Guido Castelnuovo and Francesco Severi: Two Personalities, Two Letters

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The Italian school of algebraic geometry flourished from the latter part of the nineteenth century through the early part of the twentieth century. Some of the main contributors were Luigi Cremona, Eugenio Bertini, Giuseppe Veronese, Corrado Segre, Guido Castelnuovo, Federigo Enriques, and Francesco Severi. There were, of course, other important schools of algebraic geometry in other countries, but the Italian school stood out because of its unique mathematical style, especially its strong appeal to geometric intuition. Between 1896 and 1900 two members of this school, Guido Castelnuovo and Federigo Enriques, developed the classification of algebraic surfaces, one of the great achievements of algebraic geometry.[1] A few years later (1904–1908), together with Francesco Severi, they significantly deepened that understanding of surfaces.

In this article, we present excerpts[2] from two letters to Beniamino Segre, a distinguished algebraic geometer in his own right and a distant relative of Corrado Segre: one from Severi in 1932 and the other from Castelnuovo in 1938. Severi's letter provides his frank assessment of his own and others' contributions to algebraic geometry, including those of several of the Italian geometers mentioned above. Castelnuovo's letter discusses his collaborations with Enriques and Severi in the 1904–1908 period and assesses the contributions due solely to Severi. The tone and content of the letters reflect remarkably the enormous personality differences between these two giants of Italian mathematics.

Guido Castelnuovo (1865–1952) was born and raised in Venice, the son of Enrico Castelnuovo, director of the Scuola Superiore di Commercio and a popular nineteenth-century author of novels and short stories. He completed his doctor's degree at the University of Padua in 1886 under the direction of Giuseppe Veronese, one of the leading algebraic geometers of that period. On the advice of Veronese, Castelnuovo spent the following year in Rome on a postgraduate scholarship and then spent three years as assistant to geometer Enrico D'Ovidio at the University of Turin. In 1890, Castelnuovo won a *concorso*, or national competition, for a new chair of analytical and projective geometry at the University of Rome—an award that was subsequently withdrawn by the Italian Ministry of Public Instruction on the grounds that the candidate's publications did not match the subject matter covered by the chair, although the ministry had judged his work itself to be of higher quality than that of the competition. Thus, Castelnuovo remained in Turin for another year as D'Ovidio's assistant, during which time he broadened his research interests to include linear systems of curves in a plane and the geometry of algebraic surfaces. He won the next *concorso* handily, and in 1891, at age twenty-six, he was appointed to the Rome chair, which he held until his retirement in 1935. A turning point in Castelnuovo's scientific life occurred early in his tenure at Rome, in 1892, when Federigo Enriques, a gifted geometer who had earned his degree in mathematics at the Scuola Normale in Pisa, came to Rome to attend a course in higher geometry taught by Luigi Cremona, the first occupant of the chair of higher geometry at Bologna and founder of the Italian school of geometry. Enriques deemed Castelnuovo, only five and a half years his senior, much

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[1] *An accessible account of the classification is given in [Gray99].*

[2] *Translated by Elisa Piccio and edited by the authors.*

Guido Castelnuovo, ca. 1890.

Fonti iconografiche, Biblioteca Matematica Giuseppe Peano, Department of Mathematics, University of Turin.

more open and welcoming than Cremona, whose lectures completely befuddled him. The two young mathematicians quickly became friends and several years later, when Castelnuovo married Elbina Enriques, brothers-in-law. As they strolled the streets of Rome, talking mainly about algebraic geometry, Enriques would update his new friend daily on his progress, while Castelnuovo listened attentively and offered critical comments. "It is probably not an exaggeration to assert that the theory of algebraic surfaces from the Italian point of view was created during these conversations," Castelnuovo notes in a eulogy delivered at the Accademia Nazionale dei Lincei following Enriques' death in 1946 [Cast47]. The Castelnuovo-Enriques collaboration culminated in their classification of algebraic surfaces, which has been hailed as "one of the lasting contributions to mathematics made by the Italian geometers of a century ago" [Gray99].

