

The Strong Free Will Theorem

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The two theories that revolutionized physics in the twentieth century, relativity and quantum mechanics, are full of predictions that defy common sense. Recently, we used three such paradoxical ideas to prove “The Free Will Theorem” (strengthened here), which is the culmination of a series of theorems about quantum mechanics that began in the 1960s. It asserts, roughly, that if indeed we humans have free will, then elementary particles already have their own small share of this valuable commodity. More precisely, if the experimenter can freely choose the directions in which to orient his apparatus in a certain measurement, then the particle’s response (to be pedantic—the universe’s response near the particle) is not determined by the entire previous history of the universe.

Our argument combines the well-known consequence of relativity theory, that the time order of space-like separated events is not absolute, with the EPR paradox discovered by Einstein, Podolsky, and Rosen in 1935, and the Kochen-Specker Paradox of 1967 (See [2].) We follow Bohm in using a spin version of EPR and Peres in using his set of 33 directions, rather than the original configuration used by Kochen and Specker. More contentiously, the argument also involves the notion of free will, but we postpone further discussion of this to the last section of the article.

Note that our proof does not mention “probabilities” or the “states” that determine them, which is

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fortunate because these theoretical notions have led to much confusion. For instance, it is often said that the probabilities of events at one location can be instantaneously changed by happenings at another space-like separated location, but whether that is true or even meaningful is irrelevant to our proof, which never refers to the notion of probability.

For readers of the original version [1] of our theorem, we note that we have strengthened it by replacing the axiom FIN together with the assumption of the experimenters’ free choice and temporal causality by a single weaker axiom MIN. The earlier axiom FIN of [1], that there is a finite upper bound to the speed with which information can be transmitted, has been objected to by several authors. Bassi and Ghirardi asked in [3]: what precisely is “information”, and do the “hits” and “flashes” of GRW theories (discussed in the Appendix) count as information? Why cannot hits be transmitted instantaneously, but not count as signals? These objections miss the point. The only information to which we applied FIN is the choice made by the experimenter and the response of the particle, as signaled by the orientation of the apparatus and the spot on the screen. The speed of transmission of any other information is irrelevant to our argument. The replacement of FIN by MIN has made this fact explicit. The theorem has been further strengthened by allowing the particles’ responses to depend on past half-spaces rather than just the past light cones of [1].

The Axioms

We now present and discuss the three axioms on which the theorem rests.

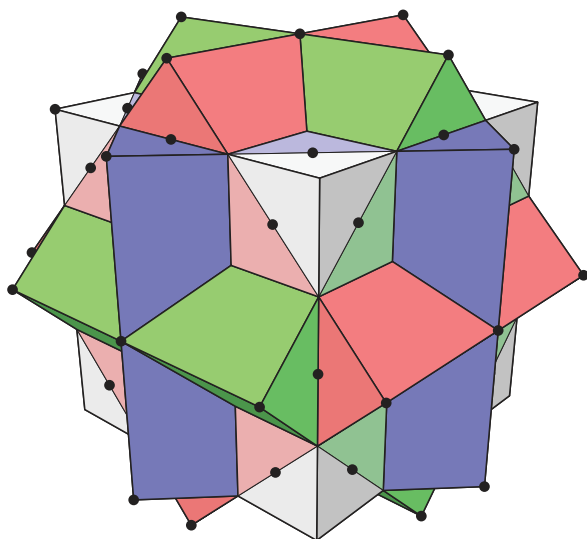


Figure 1a

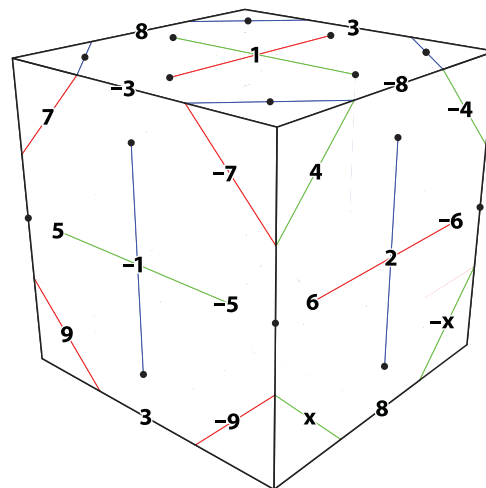


Figure 1b

Figure 1. The three colored cubes in Figure 1a are obtained by rotating the white cube through 45° about its coordinate axes. The 33 directions are the symmetry axes of the colored cubes and pass through the spots in Figure 1a. Figure 1b shows where these directions meet the white cube.

