The Mathematician's Brain

Reviewed by David Corfield

The Mathematician's Brain
David Ruelle
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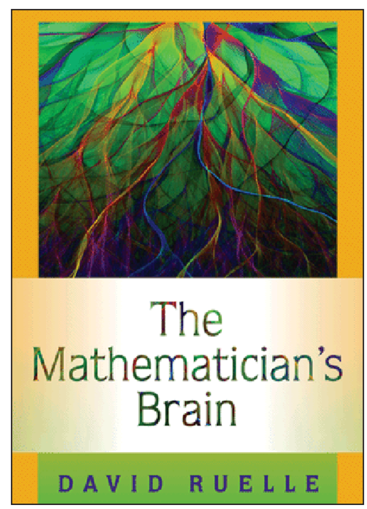
The Mathematician's Brain may be seen as having several intentions. It could be taken as an account written by a professional mathematician to apprise the man-in-the-street of the nature of mathematics. Since I have no insight into what such a person might require, I shall instead restrict my review to considering this book solely as a contribution to philosophy, broadly construed.

As a philosopher, I consider some of the most important literature written about mathematics to have come from the pens of mathematical practitioners. Indeed this book faces the challenge of finding its place in a set of works which includes such notable predecessors as Hermann Weyl's *Philosophy of Mathematics and Natural Science*, Saunders Mac Lane's *Mathematics: Form and Function* and Gian-Carlo Rota's *Indiscrete Thoughts*.

That one of Ruelle's intentions is to contribute to philosophy is made clear from the opening page, where after reciting that old chestnut about the famous inscription on Plato's Academy, dictating its entrance requirements in geometry, he writes:

> Today mathematics still is, in more ways than one, an essential preparation for those who want to understand the nature of things. But can one enter the world of mathematics without long and arid studies? Yes, one can to some extent, because what interests the curious and cultivated person (in older days called a philosopher) is not an extensive technical knowledge. Rather, the old-style philosopher (i.e., you and me) would like to see how the human mind, or we may say the mathematician's brain, comes to grips with reality. (p. vii)

Now, already something of the timbre of the book is apparent here, an enthusiasm blended with a lack of precision. I struggle to think of an age in which a "curious and cultivated person" has acted as the definition of the term "philosopher", and I think Ruelle would have been better advised to stick with the word "mind" rather than "brain" for his title. There is a philosophical position which might justify this substitution, but the only relevant chapter in the book—"The computer and the brain"—does not suggest that the author wishes to adopt such a reductionist position. Indeed, the book is precisely about how the mathematician's *mind* comes to grip with mathematical reality, an infinite-dimensional labyrinth, as Ruelle describes it.



There is an enormous amount to admire in the book. It is good to see the Erlanger Program given its due place, followed by an enjoyable example—The Butterfly Theorem—which gives an excellent illustration of how, in order to solve a problem, one needs to view it in its right setting, projective rather than Euclidean geometry here. The range of topics treated is very generous. Alongside standard subjects, such as foundations, proof, and the infinite, Ruelle treats us to his views on the mathematical reward system, beauty, Grothendieck, computers, emergence, psychoanalysis, and mathematical physics. I am happy to admit the bearing of all he writes on philosophy, which I rather imagine puts me in a small minority within my small field, even down to the incident he discusses of a mathematician's slip of the tongue where "anti-Semitic" is uttered

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for "anti-symmetric". But with this praise comes a serious reservation: there's little sense of how this work stands, or could stand, in relation to other works.

There is, in my opinion, something of a hole in academic space at the present time which needs to be filled by a disciplined approach to the understanding of the place of mathematics in the system of human thought. Clearly, Ruelle shares something of this sense that something is missing:

> How does a problem arise? How does it get solved? What is the nature of scientific thinking? Many people have asked these sorts of questions. Their answers fill many books and come under many labels: epistemology, cognitive science, neurophysiology, history of science and so on. I have read a number of these books and have been in part gratified, in part disappointed. (p. 1)

But to take steps to fill the void, we surely cannot respond to this disappointment by ignoring what is well done in these fields. The history of mathematics has changed significantly over recent decades and offers us impressive views of the changes which have transformed the field. So when Ruelle writes

> Between Euclid and the nineteenth century the proper way to handle real numbers was through geometry: a real number was represented as a ratio of the length of two line segments. (p. 24)

and

> …the remarkable thing is that modern mathematics is done precisely in the way that Euclid presented geometry. (p. 8),

you know what he means, but I can hear my historian friends' teeth grating from many a mile away. Regarding the second claim, should we not admit at the very least that styles of definition have changed from a time when it was thought proper to write "A line is a breadthless length" and "A straight line is a line which lies evenly with the points on itself"? We could take Ruelle's pronouncements as broad brush comments, strictly false yet morally true, just as, since a truth about Plato is conveyed by the story, we could little care that the documentary evidence for the inscription "Let none enter who is ignorant of mathematics" coming from the best part of a millennium after Plato flourished provides minimal support for its veracity. But my sense is that we do now need a disciplined accuracy.

