

Is the Sky Still Falling?

David M. Bressoud

In the 1998 *Notices* article “The Sky is Falling” [4], Garfunkel and Young drew attention to the alarming decrease in the number of students who study mathematics in college. In their words, “Our profession is in desperate trouble—immediate and present danger [...] If something is not done soon, we will see mathematics department faculties decimated and an already dismal job market completely collapse.” In the past ten years the situation seems to have reversed. The mathematical community is not in the desperate straits that Garfunkel and Young predicted. Yet, as this article will show, the situation is far from healthy, and in many respects we are worse off now than we were in 1995. Today we teach a smaller percentage of the total enrollment than ever before. The growth that has occurred has been entirely within our research universities, and there it can be explained by a short-term increase in the number of engineering students. This article concludes with three action items that the mathematical community needs to undertake if we are to reverse this decline.

Garfunkel and Young’s argument rested on data from the Conference Board of the Mathematical Sciences (CBMS) showing a drop in enrollments from 1985 to 1995. As Table 1 shows, the situation in 1995 looked far worse than it does today. Enrollment in precollege (remedial) mathematics has continued to decline at 4-year colleges.¹ For all other categories of courses,² enrollments are up

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¹Most precollege or remedial mathematics is taught at 2-year colleges where it now accounts for 61% of all mathematics taught at these colleges.

²Introductory level includes *College Algebra*, *Precalculus*, and *Math for Liberal Arts*. Calculus level is *Calculus I through Differential Equations*, *Linear Algebra*, and *Discrete Math*. Advanced is everything above calculus level including *Introduction to Proofs*. Statistics courses are not included in these numbers.

over the 1995 numbers by between 9% and 17%. While we are still well below the 1985 numbers for courses at the level of calculus and above, whatever was going wrong in the early 1990s seems to have been corrected.

But if we compare the number of students studying mathematics to the number of students enrolled in our 4-year undergraduate programs, we see that mathematics has been accounting for an ever-decreasing slice of the pie. The figures for 1995 were bad, but the percentages for 2005 are considerably worse (see Table 2).

These percentages should be alarming. The true situation is revealed to be even more discouraging once we unpack these numbers and look at what is happening in individual courses and at specific types of institutions. Because of its central role in the undergraduate curriculum, I will focus on calculus.

Calculus in High School

In the spring of 1985, 46,000 students took the Advanced Placement Calculus exam. In spring 2008, the number was 292,000. By 2009, it will be well over 300,000. In fact, the number of AP Calculus exams given each year has grown steadily over the past decade at an average rate of over 7% per year with no sign yet that it is approaching its inflection point (see Figure 1). AP Calculus exam takers are only a piece of the broader population of students who study calculus while in high school, a population that includes those who take an AP Calculus course but not the exam as well as those in the International Baccalaureate program, dual enrollment programs, registration in 2- or 4-year college calculus classes, and the many students who are given a soft introduction to calculus while in high school in the hope of easing the transition to college calculus. Based on the NELS study from spring 2004 [3], the NAEP transcript study from 2005 [12], and the growth of AP Calculus since then, it is safe to conclude that we have reached the point where each year over half a million high school students study calculus.

	1985	1990	1995	2000	2005
precollege level	251	261	222	219	201
introductory level	593	592	613	723	706
calculus level	637	647	538	570	587
advanced	138	119	96	102	112

Table 1. Mathematics enrollments (thousands) for fall term at 4-year colleges and universities in the United States. Sources: [1, 5–7].

	1985	1990	1995	2000	2005
precollege level	3.25%	3.04%	2.53%	2.34%	1.83%
introductory level	7.69%	6.90%	6.99%	7.72%	6.42%
calculus level	8.26%	7.54%	6.14%	6.09%	5.34%
advanced	1.79%	1.39%	1.09%	1.09%	1.02%

Table 2. Mathematics enrollments at 4-year colleges as a percentage of total number of students enrolled in fall term. Sources: [1, 5–7, 13].