(i) The SPIN Axiom and the Kochen-Specker Paradox

Richard Feynman once said that “If someone tells you they understand quantum mechanics, then all you’ve learned is that you’ve met a liar.” Our first axiom initially seems easy to understand, but beware—Feynman’s remark applies! The axiom involves the operation called “measuring the squared spin of a spin 1 particle”, which always produces the result 0 or 1.

SPIN Axiom: Measurements of the squared (components of) spin of a spin 1 particle in three orthogonal directions always give the answers 1, 0, 1 in some order.

Quantum mechanics predicts this axiom since for a spin 1 particle the squared spin operators s_x^2, s_y^2, s_z^2 commute and have sum 2.

This “101 property” is paradoxical because it already implies that the quantity that is supposedly being measured cannot in fact exist before its “measurement”. For otherwise there would be a function defined on the sphere of possible directions taking each orthogonal triple to 1, 0, 1 in some order. It follows from this that it takes the same value on pairs of opposite directions, and never takes two orthogonal directions to 0.

We call a function defined on a set of directions that has all three of these properties a “101 function” for that set. But unfortunately we have:

The Kochen-Specker Paradox: There does not exist a 101 function for the 33 pairs of directions of Figure 1 (the Peres configuration).

Proof. We shall call a node even or odd according as the putative 101 function is supposed to take the value 0 or 1 at it, and we progressively assign even or odd numbers to the nodes in Figure 1b as we establish the contradiction.

We shall use some easily justified orthogonalities—for instance the coordinate triple rotates to the triple $(2, 3, -3)$ that starts our proof, which in turn rotates (about -1) to the triples $(8, -7, 9)$ and $(-8, 7, -9)$ that finish it.

Without loss of generality nodes 1 and -1 are odd and node 2 even, forcing 3 and -3 to be odd. Now nodes 4 and $-x$ form a triple with 3, so one of them (without loss of generality 4) is even. In view of the reflection that interchanges -4 and x while fixing 4 and $-x$, we can without loss of generality suppose that -4 is also even.

There is a 90° rotation about 1 that moves 7, 5, 9 to 4, 6, x , showing that 5 is orthogonal to 4, while 1, 5, 6 is a triple, and also that 6 is orthogonal to both 7 and 9. Thus 5 is odd, 6 even, and 7, 9 odd. A similar argument applies to nodes $-5, -6, -7, -9$.

Finally, 8 forms a triple with -7 and 9, as does -8 with 7 and -9 . So both these nodes must be even, and since they are orthogonal, this is a contradiction that completes the proof. \square

Despite the Kochen-Specker paradox, no physicist would question the truth of our SPIN axiom, since it follows from quantum mechanics, which is one of the most strongly substantiated scientific theories of all time. However, it is important to realize that we do not in fact suppose all of

quantum mechanics, but only two of its testable consequences, namely this axiom SPIN and the axiom TWIN of the next section.

It is true that these two axioms deal only with idealized forms of experimentally verifiable predictions, since they refer to exact orthogonal and parallel directions in space. However, as we have shown in [1], the theorem is robust, in that approximate forms of these axioms still lead to a similar conclusion. At the same time, this shows that any more accurate modifications of special relativity (such as general relativity) and of quantum theory will not affect the conclusions of the theorem.

(ii) The TWIN Axiom and the EPR Paradox

One of the most curious facts about quantum mechanics was pointed out by Einstein, Podolsky, and Rosen in 1935. This says that even though the results of certain remotely separated observations cannot be individually predicted ahead of time, they can be correlated.

