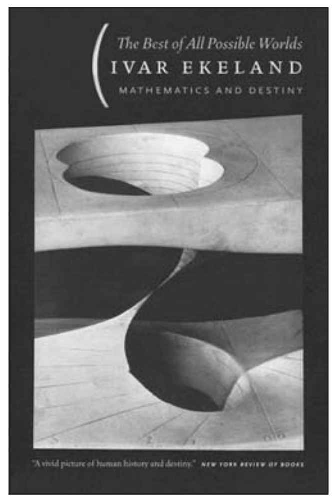
Book Review

The Best of All Possible Worlds: Mathematics and Destiny

Reviewed by Hector Sussmann

The Best of All Possible Worlds: Mathematics and Destiny
Ivar Ekeland
University of Chicago Press
Cloth US$25.00, 214 pages (2006)
ISBN-13:978-0-226-19994-8
Paper US$14.00, 207 pages (2007)
ISBN-13:978-0-226-19995-5

Leibniz held that we are living in the best of all possible worlds. Maupertuis stated the least action principle, according to which "the quantity of action necessary to cause any change in Nature always is the smallest possible", and then went on to claim, in his 1752 *Essay in Cosmology*, that he had "discovered a principle underlying all the laws of motion, which applies to hard bodies as well as elastic ones, from which all motions of all corporeal substances depend.… Our principle…leaves the world in its natural need of the Creator's power, and follows naturally from the use of that power". In Ivar Ekeland's words, Maupertuis claimed "a grand unification of his own, the unification of physics with metaphysics, and even with morals. In later work, he claims that a certain quantity of good (or bad) is attached to each of our actions, and that God has ordained the world so that, adding up all the good and subtracting all the bad, the balance will be found to be the greatest possible. In other words, this is the best of all possible worlds." These ideas would become known to future generations through Voltaire's ferociously satirical novel *Candide*, in which the optimistic Doctor Pangloss perseveres in concluding, in the midst of terrible disasters and catastrophes, that "all is well that ends well in the best of all possible worlds."



Ekeland sets the tone of his beautifully written and enormously enjoyable book with its opening sentence:[1] "The optimist believes that this is the best of all possible worlds, and the pessimist fears that this might be the case." In the book, he tells us about Leibniz's "much ridiculed idea", and about Maupertuis's "eventful life", which was "full of impressive achievements" (such as holding "the honor of being the first scientist ever to have stated the idea that animal and plant species are not immutable"), as well as his "exasperating arrogance, which did little to make him popular among his peers". The reader is given a clear and precise explanation of what Leibniz meant by the assertion that from the infinity of all possible worlds God must have chosen "the best", and of how he attempted to establish that this was true. (In the *Monadology*, Leibniz writes that "the existence of the best, which is disclosed to God by His wisdom, determines His choice by His goodness, and is produced by His

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[1] *The sources found by my own Google search attribute these words to the writer James Branch Cabell (1879–1958).*

power." To explain what Leibniz meant by "the best", Ekeland rephrases Leibniz's statements in his own words: "perfection consists of two things: variety on the one hand, that is the inexhaustible profusion of natural phenomena, and order on the other, that is the interrelatedness of all things and the basic simplicity of natural laws." And, later, "Leibniz belongs to this category of philosophers who claim that happiness lies in contemplating the wonders of God in His creation, an idea that is certainly far away from the everyday concerns of most humans beings. All in all, to say that this world is the best of all possible worlds does not imply that it is a pleasant one to live in." In other words, the great philosopher did indeed say that ours is the best of all possible worlds, but what he meant by this was "far from the crude philosophy 'All's well that ends well' incorrectly attributed to Leibniz.") He discusses Maupertuis's principle of least action, the controversy between Maupertuis and Koenig about whether Leibniz had discovered the principle before Maupertuis did, and the way Euler, Lagrange, Hamilton, and Jacobi formulated "Maupertuis's idea in a precise and workable way". He sketches a clear, plausible explanation of how the least action principle—which in reality is a principle of stationary action, namely, the assertion that the motion follows a path which is a critical point of the action—may come about. (The explanation—which, of course, is nonrigorous—is based on quantum mechanics: the quantum mechanical amplitude of a transition from point x at time t to point y at time s is given by the Feynman path integral of $e^{\frac{2\pi i}{h}S}$ over all paths that go from x at time t to y at time s, where S is the action, and the method of stationary phase suggests that, in the classical limit as $h \to 0$, the leading term in the asymptotic behavior of the integral is the sum of the contributions of the critical points of S.) And he does not forget to tell us about Voltaire's hostility toward Maupertuis. ("When Maupertuis came back from his northern expedition [to Lapland in 1736–1737], and all of Paris sang his praises, Voltaire chimed in with these verses: You have gone to confirm in places far and lonesome\\What Newton always knew without leaving his desk." And later: "[Voltaire's] book *Story of Doctor Akakia and the Native of Saint-Malo* is a collection of pamphlets against Maupertuis, the general theme being that such a mass of nonsense had been published under the name of the respected president of the Berlin Academy of Sciences that it is simply not possible that they were authentic: they had to be the work of a young impersonator, whom Voltaire proceeds to unmask.")

