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Differential Geometry Issue
(see page 266)

AMERICAN MATHEMATICAL SOCIETY

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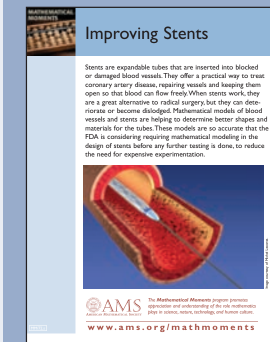

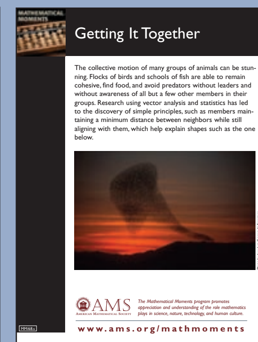

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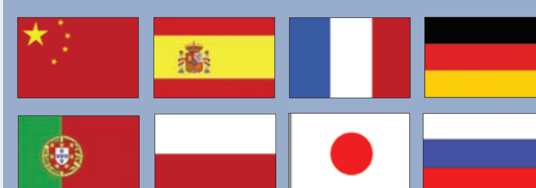

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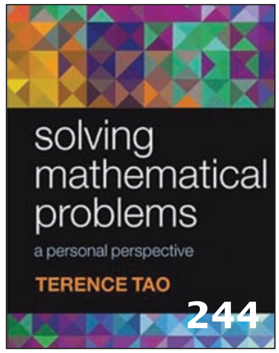

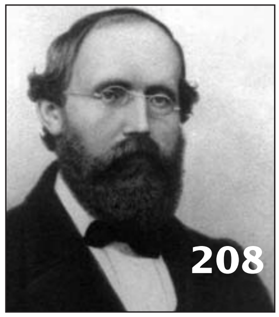

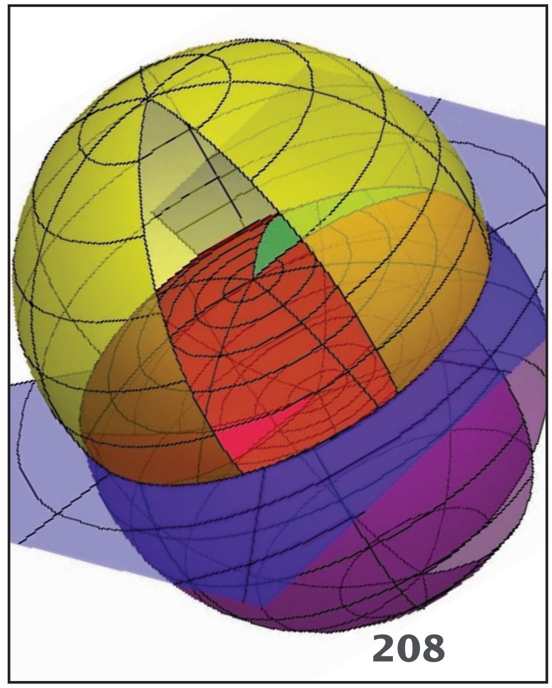

Ideas of differential geometry have permeated all parts of mathematics. Partial differential equations, topology, analysis, and many other branches of our subject have taken on a geometric patina. The recent solution of the Poincaré conjecture used geometry in surprising ways. This issue of the *Notices* showcases some recent developments in the geometry of Riemann.

—*Steven G. Krantz*
Editor

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Notices
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From the AMS Secretary

I thank Randi D. Ruden for her splendid editorial work, and for helping to assemble this issue. She is essential to everything that I do.

—Steven G. Krantz
Editor

Collaborating on Research with Mathematicians from Less Developed Countries

For a decade, I have collaborated on research with mathematicians from Jordan, Iran, Pakistan, India, and South Africa. Hardly any of my collaborators have high mathematical prestige or have positions at institutions that have it. By and large, these mathematicians make up for deficiencies that include high teaching loads and very poor libraries with enthusiasm, patience, goodwill, and hard work. Most, though not all, such efforts have led to publication in respectable journals published in western countries.

Common sense told me to expect that collaborating with mathematicians from other cultures using different technologies would bring me problems, but these problems were far more difficult than I expected.

Since the end of the Second World War, most published mathematics is written in English, which is a second language for most people. Even if your foreign co-author's knowledge of English is very good, it is often not idiomatic and needs to be corrected in a way that is effective without being pedantic or patronizing. Treat your co-author with respect and make it easy for your co-author to be critical of what you have written. A co-author's English may be less than perfect, a co-author from a poor country may know less mathematics than you do, but still may be extremely gifted. Once I overcame their reluctance to criticize my writing or mathematics because of their fear of being discourteous, my Third World co-authors could be very good at detecting errors.

Avoid political discussions or comparison of governments. Ignoring this, especially in writing, can get you into trouble. Stick to mathematics.

Keep in mind two differences in how mathematicians are "rewarded for publishing papers" in some countries. The author whose name appears first gets more of a reward (sometimes financial) than the remaining authors. There is also an Impact Factor assigned to journals that is ill understood. Publishing in a journal with a high impact factor gets a bigger reward.

Communication, electronic or otherwise, is difficult for a number of reasons. Email works most of the time, but power failures are much more frequent than in North America or Europe. Regular mail tends to be slow, and fax machines are usually turned off at night. Acknowledge all emails and encourage your co-authors to do the same. If possible, get a list of PDF files of your own publications to potential co-authors. Even governments you may not like encourage their citizens to study and earn advanced degrees abroad. Should you visit your co-author, there are many books named "Culture Shock (name of country)" written to describe local customs with a view to warn you of things that might insult your hosts. For example, in many countries your host will feel obligated to urge you to accept as a gift something that you admire—and resent it when you accept.

One communication difficulty pertaining to mathematics is that few of these foreign co-authors have access to good library facilities or even an effective interlibrary loan system. References mailed to potential collaborators often result in a polite reply saying that some book or journal is unavailable. This is a serious problem everywhere because the number of distinct mathematics journals and books has grown substantially and gotten both more specialized and more expensive.

The best substitute for a poor library is a subscription to MathSciNet, which is an electronic version of *Math Reviews*. Its reviews and papers can be read and printed mostly, but not always, sometimes for an additional charge. Were it up to me, I would provide open access to MathSciNet to all Third World AMS members even if some dues had to be increased. The main problem for these mathematicians is cost.

I am most fearful at this prospect: The quantity of research in the less developed countries is growing rapidly, while many journals' articles are becoming available only in electronic form. In some countries ideology, economics, or technological limitations restrict a mathematician's access to Web portals. Commercial publisher-maintained sites are only available to subscribers at costs that some institutions cannot afford. Ignoring this trend may result in serious negative consequences for Third World authors, indeed for all of us.

We want a worldwide community of research mathematicians. Quality of published research in mathematics that is independent of geographic location or financial wealth should be our common goal. Perhaps there are less expensive methods for making mathematical research and thoughts readily available. What is most required is a desire to reach out to all mathematicians in all countries as a way to make our combined research efforts easier to implement.

Collaborating with international mathematicians is both important and rewarding. It is difficult to exaggerate the enjoyment I feel after a successful collaboration with someone from a different culture. I hope that a committee of the AMS will study this important problem.

—Melvin Henriksen

Editor's Note: Professor Henriksen died after a long illness on October 14, 2009.

Letters to the Editor

Andrew Gleason

Many readers of the *Notices* are, I am sure, grateful for the fine articles about the late Andrew Gleason and his accomplishments, in the November 2009 issue. I myself was particularly interested in Paul Chernoff's article, for I once spent much effort to completely master Gleason's profound theorem characterizing abstractly defined "states" in quantum mechanics, and I even hoped to be able to shorten the proof, something in which, however, I failed. Because Gleason is probably most widely known for his path-breaking contribution to solving Hilbert's Fifth Problem, I thought that a personal experience of mine may be of interest.

Gleason once visited Indiana University in Bloomington, and I met him at a post-lecture party. I mentioned to him my admiration for "Gleason's Theorem". "Which one?" he asked. Upon my replying that I meant the one about states in quantum theory, his face darkened, he thought for a few moments, and then said "You know, that was the most difficult thing I ever did in my life."

—Andrew Lenard
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(Received October 2009)

As a former physics major, now physician and neuroscientist, and one of Professor Gleason's "last students" in a couple of different ways, I would just like to add to the recent recountings in the *Notices*.

I was the only nonmathematician taking Gleason's graduate real analysis class. Gleason passed out a set of notes from a prior time he had taught the class and requested that we bring any errors to his attention—there were less than a handful. I mentioned that he must be pleased that when he taught the class again, the notes would be error free! He responded that he would never teach the class again, but he continued to collect errors with interest and concern. The final for the class was half true/false questions, scored +1 for a correct answer, -3 for a wrong answer, zero for no answer. One question was so tricky that no one among the crew of past, present, and future deep-thinking and Putnam Prize-winning students dared answer! The professor still knew best!

Later I wanted to enter a team in the COMAP Mathematical Contest in Modeling. Knowing that Gleason was a three-year Putnam champion and literally wrote the book on this pure math contest, I approached him about being our faculty advisor with some trepidation, but he immediately agreed.

We solved the discrete problem for the specific given set of conditions, and also showed that with general conditions the problem was NP-complete. On Monday morning after Gleason mailed our sealed write-up, we proudly told him our results. He listened with a knowing look. He too had found the same, but didn't look nearly as tired as we were! We were awarded an Outstanding Paper. Gleason's legacy is that Harvard now regularly competes in this applied math contest.

I have done some work on J. J. Thomson's century-old problem of the minimum energy configuration of charges on the surface of a sphere. I discussed with Gleason one paper in which we were studying the difference in energies of configurations for certain numbers of charges that we had found admitted more than one nice lattice configuration. He asked why we hadn't given the general formula for such N—his legendary ability to generalize your results on the spot! My co-author Richard Stong was then able to produce the formula. I have never seen reviewers so impressed! I continued to have the pleasure and good fortune of discussing work on other problems in physics and neuroscience with Andrew Gleason.

From his lectures and comments, it seems to me that Andrew Gleason was never fully comfortable with the notion of completeness of the real numbers. When last we talked Gleason mentioned to me that he was studying Hilbert's axioms of geometry, in particular the axioms on order.

Our teacher and friend is gone. But the example he set for us lives on for us to follow.

—Eric Altschuler, M.D., Ph.D.
New Jersey Medical School–University of Medicine & Dentistry of New Jersey

(Received October 2009)

Submitting Letters to the Editor

The *Notices* invites readers to submit letters and opinion pieces on topics related to mathematics. Electronic submissions are preferred (`notices-letters@ams.org`); see the masthead for postal mail addresses. Opinion pieces are usually one printed page in length (about 800 words). Letters are normally less than one page long, and shorter letters are preferred.

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An Invitation to Cauchy-Riemann and Sub-Riemannian Geometries

John P. D'Angelo and Jeremy T. Tyson

A certain amount of complex analysis, in both one and several variables, combines with some differential geometry to form the basis of research in diverse parts of mathematics. We first describe that foundation and then invite the reader to follow specific geometric paths based upon it. We discuss such topics as the failure of the Riemann mapping theorem in several variables, the geometry of real hypersurfaces in complex Euclidean space, the Heisenberg group, sub-Riemannian manifolds and their metric space structure, the Hopf fibration, sub-Riemannian geodesics on the three-sphere, CR mappings invariant under a finite group, and finite type conditions. All of these topics fall within the branch of mathematics described by the title, but of course they form only a small part of it. We hope that the chosen topics are representative and appealing. The connections among them provide a fertile ground for future research. We modestly hope that this article inspires others to develop these connections further. To this end, our reference list includes a diverse collection of books and accessible articles.

Given Riemann's contributions to geometry, it is not surprising that his name occurs twice in the title. It is more surprising that the uses of his name here come from different parts of mathematics. We find it delightful that contemporary mathematics is forging connections that even Riemann could not have anticipated.

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Figure 1. Georg Friedrich Bernhard Riemann.

There Is No Riemann Mapping Theorem in Higher Dimensions

In an attempt to understand complex analysis in several variables, especially questions revolving around the failure of the Riemann mapping theorem, many mathematicians have focused attention on geometric properties of the boundaries of domains and how those geometric properties influence the complex analysis on the domain. Such investigations have led to the fields of CR geometry and sub-Riemannian geometry.

We begin with the Riemann mapping theorem. Let Ω be an open, connected, and simply connected proper subset of $\mathbb{C}$. Then there is a bijective holomorphic mapping f from Ω to the unit disc $\mathbb{D}$. The inverse mapping is also holomorphic. We say that Ω and $\mathbb{D}$ are *biholomorphically equivalent*. The entire plane must of course be excluded, because a bounded holomorphic function on $\mathbb{C}$ reduces to a constant. This theorem has applications throughout both pure and applied mathematics. In many of these applications, such as uniform fluid flow, the upper half plane substitutes for the unit disc. The explicit linear fractional transformation f (the *Cayley transformation*) defined by

$$z = f(w) = \frac{i - w}{i + w} \tag{1}$$

maps the upper half plane biholomorphically to the unit disc.

We mention in passing one of the most fundamental geometric generalizations of the Riemann mapping theorem, although we will not head in that direction. The *uniformization theorem* states that the universal cover of a compact Riemann surface must be one of three objects: the unit disc $\mathbb{D}$, the complex plane $\mathbb{C}$, or the Riemann sphere $\overline{\mathbb{C}}$. As a reference for analysis in one complex variable, we need only mention the classical text by Ahlfors [Ahl].

Let us now consider complex Euclidean space $\mathbb{C}^n$ of arbitrary dimension. We equip $\mathbb{C}^n$ with the usual Hermitian inner product given by $\langle z, w\rangle = \sum z_j \overline{w}_j$ and the corresponding norm given by $|z| = \langle z, z\rangle^{1/2}$. The unitary group $\mathcal{U}(n)$ consists of linear mappings preserving the inner product. The unit ball B_n is the set of z for which $|z| < 1$.

In dimension at least two the unit ball is not biholomorphically equivalent to a half space. There are many possible proofs. One approach takes into account the geometry of the boundaries. When the boundaries are smooth manifolds (as they are in this case), the *Levi form* plays a crucial role. The Levi form for a real hypersurface in $\mathbb{C}^n$ (for $n \geq 2$) is the complex variable analogue of the second fundamental form for a real hypersurface in $\mathbb{R}^n$. We define and discuss the Levi form on page 211.

For now we simply note that the Levi form is a Hermitian form defined on part of the tangent spaces of the boundary. There is no Levi form in one dimension, because the tangent space is one real dimension and the part on which the Levi form acts does not exist. For a half plane in dimension at least 2 the Levi form is identically zero. For the unit sphere it is positive definite. As a result the boundary geometries are so different that no biholomorphic mapping can exist. By contrast, in one dimension, different boundary curves are indistinguishable from this point of view.

A polydisc is a Cartesian product of discs. For example, the subset of $\mathbb{C}^2$ defined by $|z_1| < 1$ and $|z_2| < 1$ is a polydisc. It is also not biholomorphically equivalent to B_2. In fact, for $n \geq 2$ there is no biholomorphic map from any polydisc to any ball. One classical proof of this statement involves showing that the groups of biholomorphic automorphisms of the ball and of the polydisc have different dimensions.

A second proof (see [D1, page 14]) proceeds in the following manner. First consider a biholomorphic (or even proper) image Ω of the Cartesian product of bounded domains; one shows that $b\Omega$ must contain complex analytic sets. Then an easy computation, given in Lemma 6, shows that the definiteness of the Levi form precludes the existence of positive-dimensional complex analytic subsets of $b\Omega$. This computation helps unify several of the ideas in this article. For example, higher order commutators of complex vector fields can determine obstructions to the existence of complex analytic sets in a real hypersurface. Such obstructions lead to geometric finite type conditions for subelliptic estimates. In two complex dimensions a subelliptic estimate (see page 217) holds if and only if the bracket-generating property (see page 212) holds.

Consider a domain Ω with smooth boundary $b\Omega$. In order that geometric results about $b\Omega$ be meaningful for complex analysis on Ω, one requires results concerning the boundary smoothness of biholomorphic maps. Two early references in this area are [Bel] and [Fef]; we do not attempt to list any of the many newer references. In general, smoothly bounded domains, topologically equivalent to the unit ball B_n, are seldom biholomorphically equivalent to it. On the other hand, Fridman [Fri] has established a wonderful approximate Riemann mapping theorem: given any topological ball and a positive ϵ, there is a biholomorphic mapping to a domain whose boundary is always within distance ϵ of the unit sphere.

The Unit Sphere and the Heisenberg Group

The quintessential principle connecting CR geometry and sub-Riemannian geometry arises from the correspondence between the unit sphere bB_n and the Heisenberg group, which bounds the unbounded Siegel upper half space H_n. We will obtain a useful identification of bH_n with $\mathbb{C}^{n-1} \times \mathbb{R}$, on which we will impose a nonabelian group law arising from the biholomorphic automorphisms of H_n. The equivalence between H_n and B_n generalizes the one-dimensional Cayley transformation.

The Siegel upper half space H_n is the set of w in $\mathbb{C}^n$ such that

$$\mathrm{Im}(w_n) > \sum_{j=1}^{n-1} |w_j|^2. \tag{2}$$

The following elementary but useful identity is the key to discovering the biholomorphic map:

$$\mathrm{Im}(\zeta) = \left|\frac{i + \zeta}{2}\right|^2 - \left|\frac{i - \zeta}{2}\right|^2. \tag{3}$$

With $\zeta = w_n$ we plug (3) into (2) and rewrite the result to obtain

$$(4) \qquad \left|\frac{i+w_n}{2}\right|^2 > \sum_{j=1}^{n-1} |w_j|^2 + \left|\frac{i-w_n}{2}\right|^2.$$

After dividing by $|\frac{i+w_n}{2}|^2$ and changing notation, inequality (4) becomes

$$1 > \sum_{j=1}^{n} |z_j|^2.$$

Here $z_j = \frac{2w_j}{i+w_n}$ for $j < n$ and $z_n = \frac{i-w_n}{i+w_n}$. The reader should check that this transformation $f : w \mapsto z$ is biholomorphic from H_n to B_n and also compare it with the transformation in (1).

The automorphism group of H_n

We alluded earlier to the biholomorphic automorphism groups of the ball and the polydisc. For any complex manifold the automorphism group is a fundamental algebraic invariant. For example, the automorphism group of the upper half plane in $\mathbb{C}$ is $\mathrm{PSL}(2,\mathbb{R})$. Its elements act on the (one-point compactification of the) boundary of the upper half plane as linear fractional transformations $x \mapsto \frac{ax+b}{cx+d}$, for $ad - bc = 1$. Any such transformation can be represented as the composition of dilations $x \mapsto rx$, translations $x \mapsto x + x_0$, and possibly the inversion $x \mapsto \frac{1}{x}$. Automorphisms fixing the point at infinity do not require the inversion.

We now describe the analogous situation on bH_n, defined by equality in (2). The last variable in (2) plays a different role, and hence for $w \in \mathbb{C}^n$ we write $w = (w', w_n)$. Two natural families of biholomorphic self-maps of H_n are the *dilations* $\delta_r : H_n \to H_n$, for $r > 0$, given by $\delta_r(w', w_n) = (rw', r^2 w_n)$, and the *rotations* $R_A : H_n \to H_n$, for $A \in \mathcal{U}(n-1)$, given by $R_A(w', w_n) = (A(w'), w_n)$. To introduce an analogue of translation we consider the biholomorphisms $\tau_p : H_n \to H_n$, for $p = (\zeta, t) \in \mathbb{C}^{n-1} \times \mathbb{R}$, given by

$$(5) \quad \tau_p(w', w_n) = (w' + \zeta, w_n + t + 2i\langle w', \zeta\rangle + i|\zeta|^2).$$

All of the preceding maps extend to self-maps of $bH_n = \{(w', w_n) : w_n = |w'|^2\}$. The action of the family $\{\tau_p : p = (\zeta, t) \in \mathbb{C}^{n-1} \times \mathbb{R}\}$ on $H_n \cup bH_n$ is faithful and the action on bH_n is simply transitive. By this method we equip $\mathbb{C}^{n-1} \times \mathbb{R}$ with a group law $(p, q) \mapsto p \cdot q$, characterized by the identity $\tau_p \circ \tau_q = \tau_{p\cdot q}$. The resulting space is the *Heisenberg group*. Often in the literature $\mathbb{C}^{n-1}$ is replaced by $\mathbb{R}^{2n-2}$ and the group law is expressed using real variables. Finally we observe that the group of biholomorphic automorphisms of H_n which fix the point at infinity is generated by dilations, rotations, and translations. See Stein's textbook in harmonic analysis [Ste, Chapters XII and XIII] for the complete story.

CR Structures

We next discuss the geometry of a general real hypersurface M in $\mathbb{C}^n$. The beautiful interplay between real and complex geometry dominates the discussion. See [BER], [D1], [DT], [Jac], [Tre] and their references for more about CR geometry.

We wish to consider complex vector fields and therefore start by considering the complexified tangent bundle

$$(6) \qquad \mathbb{C}T(\mathbb{C}^n) = T(\mathbb{C}^n) \otimes \mathbb{C}.$$

As usual in complex analysis we define the complex partial derivative operators

$$\frac{\partial}{\partial z_j} = \frac{1}{2}\left(\frac{\partial}{\partial x_j} - i\frac{\partial}{\partial y_j}\right)$$

and

$$\frac{\partial}{\partial \overline{z_j}} = \frac{1}{2}\left(\frac{\partial}{\partial x_j} + i\frac{\partial}{\partial y_j}\right).$$

Using these operators we decompose the exterior derivative d as

$$(7) \quad df = \partial f + \overline{\partial} f = \sum_{j=1}^{n} \frac{\partial f}{\partial z_j} dz_j + \sum_{k=1}^{n} \frac{\partial f}{\partial \overline{z_k}} d\overline{z_k}.$$

Since $d^2 = 0$, it follows that $\partial^2 = \overline{\partial}^2 = \partial\overline{\partial} + \overline{\partial}\partial = 0$.

A section of the bundle (6) is a complex vector field. Each such vector field is a combination of both z and $\overline{z}$ derivatives with smooth coefficients:

$$L = \sum_{j=1}^{n} a_j(z)\frac{\partial}{\partial z_j} + \sum_{j=1}^{n} b_j(z)\frac{\partial}{\partial \overline{z}_j}.$$

We obtain two naturally defined integrable subbundles of $\mathbb{C}T(\mathbb{C}^n)$. Let $T^{10}(\mathbb{C}^n)$ denote the bundle whose sections are vector fields L of the form

$$(8) \qquad L = \sum_{j=1}^{n} a_j(z)\frac{\partial}{\partial z_j},$$

where the a_j are smooth complex-valued functions. We note that this bundle is integrable in the sense of Frobenius: if K, L are sections of $T^{10}(\mathbb{C}^n)$, then so is their commutator, or Lie bracket, given by $[K, L] = KL - LK$. The complex conjugate bundle, denoted $T^{01}(\mathbb{C}^n)$, is also integrable. Since the zero section is the only section of both bundles, we have $T^{10}(\mathbb{C}^n) \cap T^{01}(\mathbb{C}^n) = 0$. We obtain a splitting

$$\mathbb{C}T(\mathbb{C}^n) = T^{10}(\mathbb{C}^n) \oplus T^{01}(\mathbb{C}^n).$$

This splitting of the tangent bundle plays a crucial role in all aspects of complex geometry. CR geometry studies the extent to which this splitting holds on real manifolds.

Let M be a smooth real hypersurface of $\mathbb{C}^n$, or more generally, of a complex manifold. The tangent spaces of M then inherit some of the ambient complex structure. We write $\mathbb{C}TM$ for $T(M) \otimes \mathbb{C}$. We define $T^{10}(M) = T^{10}(\mathbb{C}^n) \cap \mathbb{C}TM$, and denote its complex conjugate bundle by $T^{01}(M)$. Again we have

$$(9) \qquad T^{10}(M) \cap T^{01}(M) = 0.$$

Let M be an abstract real manifold M with complexified tangent bundle $\mathbb{C}TM$. We say that a subbundle $T^{10}(M)$ of $\mathbb{C}TM$ defines a *CR structure* on M if it is integrable and its intersection with its conjugate bundle satisfies (9). We then call M a *CR manifold*. Its *horizontal bundle* is the direct sum

$$\mathcal{H}(M) = T^{10}(M) \oplus T^{01}(M). \tag{10}$$

We say that M is of *hypersurface type* if the fibers of $\mathcal{H}(M)$ have codimension one in $\mathbb{C}TM$.

In general, the *CR codimension* of M is the codimension of $\mathcal{H}(M)$ in $\mathbb{C}TM$. Real submanifolds of $\mathbb{C}^n$ of arbitrary dimension define CR manifolds of various CR codimensions. Consider a k-dimensional real subspace of $\mathbb{C}^n$. As a real submanifold it has codimension $2n-k$. On the other hand, its CR codimension can be any integer ℓ between 0 and k such that $k-\ell$ is even. The subspace has maximal CR codimension (equal to k) if it is totally real, and it has minimal CR codimension (equal to zero) if it is complex. Any complex manifold can be regarded as a CR manifold of CR codimension zero. Such manifolds are precisely the integrable almost complex manifolds.

We focus on CR manifolds of hypersurface type. On such manifolds there is a nonvanishing differential one-form η, defined up to a multiple, annihilating $\mathcal{H}(M)$. We may assume that η is purely imaginary. Each of the summands in (9) is integrable, but something new happens; their sum is not integrable. The Levi form measures the failure of integrability of the sum and leads to both CR geometry and sub-Riemannian geometry.

Definition 1. Let M be a CR manifold of hypersurface type. The Levi form λ is the Hermitian form on $T^{10}(M)$ defined by

$$\lambda(L, \overline{K}) = \langle \eta, [L, \overline{K}] \rangle.$$

Any vector field T for which $\langle \eta, T \rangle \neq 0$ has a component in the *missing direction*. The Levi form $\lambda(L, \overline{L}\rangle$ gives the component of $[L, \overline{L}]$ in the missing direction. In the following definition we identify λ with a Hermitian linear transformation of $T^{10}(M)$.

Definition 2. A CR manifold of hypersurface type is *pseudoconvex* if all nonzero eigenvalues of λ have the same sign. It is called *strongly pseudoconvex* if λ is definite, that is, all eigenvalues have the same nonzero sign.

Observe that there is an ambiguity of sign in the definition of the Levi form. Even for a real hypersurface in real Euclidean space the sign of the second fundamental form is defined only modulo the choice of normal. For a compact hypersurface in $\mathbb{R}^n$ consider a point p farthest from the origin. Since the sphere centered at the origin osculates M to order two at p, the second fundamental form at p agrees with that of the sphere, thus determining its sign. The story is the same for real hypersurfaces in $\mathbb{C}^n$ and the Levi form. On the other hand, there is no natural way to resolve the ambiguity of sign for an abstract CR manifold.

Next we express the Levi form on a hypersurface in terms of partial derivatives. In a neighborhood of a given point we suppose that M is the zero set of r, where r is smooth and $dr \neq 0$ where $r = 0$. A complex vector field L is tangent to M if and only if $L(r) = \langle dr, L \rangle = 0$ on M. For the one-form η we may use $\frac{1}{2}(\partial - \overline{\partial})(r)$. In this case $d\eta = -\partial\overline{\partial}r$.

The Cartan formula for the exterior derivative of a 1-form η states that

$$\langle d\eta, A \wedge B \rangle = A\langle \eta, B \rangle - B\langle \eta, A \rangle - \langle \eta, [A, B] \rangle. \tag{11}$$

The commutator term ensures that $d\eta$ is linear over the module of smooth functions. Let L and K be local sections of $T^{10}(M)$. Then $\langle \eta, L \rangle = \langle \eta, \overline{K} \rangle = 0$. By the Cartan formula,

$$\lambda(L, \overline{K}) = \langle \eta, [L, \overline{K}] \rangle = \langle -d\eta, L \wedge \overline{K} \rangle = \langle \partial\overline{\partial}r, L \wedge \overline{K} \rangle. \tag{12}$$

We have interpreted the Levi form as the restriction of the complex Hessian of r to sections of $T^{10}(M)$. It is therefore analogous to the real Hessian, which governs Euclidean convexity. Euclidean convexity implies pseudoconvexity, but the converse fails. See [Hör] and [Kra] for lengthy discussion, especially for the meaning of pseudoconvexity on a domain whose boundary is not smooth.