In particular, it is possible to produce a pair of “twinned” spin 1 particles (by putting them into the “singleton state” of total spin zero) that will give the same answers to the above squared spin measurements in parallel directions. Our “TWIN” axiom is part of this assertion.

The TWIN Axiom: *For twinned spin 1 particles, suppose experimenter A performs a triple experiment of measuring the squared spin component of particle a in three orthogonal directions x, y, z, while experimenter B measures the twinned particle b in one direction, w. Then if w happens to be in the same direction as one of x, y, z, experimenter B’s measurement will necessarily yield the same answer as the corresponding measurement by A.*

In fact we will restrict w to be one of the 33 directions in the Peres configuration of the previous section, and x, y, z to be one of 40 particular orthogonal triples, namely the 16 such triples of that configuration and the 24 further triples obtained by completing its remaining orthogonal pairs.

(iii) The MIN Axiom, Relativity, and Free Will

One of the paradoxes introduced by relativity was the fact that temporal order depends on the choice of inertial frame. If two events are space-like separated, then they will appear in one time order with respect to some inertial frames, but in the reverse order with respect to others. The two events we use will be the above twinned spin measurements.

It is usual tacitly to assume the temporal causality principle that the future cannot alter the past. Its relativistic form is that an event cannot be influenced by what happens later in any given inertial frame. Another customarily tacit assumption is that experimenters are free to choose between possible experiments. To be precise, we mean that

the choice an experimenter makes is not a function of the past. We explicitly use only some very special cases of these assumptions in justifying our final axiom.

The MIN Axiom: *Assume that the experiments performed by A and B are space-like separated. Then experimenter B can freely choose any one of the 33 particular directions w, and a’s response is independent of this choice. Similarly and independently, A can freely choose any one of the 40 triples x, y, z, and b’s response is independent of that choice.*

It is the experimenters’ free will that allows the free and independent choices of x, y, z, and w. But in one inertial frame—call it the “A-first” frame—B’s experiment will only happen some time later than A’s, and so a’s response cannot, by temporal causality, be affected by B’s later choice of w. In a B-first frame, the situation is reversed, justifying the final part of MIN. (We shall discuss the meaning of the term “independent” more fully in the Appendix.)

The (Strong) Free Will Theorem

Our theorem is a strengthened form of the original version of [1]. Before stating it, we make our terms more precise. We use the words “properties”, “events”, and “information” almost interchangeably: whether an event has happened is a property, and whether a property obtains can be coded by an information-bit. The exact general meaning of these terms, which may vary with some theory that may be considered, is not important, since we only use them in the specific context of our three axioms.

To say that A’s choice of x, y, z is free means more precisely that it is not determined by (i.e., is not a function of) what has happened at earlier times (in any inertial frame). Our theorem is the surprising consequence that particle a’s response must be free in exactly the same sense, that it is not a function of what has happened earlier (with respect to any inertial frame).

The Free Will Theorem. *The axioms SPIN, TWIN and MIN imply that the response of a spin 1 particle to a triple experiment is free—that is to say, is not a function of properties of that part of the universe that is earlier than this response with respect to any given inertial frame.*

Proof. We suppose to the contrary—this is the “functional hypothesis” of [1]—that particle a’s response (i, j, k) to the triple experiment with directions x, y, z is given by a function of properties α, \dots that are earlier than this response with respect to some inertial frame F. We write this as

$$\theta_a^F(\alpha) = \text{one of } (0, 1, 1), (1, 0, 1), (1, 1, 0)$$

(in which only a typical one of the properties α is indicated).

Similarly we suppose that b 's response 0 or 1 for the direction w is given by a function

$$\theta_b^G(\beta) = \text{one of 0 or 1}$$

of properties β, \dots that are earlier with respect to a possibly different inertial frame G .