By way of comparison, let us consider what sixty years ago the same publisher, Princeton University Press, saw fit to publish by way of a mathematical physicist turning to philosophy. Dip into Weyl's *Philosophy of Mathematics and Natural Science*, a book much of which had been written in 1926, and you meet many very challenging passages, phrased in the philosophical language of his day. Weyl, conversant with the writings of Leibniz, Locke, Hobbes, Hume, Kant, Fichte, Mach, and Husserl, could see himself as taking the next step in a flourishing discipline. We are not likely to see the equal of him again for a very long time. Besides his mathematical brilliance he had the good fortune to mature in an exceptional environment, where an educated person was versed in philosophy as a matter of course. Weyl comes from a time when a student of Weierstrass, Edmund Husserl, could turn to philosophy and be taken on by Hilbert at Göttingen.

Saunders Mac Lane caught the tail end of the mathematical and philosophical activity of prewar Göttingen (McLarty 2007), something which shows in his 1986 book *Mathematics: Form and Function*. Its author, too, perceives something very much lacking in the academic treatment of mathematics and is highly critical of the professional philosophical work on mathematics in the preceding half century. It could be said that this book has not been especially influential, but with its inner coherent vision it does stand a chance of proving a pile on which to build a new discipline. Ruelle's aspirations in writing *The Mathematician's Brain* were lower, but we may still ask of it whether it provides us with any useful materials.

As I have said, I take every one of the dominant themes of its twenty three chapters to be relevant to philosophy. But following the argument of the book is like following a butterfly flit, apparently purposelessly, from plant to plant. Even within a chapter, each of which comes to a close after a near regular six pages, there are frequent minor excursions. Take Chapter 21, "The strategy of mathematical invention", as an example. The thrust of the chapter is to recognize a form of intuition which governs the invention of a theory, although one which needs to ground itself via a formalism. Now this is a topic about which there is much to say, and indeed much has been said. In successive single paragraphs we hear about the role of *Mathematica* in grinding out facts; the drive to use "structural ideas", exemplified by the Grothendieck group construction in K-theory; and the use of analogy, any one of which could be the subject of a lengthy article. Any coherence of a thread in the chapter is then finally dispersed by a paragraph on the greater degree of religious belief found on average in mathematicians than in physicists.

If inner coherence is lacking, perhaps some stability could have been engendered by indicating points of attachment to the existing literature. But little is provided along these lines either in the text or in the endnotes. For even Ruelle's most obscure topics there is a body of work worth consulting. For instance, there is a tradition of thinking about mathematics in psychoanalytic terms, one in which Imre Hermann's *Parallélismes* (Hermann 1980) features, published in Ruelle's own country. As a mathematician Ruelle would not publish without a thorough literature search, why not then in his chapter on beauty compare his views to those of his fellow mathematician Gian-Carlo Rota in "The phenomenology of mathematical beauty", Chapter X of *Indiscrete Thoughts* (Rota 1996)?

Perhaps I have been too severe in this review through the disappointment of excessive expectations. As a doctoral student, I read thoroughly and many times over Ruelle's "Is our mathematics natural?" (1988). There was, I recall thinking at the time, an unresolved tension in the paper between the claim that parts of our mathematics would not have been devised had it not been for the fortuitous boost provided by the needs of parts of physics, such as equilibrium statistical mechanics, and the claim that the same pieces of mathematics may find use in many situations and may be integrated well into the rest of mathematics, suggesting multiple potential routes to their discovery. But where "Is our mathematics natural?" made me think very hard, inspired a chapter of my book (Corfield 2003), and provided me with an excellent case study for another, I don't see that *The Mathematician's Brain* can do much more than furnish me with a checklist of features of mathematics I might want to assure myself I had taken into account, if ever I felt I had reached some sort of complete philosophy of mathematics. No doubt the timing of one's encounter with a book is all-important to the opinion one forms of it. Over the years I have read an enormous amount of mathematicians' informal writings about their discipline. Daily I converse with mathematicians on the blog I jointly run. So, while little in the book struck me as new, perhaps those at earlier stages of their careers will be stimulated by the breadth of Ruelle's reach.

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