	1985*	1990	1995	2000	2005
Calculus I	217	201	192	192	201
Calculus II	95	88	83	87	85
Calculus III & IV	90	84	62	73	74

Table 3. Mainstream calculus enrollments (thousands for fall term in 4-year colleges). *1985 breakdown is estimate based on total number of students in all mainstream calculus classes. Sources: [1, 5–7, 13].

Most of the students who study calculus in high school do not receive college credit for this course. Morgan [8] estimates that about half of the students who take the AP Calculus exam are entitled to and choose to use credit for Calculus I. Perhaps another 30,000 receive college credit via dual enrollment, IB, or enrollment in a college class. A reasonable estimate is that between 150,000 and 200,000 students arrive at college each fall bringing with them credit for calculus. That suggests that we should be seeing dramatically increasing numbers of students taking Calculus II in the fall term. As Table 3 shows, this is not the case. In fact, Calculus II enrollments in the fall term actually *dropped* over the period 2000–05.

What about the other 300,000–350,000 students who took calculus but not for college credit? One would hope that the increasing numbers of these students would translate into increasing numbers of students taking calculus in college. But combining all mainstream Calculus I classes in all 2- and 4-year colleges in the United States, fall enrollments have been stuck at very close to 250,000 over the past quarter century.

Calculus in College

At least we have seen some increase in the Calculus I, III, and IV enrollments over the period 2000–05. In fact, even that is less robust than it seems. When we break down calculus enrollments

by type of institution,³ we see that the growth is occurring entirely at the research universities (see Figures 2–4).

For all levels of college calculus, the increase since 1995 is entirely within the research universities. Everywhere else, enrollment has declined. There is a distinctive pattern of enrollments across all levels of calculus that occurred at the research universities and at no other type of institution: a five-year decline from 1990 to 1995 followed by steady growth. This pattern can be explained by the fact that most large research universities have large engineering programs. If we consider the number of incoming freshmen who intend to major in engineering (Figure 5 and Table 4), we see that it also decreased from 1990 to 1995, then grew. The scatterplot in Figure 6 shows a high correlation (correlation coefficient of 0.99) between the number of entering freshmen who intend to major in engineering and the total number of students in research universities each fall who enroll in any level of calculus. The downturn at the end of the graph in Figure 5 suggests that the 2005 CBMS numbers may be overly optimistic. As Table 4 shows, the number of students who intend to major in engineering began a steady decrease following a record large number in 2004.

It is interesting to compare the number of intended majors in engineering with those in the other STEM (science, technology, engineering, university”, and “undergraduate college” are substituted as descriptive of the general type of institution and to clarify that the categorization is by type of institution.

³The CBMS categories of 4-year institutions are based on the highest degree offered in mathematics: Ph.D., M.A., or B.A. The labels “research university”, “comprehensive

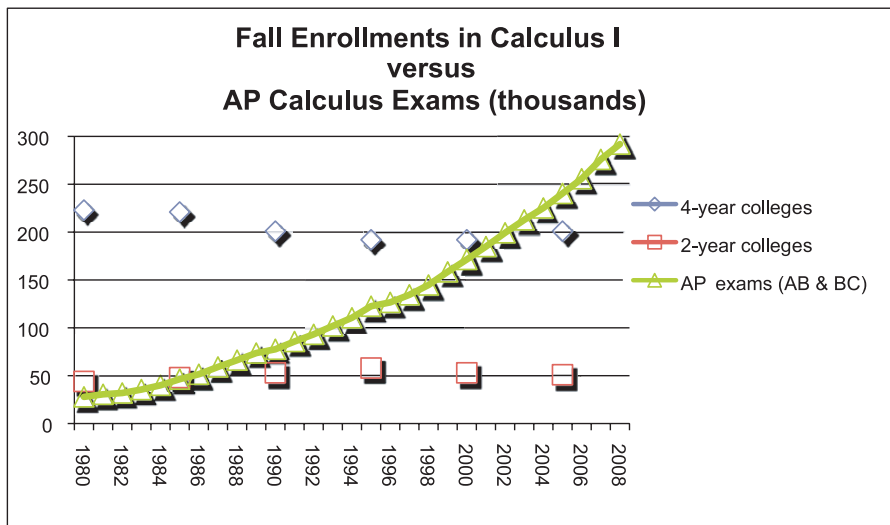


Figure 1. Fall enrollments in mainstream Calculus I and number of AP Calculus exams (thousands). Sources [1, 2, 5-7, 11].