(i) If either one of these functions, say θ_a^F , is influenced by some information that is free in the above sense (i.e., not a function of A 's choice of directions and events F-earlier than that choice), then there must be an earliest ("infimum") F-time t_0 after which all such information is available to a . Since the non-free information is also available at t_0 , all these information bits, free and non-free, must have a value 0 or 1 to enter as arguments in the function θ_a^F . So we regard a 's response as having started at t_0 .

If indeed, there is *any* free bit that influences a , the universe has by definition taken a free decision near a by time t_0 , and we remove the pedantry by ascribing this decision to particle a . (This is discussed more fully in the section "Free Will Versus Determinism".)

(ii) From now on we can suppose that no such new information bits influence the particles' responses, and therefore that α and β are functions of the respective experimenters' choices and of events earlier than those choices.

Now an α can be expected to vary with x, y, z and may or may not vary with w . However, whether the function varies with them or not, we can introduce all of x, y, z, w as new arguments and rewrite θ_a^F as a new function (which for convenience we give the same name)

$$\theta_a^F(x, y, z, w; \alpha') \quad (\star)$$

of x, y, z, w and properties α' independent of x, y, z, w .

To see this, replace any α that does depend on x, y, z, w by the constant values $\alpha_1, \dots, \alpha_{1320}$ it takes for the $40 \times 33 = 1320$ particular quadruples x, y, z, w we shall use. Alternatively, if each α is some function $\alpha(x, y, z, w)$ of x, y, z, w , we may substitute these functions in (\star) to obtain information bits independent of x, y, z, w .

Similarly, we can rewrite θ_b^G as a function

$$\theta_b^G(x, y, z, w; \beta')$$

of x, y, z, w and properties β' independent of x, y, z, w .

Now for the particular choice of w that B will make, there is a value β_0 for β' for which

$$\theta_b^G(x, y, z, w; \beta_0)$$

is defined. By the above independence of β' from w , the function $\theta_b^G(x, y, z, w; \beta_0)$ is defined with the same value β_0 for all 33 values of w . (The fact that MIN allows B to freely vary his choice of w makes this intuitively clear.)

We now define

$$\theta_0^G(w) = \theta_b^G(x, y, z, w; \beta_0),$$

noting that since by MIN the response of b cannot vary with x, y, z , θ_0^G is a function just of w .

Similarly there is a value α_0 of α' for which the function

$$\theta_1^F(x, y, z) = \theta_a^F(x, y, z, w; \alpha_0)$$

is defined for all 40 triples x, y, z , and it is also independent of w , which argument we have therefore omitted.

But now by TWIN we have the equation

$$\theta_1^F(x, y, z) = (\theta_0^G(x), \theta_0^G(y), \theta_0^G(z)).$$

However, since by SPIN the value of the left-hand side is one of (0, 1, 1), (1, 0, 1), (1, 1, 0), this shows that θ_0^G is a 101 function, which the Kochen-Specker paradox shows does not exist. This completes the proof.

Locating the Response

We now provide a fuller discussion of some delicate points.

(i) Since the observed spot on the screen is the result of a cascade of slightly earlier events, it is hard to define just when "the response" really starts. We shall now explain why one can regard a 's response (say) as having already started at any time after A 's choice when all the free information bits that influence it have become available to a .

Let $N(a)$ and $N(b)$ be convex regions of space-time that are just big enough to be "neighborhoods of the respective experiments", by which we mean that they contain the chosen settings of the apparatus and the appropriate particle's responses. Our proof has shown that if the backward half-space $t < t_F$ determined by a given F-time t_F is disjoint from $N(a)$, then the *available* information it contains is not enough to determine a 's response. On the other hand, if each of the two such half-spaces contains the respective neighborhood, then of course they already contain the responses. By varying F and G , this suffices to locate the free decisions to the two neighborhoods, which justifies our ascribing it to the particles themselves.

(ii) We remark that not all the information in the G -backward half-space (say) need be available to b , because MIN prevents particle b 's function θ_b^G from using experimenter A 's choice of directions x, y, z . The underlying reason is of course, that relativity allows us to view the situation from a B -first frame, in which A 's choice is made only later than b 's response, so that A is still free to choose an arbitrary one of the 40 triples. However, this is our only use of relativistic invariance—the argument actually allows any information that does not reveal A 's choice to be transmitted superluminally, or even backwards in time.

