Climate Change: A Research Opportunity for Mathematics?

Patricia Clark Kenschaft

"The influence of climate change on mathematical research in the twenty-first century could be comparable to physics' a century ago," claimed Gerald North of the Department of Atmospheric Sciences at Texas A&M at the Joint Mathematics Meetings in San Diego in January 2008. His was the introductory talk at the second large gathering of scientists and economists with mathematicians to learn from each other about their climate change research and to recruit more mathematicians to become involved. The symposium, cosponsored by the AMS and the Society for Industrial and Applied Mathematics (SIAM), included dozens of speakers, many of whom appealed to mathematicians to use their professional abilities to help analyze, predict, and find mitigations for climate change. The SIAM Invited Address in the ballroom, "From Global Predictions to Local Action: Mathematical Challenges in Global Warming", was delivered by Inez Fung of the University of California at Berkeley.

"We need better ways to assimilate all the data from both global and local measurements via mathematical modeling, and also better ways to think about uncertainties and risks. We need more analysis of uncertainties generally. There are many uncertainties, due both to our ignorance and perhaps to the physical uncertainties," she said. Meanwhile, we use both statistical models and constrained models, and keep weighing the advantages and disadvantages of each as we attempt to mitigate and adapt to climate change.

The complexity of the current problems is formidable. How do we analyze the dynamics of the atmosphere, the oceans, the solid earth (especially volcanic emissions) and the biosphere (the system of plants, animals, and other living things)? Scientists have studied pieces of these systems, cutting them both conceptually and geographically, but even the pieces are not tractable by current mathematics, and the challenges as we try to understand the interplay of all phenomena involved are far beyond current conceptual and computational capabilities. Some economic models already have about 20,000 parameters—and nobody believes they encompass all that need to be included.

Modeling the carbon cycle is pivotal because carbon dioxide in the air prevents easy transmission of energy, and is well known to be a major cause of climate change, along with other greenhouse gases such as methane. The 700 to 800 gigatons of carbon dioxide in the atmosphere, however, is less than that in the soil and is dwarfed by the 38,000 gt. in the ocean. The 5,000 gt. in fossil fuel is easily available for humans to put into the atmosphere and the 60 gt. in the biosphere is not negligible. The interactions in the biosphere seem impossible to even begin to understand with current mathematics and computer technology. We do know that burning fossil fuel has been a major factor in the increase of atmospheric carbon dioxide in recent centuries, as has been the disappearance of forest covers that absorb CO_2. The role and modeling of methane and other greenhouse gases provide further challenges.

Scientists from around the world have devised complex Atmosphere-Ocean General Circulation Models (AOGCMs) under forty different sets of assumptions about human behavior called "scenarios" with regard to such things as greenhouse gas release and land use. These forty scenarios are grouped into six, whose behaviors are displayed in graphs published in the reports of the Intergovernmental Panel on Climate Change (IPCC). For example, the model labeled A1FI indicates an emphasis on fossil fuels, A1B uses a balanced energy portfolio, and A1T emphasizes non-fossil energy. All three anticipate rapid economic growth,

---

Patricia Clark Kenschaft is professor emerita of mathematics at Montclair State University. Her email address is `kenschaft@pegasus.montclair.edu`.

a unified world economy, a quick spread of new and efficient technologies, and a peak population of 9 billion in 2050 with gradual decline afterward. The other three groups alter some of these assumptions.

Each of these six composite scenarios generates different consequences for twenty-first century emissions of CO_2 and other greenhouse gases. The six are graphed together on a repeatedly displayed summary from the fourth report of the IPCC (`http://www.ipcc.ch`). All six graphs rise similarly with increasing slope until 2040. After that the graphs form a fan; the bottom one continues the bowl-shaped pattern begun earlier, and the others increase rapidly, the most pessimistic seeming to shoot essentially straight upward.