Example 3. The zero set of $r(z) = \mathrm{Im}(z_n)$ is a half-space Σ. Its horizontal space $\mathcal{H}_z(\Sigma)$ decomposes into holomorphic and conjugate holomorphic subspaces, spanned respectively by the vector fields $L_j = \partial/\partial z_j$, $j = 1, \dots, n-1$, and their conjugates. In this case, the complex Hessian of the defining function is identically equal to zero, and hence the Levi form also vanishes identically.

Example 4. The zero set of $r(z) = \sum_{j=1}^{n} |z_j|^2 - 1$ is the sphere $\mathbb{S}^{2n-1}$. The horizontal space $\mathcal{H}_z(\mathbb{S}^{2n-1})$ decomposes into the holomorphic subspace $T_z^{10}(\mathbb{S}^{2n-1}) = \mathrm{span}\{L_1, \dots, L_{n-1}\}$ and its conjugate $T_z^{01}(\mathbb{S}^{2n-1})$, where

$$L_j = \overline{z_j}\frac{\partial}{\partial z_n} - \overline{z_n}\frac{\partial}{\partial z_j}. \tag{13}$$

The annihilating one-form can be taken to be

$$\eta = \frac{1}{2}\sum_{j=1}^{n} \overline{z_j}dz_j - z_j d\overline{z_j}. \tag{14}$$

The Levi form, after dividing out a nonzero factor, satisfies $\lambda(L_j, \overline{L_k}) = \delta_{jk} + \overline{z}_j z_k$. Hence the unit sphere is strongly pseudoconvex. The Heisenberg group is also strongly pseudoconvex. Its Levi form, computed similarly, is $\lambda(L_j, \overline{L_k}) = \delta_{jk}$.

Remark 5 (Hans Lewy equation)**.** In 1957 Hans Lewy produced his famous example of an *unsolvable* first-order linear PDE. His equation is $\overline{L}u = f$, where L is the type $(1,0)$ vector field tangent to $\mathbb{S}^3$ given in (13). See [Tre] for an elegant and readable account of developments in PDE and CR geometry based on this operator.

Positive definiteness of the second fundamental form for a real hypersurface M in $\mathbb{R}^n$ is a curvature condition: it precludes the existence of straight line segments in M. The next lemma provides a subtle analogue for the Levi form.

Lemma 6. *Let M be a strongly pseudoconvex real hypersurface in $\mathbb{C}^n$. Then M contains no complex analytic sets of positive dimension.*

Proof. We prove the contrapositive statement: if M contains such sets, then M cannot be strongly pseudoconvex. Assume M contains a complex analytic set V of positive dimension. Let p be a nonsingular point of V. We can then find a one-dimensional nonsingular holomorphic curve $t \to z(t) \in \mathbb{C}^n$ with $z(0) = p$, with tangent vector $z'(0) \neq 0$, and with $z(t) \subset M$ for $|t| < 1$. For each local defining function r for M near p we have $r(z(t)) = 0$ for $|t|$ small.

Taking $\frac{\partial}{\partial t}$ of the identity $r(z(t)) = 0$ and evaluating at $t = 0$ gives

$$\langle \partial r(p), z'(0) \rangle = 0. \tag{15}$$

Taking the Laplacian $\frac{\partial^2}{\partial t \partial \bar{t}}$ of the same identity and evaluating at $t = 0$ gives

$$\langle \partial\bar{\partial} r, z'(0) \wedge \overline{z'(0)} \rangle = 0. \tag{16}$$

Equation (15) says that $z'(0) \in T_p^{10}(M)$ and equation (16) says that $z'(0)$ is in the null space of the Levi form. Since $z'(0) \neq 0$, M is not strongly pseudoconvex at p. □

Sub-Riemannian Geometry

It has been evident so far that the stratification of the complexified tangent spaces of a CR manifold given by the horizontal bundle together with the missing direction is at the heart of CR geometry. In this section we will consider real manifolds equipped with a similar horizontal subbundle of the (uncomplexified) tangent bundle. In both cases there are horizontal directions which are infinitesimally accessible and missing directions. When the Levi form is nonzero we can recover the missing direction by commutators. The ability to recover missing directions via commutators of horizontal vector fields is the defining property of a sub-Riemannian manifold. This nonintegrability condition translates to a connectivity condition whereby such manifolds are equipped with a singular metric. See Theorem 9.

CR manifolds provide a natural class of examples, but the general theory covers a much wider class of spaces arising in PDE, control theory, geometric group theory, and many other settings. Example 8 puts a sub-Riemannian structure on spaces of kth order Taylor polynomials (jets). This example formalizes the well-known procedure for reducing a differential equation of high order to a first-order system. The references [Mon], [Str], [Gro], [Bel], [FS], [CCG], [CDPT], and [CC] provide many examples and discussion of other sub-Riemannian manifolds.

Differential geometric aspects

Let M be a smooth real manifold and let $\mathcal{H}(M)$ be a distribution (subbundle) in the tangent bundle $T(M)$. The classical Frobenius theorem deals with the case when $\mathcal{H}(M)$ is integrable, that is, closed under the Lie bracket. In this case M is foliated by $\mathcal{H}(M)$-integral submanifolds. We consider the opposite extreme, when $\mathcal{H}(M)$ is *completely nonintegrable*: the Lie bracket span of $\mathcal{H}(M)$, at any point $p \in M$, coincides with $T_p(M)$. In this case we say that the pair $(M, \mathcal{H}(M))$ satisfies the *bracket-generating property*, also known as *Hörmander's condition*. Put another way, for each $p \in M$ there exists an integer $s = s(p) < \infty$ so that the values at p of all s-fold iterated Lie brackets of vector fields valued in $\mathcal{H}(M)$ fill out the entire tangent space $T_p(M)$. We call $s(p)$ the *step* of M at p. We emphasize that $s(p)$ may depend on the point p. In the case when s is uniformly bounded on M, we call $\sup_{p \in M} s(p)$ the *step* of M.

We elaborate with some examples. If M is a CR manifold of hypersurface type, we define $\mathcal{H}(M)$ as in (10). The missing direction might or might not be obtained via iterated Lie brackets of horizontal vector fields. Example 3 shows that Levi flat hypersurfaces do not satisfy the bracket-generating property. The opposite extreme is the strongly pseudoconvex case. If L is a $(1,0)$ vector field and not zero at p, then the bracket $[L, \bar{L}]$ has a component in the missing direction. In this case the missing direction arises upon taking a single Lie bracket of horizontal vector fields, and the induced sub-Riemannian structure is of step two. To be step two we need only one such vector field; we recover the missing direction at p with a single Lie bracket whenever the Levi form is not zero at p. The simplest example of a higher step structure is the pseudoconvex hypersurface defined by $\text{Re}(z_2) = |z_1|^{2m}$ in $\mathbb{C}^2$ near the origin. One requires an iterated commutator

$$[\dots[L, \bar{L}], \dots, \bar{L}]$$

with $2m$ total brackets to obtain the missing direction. In this case the origin is called a point of type $2m$, and one says that the vector field L is of type $2m$ there. Notice in this case that the step at most points is 2. See page 217 for related information.

Example 7. The Heisenberg group bH_n provides the canonical example of a sub-Riemannian manifold. The group law was discussed in the paragraph following (5). As usual, the left invariant vector fields define the Heisenberg Lie algebra. As a basis for the horizontal distribution $\mathcal{H}(bH_n)$ in $T(bH_n)$ we take the vector fields

$$X_j = \frac{\partial}{\partial x_j} + 2y_j \frac{\partial}{\partial t}, Y_j = \frac{\partial}{\partial y_j} - 2x_j \frac{\partial}{\partial t}, j = 1, \ldots, n-1, \tag{17}$$

where $w_j = x_j + iy_j$ and $t = \mathrm{Re}(w_n)$. Set $T = \frac{\partial}{\partial t}$. Then $[X_j, Y_k] = -4T\delta_{jk}$ for $j, k = 1, \ldots, n-1$. By putting $L_j = \frac{1}{2}(X_j - iY_j)$ and $\overline{L_j} = \frac{1}{2}(X_j + iY_j)$ and regarding them as sections of $\mathbb{C}T(bH_n)$, we return precisely to the CR setting.

Example 8. Jet spaces provide a geometric interpretation for Taylor polynomials, by viewing the equivalence classes of C^m functions modulo m-jets (mth order Taylor approximations) as points in an abstract space. We illustrate in the one-dimensional case. Consider C^m functions $f : \mathbb{R} \to \mathbb{R}$. The underlying space M is $\mathbb{R}^{m+2}$, with coordinates x (corresponding to the base point x_0 for the Taylor approximation of f) and $u_m, \ldots, u_0$ (corresponding to $f^{(m)}(x_0), \ldots, f(x_0)$). The sub-Riemannian structure is defined by a family of smooth 1-forms $\eta_j = du_j - u_{j+1}\,dx$, $j = 0, \ldots, m-1$, corresponding to the differential equalities $df^{(j-1)}(x) = f^{(j)}(x)\,dx$. These forms, together with dx and du_m, frame the cotangent bundle $T^*(M)$. Introduce the dual frame $X, U_m, \ldots, U_0$ for $T(M)$. Let $\mathcal{H}(M)$ be spanned by X and U_m. The Cartan formula (11) gives

$$\begin{aligned} \langle \eta_{j-1}, [U_k, X] \rangle &= -\langle d\eta_{j-1}, X \wedge U_k \rangle \\ &= \langle du_j \wedge dx, U_k \wedge X \rangle = \delta_{jk}, \end{aligned}$$

which yields the Lie bracket identities $[U_j, X] = U_{j-1}$, $j = 1, \ldots, m$. We observe that $\mathcal{H}(M)$ is a bracket-generating horizontal distribution of rank two inducing a sub-Riemannian structure on $M = \mathbb{R}^{m+2}$ of step $m+1$, called the *mth order jet space* (on the real line). The first-order jet space is isomorphic (as a Lie group) to the Heisenberg group bH_2. For additional details see [CDPT, Chapter 3.1] or [Mon, Chapter 6].

The fundamental theorem of sub-Riemannian geometry

Recall that a Riemannian metric on a smooth manifold M is a smoothly varying family of inner products defined on the tangent spaces. We say that a pair $(M, \mathcal{H}(M))$ satisfying the Hörmander condition is a *sub-Riemannian manifold* if it is equipped with a smoothly varying family of inner products $\langle \cdot, \cdot \rangle_p$ (the *sub-Riemannian metric*) defined on the horizontal tangent spaces $H_p(M)$. Such a metric permits us to define notions of length, volume, angle and other geometric concepts for objects taking values in the horizontal subbundle. For instance, a smooth curve $\gamma : (a, b) \to M$ is termed *horizontal* if $\gamma'(t) \in \mathcal{H}_{\gamma(t)}M$ for all t.

As in the Riemannian case, the sub-Riemannian metric induces a distance function $d : M \times M \to [0, \infty)$ (the so-called *Carnot-Carathéodory distance*): for $p, q \in M$, $d(p, q)$ is the infimum of

$$\int_a^b \langle \gamma'(t), \gamma'(t) \rangle^{1/2}\, dt \tag{18}$$

over all smooth horizontal curves $\gamma : [a, b] \to M$ with $\gamma(a) = p$ and $\gamma(b) = q$. The fundamental result is Theorem 9 below, the Chow–Rashevsky theorem. See the preface in [Mon] for the history leading to this theorem.

Theorem 9. *Assume that Hörmander's condition is satisfied for the pair $(M, \mathcal{H}(M))$. Then all pairs of points in M are horizontally connected. Consequently, the Carnot-Carathéodory distance function on a sub-Riemannian manifold M endows M with the structure of a metric space.*

An *adapted Riemannian metric* on M is an extension of the sub-Riemannian metric to the full tangent bundle. If the step of M is finite, then the topologies defined by the sub-Riemannian metric and any adapted Riemannian metric coincide. However, the geometric and analytic properties of these two metrics differ substantially, unless the step is equal to one. For instance, if M has step at least two, then these two metrics are never bi-Lipschitz equivalent. (They are bi-Hölder equivalent with Hölder exponent $\frac{1}{s}$, where s is the step of M.)

Sub-Riemannian geodesics are smooth horizontal curves γ in M which locally minimize the length $\ell(\gamma)$ defined in (18). The horizontality condition is a family of nonlinear constraints which can be understood in the language of control theory. The path planning problem for wheeled motion (including such concrete subproblems as parallel parking an automobile) can be reinterpreted as the problem of finding geodesics joining prespecified points on certain sub-Riemannian manifolds modeled locally on the first Heisenberg group. Again, we reference [Mon] for more details.

Sub-Riemannian geodesics behave rather differently from their Riemannian counterparts. *Abnormal geodesics* are locally length minimizing curves which fail to satisfy the geodesic equations. Such curves cannot exist in Riemannian geometry. We refer to [Mon] for a thorough discussion of the geodesics in sub-Riemannian spaces and especially the issue of abnormal geodesics. We remark, however, that all geodesics in the sub-Riemannian geometries arising from CR structures on the boundaries of strictly pseudoconvex domains are necessarily normal. Additional information on the structure of sub-Riemannian geodesics can be found in [CCG] and the recent publication [CC].

A rich vein of current research activity in sub-Riemannian geometry that we do not address is the study of the Carnot-Carathéodory geometry of submanifolds of sub-Riemannian manifolds. For more information see [CDPT, Chapter 4].

Example 10. We equip the Heisenberg group bH_n with a Carnot-Carathéodory metric as follows. The left invariant vector fields from (17) define the horizontal distribution. We declare them to be an orthonormal basis. The geodesics for this metric have a beautiful geometric description. For simplicity, we restrict to the case $n = 2$. In view of the group structure it suffices to describe the geodesics from the identity element $o = (0,0,0)$ to an arbitrary point $p = (x_1, y_1, t_1)$ with $t_1 \le 0$. Each geodesic is the horizontal lift of a circular arc in the (x, y) plane with endpoints $(0,0)$ and (x_1, y_1) so that the area of the planar region bounded by the arc together with the line segment joining these two points is equal to $-4t_1$. The sub-Riemannian distance from o to p is just the Euclidean length of the projected arc. When $t_1 = 0$ the geodesic is just a ray in the $t = 0$ plane from o to p. When $(x_1, y_1) = (0,0)$ geodesics are not unique; the family of geodesics from o to $(0,0,t_1)$ is rotationally symmetric about the t-axis.

The Unit 3 Sphere

> *Just like the circle and the two-sphere, the three-sphere is very round. But there are some beautiful, classical aspects to its roundness that are not easy to guess from its lower-dimensional sisters.*
>
> William Thurston [Thu]

We illustrate the preceding circle of ideas by describing the sub-Riemannian geodesics in the three-sphere $\mathbb{S}^3$ equipped with the Carnot-Carathéodory metric arising from its canonical CR structure. These geodesics have recently been computed by several authors using different techniques. See [HR], [BR], and [CC, Chapter 10]. We loosely follow the presentation by Hurtado and Rosales [HR]. The geodesics admit a simple and elegant description in terms of the Hopf fibration.

$\mathbb{S}^3$ as a Lie group and the Hopf fibration

Hamilton's search for an "algebra of vectors in $\mathbb{R}^3$", parallel to the identification of plane vectors with complex numbers, is mathematical folklore. His search led him to develop the theory of the *quaternions*, a noncommutative algebraic structure for vectors in $\mathbb{R}^4$. An arbitrary quaternion takes the form $a + bi + cj + dk$, where $a, b, c, d \in \mathbb{R}$ and i, j, and k are indeterminates satisfying $i^2 = j^2 = k^2 = ijk = -1$. We will identify quaternions with pairs of complex numbers by the rule $p = z_1 + z_2 j \leftrightarrow (z_1, z_2) \in \mathbb{C}^2$. Thus we identify $\mathbb{C}$ with the subspace of the quaternions $\mathbb{H}$ obtained by setting the coefficients of j and k equal to zero. Note that the quaternionic conjugate $\overline{p}$ is equal to $\overline{z_1} - z_2 \cdot j$ in this presentation. We point out the

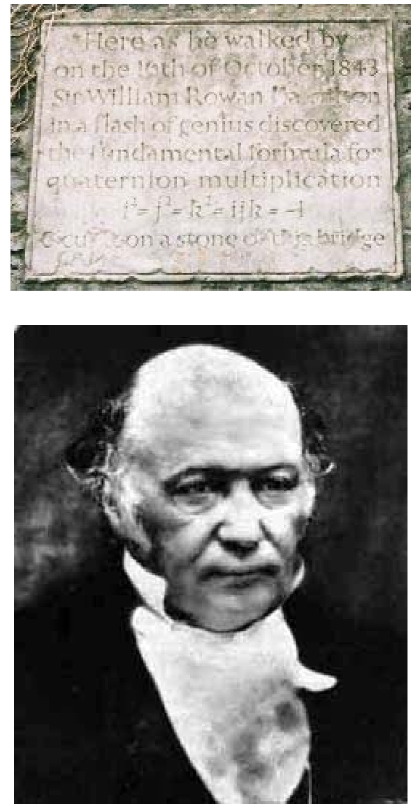

Figure 2. Sir William Rowan Hamilton and the plaque on Dublin's Brougham Bridge honoring Hamilton's invention of the quaternions. The Mathematics Department of the National University of Ireland at Maynooth commemorates the occasion with an annual walk to the site of this plaque on October 16, the anniversary of Hamilton's discovery.

elementary identities $z \cdot j = j \cdot \overline{z}$ and $z \cdot k = k \cdot \overline{z}$ for $z \in \mathbb{C}$.

We view $\mathbb{S}^3$ as the set of unit quaternions $p = z_1 + z_2 j$, where $|z_1|^2 + |z_2|^2 = 1$. With the preceding conventions in place, $\mathbb{S}^3$ is equipped with the Lie group law

$$\begin{aligned} p \cdot p' &= (z_1 + z_2 j) \cdot (z_1' + z_2' j) \\ &= (z_1 z_1' - z_2 \overline{z_2'}) + (z_1 z_2' + \overline{z_1'} z_2) j. \end{aligned}$$

It is well known that $\mathbb{S}^3$ is parallelizable: its tangent bundle admits a smoothly varying orthonormal frame. The quaternionic presentation provides an elegant description for such a frame: just consider the vector fields U, V, and W, where $U(p) = i \cdot p$, $V(p) = j \cdot p$, and $W(p) = k \cdot p$. These vector fields are linked by the bracket relations

$$[U,V] = -2W, \quad [W,U] = -2V, \quad [V,W] = -2U. \tag{19}$$

In particular, setting $\mathcal{H}(\mathbb{S}^3) = \operatorname{span}\{V, W\}$ defines a bracket-generating distribution on $\mathbb{S}^3$. If we set $p = z_1 + z_2 \cdot j$, then $V(p) = j \cdot (z_1 + z_2 j) =$

$-\overline{z_2} + \overline{z_1} j$, which coincides with the CR vector field L from (13). We have already observed that $\mathbb{S}^3$ is strongly pseudoconvex, hence step two. Formula (19) provides the same information. We define a sub-Riemannian structure by introducing a family of inner products on the horizontal bundle for which V and W are orthonormal. Our goal is to describe the geodesics of this sub-Riemannian structure.

The Lie group $\mathbb{S}^3$ is isomorphic to the matrix group $\mathcal{SU}(2)$ via the identification

$$z_1 + z_2 \cdot j \quad \longleftrightarrow \quad \begin{pmatrix} z_1 & z_2 \\ -\overline{z_2} & \overline{z_1} \end{pmatrix}.$$

We can realize $\mathcal{SU}(2)$ as the double cover of the three-dimensional real rotation group $SO(3)$. The most elegant description, which dates back to Cayley, uses the quaternionic presentation: identify $\mathbb{R}^3$ with the space of purely imaginary quaternions via the basis $\{i, j, k\}$, and observe that the map $q \mapsto p \cdot q \cdot \overline{p}$ defines a rotation of $\mathbb{R}^3$. In the coordinates for the quaternions $\mathbb{H}$ described above, the map in question takes the form

$$\begin{aligned} p = \begin{pmatrix} z_1 & z_2 \\ -\overline{z_2} & \overline{z_1} \end{pmatrix} \in \mathcal{SU}(2) \mapsto R_p \\ := \begin{pmatrix} |z_1|^2 - |z_2|^2 & -2\,\mathrm{Im}(\overline{z_1} z_2) & 2\,\mathrm{Re}(\overline{z_1} z_2) \\ 2\,\mathrm{Im}(z_1 z_2) & \mathrm{Re}(z_1^2 + z_2^2) & -\mathrm{Im}(z_1^2 + z_2^2) \\ -2\,\mathrm{Re}(z_1 z_2) & \mathrm{Im}(z_1^2 + z_2^2) & \mathrm{Re}(z_1^2 + z_2^2) \end{pmatrix} \\ \in SO(3). \end{aligned}$$

This map is two-to-one, as it is invariant under the antipodal map $p \mapsto -p$ on $\mathbb{S}^3$.

Observe that the first row vector of R_p lies in $\mathbb{S}^2$ for every p. We obtain a map $\pi : \mathbb{S}^3 \to \mathbb{S}^2$. The circle group $\mathbb{S}^1$ acts on $\mathbb{S}^3$ via the diagonal action $e^{i\theta} \cdot (z_1, z_2) = (e^{i\theta} z_1, e^{i\theta} z_2)$, leaving the first row of the corresponding rotation matrix invariant. Thus we have exhibited $\mathbb{S}^3$ as an $\mathbb{S}^1$-bundle over $\mathbb{S}^2$, called the *Hopf bundle*. The induced fibration of $\mathbb{S}^3$ by (geometric) circles is the *Hopf fibration*.

The sub-Riemannian geodesics on $\mathbb{S}^3$

We begin by observing that the annihilating one-form η from (14) and the corresponding horizontal distribution $\mathcal{H}(\mathbb{S}^3)$ are invariant under the $\mathbb{S}^1$ action. This remark suggests that the structure of the sub-Riemannian geodesics should be related to the Hopf fibration. We now confirm this suggestion.

Extend the inner product to a Riemannian metric g on $\mathbb{S}^3$ by declaring U, V and W to be orthonormal. Let D be the Levi-Civitá connection for this metric g. Define a bundle isomorphism $J : \mathcal{H}(\mathbb{S}^3) \to \mathcal{H}(\mathbb{S}^3)$ by setting $J(V) = W$ and $J(W) = -V$. The data $(\mathbb{S}^3, \mathcal{H}(\mathbb{S}^3), \eta, J)$ defines a *pseudo-Hermitian structure* on $\mathbb{S}^3$, and the ensuing discussion can be framed in the language of pseudo-Hermitian geometry. See, for instance, [CHMY] and [DT].

The sub-Riemannian geodesics are critical points for the first variation of (horizontal) length by horizontal curves with fixed endpoints. Let $\gamma : I \to \mathbb{S}^3$ be a C^2 horizontal curve defined on a compact interval I. An *admissible variation* of γ is a C^2 map $G : I \times (-\epsilon_0, \epsilon_0) \to \mathbb{S}^3$ satisfying

(1) $G(s, 0) = \gamma(s)$ for all $s \in I$,
(2) $G(s, \epsilon) = G(s, 0)$ for all ϵ and all $s \in \partial I$,
(3) for each ϵ, $\gamma_\epsilon(s) := G(s, \epsilon)$ is horizontal.

Let X be the vector field along γ whose value at $\gamma(s)$ is equal to $(\partial G/\partial\epsilon)(s, \epsilon)|_{\epsilon=0}$. Admissibility of the variation can be characterized in terms of X: G is admissible if and only if X vanishes at the endpoints of γ and

$$\gamma'(\langle X, U\rangle) = 2\langle \pi_H X, J(\gamma')\rangle, \tag{20}$$

where π_H denotes projection into the horizontal space.

The classical formula for variation of length in $(\mathbb{S}^3, g)$, specialized to admissible variations, reads

$$\frac{\partial}{\partial\epsilon}\ell(\gamma_\epsilon)|_{\epsilon=0} = -\int_I \langle D_{\gamma'}\gamma', U\rangle.$$

Considering variations of the form $X = f J(\gamma')$ for C^1 functions $f : I \to \mathbb{R}$ with $f|_{\partial I} = 0$ and $\int_I f = 0$, and taking into account (20), one finds that the local minimizers for the sub-Riemannian length functional (18) are characterized by the Euler–Lagrange equation

$$D_{\gamma'}\gamma' + 2\kappa J(\gamma') = 0, \tag{21}$$

where κ is a real constant. This constant κ can be interpreted as a curvature of the geodesic γ. Geodesics of curvature zero are just great circles on $\mathbb{S}^3$. Geodesics of nonzero curvature are horizontal lifts of non-great circles on $\mathbb{S}^2$ through the Hopf fibration. Indeed, consider a smooth curve γ taking values in $\mathbb{S}^3$ and parameterized by arc length. The tangent space to $\mathbb{C}^2$ at $p = \gamma(t)$ splits as $T_p(\mathbb{C}^2) = T_p(\mathbb{S}^3) \oplus T_p(\mathbb{S}^3)^\perp$, which yields the following decomposition for the acceleration vector:

$$\gamma''(t) = D_{\gamma'(t)}\gamma'(t) - \gamma(t). \tag{22}$$

Then (21) reads $\gamma'' + 2\kappa J(\gamma') + \gamma = 0$. Viewing $\mathbb{S}^3$ as a subset of $\mathbb{C}^2$, this equation takes the form

$$\gamma'' + 2i\kappa\gamma' + \gamma = 0. \tag{23}$$

Equation (23) is a system of constant coefficient second-order ODE's whose explicit solution is easily obtained. Rather than presenting this solution, however, we focus on its geometric content.

Theorem 11 (Hurtado–Rosales)**.** *Let γ be a C^2 horizontal curve on $\mathbb{S}^3$ parameterized by arc length. Then the following are equivalent:*

(1) *γ is a geodesic of curvature κ in the sub-Riemannian metric on $\mathbb{S}^3$,*
(2) $\langle \gamma'', J(\gamma')\rangle = -2\kappa$,

(3) *γ is the horizontal lift under the Hopf fibration $\mathbb{S}^3 \to \mathbb{S}^2$ of a piece of a circle in $\mathbb{S}^2$ of constant geodesic curvature κ.*

We recall that the *geodesic curvature* of a space curve γ contained in $\mathbb{S}^2$ is its curvature computed relative to the embedding in $\mathbb{S}^2$. For a curve γ parameterized at arbitrary speed, this curvature is given explicitly by the formula $\gamma \cdot (\gamma' \times \gamma'')/|\gamma'|^3$. For $|h| < 1$, the geodesic curvature of the circle $\gamma = \mathbb{S}^2 \cap \{z = h\}$ is $h/\sqrt{1-h^2}$.

We sketch the proof of Theorem 11. To see that (1) and (2) are equivalent, note that the triple $\{\gamma', J(\gamma'), U(\gamma)\}$ forms an orthonormal frame for $T\mathbb{S}^3$ along γ. The fact that γ is horizontal and arc length parameterized immediately gives that the components of $D_{\gamma'}\gamma'$ in the directions γ' and $U(\gamma)$ are equal to zero. Thus $D_{\gamma'}\gamma'$ is a multiple of $J(\gamma')$ and (21) can be rewritten $\langle D_{\gamma'}\gamma', J(\gamma')\rangle = -2\kappa$. Equation (22) shows that $\langle \gamma'', J(\gamma')\rangle = \langle D_{\gamma'}\gamma', J(\gamma')\rangle$, since $\gamma(t)$ is normal to the sphere and $J(\gamma'(t))$ is tangent to the sphere. To see the equivalence of (1) and (3), one calculates the binormal of the Hopf projection of γ, which turns out to be a constant. Thus this projection is a circle on $\mathbb{S}^2$. Another calculation shows that its geodesic curvature agrees with the curvature κ from (21).

The topological structure of these geodesics is quite interesting.

Proposition 12 (Hurtado–Rosales). *If $h := \frac{\kappa}{\sqrt{1+\kappa^2}}$ is a rational number, then γ is a closed curve, isotopic to a torus knot inside a group translate of the Clifford torus T_h. If h is irrational, then γ is diffeomorphic to $\mathbb{R}$ and is dense in some group translate of T_h.*

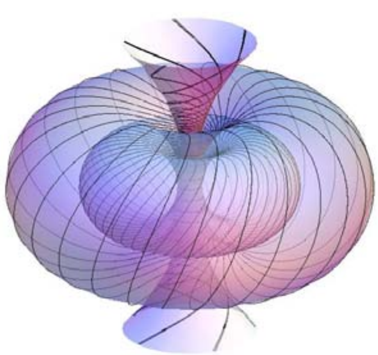

Figure 3. The Hopf fibration and sub-Riemannian geodesics.

