2008 Bôcher Prize

The 2008 Maxime Bôcher Memorial Prize was awarded at the 114th Annual Meeting of the AMS in San Diego in January 2008.

Established in 1923, the prize honors the memory of Maxime Bôcher (1867–1918), who was the Society's second Colloquium Lecturer in 1896 and who served as AMS president during 1909–1910. Bôcher was also one of the founding editors of *Transactions of the AMS*. The original endowment was contributed by members of the Society. The prize is awarded for a notable paper in analysis published during the preceding six years. To be eligible, the author should be a member of the AMS or the paper should have been published in a recognized North American journal. The prize is given every three years and carries a cash award of US$5,000.

The Bôcher Prize is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2008 prize the members of the selection committee were: Peter S. Constantin, Tai-Ping Liu (chair), and Elias M. Stein.

Previous recipients of the Bôcher Prize are: G. D. Birkhoff (1923), E. T. Bell (1924), Solomon Lefschetz (1924), J. W. Alexander (1928), Marston Morse (1933), Norbert Wiener (1933), John von Neumann (1938), Jesse Douglas (1943), A. C. Schaeffer and D. C. Spencer (1948), Norman Levinson (1953), Louis Nirenberg (1959), Paul J. Cohen (1964), I. M. Singer (1969), Donald S. Ornstein (1974), Alberto P. Calderón (1979), Luis A. Caffarelli (1984), Richard B. Melrose (1984), Richard M. Schoen (1989), Leon Simon (1994), Demetrios Christodoulou (1999), Sergiu Klainerman (1999), Thomas Wolff (1999), Daniel Tataru (2002), Terence Tao (2002), Fanghua Lin (2002), and Frank Merle (2005).

The 2008 Bôcher Prize was awarded to Alberto Bressan, Charles Fefferman, and Carlos Kenig. The text that follows presents, for each awardee, the selection committee's citation, a brief biographical sketch, and the awardee's response upon receiving the prize.

Alberto Bressan

Citation

Alberto Bressan of Penn State University is awarded the Bôcher Prize for his fundamental works on hyperbolic conservation laws. Professor Bressan has made important contributions to the well-posedness theory; the results have been summarized in his monograph *Hyperbolic Systems of Conservation Laws. The One-Dimensional Cauchy Problem* (Oxford Lecture Series in Mathematics and Its Applications, 20, Oxford University Press, Oxford, 2000, xii + 250 pp.). Another landmark achievement is the work on zero dissipation limit (with Stefano Bianchini), "Vanishing viscosity solutions of nonlinear hyperbolic systems", *Ann. of Math. (2)* **161** (2005), no. 1, 223–342.

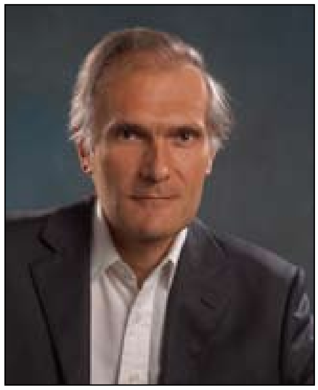
Biographical Sketch

Alberto Bressan was born in Venice, Italy. He completed his undergraduate studies at the University of Padova, Italy, and received a Ph.D. from the University of Colorado, Boulder, in 1982. He has held faculty positions at the University of Colorado and at the International School for Advanced Studies in Trieste, Italy. Presently he holds the Eberly Chair Professor of Mathematics at Pennsylvania State University. His scientific interests lie in the areas of differential inclusions, control theory, differential games, partial differential equations, and hyperbolic systems of conservation laws. He gave a plenary lecture at the International Congress of Mathematicians, Beijing, 2002. In 2006 he received the A. Feltrinelli Prize for Mathematics, Mechanics, and Applications from the Accademia Nazionale dei Lincei in Rome. Besides mathematics he enjoys playing piano and flute. He lives in State College, Pennsylvania, with his wife, Wen Shen, and two daughters, Luisa Mei and Maria Lan.

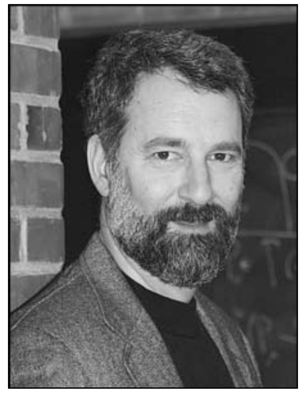
Response

It is a great honor for me to receive this prize. It was also a pleasant surprise to discover that my name is now listed among the 1,631 direct descendents of Maxime Bôcher listed in the Math Genealogy Project.

When I first became interested in hyperbolic conservation laws in the 1980s, my main training had been in other fields: parabolic equations, differential inclusions, and control theory. But as a fresh Ph.D. recipient, I was intrigued by the fact that something apparently so basic as the well-posedness of the equations for gas dynamics could have remained an open problem for so many years.