The basic dynamics of the atmosphere and ocean have been modeled by Richard P. McGehee, a mathematician at the University of Minnesota. His models describe the basic interactions among (1) the atmosphere, (2) the shallow ocean, and (3) the deep ocean, and are accessible to undergraduates. There are only three differential equations, but these vastly simplified models make predictions within the range of the predictions described in the IPCC report.

McGehee made a linear model of the transmission among these three and diagonalized it. Half of the atmospheric CO_2 "goes away" into the ocean "quickly"—in a few decades. Half of the CO_2 remaining in the atmosphere goes away in a few centuries. The twentieth century fits a variety of parameter choices for this model.

What effect does the CO_2 concentration have on global and local temperatures? How do we find out? The pre-industrial concentration of CO_2 in the atmosphere for the past thousand years was 280 ppm (parts per million). If it doubles to 560 ppm, what would the temperature be? The current level is about 380 ppm and is increasing at about 1.5 ppm per year. This increase in temperature due to increased concentration of CO_2 in the atmosphere is called the "climate sensitivity" and is predicted by the IPCC to be in the range of 2 to 4.5 degrees centigrade with 66% confidence. Using the standard assumption that the increased temperature would be a logarithmic function of the CO_2 concentration, and assuming a value of 3.3 for the climate sensitivity, McGehee found that this simple model not only agrees well with the twentieth century, but falls within the range of the IPCC predictions for the twenty-first century global mean temperatures.

McGehee concluded that to predict the magnitude of the climate change under various emission scenarios, extremely simple models can be useful in situations where AOGCMs are too complex to be useful.

However, more detailed models are clearly needed. Gordon Swaters, an applied mathematician and theoretical physical oceanographer at the University of Alberta, has used much more sophisticated methods including the Navier-Stokes equations to model deep ocean flow patterns. He is trying to understand the dynamics of deep (called "abyssal") ocean currents in the North Atlantic ocean beginning in the East Greenland and Labrador Seas and then determining the path they take as they flow toward the equator. These analyses include their interactions with the ocean floor and the surrounding ocean.

On a planetary scale, after the surface water of the ocean is heated in the tropics, this equatorial heat is transported poleward by relatively swift wind-generated surface-intensified currents such as the Gulf Stream. At the high latitudes the water cools, thereby becoming more dense, and drops to the bottom. The deep currents return these cold, dense waters back toward the equator. This planetary ocean circulation pattern is called "the convective overturning of the oceans" and is referred to as the "thermohaline" or "meridionally" overturning circulation, and in the popular press as the "global conveyor belt". Understanding climate dynamics requires understanding the dynamical properties of this circulation. The goal of Swaters's research is to better understand the planetary scale dynamics of these abyssal ocean currents in order to improve their representation in numerical climate models. This is a computationally enormous task.

At the San Diego symposium, Swaters introduced the audience to a "simple" mathematical model that describes the most important aspects of the dynamics of abyssal ocean currents. He displayed nine nonlinear PDEs with two related algebraic equations, and described these governing equations as "hopelessly simple". Swaters added, "I'm not truly modeling climate change and these equations are not a climate model, but they do get a large chunk of the large scale physics for these abyssal flows correct. In that sense what I am describing here is a process-study that is trying to determine the dominant physics that must be captured in climate models if they are to improve their representation of these flows." He likened being an applied mathematician working in this area as being "between a rock and a hard place. The numerical ocean modeler is working with large sets of complicated PDEs that appear all but hopelessly intractable to classical mathematical analysis. On the other hand, mathematical reductions that result in tractable equations often seem to be completely missing the point physically to the computational oceanographer."

As he described the spatial structure that his model predicts for equatorward flowing abyssal currents as well as their instability characteristics and mixing properties with the surrounding ocean, Swaters incorporated a wide range of applied mathematical themes including asymptotic reduction, physical modeling, Lyapunov techniques, variational principles for Hamiltonian PDEs, and hydrodynamic stability theory for non-parallel shear flows. Swaters suggested that further improvements in the predictions of numerical ocean climate models will depend in part on their ability to model accurately the physical processes his study has described.

