2008 Eisenbud Prize

The 2008 Leonard Eisenbud Prize for Mathematics and Physics was awarded at the 114th Annual Meeting of the AMS in San Diego in January 2008.

The Eisenbud Prize was established in 2006 in memory of the mathematical physicist Leonard Eisenbud (1913–2004) by his son and daughter-in-law, David and Monika Eisenbud. Leonard Eisenbud, who was a student of Eugene Wigner, was particularly known for the book *Nuclear Structure* (1958), which he coauthored with Wigner. A friend of Paul Erdős, he once threatened to write a dictionary of "English to Erdős and Erdős to English". He was one of the founders of the physics department at the State University of New York, Stony Brook, where he taught from 1957 until his retirement in 1983. His son David was president of the American Mathematical Society in 2003–2004. The Eisenbud Prize for Mathematics and Physics honors a work or group of works that brings the two fields closer together. Thus, for example, the prize might be given for a contribution to mathematics inspired by modern developments in physics or for the development of a physical theory exploiting modern mathematics in a novel way. The US$5,000 prize will be awarded every three years for a work published in the preceding six years. This is the first time the prize has been awarded.

The Eisenbud Prize is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2008 prize, the members of the selection committee were Joel L. Lebowitz, David R. Morrison (chair), and Edward Witten.

The 2008 Eisenbud Prize was awarded to HIROSI OOGURI, ANDREW STROMINGER, and CUMRUN VAFA. The text that follows presents the selection committee's citation, brief biographical sketches of the awardees, and their responses upon receiving the prize.

Citation

The Eisenbud Prize for 2008 is awarded to Hirosi Ooguri, Andrew Strominger, and Cumrun Vafa for their paper "Black hole attractors and the topological string" (*Physical Review D (3)* **70** (2004), 106007). This paper contains a beautiful and highly unexpected proposal: that the counting of black hole states, in certain string theories obtained by compactification on a Calabi-Yau manifold X, can be expressed in terms of the topological string partition function of X (i.e., in terms of the so-called Gromov-Witten invariants of X). The proposal explains some mysterious earlier results to the effect that certain scattering amplitudes in physical string theory can be expressed in terms of the topological string; the authors here argue that these amplitudes control the counting of microscopic states of certain electrically and magnetically charged black holes. Black holes and enumerative invariants such as Gromov-Witten invariants are both intensively studied but had not been significantly related to each other prior to this work.

Biographical Sketch: Hirosi Ooguri

Hirosi Ooguri was born on March 13, 1962, in Japan. He attended Gifu High School, whose notable alumni include Teiji Takagi, who developed class field theory. Ooguri received a B.A. in 1984 and an M.S. in 1986 from Kyoto University.

In 1986 Ooguri became an assistant professor at the University of Tokyo. After a year at the Institute for Advanced Study in Princeton, he moved to the University of Chicago as an assistant professor in 1989. In the same year he was awarded an Sc.D. from the University of Tokyo. A year later he returned to Japan as an associate professor at the Research Institute for Mathematical Sciences in Kyoto University. In 1994 he became a professor at the University of California at Berkeley and was appointed a faculty senior scientist at the Lawrence Berkeley National Laboratory in 1996. Since 2000 he has been at Caltech, where he is now Fred Kavli Professor of Theoretical Physics.

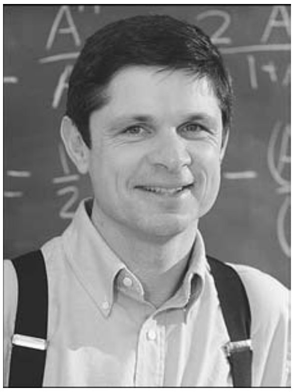
In 2007 Ooguri and his friends in Japan proposed establishing the Institute for the Physics and Mathematics of the Universe at the University of Tokyo. The proposal was approved with funding for two hundred staff scientists and visitors for the next ten years. Ooguri will continue to keep his intellectual base at Caltech, but he will spend a few months a year in Tokyo as a principal investigator at the new institute to lead activities at the interface of mathematics and physics.

Response: Hirosi Ooguri

I am deeply honored to share the Leonard Eisenbud Prize for Mathematics and Physics with such outstanding physicists as Andy Strominger and Cumrun Vafa.