But the book is not just about Leibniz, Maupertuis, Voltaire, and the least action principle. Ekeland—himself a distinguished mathematician, well known for his important contributions to variational analysis and optimization theory—is the author of several nontechnical volumes, such as *The Broken Dice and other Mathematical Tales of Chance*, *Mathematics and the Unexpected*, and *The Cat in Numberland*, in which he has developed a unique style for turning stories about mathematics into engaging narratives filled with interesting details and explanations of fundamental concepts that are understandable as well as accurate. And in this book he uses his unique approach to scientific storytelling to offer us a guided tour through the successes and failures, during four centuries of modern science, of efforts by scientists and philosophers to understand the world by means of extremal principles. (In this review, I use "extremal principles" to refer to optimization as well as critical point principles.) The result is a book that provides much information about the ideas of optimization and critical points and is full of details about a large cast of characters, such as Galileo, Huygens, Descartes, Newton, Fermat, Leibniz, Maupertuis, and Voltaire, to name just a few.

Optimizing behavior can be that of an intelligent optimizer, such as the creator God who chose to make the best of all possible worlds, or a rational utility-maximizing human being, or an engineer seeking to build a device or to design a plant so as to minimize some cost functional, or the management of an airline looking for a schedule that minimizes cost subject to a large number of constraints, or a social planner maximizing society's welfare. Or it can arise from a natural process, as in the maximization of fitness of a species by natural selection (which is really an ascent process, famously described by R. Dawkins as "climbing Mount Improbable"), in which a fitness function that can vary in time due to changes in the environment (including changes in other species) is required to increase steadily, but may get stuck at a local maximum, and need not even attain a local maximum at all. Stationarity principles occur as basic physical laws, such as the least action principle and Fermat's minimum time principle in geometrical optics, both of which assert that a physical system follows a path that is a critical point of some functional, such as action or travel time.

All the above types of extremal principles are described by Ekeland in the various chapters of the book. Furthermore, as the guided tour proceeds and the extremal principles are presented, other topics naturally appear, and the author spends time on them, always elegantly working his way from secondary topic to secondary topic and eventually back to the main theme in a continuous narrative. For example, he opens Chapter 3 on the least action principle with the condemnation of Galileo by the Inquisition on June 22, 1633. He then makes the transition from Galileo to Descartes by telling us that Galileo's condemnation "was a lesson for others as well", and that Descartes learned of it in

