The Wraparound Universe
Jean-Pierre Luminet
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The standard models of relativistic cosmology have preferred 3-dimensional spatial sections of constant curvature; these are surfaces of spatial homogeneity in the expanding universe. It has been known since the earliest days of relativistic cosmology that they need not have the obvious simply connected topology, and indeed it was already known in the 1930s that their topology could be extremely complex (with an infinite number of possibilities in the negatively curved case).

What is new in the last few decades is that observational investigation of this spatial topology has become an active field of research in cosmology. While the simply connected topologies ($S^3$ in the case of positive curvature, $R^3$ in the cases of zero and negative curvature) are usually assumed in most cosmological studies, there is in fact no known physical mechanism that determines this spatial topology (the Einstein field equations are differential equations that determine the spacetime curvature locally but do not directly specify global connectivity; and no mechanism related to quantum gravity has so far been proposed that will determine this topology). Furthermore since the advent of string theory, the idea that immensely complex spatial topologies may occur in physics has become commonplace. The mathematical investigation of the possible spatial topologies relevant to cosmology has progressed greatly through the work of William Thurston and others.

Jean-Pierre Luminet is one of those who has been studying the way the different possible spatial topologies may be observationally investigated. One specific proposal he and colleagues have put forward is that the spatial topology may be that of a Poincaré dodecahedral space. This well-written book is a comprehensive introduction to this topic for the lay reader (it contains no equations but has many geometric diagrams illustrating the concepts used). It gives a sound introduction to the relevant cosmological theory and data and discusses in detail the possibilities of complex topologies in a universe where the resulting scale of spatial closure is so small that we have seen right around the universe since the time the universe became transparent, hence seeing many images of the same distant objects in different directions in the night sky. One can in principle study the topology by identifying such multiple images, but this is likely to be very difficult (it is not easy to identify multiple images as being due to the same object, for they will be seen at different distances and at different times in their histories, and effectively from different directions). One can also analyze the statistics of observations of distant sources, but this again may not be conclusive, for example, due to source evolution effects. What is easier in principle is to determine identical circles in the sky in the temperature fluctuations in the cosmic blackbody radiation that comes to us from all directions at an average temperature of 2.75K. This is a difficult task for statistical reasons, but such searches are under way and can in principle eventually be conclusive.

This book introduces all this in a clear way that will appeal to any reader interested in the topic at the Scientific American level. It will thereby introduce a very interesting branch of mathematics to nonmathematicians and show how it may relate to the real universe in which we live.

George F. R. Ellis is professor of mathematics at the University of Cape Town, South Africa. His email address is George.Ellis@uct.ac.za.