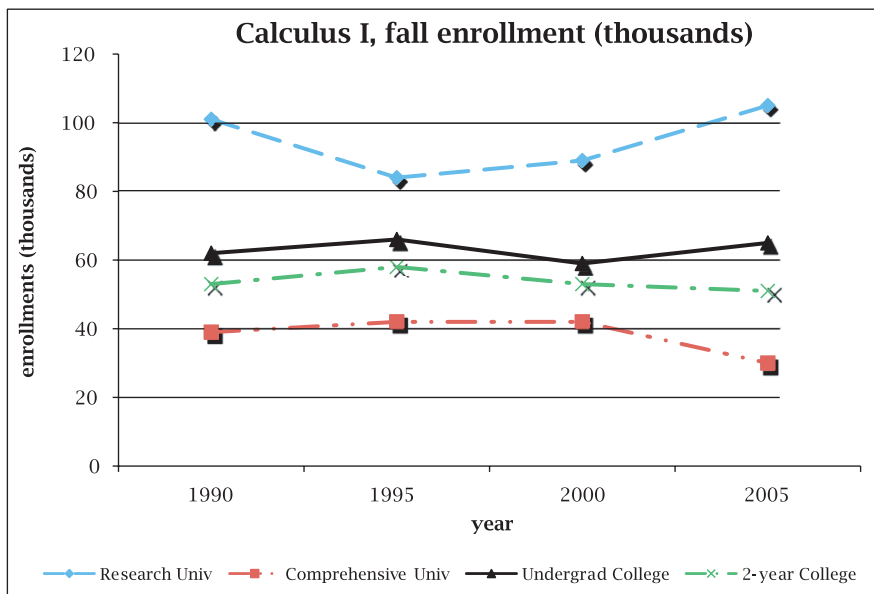


Figure 2. Fall enrollments in mainstream Calculus I by type of institution (thousands). Sources: [1, 5-7].

mathematics) disciplines: the biological sciences in Figure 7 and the physical sciences (including mathematics⁴) in Figure 8.

Conclusions

We have seen growth in enrollments in mathematics courses over the past ten years, but that growth is well below the rate of increase in total enrollments. The only place where it has been robust has been where it is tied to the increase in engineering

⁴The number of freshmen intending to major in mathematics dropped from 1.1% in 1985 to 0.5% in 2000. It has since grown to 0.8%, approximately the current percentage of graduates who earn majors in mathematics.

majors, a phenomenon that appears to be cyclical and has now entered a downturn.

The mathematical community needs to look at what it can do to strengthen enrollments. One solution is to get a lot more high school students to plan careers in engineering. It would be interesting to know what caused the reversal in engineering enrollments in the mid-1990s. These projections of intent to major in engineering were measured during freshman orientation, and thus the increase after 1995 was the result of something that happened in high school. What role did the introduction and widespread acceptance of graphing calculators and reform teaching methods within high schools have on the increased interest in and willingness to pursue highly technical majors? What is causing the current downturn in interest in engineering?

Engineering has served us well, but there is no reason why the fate of mathematics should be so dependent on just this discipline. The key to getting students into our advanced courses is to first get them into first-year courses that teach solid mathematics and pique their interest to continue in mathematics. This does not have to be a course tied to the engineering curriculum. Nevertheless, calculus is at the heart of the mathematics curriculum, and we must begin by taking a serious look at what is happening in college calculus and how well it articulates with the experiences that today's students have in high school. This is the basis for my first two recommendations.

Recommendation 1: *We need to understand what happens in college to students who study calculus in high school.*

The half million students who study calculus in high school are a reasonable approximation of the top 15% of all high school graduates. They should be swelling the ranks of the students taking calculus- and advanced-level mathematics. We need a better understanding of what happens to these students after they enter college. For the 150,000 to 200,000 who arrive with and use credit for calculus taken in high school, how many continue to pursue mathematics and how well do they succeed? What happens to the other 300,000 to 350,000? For all of these students, what are the programs that most

effectively engage them, preparing and encouraging them into the further study of mathematics?

