his by now famous Regularity Lemma, has become a foundation of modern combinatorics. Its statement of striking simplicity asserts roughly that any sufficiently large dense graph can be approximated by a union of a bounded number of very regular subgraphs of almost equal size, looking in pairs like very regular bipartite graphs; the upper and lower bounds for the number of subgraphs are determined only by the desired quality of approximation and are independent of the size of the graph. In essence, every large dense graph is well approximated by a controlled bounded union of quasirandom bipartite graphs of almost equal size. This is a very surprising result, far from intuitive. The proof is short but very subtle, leading to bounds for the number of components larger than any tower of exponentials. The subtlety of the statement has been confirmed by recent work by Gowers, showing that these gigantic bounds are indeed necessary for the validity of the Regularity Lemma in all cases.

The impact in combinatorics of the Regularity Lemma and of the numerous variants that followed it is due to the fact that there are many techniques available for studying random graphs and, via the Regularity Lemma, they can be transferred to the study of completely arbitrary graphs. It is fair to say that the Regularity Lemma has transformed the focus of graph theory from the study of special graphs and of extremal problems to the study of general graphs and random graphs. Beyond combinatorics it has found applications in number theory and in computer science, in particular in complexity theory.

However, the impact of Szemerédi's paper goes beyond this. The solution of the Erdős-Turán conjecture stimulated other mathematicians to find other lines of attack. In 1977 Furstenberg found a new proof of Szemerédi's theorem using deep methods of ergodic theory, together with a correspondence principle showing the equivalence of Szemerédi's theorem with his new ergodic theorem. Furstenberg's new method could then be used to attack multidimensional versions of the theorem as well as nonlinear versions. In 2001 Gowers obtained a new proof of Szemerédi's theorem, based on his novel idea of a Fourier analysis with nonlinear phases. More recently, Green and Tao were able to replace the positive density condition in Szemerédi's theorem by other arithmetical conditions, which allowed them, using again a suitable transference principle, to prove the same result for any sequence of primes of relative positive density, thereby solving another famous conjecture of Erdős considered inaccessible by standard methods of analytic number theory.

Recent work by many authors strongly indicates that these different approaches to Szemerédi's theorem are all interrelated. There is no doubt that Szemerédi's landmark paper is the source of these beautiful developments in mathematics.

Biographical Sketch

Endre Szemerédi was born in Budapest in 1940. He finished university in Budapest, at ELTE University. He received his Ph.D. at the Moscow State University. He has been a member of the Renyi Institute of the Hungarian Academy of Sciences since 1970. Currently he is a professor in the Department of Computer Sciences, Rutgers University. He is a member of the Hungarian Academy of Sciences. In 1976 he received the Pólya Prize.

Response

I am really grateful to the AMS, to the Steele Prize Committee, and to those people who recommended me. This prize is a great honor.

Here is what actually sparked my work on $R_4(n)$. Assuming that it was a well-known fact that dense sets of integers have arithmetic progressions of length four, I proudly showed Paul Erdős a proof that no positive fraction of elements in a long arithmetic progression could be squares. Erdős pointed out a flaw in the argument, namely that $R_4(n)$ was actually an open problem and that the rest of my proof was in fact already known to Euler. So now I really had to work on $R_4(n)$. Once $R_4(n)$ was settled, so was the original problem about squares. Later, Bombieri, Granville, and Pintz greatly improved my result. Luckily for me this occurred several years after $R_4(n)$; otherwise I would never have worked on it.

It is my opinion (and maybe only mine) that the Regularity Lemma was born after the $R_k(n)$ result, though certainly inspired by ideas from that paper. It is necessary to acknowledge Andras Hajnal for the $R_k(n)$ paper and Vasek Chvatal for the Regularity Lemma paper. These friends literally wrote every word of the papers based on my explanations. I also want to express my gratitude to Paul

Erdős and to K. F. Roth for their encouragement to persevere with $R_k(n)$.

This award could not have occurred were it not for the fundamental work of other mathematicians who developed the field of additive combinatorics and established its relations with many other areas. Without them my theorem is only a fairly strong result, but no "seminal contribution to research". I acknowledge my debt to them. Finally, I want to thank my wife, Anna, for all her patience, good humor, and support.