In an early stage of my career I had the good fortune to work with Tohru Eguchi in Tokyo and to experience the power of quantum field theory in revealing new connections between different areas of mathematics. I have collaborated with Cumrun Vafa for over eighteen years on various aspects of gauge theory and string theory, including $N = 2$

Hirosi Ooguri



Andrew Strominger

Cumrun Vafa

string theory, topological string theory, gauge theories on D-branes, and their geometric engineering. Our collaborations have almost always aimed to discover hidden geometric structures in physical problems and to exploit them to develop new theoretical tools. Cumrun brims over with ideas that he has generously shared with me and many others. I thank him for the collaboration and friendship. I have always admired Andy Strominger for his creative insights, and I am happy to have had the chance to collaborate with both Andy and Cumrun in the academic year of 2003–2004, which led to the paper cited above. In this work we formulated a conjecture that relates two different concepts: topological string theory, which computes the Gromov-Witten invariants, and the counting of quantum states of black holes, which has to do with topological invariants of gauge theories in various dimensions. I would like to make a brief comment on each of them.

Topological string theory was introduced by Edward Witten. The construction of mirror pairs of Calabi-Yau manifolds by Brian Greene and Ronen Plesser and their application to the computation of the genus-zero Gromov-Witten invariants by Philip Candelas, Xenia De La Ossa, Paul Green, and Linda Parkes sparked interest in the mathematics community. I spent the academic year of 1991–1992 at Harvard University and collaborated with Michael Bershadsky, Sergio Cecotti, and Cumrun Vafa to generalize their results to higher genus. We found that the higher genus topological string partition functions can be used to compute certain scattering amplitudes in superstring theory compactified on a Calabi-Yau manifold. It took another twelve years to find the compelling question in physics, i.e., the counting of quantum states of black holes, to which these amplitudes give an answer. We also derived the holomorphic anomaly equations for the topological string partition functions and developed a method to solve them recursively in the genus. In this work we made several mathematical conjectures. Recently, the conjecture on the genus-one Gromov-Witten invariants for a quintic threefold was proven by Aleksey Zinger, and the conjecture on the so-called BCOV torsion for the mirror of the quintic was proven by Hao Fang, Zhiqin Lu, and Ken-ichi Yoshikawa. The conjectures for genus greater than one remain open.

The black hole entropy formula was proposed by Jacob Bekenstein and Stephen Hawking based on a remarkable mathematical analogy between thermodynamics and black hole mechanics and on the semiclassical theory of black hole radiance. It was expected that if there is a theory that successfully unifies quantum mechanics and general relativity, in such a theory the Bekenstein-Hawking formula can be derived as the statistical entropy of quantum states of black holes. Thanks to the D-brane construction by Joseph Polchinski for a certain class of black holes in string theory, it has become possible to count quantum states by evaluating topological invariants of gauge theory on D-branes, such as the Euler characteristic of instanton moduli space. The counting was carried out by Strominger and Vafa in 1996, and they found a perfect agreement with the Bekenstein-Hawking formula in the limit of large black holes, for which the approximation used by Bekenstein and Hawking becomes precise. Our paper cited above showed that this approximation can be significantly improved by using topological string theory. I was surprised and delighted to find the application of topological string theory to the counting of quantum states of black holes. This reaffirmed my belief that exact results in quantum field theory and string theory have enduring value and unintended applications.

When I was a high school student, physics was my least favorite subject until I learned calculus. Clearly, physicists need mathematics to formulate fundamental laws of nature. In return, physicists' search for fundamental laws has inspired many important developments in mathematics. In the past couple of decades interactions of mathematicians and physicists have been particularly intense and productive in the area involving quantum field theory and string theory. Since neither of them has a proper definition, mathematicians often view them as black boxes from which interesting conjectures materialize. I think that collaborations of mathematicians and physicists can be elevated to an even higher level if these physical theories are placed on more solid mathematical foundations.

I would like to thank Andy Strominger and Cumrun Vafa for the wonderful collaboration. Topological string theory has been developed by many people. In particular, I would like to acknowledge the influence of the earlier work by Gabriel Lopes Cardoso, Bernard de Wit, and Thomas Mohaupt.