He then introduced a simpler model for abyssal ocean currents that includes only three nonlinear "potential vorticity" PDEs—but over twenty variables were visible. Swaters's talk culminated with a description of a high-resolution numerical simulation based on his model for the flow of the abyssal ocean currents in an idealized North Atlantic Ocean. "It ran for thirty days of computer time to simulate about fifty years. The bottleneck in the computation is the coupled elliptic inversions required at each time step." Fortunately, the results were remarkably true to observations.

Climate models involve more than interactions between atmosphere, ocean, and soil. In his introductory talk Gerald North introduced us to four "forcings" that might play a role: the sun, volcanoes, aerosols, and greenhouse gases (GHGs). Over the past thirty years we have excellent records of temperatures, not just at the surface of the earth, but also records of all the forcings affecting the CO_2 concentrations, including the sun. Satellites tell us about the sun and sun spots. The detailed measurements of these four forcings allow us to check the cause-and-effect connections, forming a deeper understanding of the connections.

The evidence is very convincing that it is the greenhouse gases, not the natural forcings, that are causing the dramatic increase in the CO_2 concentrations. Indeed, with the records of the past 100 years, all the models indicate that with all the forcings except the aerosols and GHGs, the band of temperatures is fairly steady, oscillating around a steady average. We are now reconstructing data for the past 2,000 years with the help of tree rings, earthquake faults, and ice slices. We now know that the average temperatures in the past thirty years have been the highest in the past 400 years.

In her talk "Climate Science and Adaptation Strategies", Emily F. Shuckburgh of the British Antarctic Survey, a Fellow in Applied Mathematics, Cambridge University, said that there is an important role for mathematicians in improving climate predictions, helping to quantify the threats, and informing adaptation strategies. "We especially need to move from global-average predictions to develop better local predictions. How can we refine our climate models? How can we predict high-impact, low-probability events? How can we characterize uncertainty? Mathematicians can help address these questions. We need the answers to devise adaptation strategies for threats such as flooding, droughts, rapid climate change, extreme weather, and endangered societies and habitats."

Gerald North said that one "big issue" is the response of snow to just a small temperature change. Places that have been snow-covered all year are now having extended periods without snow. This is affecting even California, but is serious at the poles. One degree in temperature can make a huge difference.

Every year the grid on which the data are based gets finer, as people make measurements ever more closely and as computers improve their speed and capacity. We gather data from ground observations, satellites, the oceans, the atmosphere, and the biosphere. Our models are now based on grids with a much finer resolution than previously. In 1990 the grid resolution was 500 km and by 2005 it was 110 km, an increase of almost a factor of five. Atmospheric winds' resolution has increased by a factor of six since 1992. These data enable us check the models not only for their output but also for the proper magnitudes of the forcings that go into the simulations.

However, this improvement in data collection is not having the desired result, and the appeal was strong for better use of the data—in other words, for more mathematical work. William Collins, a senior scientist in the Earth Science Division of Lawrence Berkeley National Laboratory, has been studying "scaling": whether climate models converge as we take a finer and finer grid resolution. His answer for *local* weather systems is "no"—just the opposite. It is local weather that interests each of us, not global averages.

In particular, we want to predict extreme phenomena. For example, how hard is it raining? This is measured in (amount of rain) divided by time. It will rain much, *much* harder by the end of the century. Also, we will have more heat waves. Europe in the summer of 2003 was a beginning; the 2003 European temperatures were several standard deviations away from the mean of the past century. France alone reported 15,000 deaths due to the heat wave.

Humans can adapt to slow change in the developed world, but we would cope better if we could predict the sudden changes. Even in the twentieth century there were increasing numbers of heat waves, and decreasing numbers of frost days per year. A heat wave is defined as five days or more of temperatures 5 degrees centigrade hotter than expected. By this definition, every year recently there have been many heat waves in North America and around the Mediterranean. There have also been more dry days. However, as mentioned above, as we have finer and finer grid resolution, our models' predictions of increased rainfall have *more* variability!