Lifetime Achievement: George Lusztig

George Lusztig

Citation

The work of George Lusztig has entirely reshaped representation theory and in the process changed much of mathematics.

Here is how representation theory looked before Lusztig entered the field in 1973. A central goal of the subject is to describe the irreducible representations of a group. The case of reductive groups over locally compact fields is classically one of the most difficult and important parts. There were three more or less separate subjects, corresponding to groups over $\mathbb{R}$ (Lie groups), $\mathbb{Q}_p$ (p-adic groups), and finite fields (finite Chevalley groups).

Lusztig's first great contribution was to the representation theory of groups over finite fields. In a 1974 book he showed how to construct "standard" representations—the building blocks of the theory—in the case of general linear groups. Then, working with Deligne, he defined standard representations for all finite Chevalley groups. This was mathematics that had been studied for nearly a hundred years; Lusztig and Deligne did more in one paper than everything that had gone before.

With the standard representations in hand (in the finite field case), Lusztig turned to describing irreducible representations. The first step is simply to get a list of irreducible representations. This he did almost immediately for the "classical groups", like the orthogonal groups over a finite field. The general case required deep new ideas about connections among three topics: irreducible representations of reductive groups, the representations of the Weyl group, and the geometry of the unipotent cone. Although some key results were contributed by other (great!) mathematicians like T. Springer, the deepest new ideas about these connections came from Lusztig, sometimes in work with Kazhdan.

Lusztig's results allowed him to translate the problem of describing irreducible representations of a finite Chevalley group into a problem about the Weyl group. This allowed results about the symmetric group (like the Robinson-Schensted algorithm and the character theory of Frobenius and Schur) to be translated into descriptions of the irreducible representations of finite classical groups. For the exceptional groups, Lusztig was asking an entirely new family of questions about the Weyl groups, and considerable insight was needed to arrive at complete answers, but eventually he did so.

Lusztig's new questions about Weyl groups originate in his 1979 paper with Kazhdan. The little that was known about irreducible representations first becomes badly behaved in some very specific examples in $SL(4, \mathbb{C})$. Kazhdan and Lusztig noticed that their new questions about Weyl groups first had nontrivial answers in exactly these same examples (for the symmetric group on four letters). In an incredible leap of imagination, they conjectured a complete and detailed description of singular irreducible representations (for reductive groups over the complex numbers) in terms of their new ideas about Weyl groups. This (in its earliest incarnation) is the Kazhdan-Lusztig conjecture. The first half of the proof was given by Kazhdan and Lusztig themselves, and the second half by Beilinson-Bernstein and Brylinski-Kashiwara independently.

The structure of the proof is now a paradigm for representation theory: use combinatorics on a Weyl group to calculate some geometric invariants, relate the geometry to representation theory, and draw conclusions about irreducible representations. Lusztig has used this paradigm in an unbelievably wide variety of settings. One striking case is that of groups over p-adic fields. In that setting Langlands formulated a conjectural parametrization of irreducible representations around 1970. Deligne refined this conjecture substantially, and many more mathematicians have worked on it. Lusztig (jointly with Kazhdan) showed how to prove the Deligne-Langlands conjecture in an enormous family of new cases. This work has given new direction to the representation theory of p-adic groups.

There is much more to say: about Lusztig's work on quantum groups, on modular representation theory, and on affine Hecke algebras, for instance. His work has touched widely separated parts of mathematics, reshaping them and knitting them together. He has built new bridges to combinatorics and algebraic geometry, solving classical problems in those disciplines and creating exciting new ones. This is a remarkable career and as exciting to watch today as it was at the beginning more than thirty years ago.