Curves of the latter type are the well-known *skew lines* on the torus. The *Clifford tori* in $\mathbb{S}^3$ are tori of the form $T_\rho := \{(z_1, z_2) \in \mathbb{S}^3 : |z_1|^2 = (1+\rho)/2, |z_2|^2 = (1-\rho)/2\}$, where $|\rho| < 1$. Figure 3 illustrates the Hopf fibration, sub-Riemannian geodesics, and the Clifford tori.

Group-invariant Mappings between Spheres in Complex Spaces

The unit sphere $\mathbb{S}^3$ has dominated our discussion, because of its symmetry and its role in both CR and sub-Riemannian geometry. Earlier we observed an action of the circle on $\mathbb{S}^3$ defined by $(z_1, z_2) \to (e^{i\theta}z_1, e^{i\theta}z_2)$. In this section we study symmetries of $\mathbb{S}^3$ via actions of finite subgroups of $\mathcal{U}(2)$. The simplest case is when the group is cyclic; even in the cyclic case there is a remarkable depth and breadth of ideas. Given a positive integer p and an integer q with $1 \le q < p$, we choose a primitive p-th root of unity α, and consider the group action

$$(z_1, z_2) \to (\alpha z_1, \alpha^q z_2). \tag{24}$$

The quotient of the sphere by this group is called a *lens space*, written $L(p,q)$. Lens spaces, introduced by Tietze in 1908, are important in topology. The fundamental group of $L(p,q)$ is cyclic of order p for all q, and thus lens spaces with different p are not homotopy equivalent. Famous work of Alexander in 1919 showed that the lens spaces $L(5,1)$ and $L(5,2)$ are not homeomorphic even though they have isomorphic fundamental groups and the same homology. Necessary and sufficient conditions on p and q for being homeomorphic are known; similarly, necessary and sufficient conditions on p and q for being homotopy equivalent are also known. In Figure 4, the colors represent successive iterations of the group action (24). See http://lukyanenko.net/math/heisenberg09/lens.html for animation and additional details.

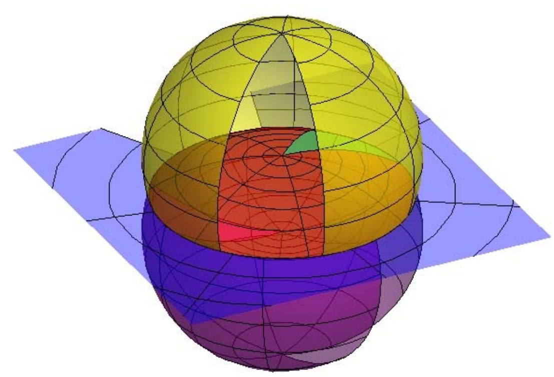

Figure 4. A provocative view of the lens space $L(5,2)$.

Lens spaces also arise in CR geometry. We naturally ask the following question. Given a finite subgroup Γ of $\mathcal{U}(n)$, is there an integer N and a nonconstant rational mapping, invariant under Γ, from $\mathbb{S}^{2n-1}$ to $\mathbb{S}^{2N-1}$? The answer is, in general, no. In fact, according to a theorem of Lichtblau [Lic], such a map can exist only when Γ is cyclic. Furthermore, (see [D1]), most representations of cyclic groups are also ruled out. We describe the results in the special case when $n = 2$. Roughly speaking, the only possibilities for nonconstant invariant rational maps to spheres are the special cases when the quotient space is $L(p,1)$ or $L(p,2)$. One amazing consequence is that there are homeomorphic lens spaces, for example $L(4,1)$ and $L(4,3)$, only one of which (in this case $L(4,1)$) admits a nonconstant rational CR mapping to a sphere.

We do not give up. In order to get more groups involved, we can proceed in two ways. First, we can weaken the assumptions on the mapping. Smooth CR mappings in this context must be rational. Second, we could allow continuous functions satisfying the tangential Cauchy-Riemann equations in the sense of distributions. We then obtain spheres as targets of group-invariant maps for any fixed-point free subgroup of $\mathcal{U}(n)$. Such results require considerable machinery in function theory of several complex variables. If we weaken the assumption on the target by allowing hyperquadrics, then more elementary but fascinating ideas appear. Given a finite subgroup Γ of $\mathcal{U}(n)$, by [D1] we can always find an N and a nonconstant polynomial mapping $p : \mathbb{C}^n \to \mathbb{C}^N$ such that p is Γ-invariant, and p maps the unit sphere to a hyperquadric. The polynomial p arises from the following construction. First, one defines the Hermitian symmetric polynomial Φ_Γ as follows:

$$\Phi_\Gamma(z, \overline{w}) = 1 - \prod_{\gamma \in \Gamma} (1 - \langle \gamma z, w \rangle).$$

By elementary linear algebra, there exist holomorphic vector-valued Γ-invariant polynomial mappings g, h such that

$$\Phi_\Gamma(z, \overline{z}) = \sum_{j=1}^{N_+} |g_j(z)|^2 - \sum_{j=1}^{N_-} |h_j(z)|^2.$$

Then the polynomial map $p = (g, h)$ has the desired properties. Even for the cyclic subgroups $\Gamma(p,q)$ used to define lens spaces, the computation of $\Phi_{\Gamma(p,q)}$ is interesting. For $\Gamma(p,1)$ the result is $(|z_1|^2 + |z_2|^2)^p$. For $\Gamma(p,2)$ and $\Gamma(p,p-1)$ explicit formulas exist. For all q, the formula for $\Gamma(p,q)$ is a polynomial $f_{p,q}$ in the variables $|z_1|^2$ and $|z_2|^2$ with integer coefficients. In general these polynomials have many interesting properties; we give here a few examples. By [D2], for each q, the congruence

$$f_{p,q}(|z_1|^2, |z_2|^2) \cong |z_1|^{2p} + |z_2|^{2p} \pmod{p}$$

holds if and only if p is prime. A combinatorial interpretation of the integer coefficients appears in [LWW]. See [Mus] for an application to counting points on elliptic curves. See [D2] for many additional properties of these polynomials. See [BR] for a study of sub-Riemannian geometries on the lens spaces $L(p,q)$; these geometries are obtained by projecting the horizontal distribution from $\mathbb{S}^3$ to $L(p,q)$ via the natural quotient map.

Finite Type and Higher Brackets

Iterated commutators of vector fields provide a systematic method for dealing with higher order conditions, and their role in sub-Riemannian geometry has been evident in this article. Such conditions are also significant in partial differential equations. Given a finite set of vector fields $X_1, \dots, X_k$ in a neighborhood of a point in $\mathbb{R}^d$, consider the operator $T = \sum_j X_j^2$. Such *sums of squares* of vector fields arise throughout partial differential equations and probability. When the X_j form a basis for the tangent space, T is elliptic, and hence hypoelliptic. Hypoellipticity means that solutions u to the equation $Tu = f$ are smooth when f is smooth. Subellipticity is almost as useful as ellipticity, because it also implies hypoellipticity. An elliptic operator of order two gains two derivatives in the Sobolev scale; a subelliptic operator gains fewer derivatives than its order, but the gain suffices to establish regularity results. The Hörmander condition, applied to the span of the X_j's, provides the necessary and sufficient condition for subellipticity of T.

On the other hand, subellipticity for systems is much more difficult. In two dimensions, subellipticity for the Cauchy-Riemann equations reduces to estimates for a scalar sum of squares operator. The following theorem precisely relates the gain in a subelliptic estimate with the step of the horizontal distribution on the boundary. We omit the technical definition of a subelliptic estimate with gain ϵ. Kohn established the first subelliptic estimate for weakly pseudoconvex domains in $\mathbb{C}^2$ under the condition of finite step, and Greiner established the necessity of this condition. The sharp result in Theorem 13 relies on a sophisticated theory of estimates developed by Rothschild and Stein [RS] based on the sub-Riemannian geometry of nilpotent Lie groups; their theory made it possible to relate the geometry and the estimates in a precise fashion.

Theorem 13. *Let Ω be a smoothly bounded pseudoconvex domain in $\mathbb{C}^2$, and suppose $p \in b\Omega$. The following are equivalent:*

- *There is a subelliptic estimate at p with gain $\epsilon = \frac{1}{2m}$, but for no larger value of ϵ.*
- *For a $(1,0)$ vector field L on $b\Omega$ with $L(p) \neq 0$, we have type$(L,p) = 2m$.*
- *The bundle $\mathcal{H}(M) = T^{10}(M) \oplus T^{01}(M)$ has step $2m$ at p.*
- *The maximum order of contact at p of a complex analytic curve with $b\Omega$ equals $2m$.*

In higher dimensions things are more subtle; algebraic geometry enters the story. Consider the hypersurface M in $\mathbb{C}^3$ defined by

$$\mathrm{Re}(z_3) = |z_1^2 - z_2^3|^2.$$

The step at the origin is 4, and each $(1,0)$ vector field has type either 4 or 6 there. Thus each such vector field is of finite type at the origin. On the other hand, M contains the complex analytic variety V defined by $z_1^2 - z_2^3 = z_3 = 0$, which has a cusp at the origin. (See Figure 5.) At all nonsingular points of V, the step is 2, but there is a $(1, 0)$ vector field (tangent to V) of infinite type. In particular the condition that each $(1, 0)$ vector field be of finite type holds at the origin but fails at nearby points. This failure of openness has consequences in the study of subelliptic estimates. The condition for subellipticity must be an *open* finite-type condition. The answer turns out to be that there is a bound on the order of contact of (possibly singular) complex analytic one-dimensional varieties with M at p. See [Cat], [CD], [D1], [D3], [DK], and [Koh1].

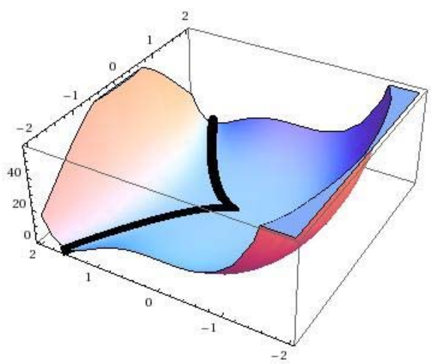

Figure 5. A singular variety in the hypersurface $M = \{z \in \mathbb{C}^3 : \mathrm{Re}(z_3) = |z_1^2 - z_2^3|^2\}$.

Concluding Remarks

Math 499, a required course for first-year graduate students at the University of Illinois, exposes students to a wide variety of mathematical points of view through engaging invitations to the research areas of the speakers. This note arose from discussing the background common to talks we have given in this course but includes material well beyond what we actually presented. We hope that we have provided sufficient insights and references to motivate further study.

Much of the material here is only tangentially related to differential geometry, especially if one appeals to the hilarious definition *"Differential geometry is the study of those properties invariant under change of notation"* sometimes attributed to Calabi. Regardless of the definition, differential geometry encompasses a diverse collection of subjects, including both CR geometry and sub-Riemannian geometry. While both these subjects have their roots in mapping theorems from complex analysis, they have diverged considerably over the past forty years. Perhaps this article is a small step toward their eventual convergence.

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Mathematics of the Gateway Arch

Robert Osserman

One of the oldest problems to have been solved using the calculus of variations was to find the equation for the shape formed by a hanging chain or flexible cord. Most often it is said that Galileo was the first to pose the problem and that he claimed that the resulting curve was a parabola. (See, for example, Goldstine [5], p. 32, footnote 42, and Truesdell [18], pp. 43–44.) Both of these statements are at best half true. Galileo's exact words (in a classic English translation of his book *Two New Sciences* [4], p. 290) are

> …a cord stretched more or less tightly assumes a curve which closely approximates the parabola.…the coincidence is more exact in proportion as the parabola is drawn with less curvature or, so to speak, more stretched; so that using parabolas described with elevations less than 45° the chain fits its parabola almost perfectly.…

Nowhere, to my knowledge, does Galileo address the question of finding the exact shape of the curve formed by the hanging chain. And he is exactly right that for a parabola with elevation less than 45° the curve formed by the chain is an extremely close approximation to the parabola. (See Figure 1.)

This passage follows immediately after an extended discussion during which Galileo presents one of his most famous discoveries: that the path of a projectile in a vacuum under the influence of gravity is a parabola ([4], pp. 244–290). In the course of the discussion, he points out that for a fixed initial velocity, the range will be maximized by an initial elevation of 45°. Finally he addresses the question of how to quickly draw a number of parabolas, leading to the passage cited above.

The reason that one might mistake Galileo's intentions and conclusions is that elsewhere in *Two New Sciences* ([4], p. 149) Galileo addresses the same problem and says simply that the "chain will assume the form of a parabola." The reason that virtually everyone refers to this passage rather than the one in which Galileo spells out his conclusions in far greater detail and complete accuracy is not as clear.

Some of Galileo's earliest experiments with hanging chains apparently date from the summer of 1602, when he was working with Guidobaldo del Monte. Had he thought to pose and try to answer the question of the exact equation for the shape of a hanging chain, he would have encountered two serious obstacles. First, the essential tool needed—the calculus—had not yet been invented. Second, the equation for the curve involves logarithms, which had also not yet been invented.

After the problem was finally explicitly stated, it was treated frequently by a number of authors between 1690 and 1720 and the correct answer derived, although some of the initial reasoning used was decidedly suspect. The curve became known as a *catenary*. Its equation is $y = \cosh x$, the hyperbolic cosine, up to change of coordinates.

One method used to arrive at this result was the calculus of variations, finding the shape of a curve whose center of gravity is the lowest among all curves having a prescribed length and prescribed endpoints.

The Gateway Arch

It was Robert Hooke who in 1675 made the connection between the ideal shape of an arch and that of a hanging chain in an aphorism that says, in abbreviated form, "As hangs the chain, so stands the arch." In other words, the geometry of a standing arch should mirror that of a hanging chain. The horizontal and vertical forces in a hanging chain must add to a force directed along the chain, since any component perpendicular to the chain would cause it to move in that direction to gain equilibrium. Similarly, one wants the combined forces at each point of an arch to add up to a vector tangent to the arch. In both cases, the horizontal component of the force is constant and simply transmitted along the arch or chain, while the vertical forces are mirror images.

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Figure 1. Left: 45° parabola and catenary; Right: 30° parabola and catenary.

At least in part for these reasons, the shape of the Gateway Arch is often described mistakenly as a catenary (when not even more mistakenly as a parabola). In fact, the equation on which the arch is based is

$$y = A \cosh Bx + C, \tag{1}$$

which is a catenary only if $A = 1/B$. For the Gateway Arch, A, B, and C are numerical constants, with A approximately equal to $.69(1/B)$. As a result, those who wish to be more accurate describe the shape of the Arch as a "modified catenary" or, more often, a "weighted catenary". The idea of the weighted catenary is to use either a chain with literal weights attached at various points or else a continuous version, such as a cord of varying density. (Both of those methods, incidentally, were used by Saarinen to find a shape for the Gateway Arch that appealed to him esthetically.)

Some natural questions to ask are:

1. Is there a density function that yields the curve defined in equation (1) above, and if so, what is that function?
2. How descriptive is the term "weighted catenary"? In other words, what are the curves that fall in this category?

In order to answer these questions, we start with some precise definitions.

Definitions. (1) • A *weighted chain* **C** is a pair $(f(x), \rho(s))$ where $f(x)$ is a suitably smooth function on an interval $x_1 \leq x \leq x_2$, s is arclength along the curve $y = f(x)$, and $\rho(s)$ is a positive continuous function of s.

• The function $\rho(s)$ is called the *density function* of **C**.

• A *weighted catenary* is a weighted chain **C** in which

(i) $-\infty < f'(x_1) < 0 < f'(x_2) < \infty$,

(ii) $f''(x)$ is continuous and positive,

(iii) for $x_1 \leq X \leq x_2$,

$$f'(X) = V(X)/H, \tag{2}$$

with

$$V(X) = \int_{x_0}^{X} \rho(s(x)) \frac{ds}{dx}\, dx \tag{3}$$

where $f'(x_0) = 0$, and H is a constant that depends on both f and ρ:

$$H = V(x_2)/f'(x_2). \tag{4}$$

There is a certain amount of arbitrariness in these definitions. The smoothness of the function $f(x)$ is deliberately left vague, since one might well want to allow different classes of functions in different situations. Similarly, one could allow the density function to have point masses or to be even more general.

Nonetheless, conditions (i)–(iii) are natural ones based on the physics underlying the problem. As noted earlier, for equilibrium we want the sum of the horizontal and vertical forces acting on the chain to be a vector tangent to the curve. The horizontal force is constant and simply transmitted along the curve, while the vertical force is due to gravity acting on the part of the chain from the lowest point to the point that we are considering on the curve and is equal to the total weight of that part of the chain.

Note that we do not require that the endpoints be at the same height—in other words, that $f(x_1) = f(x_2)$—although that will be true in all the examples of interest to us. In that case, condition (i) would follow from the physics of the situation. However, condition (i) does guarantee the existence of an interior point x_0 of the interval where $f' = 0$, while condition (ii) implies that there is a unique such point.

Note also that the fact that there is a constant nonzero horizontal component to the force acting on the curve implies that the slope cannot be infinite at the endpoints. For example, a weighted catenary cannot be in the form of a semicircle.

Note finally that if $\rho(s)$ is multiplied by a positive constant, then both H and $V(x)$ are multiplied by the same constant. It follows that if the pair $(f(x), \rho(s))$ defines a weighted catenary, then so does the pair $(f(x), c\rho(s))$ for any positive constant c.

We are now able to state our first result.

Theorem 1. *If $f(x)$ satisfies conditions (i) and (ii) above, then there is a function $\rho(s)$, unique up to a multiplicative constant, such that the pair $(f(x), \rho(s))$ is a weighted catenary. Namely*

$$\rho(s(x)) = \frac{Hf''(x)}{\sqrt{1 + f'(x)^2}}, \tag{5}$$

where H is an arbitrary positive constant. The value of H is then given by

$$H = \frac{W}{|f'(x_1)| + |f'(x_2)|}, \tag{6}$$

with

$$W = \int_{x_1}^{x_2} \rho(s(x)) \frac{ds}{dx}\, dx$$

equal to the total weight of the chain.

Proof. Suppose first that there exists a function $\rho(s)$ such that the pair $(f(x), \rho(s))$ is a weighted catenary. Then by equations (2) and (3), for $X \geq x_0$,

$$Hf''(X) = V'(X) = \rho(s(X))(ds/dx)|_{x=X}.$$

Since $ds/dx = \sqrt(1 + f'(x)^2)$, it follows that $\rho(s)$ must have the form given in equation (5). For $X < x_0$,

$$V(X) = -\int_X^{x_0} \rho(s(x))(ds/dx)dx$$

and

$$Hf''(X) = V'(X) = \rho(s(X))(ds/dx)|_{x=X}$$

as before. Finally, the total weight W is

$$W = \int_{x_1}^{x_2} \rho(s)ds = \int_{x_1}^{x_0} + \int_{x_0}^{x_2} = V(x_2) - V(x_1).$$

But $V(x_2) = Hf'(x_2) > 0$, and $V(x_1) = Hf'(x_1) < 0$, so that H takes the form indicated.

This proves that the density function ρ is uniquely determined by the function $f(x)$ up to the choice of the constant H.

For the existence of a suitable density function $\rho(s)$, given $f(x)$, define $\rho(s)$ by equation (5). Then

$$\begin{aligned} V(X) &= \int_{x_0}^{X} \rho(s(x)) \frac{ds}{dx}\, dx = \int_{x_0}^{X} Hf''(x)\, dx \\ &= Hf'(X) - Hf'(x_0) = Hf'(X), \end{aligned}$$

so that the pair $(f(x), \rho(s))$ form a weighted catenary, and then the constant H must have the form indicated in equation (6). □

Example 1 (catenary)**.**

$$f(x) = \frac{1}{B}\cosh Bx + C.$$

Then

$$f'(x) = \sinh Bx, \quad \sqrt{1 + f'(x)^2} = \cosh Bx$$

and $f''(x) = B\cosh Bx$, so

$$\rho(s(x)) = HB, \quad \text{constant.}$$

Comment: This is a kind of inverse to the original catenary problem of asking for the shape taken by a uniformly weighted hanging chain. It tells us that *only* for a uniform chain will the resulting curve be a catenary.

Example 2 (parabola)**.**

$$f(x) = ax^2 + b, \quad f'(x) = 2ax, \quad f''(x) = 2a.$$

Then

$$\rho(s(x)) = 2aH \Big/ \frac{ds}{dx}$$

and

$$\int \rho(s)ds = \int \rho(s(x))\frac{ds}{dx}dx = 2aH \int dx,$$

and

$$\int_{s(x_1)}^{s(x_2)} \rho(s)ds = 2aH(x_2 - x_1).$$

Comment: This is in a sense the answer to Galileo's original question of finding a mechanical method for drawing a parabola. It tells us exactly how we have to weight a chain so that it will hang in the form of a parabola. The key is that the weight distribution has to be uniform in the horizontal direction. For this reason, it is usually stated that the cables on a suspension bridge will hang in the shape of a parabola, since the weight of the cables themselves, together with that of the vertical support cables holding up the roadway, is small compared to the weight of the roadway, which has (in essence) a uniform horizontal weight distribution.

A curious by-product of the equation for the density ρ in the case of a parabola is that, since $ds/dx \geq 1$, with equality only where $f'(x) = 0$, it follows that $\rho(s(x))$ is maximum at the vertex and then decreases monotonically as $|f'(x)|$ increases. In other words, the chain has to be weighted the most near the vertex and then decrease as the steepness of the curve increases. As a result, if Saarinen had decided that he found a parabolic arch most pleasing esthetically, he would have been faced with the paradox that in order to have the line of thrust be everywhere directed along the arch, the arch would have to be thickest at the top and taper *down* toward the bottom, which would be both ungainly esthetically and potentially disastrous structurally.

Example 3 (circular arc)**.**

$$x = R\cos\theta, \quad y = R\sin\theta, \quad \pi < \theta_1 \leq \theta \leq \theta_2 < 2\pi.$$

Then

$$f'(x) = \frac{dy}{d\theta}\Big/\frac{dx}{d\theta} = -\cot\theta, \quad f''(x) = -\frac{1}{R}\csc^3\theta,$$

while

$$s = R\theta, \quad ds/dx = -\csc\theta > 0.$$

So

$$\rho(s(x)) = \frac{H}{R}\csc^2\theta = HR/y^2.$$

Comment: As noted earlier, a full semicircle cannot be a weighted catenary. We see explicitly that the density function would tend to infinity. However, any circular arc that is short of a semicircle at both ends can be realized (at least in theory) as a weighted catenary. The weighting is simply

$$\rho(s) = \frac{H}{R\cos^2(s/R)}, \quad \pi R < s_1 \leq s = R\theta \leq s_2 < 2\pi R.$$

Although this example may seem a typical "textbook example", of no practical interest but merely a case that can be worked out explicitly, the opposite is true. A monumental "Gateway Arch" had been proposed as an entryway to a planned 1942 International Exposition in Rome, but it was derailed by the war. The arch was to be in the form of a semicircle, and the question of exactly how much it should be tapered was potentially critical. (See [12] for more on this subject.)

Note that this theorem answers the first part of Question 1 above about the existence of a density function yielding the particular "weighted catenary" shape of the Gateway Arch. It also answers Question 2 concerning the term "weighted catenary" itself, which turns out to have essentially no content beyond "convex curve". Before turning to the second part of Question 1 regarding the exact form of the density function that yields the curve underlying the Gateway Arch, we prove a strong form of the converse to the theorem.

Converse of Theorem 1. *Let* **C** *be a curve with endpoints* $P_1 = (x_1, y_1, 0)$ *and* $P_2 = (x_2, y_2, 0)$*, where* $x_1 < x_2$*. Assume that* **C** *is parameterized by arc length* s *in the form*

$$\mathbf{X}(s) = (x(s), y(s), z(s)), \quad P_1 = \mathbf{X}(s_1), \quad P_2 = \mathbf{X}(s_2).$$

Let $\rho(s) > 0$ *be a continuous function along* **C***, taken as a density function. Assume gravity acts in the negative* y*-direction, and the pair* $(\mathbf{C}, \rho(s))$ *is in equilibrium. Then:*

1. **C** *lies entirely in the* x, y*-plane.*
2. **C** *is in the form of a graph* $y = f(x)$*,* $x_1 \le x \le x_2$*.*
3. $f'(x)$ *is finite and monotone increasing for* $x_1 \le x \le x_2$*.*

The idea underlying the proof is that if we know that **C** is a graph and $(\mathbf{C}, \rho(s))$ is in equilibrium, so that equation (2) above holds, where $V(X)$ is defined by (3), then

$$-f'(x_1) = \frac{1}{H}\int_{x_1}^{x_0} \rho(s(x))(ds/dx)dx < \infty,$$

$$f'(x_2) = \frac{1}{H}\int_{x_0}^{x_2} \rho(s(x))(ds/dx)dx < \infty,$$

and for $x_1 \le x_3 < x_4 \le x_2$,

$$f'(x_4) - f'(x_3) = \frac{1}{H}\int_{x_3}^{x_4} \rho(s(x))(ds/dx)dx > 0.$$

Hence condition 3 in the statement of the converse must hold.

In terms of the representation above for **C** as $\mathbf{X}(s)$, the unit tangent vector to **C** will be given by $\mathbf{T} = d\mathbf{X}/ds$. We denote the tension at the point $\mathbf{X}(s)$ by τ, and for the case considered here, the tension vector will be

$$\tau\mathbf{T} = (H, V, W), \tag{7}$$

where H is constant and $W = 0$, while V is given by (3). Then

(8)
$$\frac{d}{ds}\tau\mathbf{T} = \left(0, \frac{dV}{ds}, 0\right) = \left(0, \frac{dV}{dx}\Big/\frac{ds}{dx}, 0\right) = (0, \rho(s), 0)$$

by (3), or

$$\frac{d}{ds}(\tau\mathbf{T}) = \rho(s)\mathbf{j}, \tag{9}$$

where **j** is the unit vector directed along the positive y-axis.

The proof of the above converse depends on the fact that equation (9) is the equation of equilibrium for an arbitrary curve joining the points P_1, P_2 in the x, y-plane, with no special assumptions about its shape. (See [2], Chapter III, sections 1–3, for a detailed discussion of these questions in maximum generality. I would like to thank Joe Keller for providing this reference, as well as further helpful information. Note that in contrast to [2], we take the density ρ to mean weight, rather than mass, per unit length, so that the acceleration of gravity is absorbed into it.)

Proof of the converse. Let **C** be defined by

$$\mathbf{X}(s) = (x(s), y(s), z(s)), \quad s_1 \le s \le s_2,$$

where

$$\mathbf{X}(s_1) = (x_1, y_1, 0), \quad \mathbf{X}(s_2) = (x_2, y_2, 0), \quad x_1 < x_2,$$

and let $\rho(s) > 0$ be a continuous density function along **C**. Then the pair $(\mathbf{C}, \rho(s))$ will be in equilibrium if and only if (9) holds; that is,

$$\frac{d}{ds}\left(\tau(s)\frac{dx}{ds}\right) = 0, \quad \frac{d}{ds}\left(\tau(s)\frac{dy}{ds}\right) = \rho(s), \quad \frac{d}{ds}\left(\tau(s)\frac{dz}{ds}\right) = 0. \tag{10}$$

Hence

$$\tau(s)\frac{dx}{ds}(s) = c, \tag{11}$$

$$\tau(s)\frac{dy}{ds}(s) = \tau(s_1)\frac{dy}{ds}(s_1) + \int_{s_1}^{s} \rho(\sigma)d\sigma, \tag{12}$$

$$\tau(s)\frac{dz}{ds}(s) = d, \tag{13}$$

where c and d are constants. We claim first that **C** must lie completely in the x, y-plane. If not, there must be a value $s_0, s_1 < s_0 < s_2$, where $|z(s)|$ attains its maximum $|z(s_0)| > 0$. Then $(dz/ds)(s_0) = 0$, and the constant d in (13) must vanish. But if $z(s) \not\equiv 0$, there must be some value s_3 such that dz/ds is nonzero at s_3 and hence in an interval around s_3. But then, since $d = 0$, it follows from (13) that $\tau(s) \equiv 0$ on that interval. But that contradicts equation (9), since $\rho(s) > 0$. Hence we must have $z(s) \equiv 0$, and the curve lies entirely in the x, y-plane.