Alberto Bressan



Charles Fefferman

Carlos Kenig

The key estimates needed to establish continuous dependence of solutions were something I could figure out fairly quickly. However, it took me nearly ten years to fix details and achieve a rigorous proof in some significant cases. When I attended my first hyperbolic meeting in Stony Brook in 1994, I was still an outsider. Within the research community on hyperbolic problems I found very friendly and encouraging people. One can now say that the well-posedness for hyperbolic conservation laws in one space dimension has really been a cooperative accomplishment. In particular, the ideas contributed by Tai Ping Liu and Tong Yang have been instrumental in creating the polished theory we now have.

Understanding vanishing viscosity approximations was a second major challenge. This was achieved in 2001 in joint work with Stefano Bianchini at the International School for Advanced Studies in Trieste. Bianchini was the kind of student that you can call yourself fortunate if you find one in a lifetime. He took up my research program and contributed a new and fundamental idea: using the center manifold theorem to decompose a solution as local superposition of traveling waves. He also found the energy and determination to push his way through an incredible amount of computational details, eventually completing the proof.

In the end, all this is far beyond anything I could have hoped for when I first started reading about conservation laws and the Glimm scheme in Joel Smoller's book. I am delighted to receive this prize, and I thank the American Mathematical Society for the award.

Charles Fefferman

Citation

Charles Fefferman of Princeton University is awarded the Bôcher Prize for his many fundamental contributions to different areas of analysis, including his recent work on the Whitney extension problem. His important work in this area is contained in his papers "A sharp form of Whitney's extension theorem", *Annals of Math.* **161** (2005), 509–577, and "Whitney's extension problem for C^m", *Annals of Math.* **164** (2006), 313–359.

Biographical Sketch

Charles Fefferman was born in Washington, D.C., in 1949. He received his B.S. at the University of Maryland in 1966 and his Ph.D. at Princeton in 1969 under E. M. Stein. He taught at Princeton from 1969 to 1970, at the University of Chicago from 1970 to 1974, and again at Princeton since 1974. Fefferman has worked in classical Fourier analysis, partial differential equations, several complex variables, conformal geometry, quantum mechanics, fluid mechanics, and computational geometry. His honors include the Salem Prize, the Waterman Award, the Fields Medal, the Bergman Prize, and several honorary doctorates. He has served as chairman of the Princeton mathematics department and currently chairs the board of trustees of the Mathematical Sciences Research Institute in Berkeley. He is a member of the American Academy of Arts and Sciences, the National Academy of Sciences, and the American Philosophical Society.

Response

I am grateful for my selection for the Bôcher Prize and for the recognition of my work on Whitney's problem. That question and its close relatives have fascinated me for years. In solving them, I've had crucial help in the form of beautiful, highly original ideas due to several people. Let me mention especially G. Glaeser, who invented a key geometric construction; E. Bierstone, P. Milman, and W. Pawłucki, who discovered a general form of Glaeser's construction; and Y. Brudnyi and P. Shvartsman, who conjectured a basic finiteness principle and proved it in the first hard case.

It has been a joy to collaborate with Bo'az Klartag on the effective finite version of Whitney's problem, which I hope will one day connect to applied problems. Bo'az's brilliant ideas (he insists they are obvious) have gotten us out of many an impasse.

Most of all, I am grateful that I can share the pleasure of this occasion with my wife, Julie.

Carlos Kenig

Citation

Carlos Kenig of the University of Chicago is awarded the Bôcher Prize for his important contributions to harmonic analysis, partial differential equations, and in particular to nonlinear dispersive PDE. Kenig's work has been influential in the analysis of well-posedness under minimal regularity assumptions for physical equations. Examples of this work include his seminal paper with G. Ponce and L. Vega, "Well-posedness and scattering results for generalized Korteweg-de Vries equations via the contraction principle", *Comm. Pure Appl. Math.* **46** (1993), 527–620; his remarkable work with A. Ionescu, "Global well-posedness of the Benjamin-Ono equation in low regularity spaces", *J. Amer. Math. Soc.* **20** (2007), 3, 753–798; and his outstanding work with F. Merle, "Global well-posedness, scattering and blow-up for the energy critical focusing nonlinear wave equation", to appear, *Acta Math.*

Biographical Sketch

Carlos E. Kenig was born on November 25, 1953, in Buenos Aires, Argentina, where he received his early education. He obtained his Ph.D. at the University of Chicago in 1978 under the direction of Alberto Calderón. From 1978 to 1980 he was an instructor at Princeton University, after which he held positions at the University of Minnesota, becoming professor in 1983. In 1985 he returned to the University of Chicago, where he now is the Louis Block Distinguished Service Professor.

Kenig has been a recipient of Sloan and Guggenheim Fellowships. In 1984 he was awarded the Salem Prize. He was an invited speaker at the International Congress of Mathematicians in Berkeley (1986) and in Beijing (2002). Since 2002 he has been a fellow of the American Academy of Arts and Sciences.

Kenig's current research interests include boundary value problems under minimal regularity conditions, degenerate diffusions, free boundary problems, inverse problems, and nonlinear dispersive equations.

