Mathematics and the Aesthetic

Reviewed by Annalisa Crannell

Mathematics and the Aesthetic
Nathalie Sinclair, David Pimm, and William Higginson, Editors
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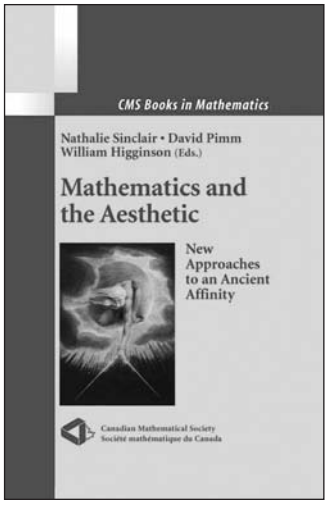
Mathematics and the Aesthetic is a heavy book. I mean this in the literal sense: at 288 pages, it weighs 721 grams. The book has the same dimensions as the little 206-page book on Fourier series that my mother studied from in 1959 [1], but into that small space it packs almost half again the number of pages and one-third again the mass. This weight is a tangible sign of the care that the authors, editors, and publisher used in putting the book together. The high-quality paper is matched by a sense of attention to detail throughout. The color pictures are carefully chosen and well presented; the text is easy to read.

Which is all as it should be for a book with the word "Aesthetic" in the title.

This is a daunting project to pull together, and there are many possible ways to botch the job. When we try to describe the aesthetics of mathematics—the discipline that gives us cohomology and nonlinear fourth-order partial differential equations—it's far too easy to become pretentious and wordy. I've often caught myself at the other extreme of sophistication, figuratively burbling in my beer: "No, man, it's beautiful. No, really!" And the middle ground, that space between pretension and exuberance, has been so famously traveled by the likes of Hardy and Poincaré, that it seems like hubris or folly (or both) to trod this ground once again. Who could touch their eloquence? What more could a body say?



Enter Sinclair, Pimm, and Higginson. This trio of editors/authors assembled a group of scholars with diverse academic credentials: philosophers, math educators, and (yes) research mathematicians. The authors' combined contributions are not (as I might have originally supposed) additional propaganda to convince the masses that math is beautiful. Rather, each author explores the ways in which mathematics is an aesthetic experience. For some authors, "aesthetic" translates into positive notions (like "elegance" or "beauty"); other authors use "aesthetic" in a more technical, neutral meaning of "relating to the senses and sensibility". If the second sense seems far from logic and from the abstract reasoning of mathematics, we might use instead the definition proposed by Judith Wechsler [3, page 2]:

> In its dealings with "things perceptible to the senses", aesthetics comes to grips with *relations*: structure, context, schemata, similarity/dissimilarity, consonance/dissonance. (About isolated,

Annalisa Crannell is professor of mathematics at Franklin & Marshall College. Her email address is annalisa.crannell@fandm.edu.

> simple, sense impressions aesthetics has little to say.)

There are ten authors of nine chapters (eleven chapters, if you count the forward and afterward), and often this mix would lend a hodge-podge feel to a volume. It's true that this work didn't escape completely from the bane of variable quality. Still, more than in any other collection of essays I've read, the editors clearly worked hard with the authors to ensure a sense of the whole; authors from one chapter often compare their own descriptions to those that appear in other chapters, and navigating through the book is easy to do.

Are there general themes that emerge?

One of the things that struck me as I read through these chapters—particularly through the first half of the book—was the recurring theme of describing the *doing* of mathematics, rather than describing the mathematics itself. We often articulate the *usefulness* of doing mathematics (for example, telling our students that "mathematics is not a spectator sport"). But these authors move beyond utility to aesthetics, in ways I'll elaborate below. Their collective perspectives made me wish for a few new good verbs. There is no good verb for "to do beautiful things", for example: *beautify*, as a transitive verb, doesn't quite work, since it requires that something else get beautified, rather than implying the action itself is beautiful. And similarly, there's no good verb for "to do mathematics".