Exactly analogous reasoning using equation (11) shows that there cannot be a value s_4 where $dx/ds = 0$. In fact, since $x(s_2) = x_2 > x_1 = x(s_1)$, there must be a point where $dx/ds > 0$. It follows that $dx/ds > 0$ on the whole interval $[s_1, s_2]$, and hence x is a strictly monotone increasing function of s on that interval. Thus we have a monotone increasing inverse $s(x)$, and we can set

$$y(s(x)) = f(x), \quad x_1 \le x \le x_2,$$

defining **C** as a graph. It follows from equations (11) and (12) that

(14)
$$f'(x) = \frac{dy}{ds}\Big/\frac{dx}{ds} = \frac{1}{c}\tau(s_1)\frac{dy}{ds}(s_1) + \frac{1}{c}\int_{s_1}^{s}\rho(\sigma)d\sigma.$$

Hence $f'(x)$ is a monotone increasing function of s and therefore of x. This completes the proof of the converse. □

Note that it follows from the above that

$$f''(x) = \frac{d}{ds}f'(x)\Big/\frac{dx}{ds} = \frac{1}{c}\rho(s(x))\frac{ds}{dx} > 0. \tag{15}$$

Since c is the horizontal component of the tension vector, it is the quantity we denoted earlier by H, and equation (15) is the same as equation (5) above.

We now come to our principal example of the theorem.

Definition 1. A *flattened catenary* is a curve of the form $y = f(x) = A\cosh Bx + C$, or

$$y = Dg(x) + C, \quad D = AB,$$

where $0 < D < 1$, and

$$g(x) = \frac{1}{B}\cosh Bx$$

is a catenary.

Example 4 (flattened catenary)**.** We examine this case in the following theorem.

Theorem 2. *Let* **C** *be a weighted catenary defined by the pair* $(f(x), \rho(s))$*, and let* $p(x) = \rho(s(x))(ds/dx)$*. Assume that coordinates are chosen so that* $f'(0) = 0$*. Then* $p(x)$ *will be of the form*

$$p(x) = ay + b = af(x) + b$$

for some constants $a > 0$ *and* b *if and only* $f(x)$ *is a flattened catenary:*

$$f(x) = A\cosh Bx + C,$$

for constants A, B, C.

Proof. We have for all X in $[x_1, x_2]$ that $f'(X) = V(X)/H$, where

$$V(X) = \int_{x_0}^{X} p(x)dx, \ H = \int_{x_1}^{x_2}\frac{p(x)}{|f'(x_1)| + |f'(x_2)|}dx.$$

Then

$$f''(X) = V'(X)/H = p(X)/H.$$

Suppose first that

$$y = f(x) = A\cosh Bx + C. \tag{16}$$

Then $f'(x) = AB\sinh Bx$ and

$$f''(x) = AB^2\cosh Bx = B^2(f(x) - C).$$

Then

$$p(x) = Hf''(x) = af(x) + b, \tag{17}$$

where

$$a = HB^2 > 0, \quad b = -HB^2C. \tag{18}$$

Conversely, suppose that $p(x) = af(x) + b$. Then $f''(x) = (af(x) + b)/H$. Let

$$g(x) = f(x) + b/a.$$

Then $g'(0) = f'(0) = 0$, and

$$g''(x) = f''(x) = ag(x)/H.$$

It follows that $g(x) = A\cosh Bx$ where $B^2 = a/H$. Hence

$$f(x) = g(x) - b/a = A\cosh Bx + C,$$

where

$$A = f(0) + b/a, \quad B = \sqrt{a/H}, \quad C = -b/a. \tag{19}$$

□

Historical Note

The flattened catenary was studied in the nineteenth century by a number of authors in France, England, Ireland, and Scotland. The first may have been Villarceau [19]; see Heyman [6], p. 48. Others include W. J. M. Rankine [13] and A. M. Howe [9]. (I thank William Thayer for these latter references and for alerting me to the connection between the equation for the Gateway Arch and the nineteenth-century application discussed here.) In those works, the flattened catenary is often referred to as a "transformed catenary", and the term "two-nosed catenary" is used for a highly flattened catenary, for reasons explained below. The context in which it arises is in answer to the question: What is the ideal shape of an arch that supports a horizontal roadway held up by evenly packed dirt on top of the arch? The vertical load applied to each point on the arch is then proportional to the distance from that point up to the level of the roadway. If we write the equation of the arch as $y = f(x)$, where y is the distance from the roadway down to a given point on the arch, then the total vertical thrust acting on the part of the arch from the apex to an arbitrary point of the arch will correspond to the weight of a hanging chain from the minimum point to a given point, and it will be of the form $\int p(x)dx$, where $p(x)$ is a linear function of y. The same reasoning as in the above theorem then shows that $f(x)$ should be of the form (16): a flattened catenary. (See, for example, Heyman [6], pp. 48-49.)

We now come to the Gateway Arch itself. At first sight it would appear to be a very different situation from that of an arch supporting a bridge, since it supports nothing except its own weight. On closer look, however, the fact that the Arch is tapered means that the weight supported at each level is not determined just by the length of the arch above it but by an integral with respect to arclength of a function exactly analogous to the density function for a weighted chain. The physical Gateway Arch is reasonably well modeled as one of a class of geometric shapes that are of interest in their own right.

Tube-like Domains

Two chance meetings in 1939 led to one of the major advances in differential geometry, the intrinsic proof by Chern of an n-dimensional Gauss–Bonnet theorem. The first occurred when mathematician Hermann Weyl attended a lecture by statistician Harold Hotelling in which Hotelling described a new result of his and posed an open question. Hotelling had been led by a statistical problem to pose the question: Given $r > 0$, what is the probability that a point chosen at random on the n-sphere would lie within a distance r of a given curve **C**? The answer amounts to computing the volume of the domain consisting of all points whose distance to **C** is less than r and comparing that to the total volume of the sphere. Hotelling was able to answer the question in both Euclidean n-space and the n-sphere for r sufficiently small [8]. The answer in Euclidean space was that the volume is equal to the length of **C** times the volume of an $(n-1)$-ball of radius r, with an analogous result for the n-sphere. The answer was perhaps a surprise, since one might think that the volume of the "tube" around the curve **C** would depend somehow on the twists and turns in the curve, and not simply on its total length.

What Hotelling wanted to know, and was unable to derive, was a formula for the case that the curve **C** was replaced by a surface of two or more dimensions. Hermann Weyl [22] provided the answer in spectacular fashion: for a compact smooth submanifold **S** of Euclidean space, the volume of the domain consisting of all points lying within a fixed distance r of **S** is given, for r sufficiently small, by a polynomial in r whose coefficients are *intrinsic* quantities associated with **S** , and do not depend on the way that **S** is immersed in space. Most strikingly, the top coefficient does not even depend on the geometry of **S**, but only its topology; it is just a constant times the Euler characteristic χ of **S**.

The other chance meeting in 1939 took place between André Weil and Lars Ahlfors in Finland. Weil first learned about the classical Gauss–Bonnet theorem from Ahlfors, who expressed an interest in having a generalization to higher dimensions in order to develop a higher-dimensional Nevanlinna theory of value distribution ([20], p. 558, [21]. When Weil came to America several years later he met Carl Allendoerfer and learned that both Allendoerfer and Werner Fenchel had independently proved a Gauss–Bonnet theorem for manifolds embedded in Euclidean space, using Weyl's formulas. Remembering Ahlfors's desire for a general Gauss–Bonnet theorem, Weil was able, together with Allendoerfer, to prove the Gauss–Bonnet theorem for all Riemannian manifolds ([1], [20], p. 299). Their proof went via Weyl's formulas together with local embedding theorems. Shortly after, using the appropriate form of the integrand from these earlier approaches, Chern was able to provide an intrinsic proof of the Gauss–Bonnet theorem without the detour through embeddings.

Our interest here is in a generalization of Hotelling's result in a different direction. We go back to the case of a curve **C** in Euclidean space, but are interested in much more general domains.

Notation 1. A curve **C** in $\mathbb{R}^n$ will be denoted by

$$X(s) = (x_1(s), \dots, x_n(s)), 0 \le s \le L,$$

s = arc length parameter,

$$e_1(s) = dX/ds$$

the unit tangent vector,

$$e_1(s), e_2(s), \dots, e_n(s)$$

a positively oriented orthonormal frame field along **C**.

Definition 2. A *tube-like domain* T with centerline **C** is the image under a diffeomorphism F of a domain D in $\mathbb{R}^n$, with coordinates $u_1, \dots, u_n$, such that $0 < u_1 < L$, and for any t, $0 < t < L$, the intersection D_t of D with the hyperplane $u_1 = t$ contains the point $(t, 0, \dots, 0)$. The map F satisfies $F(u_1, u_2, \dots, u_n) = X(u_1) + u_2 e_2(u_1) + \cdots + u_n e_n(u_1)$, with $u_1 = s$ = arc length parameter along **C**.

For $0 < s < L$, let T_s be the intersection of T with the normal hyperplane to **C** at the point $X(s)$. Then $F : D_s \to T_s$ is a linear transformation mapping D_s isometrically to T_s.

Theorem 3. *Let T be a tube-like domain in $\mathbb{R}^n$ with centerline* **C**. *Then the volume V of T is given by*

$$(20) \quad V = \int_0^L V(T_s) ds - \int_0^L \left[\int_{D_s} \vec{k}(s) \cdot (F(s, u_2, \dots, u_n) - X(s)) du_2 \dots du_n \right] ds,$$

where $\vec{k}(s) = d^2X/ds^2$ is the curvature vector, and for each fixed value of s, the vector

$$F(s, u_2, \dots, u_n) - X(s) = u_2 e_2(s) + \cdots + u_n e_n(s)$$

traces out the normal section T_s of T as the point $(s, u_2, \dots, u_n)$ ranges over the parameter domain D_s.

Proof. Let $J = \det dF$ be the Jacobian of the map F. If $Y = (y_1, \dots, y_n) = F(u_1, \dots, u_n)$, then

$$J = \frac{\partial(y_1, \dots, y_n)}{\partial(u_1, \dots, u_n)}, \quad V(T) = \int_D J\, du_1 du_2 \dots du_n = \int_0^L \left[\int_{D_s} J\, du_2 \dots du_n \right] ds.$$

Now write

$$Y = X(u_1) + u_2 e_2 + \cdots + u_n e_n,$$

with $e_j = e_j(u_1),\ 0 \le u_1 \le L$. Then

$$\frac{\partial Y}{\partial u_1} = e_1 + u_2\frac{de_2}{du_1} + \cdots + u_n\frac{de_n}{du_1} \text{ and } \frac{\partial Y}{\partial u_i} = e_i, \quad i = 2, \dots, n,$$

and

$$\frac{\partial Y}{\partial u_1}\wedge\cdots\wedge\frac{\partial Y}{\partial u_n} = \frac{\partial Y}{\partial u_1}\wedge e_2\wedge\cdots\wedge e_n = Je_1\wedge\cdots\wedge e_n.$$

Write

$$\frac{\partial e_i}{\partial u_1} = \sum_{j=1}^{n} a_{ij}e_j = a_{i1}e_1 + \cdots,$$

where the coefficients are given by

$$a_{i1} = \frac{de_i}{du_1}\cdot e_1 = -\frac{de_1}{du_1}\cdot e_i = -\vec{k}\cdot e_i,$$

since the curvature vector $\vec{k}(s) = \dfrac{d^2X}{ds^2}$ equals $\dfrac{de_1}{ds}$. So

$$\sum_{i=2}^{n} a_{i1}u_i = -\vec{k}\cdot\sum_{i=2}^{n} u_ie_i = -\vec{k}(u_1)\cdot(Y - X),$$

where $a_{ij} = a_{ij}(u_1)$, $X = X(u_1)$, $Y = Y(u_1, \dots, u_n)$.

Finally,

$$\frac{\partial Y}{\partial u_1} = \Big(1 + \sum_{i=2}^{n} a_{i1}u_i\Big)e_1 + \cdots,$$

where the omitted terms involve the vectors $e_2, \dots, e_n$, and will vanish after taking the wedge product with e_2 through e_n. Hence,

$$J = 1 + \sum_{i=2}^{n} a_{i1}u_i = 1 - \vec{k}(u_1)\cdot(Y - X(u_1)),$$

as asserted. □

Note. Different versions of this theorem have been known at least since 1978, when Michael Raugh [14] studied the case $n = 3$ in the context of the growth and structure of tendrils. Another discussion of the case $n = 3$ appears in Chapter 3 (*Cálculo de várias variáveis*) of [3], pp. 202–210. Both of those discussions assume the existence of a Frenet frame field for the centerline of the domain. A recent paper of Raugh [15] also treats the n-dimensional case.

Note. For the proof to be valid, one needs the Jacobian to be positive. If it were to change sign, then there would be cancellations, and the integral would not represent the actual volume. Since our definition of a tube-like domain includes the assumption that the defining map is a diffeomorphism, the Jacobian cannot change sign. However, if one starts with a domain D and tries to represent it as a tube domain, then one must check the positivity of the Jacobian. We shall return to this point later on.

Corollary 1. *If the point X_s is the centroid of T_s for each $s \in [0, L]$, then*

$$V(T) = \int_0^L V(T_s)\,ds. \tag{21}$$

Note that it suffices to consider points where $\vec{k}(s) \ne 0$.

Proof. For each value of s, the inner integral in the second term of the formula for $V(T)$ is a sum of terms consisting of integrals over D_s of terms of the form c_ju_j, where c_j is a constant. But if $X(s)$ is the centroid of T_s, then the origin is the centroid of D_s, and each of those integrals must vanish. □

Definition 3. Under the hypotheses of Corollary 1, the centerline **C** is called a *centroid curve* of T.

A given domain may have more than one centroid curve; for example, a Euclidean ball can be viewed as a tube-like domain with respect to any diameter as centerline, and that diameter will be a centroid curve for the ball.

Corollary 2 (Hotelling's theorem)**.** *If T is a tube domain in $\mathbb{R}^n$ of radius r around a curve* **C** *of length L, then*

$$V(T) = \omega_{n-1}r^{n-1}L,$$

where ω_k is the volume of the ball of radius r in $\mathbb{R}^k$.

Proof. In this case D is a cylinder of radius r and height L. For $0 < s < L$, the normal section T_s of T is an $(n-1)$-ball of radius r centered at the point $X(s)$ of **C**. □

A number of statisticians returned to questions related to Hotelling's theorem after a lapse of many years. See the paper [11] and further references given there.

Special Cases

<u>$n = 2$.</u> Then e_2 is uniquely determined by $e_1 = dX/ds$. The orthogonal section T_s is an interval I_s, and the second term vanishes if $X(s)$ is the midpoint of I_s at all points where the curvature is nonzero.

Note. For $n = 2$, $\vec{k}(s) = \kappa(s)e_2(s)$, where the curvature κ can be positive, negative, or zero.

<u>$n = 3$.</u> If $\vec{k}(s) \ne 0$ at a point, then the same holds in an interval around the point, and we may define $e_2 = \vec{k}(s)/\kappa(s)$, where $\kappa(s) = |\vec{k}(s)|$ is the curvature. Then $e_2(s)$ is a smooth unit vector field on the interval, and e_1, e_2, together determine e_3. Since $de_1/ds = \vec{k} = \kappa e_2$, we have $a_{21} = -\kappa$, $a_{31} = 0$, and $J = 1 - \kappa(s)u_2$. As already noted, this equation also holds whenever $\kappa = 0$. Therefore, for $n = 3$,

$$V(T) = \int_0^L \left[\int_{T_s} (1 - \kappa(s)u_2)\,du_2\,du_3\right] ds = \int_0^L V(T_s)\,ds - \int_0^L \kappa(s)M_s\,ds,$$

where M_s is the moment of T_s about the line through $X(s)$ in the e_3 (binormal) direction. It follows that for (21) to hold, one does not need $X(s)$ to be the centroid of the orthogonal section, but only that the moment of that section be zero with respect to the line through $X(s)$ in the binormal direction, at all points where $\kappa \neq 0$. (See Raugh [15].)

An analogous statement holds for arbitrary n.

Another formulation is

$$\int_{D_s} J\, du_2 \ldots du_n = \int_{D_s} (1 + \sum_{i=2}^{n} a_{i1} u_i)\, du_2 \ldots du_n$$
$$= V(T_s) + \sum_{i=2}^{n} a_{i1} M_i,$$

where M_i is the i-th moment of D_s, and $a_{i1} = a_{i1}(s)$.

Gateway Arch Specifications

The *shape of the Gateway Arch* is a polyhedral surface, the piecewise linear approximation by quadrilaterals to the surface S of the theoretical Gateway Arch, with two of the sides of each quadrilateral lying on designated straight-line segments of S.

Note. The surface just described is an abstract mathematical representation of the surface of the physical Gateway Arch, that surface consisting of the exterior of a set of stainless steel plates subject to a variety of distortions. Among those are the (uneven) expansion and contraction with changes of temperature, deflection due to wind forces, and deformation under the moving weight distribution of the internal transportation system carrying passengers to the top.

The *theoretical Gateway Arch* is a tube-like domain T whose centerline **C** is the reflection in a horizontal plane of a flattened catenary. The curve **C** is given by the equation

$$y = 693.8597 - A \cosh Bx, \tag{22}$$

where A and B are the numerical constants

$$A = 68.7672, B = .0100333, \tag{23}$$

$$-299.2239 \leq x \leq 299.2239, \tag{24}$$

and x, y represent distances measured in feet. We may also write the equation of **C** as

$$y = 625.0925 - Y \tag{25}$$

where

$$Y = A \cosh Bx + C = A(\cosh Bx - 1) \tag{26}$$

represents the vertical distance down from the vertex of **C**.

The orthogonal section $T(x,y)$ at the point (x,y) of **C** is in the form of an equilateral triangle of area $Q(x,y)$, where the sides of $T(x,y)$ when $x = 0$ (at the vertex of **C**) have length 17 feet and when $y = 0$ (at the base of the arch) have length 54 feet. The area $Q(x,y)$ is defined by interpolating linearly in Y between the corresponding areas so defined:

$$Q(x,y) = Q_v + (Q_b - Q_v)Y/Y_b,$$

where Q_v is the cross-sectional area at the vertex, and Q_b is the corresponding area at the base. Since the area Q of an equilateral triangle is equal to

$$Q = \frac{\sqrt{3}}{4} d^2,$$

where d is the length of a side, and since $d = 17$ at the vertex v and $d = 54$ at the base, we find

$$Q_v = 125.1406, \quad Q_b = 1262.6651,$$

while $Y_b = Y|_{y=0} = 625.0925$. It follows that

$$Q(x,y) = 125.1406 + 1.81977Y$$
$$= 1262.6651 - 1.81977y.$$

Using the relation above between the area Q and the side d of an equilateral triangle, we obtain the exact dimensions of the cross-sections at each height y. It remains to specify that the positions of the triangles in the planes orthogonal to the curve **C** are symmetric with respect to the x, y-plane, with one vertex lying in that plane and interior to the curve **C**, and that the centroid of each of those triangles lies on **C**, so that **C** is a centroid curve of the Arch.

It follows from these specifications that the surface of the theoretical Gateway Arch is the union of three surfaces: a pair of ruled surfaces that are symmetric images with respect to the x, y-plane and meet in a curve lying in the x, y-plane, called the *intrados* of the Arch, together with a cylindrical surface orthogonal to that plane: the *extrados.* The maximum height of the extrados occurs at the point directly above the vertex of **C** where $x = 0$ and $y = y_v = 625.0925$. At this point the orthogonal plane is vertical, and the distance from the vertex of **C** to the base of the triangle is one third of the length of the median, or $17\sqrt{3}/6 = 4.90748$. Adding this to y_v gives for the maximum height of the Arch $h = 629.99998$.

The width of the Arch is a bit more subtle. We take the points where $y = 0$ on the centroid curve **C** to represent ground level. At those points, $x = 299.2239$ and -299.2239 so that the width of the curve at ground level is 598.4478. The equilateral triangle representing the cross-section at those points has side 54, and hence the distance to the outer curve of the Arch is

$$54\sqrt{3}/6 = 9\sqrt{3} = 15.58846.$$

Adding this on each side to the width of **C** at ground level gives a total width for the Arch of 629.6247. However, since the curve **C**, although steep, is not vertical at ground level, the cross-section of the Arch is not horizontal, and the actual outer width is slightly larger. However, one sees that the dimensions of the centroid curve together with the size and shape of the cross-sections produce an arch that for all practical purposes has exactly the same total height as width. It may be worth noting, however, since it is sometimes a source of confusion, that the centroid curve is distinctly taller than wide, and the same is even more true of the inner curve of the Arch, whose height is 615.3 feet, and width is 536.1. For more on this and related matters, see [12].

Curvature Computations

Let $y = f(x) = A \cosh Bx = D(\frac{1}{B} \cosh Bx)$ define a flattened catenary, where A, B are positive constants, and $D = AB$. Then $f''(x) = B^2 y > 0$ so that f is convex and the curvature $\kappa > 0$. Specifically,

$$\kappa = \frac{B^2 y}{(B^2 y^2 + 1 - D^2)^{3/2}}.$$

For a catenary, $D = 1$ and $\kappa = 1/By^2$, with a maximum of B at the vertex $(0, 1/B)$ and decreasing monotonically to zero as $y \to \infty$.

In general, let $g(y) = (B^2/\kappa)^2 = B^4 R^2$, $R = 1/\kappa$. Then $dR/dy \geq 0 \Leftrightarrow g'(y) \geq 0$. But one finds

$$g'(y) = \frac{2}{y^3}(B^2 y^2 + 1 - D^2)^2 (2B^2 y^2 - 1 + D^2)$$

and since $y \geq A$, $B^2 y^2 + 1 - D^2 \geq 1$, and

$$g'(y) > 0 \Leftrightarrow 2B^2 y^2 > 1 - D^2.$$

Case 1: $D > 1$. An "elongated" catenary, obtained by stretching a catenary uniformly in the vertical direction. Then the right-hand side is negative, $g'(y) > 0$ along the whole curve, so that R has a minimum and κ a maximum at the vertex, where $x = 0, y = D/B$, and $\kappa = BD$.

Case 2: $D = 1$. A catenary, for which, as we have seen, R has a minimum value of $1/B$ at the vertex and $R \to \infty$ as $y \to \infty$.

Case 3: $0 < D < 1$. Then there are two possibilities.

Case 3a: $D^2 \geq 1/3$. Since y takes its minimum value A at the vertex, we have

$$2B^2 y^2 + D^2 \geq 2B^2 A^2 + D^2 = 3D^2 \geq 1$$

so that $g'(y) \geq 0$ along the whole curve, and as in cases 1 and 2, the curvature has its maximum value of BD at the vertex.

Case 3b: $D^2 < 1/3$. Then $1 - D^2 > 2D^2 = 2B^2 A^2$, and we have $g'(y) < 0$ in the interval

$$y|_{x=0} = A \leq y < \left(\frac{1 - D^2}{2B^2}\right)^{1/2}.$$

In this case, the vertex of the curve will be a local *minimum* of the curvature, which will increase monotonically until y reaches the value indicated on the right, where the curvature will reach its maximum. The curve is in this case sometimes referred to as a *two-nosed catenary.*

Figure 2 shows a two-nosed catenary for which $D = 1/3$, obtained by stretching a catenary uniformly in the horizontal direction by a factor of 3, which is equivalent, up to a similarity, to compressing it vertically by a factor of 3.

It may be worth observing that when illustrating geometric graphs, one often sees different scales used on the two axes. For a parabola, the resulting curve will still be a parabola, but for a catenary, the result will be a flattened or elongated catenary. As the figure, as well as the above calculation, makes clear, the shape may be very different geometrically from a true catenary. Such differences are obvious when different scales in the horizontal and vertical directions turn a circle into an ellipse, but they are often overlooked for noncompact curves such as the catenary.

Finally, we note that for the centroid curve of the Gateway Arch, we have $D = AB = .69$, well above the critical value $D = \sqrt{3}/3 = .577\ldots$, so that the curvature will take its maximum value at the vertex, where it is equal to $DB = .0069$ so that the radius of curvature at that point is $R = 145$ feet, and it grows monotonically from there. Since all points of the cross-sectional equilateral triangles that define the Gateway Arch are less than 54 feet (and in fact, $< 54\sqrt{3}/3 < 32$ feet) from the centroid curve, it follows that the Jacobians entering into the computations leading up to equation (21) are all positive, where the Gateway Arch is considered as a tube-like domain.

Note. This paper is an expanded version of the Roever Lecture given by the author at Washington University in St. Louis on March 24, 2009. William H. Roever was born in St. Louis, and after receiving his Ph.D. at Harvard University and teaching at MIT, he returned to Washington University, where he served as chairman of the mathematics department. He wrote two papers on the curves of light formed by reflection under particular circumstances [16], [17]. He would undoubtedly have been fascinated by the light curve that forms on the Gateway Arch when the sun is at a certain angle.

Final Note

Although the essence of Hooke's dictum relating the standing arch to the hanging chain seems clear, there are subtle issues that arise, some involving the fact that a standing arch is a three-dimensional body, whereas the hanging chain is modeled just as a curve. In fact, a careful analysis of the stability of an arch and its relation to the geometry of the arch is a subject about which whole books could be (and have been) written. (See, for example, [6].)

Here is an example of a seemingly paradoxical fact that does not seem to have been mentioned anywhere. As we noted at the outset, the fact that the shape of a hanging chain is a catenary is often derived by using the calculus of variations, finding the curve of given length that has the lowest center of gravity. It follows then, according to Hooke, that the ideal shape of an arch would be an upside-down

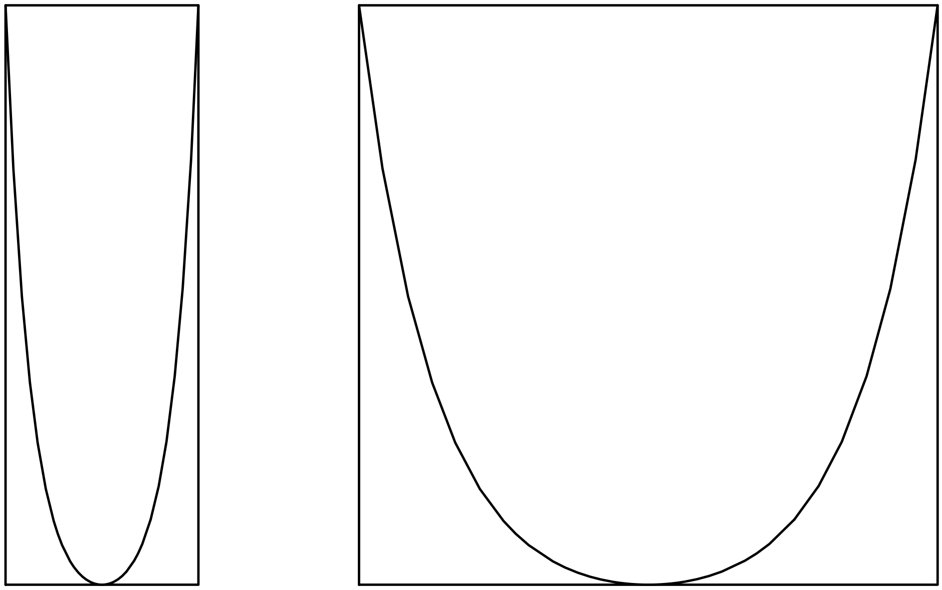

Figure 2. Left: catenary; right: 2-nosed catenary.

catenary, which would have the *highest* center of gravity among all arches of the same length. But that would appear to make it maximally unstable.

Some of these and related issues are discussed in the paper [12]. We hope to return to the subject for a fuller discussion at a later date.

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Model Theory and Complex Geometry

Rahim Moosa

Model theory is a branch of mathematical logic whose techniques have proven to be useful in several disciplines, including algebra, algebraic geometry, and number theory. The last fifteen years have also seen the application of model theory to *bimeromorphic geometry*, which is the study of compact complex manifolds up to bimeromorphic equivalence. In this article I will try to explain why logic should have anything to say about compact complex manifolds. My primary focus will be on the results in bimeromorphic geometry obtained by model-theoretic methods and the questions about compact complex manifolds that model theory poses.

Structures and Definable Sets

Besides being a discipline in its own right, model theory is also a *way* of doing mathematics. Given a mathematical object, such as a ring or a manifold, we begin by stating explicitly what structure on that object we wish to investigate. We then study those sets that can be described using formal expressions that refer only to the declared structure and whose syntax is dictated by first-order logic. Let me give a few details.