(iii) Although we've precluded the possibility that θ_b^G can vary with A 's choice of directions, it is conceivable that it might nevertheless vary with a 's (future!) response. However, θ_b^G cannot be affected by a 's response to an unknown triple chosen by A , since the same information is conveyed by the responses $(0, 1, 1)$, to (x, y, z) , $(1, 0, 1)$ to (z, x, y) , and $(1, 1, 0)$ to (y, z, x) . For a similar reason θ_a^F cannot use b 's response, since B 's experiment might be to investigate some orthogonal triple u, v, w and discard the responses corresponding to u and v .

(iv) It might be objected that free will itself might in some sense be frame-dependent. However, the only instance used in our proof is the choice of directions, which, since it becomes manifest in the orientation of some macroscopic apparatus, must be the same as seen from arbitrary frames.

(v) Finally, we note that the new proof involves four inertial frames— A -first, B -first, F , and G . This number cannot be reduced without weakening our theorem, since we want it to apply to arbitrary frames F and G , including for example those in which the two experiments are nearly simultaneous.

Free Will Versus Determinism

We conclude with brief comments on some of the more philosophical consequences of the Free Will Theorem (abbreviated to FWT).

Some readers may object to our use of the term "free will" to describe the indeterminism of particle responses. Our provocative ascription of free will to elementary particles is deliberate, since our theorem asserts that if experimenters have a certain freedom, then particles have exactly the same kind of freedom. Indeed, it is natural to suppose that this latter freedom is the ultimate explanation of our own.

The humans who choose x, y, z , and w may of course be replaced by a computer program containing a pseudo-random number generator. If we dismiss as ridiculous the idea that the particles might be privy to this program, our proof would remain valid. However, as we remark in [1], free will would still be needed to choose the random number generator, since a determined determinist could maintain that this choice was fixed from the dawn of time.

We have supposed that the experimenters' choices of directions from the Peres configuration are totally free and independent. However, the freedom we have deduced for particles is more constrained, since it is restricted by the TWIN axiom. We introduced the term "semi-free" in [1] to indicate that it is really the pair of particles that jointly makes a free decision.

Historically, this kind of correlation was a great surprise, which many authors have tried to explain away by saying that one particle influences the other. However, as we argue in detail in [1], the correlation is relativistically invariant, unlike any such explanation. Our attitude is different: following Newton's famous dictum "Hypotheses non fingo", we attempt no explanation, but accept the correlation as a fact of life.

Some believe that the alternative to determinism is randomness, and go on to say that "allowing randomness into the world does not really help in understanding free will." However, this objection does not apply to the free responses of the particles that we have described. It may well be true that classically stochastic processes such as tossing a (true) coin do not help in explaining free will, but, as we show in the Appendix and in §10.1 of [1], adding randomness also does not explain the quantum mechanical effects described in our theorem. It is precisely the "semi-free" nature of twinned particles, and more generally of entanglement, that shows that something very different from classical stochasticism is at play here.

Although the FWT suggests to us that determinism is not a viable option, it nevertheless enables us to agree with Einstein that "God does not play dice with the Universe." In the present state of knowledge, it is certainly beyond our capabilities to understand the connection between the free decisions of particles and humans, but the free will of neither of these is accounted for by mere randomness.

The tension between human free will and physical determinism has a long history. Long ago, Lucretius made his otherwise deterministic particles "swerve" unpredictably to allow for free will. It was largely the great success of deterministic classical physics that led to the adoption of determinism by so many philosophers and scientists, particularly those in fields remote from current physics. (This remark also applies to "compatibilism", a now unnecessary attempt to allow for human free will in a deterministic world.)

Although, as we show in [1], determinism may formally be shown to be consistent, there is no longer any evidence that supports it, in view of the fact that classical physics has been superseded by quantum mechanics, a non-deterministic theory. The import of the free will theorem is that it is not only current quantum theory, but the world itself that is non-deterministic, so that no future theory can return us to a clockwork universe.

