Opinion

Mathematicians and the Tower of Babel

Mathematics has been flourishing for the last fifty years; classical problems have been solved, many areas have made major progress, and mathematics and physics have rediscovered each other, through the efforts of Atiyah, Singer, Witten, Seiberg, and many others.

At the same time the mathematical community is becoming increasingly fragmented. Looking at the titles on the covers of mathematical journals, or at the topics of lectures at conferences and colloquia, it seems that mathematics has broken into dozens of separate fields. Participation at specialty seminars is growing and attendance at departmental colloquia is dwindling. We are in danger of becoming residents of a Tower of Babel.

This is paradoxical because, to any mathematician who is watching, it is clear that the different branches of mathematics—analysis, algebra, geometry, topology, etc.— are enormously intertwined. Some of the most striking new results occurred at the interface between different areas. Leray's sheaves, beginning as a tool in topology, in the hands of H. Cartan and J. P. Serre revolutionized the study of complex manifolds and had a similar effect on algebraic geometry. Within analysis, a 1949 paper of Arne Beurling brought together Hardy space theory in the disk and the study of operators on Hilbert space, having a profound effect on that subject. Other examples abound.

The departmental colloquium used to be an occasion where individual specialists enlightened their colleagues in other fields about what was happening in their area. Most of the department members came to listen. As a graduate student at Harvard, I regularly attended colloquia. Despite being pretty ignorant, I got a lot out of the talks. I particularly remember, from the late 1940s, a colloquium by Heinz Hopf on almost complex structures on manifolds. I did not know what a complex manifold was, nor know any serious topology, but somehow he managed to explain things, and I found it very interesting. The same happy situation continued for me at Yale, where I was an instructor for some years, and then at Brown for many years.

What has happened? I once heard about a seminar given by Grothendieck, which was described as "A telegram by Grothendieck to Serre". This should not serve as a model for our colloquium speakers. A person giving a colloquium should have a realistic image of the audience in front of him or her. He or she should not generalize from the fact that two people in the front row are asking penetrating questions. This enlivens the lecture, but is misleading about the state of knowledge of the rest of the audience.

The innocent (nonexpert) listener at a mathematics lecture wants certain things: he or she would like an idea of the background of the problem, some of the objectives of the area, to see some simple examples illustrating the problem, and he or she wants to see some of the key steps in the argument. What he or she cannot handle is a massive amount of technical details and so many formulas on the board he or she cannot possibly absorb them.

It comes down to the fact that there are both a transmitter (the speaker) and a receiver (the listener), and the former can send much more information than the latter can absorb.

Here are some things that might improve the situation: more colloquia should inform people outside the field about interesting developments in the field. I remember some excellent talks on the proof of Fermat's Last Theorem which concentrated on explaining main elements in the argument and similar successful talks given about the Seiberg-Witten theory. You may say: "But a speaker traveling to another institution wants to give details of exciting recent developments, especially by him- or herself or coworkers." Of course! But this should be done before and after the colloquium, in the form of a conversation with interested parties.

Also, some schools have programs where speakers are invited for several days, which facilitates the above. In a similar spirit, a mathematician writing an expository article on a subject needs to concentrate on the central issues, and not overwhelm the reader with details. References at the end of the paper can lead the interested reader to continue his or her pursuit of the subject. As well, the writer or lecturer needs to remember that terminology current in his or her field for the last ten years (and in his or her mind part of the English language) may be only fuzzily, or not at all, familiar to the reader or listener.

The problems discussed above have been widely known for many years. It is high time that we tackled them.

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