Castelnuovo eventually stopped working actively in algebraic geometry. After 1906 he published only two original papers relating to the field, including his notable 1921 paper on Abelian functions [Cast21]. Although best known outside Italy for his contributions to algebraic geometry, Castelnuovo explored other fields, including probability, mathematical pedagogy, and the philosophical implications of Einstein's theories of special and general relativity (an interest he shared with Enriques)—lecturing, writing, and publishing on all these topics. Nevertheless, he continued to keenly follow the developments of algebraic geometry at home and abroad and made penetrating judgments on them throughout his life.

Castelnuovo was an unabashed champion of the role of intuition in the success of the Italian school. At the 1928 International Congress of Mathematicians held in Bologna, he delivered one of the major addresses, an overview of the work in algebraic geometry not just in his own country but in Germany, France, and the United States. At the end of his talk, he issued the following warning with regard to its future development: "[A]bandoning geometric intuition—the only means that so far has allowed us to find the way in this tangled territory—would mean extinguishing the feeble flame that can lead us into the dark forest" [Cast28]. This may have been a criticism of the then-current "algebraizing of algebraic geometry" by Emmy Noether and B. L. van der Waerden [Sch07].

A student of his, Oscar Zariski, offers the following sketch of the great man [Parikh91].

> Castelnuovo was a somewhat distant fellow, he wouldn't be chummy with you, he was not that type. He was very dignified, long beard. He looked like the Moses of Michelangelo. When he smiled, his face was transformed. But mostly he was very serious.

Beniamino Segre paints a rather more attractive picture, describing Castelnuovo's house in the Via Veneto section of Rome as "modest but welcoming", and as an academic gathering place [Segre54].

> … a center where every Saturday for several years colleagues and students, Italians or foreign visitors, gathered for friendly conversation on a wide variety of subjects; his influence on those present was enormous, with his display of calm wisdom, his interest in each idea expressed, his offering of serene and objective opinions and thoughtful advice, his courtly presence, his genuine modesty. These qualities, even if chastely veiled by a certain reserve, made him loved and appreciated by everybody: you can be certain that he did not have any enemies or detractors.

Francesco Severi (1879–1961) was a man of a very different stripe. His personality is described thus in an obituary in the *Journal of the London Mathematical Society* [Roth63].

> Personal relationships with Severi, however complicated in appearance, were always reducible to two basically simple situations: either he had just taken offence or else he was in the process of giving it—and quite often genuinely unaware that he was doing so. Paradoxically, endowed as he was with even more wit than most of his fellow Tuscans, he showed a childlike incapacity either for self-criticism or for cool judgment. Thus he meddled in politics, whereas it would have been far better had he left them alone.

Oscar Zariski's biographer, Carol Parikh, describes her subject's relations with Severi [Parikh91].

A tall heavy man from Tuscany, he lectured in a way that was particularly disquieting to Zariski. Lacking both the playfulness of Enriques…and the meticulous formality of Castelnuovo, Severi's dictatorial style seemed designed to make it impossible for his students to distinguish between guesses and assertions, hunches and hypotheses.

Outside of mathematics Severi was also a forceful and disquieting presence. 'I love you, Zariski, but you don't love me,' he once said, a surprising statement from a man as vain as he seemed to be. His wild driving was legendary; oblivious to the pleading of his passengers, he would careen through the hills above Rome. Even old age seems not to have slowed him down behind the wheel; Zariski remembered with terror being driven through Rome by Severi, when Severi was already eighty-one.