Biographical Sketch

George Lusztig was born in Timisoara, Romania, in 1946. After graduating from the University of Bucharest in 1968, he was an assistant at the University of Timisoara and then a member of the Institute for Advanced Study in Princeton, where he studied with Michael Atiyah. During his second year at IAS he was also a graduate student at Princeton University and received a Ph.D. degree (1971) for work on Novikov's higher signature and families of elliptic operators. He then moved to the University of Warwick, U.K., becoming a professor in 1974. For the last thirty years he has been a professor at the Massachusetts Institute of Technology. He has been a frequent visitor to the IHÉS (Institut des Hautes Études Scientifiques) and spent the academic year 1985–86 at the University of Rome. Lusztig received the Berwick Prize (London Mathematical Society, 1977), the Cole Prize in Algebra (American Mathematical Society, 1985), and the Brouwer Medal (Dutch Mathematical Society, 1999). He is a fellow of the Royal Society of London, a fellow of the American Academy of Arts and Sciences, and a member of the National Academy of Sciences.

Response

When writing a response it is very difficult to say something that has not been said before. Therefore, I thought that I might give some quotes from responses of previous Steele Prize recipients which very accurately describe my sentiments.

"What a pleasant surprise!" (Y. Katznelson, 2002). "I feel honored and pleased to receive the Steele prize—with a small nuance, that it is awarded for work done up to now" (D. Sullivan, 2006). "I always thought this prize was for an old person, certainly someone older than I, and so it was a surprise to me, if a pleasant one, to learn that I was chosen a recipient" (G. Shimura, 1996). "But if ideas tumble out in such a profusion, then why aren't they here now when I need them to write this little acceptance?" (J. H. Conway, 2000).

Now, I thank the Steele Prize Committee for selecting me for this prize. It is an unexpected honor, and I am delighted to accept it. I am indebted to my teachers, collaborators, colleagues at MIT, and students for their encouragement and inspiration over the years.

Around the time of my Ph.D., I switched from being a topologist with a strong interest in Lie theory to being a representation theorist with a strong interest in topology. (The switch happened with some coaching by Michael Atiyah and later by Roger Carter.) After that most of my research was concerned with the study of representations of Chevalley groups over a finite field or used the experience I gained from groups over a finite field to explore neighboring areas such as p-adic groups (which can be viewed as groups over a finite field that are infinite dimensional) or quantum groups (which can be viewed as analogues of the Iwahori-Hecke algebras, familiar from the finite group case).

Here are three topics from my research which I am particularly fond of:

(i) the classification of complex irreducible representations of a finite Chevalley group;

(ii) the theory of character sheaves, which helps in computing the irreducible characters in (i);

(iii) the theory of canonical bases arising from quantum groups, which unexpectedly provides a very rigid structure with coefficients in the natural numbers for several of the known objects in Lie theory.

I would like to make some comments on the period in which I focused on topic (i) above, from late 1975 (when my paper with Deligne (DL) was just completed) to the spring of 1978. In the first few months of that period I worked on the "Coxeter paper" (CP), in which I studied in detail the cohomology with compact support of the variety attached in (DL) to a Coxeter element in the Weyl group. Luckily, in this case the eigenvalues of Frobenius could be explicitly computed, and the eigenspaces provided a complete decomposition into irreducible representations, giving several new key examples of cuspidal representations. Then during the next year I found the classification and degrees of the irreducible representations of classical groups over a finite field using an extension of the method of (DL). After this (in 1977), as I wrote the notes for my lectures in the CBMS Regional Conference Series, No. 39, I found the classification and degrees of the irreducible unipotent representations of the finite exceptional groups of type other than E_8, based on (DL) and (CP). Towards the end of 1977 I discovered the nonabelian Fourier transform attached to any finite group H (which in the case where H is abelian reduces to the ordinary Fourier transform for functions on H times its dual). This new Fourier transform allowed me to find (in the spring of 1978) the classification and degrees of the irreducible unipotent representations for E_8. The same (or somewhat easier) methods can be used to obtain the classification and degrees of nonunipotent irreducible representations of finite exceptional groups. Thus, contrary to what the citation says, the classification of irreducible representations of finite exceptional groups does not depend on the "geometry of the unipotent cone" or on my work with Kazhdan done in 1979 (KL). On the other hand, the latter (KL) did play a role in my work (1981, 1982) on computing the values of irreducible characters on semisimple elements, and the former played a role in my work (1983–1986) on character sheaves. Moreover, the use of (KL) simplifies some of the arguments in the classification, as I showed in my 1984 book.