Response

It is a great honor to be a corecipient of this year's Bôcher Memorial Prize. I am grateful to the American Mathematical Society and to the selection committee for their recognition of my research. I would like to thank my family—my wife, Sarah, and my daughters, Lucy and Anna—for their love and support throughout the years. I would also like to thank my teachers, my many collaborators, and my students, all of whom have shared many insights with me. I am especially indebted to my long-time collaborators Gustavo Ponce and Luis Vega for more than twenty years (and still counting) of joint work, friendship, and shared fun.

There are many people who have influenced my mathematical career to whom I owe thanks, beginning with Alberto Calderón, my advisor, and Antoni Zygmund (both now deceased), who introduced me as a graduate student to the Calderón-Zygmund school of analysis. Eli Stein was my postdoctoral mentor, and I have greatly profited from many mathematical discussions with him and from his continued support and encouragement. The late Gene Fabes introduced me to research in partial differential equations; he was my mentor, collaborator, and dear friend. I continue to miss him. I am also particularly indebted to David Jerison and to the late Björn Dahlberg for their influence on me early on in my career. The three papers cited by the selection committee are joint works. I am very thankful to Gustavo Ponce, Luis Vega, Alex Ionescu, and Frank Merle, my coauthors in the cited papers, for their fundamental contributions to these joint works, without which these projects could not have been carried out. Finally, I would like to thank the University of Chicago, my home institution for more than twenty years, for providing me with the excellent working conditions in which my research is carried out.

The use of harmonic analysis techniques in the study of nonlinear dispersive equations was pioneered in works of I. Sigal, R. Strichartz, J. Ginibre-G. Velo, and T. Kato. In the late 1980s in joint work with Ponce and Vega, we introduced the use of the machinery of modern harmonic analysis for the study of nonlinear dispersive equations with derivatives in the nonlinearity. We showed for the first time that the initial value problem for the generalized Korteweg-de Vries equation with data in Sobolev spaces can be solved by the contraction mapping principle. In doing so, we obtained results that (for many powers in the nonlinearity) turned out to give the minimal regularity assumptions on the data for which this can be done. This was not the case with our first results for the quadratic nonlinearity in the KdV equation. Here, fundamental work of J. Bourgain (1993) expanded the functional framework for the use of the contraction mapping principle in this setting. This eventually led, in joint work with Ponce and Vega (1996), to the minimal regularity result for this case too. The resulting body of techniques (with refinements and extensions by many authors) has proved extremely powerful in many problems and settings and has attracted the attention of a large community of researchers.

In recent years I have been interested in some natural equations for which there is an exact balance between the smoothing properties of the linear part and the strength of the nonlinearity, which precludes the direct application of the techniques described before. The Benjamin-Ono equation is one such model. For this equation, examples of Molinet-Saut-Tzvetkov (2001) show that it is not possible to use the contraction mapping principle on any Sobolev space. After an important contribution by Tao (2004), who introduced a gauge transform into the problem (with a further extension by Burq-Planchon (2005) simultaneous to our work), Ionescu and I were able to obtain the conjectured global well-posedness for data of finite mass. This was achieved by combining the gauge transform of Tao with some new function spaces inspired by earlier work of Tataru in the wave map problem. These new functional structures have since proved useful for Schrödinger maps in joint works with Ionescu and with Bejenaru and Ionescu.

Lately there has been considerable interest in the study (for nonlinear dispersive and wave equations) of the long-time behavior of solutions. Issues like blow-up, global existence, and scattering have come to the forefront, especially in critical problems. The case of the energy critical, defocusing nonlinear wave equation was studied in pioneering works of many researchers in the 1980s and 1990s. (For instance M. Struwe (radial case), M. Grillakis (general case), J. Shatah-M. Struwe, H. Bahouri-J. Shatah, H. Bahouri-P. Gerard, and others.) These works show that for general data in the energy space we have global existence and scattering. Corresponding results for the energy critical, defocusing nonlinear Schrödinger equations were obtained in groundbreaking works of Bourgain (radial case, 1998), Colliander-Keel-Staffilani-Takaoka-Tao (general three-dimensional case), with higher-dimensional extensions due to Ryckman-Visan and to Visan (2005). For the corresponding focusing problems, say in the case of the wave equation, H. Levine (1974) had shown that blow-up in finite time can occur. Moreover, there is a stationary solution W (which solves the corresponding elliptic problem and plays an important role in the Yamabe problem). For this solution, scattering obviously does not occur. In a series of joint works with Merle, partly inspired by the elliptic case and also by works of Merle and Martel-Merle in mass critical problems, we have developed an approach to critical dispersive problems that applies to defocusing and for the first time also to focusing problems. The approach goes through a concentrated compactness procedure that reduces matters to a rigidity theorem. For instance, for the case of the energy critical focusing nonlinear wave equation, we show that the energy of W is a threshold. For data of energy smaller than that of W, if the critical Sobolev norm is smaller than the one of W, we have global existence and scattering; while if it is bigger, there is finite time blow-up.

There are many natural directions for future research in the areas just described. I look forward to continued research in them. I thank the selection committee once more for honoring these lines of research.