Recommendation 2: *We need to know more about the preparation of the students who take calculus in college and what they need in order to succeed once they get to our classes.*

We must have a better sense of who these students are who sit in our college calculus classes. What is the preparation that has gotten them to this point? How can we modify our courses so as to capitalize on the strengths and correct the weaknesses that these students bring? The answers to these questions will necessarily be local, highly dependent on the nature of a given college or university, but the entire mathematical community should be able to identify commonalities among similar types of institutions. The entire community should also promote programmatic and course structures that are particularly effective for each of the different populations we encounter.

Recommendation 3: *Mainstream calculus should not be the only entry to good college-level mathematics.*

The department of mathematics should be at the core of its college or university, interacting with every other department and working collaboratively to develop courses that meet the needs of each group of students. These should be courses that involve real mathematics and that open the way to the further, deeper study of mathematics. This conviction should be part of the vision of every department of mathematics.

I look with longing at those 120,000 prospective biological science majors coming in each year. We need courses that are attractive to them, courses that give them the tools from linear algebra that they will need for sophisticated statistical modeling, courses that enable them to read and write differential equations and turn them into computer simulations. We are not going to be able to convince the biologists that their students need to take more of the courses that we have created for the engineers, nor is it enough to take an engineering course and throw in some biological examples. These courses must be designed from the ground

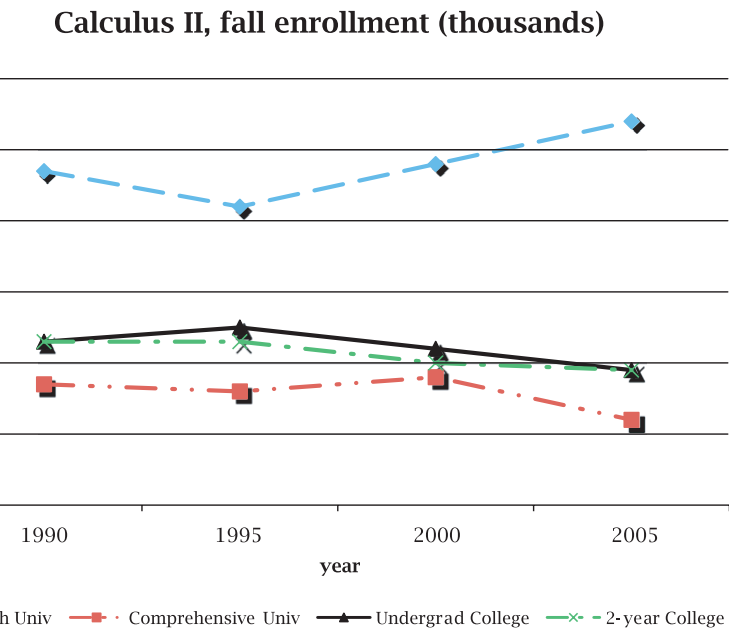


Figure 3. Fall enrollments in mainstream Calculus II and number of AP Calculus exams (thousands). Sources: [1, 2, 5-7].