A *structure* consists of an underlying set M together with a set of distinguished subsets of various cartesian powers of M called the *basic relations*. It is assumed that equality is a basic (binary) relation in every structure. One could also allow *basic functions* from various cartesian powers of M to M, but by replacing them with their graphs I will, without loss of generality, restrict myself to relational structures. For example, a ring can be viewed as a structure where the underlying set is the set of elements of the ring and there are, besides equality, two basic relations: the ternary relations given by the graphs of addition and multiplication. If the ring also admits an ordering that we are interested in, then we can consider the new structure where we add the ordering as another basic binary relation. The *definable sets* of a structure are those subsets of cartesian powers of M that are obtained from the basic relations in finitely many steps using the following operations:

- intersection,
- union,
- complement,
- cartesian product,
- image under a coordinate projection, and
- fibre of a coordinate projection.

I am avoiding talking about the logical syntax here, but the reader familiar with some first-order logic will recognize that the basic relations form the *language* of the structure and the various operations correspond to *logical symbols* such as conjunction, disjunction, and negation. The operation of taking the image under a coordinate projection corresponds to existential quantification, while that of taking a fibre of a coordinate projection corresponds to substituting parameters for variables (that is, *specialisation*). The definable sets are then the sets described by *first-order formulae*. This way of viewing definable sets is an essential feature of model theory, even though many expositions (including this one) avoid formulae by introducing definability set-theoretically, just as I have done here.

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In any case, given a structure we have an associated collection of definable sets. When $(R, +, \times)$ is a commutative unitary ring, for example, it is not hard to see that if $f_1, \ldots, f_\ell$ are polynomials in $R[x_1, \ldots, x_n]$, then the *algebraic set* they define, namely their set of common zeros in R^n, is definable. Hence the finite boolean combinations of such sets, that is, the *Zariski constructible* sets, are all definable. It is an important fact that if R is an algebraically closed field, then these are the only definable sets.[1]

But we are interested in a somewhat different sort of example. Fix a compact complex manifold X and consider the structure $\mathcal{A}(X)$ where the basic relations are the complex analytic subsets of X^n, for all $n > 0$. By a *complex analytic subset*, or just *analytic* subset for short, I mean a subset A such that for all $p \in X^n$ there is a neighbourhood U of p and finitely many holomorphic functions $f_1, \ldots, f_\ell$ on U such that $A \cap U$ is the common zero set of $\{f_1, \ldots, f_\ell\}$. Note that the local data of U and $f_1, \ldots, f_\ell$ are not part of our structure; only the global set A is named as a basic relation. The model theory of compact complex manifolds was begun by Zilber's [14] observation in the early 1990s that $\mathcal{A}(X)$ is "tame". In particular, as a consequence of Remmert's proper mapping theorem, Zilber shows that every definable set in $\mathcal{A}(X)$ is a finite boolean combination of analytic subsets. But the tameness goes much further, making the geometry of analytic sets susceptible to a vast array of model-theoretic techniques.

Zilber's analysis of individual compact complex manifolds extends to the *many-sorted* structure $\mathcal{A}$, which includes all compact complex manifolds at once, and where all complex analytic subsets are named as basic relations. Note that algebraic geometry is part of this structure; amongst the compact complex manifolds in $\mathcal{A}$ are the complex projective spaces, and so all quasi-projective algebraic varieties are definable in $\mathcal{A}$. But there are also nonalgebraic compact complex manifolds, and in some sense the model-theoretic analysis focuses on those. Let me discuss two examples that we will see again later.

Suppose $\{\alpha_1, \ldots, \alpha_{2n}\}$ is an $\mathbb{R}$-basis for $\mathbb{C}^n$, and $\Lambda = \mathbb{Z}\alpha_1 + \cdots + \mathbb{Z}\alpha_{2n}$ is the real $2n$-dimensional lattice it generates. Then the quotient $T = \mathbb{C}^n/\Lambda$ is a compact complex manifold of dimension n, called a *complex torus*, that inherits a holomorphic group structure from $(\mathbb{C}^n, +)$. While some complex tori can be embedded into projective space, if Λ is chosen sufficiently generally, then the resulting torus is nonalgebraic. For example, if the real and imaginary parts of $\{\alpha_1, \ldots, \alpha_{2n}\}$ form an algebraically independent set over $\mathbb{Q}$, then T will not contain any proper infinite analytic subsets. In particular, such tori, which are called *generic complex tori*, cannot be projective varieties if $n > 1$. Another example of a nonalgebraic compact complex manifold is the following *Hopf surface*: fix a pair of complex numbers $\alpha = (\alpha_1, \alpha_2)$ with $0 < |\alpha_1| \leq |\alpha_2| < 1$, and consider the infinite cyclic group Γ of automorphisms of $\mathbb{C}^2 \setminus \{(0,0)\}$ generated by $(u, v) \mapsto (\alpha_1 u, \alpha_2 v)$. The quotient $H_\alpha = \mathbb{C}^2 \setminus \{(0,0)\}/\Gamma$ is a compact complex surface that is never algebraic. Indeed, this can already be deduced from its underlying differentiable structure; H_α is diffeomorphic to $S^1 \times S^3$, something that is never the case for a projective surface. Unlike tori, Hopf surfaces always contain proper infinite analytic subsets: the images of the punctured axes in $\mathbb{C}^2 \setminus \{(0,0)\}$ give us at least two irreducible curves on H_α. If α is chosen sufficiently generally, then these will be the only curves. In that case, like generic complex tori, H_α will have no nonconstant meromorphic functions.

In model theory there are several notions, of varying resolution, that describe the possible interaction between two definable sets. These notions, specialised to the structure $\mathcal{A}$, have relevant counterparts in bimeromorphic geometry. Suppose that X and Y are irreducible complex analytic sets in $\mathcal{A}$. The strongest possible interaction is if X and Y are biholomorphic. This can be weakened to *bimeromorphic equivalence*: there exists an irreducible analytic subset $G \subset X \times Y$ such that both projections $G \to X$ and $G \to Y$ are generically bijective. By "generically bijective" I mean that off a countable union of proper analytic subsets of X the fibres of $G \to X$ are singletons, and similarly for $G \to Y$. If $G \to X$ and $G \to Y$ are only assumed to be generically finite-to-one, then we say that G is a *generically finite-to-finite correspondence* between X and Y. Finally, we can weaken the condition much further by merely asking that there exist some proper analytic subset of $X \times Y$ that projects onto both X and Y. In that case we say that X and Y are *nonorthogonal*. For example, any two projective varieties are nonorthogonal. On the other hand, any compact complex manifold without nonconstant meromorphic functions—such as a generic complex torus or a generic Hopf surface—is *orthogonal* to any projective variety. Indeed, nonorthogonality to some projective variety implies nonorthogonality to the projective line $\mathbb{P}_1$, and if $G \subset X \times \mathbb{P}_1$ witnesses this, then G has nonconstant meromorphic functions, and as $G \to X$ will necessarily be generically finite-to-one, so does X.

[1] *This is* quantifier elimination *for algebraically closed fields, or equivalently Chevellay's theorem that over an algebraically closed field the projection of a constructible set is constructible.*

Classifying Simple Compact Complex Manifolds

One way for a structure to be considered "tame" is if the definable sets have a *dimension theory*, that is, if a certain intrinsic model-theoretically defined dimension function, called the *rank*, takes on ordinal values on all definable sets. Zilber's initial analysis of $\mathcal{A}$ showed that every analytic subset of a compact complex manifold is of *finite* rank. In fact, this rank is bounded by, but typically not equal to, the complex dimension. I will not define rank here, but I will in a moment explain what it means for a compact complex manifold to have rank 1. This reticence is partially justified by the fact that there is general model-theoretic machinery available, due mostly to Shelah, that analyses arbitrary finite-rank definable sets in terms of rank 1 sets. In particular, and this is only the starting point of such an analysis in $\mathcal{A}$, every analytic set will be nonorthogonal to one of rank 1. It follows that the study of rank 1 sets is central to understanding compact complex manifolds in general. Note that this is not true of complex dimension: understanding compact complex curves tells us essentially nothing about compact complex surfaces that contain no curves.

So what does it mean for a compact complex manifold X to be of "rank 1"? Here is a geometric characterisation: X is not covered by a definable family of proper infinite analytic subsets. Pillay [10] observed that such manifolds were already of interest to complex geometers and were called *simple*. More precisely:

Definition. A compact complex manifold X is said to be *simple* if $\dim X > 0$, and whenever Y is a compact complex manifold, $A \subset Y \times X$ is an analytic subset, and $E \subset Y$ is a proper analytic subset such that the fibres of A above $Y \setminus E$ are proper infinite subsets of X, then the union of all the fibres of A above $Y \setminus E$ is contained in a proper analytic subset of X.

Projective curves are simple; they are the only simple projective varieties. But so is a generic complex torus, or indeed any compact complex manifold without proper infinite analytic subsets. The generic Hopf surfaces, having precisely two curves on them, are also simple. It is not hard to see that simplicity is a bimeromorphic invariant; in fact, it is preserved by generically finite-to-finite correspondences. In a sense that I have hinted at above and will return to again later, simple manifolds are the building blocks for all compact complex manifolds.

The contribution of model theory to bimeromorphic geometry begins with the following dichotomy for simple compact complex manifolds, which is a consequence of the deep results of Hrushovski and Zilber from [5]. It says that a simple compact complex manifold is either algebraic or its cartesian powers have no "rich" definable families of analytic subsets. More precisely, if X is a simple compact complex manifold, then exactly one of the following holds:

I. X is a projective curve, or

II. X is *modular:* whenever Y is a compact complex manifold with $\dim Y > 0$, and $A \subseteq Y \times X^2$ is an analytic subset whose generic fibres over Y are distinct proper infinite irreducible analytic subsets of X^2 that project onto each coordinate, then Y is simple.

While the condition of modularity only explicitly mentions X^2, it actually restricts the rank of families of analytic subsets of X^n, for all n. All projective curves are nonmodular. For example, the family of lines $y = ax + b$ is a two-parameter algebraic family; it gives rise to a family of subvarieties of $\mathbb{P}_1 \times \mathbb{P}_1$ that is parameterised by a two-dimensional projective variety, and two-dimensional projective varieties are not simple.

So, by the above dichotomy, every simple manifold of dimension at least two is modular. As we have seen, examples can be found among the complex tori and the Hopf surfaces. In fact the model-theoretic analysis provides us with a further dichotomy which distinguishes sharply between these two examples. Very roughly speaking, a modular manifold that is not a complex torus admits only binary definable relations. More precisely, if X is a simple modular compact complex manifold, then exactly one of the following holds:

I. X is in generically finite-to-finite correspondence with a complex torus, or

II. X is *relationally trivial:* if $A \subseteq X^n$ is an irreducible analytic subset that projects onto X in each coordinate, and if $\{\pi_1, \dots, \pi_\ell\}$ is an enumeration of all the coordinate projections from X^n to X^2, then A is an irreducible component of $\bigcap_{i=1}^{\ell} \pi_i^{-1}(\pi_i(A))$.

Note that complex tori are not relationally trivial: being complex Lie groups they admit a truly ternary analytic relation, namely the graph of the group law. Simple Hopf surfaces are relationally trivial. Actually one can be more precise in case I: there exists a generically finite-to-one meromorphic surjection from X to a *Kummer manifold*, a manifold of the form T/Γ where T is a complex torus and Γ is a finite group of holomorphic automorphisms of T.

The above characterisation of simple modular compact complex manifolds was suggested by Hrushovski in his 1998 address to the International Congress of Mathematicians [4]. It was proved by Pillay and Scanlon in [12], building on some unpublished work of Scanlon. The result is obtained as a corollary to the main theorem in that paper which I would at least like to state in brief. *Meromorphic groups* are a natural generalisation of algebraic groups to the complex analytic category; they are complex Lie groups that are "compactifiable" in an appropriate sense. What Pillay and Scanlon actually prove, using model-theoretic methods and building on work of Fujiki, is that every meromorphic group is the extension of a complex torus by a linear algebraic group.

In any case, putting the two dichotomies together, we get:

Theorem 1. *If X is a simple compact complex manifold, then*

- *X is a projective curve, or*
- *X is in generically finite-to-finite correspondence with a simple complex torus of dimension > 1, or*
- *X is relationally trivial.*

It remains then to understand the relationally trivial compact complex manifolds. In dimension one there are none because all compact curves are algebraic (this is by the Riemann existence theorem), and it is not hard to see that projective curves are not relationally trivial. Besides Hopf surfaces, examples of relationally trivial surfaces can also be found among the $K3$ and Inoue surfaces. Relationally trivial manifolds remain quite elusive, and, except for the case of surfaces, there are only conjectural results. These conjectures are restricted to Kähler manifolds, which also play a special role from the model-theoretic point of view, and to which I now turn.

Compact Cycle Spaces and Kähler Manifolds

One of the obstacles to the full application of model-theoretic techniques to compact complex manifolds is the size of the language, the fact that *every* analytic set is a basic relation. Let me point out that in the algebraic case there is a more economical choice of structure. Consider, for example, the compact complex manifold $X = \mathbb{P}_m$, projective m-space over the complex numbers. Instead of $\mathcal{A}(X)$ we can consider the structure $\mathcal{A}_{\mathbb{Q}}(X)$, where we only include as basic relations the algebraic subsets of X^n that are defined by polynomials with *rational* coefficients. Then $\mathcal{A}_{\mathbb{Q}}(X)$ has only countably many basic relations. But all analytic subsets of X^n are still *definable* in $\mathcal{A}_{\mathbb{Q}}(X)$. This is because every complex analytic subset of projective space is algebraic (Chow's theorem) and every algebraic set is obtained by specialisation from an algebraic set over $\mathbb{Q}$. Hence $\mathcal{A}(X)$ and $\mathcal{A}_{\mathbb{Q}}(X)$ have the same definable sets, even though the latter is equipped with a much reduced collection of basic relations.

This motivates the following definition: A compact complex manifold X is called *essentially saturated* if there exists a countable collection $\mathcal{L}_0$ of analytic subsets of cartesian powers of X such that every analytic set is definable in the reduct where only the sets in $\mathcal{L}_0$ are named as basic relations. Essentially saturated compact complex manifolds are significantly more amenable to a model-theoretic analysis. Exactly why is described in [7] and is somewhat beyond the scope of this article. Instead, I would like to focus on a very suggestive geometric characterisation of essential saturation.

Recall that a *k-cycle* of a compact complex manifold X is a finite linear combination $Z = \sum_i n_i Z_i$ where the Z_i are distinct k-dimensional irreducible complex analytic subsets of X and each n_i is a positive integer. In particular, every irreducible analytic subset is itself a cycle. The set of all k-cycles is denoted by $\mathcal{B}_k(X)$, and $\mathcal{B}(X)$ denotes the disjoint union of all the $\mathcal{B}_k(X)$. In the 1970s Barlet endowed $\mathcal{B}(X)$ with a natural structure of a complex analytic space. If X is algebraic, then $\mathcal{B}(X)$ coincides with the Chow scheme. With this terminology in place, we can characterise essential saturation as follows: *X is essentially saturated if and only if all the irreducible components of $\mathcal{B}(X^n)$ are compact, for all $n \geq 0$.* This follows from [7], in which the universal family of complex analytic subspaces (the *Douady spaces*) were used instead of Barlet's cycle spaces. In any case, it is important here that the condition is on all the cartesian powers; if H is a Hopf surface, then $\mathcal{B}(H)$ has only compact components, but $\mathcal{B}(H \times H)$ has a noncompact component. In particular, Hopf surfaces are not essentially saturated. While the property of $\mathcal{B}(X)$ having only compact components is one that appears often in bimeromorphic geometry, it seems that the extension of this condition to all cartesian powers of X has not been studied. For example, is essential saturation preserved under cartesian products and bimeromorphic equivalence?

How does one check if a component of the Barlet space is compact? In the late 1970s Lieberman [6] gave the following differential geometric criterion: a set of cycles is relatively compact in the cycle space if and only if the volume of the cycles in that set (with respect to some hermitian metric) is bounded. This gives us many examples of essentially saturated manifolds. Recall that a *Kähler manifold* is one that admits a (hermitian) metric which locally approximates to order 2 the standard euclidean metric on $\mathbb{C}^n$. With respect to such a metric, the volume of the cycles in any given component of the Barlet space is constant, and this remains true of cartesian powers because kählerianity is preserved under cartesian products. It follows from Lieberman's theorem that all compact Kähler manifolds are essentially saturated. Moroever, any compact complex analytic space that is bimeromorphic to a Kähler manifold—this is Fujiki's class C—is also essentially saturated.

While C does not include all essentially saturated manifolds (Inoue surfaces of type S_M are counterexamples, see [8]), it is a large class of manifolds that contains all projective varieties and complex tori and that is preserved under various operations including meromorphic images and generically finite-to-finite correspondences. Compact Kähler manifolds play a prominent role in bimeromorphic geometry largely because they are susceptible to many of the techniques of algebraic geometry. Because of essential saturation, they are also important to the model-theoretic approach.

Let us return to the classification problem for simple compact complex manifolds. In the previous section I explained how this reduces to understanding the relationally trivial manifolds. We can restrict the problem further and ask: What are the relationally trivial simple *Kähler* manifolds? We have already seen that there are none in dimension one. In dimension two, inspecting the Enriques-Kodaira classification, we see that all relationally trivial simple Kähler surfaces are bimeromorphic to $K3$ surfaces. These surfaces (introduced by Weil and named in honour of Kummer, Kodaira, Kähler, and the mountain K2) are by definition simply-connected compact surfaces with trivial canonical bundle. In higher dimensions the correct generalisation of $K3$ seems to be *irreducible hyperkähler:* simply connected compact Kähler manifolds with the property that their space of holomorphic 2-forms is spanned by an everywhere nondegenerate form. Pillay has conjectured that *all relationally trivial simple Kähler manifolds are in generically finite-to-finite correspondence with an irreducible hyperkähler manifold.* Since irreducible hyperkähler manifolds are always even-dimensional, Pillay's conjecture, coupled with Theorem 1 above, would imply that every odd-dimensional simple Kähler manifold is in generically finite-to-finite correspondence with a complex torus. In dimension three this is essentially a conjecture of Campana and Peternell, namely that every simple Kähler threefold is Kummer.

Variation in Families

In justifying our focus on simple manifolds I have already mentioned the fact that every compact complex manifold is nonorthogonal to a simple one. From this one can deduce that for every compact complex manifold X there exists a meromorphic surjection $f : X \to Y$, where $\dim Y > 0$ and Y is in generically finite-to-finite correspondence with some cartesian power of a simple manifold. The same is then also true of each generic fibre X_y of f. At least in the Kähler case, using essential saturation, one can show that the corresponding meromorphic surjections on X_y vary uniformly in the parameter y. Since the dimension of X_y is strictly less than that of X, such an analysis must stop after finitely many iterations. It is in this sense, via sequences of meromorphic fibrations, that simple manifolds control the structure $\mathcal{A}$. Of course this does not reduce the classification problem to the case of simple manifolds; at the very least one needs to also understand how such manifolds fit into meromorphic fibrations and how they interact with each other. I want to state one conjecture that is central to this question.

First of all, from the definition of simplicity it follows that two simple manifolds are nonorthogonal to each other if and only if they are in generically finite-to-finite correspondence. The three classes of compact complex manifolds appearing in Theorem 1—projective curves, simple complex tori of dimension > 1, and simple relationally trivial manifolds—are all mutually orthogonal in the sense that any two manifolds coming from different classes will be orthogonal. Moreover, while all curves are nonorthogonal to each other, there exist orthogonal pairs within each of the other two classes.

One fundamental question is whether or not there exist entire definable families of simple manifolds any two of which are orthogonal. Actually, it follows from observations of Pillay and Scanlon that such families do exist, but not, conjecturally, among the Kähler manifolds. More precisely, the conjecture is that *if $f : X \to Y$ is a meromorphic surjection between compact Kähler manifolds with simple generic fibres, then any two generic fibres are in generically finite-to-finite correspondence.* In 2005 Campana [2] proved an isotriviality result which proves the conjecture, with the stronger conclusion of isomorphism rather than correspondence, in the following cases: when the generic fibres are surfaces, when the generic fibres are "general" complex tori, and when the generic fibres are irreducible hyperkähler.

Analogies

By way of conclusion, I want to discuss the special role of model theory as a medium between different geometric contexts. Certain results from bimeromorphic geometry have informed advances in abstract model theory which can then be reapplied in other areas. In this way model theory acts as a conduit between bimeromorphic geometry and, in the case I want to discuss, *differential-algebraic geometry*. I need to say a few words about differential-algebraic geometry.

Like bimeromorphic geometry, differential-algebraic geometry is an "expansion" of algebraic geometry. A *differential field* is a field K equipped with a *derivation*; an additive map $\delta : K \to K$ such that $\delta(xy) = x\delta(y) + y\delta(x)$. For example, $(\mathbb{C}(t), \frac{d}{dt})$ is a differential field. *Differential algebra* is then commutative algebra in the presence of this derivation. The role of polynomials is played by *differential polynomials*: functions of the form $P(x, \delta(x), \ldots, \delta^r(x))$, where $P \in K[X_0, \ldots, X_r]$ is an ordinary polynomial. A differential field (K, δ) is *differentially closed* if any system of differential polynomial equations and inequations having a solution in some differential field extension already has a solution in K. The model theory of differentially closed fields of characteristic zero is also tame. In particular, every definable set in $(K, +, \times, \delta)$ is a finite boolean combination of *differential-algebraic sets*: zero sets of systems of differential polynomial equations. These are the objects of study in differential-algebraic geometry.

Many of the model-theoretic techniques that apply to the structure $\mathcal{A}$ also apply to $(K, +, \times, \delta)$. In fact, there is a fruitful analogy between these structures whereby complex analytic sets correspond to finite-rank differential-algebraic sets (see [10]). This analogy can lead to transferring results between the two disciplines that these structures represent. The example I have in mind is based on the following theorem of Campana [1], due also independently to Fujiki [3], from the early 1980s. Suppose X is a compact complex manifold and C is a compact analytic subset of the cycle space $\mathcal{B}(X)$. Then for any $a \in X$, the set of cycles in C that pass through a is (up to bimeromorphism) an algebraic set. Moreover this happens uniformly as a varies. In 2001 Pillay [11] observed that this theorem could be used to give a more direct proof of the first dichotomy; the fact that every simple manifold is either a curve or modular. Pillay's argument involves formulating an abstract model-theoretic counterpart to Campana's theorem, which replaces the difficult and much more general results of Hrushovski and Zilber [5]. Pillay and Ziegler [13] then show that this model-theoretic counterpart also holds in differentially closed fields. As a result one has an analogue of Campana's theorem in differential-algebraic geometry, as well as a direct proof of the corresponding dichotomy for rank 1 differential-algebraic sets. Another related example of this phenomenon can be found in [9], in which Campana's "algebraic connectedness" is abstracted from bimeromorphic geometry to model theory and then applied to differentially closed fields. The outcome in that case is a criterion for when a finite-rank differential-algebraic set is in generically finite-to-finite correspondence with the C_K-points of an algebraic variety, where $C_K = \{x \in K : \delta(x) = 0\}$.

The kind of transfer of ideas that we see in the above examples, and the role that model theory plays here of recognising, formalising, and facilitating analogies between different geometric settings, is not something new or unique to its interaction with bimeromorphic geometry. This has been a defining feature of model theory since the 1970s and continues to fuel the internal development of the subject.

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WHAT IS...
a Halting Probability?

Cristian S. Calude and G. J. Chaitin

Turing's famous 1936 paper "On computable numbers, with an application to the *Entscheidungsproblem*" defines a computable real number and uses Cantor's diagonal argument to exhibit an uncomputable real.

Roughly speaking, a computable real is one that can be calculated digit by digit, one for which there is an algorithm for approximating as closely as one may wish. All the reals one normally encounters in analysis are computable, like π, $\sqrt{2}$, and e. But they are much scarcer than the uncomputable reals because, as Turing points out, the computable reals are countable, whilst the uncomputable reals have the power of the continuum. Furthermore, any countable set of reals has measure zero, so the computable reals have measure zero. In other words, if one picks a real at random in the unit interval with uniform probability distribution, the probability of obtaining an uncomputable real is unity. One may obtain a computable real, but that is infinitely improbable.

But how about individual examples of uncomputable reals? We will show two: H and the halting probability Ω, both contained in the unit interval. Their construction was anticipated in 1927 by Émile Borel, who "defined" a real number $0 < B < 1$ in the following way. The Nth digit b_N of B answers the Nth question in an enumeration of all possible yes/no questions that one can write in French. Borel argued that knowledge of every point of the continuum would have strange consequences, and B is a case in point. The number B is an oracle that "would give answers to all past, present, and future enigmas of science, history, and curiosity."

Let's change this into something more mathematical, more realistic: an oracle for the halting problem H.

Systematically enumerate all programs with no input (say, lexicographically). The Nth bit h_N of the real number H tells us whether or not the Nth computer program ever halts. H is an oracle for the halting problem, which would be extremely useful to have, as we will show later.

However, this oracle H for the halting problem is extremely wasteful, because N instances of the halting problem are not N bits of mathematical information, they are only $\log_2 N$ bits of mathematical information. One only needs to know *how many* of these N programs halt—a number expressed in less than or equal to $1 + \log_2 N$ bits—to be able to determine which ones halt. Indeed, one just runs in parallel all programs till exactly the known number of programs that halt have stopped: all the remaining programs won't halt.

Using a slightly more sophisticated version of this idea, we finally arrive at the **halting probability** Ω, which is defined by the following formula:

$$0 < \Omega = \sum_{\text{program } p \text{ halts}} 2^{-(\text{size of } p \text{ in bits})} < 1.$$

This is the probability that a computer program whose bits are generated one by one by independent tosses of a fair coin will eventually halt. There are actually many halting probabilities, not one, because the precise numerical value of $\Omega = \Omega_L$ depends on the choice L of programming language for the programs p.

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Nota Bene: For this definition of a halting probability to work properly, the computer programming language L that the programs p are written in has to satisfy certain requirements:

- The programs p have to be self-delimiting (no extension of a valid program is a valid program) or the sum won't converge. This enables one to construct large programs by concatenating (abutting) subroutines.
- The language L has to be universal and concise. More precisely, if any other language L' can program something in K bits, then L can do it in $\leq K + c_{L'}$ bits.

For historical reasons, such a programming language L is often referred to as a *universal self-delimiting Turing machine,* and we shall do so below.

What are the key properties of a halting probability Ω?

By writing Ω in binary one can show that given the first N bits $\omega_1 \dots \omega_N$ of

$$\Omega = \sum_{K=1,2,3,\dots} \omega_K / 2^K,$$

one can solve the halting problem for all programs up to N bits in size. This implies that the bits of Ω are algorithmically and logically irreducible:

- The smallest program for computing the first N bits of Ω has $\geq N - O(1)$ bits. (This algorithmic irreducibility property is normally called (*algorithmic*) *randomness.*)
- Any formal axiomatic theory, like ZFC, that enables one to prove what are the values of the first N bits of Ω must have $\geq N - O(1)$ bits of axioms. (This is *logical irreducibility.*)

In other words, the bits of Ω are the most concise oracle for solving the halting problem, the best possible compression of all the answers to individual instances of the halting problem.

The philosophical and epistemological significance of Ω is primarily due to the following surprising fact:

> *The bits $\omega_1\omega_2\omega_3\dots$ of the halting probability Ω provide a perfect simulation within pure mathematics, where all truths are necessary, of an infinite stream of contingent, accidental yes/no facts.*

For instance, Ω provides us with a natural example of what É. Borel termed an *absolutely normal real number.* This means that if Ω is written in any base $b \geq 2$, then all blocks of K base-b digits will occur with equal limiting relative frequency b^{-K}.

It is an immediate corollary of Ω's algorithmic and logical irreducibility that Ω is uncomputable and that, in fact, no algorithm can compute more than finitely many scattered bits of Ω. Some of these bits have actually been determined. For a natural choice of the programming language L, the first 40 bits of $\Omega = \Omega_L$ are[1]

0001000000010000101001110111000011111010.

If we knew the first 7,780 bits,[2] which is less than one quarter of this note's size in bits, we would know whether the Riemann hypothesis is correct.

Now for some more advanced results.

Although uncomputable, Ω is the most "computable" among all algorithmically random reals: any Ω_L is *left-computable,* i.e., the least upper bound of a computable sequence of rationals. The set $\{\Omega_L\}$ of all halting probabilities of universal self-delimiting Turing machines L coincides with the set of all left-computable algorithmically random reals.