This impoverishment of vocabulary strikes me as a shame. The authors in this volume frequently used theorems and proofs (well-known, surprising, or personally dear) to illustrate their points, but they just as often described the experiences of grappling with mathematics, of puzzling through it, and of making sudden sense of previously confusing concepts. Poincaré famously described his own epiphanic experience of completing a perplexing proof, and many mathematicians are familiar with that kind of "aha" gift of insight. To me an even more touching description of the doing of mathematics is Doris Schattschneider's description (page 56) of pleasure in hammering together some not-so-beautiful mathematics. After describing in some particular detail why the proof she had worked on was neither elegant, nor ingenious, nor generalizable, nor insightful, nor illuminating, she continued on:

> Yet, I received a measure of satisfaction in proving this result. I had believed it was true and now it was proven to be true. And, along the way, I saw some connections that I could not have seen without actually going through the process …

And then she delightedly shares some of the connections she discovered.

The aesthetic of "doing" arises in other chapters: Sinclair (Chapter 4) divides her analysis of "The aesthetic sensibilities of mathematicians" into three parts: how we evaluate mathematics, how we generate new mathematics, and how we are motivated to do various kinds of mathematics. Schiralli (Chapter 5) creates parallels between mathematics and art (and the aesthetics of each) by creating a page-long, bulleted list, each bullet beginning with an italicized verb (*e.g.*, "*create* novel or original symbolic patterns"). Jackiw (Chapter 7) devotes his entire chapter to "dynamic geometry" and to ways in which mathematicians understand a figure by moving its parts. Henderson and Taimina (Chapter 3) address the noun/verb issue explicitly in their essay on "Experiencing meanings in geometry" (page 83):

> …In fact, rarely do we find proofs, in and of themselves, to be aesthetic objects. Instead, we locate the aesthetic value of mathematics in the coming-to-understanding, in the integration of experience and meaning.

But this emphasis on verbs is a motif that I myself have imposed on the reading of this book. It would be fairer to the book for me to sketch its outline and give a quick tour of the content of its chapters. Here is an overview, which I'll follow with a deeper examination of one or two of the more provocative chapters.

The editors divide the main body of the work into three-chapter groupings, which fit into three sections: first, mathematics as an aesthetic subject; second, an explanation of "the aesthetic dimension of mathematics"; and third, a description of how both of these subjects fit into the study of aesthetics in other disciplines. If the first two sections seem similar, I will add that locating the emphasis on mathematics (in the first case) or on aesthetics (in the second) is key: Section A is authored by research mathematicians who look at mathematics through their own perspectives; Section B is authored by mathematics educators and philosophers who look at mathematics, so to speak, through other people's eyes. Each of the three sections comes with a one-page introduction from the editors.

In addition, the editors include Chapters Alpha and Omega to introduce and close out the book. Chapter Alpha was for me a welcome introduction to the history that precedes this book, telling the story of mathematical paradigms from the time of the Pythagoreans through St. Augustine, Gauss, Hadamard, Russell, and others, and then pointing toward the future. (I'll pick a fight with Chapter Omega in due time.)

Section A ("The mathematician's art") is probably the most familiar and comfortable material to readers of the *Notices*. Jonathan Borwein leads

off with a chapter that takes a, "Hey! Look at this cool proof!" approach. He doesn't use those words or even exclamation points, which would be too campy, but his enthusiasm shines through. He shares some of his favorite insights into the number *pi*, and he demonstrates some elegant geometric proofs of the almost-without-words variety.

Then Doris Schattschneider takes over with a chapter that—in true mathematical style—leads off with illustrative examples that provide the context and motivation for a more general aesthetic framework. She starts with some of the historical, classical theorems that have shaped our own mathematical culture and then lists five attributes of mathematical work that we claim are beautiful (page 43): elegance, ingenuity, insight, connections, and paradigm. (My art colleagues, I think, would appreciate this list.) Throughout, she uses specific pieces of mathematics (like an elegant proof of the arithmetic-geometric mean inequality) to illustrate these ideas in context.