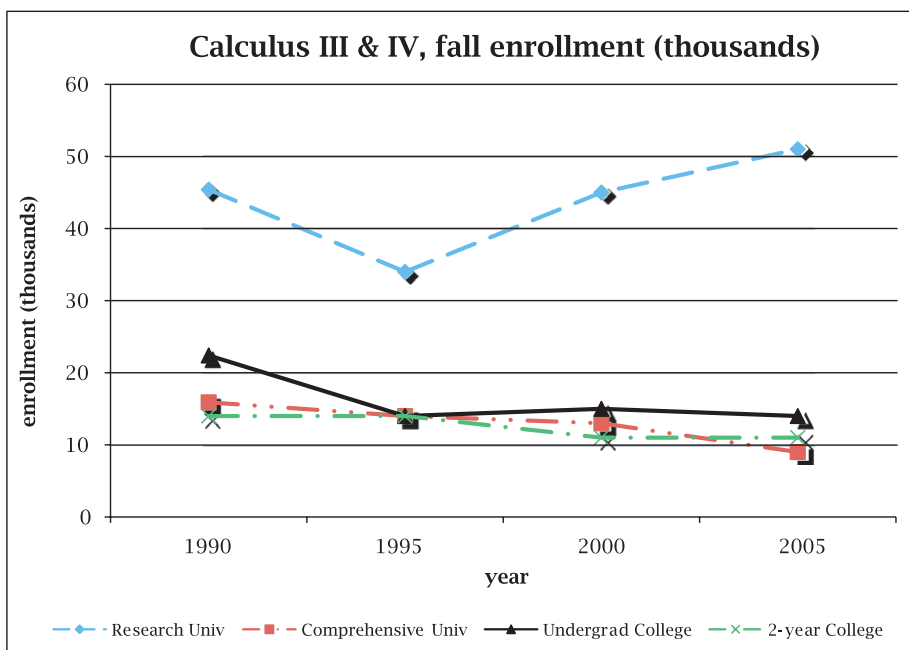


Figure 4. Fall enrollments in mainstream Calculus III & IV and number of AP Calculus exams (thousands). Sources: [1, 2, 5-7].

up in collaboration with biologists. Many colleges and universities have begun this process. See, for example, *Math & Bio: 2010* [14]. Doing this for biology is just the beginning of what should be a broad program of outreach and development.

The surge in engineering enrollments since 1995 coupled with the growth in physical science enrollments over the past five years has given us

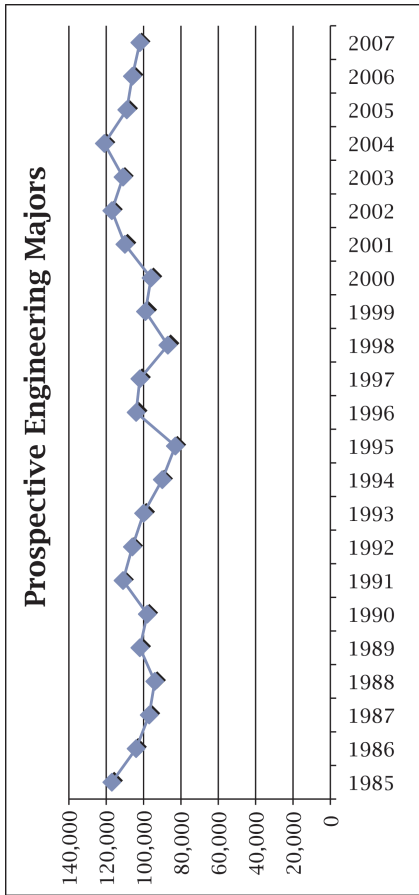


Figure 5. Number of freshmen in 4-year undergraduate programs who intend to major in engineering. Sources: [9, 10].

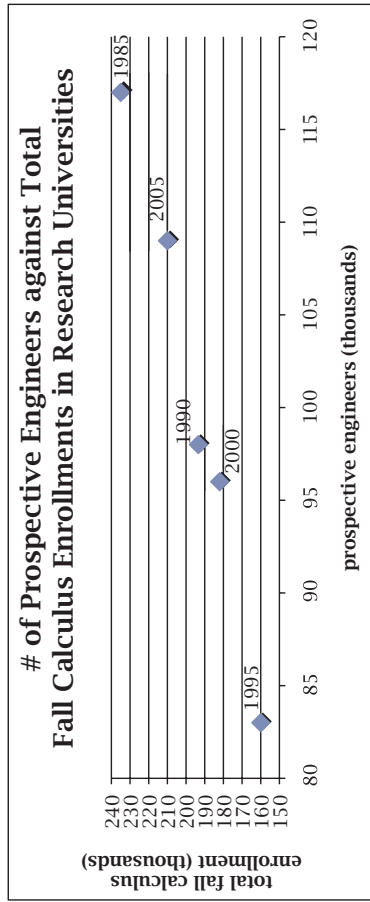


Figure 6. Number of prospective engineering majors against total fall calculus enrollment at research universities. Sources: [1, 5-7, 9, 10].

