

### On the Graph (10-3)-a

In the March 2008 *Notices* (55, No. 3), Toshikazu Sunada, quoting Stephen T. Hyde, suggested that it was in 1977 that A. F. Wells—in his book *Three Dimensional Nets and Polyhedra*, Wiley, 1977—first described the triply-periodic chiral graph “(10-3)-a”. But twenty-one years earlier, Wells had already published stereoscopic drawings of (10-3)-a in *The Third Dimension in Chemistry*, Oxford Clarendon Press, 1956. In the 1970s, (10-3)-a (which I call the *Laves graph* for reasons explained below) gradually became widely known among materials scientists because of the steadily increasing variety of putative physical manifestations of the gyroid triply-periodic minimal surface.

I was intrigued by the Laves graph when I first saw it in 1958 depicted in Wells’ book. A few years later I learned that in 1955 Donald Coxeter had described—in “On Laves’ graph of girth ten”, *Can. J. Math.* 7 (1955)—how the Laves graph can be inscribed in the square faces of an infinite regular skew polyhedron discovered in 1926 by his close friend J. F. Petrie (“Regular skew polyhedra in three and four dimensions, and their topological analogues”, *Proc. Lond. Math. Soc.* 2 (43), (1937). I believe that the article by Fritz Laves in “Zur Klassifikation der Silikate”, *Z. Kristallogr.* 82 (1932) is probably the first publication that mentions the Laves graph.

In 1958 I derived the shape of the 17-faced Voronoï cell of the vertices of the Laves graph and made a dozen wooden models of this polyhedron, in four colors, with a magnet centered in each of its three ten-sided faces. In 1966, while experimenting with a toy vacuum-forming machine, I constructed plastic models of two triply-periodic surfaces. When I described them to Hans Nitsche, he suggested that I might have stumbled onto the adjoint minimal surfaces  $P$  and  $D$  analyzed in the 1860s by Schwarz, Riemann, and Weierstrass (SRW). I recognized three special properties of  $P$  and  $D$  not mentioned by SRW: (a) they are [infinite] regular

polyhedra whose faces and vertex figures happen to be skew polygons; (b) they can be generated by “eliminating the folds at the edges of the faces” of the polyhedra described by Coxeter and Petrie, i.e., by replacing the plane faces by skew polygons that span soap films; and (c) the pair of intertwined infinite labyrinths into which each of these surfaces partitions  $R^3$  have the symmetry and combinatorial structure of either the simple cubic ( $P$ ) or diamond ( $D$ ) skeletal graphs. I wondered about the possible existence of a third minimal surface (I dubbed it the *Laves surface L*), in which the two oppositely congruent labyrinths are represented by a pair of enantiomorphic skeletal Laves graphs. After all, I thought, the Laves graph is composed of regular polygons, just like  $P$  and  $D$ . The only difference is that the polygons in the Laves graph happen to be infinite and helical. But I was unable to imagine how to construct the Laves minimal surface, because it has no reflection symmetries.

In 1968 at the NASA Electronic Research Center, I “accidentally” constructed a plausible physical model of this hypothetical new surface. The skeletal graphs of its labyrinths are the two enantiomorphic forms of the Laves graph. I shared my excitement with experts, including Fred Almgren, Blaine Lawson, and Bob Osserman, and I even delivered a ten-minute talk about the conjectured surface at a Madison summer meeting of the AMS. A week after Madison, I realized that this surface, which I renamed the “gyroid”, is nothing more than a Bonnet associate of  $P$  and  $D$  that manages—like them—to be embedded in  $R^3$ . (K. Grosse-Brauckmann and M. Wohlgemuth proved this embeddedness in 1996.)

I visited the very cordial “Jumbo” Wells at his home in Storrs in early 1969 and inquired whether he knew of any chemical compounds whose space group is the same as that of the gyroid ( $Ia3d$ ). Wells replied that he did not. I then telephoned the structural biologist Donald L. D. Caspar to ask him the same question. He

instantly referred me to “Polymorphism of lipids”, *Nature* 215, August 12, 1967, a paper by V. Luzzati and A. Spegt, in which a high-temperature phase of some divalent cation soaps is shown to have space group  $Ia3d$ . I suspect that Fritz Laves would have been pleased to learn about the eventual near-ubiquity of his graph. (I know that Coxeter was.)