David Henderson and Daina Taimina close out this section by examining our "need" for meaning. They illustrate this need through mathematical examples and paradoxes, comparing various definitions of the concept "line" (e.g., "shortest distance" versus "no wiggles" versus "circle of infinite radius"). Correctness of the results alone, they maintain, feeds the belly but not the soul:

> For example, [a 1973 paper by Henderson] has a very concise, simple (half-page) proof. This proof has provoked more questions from other mathematicians than any of his other research papers, and most of the questions were of the sort: "Why is it true?", "Where did it come from?", "How did you see it?" They accepted the proof logically, yet were not satisfied. (page 66)

If Section A starts with the math and then describes where aesthetics fits in, then Section B ("A sense for mathematics") does the reverse. Nathalie Sinclair uses interviews with a half-dozen living mathematicians, co-mingled with the writings of many other prolific mathematicians, to explore aesthetic considerations in mathematics. She is methodical and clear, and I enjoyed her insights, even as I felt a bit like a biological specimen displayed under a microscope. For example, when she describes standard ways that mathematicians approach solving problems, she includes "playing" and "establishing intimacy", perhaps by naming or claiming the objects involved: *my* triangle. Maybe next time I'll lower the blinds.

Martin Schiralli builds on Keith Devlin's book on mathematics and pattern [2] by adding a consideration of the aesthetic connections between these two concepts. He notes that the Pythagoreans' conception of arithmetic (as understood to represent ordered arrangements of numbers) connected the subject with "the very fabric of the knowable world and their efforts to find the principles unifying that fabric in the mathematical-aesthetic ratio of harmonic intervals" (page 119). He continues in this vein to describe how later philosophers such as Wittgenstein and Kant used "numbers as cognitive scaffolding" for understanding natural phenomena. William Higginson finishes the section with a chapter that flits among stereotypes of mathematicians, popularizations of mathematics, and the anthropological and historical basis for mathematical capacity.

Section C (oddly titled "Mathematical agency") contains three essays that take interdisciplinary approaches to examining aesthetics. This section is a wild ride. It begins with an essay on the didactics of dynamic geometry by Nicholas Jackiw (the chief technology officer of KCP Technologies and original designer of the Geometer's Sketchpad). It concludes with an essay by the late Dick Tahta that takes a Freudian approach to "the perennial debate about the nature of mathematical objects" (page 204) and all that this implies about our anxiety regarding our mothers. Sandwiched between those two pieces sits something pure and holy.

David Pimm's essay ("Drawing on the image in mathematics and art") caught me by surprise and kept me up late reading it. In it, he describes how one particular aesthetic (the purity of iconoclasm) permeated and became ritual in both the Bourbaki movement and in the reformed English Church. He contends that notions of "purity" (in the religious and also in the "pure math" sense) led to the same sense of iconoclasm: rejection of figures (geometric or human) and an attempt to get at Truth directly, without any mediating interference. There is more; it is worth reading. I am unabashedly unfamiliar with the subject and don't pretend to have the slightest idea of the correctness of his argument. And yet I very much appreciated this one essay which homed in so narrowly on a single historical mathematical movement and which chose one single particular aesthetic ideal, and tied the two together so neatly. And fittingly, if surprise and insight are aesthetic qualities, then I have all the philosophical justification I'd need to say this essay is an aesthetic work.