Appendix. Can There Be a Mechanism for Wave Function Collapse?

Granted our three axioms, the FWT shows that nature itself is non-deterministic. It follows that there can be no correct relativistic deterministic

theory of nature. In particular, no relativistic version of a hidden variable theory such as Bohm's well-known theory [4] can exist.

Moreover, the FWT has the stronger implication that there can be no relativistic theory that provides a mechanism for reduction. There are nonlinear extensions of quantum mechanics, which we shall call collectively GRW theories (after Ghirardi, Rimini, and Weber, see [5]) that attempt to give such a mechanism. The original theories were not relativistic, but some newer versions make that claim. We shall focus here on Tumulka's theory rGRWf (see [6]), but our argument below applies, *mutatis mutandis*, to other relativistic GRW theories. We disagree with Tumulka's claim in [7] that the FWT does not apply to rGRWf, for reasons we now examine.

(i) As it is presented in [6], rGRWf is not a deterministic theory. It includes stochastic "flashes" that determine the particles' responses. However, in [1] we claim that adding randomness, or a stochastic element, to a deterministic theory does not help:

"To see why, let the stochastic element in a putatively relativistic GRW theory be a sequence of random numbers (not all of which need be used by both particles). Although these might only be generated as needed, it will plainly make no difference to let them be given in advance. But then the behavior of the particles in such a theory would in fact be a function of the information available to them (including this stochastic element)."

Tumulka writes in [7] that this "recipe" does not apply to rGRWf:

"Since the random element in rGRWf is the set of flashes, nature should, according to this recipe, make at the initial time the decision where-when flashes will occur, make this decision 'available' to every space-time location, and have the flashes just carry out the pre-determined plan. The problem is that the distribution of the flashes depends on the external fields, and thus on the free decision of the experimenters. In particular, the correlation between the flashes in *A* and those in *B* depends on both external fields. Thus, to let the randomness 'be given in advance' would make a big difference indeed, as it would require nature to know in advance the decision of both experimenters, and would thus require the theory either to give up freedom or to allow influences to the past."

Thus, he denies that our "functional hypothesis", and so also the FWT, apply to rGRWf. However, we can easily deal with the dependence of the distribution of flashes on the external fields F_A and F_B , which arise from the two experimenters'

choices of directions x, y, z , and w .¹ There are $40 \times 33 = 1320$ possible fields in question. For each such choice, we have a distribution $X(F_A, F_B)$ of flashes, i.e., we have different distributions $X_1, X_2, \dots, X_{1320}$. Let us be given "in advance" all such random sequences, with their different weightings as determined by the different fields. Note that for this to be given, nature does not have to know in advance the actual free choices F_A (i.e., x, y, z) and F_B (i.e., w) of the experimenters. Once the choices are made, nature need only refer to the relevant random sequence X_k in order to emit the flashes in accord with rGRWf.

If we refer to the proof of the FWT, we can see that we are here simply treating the distributions $X(F_A, F_B) [= X(x, y, z, w)]$ in exactly the same way we treated any other information-bit α that depended on x, y, z, w . There we substituted all the values $\alpha_1, \dots, \alpha_{1320}$ for α in the response function $\theta_a(x, y, z, w; \alpha)$. Thus, the functional hypothesis does apply to rGRWf, as modified in this way by the recipe.

Tumulka [7] grants that if that is the case, then rGRWf acquires some nasty properties: In some frame Λ , "[the flash] f_y^Λ will entail influences to the past." Actually, admitting that the functional hypothesis applies to rGRWf has more dire consequences—it leads to a contradiction. For if, as we just showed, the functional hypothesis applies to the flashes, and the first flashes determine the particles' responses, then it also applies to these responses, which by the FWT leads to a contradiction.

(ii) Another possible objection is that in our statement of the MIN axiom, the assertion that *a*'s response is independent of *B*'s choice was insufficiently precise. Our view is that the statement must be true whatever precise definition is given to the term "independent", because in no inertial frame can the past appearance of a macroscopic spot on a screen depend on a future free decision.