I would like to thank the American Mathematical Society and the Eisenbud Prize Committee for recognizing the progress in this line of research. I am grateful to my teachers, collaborators, and friends for helping me make contributions to this area. Finally, I would like to thank my wife, Kyoko, for her love and support and my daughter, Tomoko, for adding extra dimensions to my life.

Biographical Sketch: Andrew Strominger

Andrew Strominger, the son of biochemist Jack Strominger, is an American theoretical physicist whose research centers around string theory. He is currently a professor at Harvard University, cofounder of the Center for the Fundamental Laws of Nature at Harvard, and a senior fellow at the Society of Fellows. He received his undergraduate degree from Harvard University in 1977 and his Ph.D. from the Massachusetts Institute of Technology in 1982 under the supervision of Roman Jackiw. His wide and varied contributions to physics include:

- a paper with Cumrun Vafa that explains the microscopic origin of the black hole entropy, originally calculated thermodynamically by Stephen Hawking and Jacob Bekenstein from string theory;
- a paper with Philip Candelas, Gary Horowitz, and Edward Witten about the relevance of Calabi-Yau manifolds for obtaining the Standard Model from string theory;
- other articles discussing the dS/CFT correspondence (a variation of AdS/CFT correspondence), S-branes (a variation of D-branes), and OM-theory (with Shiraz Minwalla and Nathan Seiberg);
- research on massless black holes in the form of wrapped D3-branes that regulate the physics of a conifold and allow topology change interpretation of mirror symmetry as a special case of T-duality (with Eric Zaslow and Shing-Tung Yau).

The fundamental laws of nature as we currently understand them are both incomplete and contradictory. Unsolved problems concerning these laws include the incompatibility of quantum mechanics and Einstein's theory of gravity, the origin of the universe, and the origin of the masses of the elementary particles. Strominger's research has concerned various aspects of these problems. The emergence of string theory as the most promising approach to these problems began with Strominger's 1985 codiscovery of so-called Calabi-Yau compactifications. This construction demonstrated that string theory not only reconciles quantum mechanics and gravity but can also contain within it electrons, protons, photons, and all the other observed particles and forces and hence is a viable candidate for a complete unified theory of nature. In 1991 Strominger codiscovered the brane solutions of string theory, which have played a crucial role in unraveling the beautiful mathematical structure and duality symmetries of the theory. The branes were eventually used by Strominger and collaborators to give a microscopic explanation of how black holes are able to store information, finally resolving a deep paradox uncovered by Hawking and Bekenstein a quarter century earlier. He and coworkers also used the branes to derive new relations in algebraic geometry, equating the moduli space of a brane in a Calabi-Yau space to the mirror Calabi-Yau. Preliminary attempts have been made to apply these insights to cosmology. Current research continues attempts to better understand the fundamental laws of nature.

Response: Andrew Strominger

I am greatly honored to receive, along with my collaborators Cumrun Vafa and Hirosi Ooguri, the first Leonard Eisenbud Prize of the American Mathematical Society for our work demonstrating a connection between Gromov-Witten invariants and microstate degeneracies of black hole attractors. Our success in discovering this connection relied on the uncanny ability of physical reasoning to lead to insights into pure mathematics.

Biographical Sketch: Cumrun Vafa

Cumrun Vafa is a Donner Professor of Science at Harvard University, where he teaches and does research on theoretical physics.

Vafa was born in Tehran, Iran, in 1960 and came to the U.S. for continuation of his education in 1977. He earned his B.S. in mathematics and physics from the Massachusetts Institute of Technology in 1981. He went on to earn his Ph.D. in physics from Princeton University in 1985 under the supervision of Edward Witten. He came to Harvard University in 1985 as a junior fellow of the Harvard Society of Fellows and has been on the Harvard faculty since 1988. He is married to Afarin Sadr, and they are the proud parents of three sons: Farzan, Keyon, and Neekon.

Response: Cumrun Vafa

It is a great pleasure to receive the 2008 Leonard Eisenbud Prize, together with my collaborators. I view this not only as an acknowledgment of a single paper but also as an appreciation of the work of so many physicists and mathematicians that led to this work. With the intrinsic beauty of the connection between mathematics and physics and with so many talented researchers, I hope to witness the continuing development of this remarkable area of science.

I am greatly indebted for the support I have received from my family and my parents, as well as my teachers over the years.