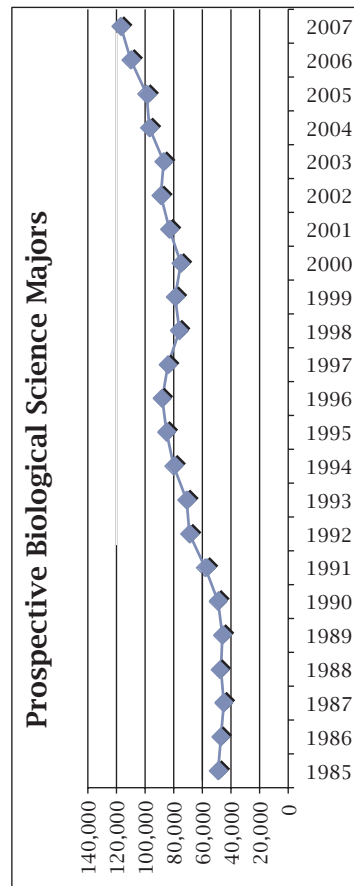


Figure 7. Number of freshmen in 4-year undergraduate programs who intend to major in biological sciences. Sources: [9, 10].

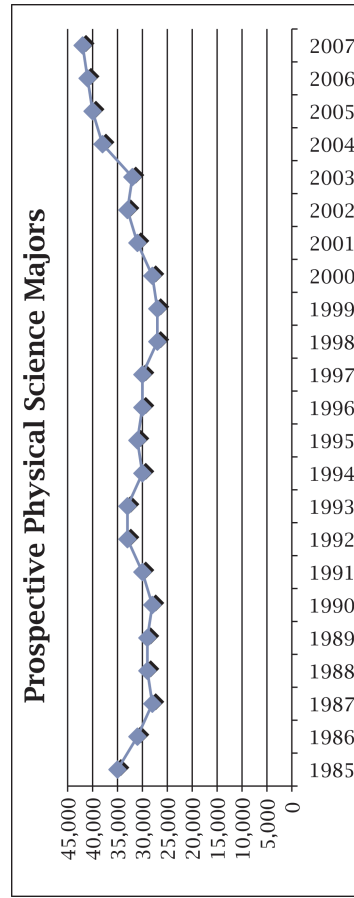


Figure 8. Number of freshmen in 4-year undergraduate programs who intend to major in physical sciences. Sources: [9, 10].

a reprieve. Yet, unless we address fundamental weaknesses, the long-term prognosis for the health of undergraduate mathematics is not good. I am still optimistic. Many talented people are working hard to improve the undergraduate program in mathematics. With a better sense of where we are and widespread dissemination of what works, we can build a foundation for the future.

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year	4-year freshman enrollment	% engineering	# prospective engineers
1985	1,067,928	11.0%	117,000
1986	1,023,762	10.2%	104,000
1987	1,031,968	9.4%	97,000
1988	1,076,036	8.7%	94,000
1989	1,028,143	9.9%	102,000
1990	1,010,548	9.7%	98,000
1991	1,024,976	10.8%	111,000
1992	1,060,087	10.0%	106,000
1993	996,690	10.0%	100,000
1994	1,017,725	8.8%	90,000
1995	1,024,550	8.1%	83,000
1996	1,076,035	9.7%	104,000
1997	1,054,500	9.7%	102,000
1998	1,066,679	8.2%	87,000
1999	1,098,833	9.0%	99,000
2000	1,101,817	8.7%	96,000
2001	1,204,240	9.1%	110,000
2002	1,234,968	9.5%	117,000
2003	1,196,089	9.3%	111,000
2004	1,258,333	9.6%	121,000
2005	1,298,093	8.4%	109,000
2006	1,320,824	8.0%	106,000
2007	1,354,958	7.5%	102,000

Table 4. Number of freshmen in 4-year undergraduate programs who intend to major in engineering. Sources: [9, 10].

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