Can a halting probability be formally proved algorithmically random in Peano arithmetic (PA)? The answer depends on the representation: if $\Omega = \Omega_L$ is given by a self-delimiting Turing machine L whose universality (and conciseness) is proved in PA, then PA proves that Ω is algorithmically random. Every Ω can be written as $\Omega = \Omega_L$, for some L satisfying the above requirements, so it is provably random in PA.[3]

Further Reading. For the intellectual history leading up to Ω, including the role played by Leibniz, see the second author's *Meta Math!*. For a technical treatment, see the first author's *Information and Randomness*, or *An Introduction to Kolmogorov Complexity and Its Applications* by M. Li and P. Vitányi. For halting probabilities for versions of a real programming language, LISP, and a plethora of other nonstandard halting probabilities and their applications, see the second author's *Information-Theoretic Incompleteness.* For some of the latest results, see *Computability and Randomness* by A. Nies and *Algorithmic Randomness and Complexity* by R. G. Downey and D. Hirschfeldt (forthcoming). For discussions of the philosophical impact and additional historical material, see the second author's *Thinking about Gödel and Turing.*

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What Then? Plato's Ghost: The Modernist Transformation of Mathematics

Reviewed by Yuri Manin

Plato's Ghost: The Modernist Transformation of Mathematics
Jeremy Gray
Princeton University Press, 2008
US$45.00, 526 pages
ISBN-13: 978-0691136703

The new book of the renowned historian of mathematics Jeremy Gray can be succinctly described as a rich and thorough study of Western ethnomathematics in the period of five decades or so (1880–1930) preceding and following World War I. What distinguishes such a project from more conventional Bourbaki–style enterprises is the stress on extra–mathematical background: social and political structures of society, economic practices, forms of professional self–organization, and culture.

In fact, culture (represented by visual arts, music, philosophy) dominates the discourse at several key points, and culture furnished the basic metaphor for the book. The time span in question is characterized as the period of modernist transformation in mathematics, and the first quotation in the Introduction is taken from Guillaume Apollinaire's "The Beginning of Cubism" (1912).

The scope of the book is already ambitious. Chronologically, it starts well before the arrival of "modernism": There are brief essays on Monge, Poncelet, and projective geometry, followed by a discussion of Lobachevsky, Bolyai, and non-Euclidean geometry, including a description of Gauss's role in its formation and the subsequent contributions of Riemann and Poincaré that led to a new vision of what geometry is and what it should be. Similar essays on algebra, analysis, "British Algebra and Logic", and philosophy follow, all of this interspersed with information about professional organizations, teaching, journals, and quotations from contemporary articles and letters. (I cannot resist the temptation to reproduce a delightfully funny sentence from a letter by C. G. J. Jacobi to A. von Humboldt, although in the book it appears only considerably later, on page 268:

> If Gauss says he has proved something, it seems very probable to me; if Cauchy says so, it is about as likely as not; if Dirichlet says so, it is certain.)

Chapter 2, containing this wealth of information, is called "Before Modernism", and ends with a brief synopsis "Consensus in 1880".

The arrival of mathematical modernism is addressed in Chapter 3. Again, the discussion is consecutively focused on geometry (in particular, reemerging connections to physics and Schwarzschild's paper of 1900); then analysis, algebra, and logic. With hindsight, the central modernizing figure appears in the person of Georg Cantor. Especially interesting for me were pages discussing religious overtones of Cantor's transcendental flights in the infinity of infinities and the related subsection on "Catholic Modernism". (As I have written elsewhere, I discern a subtle mockery in the famous and often quoted sentence of Hilbert

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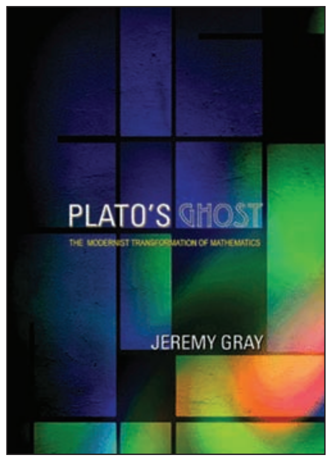
about "Cantor's Paradise", usually perceived only as unconditional support for set theory).

The subsequent chapters are called "Modernism Avowed", "Faces of Mathematics", "Mathematics, Language and Psychology", and "After the War" and are bursting with information, insight, and scholarship. As far as I know, nobody before Jeremy Gray discussed in this context "Popularizing Mathematics around 1900" (section 5.4), or "Languages Natural and Artificial" (section 6.1).

Constrained by the restrictions of space, I will end this brief survey of the detailed contents of this remarkable book and turn to the discussion of several of its grand themes, sometimes underlying the exposition and sometimes explicitly invoked in it.



Mathematics is a product of civilization: it is discovered or created (see discussion below) by a very limited number of human beings in each generation, then encoded in a mixture of words, formulas, and pictures (nowadays software and hardware might be added to this list), and in this way transmitted to the next generation, which continues this process. To estimate the number of persons creating mathematics these days, one can refer to the attendance of International Congresses of Mathematicians that are held every four years: recently it was about three to five thousand. Undoubtedly, this should be viewed as a great explosion in comparison with previous centuries. In Newton's time the respective number probably did not exceed a few dozen.

One remarkable feature of mathematical knowledge is this: we learn more and more about the same objects that ancient mathematicians already started to see with their mental eyes: integers and prime numbers, real numbers, polynomial equations in one or many variables, space and various space forms… To illustrate this statement, I will mention just one string of events. Some time around 300 BC, the Hellenistic scholar Pappus discovered a remarkable and, in the Euclidean context, quite unusual theorem (see the modernized statement and the picture on page 463 of Gray's book, and beware of a small misprint: $B'A$ must be BA'). Pappus's Theorem does not conform to the spirit of Euclidean geometry, because lengths, angles, and rigid plane motions are quite irrelevant here: only points, lines, and incidence relations "a point x lies on a line X" play a role. In the nineteenth century it was understood that Pappus's Theorem characterizes projective geometry of the real plane. When an abstract projective plane—as a set of points P, with a set of subsets $l \subset P$ called lines—was introduced at the beginning of the twentieth century, it turned out that Pappus's Theorem, now taken as an axiom, is a necessary and sufficient condition for the possibility of introducing projective coordinates in P with values in an abstract field (at least, if P is infinite). Several more decades elapsed, and it was discovered that this latter statement can be vastly generalized in the theory of models of formal languages: "Zariski geometries" of Zilber and Hrushovski describe when abstract sets with subsets satisfying certain combinatorial requirements can be conceived as sets of solutions of arbitrary polynomial equations over a field.

A *sine qua non* condition of such permanency, continuity of preoccupation, is the creation and maintenance of a very safe, reliable, intergenerational flow of information. Linguistic resources of mathematics by which this information is encoded and transferred are arguably much more variable, fickle, subject to sweeping winds of history, than the content of the information itself.

The whole bulk of discoveries of one generation, or one of its active but geographically restricted subgroups, might in a few decades become almost incomprehensible to another generation, unless new expressive means, with their new intuitions and hygienic rules, come to the rescue. The old arguments are translated into a new language, rewritten, purged of perceived obscurities and errors, in the process of mastering old results and moving to new discoveries.

My professional youth was spent in this exhilarating process of assimilation of glorious Italian algebraic geometry and recasting it using schemes, coherent sheaves, and homological algebra of the emerging "new age" algebraic geometry, which in two to three decades was to become one of the powerful vehicles not only in pure mathematics but also in theoretical physics and quantum field theory. Perhaps for this personal reason, I became perceptive of similar events that took place in other times and locations.

From this perspective, "the modernist transformation of mathematics", which is the subtitle of Jeremy Gray's book, was one such periodically occurring refurbishing of basic vocabulary, grammar, and, yes, aesthetic requirements for mathematical thought and mathematical texts. It took about thirty years, and its outcomes dominated the next thirty years: all in all, just about the span of three generations. The fact that during the lives of these generations two world wars swept Europe (or, according to some accounts, one Thirty Years' War 1914–1945) influenced mathematics and the wider culture in many ways.

The Bourbaki group formed itself in the 1930s in a conscious attempt to fill the void left by the "lost generation": not in the somewhat egocentric sense of the phrase coined by Gertrude Stein, who referred to bohemian American émigrés in Paris, but a real and painful void. About 40 percent of French mathematicians died in the first world war.

Although my argument posits "the modernist transformation" as one of the series of structurally similar transformations that occurred in other times and places, both the choice of the time span and the use of the adjective "modernist" for it seem to me very felicitous. After formulating what he sees as characteristic of modernist aesthetics, creative psychology, the relationship with society, and self–perception, shared by, say, painting and mathematics of that period, Jeremy Gray himself warns that this looks like a case of "convergent evolution" (page 8) rather than an effect of "common genes".

In his own sober words: "I am not sure there is more than a resonance that links mathematical modernism to the arrival of modernity, modern capitalism, and its horrific opponent, the Nazi state. We are far from knowing much about the societies we inhabit."

In a book largely dedicated to the history and philosophy of mathematics, Plato's name might have been invoked in many contexts, but the first that comes to mind, that of "Mathematical Platonism", is relegated to the very end of the discussion, section 7.3, pages 440–452. Besides this section 7.3, Jeremy Gray, according to the Index, mentions Plato or Platonism on twelve more pages scattered throughout the text. By contrast, more than fifty entries in the Index invoke Poincaré and only slightly fewer, Bertrand Russell.

Why, then, is this monograph called "Plato's Ghost" ? The explanation is on the very first page, which quotes in its entirety Yeats's "What Then?", a poem of four five-line stanzas about an accomplished life of an accomplished human being:

> Everything he wrote was read,
> After certain years he won
> Sufficient money for his need;
> Friends that have been friends indeed;
> *"What then?" sang Plato's ghost, "What then?"*

The most direct reading of these stanzas suggests to me a simple and universal human emotion, *existential anxiety,* as beautifully expressed by Yeats as by many others before and after him.

Jeremy Gray himself hears in Plato's voice denial of a claim to perfection (page 14). This interpretation is readily accommodated in a chronicle of (para)philosophical controversies around mathematics. But, significantly, the word "anxious" appears already in the first paragraph of the Introduction, and "anxiety" pops up on page 4, where it is first introduced as "a well established theme in writing about modernism". Later this theme is developed in the section 4.8, "Anxiety", already fully in the context of the mathematics of the first half of the twentieth century, against the background of the "crisis of foundations".

Is it conceivable that Gray's whole discourse on modernism in mathematics is informed by existential anxiety and that the last section, 7.5, "The Work Is Done", is as self–referential as it sounds?

Of course it is, as the present author willingly testifies, relying upon his own experience, and as Jeremy Gray himself hints in the last lines of page 4.

But before bidding farewell to the reader, let's brace ourselves and try to face the Ghost question "What then?", which I will interpret in the most prosaic way, as a question about the direction in which self–consciousness of mathematicians moved during the last few decades, separated by more than a half–century from the 1930s and 1940s where the detailed analysis of Gray stops.

So far as I can judge, "Platonism" of working mathematicians is based on a feeling that important mathematical facts are *discoveries* rather than *inventions.*

The Bering Strait was named after Vitus Bering who discovered it, whereas the diesel engine was named after Rudolf Diesel who invented the engine. What about Galois groups? If you feel that they were discovered by Évariste Galois, rather than invented by him, you are in a sense a Platonist.

I will call such an attitude *emotional Platonism* in order to stress that (in my view) it is intellectually indefensible, but not to the least degree invalidated by this fact, since our emotions happily resist rational arguments.

Being such an emotional Platonist myself, I do not want to say that *all* mathematics is a discovery of a Platonic world, whatever that could mean. Certainly, the history of mathematics is also marked by inventions of marvelous intellectual telescopes and efficient vehicles, allowing people to travel from one discovery to another. Moreover, there are mathematicians whose oeuvre deserves comparison to the *Odyssey* rather than to Columbus's travelogue, insofar as mathematics is exteriorized in texts forming a part of the much vaster general culture of the written word.

Elaborating the latter metaphor, one can represent the history of the "modernist transformation of mathematics" simply as a tale about the birth and development of a certain style of thought, expression, and teaching, starting with Georg Cantor and culminating with Bourbaki. Much will have to be left out of such a tale, but it could serve as a good starting point for guessing "What then?"

This style is formed around a system of pretty explicit prescriptions: how a mathematician is supposed to introduce his or her object of study ("what

I am talking about"), how his or her results should be stated ("what I am saying"), and finally how one should write arguments convincing her/himself and potential readers that the stated results are correct ("proofs").

Briefly, the object of study (group, space with measure, topological space…) is introduced by a *definition*, presenting it as a Bourbaki–style *structure*, which in turn is a collection of Cantorian sets satisfying certain relationships stated in terms of the basic relation "being an element of" and standard logical means. The results are stated as *theorems*, statements of the type "if a structure has a certain property P, then it has another property Q as well" etc. Finally, proofs are texts written in a mixture of natural language, formulas, and sometimes pictures (although the latter are considered "bad taste" in the Bourbaki aesthetics). Such a text can be considered as a valid proof only if in principle it can be *replaced by a sequence of statements* such that each term of this sequence *is valid either by definition, or can be obtained by an elementary logical step from previously obtained statements.*

There are several more or less hidden or explicit sources of self–referentiality in this picture, of which I will mention two.

One is that the notion of mathematical reasoning invoked in the previous paragraphs can itself be rigidified to become a mathematical structure. This was of course the main discovery of the formalist program; as soon as it had crystallized, Gödel's and Tarski's theorems formalizing the "liar's paradox" or Cantor's diagonal argument became inevitable.

Another source of self–referentiality is less formal. Namely, a "proof" presented in a mathematical paper must convince the reader not just that the stated theorem is true, but that it by itself is a PROOF, the incarnation of an ideal object residing in the Platonic territory of formalized mathematics. Severe rules of hygiene are imposed upon mathematical exposition in order to ensure this. The reader must be alerted even to the occurrence of an "empty proof" (empty in the set–theoretical sense), as Edmund Landau used to practice with his inimitable "*Beweis:* Klar. (*Proof:* Clear.)".

The imposition of the hygienic restrictions was a part of the reaction to the perceived crisis of foundations, but in the real life of mathematics of the twentieth century it played another, and probably unexpected, role: that of unification of many diverse fields of mathematical studies. Algebraists, analysts, geometers, probabilists, number theorists, logicians could now speak in the same language, even when speaking about different structures.

Logicists stubbornly struggled in the losing battle against this loss of their dominance as Keepers of Foundations. Nicolas Bourbaki did not really understand the structures emerging in new mathematical logic and botched the respective chapters of his Treatise. As a result, the basic new discoveries in logic, the Turing–Church Thesis and the theories of computability and of complexity, suffered because their ties to the mainstream mathematics were loosened. Fortunately, such researchers as A. N. Kolmogorov helped tighten these ties.

What we see now in the flow of mathematical research can be termed loosely as a "post-modern" period, but the term can be taken literally only if we adopt Jeremy Gray's concept of "modernism". Contemporary mathematics bears no traces or connotations of Lyotard's description of the post-modern condition, whose main characteristic is the loss of credibility of all grand narratives. The grand narrative of mathematics is steadily developing, reaching new depths and new sophistication.

One trend that visibly changes the face of set theory as the foundational language of mathematics is the current popularity of categories (functors, enriched categories, polycategories…) as the dialect in which basic definitions of mathematical objects are formulated. Another related, but not identical, trend is the evolution of homotopy theory, which has gradually become a kind of new language for postmodern mathematics (for an initiated reader, I can mention "brave new rings").

Briefly, these two trends together revolutionize our collective vision of both semantics and syntax of the language(s) of mathematics.

First, basic new objects "category" (up to equivalence) and "homotopy type" are *not* Bourbaki structures: they are formed by a class of Bourbaki structures that are related by certain equivalence relations. Both "class" and "relation" in this statement can be represented set–theoretically by "indefinitely large" collections of sets.

Accepting this, a working mathematician not only sheds the primeval horror of large infinities but opens his/her imagination to a radically new vision even of common objects. For example, natural numbers "simply" count things (or cardinalities of finite sets), but the history of mathematics teaches us how late zero and negative integers were introduced and accepted. External references such as the notion of "debt" in trade were necessary in order to legitimate negatives. Nowadays integers are homotopy classes of pointed loops in a real plane with a deleted point: this is a germ of the idea of "brave new rings". Even more generally, "small sets" that in the Cantor/Bourbaki paradigms could be turned into topological spaces by imposing an additional structure now become secondary/derived objects: say π_0 of a homotopy type. The traditional view "continuous from discrete" gives way to the inverted paradigm: "discrete from continuous".

Second, the notion of formal language that crystallized in the 1930s as a purified written form of natural languages, after it became treated as a Bourbaki structure, could be vastly extended. Seemingly, there are other Bourbaki structures (or better, categories) that have "language–like properties", such as categories of graphs. The intuition behind considering, say, a directed graph as a potential linguistic entity is that of "flowcharts". Generally, computer science, that no–nonsense child of mathematical logic, will exert growing influence on our thinking about the languages by which we express our vision of mathematics.

Finally, I want to discuss briefly the collaboration of mathematicians and physicists, which became dormant during the several decades of "mathematical modernism" but renewed with new vigor after 1950s and 1960s. I will primarily stress the benefits of this collaboration for mathematics and will describe these benefits as the emergence of a vast research program that could be called "quantization of classical mathematics".

This program is historically related to the fact that when physicists started to see quantum phenomena as the basic natural causes underlying observable classical behavior of matter and fields, they had to gradually discover what kind of changes must be made in order to proceed from a known mathematical description of a classical system to a new, quantum description ("quantization"), and back ("classical approximation"). Some of the discovered prescriptions, such as "deformation quantization" involving Planck's constant as a small parameter, turned out to be much more universal mathematically than suggested just by their initial uses.

Another prescription stressing quantum observables as operators in (generally infinite–dimensional) Hilbert spaces led to "non–commutative geometry", one germ of which was the Heisenberg commutation relation, and later to "quantum groups".

Philosophical problems related to the changed role of observation and measurement led to discussions of "quantum logic" and later, in a more applied vein, "quantum computation".

In the 1940s a development started that produced some mathematical miracles. Richard Feynman developed path integrals, a notion that is highly intuitively appealing though mathematically vague, as well as the powerful machinery of Feynman diagrams, which are well understood mathematically but are motivated only by the idea that they somehow capture path integrals. It was a great success in quantum field theory and elementary particles, but nobody could foresee how, in mathematics, it would backfire.

This became reality after many physics papers of Witten and his collaborators, which mathematicians could interpret as rich and powerful heuristic tools to guess precise and striking new mathematical facts, such as "quantum invariants of knots", from the physical intuition related to, say, "topological quantum field theories" and the respective formalism of path integrals. These facts could afterwards be investigated and put on firm ground by mathematicians: we got "quantum topology", "quantum cohomology" (from string theory), and much more.

If you search in the arXiv, MathSciNet, and Google for the terms I put in quotation marks above, you will find oceans of information about this stuff.

At this point, Plato's apparition intervenes again and sings *"What then?"*

And I respond: *"For us, there is only the trying. The rest is not our business."*[1]

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Book Review

Solving Mathematical Problems: A Personal Perspective

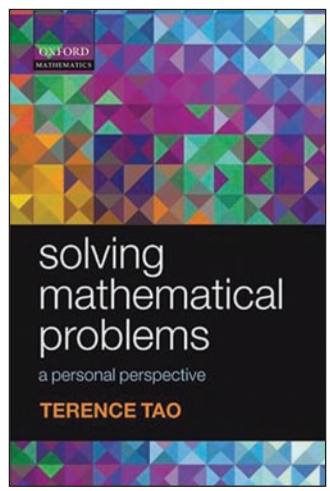
Reviewed by Loren Larson

Solving Mathematical Problems: A Personal Perspective
Terence Tao
Oxford University Press, September 2006
US$34.99, 128 pages
ISBN-13:978-0199205608

In 1980 Paul Halmos concluded his *Monthly* article "The Heart of Mathematics" [1], with this thought:

> I do believe that problems are the heart of mathematics, and I hope that as teachers, in the classroom, in seminars, and in the books and articles we write, we will emphasize them more and more, and that we will train our students to be better problem posers and problem solvers than we are.

Now, thirty years later, I think he would be pleased with what has transpired. Problem solving is central in current mathematics education, from kindergarten through middle school, from high school through college and beyond. Learning mathematics means *doing* mathematics, a mix of practice *exercises* to develop skills and *problems* for deeper understanding. The proportion of each depends on the context, anything from 100/0 for crash courses where the problems will come later (in more than one sense?) to 0/100 for Moore-method courses where mastering techniques will come later. Both are necessary, but from a pedagogical point of view, appropriately chosen problems under the right conditions are more fun and offer more satisfaction, at least for mathematically inclined students. To paraphrase Albrecht Dürer, "love and delight are better teachers than compulsion."



One means of promoting problem solving is by organizing math contests, and there are now a variety of contests and plenty of opportunities for participation. These contests are supported by a vast literature of preparatory supplements: anthologies, compilations, how-to-solve books, special topics, problem-solving strategies, online classes, forums, summer math camps, and videos (e.g., see `http://www.artofproblemsolving.com`). Contests have a proven record for fostering interest in mathematics, but they aren't for everyone. Some students are turned off by competition, or have mathematical interests that aren't particularly amenable to a contest format, or feel that contests favor speed over power (not that they can't coexist). But there are other ways of promoting problem solving and personal involvement, such as problem seminars, independent research, and interdisciplinary projects.

Finding suitable problems is a time-intensive task. Problem books can provide ideas and starting points, for example nontrivial tweaks or special cases of published results. But this approach can lead to stagnation and irrelevance. A better source for keeping ideas meaningful and up-to-date is the

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research community—through personal communication, networking, newsgroups, problem sections of professional journals, and journal articles. Problems for high-level math competitions not only need to be original, but they should also be presented with a certain flair. Composers of such problems have to indulge their feel for language, their artistic temperament, and even their sense of humor. It's in the best interest of our community to encourage this kind of effort, as getting students interested in current research through problems is an important means of renewal. Paul Erdős was a master at introducing rich mathematical ideas by way of simply stated problems. Putnam problems have stayed fresh over the years because of contributions from researchers and inveterate problem enthusiasts. A book by Paul and Judith Sally [4] is helpful in showing how a single mathematical concept can sometimes be adapted and framed into an interesting problem at virtually every level of mathematical expertise.

Terence Tao, the author of the book under review, is one of math's luminaries, a winner of the Fields Medal in 2006. If you aren't familiar with his prodigious accomplishments and life as a child prodigy, by all means check out his amazing story. This book was written when he was just fifteen years old, but even by this age he was well-qualified for the task: he had already competed in three International Mathematical Olympiads, winning a bronze medal, a silver medal, and then a gold medal just days after his thirteenth birthday.

In the narrowest sense, the book is aimed at showing high school students how to solve Math-Olympiad-like problems. Almost all the problems are taken from published collections of problem sets for mathematics competitions. In the Preface to this second edition, the mature Tao at age thirty comments that if he were to write a problems book now it would be very different. But he resisted the temptation to tamper with it except for a few organizational changes, noting that "his younger self was almost certainly more attuned to the world of the high-school problem solver." Indeed, one of the most charming features of the book is the exuberant exposition, which the elder Tao points out "[sometimes] has a certain innocence, or even naiveté." Here are some delightful excerpts taken from chapter previews (and if you've worked with precocious young people, you'll recognize the voice):

> **Algebra:** Algebra is the basic foundation of a large part of applied mathematics. Problems of mechanics, economics, chemistry, electronics, optimization, and so on are answered by algebra and differential calculus, which is an advanced form of algebra. In fact, algebra is so important that most of its secrets have been discovered—so it can be safely put into a high school curriculum. However, a few gems can still be found here and there.
>
> **Number Theory:** Unlike algebra, which has as its backbone the laws of manipulating equations, number theory seems to derive its results from a source unknown. Take, for example, Lagrange's Theorem, … . Number theory is a fundamental cornerstone which supports a sizeable chunk of mathematics.
>
> **Analysis:** Analysis is the study of functions and their properties. …Functional equations form a sort of "pocket mathematics", where instead of the three dozen or so axioms and countless thousands of theorems, one has only a handful of "axioms" (i.e., data) to use and there is a clear direction in which to go. And yet, it still has surprises.
>
> **Geometry:** The true beauty of geometry is in how a non-obvious-looking fact can be shown to be undeniably true by repeated application of obvious facts. As an example: the midpoints of the four sides of a quadrilateral always make up a parallelogram. These facts—they have a certain something about them.

Statements like these have to be put into context. Remember that his target reader is someone like himself at age nine, intensely eager and capable and focused on learning as much as possible about Olympiad-level math problems. A certain panache and dash, even some exaggeration and eccentricity, is expected at this age. I was all ears, a teenager again, fired-up and flattered to be treated as an equal. His intentions for the book are laid out in the Preface:

> I will try to demonstrate some tricks of the trade when problem solving. Two of the main weapons—experience and knowledge—are not easy to put into a book: they have to be acquired over time. But there are many simpler tricks that take less time to learn. There are ways of looking at a problem that make it easier to find a feasible attack plan. There are systematic ways of reducing a problem into successively easier subproblems.

Math "how-to" books usually focus on specific applications of methods given by George Pólya [3], and this book is no exception—the intention is to motivate the solution. But the manner in which Tao does it is the book's most distinguishing feature. For not only does Tao motivate the solution but he also walks through his entire thinking process, including rejected ideas and false starts. He seems to be thinking aloud, explaining as he goes—each journey an apparently effortless flow of ideas. It's a slim book, about 100 pages, but the problems are well chosen, only twenty-six of them, with about the same number of instructive exercises, without solutions. The solutions to the problems are almost incidental to Tao's observations and reformulations along the way. He shows us how to think like mathematicians. For example, the actual solution to Problem 2.2 (Is there a power of 2 whose digits could be rearranged and made into another power of 2?) takes only six *lines*, but it is preceded by five *pages* of discussion.

The defining characteristic of Tao's perspective on problem solving is based on Pólya's dictum, nicely stated by Halmos [1]:

> Make it easier. In slightly greater detail: if you cannot solve a problem, then there is an easier problem that you cannot solve, and your first job is to find it! Make it sharp. By that I mean: do not insist immediately on asking the natural question ("what is…?", "when is…?", "how much is…?"), but focus first on an easy (but nontrivial) yes-or-no question ("is it…?").

Essentially, this approach is analogous to the *Bolzano-Weierstrass Method* for catching a lion in the desert [2]: Repeatedly bisect the area with alternating vertical and horizontal lines, choosing the half that contains the lion. At each stage, build a fence. The lion is ultimately enclosed by a fence of arbitrarily small perimeter.

In other words, first you have to survey the desert, that is, understand the problem and write down *everything* you can think of that might be relevant. This is especially important in problems at this level because the most obvious beginnings probably won't work. Then, with this list in hand, you start bisecting, eliminating ideas that appear to make things more difficult. Iterate on this process, systematically asking questions and reducing it into easier subproblems until the tricky parts are no longer tricky. "Make it easy" is the theme of the book, which also makes it fun to read. Expressed more flamboyantly by Tao:

> So that is it. We keep reducing the equation into simpler and simpler formulas, until it just collapses into nothing. A bit of a long haul, but sometimes it is the only way to resolve these very complicated questions: step by step reduction.
>
> Simplify repeatedly until the more unusable and unfriendly parts of the problem are exchanged with more natural, flexible, and cooperative methods.
>
> It is best to try elementary techniques first, as it may save a lot of dashing about in circles later.
>
> As long as one always tries to simplify and connect, chances are that the solution will soon fall into place. (Assuming, of course, that there is one—and most problems are not trying to pull your leg.)

Here's an example of how it plays out: One of the geometry problems asks you to prove that either the given triangle is isosceles or a particular angle is 60°. Tao quickly rules out coordinate geometry: "Hack-and-slash coordinate geometry is one long and boring way that is prone to abysmal complications and huge errors. Let us try that as a last resort." Realizing that the conclusion involves angles and the given data involves equal line segments, he needs results that relate length and angle. So he writes down relevant facts about right triangles, isosceles triangles, the law of cosines, the law of sines. There aren't a lot of right triangles in the figure so let's not create any yet. The equal line segments aren't part of an isosceles triangle that directly connects to the conclusion, so we'll pass on that for now. He continues: "The law of cosines usually complicates rather than simplifies, and it just creates more unknown lengths. This leaves only the law of sines as a feasible alternative." This leads to equations, then to further manipulations and reformulations and simplifications driven by the overall objective, and finally to the relevant angles.