It is possible to give a more precise form of MIN by replacing the phrase "particle *b*'s response is independent of *A*'s choice" by "if *a*'s response is determined by *B*'s choice, then its value does not vary with that choice." However, we actually need precision only in the presence of the functional hypothesis, when it takes the mathematical form that *a*'s putative response function θ_a^F cannot in fact vary with *B*'s choice. To accept relativity but deny MIN is therefore to suppose that an experimenter can freely make a choice that will alter the past, by changing the location on a screen of a spot that has already been observed.

¹This unfortunately makes rGRWf non-predictive—it can only find the flash distribution that "explains" either particle's behavior when both experimenters' fields are given.

Tumulka claims in [7] that since in the twinning experiment the question of which one of the first flashes at A and B is earlier is frame-dependent, it follows that the determination of which flash influences the other is also frame-dependent. However, MIN does not deal with flashes or other occult events, but only with the particles' responses as indicated by macroscopic spots on a screen, and these are surely not frame-dependent.

In any case, we may avoid any such questions about the term "independent" by modifying MIN to prove a weaker version of the FWT, which nevertheless still yields a contradiction for relativistic GRW theories, as follows.

MIN': *In an A-first frame, B can freely choose any one of the 33 directions w , and a 's prior response is independent of B's choice. Similarly, in a B-first frame, A can independently freely choose any one of the 40 triples x, y, z , and b 's prior response is independent of A's choice.*

To justify MIN' note that a 's response, signaled by a spot on the screen, has already happened in an A-first frame, and cannot be altered by the later free choice of w by B ; a similar remark applies to b 's response. In [7], Tumulka apparently accepts this justification for MIN' in rGRWf: "... the first flash f_A does not depend on the field F_B in a frame in which the points of B are later than those of A ."

This weakening of MIN allows us to prove a weaker form of the FWT:

FWT': *The axioms SPIN, TWIN, and MIN' imply that there exists an inertial frame such that the response of a spin 1 particle to a triple experiment is not a function of properties of that part of the universe that is earlier than the response with respect to this frame.*

This result follows without change from our present proof of the FWT by taking F to be an A-first frame and G a B-first frame, and applying MIN' in place of MIN to eliminate θ_a^F 's dependence on w and θ_b^G 's dependence on x, y, z .

We can now apply FWT' to show that rGRWf's first flash function (f_y^A of [4]), which determines a 's response, cannot exist, by choosing Λ to be the frame named in FWT'.

The Free Will Theorem thus shows that any such theory, even if it involves a stochastic element, must walk the fine line of predicting that for certain interactions the wave function collapses to some eigenfunction of the Hamiltonian, without being able to specify which eigenfunction this is. If such a theory exists, the authors have no idea what form it might take.

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Authors' Note: We thank Eileen Olszewski for typesetting the paper and Frank Swenton for the graphics.



Noticed

Mathematicians rarely make it into the newspaper, much less into fashion spreads. But on September 21, 2008, the fashion section of the *New York Times* Sunday magazine carried a picture of Annalisa Crannell, a professor of mathematics at Franklin & Marshall College (and a book reviewer in this issue of the *Notices*). The magazine contacted Crannell saying that it was doing a photo portfolio about academics and had chosen her because she had received the Haimo Award from the Mathematical Association of America. "By phone, I learned that the 'photo portfolio' was really about fashion, which was pretty funny because part of my Haimo Award speech included the price tags of the clothes I usually wear (US\$1 or less)," she said.

A crew of eight spent three hours dressing Crannell and doing her hair and makeup. One of them told Crannell he loves mathematics and asked her some questions about fractals. The picture of Crannell, which can be found on the Web, does not show the US\$2,500 Gucci boots, which Crannell called "pretty darned uncomfortable". "I was wearing my blue AMS 'I love math' bracelet that I'd picked up at MathFest, and the guy admired it so I gave it to him," Crannell recalled. "I didn't get to keep any of the clothes they brought, but he got to keep my bracelet!"