Severi was born in Arezzo, the last of nine children, to a family with deep roots in Tuscany. His father, Cosimo Severi, a notary who also wrote and published poetry and hymns, committed suicide when Severi was nine, leaving his widow broke and too proud to ask for help raising the four surviving children who were still living at home. During an impoverished childhood, Severi held down a variety of tutoring jobs to help support the family and did not abandon the tutoring until he had graduated with a doctorate in pure mathematics from the University of Turin in 1900. A prodigious and frenetic worker throughout his life, Severi later joked with Beniamino Segre, his student, that he had been "sentenced to a life of hard labor in a penal colony" [Segre62].

At Turin, Severi came under the spell of the geometer Corrado Segre and dedicated his first mathematical work, self-published while he was still an undergraduate, to Segre, calling him an "incomparable teacher", the one who "trained my intellect", taught him to appreciate "rigorous scientific investigations", and stirred his "heart to the highest filial sentiments" [Sev59]. Severi would apparently disavow these sentiments by the 1930s, as we shall see.

He spent several years as an assistant, first in Turin with D'Ovidio, then in Bologna with Enriques, and finally in Pisa with the geometer Eugenio Bertini, before moving in 1904 to Parma following his appointment there as professor of projective and descriptive geometry. A year later, Severi transferred to Padua, where he joined the Socialist Party; there he remained until the call from the faculty of mathematical sciences at Rome came in 1922. At Rome, Severi taught a variety of courses, from calculus to higher geometry, and in 1923 he became rector of the university as well, a position he resigned to protest the assassination of the Socialist deputy Giacomo Matteotti by Fascist thugs in June of the following year.

Photograph of Francesco Severi previously published in *Civiltà delle Macchine*, 1957 (furnished by authors).

Francesco Severi, 1915, the year that he received the Accademia dei Lincei Mathematics Prize.

Severi also signed (as did his Rome colleagues Vito Volterra, Castelnuovo, and Tullio Levi-Civita) the philosopher Benedetto Croce's anti-Fascist manifesto in 1925. Taking aim at the philosopher Giovanni Gentile's manifesto of the Fascist intellectuals published ten days earlier, Croce's manifesto advocated acceptance of a universal culture, not one confined to a particular political system. Soon after, however, perhaps because of his outsized ambition, Severi began to ingratiate himself with Benito Mussolini's regime. Severi's election in 1929 to Mussolini's new Academy of Italy as a last-minute substitution for Federigo Enriques—Enriques, now a member of the Rome faculty, had not signed either manifesto—thrust Severi into the limelight as the regime's spokesman for Italian mathematics. Like Castelnuovo, Levi-Civita, and Volterra, Enriques was Jewish, which explains his name's deletion from the list of candidates forwarded to the Academy's president-elect. The Fascists denied that there was any ban against Jewish members at the outset. In its fourteen years of existence, the Fascist academy never admitted any Jews to its ranks. Situated directly across the street from the venerable and anti-Fascist-leaning Accademia dei Lincei, its rise was a highly visible first step in the chain of events leading up to the formal annexation of the Lincei in 1939.

Beniamino Segre (1903–1977) entered Severi's life in 1927, two years after Mussolini had turned Italy into a dictatorship. A native of Turin who trained as a geometer there (he counted Guido Fubini, Gino Fano, and his distant cousin Corrado among his teachers), Segre moved briskly through the academic ranks after receiving a doctor's degree in mathematics in 1923 with a dissertation in algebraic geometry. After holding several positions in Turin (assistant to the chair of rational mechanics; assistant to the chair of analytical, projective,

Beniamino Segre in Venice, 1932. Photograph courtesy of Sergio Segre.

and descriptive geometry; and associate of analytical mathematics at the Military Academy of Artillery and Engineers), and studying with Élie Cartan and Émile Picard in Paris on a Rockefeller fellowship, in 1927 the twenty-four-year-old Segre accepted Severi's invitation to become his assistant in Rome. By then, Segre had also obtained the libera docenza, a license to teach at the university level, in analytic and projective geometry.

