Book Review

Mathematics and Common Sense: A Case of Creative Tension

Reviewed by Chelluri C. A. Sastri

Mathematics and Common Sense: A Case of Creative Tension
Philip J. Davis
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A common theme of several recent books about mathematics and mathematicians is an exploration into the nature of mathematics, the motivations of mathematicians, and modes of mathematical thought. Besides the book under review, the ones that come to mind are those by Ivar Ekeland (*The Best of All Possible Worlds: Mathematics and Destiny*) and David Ruelle (*The Mathematician's Brain: A Personal Tour through the Essentials of Mathematics and Some of the Great Minds Behind Them*).[1] What distinguishes them from one another is mainly the audience to which each book is addressed. The one by Davis has a leitmotif: common sense—the extent to which mathematics is informed by it and the ways in which mathematics transcends it. The book begins with an exchange of letters between Davis and Christina, a real person with a fictitious name, who is intelligent and curious and whose mathematical background is somewhat above that of a high school graduate. Thus it is safe to assume that the book is aimed at

people like her. The first questions that Christina asks are the basic ones: What is mathematics? What is the nature of mathematical talent? Why should one study mathematics as opposed to, say, literature, art, history, or, for that matter, physics? Davis answers them well, but he could do even better. This is a golden opportunity for him to make the best and most effective use of his pedagogic and storytelling abilities—he *has* written some interesting fiction—to convince the lay reader that studying mathematics at least to some extent, say, at the college level, not only is necessary for becoming a well-informed citizen but can be great fun. In addition, he could make a case for the usefulness, discovered in unexpected ways sometimes, of such study, even if the student never again looks at a math book. Unfortunately, by being succinct and implicit rather than explicit, Davis doesn't take full advantage of this opportunity.

Let us consider the first question, which is actually very difficult. Davis gives a good answer, pointing out the salient features of mathematics, particularly of the pure variety, including its logical structure, beauty, and usefulness. He does mention applied mathematics but avoids getting drawn into the controversy surrounding the relative merits of pure and applied mathematics, which is unfortunate, since he is very well qualified, being a

Chelluri C. A. Sastri is a visiting professor of mathematics at Missouri University of Science and Technology. Previously he was professor of mathematics at Dalhousie University, Halifax, Nova Scotia, from which he retired in 2006. His email address is sastric@mst.edu.

[1] *Editor's Note: Both books have been reviewed in the* Notices. The Best of All Possible Worlds *was reviewed by Hector Sussman in the March 2009 issue, and* The Mathematician's Brain *was reviewed by David Corfield in the November 2008 issue.*

well-known applied mathematician, to delve into that difficult subject. On the question of mathematical talent, he doesn't draw a distinction between the ability to learn, understand, and use mathematics and the ability to create new mathematics. The distinction is an important one, for while people with the latter ability form a relatively small group of talented individuals, people with the other type of ability are essential for teaching, clarifying, and, in general, exegesis.

The book is divided into thirty-three chapters, each dealing with a different topic. Some of the topics discussed are difficult and thorny, but Davis deals with them deftly and in a balanced way. The interplay between common sense and mathematics comes into focus early in a chapter provocatively titled "Why Counting Is Impossible". In it, Davis shows that although the laws of arithmetic are precise and simple, when the numbers are large enough, counting becomes impossible if absolute precision is demanded. He uses two interesting examples—the 2000 presidential election and Bertleson's number (the number of prime numbers less than 10^9)—to illustrate this and then describes methods for arriving at satisfactory estimates once a preset level of accuracy is agreed upon. In another chapter he discusses the deductive method, starting with the common-sense understanding of axioms as self-evident truths. He constructs an example with a set of axioms and proves a theorem. The proof, though, involves not just a formalism but also human interaction. This interaction between formalism and human beings is a topic Davis discusses at some length, making a plausible argument that it is unlikely in the foreseeable future that mathematics will be done entirely by computers. Throughout the book, Davis hews to the theme of a creative tension between mathematics and common sense.

The most interesting chapters are the ones on logic, inconsistency, ambiguity, and randomness, all very well done. Concerning proof, an example Davis cites is the work of Thomas Hales on Kepler's sphere-packing conjecture. He tells the story of how, in spite of the herculean efforts of a team of twelve referees, Hales's proof could not be completely checked and concludes that there will be more such instances in the future. In the discussion about logic, he describes examples of strange objects to which the study of logic sometimes leads. Included among them is what is generally called the Banach-Tarski paradox. (Davis calls it the Banach-Tarski-Hausdorff paradox.) He gives a convincing resolution of the paradox by pointing out that a mathematical object is not the same as a physical one even when the mathematical description of the two objects is the same, thus making their images in our minds the same, and by alluding to the notion of nonmeasurability. This is one of the occasions when a lay reader might benefit by referring to a glossary, if there were one. In the discussion about probability, Davis explains how difficult it is to come up with a proper definition of randomness and muses on the reasons why people gamble even when they know their chances of winning are extremely small. In this connection, he mentions two examples, the first of which trips people up because they don't know or understand the monotonic nature of probability (if $A \subset B$, then $P(A) \leq P(B)$), while the second one causes trouble because in it a single event is described in twodifferent ways, leading to the misperception that two distinct events are being referred to. However, his assertion that "in most applications of probabilistic reasoning, it is necessary to make independence assumptions that, to a common-sense understanding, are not even close to true but are wildly fantastic when taken literally" cannot go unchallenged. There are many examples where such an assumption makes perfect sense. The Poisson process is one that comes to mind right away.

