

Homage to Henri Cartan (1904–2008)

Michèle Audin

On August 13, 2008, Henri Cartan died at the age of 104. He was, in more ways than one, an icon of French mathematics in the twentieth century.

In 1986 the Palais de la Découverte, France's premier science museum, presented an exposition on the "Whitehead link", an intertwining of two "knots" (the black loop and the gray figure-eight that one sees in the photograph at right), neither of which is knotted but each of which cannot be separated from the other.

This property interests mathematicians and, in the spirit of Jacques Lacan (1901–81), psychoanalysts. The exposition was thus conceived by a psychoanalyst, Jean-François Chabaud, and a mathematician, Henri Cartan. One sees Cartan in the accompanying picture at the Palais performing a demonstration of the fact that one can exchange the gray and black components, unraveling the gray to make it into a circle, a procedure that has the secondary effect of transforming the black component into a figure-eight. Cartan is accompanied by his wife (who died in December 2008); Jean Brette of the department of mathematics of the Palais is in the back. At the time, Henri Cartan was 82 years old and retired, but he continued to be interested in mathematics and, as one sees, its popularization.

Born in 1904, Henri Cartan was the eldest son of Élie Cartan (1869–1951) and Marie-Louise Bianconi (1880–1950). Élie Cartan was a mathematician, one of the founders of differential geometry,

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and the one who introduced Lie groups into this branch of mathematics. Among the four Cartan children, two became mathematicians, Henri and Hélène (1917–52), and entered the École Normale Supérieure (ENS) in rue d'Ulm; Louis became a physicist (1909–43, he was deported to Germany as a participant in the resistance and beheaded); and Jean became a composer (1906–32, he died of tuberculosis). There was a great deal of music in the Cartan home, and Henri was a very good pianist.

An Exceptional Mathematician

Henri Cartan entered the ENS in 1923; he got to know Jean Delsarte (1903–68), André Weil (1906–98), and Claude Chevalley (1909–84), as well as other members of a group that, in December 1934, began to lay the foundations for a treatise that would become *Les éléments de mathématiques* by Nicolas Bourbaki, the collective pseudonym under which the group signed its books.

Henri Cartan specialized in complex analysis. After the crucial works of Charles-Émile Picard (1856–1941), Paul Montel (1876–1975), and others, one knew "everything" about the functions of a single variable. Cartan was one of the founders of the theory of functions of several complex variables and analytic spaces, and he was the one responsible for the use of sheaves in that theory. He was also the author of important contributions to algebraic topology, homological algebra (with his American friend Samuel Eilenberg (1913–98)), and potential theory.

From 1929 to 1975 Cartan held various university posts, from Lille to Orsay by way of Strasbourg (just before the war), as well as ENS, where he taught generations of *normaliens* from 1940 to 1965. Through his contributions to the work of Bourbaki (he was, for example, the inventor of filters), through his mathematical work, through his teaching and the seminar that carried his name

Hélène Cartan

In the Cartan family, the little sister, Hélène, was, like her older brother, Henri, a very good pianist and a mathematician. Born in 1917, she entered the École Normale Supérieure in rue d'Ulm in 1937 and passed the *agrégation* in mathematics in 1940, getting the best rank of all the young women (there was no *agrégation* for young men in wartime). She became a teacher in a secondary school. In 1942, she published a *Comptes Rendus* note, which was presented to the French Academy of Sciences by her father Élie Cartan. The paper proved necessary and sufficient conditions for a connected topological space E to be homeomorphic to a circle: E minus any point is connected, E minus two points is not, and E is either compact or locally connected and contains a non-enumerable dense subset. Hélène contracted a very bad and contagious form of tuberculosis, which prohibited her from teaching, and she spent several years in a sanatorium. In 1952, after going out for a walk one day, she disappeared for a few days and was eventually found dead in the river Isère.

—M. A.

from 1948 to 1964, and through his books, he had an enormous and lasting influence on the French mathematical school: one can count the majority of living French mathematicians as his “grandchildren” or “great-grandchildren”.

A Man of His Century

Henri Cartan also had an influence on international affairs, notably in Germany, where at the end of the war he re-established scientific contact with some of his colleagues, for example Heinrich Behnke (1898–1979). Cartan first visited Germany in 1931; he returned in 1946, giving a lecture (and playing the piano) at the mathematics institute in Oberwolfach.

All of this was consistent with his European convictions. He was also a great defender of human rights. His defense of Leonid Plioutch in 1974 and of José-Luis Massera (1915–2002) during the years 1970–80 come to mind. But Cartan did not wait until his retirement to take an interest in the fate of his colleagues. Recall the battle he took on as president of the Société Mathématique de France in 1950, with McCarthyism in full swing, to get the American authorities to give visas to Jacques Hadamard (1865–1963) and Laurent Schwartz (1915–2002) to attend the International Congress of Mathematicians in Cambridge, Massachusetts (where Schwartz received the Fields Medal).

One can also recall the assistance he brought, in occupied France, to the mathematician Jacques



Photograph © Palais de la Découverte, C. Rousselin.

Henri Cartan in 1986, at age 82, with his wife Nicole, at the "Whitehead link" exposition.

Nicole Cartan

Nicole Cartan, born in 1916, was the daughter of physicist Pierre Weiss. She married Henri Cartan in 1935 and they had five children. She died in December 2008, a few months after the death of her husband.

—M. A.

Feldbau (1914–45), who was persecuted by anti-Semitic laws (persecuted even to his death).

The twentieth century has passed, Henri Cartan is dead. He leaves us his heritage, visible and invisible, in the living mathematics of the twenty-first century.

Editor's Note: The *Notices* plans to publish a more extensive obituary for Henri Cartan in the future. For further information about Cartan, see the interview conducted by Allyn Jackson in the August 1999 issue of the *Notices* (<http://www.ams.org/notices>), as well as the articles about Cartan collected by the Société Mathématique de France on the occasion of his 100th birthday (<http://smf.emath.fr/VieSociete/Rencontres/JourneeCartan/>).