Physically, the two mathematicians—Severi towered over Segre—could not have been more dissimilar. After meeting both men on a visit to Rome in 1928, W. E. Tisdale of the Rockefeller Foundation described Severi in his interview log as "a huge, bearded man, decidedly teutonic in general appearance" and Segre as "a nice appearing young fellow" who spoke decent French and seemed to rank high in Severi's estimation [RFA28]. The self-assured, flamboyant Severi showered attention on his able new assistant, delighted in calling Segre "my favorite" (a play on the double meaning of Segre's first name in Italian), and cultivated his interest in algebraic geometry, the field in which Severi had done his most significant work. In 1931, after four years as Severi's assistant, Segre won the nationwide *concorso* for the chair of higher geometry at Bologna. By then, he had become Severi's star pupil, his sounding board, the protégé who occasionally had to endure his maestro's harsh editorial judgments (on the occasion described below, certainly), but he distanced himself from Severi's pro-Fascist politics.

The Letters

When Beniamino Segre was appointed to the chair in geometry at Bologna in 1931, he was required to give an inaugural lecture, which he entitled "Italian Geometry from Cremona to the Present Day". He evidently sent a draft to Severi asking for his comments. The letter in question, dated January 2, 1932, is Severi's response.

> My dear Segre,
>
> …[T]he general outline [of B. Segre's draft–ed.] is mediocre in several places, especially where you talk about algebraic geometry.
>
> It lacks perspective so that a reader who doesn't know much will not be able to understand the hierarchy of ideas and names.
>
> 1) The work of C. Segre has been overrated…Segre, for example, did not prove anything major in the field of geometry of curves although he did carry out a very significant revision of the subject. His contributions to higher dimensional projective geometry are overrated when compared, for example, with those of Veronese. This exaggerated evaluation is probably explained by your love of him as a disciple…
>
> 2) The work by Veronese is underrated. In Italy he was the true creator of higher dimensional projective geometry.
>
> 3) The work of Castelnuovo has been overrated as has been that of Enriques, especially when compared to that of [Max] Noether, whose name you have completely neglected in your discussion of surfaces.
>
> 4) My work has been underrated, which seems odd to me since you were my student, and, in addition, your affection for your first teacher [Segre] has caused you to overestimate his work.

In elaborating on this third point, Severi lists many of the important things that were known in algebraic geometry before Castelnuovo and Enriques did their work on the classification of surfaces, some of which were essential to their classification. These include the notions of the geometric and arithmetic genus of a surface (Cayley, Zeuthen, M. Noether), the Zeuthen-C. Segre invariant, the Brill-Noether Theorem, and the work of Picard on surfaces. He thus suggests that the classification of Castelnuovo and Enriques was built on the shoulders of giants and that Segre did not appreciate the importance of this fact. Severi ends this part of his discourse by saying: "And in this article you write that before Castelnuovo-Enriques there were only 'a few developments that had only created difficulties'. Poor Noether!"

On his fourth point, he has this to say:

> Beginning in 1904 I developed new ideas that untied the Gordian knots that had been bound so tightly up to that time. I myself untied most of them, such as the characterization of irregular surfaces from both the transcendental and algebraic point of view.…Even setting aside my work on conceptual clarifications as well as my work on the hyperelliptic surfaces with Enriques, which I am willing to do, this does not justify the humiliating description of my status as "arrived" that you have written on page 12, thus putting my work and that of Castelnuovo-Segre's [Severi may have meant Enriques here–ed.] on two completely different levels. You need to weigh your words!
>
> Also forgotten by you were my Theorem of the Base [Sev06] and my work on the geometry of varieties of higher dimension. How could you have done this when you discuss Italian geometry? In addition, there is no mention of the fact that I am the only one among living Italian algebraic geometers who has created a school.
>
> You have also underestimated my contributions to enumerative geometry. If only you could understand them. All of this is not to reproach you, because you certainly have done your best. Although your mathematical knowledge is wide, you currently do not have a deep enough understanding in the vast field of algebraic geometry to allow you to have a reliable perspective on the subject. But I am also surprised that the comparative evaluations that we discussed many times in the past did not have an effect on you even though I was always very conscientious about being objective.