Davis spends some time talking about the "chipification" of mathematics and remarks on how tasks that required a lot of time and effort in the past are now accomplished with ease and speed but doesn't say too much about the negative effects. We know that chipification certainly hides the mathematics behind a gadget, device, or procedure, thus allowing the public to persist in its lack of awareness of the truth—namely, that mathematics underpins the developments in question—and in its thinking that mathematics is an arcane and useless discipline. This is a situation that calls for intervention by mathematicians, specifically by communicating mathematical ideas and their importance to the public as well as to the powers that be, for without such communication, support for mathematics would dry up. Chipification also fosters a tendency in the younger generation not to bother with "rote" skills, making it harder to train people to maintain and improve upon the technology or make new discoveries.

In his discussion of mathematics, war, and entertainment, Davis suggests that mathematics is ambiguous as far as values are concerned. It is true, of course, that mathematics, like other human activities and creations, can be used for good as well as for evil. However, there is one value that is paramount in mathematics, indeed in all of science—namely, truth. The high status it is accorded seems to yield some beneficial byproducts, such as honesty and the willingness to admit error. In a recent review of a book in the *American Mathematical Monthly*, the mathematician Ed Nelson wonders why it is that we mathematicians are so nice: by and large, we are honest, quick to say "sorry", and willing to give the other guy the benefit of the doubt. Why, though? A guess would be that these are natural consequences of the importance attached to truth. One may argue that other sciences value truth equally highly; does it follow that scientists in general are nice? Maybe, but the process appears simpler in mathematics, for all it takes for a mathematical conjecture or belief to be falsified is a single counterexample! In any case, there are many instances in which a young, unknown, and clearly vulnerable mathematician comes up with a good idea or technique, and the adviser or senior mathematician bends over backwards to give credit to the younger person. Isn't it the epitome of civilization to be fair and kind to someone over whom one has power? The more absolute the power, the harder it is to be fair. That mathematicians are decent is certainly something to cheer about. It doesn't follow, however, that there are no charlatans or crooks among mathematicians; of course there are, but the overwhelming majority appear to be honest. This is obviously a consequence of the high status truth occupies, and it goes to show that mathematics is far from being value-free. Unfortunately, Davis doesn't point this out. In another chapter he wonders if there are racial and gender differences in being hard wired for mathematics but doesn't pursue the question further. It would be interesting if he did, even though, or perhaps because, it is an explosive issue.

The question as to whether mathematicians are prone to eccentricity or weirdness, or even downright mental illness, doesn't get much attention in the book. It is of course a very difficult question to answer. What is obvious is that since mathematics is a highly cerebral activity, it would be good for a mathematician to have an interest or a hobby that engages her physically and thus balances her life. Anecdotal evidence seems to indicate that *certain* types of mental illness such as obsessive-compulsive disorder and bipolar disorder may be more prevalent among mathematicians and others who engage in similar activities than in the general population. However, a thorough scientific study of the problem would be needed to either substantiate or refute such a claim. At any rate, the matter is too important to be ignored.

It is surprising and disappointing that a book such as this, addressed to a lay audience, lacks a glossary as well as an index. The absence of an index is actually frustrating, because Davis touches upon several important and difficult questions and, for further reading, gives a wealth of excellent references at the end of each chapter. If one forgets a particular reference, one has to make quite an effort to find it again. Occasional discussions of technical examples—involving complex integration or splines, for example—make one wonder whether a glossary of some sort wouldn't have made the book much more attractive to a lay person. It is true that if there were to be a glossary, the author would have to face the vexing problem of deciding where to begin and where to end, but clearly *some* help is better than none at all.

Davis has written several books, some with collaborators and others by himself. In particular, his fiction reads well. Unfortunately, one cannot say that of the book under review. The reason is that it abounds in not just typos, but stylistic errors and downright bad constructions—misplaced modifiers and dangling participles are common. Now, cleaning up the prose does not mean that it has to be formal and pedantic—witness the writings of Bertrand Russell, Peter Medawar, and Freeman Dyson, for example. Finally, there are instances where what Davis says sounds like something one knows but one isn't sure of the connection. For example, on page 68 he refers to what is called the Weber-Fechner Principle, which says $S = k \log R$, where S = Sensory Response and R = Stimulus; k is presumably a constant. This looks a lot like Boltzmann's formula for entropy: $S = k \log W$, where S is the entropy, k the Boltzmann constant, and W the number of microstates corresponding to a given configuration. Is there a connection?

Here is a sample of the typos and other errors:

p. xxi, paragraph 3, line 5: no single group that is predominates (predominant)

p. xxiv, paragraph 2, line 2: Hilbert's reputation and influence was so great (were)

p. xxv, paragraph 3, line 1: Mathematics can and has flourished (can flourish and has)

p. xxxvii, paragraph 1, line 6: As a young researcher in mathematics, one of my professors gave me (misplaced modifier)

p. xliii, paragraph 3, line 1: Having said all this, Christina, it occurs to me (dangling participle)

p. 3, The volume of a sphere of radius r is $4\pi r^3$ ($\frac{4}{3}\pi r^3$)

p. 44, paragraph 4, line 3: Having written the book, my curiosity was saturated (dangling participle)

p. 213, paragraph 1, line 4: the principle motivation (principal)

In summary, this is a book whose content is very good and well organized. If a second edition is issued with the typos corrected and the prose cleaned up and with changes somewhat along the lines indicated, it would be not just a very good book but an excellent one and would be a welcome addition to the library of books on mathematics and mathematicians.

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