Granted, this step-by-step reduction is particularly effective for Olympiad-like problems, and Tao acknowledges this in the original Preface:

> [Olympiad-like] mathematics problems are "sanitized" mathematics, where an elegant solution has already been found, the question is stripped of all superfluousness and posed in an interesting and (hopefully) thought-provoking way. If mathematics is likened to prospecting for gold, solving a good mathematics problem is akin to a "hide-and-seek" course in gold-prospecting: you are given a nugget to find, and you know what it looks like, that it is out there somewhere, that it is not too hard to reach, that its unearthing is within your capabilities, and you have conveniently been given the right equipment (i.e., data) to get it. It may be hidden in a cunning place,

but it will require ingenuity rather than digging to get it.

Contest problems provide a setting in which to acquire the habit of thinking mathematically. Here the psychological advantage makes it ideally suited for developing a way of thinking that will transfer to "unsanitized" mathematics, the habit of doing what you can and thinking ahead about how the complicated parts might be simplified.

The book assumes that the reader has a strong background in basic arithmetic and algebra (modular arithmetic, factorization formulas, elementary trig identities), standard Euclidean geometry, including elementary vector methods, and properties of polynomials and functions, as well as mathematical induction. If Tao were fifteen today his book would probably have included more topics from discrete math, such as the pigeonhole principle, elementary graph theory (Euler's formula $V - E + F = 2$, Euler and Hamiltonian circuits), and combinatorics (counting principles, recurrences). Nevertheless, the overall problem-solving approach would be the same, and it is largely this feature, along with the thorough, caring, and energizing delivery, that makes the book a noteworthy addition to the literature on problem solving and how to teach it.

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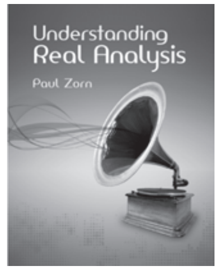

Simons Foundation Launches US$40-Million Program for Theoretical Research

Allyn Jackson

The Simons Foundation has launched a program that is now spending an estimated US$40 million a year to support research in mathematics and theoretical aspects of areas related to mathematics. The program began in fall 2009, creating seventy postdocs intended to ease the effects of the tight academic job market.

In mathematics, US$40 million is a serious chunk of change: For comparison, the yearly budget of the Division of Mathematical Sciences at the National Science Foundation (NSF) was US$226 million in fiscal year 2009. The Simons Foundation program will not be solely devoted to mathematics but will also fund "theoretical science radiating from mathematics", as David Eisenbud put it. Eisenbud, a professor at the University of California Berkeley and former director of the Mathematical Sciences Research Institute (MSRI), is heading up the program as the Simons Foundation Vice President for Mathematics and the Physical Sciences.

James Simons, of Chern-Simons invariant fame, has made billions through his investment company, Renaissance Technologies. In 1994 he and his wife Marilyn established the Simons Foundation, which supports basic science and mathematics. The Simons fortune also funds Math for America, a separate entity from the Simons Foundation that focuses on attracting and retaining outstanding individuals to teach mathematics in public secondary schools. In 2009 James Simons, seventy-one years old, announced he is retiring from day-to-day management of the firm he has run for thirty-one years and will spend more time on his philanthropic work, which also includes support for autism research.

"The mission of the Simons Foundation is to support basic scientific research," said James Simons in an email message. "There is already an active program in the life sciences, and several important but isolated activities in mathematics and the physical sciences, such as the newly founded Center for Geometry and Physics, but there is no coherently articulated program in the latter area. David Eisenbud was brought on to establish such a program, and over the next several months he will be gathering ideas and advice to guide him in carrying out this mission." (For more information on the center, see "Major Gift to Stony Brook for Simons Center in Geometry and Physics", *Notices*, June/July 2008.)

The new program does not have a name yet—nor has it been decided exactly how the US$40 million will be used. But the driving idea is to "strengthen basic research internationally and across science", Eisenbud said. The plan is to structure the program so that it complements rather than duplicates modes of support already available through existing funders of theoretical research, such as the NSF. Private sources of support for mathematics include the Clay Mathematics Institute, which provides funding for individual mathematicians and specific projects like conferences, and the electronics-chain-store magnate John Fry, who is the main funder of the American Institute of Mathematics in Palo Alto, California. The Simons Foundation program will be "substantially larger" than what these other private funders provide, Eisenbud said.

As a first step in establishing the program, the foundation decided to fund about seventy

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postdoctoral positions, called Simons Postdoctoral Fellowships. Fifteen three-year positions in mathematics and ten three-year positions in theoretical physics will be funded this year and next; in theoretical computer science, nine two-year positions will be offered for three consecutive years. According to Eisenbud, the breakdown of positions was dictated by a sense of the "size of the enterprise" in each area, partly measured by the number of new doctorates in each. With about 1,400 new doctorates last year, mathematics had by far the largest number. The funding will be given to university departments, each of which will be able to hire one or two postdocs recruited from anywhere in the world. Recruitment began in fall 2009, and the positions will start in fall 2010.

The Simons Foundation has decided not to announce publicly the full list of departments receiving the postdoc funds. The selection of mathematics departments was made by a five-member committee of "distinguished mathematicians", Eisenbud said; parallel committees in theoretical physics and theoretical computer science made the selections in those areas. The awards were made to departments that can nurture postdocs well; they were "*not* seen as rewards to great departments," Eisenbud noted. The departments were chosen on the basis of "whether the committee felt that people in a reasonably broad range of subjects would advise their graduate students to go to those departments," he explained.

These postdoctoral positions are sorely needed in the job market facing new and recent Ph.D.s, a market that Eisenbud described as "gruesome". "The number of postdoctoral positions is down at research universities" by as much as one-half, he said. "This is something we will feel for years." A survey conducted in early 2009 by an AMS task force on employment estimated that the number of academic positions open to new doctorates had declined by 39 percent from summer 2008 to summer 2009 (see `http://www.ams.org/prof-services/employtaskforce/ETF.html` for the task force report). The NSF pitched in to help in 2009 with a set of temporary postdoctoral positions managed jointly by the mathematical sciences research institutes. These postdocs will probably not be repeated in 2010, absent the flexibility provided by funds from the Obama administration's one-time stimulus package.

The Simons Foundation decided to give the funds for the postdocs to departments, rather than to individuals, because there was not time to set up a mechanism for reviewing individual applications. In the future, if it does fund more postdocs, it will have a mechanism for direct application by individuals. Will the program in fact continue to fund postdocs? That is not clear yet—and indeed only a few aspects of the program have been decided upon. One of these is the areas to be supported: mathematics and theoretical topics in areas connected to it, such as theoretical physics and theoretical computer science (but not the biological sciences, which are the focus of a different Simons Foundation program). Support for large experimental facilities is not within the scope of the program, but Eisenbud said that it might be possible to fund some experimental projects "if the right project comes along". Researchers from anywhere in the world are eligible for support. Unlike grants from the NSF, which pay overhead rates set by the institutions receiving the grants, the grants from the Simons Foundation program will pay only the foundation's standard overhead rate of 20 percent of certain direct costs.

Exactly what kinds of activities the program will fund has yet to be determined. Asked whether the money might be used for a new institute, Eisenbud said that he thought it unlikely, given that the foundation just recently launched the Simons Center. However, the foundation has been giving funds to enhance activities at some institutes, and this might continue. How about a small grants program, which many mathematicians in the United States have said is what they really need? That is a possibility. "Nothing is settled yet," Eisenbud said.

"Figuring out something really useful and really effective to do, even with a lot of money, is, I think, not so easy," Eisenbud remarked. To this end the Simons Foundation will in coming months hold several roundtable events in which mathematicians and scientists "will offer us, we hope, sage advice about how to spend this money," Eisenbud said. "We will listen hard before we make any decisions." The foundation will also work closely with organizations such as the NSF and the Clay Mathematics Institute to ensure that the new program complements rather than duplicates existing ones.

Eisenbud said he would be interested in hearing from researchers who have ideas for how best to use the funds (his email address is `de@simonsfoundation.org`). It is not appropriate at this stage to send proposals for specific research projects; what is needed at this time are ideas about what kind of grant mechanisms would be most effective. Input from a broad segment of the mathematical community can help ensure that this unusual program has a lasting and positive impact on the field.

2009 Annual Survey of the Mathematical Sciences

(First Report)

Preliminary Report on the 2008–2009 New Doctoral Recipients

Polly Phipps, James W. Maxwell, and Colleen Rose

This report presents a statistical profile of recipients of doctoral degrees awarded by departments in the mathematical sciences at universities in the United States during the period July 1, 2008, through June 30, 2009. All information in the report was provided over the summer and early fall of 2009 by the departments that awarded the degrees. The report includes a preliminary analysis of the fall 2009 employment plans of 2008–09 doctoral recipients and a demographic profile summarizing characteristics of citizenship status, gender, and racial/ethnic group. This preliminary report will be updated in the Second Report of the 2009 Annual Survey to reflect subsequent reports of additional 2008-2009 doctoral recipients from the departments that did not respond in time for this report. No adjustments have been made to the numbers in this report for the non-responding departments. The Second Report, to appear in the August 2010 issue of *Notices*, will also reflect additional information provided by the new doctoral recipients themselves, including their starting salaries.

Table 1 provides the number of departments responding to the 2009 Survey of New Doctoral Recipients in time for this report. This year's response rates were above 80% for all groups except Group IV which was 74% (up from 67% last year.) Overall, eighteen more departments responded in time for the First Report this year than responded in time for last year's First Report. Efforts continue to obtain data from as many of the non-responding departments as possible.

Table 1: Number of Departments Responding to Doctorates Granted Survey

Group I (Pu)	23 of 25 including	0 with no degrees
Group I (Pr)	21 of 23 including	0 with no degrees
Group II	50 of 56 including	3 with no degrees
Group III	72 of 81 including	18 with no degrees
Group IV	68 of 92 including	4 with no degrees
Statistics	45 of 57 including	2 with no degrees
Biostatistics	23 of 36 including	2 with no degrees
Group Va	18 of 21 including	0 with no degrees

See "Definitions of the Groups" on page 258.

Table 2: New Doctoral Degrees Awarded by Group, Preliminary Count

Group	I (Pu)	I (Pr)	II	III	IV	Va	TOTAL
1999–00	256	157	223	132	284	67	**1119**
2000–01	233	129	203	125	237	81	**1008**
2001–02	218	139	164	124	222	81	**948**
2002–03	258	138	170	121	239	91	**1017**
2003–04	195	187	215	111	243	90	**1041**
2004–05	243	146	203	153	285	86	**1116**
2005–06	307	184	216	140	287	111	**1245**
2006–07	300	119	234	138	279	87	**1157**
2007–08	234	172	290	142	291	106	**1235**
2008–09	326	211	282	171	352	88	**1430**

Doctoral Degrees Granted in 2008–09

Table 2 shows the number of new doctoral degrees granted by the different doctoral groups surveyed in the Annual Survey for the past ten years. The preliminary count of 1,430 new doctorates granted by these departments in 2008–09 is an increase of 195 from the preliminary count for 2007–08.

From Table 2 we see that all groups except Groups II and Va reported an increase in the number of doctoral recipients from the previous year. The

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decrease reported for Group II is almost certainly the result of the three departments in Group II that responded in time for last year's report but not this year's report. These departments have awarded an average of 17 doctoral degrees each year over the past four years. The final count for 2008–09 is likely to be very close to 1,500.

The 2008–09 numbers in Table 2 will be broken down in various ways, such as by gender, in later sections of this report. The names of the 1,430 new doctoral recipients are found on pages 276–99 of this issue of the *Notices*.

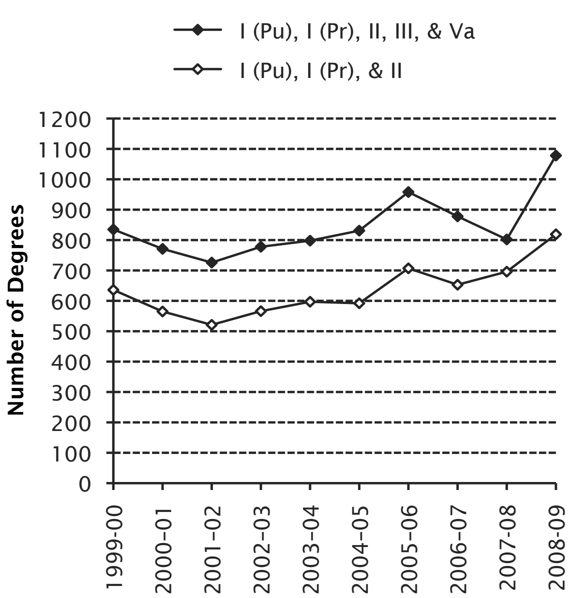
By way of background, additional information about various types of full-time graduate students is available in the Third Report of the 2008 Annual Survey (*Notices*, November 2008), Table 6B, page 1299.

Employment Plans of 2008–09 New Doctoral Recipients

Tables 4A and 4B each provide a cross-tabulation of the 1,430 new doctoral recipients in the mathematical sciences. These tables contain a wealth of information about these new doctoral recipients, some of which will be discussed in this report. Note that these tables give a breakdown by gender for type of employer and type of degree-granting department. Additional information is available on the AMS website at `www.ams.org/employment/surveyreports.html`.

The preliminary unemployment rate for these data is 7.9%. This preliminary rate will be updated later with information gathered from the

Figure 1: New Doctoral Degrees Awarded by Combined Groups, Preliminary Count



Highlights

There were 1,430 new doctoral recipients reported for 2008-09 by departments responding in time for the 2009 First Report. The number of departments responding in time for this year's report increased by 8 and in every group except Group IV. When one considers only the 208 departments that responded in time for the First Report in both years, the 2009 figure reported by these departments is up 9.7% over that reported for 2008.

There were 669 U.S. citizens reported among this year's new doctoral recipients, 47% of the total. Last year's figure was 44%.

The fall preliminary 2009 unemployment rate for the 1,231 new doctoral recipients whose employment status is known is 7.9%, up from 5.4% for fall 2008.

Seventy-two new doctoral recipients hold positions at the institution that granted their degree, although not necessarily in the same department. This is 6% of the new doctoral recipients who are currently known to have jobs and 10% of those who have academic positions in the U.S. Seventeen new doctoral recipients have part-time positions.

The number of new doctoral recipients employed in the U.S. is 987, up 101 from last year. The number of new doctoral recipients employed in academic positions in the U.S. has increased to 741, compared to 650 last year.

Of the 987 new doctoral recipients taking positions in the U.S., 184 (19%) have jobs in business and industry, a decrease of 11% from last year's figure of 207. The fall 2009 number remains up 69 (60%) from fall 2005. The number of new doctoral recipients taking jobs in government is up 30 (107%) over fall 2008.

Among the 987 new doctoral recipients having employment in the U.S., 501 (51%) are U.S. citizens (up from 426 (48%) last year). The number of non-U.S. citizens having employment in the U.S. is 486; last year it was 460.

Among the 333 new doctoral recipients hired by U.S. doctoral-granting departments, 47% are U.S. citizens (down from 49% last year). Among the 408 having other academic positions in the U.S., 62% are U.S. citizens (up from 57% last year).

Of the 1,430 new doctoral recipients, 32% (462) are female, the same percentage reported in fall 2008. Of the 669 U.S. citizen new doctoral recipients, 30% (202) are females, down from 31% in fall 2008.

Among the 669 U.S. citizen new doctoral recipients, 4 are American Indian or Alaska Native, 44 are Asian, 19 are Black or African American, 19 are Hispanic or Latino, 1 is Native Hawaiian or Other Pacific Islander, 575 are White, and 7 are of unknown race/ethnicity.

Group IV produced 352 new doctorates, of which 159 (45%) are female, compared to all other groups combined, where 303 (28%) are female. In Group IV, 92 (26%) of the new doctoral recipients are U.S. citizens (while in the other groups 54% are U.S. citizens).

Twenty-nine percent of the new doctorates had a dissertation in statistics/biostatistics (410). The next highest percentage was in algebra and number theory with 14% (203).

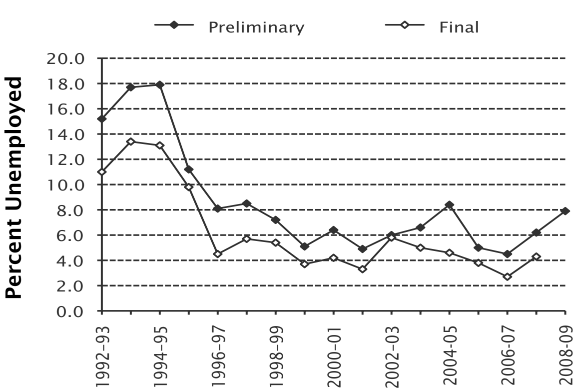
The Faculty Salary Survey report, will appear in the March issue of *Notices*.

Table 4A: Employment Status of 2008–09 New Doctoral Recipients in the Mathematical Sciences by Type of Degree-Granting Department

	TYPE OF DOCTORAL DEGREE-GRANTING DEPARTMENT							Row Subtotals	
TYPE OF EMPLOYER	Group I (Public) Math.	Group I (Private) Math.	Group II Math.	Group III Math.	Group IV Statistics	Group Va Applied Math.	TOTAL	Male	Female
Group I (Public)	41	18	16	2	0	3	**80**	67	13
Group I (Private)	22	38	7	3	1	1	**72**	56	16
Group II	24	10	20	4	4	2	**64**	44	20
Group III	9	3	13	18	0	1	**44**	31	13
Group IV	0	0	3	2	54	1	**60**	37	23
Group Va	5	1	0	0	0	7	**13**	12	1
Master's	11	3	19	10	5	2	**50**	30	20
Bachelor's	34	16	59	30	16	5	**160**	92	68
Two-Year College	3	1	4	12	0	2	**22**	16	6
Other Academic Dept.	21	10	15	15	55	18	**134**	80	54
Research Institute/ Other Nonprofit	11	4	2	1	20	4	**42**	25	17
Government	7	1	10	13	27	4	**62**	35	27
Business and Industry	31	17	23	13	85	15	**184**	124	60
Non-U.S. Academic	38	37	22	9	17	6	**129**	99	30
Non-U.S. Nonacademic	4	3	2	1	3	2	**15**	10	5
Not Seeking Employment	3	4	2	1	5	0	**15**	11	4
Still Seeking Employment	22	13	18	18	9	5	**85**	64	21
Unknown (U.S.)	21	12	29	8	31	8	**109**	74	35
Unknown (non-U.S.)*	19	20	18	11	20	2	**90**	61	29
TOTAL	**326**	**211**	**282**	**171**	**352**	**88**	**1430**	**968**	**462**
Column Subtotals Male	262	166	185	100	193	62	968		
Column Subtotals Female	64	45	97	71	159	26	462		

*Includes those whose status is reported as "unknown" or "still seeking employment".

Figure 2: Percentage of New Doctoral Recipients Unemployed in the U.S.



*Excludes those whose status is reported as "unknown" , "not seeking" or having employment outside the U.S. (Non-U.S. Academic or Non-U.S. Nonacademic). This is a change from prior reports which excluded only the unknown categories.

individual new doctoral recipients. The additional information from prior years is reflected in the final unemployment rates displayed in Figure 2. The preliminary rate for fall 2008 was 6.0%. The unemployment rates shown in Figure 2 differ from those given in previous Annual Survey reports. The rates shown are now based on only those 1,072 individuals in the U.S. labor market. For further details, see the explanatory note on unemployment rates at the end of the report. The unemployment rates, calculated by type of doctoral degree-granting department using Table 4A, vary from group to group, with a high of 12.8% for Group III and a low of 3.3% for Group IV.

There are 987 new doctoral recipients employed in the U.S. Table 5A gives a breakdown of type of employer by type of degree-granting department for these 987 new doctoral recipients. Of these, 741 (75%) hold academic positions, 62 (6%) are employed by government, and 184 (19%) hold positions in business and industry. In the First Report for 2007–08, there were 886 new doctoral recipients employed in the U.S., of which 649 (73%) held academic positions, 30 (3%) were in government, and 207 (23%) were in business and industry. The number of new doctoral recipients employed in the U.S. increased in all categories except "Business and Industry" which decreased 11%. "Government" showed the largest percentage increase, 107%.

Table 5B shows the number of new doctoral recipients who took positions in business and industry by the type of department granting their degree for fall 2005 to fall 2009. The number of new doctoral recipients taking jobs in business and industry, which had been steadily increasing, decreased from 207 to 184, an 11%

Table 4B: Field of Thesis of 2008–09 New Doctoral Recipients by Type of Degree-Granting Department

TYPE OF DOCTORAL DEGREE-GRANTING DEPARTMENT		FIELD OF THESIS												TOTAL
		Algebra/ Number Theory	Real, Comp., Funct., & Harmonic Analysis	Geometry/ Topology	Discr. Math./ Combin./ Logic/ Comp. Sci.	Probability	Statistics/ Biostat.	Applied Math.	Numerical Analysis/ Approxi-mations	Linear Nonlinear Optim./ Control	Differential, Integral, & Difference Equations	Math. Educ.	Other/ Unknown	
Group I (Public)		77	29	56	34	23	11	40	21	2	31	1	1	**326**
Group I (Private)		52	10	53	24	19	3	22	7	1	16	0	4	**211**
Group II		57	30	28	32	19	12	42	27	6	28	1	0	**282**
Group III		16	22	9	20	4	32	16	16	4	21	11	0	**171**
Group IV		0	0	0	0	4	343	4	0	0	0	0	1	**352**
Group Va		1	1	0	12	8	9	36	12	5	4	0	0	**88**
Column Total		**203**	**92**	**146**	**122**	**77**	**410**	**160**	**83**	**18**	**100**	**13**	**6**	**1430**
Column Subtotals	Male	153	68	112	86	52	227	113	62	15	73	4	3	**968**
	Female	50	24	34	36	25	183	47	21	3	27	9	3	**462**

drop. Among the 987 new doctoral recipients known to have employment in the U.S. in fall 2009, Group III has the smallest percentage taking jobs in business and industry at 11% and Group IV the highest at 32%.

Table 5C shows the number of new doctoral recipients who took academic positions in the U.S. by type of department granting their degree for fall 2005 to fall 2009. The total number of new doctoral recipients taking academic employment in fall 2009 increased 14% to 741 from 649 last year. Among the 987 new doctoral recipients employed in the U.S. in fall 2009, 75% have academic positions. This percentage is highest for Group I (Pr) at 85% and lowest for Groups IV at 58%.

Table 5D shows the number of positions filled with new doctoral recipients for each type of academic employer. Increases in positions filled by new doctoral recipients were realized by all groups. The biggest increase in hires of new doctorates into academic positions was in Groups IV and Va with 43% and 44%, respectively. Hires of new doctorates into positions at research institutes increased from 29 in fall 2008 to 42 in fall 2009.

In fall 2009, 72 new doctoral recipients held positions in the institution that granted their degree, although not necessarily in the same department. This represents 6% of new doctoral recipients who are currently employed in the U.S. and 10% of the U.S. academic positions held by new doctoral recipients. In fall 2008 there were 69 such individuals making up 7% of the new doctoral recipients who were employed at the time of the First Report. Seventeen new doctoral recipients have taken part-time positions in fall 2009 compared with 18 in fall 2008.

Information about 2008–09 Female New Doctoral Recipients

Tables 4A and 4B give the breakdown of the new doctoral recipients in 2008–09 by Field of Thesis, by Type of Degree-Granting Department and by Type of Employer.

Overall, 462 (32%) of the 1,430 new doctoral recipients in 2008–09 are female. In 2007–08, 388 (31%) of the new doctoral recipients were female. This percentage varies over the different groups, and these percentages are given in the first row of Table 5E. This year the percentage of females produced is highest again for Group IV at 45%, compared with 52% last year. The second row of Table 5E gives the percentage of the new doctoral recipients hired who are female for each of the Groups I, II, III, IV, and Va. In addition, 40% of the

Table 5A: 2008–09 New Doctoral Recipients Employed in the U.S. by Type of Degree-Granting Department

	Group						
Type of Employer in U.S.	I (Pu)	I (Pr)	II	III	IV	Va	TOTAL
Groups I, II, III, IV, and Va	101	70	59	29	59	15	**333**
Master's, Bachelor's, and 2-Year Colleges	48	20	82	52	21	9	**232**
Other Academic and Research Institutes	32	14	17	16	75	22	**176**
Government	7	1	10	13	27	4	**62**
Business and Industry	31	17	23	13	85	15	**184**
TOTAL	**219**	**122**	**191**	**123**	**267**	**65**	**987**

Table 5B: Number of New Doctoral Recipients Taking Positions in Business and Industry in the U.S. by Type of Degree-Granting Department, Fall 2005 to Fall 2009

	Group						
Year	I (Pu)	I (Pr)	II	III	IV	Va	TOTAL
Fall 2005	5	9	14	15	64	8	**115**
Fall 2006	27	14	19	9	80	18	**167**
Fall 2007	39	10	16	19	88	15	**187**
Fall 2008	24	19	32	22	87	23	**207**
Fall 2009	31	17	23	13	85	15	**184**

Table 5C: Number of New Doctoral Recipients Taking U.S. Academic Positions by Type of Degree-Granting Department, Fall 2005 to Fall 2009

	Group						
Year	I (Pu)	I (Pr)	II	III	IV	Va	**TOTAL**
Fall 2005	131	88	130	83	131	39	**602**
Fall 2006	167	108	123	86	137	50	**671**
Fall 2007	178	76	146	87	120	43	**650**
Fall 2008	126	90	174	77	133	49	**649**
Fall 2009	181	104	158	97	155	46	**741**

Table 5D: Academic Positions in U.S. Filled by New Doctoral Recipients by Type of Hiring Department, Fall 2005 to Fall 2009

	Group					
Year	I–III	IV	Va	M&B	Other*	**TOTAL**
Fall 2005	231	45	12	188	126	**602**
Fall 2006	262	69	12	185	143	**671**
Fall 2007	264	39	17	186	144	**650**
Fall 2008	256	42	9	180	162	**649**
Fall 2009	260	60	13	210	198	**741**

*Includes other academic and research institutes/nonprofit.

Table 5E: Females as a Percentage of 2008–09 New Doctoral Recipients Produced by and Hired by Doctoral-Granting Groups

	Group						
Percent	I (Pu)	I (Pr)	II	III	IV	Va	**TOTAL**
Produced	20%	21%	34%	42%	45%	30%	32%
Hired	16%	22%	31%	30%	38%	8%	26%

new doctoral recipients hired in Group M, Master's departments, are female; 43% of the new doctoral recipients hired in Group B, Bachelor's departments, are female, up from 40% last year. This year, Group IV hired the highest percentage of women with 38%, while Groups I, II, III ranged from 16% to 31%.

The unemployment rate for female new doctoral recipients is 5.8%, compared to 9.0% for males and 7.9% overall.

The percentage of female new doctoral recipients within fields of thesis ranged from 19% in linear, nonlinear optimization/control, to 69% in mathematics education and 45% in statistics.

Later sections in this First Report give more information about the female new doctoral recipients by citizenship and the female new doctoral recipients in Group IV.

Employment Information about 2008–09 New Doctoral Recipients by Citizenship and Type of Employer

Table 5F shows the pattern of employment within employer categories broken down by citizenship status of the new doctoral recipients.

The unemployment rate for the U.S. citizens is 8.6% compared to 5.8% in fall 2008. The unemployment rate for non-U.S. citizens is 7.3%. This varies by type of visa. The unemployment rate for non-U.S. citizens with a permanent visa is 8.3%, while that for non-U.S. citizens with a temporary visa is 7.2%. Among U.S. citizens whose employment status is known, 84% are employed in the U.S.

Table 5F: Employment Status of 2008–09 New Doctoral Recipients by Citizenship Status

	CITIZENSHIP				
		NON-U.S. CITIZENS			
TYPE OF EMPLOYER	U.S. CITIZENS	Permanent Visa	Temporary Visa	Unknown Visa	**TOTAL**
U.S. Employer	501	66	414	6	987
U.S. Academic	412	45	280	4	741
Groups I, II, III, and Va	135	14	122	2	273
Group IV	23	6	31	0	60
Non-Ph.D. Department	238	20	107	1	366
Research Institute/Other Nonprofit	16	5	20	1	42
U.S. Nonacademic	89	21	134	2	246
Non-U.S. Employer	39	3	99	3	144
Non-U.S. Academic	36	3	87	3	129
Non-U.S. Nonacademic	3	0	12	0	15
Not Seeking Employment	8	0	7	0	15
Still Seeking Employment	47	6	32	0	85
SUBTOTAL	595	75	552	9	1231
Unknown (U.S.)	72	8	29	0	109
Unknown (non-U.S.)*	2	1	80	7	90
TOTAL	**669**	**84**	**661**	**16**	**1430**

*Includes those who left the U.S. and whose employment status is reported as "unknown" or "still seeking employment".