We do not have the original draft that Segre sent to Severi, so we do not know how Severi's criticisms affected the content of Segre's inaugural lecture. There is, however, the paper that resulted from the lecture, which appeared in the *Annali di Matematica* in 1932 [Segre33]. In it, one notices that Cremona is mentioned sixteen times (including twice in footnotes); Max Noether is mentioned twice; Corrado Segre is mentioned six times, as is Guido Castelnuovo (and one footnote); Enriques is mentioned a total of seven times (four in footnotes). Severi is mentioned nine times, including four footnotes. The fact that Noether is now mentioned twice indicates that Segre took Severi's comments to heart at least as far as Noether is concerned. It is easy to conjecture that Segre gave Severi more attention in his lecture and in this paper than in the draft that provoked Severi's hectoring letter.

And here is the excerpt from Castelnuovo's letter to Segre in 1938. It contains comments, perhaps solicited by Segre, on a recent paper by Segre in the *Annali di Matematica Pura ed Applicata* [Segre38]. The excerpt deals with some historical commentary of Segre's in the preface on the work of Castelnuovo, Enriques, and Severi in the 1904–1908 period:

> One last comment regarding the historical issues.…The notion of a continuous system [now called an algebraic system–ed.] of curves on some special surfaces already appears in some works of Enriques and mine that precede the work of Severi.…In some special cases I suggested the definition of the characteristic series of a continuous system to Severi. But since this suggestion had been given in an unpublished letter, and subsequently Severi brilliantly developed the idea mentioned in it, it is not useful to make a claim of priority here. I only mention this matter to show you how much caution is needed when you assign scientific priorities in periods in which the research was often done in collaboration, or was suggested by elders to their more youthful colleagues. It was the good fortune of the Italian school of algebraic geometry to have this disinterested collaboration between 1890 and 1910. But this makes it necessary to smooth out certain overly clean divisions between the work of one and the other.
>
> What is undoubtedly due to Severi in the period 1904–08 are the following: the theorem that the existence of Picard integrals of the 1st and 2nd kind on an algebraic surface depends on the irregularity of the surface (1904), a theorem that was successively stated precisely by both of us; the theory of the algebraic equivalence of curves on a surface; and the Theorem of the Base [this has evolved into the Néron-Severi Theorem–ed.]. That is more than enough to show his great worth.

The first contribution to which Castelnuovo refers is contained in a beautiful and fundamental theorem due collectively to Pierre Humbert, Picard, Enriques, Castelnuovo, and Severi concerning the dimension of the space of Picard integrals of the first and second kind on an algebraic surface F over the complex numbers C. Its statement requires the notion of the irregularity q of an algebraic surface F which is defined to be the nonnegative integer $p_g - p_a$, where p_g is the usual geometric genus of F and p_a = the arithmetic genus of F (a notion somewhat more difficult to define, which we will not do here). The theorem states if F is an algebraic surface defined over C of irregularity q, then the vector space I_1 of Picard integrals of the first kind has dimension q and the vector space I_2 of Picard integrals of the second kind has dimension $2q$. Picard integrals of the first kind are ones whose integrands are closed and regular 1-forms, and Picard integrals of the second kind are those whose integrands are closed 1-forms with only polar, as opposed to logarithmic, singularities on F. It is interesting to note that F must be irregular in order to have nontrivial Picard integrals of the first or second kind.

All the mathematicians mentioned above contributed to the proof of the theorem [Roth63], [Zar34,Ch.6]. The final steps were furnished in 1905 by Severi, who showed that $\dim(I_1) \le q, \dim(I_2) \le 2q$, and by Castelnuovo, who showed that $q \le \dim(I_1)$. Castelnuovo's proof depended on a technical "Theorem" due to Enriques [Enrq04], based on a suggestion of Severi (in 1904), whose algebro-geometric proof was shown later by Severi [Sev21] to be fundamentally flawed.[3] Fortunately, Henri Poincaré gave a valid transcendental proof of the Enriques-Severi assertion in 1910, so that at least from this date on, the theorem was legitimate.