"Does it make sense to put more time, money, and effort into taking measurements in an increasingly finer grid if we can't agree on what models to use?" Collins asked. "Perhaps we need to put more effort into the intellectual work and less into the measurements."

Maximilian Auffhammer, of the Department of Agriculture and Resource Economics at the University of California, Berkeley, has made important forecasts of the carbon dioxide of China. He and his co-authors studied the rice output from 1968 to 1998 in nine states of India that produce "rainfed" rice. There are many factors that farmers can't control: support prices, prices of inputs, early rainfall, and sales prices. However, they do affect the area planted, the extent of irrigation, fertilizer used, and whether and how much labor is hired.

However, brown clouds and greenhouse gases also affect the yield of these crops. The main finding of the study is that the absence of brown clouds increases yields, as does the absence of greenhouse gases. Yields drop when the weather is warmer, dryer, or the light dimmer for any reason. Studying these interactions provides many mathematical opportunities. Auffhammer is studying the effects of fertilizer and crop substitution in multi-crop systems, but the interplay between climate and economics is complex, with great implications for human choices.

There is a major cycle of four factors: climate → short term physical impact → economics → public and private decisions → climate.

Auffhammer observed that the potential for adaptation is much less in Africa, which is already under stress from increased drought compared to here.

Roy Radner, Professor of Economics, Information Systems, and Environmental Studies, has spent most of his career as an economist after earning a doctorate in mathematical statistics. Saying, "The absence of a world government implies a need for self-enforcing treaties to curb global warming," he showed how game theory proves that such a possibility exists. He presented a model of a dynamic worldwide "climate-change game" in which the countries are the players—although he commented that countries don't act exactly as people. There are infinitely many Nash equilibria, one of which is "business as usual". The goal of the analysis is the characterization of equilibria that yield each country a higher present value of GDP by reducing greenhouse gas emissions. Such equilibria (some of which have been identified by the analysis) could be templates for self-enforcing treaties in the sense that deviation by one player would provoke sanctions by the others.

Radner and his colleagues have shown that such equilibria exist, and have begun analysis of what they are and how they might be attained. There are many research opportunities in exploring the consequences of non-uniform dependencies on current generation of greenhouse gases, varying effects of climate change on different countries (in particular sea level rise, agricultural productivity, and disease), long-term economic effects of curbing GHGs, and needed incentives for countries to collaborate. How do unequal current levels of economic development affect a self-enforcing treaty? Under what conditions would the prevention of global warming slow down the rate of economic growth? Altering the linearity and other assumptions in his and other models provides further research possibilities.

The final speaker in the symposium was Congressman Jerry McNerney, the first member of the United States Congress to hold a Ph.D. in mathematics. He has been a member of the AMS for over three decades. He urged mathematicians to become active both professionally and politically in climate change issues. He said that wind energy is the fastest growing alternative energy, and that Germany has five times the installed capacity for wind power as the U.S. His own first postdoctoral employment was in modeling commercial wind machines.

"If we rise up and meet those challenges, we can be winners. There are going to be winners and losers. Climate change can be a win for technology, for business, and diplomacy. We can create jobs and stimulate the economy." He urged mathematics professors to visit their congressmen and point out how to promote businesses in their districts that develop alternative sources of energy and other technological innovations to mitigate and adapt to climate change.

"The opportunities for cooperation are the greatest in the world's history. If we don't cooperate, we are all going to be losers… We need a national purpose similar to Sputnik to go after global warming.

"You will enjoy doing it. I'm excited about living in this period of history, when we can really make change."

Mathematicians interested in learning more can see videos of the talks at the first joint workshop of mathematicians and climate scientists in April 2007 at `http://www.msri.org/specials/climatechange/workshop`. Those who want to explore research possibilities can come to another such conference this summer: `http://www.msri.org/calendar/sgw/WorkshopInfo/453/show_sgw`.

There will be a summer graduate workshop on climate change at the Mathematical Sciences Research Institute in Berkeley, CA, July 14-25, 2008.