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### Hardy and Biology

The article about Hardy in the March 2008 issue of *Notices* says that he was “one of the greatest contributors to contemporary mathematical biology”. This can be true only in the sense that biologists were previously unaccustomed to using mathematics at all. What Hardy did might have been research-level work back in the time of De Moivre, but by 1908 it was little more than an exercise. R. C. Punnett, the biologist involved, wrote of it in his “Early days of genetics”, *Heredity* 4 (1950), 1-10:

In 1908 ... I was asked why it was that, if brown eyes were dominant to blue, the population was not becoming increasingly brown-eyed ... I could only answer that the heterozygous browns also contributed their quota of blues, and that somehow this must lead to equilibrium. On my return to Cambridge I at once sought out G. H. Hardy, with whom I was then very friendly ... we used to play cricket together. Knowing that Hardy had not the slightest interest in genetics, I put my problem to him as a mathematical one. He replied that it was quite simple and soon handed me the now well-known formula  $pr=q^2$ . Naturally pleased at getting

so neat and prompt an answer I promised him that it should be known as “Hardy’s Law”—a promise fulfilled in the next edition of my *Mendelism*.

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### Referendum Results Remain Relevant

Allyn Jackson’s “Opinion” on DARPA funding mathematicians to work on “projects that could eventually lead to technology of use to the military” and the role of the AMS (*Notices*, April 2008, p. 445) prompts me to make some comments.

She wonders “whether the U.S. mathematical community is becoming more receptive to military funding than it was, say, twenty years ago,” when a referendum was adopted by the Society indicating the “AMS concern about the large proportion of military funding of mathematics research” (in detriment of other sources of funding).

I believe that the measure approved twenty years ago is as relevant today as it was then, or even more so.

Let me mention what should be obvious, but may have been disregarded: The statement of concern and the directives stated in the ballot questions are still the official policy of the AMS, and its officers are bound by them.

Jackson repeats an old fallacious argument about members’ personal research choices that confuses the issue. Individual mathematicians have always been free to pursue whatever funding they may wish; this was never in question.

The issue debated two decades ago was the Society’s involvement in promoting research oriented to “technology of use to the military”, with its moral and political implications. Nowadays this may include data mining techniques to help the military eavesdrop on our telephone and email exchanges, and to spy on citizens’ groups opposing the Iraq war, for example.

Jackson remarks that although there was a record turnout for the referendum, and it passed by a wide margin, “the membership was not unanimously against military funding for mathematics.” During Mathematics Awareness Month, this year dedicated to the mathematics of voting, I find it ironic that the existence of a minority opinion should be invoked in order to cast doubt on the position taken by the overwhelming majority.

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### Government Funding Facts

Allyn Jackson is right to point out (*Notices*, “Opinion”, April, 2008) that there may not be a free lunch with government, in particular military, support of mathematics research. However, there are three immediate comments that should be made. First, our accustomed federal support is, in fact, a direct result of the contributions mathematicians and scientists made during World War II. Policymakers felt then, and now, that possessing a trained cadre of such specialists would be crucially important during national emergencies. Second, agencies, like NSF [National Science Foundation], are well aware that their funding decisions and initiatives can have a profound effect on the direction of science and mathematics and take care not to cut off apparently less fashionable areas of research. Finally, whether we mathematicians like it or not, mathematics will be applied to activities that may not meet with our approval.

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### Abusing Mathematical Models

In addition to the comments made by Professor Jones in his review of *Useless Arithmetic* in the April *Notices*, I would like to add the following. This book presents several cases from environmental science where quantitative mathematical models have failed,

e.g., cod fish in the great lakes, Yucca Mountain used for a nuclear waste repository, etc... In all the examples presented the criticism is given to the long range predictions of the models. But the systems presented are known to be nonlinear and open systems. Consequently long range prediction is out of the question. So why blame mathematical models? An example of a successful quantitative model used in its proper context is weather prediction which is typically done for 3-5 days.

The book points out the effect of politics on encouraging or discouraging scientific results, e.g., the global warming phenomenon. This is an important factor when science affects people’s lives. The book contains some surprising statements, e.g., on page xiii “... mathematics has become a substitute for science!!!” Also on page 201-202, “Thinking like physicists and not recognizing complexity...”. Most physicists and mathematicians studying complex adaptive systems are well aware of many of the problems mentioned in the book. I think this book should be called *Use and Abuse of Mathematical Models*.

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