The second and third contributions discussed by Castelnuovo are closely related. Again, they concern an algebraic surface F over C. We need to introduce some additional notions. The divisor group of $F, \mathrm{Div}(F)$, is the free Abelian group over the integers Z generated by the irreducible (algebraic) curves on F. Its elements are referred to as the divisors on F. Following [Mum66], we say that two curves C_1 and C_2 on F are algebraically equivalent if they are parametrized by a connected algebraic variety S. We denote by $G_a(F)$ the subgroup of $\mathrm{Div}(F)$ generated by divisors of the form $C_0 - C_1$, where C_0 and C_1 on F are algebraically equivalent curves. The Néron-Severi group is the quotient group $\mathrm{Div}(F)/G_a(F)$. The Theorem of the Base in Castelnuovo's discussion says that the Néron-Severi group is finitely generated. The paper where the proof appeared (*Math. Ann.* [Sev06]) was solicited by its editor, Max Noether. (This is perhaps why he is mentioned so solicitously in Severi's 1932 letter.) Subsequently, in 1952, Néron [Nér52] refined and extended Severi's Theorem of the Base and gave a modern (rigorous) proof of it—that is, one acceptable to the Franco-American school of algebraic geometry.

Epilogue

The Fascist racial laws enacted in the summer of 1938 barred Jewish students from attending public schools and universities; Jewish authors from publishing works under their own names; and scores of Jewish academics, including some of Italy's best and brightest mathematicians, from teaching. Vito Volterra, the dean of Italian mathematics, had already forfeited his position at the University of Rome in 1931, by refusing to sign the Fascist loyalty oath. Guido Castelnuovo had retired from teaching at Rome in 1935 at the age of seventy, capping a career spanning nearly forty-five years in the classroom. But their younger Jewish university colleagues, including Tullio Levi-Civita and Federigo Enriques in Rome, Beppo Levi and Beniamino Segre in Bologna, and Guido Fubini, Gino Fano, and Alessandro Terracini in Turin, felt the full brunt of the racial legislation. Levi and Terracini found new jobs in Argentina; Fano emigrated to Lausanne, Switzerland; and Fubini went to Princeton.

Beniamino Segre woke up that September to find that he had been dismissed from his position as director of Bologna's mathematical institute, relieved of his duties as an editor of Italy's oldest scientific journal, the *Annali di Matematica Pura ed Applicata*, expelled from numerous scientific academies and organizations, including the Italian Mathematical Union (UMI), of which he had been a founding member, and denied any form of compensation. Deeply offended by the anti-Semitic legislation, Segre immediately renounced his membership in the Fascist Party, reportedly telling Bologna's rector, Alessandro Ghigi, "Since his Excellency the Head of the Government has declared that a Jew is not an Italian, I took it as a given that I could no longer wear the fascist insignia as it might have been interpreted as an insincere gesture" [Finzi94]. Having tried and failed to find a position in the United States, Segre took refuge in England in the spring of 1939 with his wife and their three children. In September, when Britain declared war on Germany, he was interned for several months on the Isle of Man as an enemy alien, while his wife and children stayed with the mathematician Leonard Roth and his wife in London. The youngest daughter fell ill with measles, which turned into blood poisoning during an air raid attack over the city. The hospitals overflowed with emergencies, making it impossible to get the little girl admitted in time, and she died early the next morning. In 1942 the family moved to Manchester, where Segre taught for several years

[3] *Controversy over the proof of this "Theorem", especially between Enriques and Severi, raged until a valid algebro-geometric proof was obtained by Grothendieck in 1960. Grothendieck's proof utilized in a nontrivial way non-reduced schemes and other powerful machinery due to Cartier and Kodaira and Spencer [Mum66].*

before returning to the University of Bologna in 1946. Four years later, Segre succeeded Severi as professor of geometry at the University of Rome.