Why, then, did Pimm and Sinclair have to muck around in Chapter Omega? Darn it, the review would have been lovely if I could have stopped one paragraph ago—just as this book would have been lovely if it stopped one chapter earlier (or two, depending on how you feel about Freud and maternal separation anxieties). Chapter Omega comes in several unhappy pieces, three of which I lay out here (in a butchered and, I admit, unfair way). The first is that mathematics is dehumanizing (to be more specific, "decontextualized", "depersonalized", and "detemporalized"), a fact

the authors conclude from the way we write our research articles. Second, the mathematical mind is a melancholy one, for which the most striking evidence is the famous etching *Melencolia I* by Albrecht Dürer. Dürer was melancholy. True, he was an artist, but (as the authors take pains to point out) he was an artist who *liked mathematics*. Third, there is some connection between mathematics and autism, particularly in the way we (don't) relate to other people. The authors wend their way through the argument for this connection by noting that we seem to have similar modes of reasoning, but also (on pages 240–241):

- some famous mathematicians might have had autism (that is, some psychologists have written speculative articles with post hoc diagnoses of a half-dozen mathematicians);
- by implication, most mathematicians are deficient at relating to other people, and
- this might "be part of the explanation" about why women are less likely to be mathematicians.

Well, faugh.

Leaving aside completely questions of scholarship and evidence, this chapter stands at odds with the rest of this book. It certainly is no summary of the previous essays, and indeed, it contrasts most markedly with the writings of Section A (written by the research mathematicians themselves).

For me, this chapter was particularly surreal because I read it while I was at the 2008 MathFest. There, Christine Stevens spoke about the more-than-1000 Project NExT Fellows and about the impact they have had on the mathematical community—and about how they have had this impact precisely by building and strengthening communities. Erik Demaine gave a series of standing-room-only talks about the mathematics of folding, and he wove in touching stories of working with his father and with his many coauthors. "The main point of these talks," he repeated often, "is that you ought to have fun." Sixteen hundred people joined together in a celebration of mathematics, in meeting new people, and in reacquainting with old friends.

And if this book has any glaring holes, it is perhaps this. Aside from a brief nod from Sinclair (two paragraphs) that social factors can have a motivating influence on the kinds of mathematics we choose to do, there's little mention of the collaborative aspects of mathematics. Nobody writes about the aesthetics of co-authoring, the joy of mentoring junior faculty, the sheer hero-worship that impels us to follow others, nor the invigoration that can come from bouncing our ideas off of other people. Nobody writes about talking out loud about mathematics, nor of hearing others talk about it. For a book that arose—as the first sentence of the preface tells us—from a series of talks at a conference, this is an odd and paradoxical omission.

But perhaps I am being too harsh—perhaps I am complaining that I didn't get an action toy in my box of Cracker Jack, and forgetting to be grateful for the secret decoder ring that *did* come with my snack. *Mathematics and the Aesthetic* is a thoughtful collection of essays, bound together with careful organization, attention to small details, and even a bit of elegance.

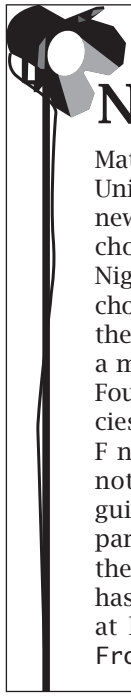
Which is all as it should be for a book with the word "Mathematics" in the title.

References

[1] RUEL CHURCHILL, *Fourier Series and Boundary Value Problems*, McGraw-Hill, New York, NY, 1941.

[2] KEITH DEVLIN, *Mathematics: The Science of Patterns: The Search for Order in Life, Mind and the Universe*, Henry Holt & Co., New York, NY, 1994.

[3] JUDITH WECHSLER, editor, *On Aesthetics in Science*, MIT Press, Cambridge, MA (1978).



Noticed

Mathematician Jason Brown of Dalhousie University in Halifax, Canada, was in the news for his work analyzing the opening chord of the Beatles' song "A Hard Day's Night". It was known that most of the chord was played by George Harrison, but the exact nature of the chord remained a mystery for forty years. Brown used a Fourier transform to analyze the frequencies and solve the mystery. He found an F note in the opening sound that could not be played on Harrison's 12-string guitar and deduced that the note was part of a piano chord played along with the guitar. The *Halifax Chronicle Herald* has more about Brown and his research at `http://thechronicleherald.ca/Front/1087119.html`.