November of 1633, and "immediately decided not to publish his magnum opus, the *Treatise of the World, or of Light*". The stage is thus set for a brief description of Descartes's treatises on philosophy, mathematics, and physics. Ekeland regrets that, with the separate publication of these works, "An essential part of [Descartes's] message has…been lost, for unity is central to his way of thinking." (This unity is that of reasoning rather than experimentation. "Descartes…believed that science rests on eternal truths. As a consequence, he held experimental results in low esteem…and less trustworthy than sound argumentation.") From here he moves on to the work of Snell, Descartes, and Fermat on the law of refraction of light and the minimum time principle, including a discussion of the conflict between two basic approaches to the formulation of physical laws, namely, that of extremal principles—which seems to involve final causes, since a system obeying the least action principle or a light ray that follows Fermat's minimum time principle appears to know that they want to go from point A to point B of configuration space and then choose their path to be extremal—and the equally widely used causal approach in which initial conditions determine the future by means of dynamical laws, such as ordinary differential equations. He illustrates this conflict with a discussion of the criticism of Fermat in 1662–1665 by the followers of Descartes (who had died in 1650) about the law of refraction of light. Descartes and Fermat derived the same law of refraction starting from mutually contradictory assumptions. Descartes compared "light traveling from air to water to a tennis ball which is accelerated in the vertical direction as it crosses the surface, the horizontal speed being unaffected". Fermat believed that the speed of light in air is larger than that in water, and that a light ray will travel from a point A in the air to a point B in the water by following a minimum-time path. Remarkably, both assumptions lead to exactly the same path, determined by Snell's law of refraction $\sin i = n \sin r$, where i and r are the angles of incidence and of refraction, and n is the index of refraction. "[Descartes and Fermat] both agreed on the value of … n, namely, 1.33 for the water-air interface, but they did not agree on its meaning. For Descartes that number meant that light travels 1.33 times faster in water than in air, and for Fermat it meant that it travels that much slower." Descartes's model is clearly causal—i.e., such that the past determines the future—whereas Fermat's approach involves, or at least appears to involve, final causes, as the light ray uses time minimization to choose which path to follow from all the paths that lead to the desired target point. Fermat was criticized by Descartes's disciple Clerselier for the seemingly teleological nature of his minimum-time principle, to which he gave the "very modern" reply that "light propagates *as if* it had both [the desire to travel fast] and [the means to compute the quickest path], and while the mathematical problem may not be an accurate description of what happens at some deeper level of reality, it is good enough to make predictions which turn out to be in agreement with experiments." Ekeland explains that Fermat was eventually proven right by the measurement of the speed of light in water by Foucault and Fizeau in 1850 and does not miss the opportunity to relate this early discussion of determinism vs. extremal principles to the later arguments by Mach and the controversy between Bohr and Einstein about the foundations of quantum mechanics. He then returns to the theme of the critique of Fermat by the Cartesians, by telling us that this was the "opening battle" of the "long, drawn-out struggle" of the Cartesians against Newtonian physics, and then turning his attention to the closing battle, which was Maupertuis's expedition to Lapland. "Newton, working on the idea that the Earth is a liquid ball that had solidified, had predicted that it would be flattened at the poles, because its rotation when it was fluid would have created a bulge around the equator. Cassini, the French astronomer royal, a loyal Cartesian, believed the opposite: the Earth should be elongated at the poles, like a lemon." The measurements that Maupertuis brought, of the arc of meridian in the far North, "compared with the arc of meridian at the latitude of Paris, showed that Newton was right and made Maupertuis a hero overnight". After entering Ekeland's book in this triumphant way, Maupertuis finally gets to occupy center stage, to which he is justly entitled in a book on "the best of all possible worlds". Thus Ekeland introduces the central theme of the book and turns to the least action principle.

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The same narrative technique of making continuous transitions to take us from topic to topic and from character to character, always finding a smooth way to return to the main theme, is displayed throughout the book. We are treated to side trips to such subjects as the measurement of time and clock making: from Galileo's ideas about the pendulum to Huygens's achievement as "the founder of modern chronometry", to John Harrison, to the overturning of Galileo's "idea of the whole universe at a given instant" by Einstein's theory on the relativity of simultaneity. We learn of the unification of algebra and geometry by Descartes, Euler's analysis of the free motion of rigid bodies, the integrable cases discovered by Lagrange and Kowalewska of the motion of a rigid body subject to gravity, and the unpredictability of the behavior of nonintegrable systems (including a detailed discussion of billiard balls and how the shape of the table determines whether the ball's motion is integrable or not, with tables that are elliptic or close to elliptic leading to integrable behavior, while chaos occurs for tables that are far away from being elliptic). Ekeland describes some of "many attempts toward a scientific theory of history, comparable to the theory of evolution" (especially the account by Thucydides of the Peloponnesian War, and that of Guicciardini of the wars in Italy between 1492 and 1534, in both of which Ekeland discerns "an echo of chaos theory", as in Guicciardini's statement that "small events that would hardly be noticed are often responsible for great ruins and successes"). We learn about the Condorcet three-voters paradox and equilibria (that is, Nash equilibria) in game theory, and are treated to a discussion of whether future global catastrophes, such as global warming or a nuclear war, are avoidable, and of the possible dangers of cloning and altering our genes ("history and mythology hand down stern warnings against tampering with such things.…Are such warnings valid for the modern world? We do not know.…Possible worlds are now crowding our doorstep.…For instance, one very real possibility is a warmer world, a planet where the environment has been profoundly altered by the greenhouse effect.…We have to shape a new world, and do it now. What a change since the time of Leibniz! In his view the choice between all possible worlds had been made once and for all, by God Himself, at the time of creation. Now this choice is ours to make").

The book can be savored in bits and pieces, by reading individual chapters, or portions of chapters dealing with particular topics such as the measurement of time or the mathematical problems posed by collective decision-making. But the reader who just chooses to start on page one and keep going will almost certainly find it impossible to put the book down, because it is densely packed with delightful items of information and is as entertaining as a fast-moving thriller.