In the wake of the 1938 racial laws, Jewish elementary and secondary schools sprang up in Rome and other major Italian cities with the permission of the authorities, who had banned any university-level coursework. In December 1941 Guido Castelnuovo organized a clandestine university, recruited a host of professors, including himself, and arranged for the students to register (in absentia) at the privately run Istituto Politecnico di Friburgo, in Switzerland. The ad-hoc university, as well as the Jewish schools, ceased operations when the Germans occupied Rome in September 1943. At the end of the war the students enrolled at the University of Rome, and their transcripts from the Fribourg Polytechnic were submitted as evidence of their advanced standing. During the occupation, Castelnuovo and his wife were sheltered briefly by Tullio Viola, a young mathematician at Rome. The couple later took refuge in a religious institute, and when that arrangement became too dangerous, they lived for many months in a small pensione off the Via Veneto, using an assumed name, Cafiero [NM04].

Photograph of Mussolini and Severi previously published in *L'Illustrazione Italiana*, 1939 (furnished by authors).

Mussolini (in front) at the University City (Città Universitaria) in Rome to visit the new building of the Royal Institute of Higher Mathematics. Severi is next to him on the right.

After the liberation of Rome in June 1944, Castelnuovo, then seventy-nine, came out of retirement to reconstitute Italy's pre-Fascist scientific organizations. Formally reinstated as professor emeritus at Rome, he served as general commissioner of Italy's National Research Council and president of its mathematics committee and played a leading role in reviving both organizations. He contributed in a major way to the rebirth of the Lincei, whose president he became in December 1946, a post he held until his death in 1952. Castelnuovo was named a Senator for Life in Italy's parliament in 1949, and soon after his death the building that houses the University of Rome's ("La Sapienza") Institute of Mathematics was named in his honor.

From late 1938 until the liberation of Rome, Francesco Severi, forever a loyal Fascist, was undeniably Italy's most prominent mathematician, especially now that any possible competitors were either in hiding or had emigrated. During most of this time, he was the president of the Italian National Institute for Higher Mathematics (INDAM), also based at "La Sapienza" near the Rome Termini train station. Shortly after the liberation, a High Commission for Sanctions against Fascism was established. The commission's specific charge was to look into allegations of wartime collaboration against party members who had been accused of taking an active part in Fascist political life or who had remained loyal to Mussolini after he was deposed in September 1943. The commissioners initially suspended Severi from university teaching. He appealed the ruling and received *sanzioni minori*, simple censures involving nothing more than a letter placed in his university personnel file. When Severi got in touch with Segre again after the war, he enclosed with his letter a "To whom it may concern" document issued by the Italian Ministry of Public Instruction stating that the ministry had not subjected him "to any sanction provided for by current legislation on the cleansing of the Italian Civil Service" [MPI46] for Fascist activities. Severi also received a clean bill of health from another commission assigned to examine the behavior of former members of the Academy of Italy, which concluded that he "had not received from Fascism anything more than was his due as a distinguished scientist." His "moral rectitude", the commission added, was never called into question [Sev45]. Severi also defended his behavior during the war, pointing out to Segre in 1945 that he had worked diligently to save the assets of the local bank in Arezzo, his hometown. However, a committee (including Castelnuovo) that was given the task of rebuilding the Lincei, refused to reelect Severi, and he regained membership only in 1948, after the government declared a general nationwide amnesty. Severi, who had lost his position as president of INDAM, also recovered that post following the amnesty and held it until his death.

Appendix

As is well known in the algebraic geometry community, there was increasing skepticism among algebraic geometers outside the Italian school about the way the Italians did algebraic geometry. In particular, there was concern about the mathematical precision of some of the key definitions and the logical rigor of the proof of some of the important theorems. This was especially the case among members of the Franco-American school of algebraic geometry, beginning with Castelnuovo's student Oscar Zariski in the mid-1930s, soon after he had written his well-known book on algebraic surfaces [Zar34]. He was later joined, most prominently, by André Weil, Claude Chevalley, and Pierre Samuel. The Italian school was very sensitive to these criticisms. Francesco Severi, especially, tried unsuccessfully to address them. (See, for example, [Sev49] and Chevalley's review in *Mathematical Reviews* [Chev52].) There was also a well-known confrontation between Severi and Weil at the 1954 International Congress of Mathematicians at Amsterdam over the rigor of Severi's theory of the intersection of subvarieties on a projective variety and rational equivalence. (See, for example, [VdW70].) Posterity has shown that Weil won the argument.

Below is a translated[4] excerpt from the preface by Guido Castelnuovo to the posthumous (1949) magnum opus of Federigo Enriques on algebraic surfaces [Enrq49] in which Castelnuovo defends the more intuitive approach to mathematics of an earlier era—which certainly included Italian algebraic geometry in its heyday—as opposed to the new stress on formal rigor espoused by the Franco-American school.

> Will someone come along soon who will continue the work of the Italian and French schools [here Castelnuovo presumably is referring to the French school prior to the dominance of the Franco-American school–ed.][5] who will succeed in developing the theory of algebraic surfaces that has already been accomplished for the theory of algebraic curves? I hope so, but I doubt it… mathematics has now taken a different course from that of the past [i.e., the nineteenth century]. Fantasy and intuition characterized research then, but now these are treated with suspicion, as there is the fear that they could lead to errors. Theories were developed by mathematicians to make more precise many ideas that were already vaguely in their mind. It was the exploration of a vast territory seen from a distant shore. In this way such jewels of mathematics as the theory of analytic functions, elliptic functions, and Abelian functions were created during this past century. Nowadays there is more interest in the road that leads to a field of exploration rather than to the field itself. And this tendency will not be short-lived, as we can also see in other fields such as in music and in the arts, where fantasy is banned and where the technique or the way of expression is more interesting than the work itself. It would be an exaggeration to extend these pessimistic judgments to the evolution that mathematics is undergoing nowadays, but if we compare these fifty years to the corresponding years of the last century, when people like Gauss, Abel, Jacobi, Cauchy, and many others rose, we certainly worry about the future of our science.
>
> One day, sooner or later, the love for the great theories will be born again and on that day people will read the treatise by Enriques as a report wherein many gems have been unearthed and many others wait to be discovered.

[4] *Translated by the authors.*

[5] *For an interesting discussion of the relations between the Italian and French schools in the early twentieth century, see [Brig84].*

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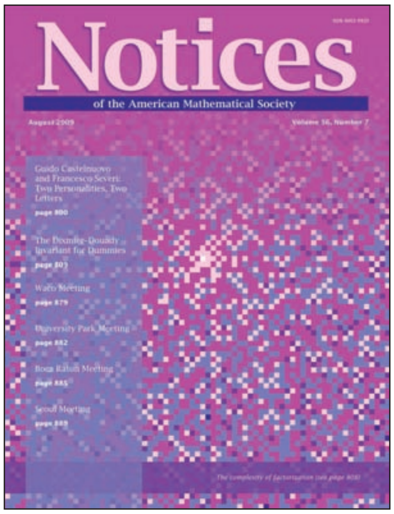
About the Cover

The complexity of factorization

This month's cover was produced by a mild variant of Eratosthenes' sieve.The fascination with the mixture of determinism and randomness associated with factorization has lasted over 2,000 years. As the WHAT IS … article by Friedlander and Iwaniec (page 817) reminds us, optimism still reigns in this field.

—Bill Casselman, Graphics Editor
(notices-covers@ams.org)



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