Table 5G: 2008–09 New Doctoral Recipients Having Employment in the U.S. by Type of Employer and Citizenship

U.S. EMPLOYER	CITIZENSHIP U.S.	CITIZENSHIP Non-U.S.	TOTAL
Academic	412	329	741
Groups I–Va	158	175	333
M, B, & 2-Year	166	66	232
Other Acad. & Research Inst.	88	88	176
Government, Business & Industry	89	157	246
TOTAL	**501**	**486**	**987**

Among non-U.S. citizens with a permanent visa whose employment status is known, 88% have jobs in the U.S. (last year this percentage was 90%), while the similar percentage for non-U.S. citizens with a temporary visa is 75% (last year the percentage was 76%). The number of non-U.S. citizens having employment in the U.S. is 486, up from 459 last year.

Table 5G is a cross-tabulation of the 987 new doctoral recipients who have employment in the U.S. by citizenship and broad employment categories, using numbers from Table 5F. Of the 987 new doctoral recipients having jobs in the U.S., 51% are U.S. citizens (up from 48% last year). Of the 333 new doctoral recipients who took jobs in U.S. doctoral-granting departments, 47% are U.S. citizens (down from 49% last year). Of the 408 who took other academic positions, 62% are U.S. citizens (up from 57% last year). Of the 246 who took nonacademic positions, 36% are U.S. citizens. Of the 501 U.S. citizens employed in the U.S., 32% have jobs in a doctoral-granting department, 51% are in other academic positions, and 18% are in nonacademic positions. For the 486 non-U.S. citizens employed in the U.S., the analogous percentages are 36%, 32%, and 32% respectively.

Gender, Race/Ethnicity, and Citizenship Status of 2008–09 New Doctoral Recipients

Table 6 presents a breakdown of new doctoral recipients according to gender, racial/ethnic group, and citizenship status. The information reported in this table was obtained in summary form from the departments granting the degrees. Additional reports on gender, race/ethncity, and citizenship is available on the Web at `www.ams.org/employment/specialreports.html`.

There were 669 (47%) U.S. citizens among the 1,430 new doctoral recipients in 2008–09. Among U.S. citizens, 4 are American Indian or Alaska Native, 44 are Asian, 19 are Black or African American, 19 are Hispanic or Latino, 1 is Native Hawaiian or Other Pacific Islander, 575 are White, and 7 are of unknown race/ethnicity. Among non-U.S. citizens, there are no American Indians or Alaska Natives, 471 Asians, 18 Blacks or African Americans, 49 Hispanics or Latinos, 217 Whites, and 6 of unknown race/ethnicity.

Table 7 gives the number of new U.S. doctoral recipients and the number of U.S. citizens back to 1998-99. The 669 U.S. citizen new doctoral recipients is up by 236 (55%) from its low in 2004–05.

Females make up 30% of the 669 U.S. citizens receiving doctoral degrees in the mathematical sciences in 2008–09. Last year this percentage was 31%. Among the 761 non-U.S. citizen new doctoral recipients, 34% (260) are female, up from last year's 32%. Figure 3 gives the historical record of U.S. citizen female new doctoral recipients and the percentage of females among U.S. citizen (full-time) graduate students beginning in fall 1989. The number of female U.S. citizen new doctoral recipients is up 15 (8%) from 187 in 1998–99; reaching an all time new high. Additional historical information on U.S. citizen doctoral recipients is available on the Web

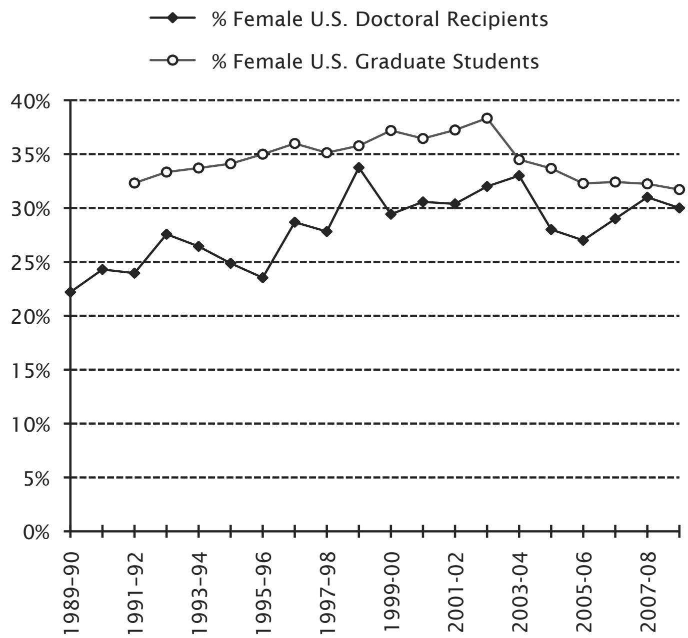
Table 6: Gender, Race/Ethnicity, and Citizenship of 2008–09 New Doctoral Recipients

	MALE					FEMALE					
		NON-U.S. CITIZENS					NON-U.S. CITIZENS				
RACIAL/ETHNIC GROUP	U.S. CITIZENS	Permanent Visa	Temporary Visa	Unknown Visa	Total Male	U.S. CITIZENS	Permanent Visa	Temporary Visa	Unknown Visa	Total Female	TOTAL
American Indian or Alaska Native	0	0	0	0	0	4	0	0	0	4	**4**
Asian	31	14	263	4	312	13	26	161	3	203	**515**
Black or African American	8	3	12	1	24	11	1	1	0	13	**37**
Hispanic or Latino	10	4	35	1	50	9	0	9	0	18	**68**
Native Hawaiian or Other Pacific Islander	0	0	0	0	0	1	0	0	0	1	**1**
White	413	24	131	3	571	162	12	43	4	221	**792**
Unknown	5	0	6	0	11	2	0	0	0	2	**13**
TOTAL	**467**	**45**	**447**	**9**	**968**	**202**	**39**	**214**	**7**	**462**	**1430**

Table 7: U.S. Citizen Doctoral Recipients, Preliminary Counts

Year	Total Doctorates Granted by U.S. Institutions	Total U.S. Citizen Doctoral Total	%
1998–99	1133	554	49%
1999–00	1119	537	48%
2000–01	1008	494	49%
2002–03	1017	489	48%
2003–04	1041	441	42%
2004–05	1116	433	39%
2005–06	1245	522	42%
2006–07	1157	500	43%
2007–08	1235	540	44%
2008-09	1430	669	47%

Figure 3: Females as a Percentage of U.S. Citizen Doctoral Recipients and Graduate Students, Preliminary Counts



at www.ams.org/employment/specialreports.html.

2008–09 New Doctoral Recipients with Dissertations in Statistics/Biostatistics and Probability

Group IV contains U.S. departments (or programs) of statistics, biostatistics, and biometrics reporting a doctoral program. In the Annual Survey Reports, Group IV is referred to as the Statistics Group. In addition, other groups in the Annual Survey produce new doctoral recipients with dissertations in statistics/biostatistics or probability. The other groups produced 140 new doctoral recipients with dissertations in statistics/biostatistics or probability in 2008–09 and have averaged 90 per year over the ten-year period reported in Table 8. Information about these 140 new doctoral recipients and the 347 new doctoral recipients in Group IV is found in this section of the report.

Table 8 contains information about new doctoral recipients in Group IV as well as those with dissertations in statistics/biostatistics and probability in other groups for this year as well as for the past nine years. In addition, the last two rows of Table 8 give a split of the 2008–09 results between the 57 statistics departments and the 35 biostatistics and biometrics departments in Group IV. This year 487 new doctorates had a dissertation in statistics/biostatistics (410) or probability (77), a 27% increase from last year's number. Those with dissertations in statistics/biostatistics and probability accounted for 34% of new doctorates in 2008–09. Quite a bit of the year-to-year variation in these numbers is due to the changes made in the departments included in Group IV over the ten years and to the response rate variation in this group.

Group IV has 92 departments for 2008–09, 11 more than the next largest doctoral group. It contains 31% of all doctoral departments surveyed, and the 68 Group IV departments responding to the Annual Survey reported 352 new doctoral recipients, 25% of all new doctoral recipients in 2008–09. The number of new doctoral recipients in Group IV is up 61 from the number reported at this time last year, while the number of departments responding is up 8 from the number responding by this time last year.

Because of its size, the data from Group IV have a large effect on the results when all doctoral groups are combined. Furthermore, Group IV results are often quite different from those for Groups I (Pu), I (Pr), II, III, and Va. Group IV results can mask important changes in the other doctoral groups. In the following paragraphs some of these differences are presented.

Group IV is producing a larger percentage of female doctorates than the other doctoral groups. Females accounted for 45% of the 352 new doctoral recipients in Group IV, while 28% of 1,078 are female in the other doctoral groups. Among U.S. citizens, females accounted for 59% of the 92 Group IV new doctoral recipients, while for the other groups 26% of 577 were female. Overall, 30% of the 669 U.S. citizen new doctoral recipients were female.

Group IV is producing a smaller percentage of U.S. citizen new doctorates than the other doctoral groups. In Group IV, 36% of the 352 new doctoral recipients are U.S. citizens, while in other groups 50% of the 1,078 are U.S. citizens. In Group IV, 66% of the 159 females were not U.S. citizens.

Group IV doctorates are more likely to take jobs in business and industry than those in other doctoral

Table 8: New Doctoral Recipients with Dissertations in Statistics/Biostatistics and Probability

Year	Group IV Depts Surveyed	Group IV Depts Responding (percent)	New Doctoral Recipients in Group IV only				New Doctoral Recipients in Statistics/Biostatistics and Probability, Group IV and Other* Groups				New Doctoral Recipients Hired by Group IV	
			Total	Female (percent)	Jobs in Bus & Ind	Percentage Unemployed	Total	Group IV	Other Groups	Percentage Unemployed	Male	Female
1999–00	89	75 (84%)	284	110 (39%)	79	2.6%	351	278	73	2.2%	24	22
2000–01	86	70 (81%)	237	98 (41%)	59	5.9%	289	221	68	6.1%	27	14
2001–02	86	72 (84%)	222	92 (41%)	56	6.6%	288	221	67	6.1%	31	15
2002–03	86	74 (86%)	239	98 (41%)	45	2.3%	302	234	68	3.6%	20	19
2003–04	87	65 (75%)	243	97 (40%)	50	3.4%	318	241	77	4.2%	48	15
2004–05	87	63 (72%)	285	126 (44%)	64	5.1%	374	283	91	6.1%	26	19
2005–06	88	60 (68%)	287	134 (47%)	80	1.8%	396	278	118	3.2%	41	28
2006–07	86	50 (58%)	279	127 (46%)	88	3.1%	380	279	101	4.2%	24	15
2007–08	89	60 (67%)	291	151 (52%)	87	1.3%	382	281	101	3.2%	21	21
2008–09	92	68 (74%)	352	159 (45%)	85	3.3%	487	347**	140***	5.0%	37	23
Statistics	57	45 (79%)	245	103 (42%)	74	3.7%					25	12
Biostatistics	35	23 (66%)	107	56 (52%)	11	1.3%					12	11

* Includes other academic departments and research institutes/other nonprofits.
** Of 347, there were 343 in statistics/biostatistics and 4 in probability. For complete details, see Table 4B.
*** Of 140, there were 67 in statistics/biostatistics and 73 in probability. For complete details, see Table 4B.

groups. Of the 267 new doctoral recipients from Group IV who found employment in the U.S., 85 (32%) took jobs in business or industry. From the other groups, 720 new doctoral recipients found employment in the U.S., of which 99 (14%) took jobs in business or industry.

Group IV doctorates have a lower unemployment rate than the other doctoral groups. The employment status for 301 Group IV new doctoral recipients is known, and 9 (3.3%) are unemployed. For the other groups, the employment status of 930 is known, and 66 (9.5%) are unemployed. Group IV is hiring a bigger percentage of females than the other doctoral groups. Twenty-three of 60 (38%) new doctoral recipients hired by Group IV departments were female, down from last year's 50%. The other doctoral groups reported that 63 of 273 (23%) new doctoral recipients hired were female, up from last year's 22%.

Group IV had 347 new doctoral recipients with fields of thesis in statistics/biostatistics (343) and the other doctoral departments had 140 with fields of thesis in statistics/biostatistics (67) and probability (73) (last year the other doctoral departments had 57 new doctorates in statistics and 44 in probability). The distribution of these degrees among the various groups can be found in Table 4B. The number of new doctoral recipients with theses in statistics/biostatistics and probability (487) is substantially larger than any other field, with algebra and number theory next with 203.

Changes in Reporting of Unemployment Rate

In the unemployment calculations provided in this report the individuals employed outside the U.S. have been removed from the denominator used in the calculation of the rate, in addition to the routine removal of all individuals whose employment status is unknown. This is a change from prior Annual Survey Reports. As a consequence, the unemployment rate now being reported more accurately reflects the U.S. labor market experienced by the new doctoral recipients. This change tends to increase the rate of unemployment over that produced in prior years.

In a further small change from prior years, those individuals reported as not seeking employment have also been removed from the denominator. The number of individuals so designated is small each year, and the impact of this change is to produce a slight increase in the rate over that reported in prior years.

The unemployment rates for years prior to 2009 shown in this report have been recalculated using this new method. One can view a comparison of the unemployment rates using the traditional method and the new method by visiting the AMS website at `www.ams.org/employment/surveyreports.html`.

Previous Annual Survey Reports

The 2008 First, Second, and Third Annual Survey Reports were published in the *Notices of the AMS* in the February, August, and November 2009 issues respectively. These reports and earlier reports, as well as a wealth of other information from these surveys, are available on the AMS website at `www.ams.org/employment/surveyreports.html`.

Acknowledgments

The Annual Survey attempts to provide an accurate appraisal and analysis of various aspects of the academic mathematical sciences scene for the use and benefit of the community and for filling the information needs of the professional organizations. Every year, college and university departments in the United States are invited to respond. The Annual Survey relies heavily on the conscientious efforts of the dedicated staff members of these departments for the quality of its information. On behalf of the Annual Survey Data Committee and the Annual Survey Staff, we thank the many secretarial and administrative staff members in the mathematical sciences departments for their cooperation and assistance in responding to the survey questionnaires.

Other Sources of Data

Vist the AMS website at `www.ams.org/employment/specialreports.html` for a listing of additional sources of data on the Mathematical Sciences.

The Annual Survey series begun in 1957 by the American Mathematical Society is currently under the direction of the Data Committee, a joint committee of the American Mathematical Society, the American Statistical Association, the Institute of Mathematical Statistics, the Mathematical Association of America, and the Society of Industrial and Applied Mathematics. The current members of this committee are Richard Cleary, Richard M. Dudley, Susan Geller, John W. Hagood, Abbe H. Herzig, Ellen Kirkman, Joanna Mitro, James W. Maxwell (ex officio), Bart Ng, Polly Phipps (chair), Douglas Ravanel, Jianguo (Tony) Sun, and Marie Vitulli. The committee is assisted by AMS survey analyst Colleen A. Rose. Comments or suggestions regarding this Survey Report may be directed to the committee.

Definitions of the Groups

As has been the case for a number of years, much of the data in these reports is presented for departments divided into groups according to several characteristics, the principal one being the highest degree offered in the mathematical sciences. Doctoral-granting departments of mathematics are further subdivided according to their ranking of "scholarly quality of program faculty" as reported in the 1995 publication *Research-Doctorate Programs in the United States: Continuity and Change.*[1] These rankings update those reported in a previous study published in 1982.[2] Consequently, the departments which now compose Groups I, II, and III differ significantly from those used prior to the 1996 survey.

The subdivision of the Group I institutions into Group I Public and Group I Private was new for the 1996 survey. With the increase in number of the Group I departments from 39 to 48, the Annual Survey Data Committee judged that a further subdivision of public and private would provide more meaningful reporting of the data for these departments.

Brief descriptions of the groupings are as follows:

Group I is composed of 48 departments with scores in the 3.00–5.00 range. Group I Public and Group I Private are Group I departments at public institutions and private institutions respectively.

Group II is composed of 56 departments with scores in the 2.00–2.99 range.

Group III contains the remaining U.S. departments reporting a doctoral program, including a number of departments not included in the 1995 ranking of program faculty.

Group IV contains U.S. departments (or programs) of statistics, biostatistics, and biometrics reporting a doctoral program.

Group V contains U.S. departments (or programs) in applied mathematics/applied science, operations research, and management science which report a doctoral program.

Group Va is applied mathematics/applied science; Group Vb, which was no longer surveyed as of 1998–99, was operations research and management science.

Group M contains U.S. departments granting a master's degree as the highest graduate degree.

Group B contains U.S. departments granting a baccalaureate degree only.

Listings of the actual departments which compose these groups are available on the AMS website at `www.ams.org/employment/groups_des.html`.

[1] Research-Doctorate Programs in the United States: Continuity and Change, *edited by Marvin L. Goldberger, Brendan A. Maher, and Pamela Ebert Flattau, National Academy Press, Washington, DC, 1995.*

[2] *These findings were published in* An Assessment of Research-Doctorate Programs in the United States: Mathematical and Physical Sciences, *edited by Lyle V. Jones, Gardner Lindzey, and Porter E. Coggeshall, National Academy Press, Washington, DC, 1982. The information on mathematics, statistics, and computer science was presented in digest form in the April 1983 issue of the* Notices *of the AMS, pages 257–67, and an analysis of the classifications was given in the June 1983* Notices *of the AMS, pages 392–3.*

Mathematics People

MacPherson Awarded Hopf Prize

ROBERT MACPHERSON of the Institute for Advanced Study has been chosen the first winner of the Heinz Hopf Prize given by ETH Zurich for outstanding scientific work in the field of pure mathematics. MacPherson, a leading expert in singularities, delivered the Heinz Hopf Lectures, titled "How Nature Tiles Space", in October 2009. The prize also carries a cash award of 30,000 Swiss francs, approximately equal to US$30,000.

The following quotation was taken from a tribute to MacPherson by Gisbert Wüstholz of ETH Zurich: "Singularities can be studied in different ways using analysis, or you can regard them as geometric phenomena. For the latter, their study demands a deep geometric intuition and profound geometric insight; this is what MacPherson masters in a most striking and extremely artful way. He combines geometric visions with algebraic rigidity. If you study his work you see that it is glowing with elegance and profound in its depth.

"He consistently was ahead of his time, developing new ideas and new approaches—ones often not shaped by the main streams of mathematical thought of the day, but rather characterized by great vision. Repeatedly the mathematical community came to embrace, extend, and apply his ideas and results as they caught up with that vision.

"MacPherson's connections with the Swiss mathematical community date back to 1983, when he participated in and significantly contributed to the famous Borel seminar, a joint seminar organized by several Swiss universities, including ETH Zurich, Lausanne, Geneva, Bern, and Basel. The seminar was initiated by Armand Borel, one of the most distinguished Swiss mathematicians of the last century. The topic of the seminar was the Goresky-MacPherson intersection homology and its use for the cohomology of arithmetic groups, one of the main research areas of Armand Borel.

"This illustrates only a small part of the work of MacPherson, for it spans a wide spectrum of contributions in many very different areas: algebraic geometry and topology; algebraic groups, group actions, and representation theory; enumerative geometry and combinatorics; locally symmetric spaces, L^2-cohomology, arithmetic groups, and the Langlands program. The list is by far not complete, and we try only to give a representative selection of his contribution to mathematics. He influenced a whole generation of mathematicians by giving them new tools to attack difficult problems and teaching them novel geometric, topological, and algebraic ways of thinking."

Robert MacPherson was born in 1944 in Lakewood, Ohio. He received his B.A. from Swarthmore College in 1966 and his Ph.D. from Harvard University in 1970. He taught at Brown University from 1970 to 1987 and at the Massachusetts Institute of Technology from 1987 to 1994. He has been at the Institute for Advanced Study in Princeton since 1994. His work has introduced radically new approaches to the topology of singular spaces and promoted investigations across a great spectrum of mathematics. MacPherson works in several fields of geometry-topology, algebraic geometry, differential geometry, and singularity theory, and he is especially interested in aspects of geometry that interact with other areas of mathematics. In 1992 he received the National Academy of Sciences Award in Mathematics, and in 2002 he received the Leroy P. Steele Prize of the AMS.

The Heinz Hopf Prize at ETH Zurich was established through a donation made by Dorothee and Alfred Aeppli and will be awarded every two years for outstanding scientific work in the field of pure mathematics.

—From an ETH announcement

Lindenstrauss and Villani Awarded Fermat Prize

ELON LINDENSTRAUSS of Princeton University and CÉDRIC VILLANI of École Normale Supérieure de Lyon have been awarded the 2009 Fermat Prize by the Institut de Mathématiques de Toulouse. Lindenstrauss was honored for his contributions to ergodic theory and their applications in number theory. Villani was selected for his contributions to the theory of optimal transport and his studies of nonlinear evolution equations.

The Fermat Prize, given every two years, recognizes outstanding research in the fields in which Pierre de Fermat made significant contributions: statements of variational principles, foundations of probability and analytical geometry, and number theory. The prize is intended to reward research that is accessible to the greatest number of professional mathematicians within these fields.

The prize carries a cash award of €20,000 (approximately US$29,700).

—Elaine Kehoe

Pujals Awarded TWAS Prize in Mathematics

Enrique Pujals of the Institute of Pure and Applied Mathematics (IMPA) in Rio de Janeiro, Brazil, has been named the winner of the 2009 TWAS Prize in mathematics, awarded by the Academy of Sciences for the Developing World (TWAS). He was honored for his contributions toward developing "a theory about robust dynamics and about the role of homoclinic bifurcation as a universal mechanism to describe the way to produce very rich and complex dynamics." Pujals will receive a cash prize of US$15,000 and will deliver a lecture at the academy's general meeting in 2010.

—From a TWAS announcement

Dereich Receives 2009 Information-Based Complexity Young Researcher Award

Steffen Dereich of the Technische Universität Berlin, Germany, has been awarded the Information-Based Complexity Award for 2009. The award is given every year for significant contributions to information-based complexity by a young researcher who has not reached his or her thirty-fifth birthday by September 30 of the year of the award. The prize consists of US$1,000 and a plaque.

The award committee this year consisted of Dirk Nuyens, Katholieke Universiteit, Leuven; Andreas Neuenkirch, University of Frankfurt; Jan Vybiral, University of Jena; Joseph F. Traub, Columbia University; and Henryk Wozniakowski, Columbia University and University of Warsaw.

—Joseph Traub, Columbia University

NSF CAREER Awards Made

The Division of Mathematical Sciences (DMS) of the National Science Foundation (NSF) has honored twenty-nine mathematicians in fiscal year 2009 with Faculty Early Career Development (CAREER) awards. The NSF established the awards to support promising scientists, mathematicians, and engineers who are committed to the integration of research and education. The grants provide funding of at least US$400,000 over a five-year period. The 2009 CAREER grant awardees and the titles of their grant projects follow.

Miklos Abert, University of Chicago, Asymptotic Invariants of Residually Finite Groups; Rafail Abramov, University of Illinois, Chicago, Predicting Global Climate Change through Fluctuation-Dissipation: A Practical Computational Strategy for Complex Multiscale Dynamics; Orly Alter, University of Texas, Austin, Integrative and Comparative Tensor Algebra Models of DNA Microarray Data from Different Studies of the Cell Cycle; Benjamin Brubaker, Massachusetts Institute of Technology, Multiple Dirichlet Series, Automorphic Forms, and Combinatorial Representation Theory; Francesco Calegari, Northwestern University, Arithmetic of Cohomological Automorphic Forms; Gautam Chinta, City College, City University of New York, Multiple Dirichlet Series and Metaplectic Groups; Tommaso de Fernex, University of Utah, Singularities in the Minimal Model Program and Birational Geometry; Ioana Dumitriu, University of Washington, Synergistic Interactions between Numerical Linear Algebra and Stochastic Eigenanalysis (Random Matrix Theory); Noureddine El Karoui, University of California Berkeley, Random Matrices and High-Dimensional Statistics; Yongtao Guan, Yale University, New Statistical Methods for Massive Spatial, Temporal, and Spatial-Temporal Processes; Jeffrey Humpherys, Brigham Young University, Interdisciplinary Mentoring Program in Analysis, Computation, and Theory (IMPACT); Marta Lewicka, University of Minnesota, Twin Cities, Thin Shells: Problems in Nonlinear Elasticity and Fluid Dynamics; Fengyan Li, Rensselaer Polytechnic Institute, Development and Applications of Discontinuous Galerkin Methods; Di Liu, Michigan State University, Modeling, Analysis, and Computation of Stochastic Intracellular Reactions; Gregory Lyng, University of Wyoming, Behavior of Solutions of Nonlinear Partial Differential Equations; Mauro Maggioni, Duke University, Multiscale Methods for High-Dimensional Data, Graphs, and Dynamical Systems; Dionisios Margetis, University of Maryland, College Park, Thermodynamic and Kinetic Approaches for Epitaxial Material Systems; Lenhard Ng, Duke University, Symplectic Field Theory and Low-Dimensional Topology; Jiawang Nie, University of California San Diego, Linear Matrix Inequality Representations in Optimization; Duane Nykamp, University of Minnesota, Twin Cities, Toward a Second-Order Description of Neuronal Networks; Jeffrey Schenker, Michigan State University, Analysis of Disordered Systems; Jian Song, Rutgers University, Canonical Metrics, Complex Monge-Ampère Equations, and Geometric Flows; Jason Starr, State University of New York, Stony Brook, Higher Rational Connectedness, Higher Fano Manifolds, and Applications; Katrin Wehrheim, Massachusetts Institute of Technology, The Symplectic Category, Floer Field Theory, and Relations to Gauge Theory and Topology; Anna Wienhard, Princeton University, Higher Teichmüller Theory; Lexing Ying, University of Texas, Austin, Fast Algorithms for Oscillatory Integrals; Ming Yuan, Georgia

Institute of Technology, Sparse Modeling and Estimation with High-Dimensional Data; Aleksey Zinger, State University of New York, Stony Brook, Holomorphic Curves in Algebraic Geometry and Symplectic Topology; and Hui Zou, University of Minnesota, Twin Cities, New Statistical Methodology and Theory for Mining High-Dimensional Data.

—Elaine Kehoe

Memories of Eddie Nussbaum

A. Edward (Eddie) Nussbaum died of congestive heart failure on October 31, 2009, at the age of eighty-four. He was a faculty member in the Department of Mathematics at Washington University in St. Louis for thirty-seven years.

Eddie Nussbaum was born in 1925 in the region Moenchen-Gladbach-Rheydt (three adjacent towns), Germany, where his parents operated a department store. His elder brother was arrested on Kristallnacht in 1938. Soon thereafter, Eddie and his sister were sent by the Kindertransport train to live in Belgium. However, conditions in Belgium were not safe, and Eddie and his sister were soon separated. Eddie fled to southern France, and when conditions there also became unsafe, he crossed into Switzerland with the help of two local women and their woodsman father. When he was quickly put in jail by the Swiss authorities, he invented a story that led to his release, and he lived for several years with a spinster and her nephew in Switzerland while studying mathematics at the University of Zurich. Although Eddie's sister also survived the Holocaust, sadly both his parents and his elder brother died in the Nazi death camp at Auschwitz.

In 1947 Eddie arrived penniless in New York and began taking courses at Brooklyn College while supporting himself by rolling clay tennis courts. Soon thereafter he was admitted to Columbia University for graduate studies in mathematics, receiving his M.A. from Columbia in 1950. The high regard in which he was held by the Columbia mathematics department is attested to by his appointment as a lecturer for the academic year 1951–52. For 1952–53 he was a staff member of the electronic computer project headed by John von Neumann at the Institute for Advanced Study in Princeton. After serving as an instructor at the University of Connecticut (1953–55) and an instructor at Rensselaer Polytechnic Institute (1955–57), he received his Ph.D. from Columbia in 1957 with a dissertation titled "The Hausdorff-Bernstein-Widder Theorem for Semigroups in Locally Compact Abelian Groups". Following a year of service as an assistant professor at RPI, he was appointed assistant professor of mathematics at Washington University in 1958; he was quickly promoted to associate professor in 1961 and to full professor in 1965.

In 1955 Eddie, with Allen Devinatz and John von Neumann, coauthored a paper published by the *Annals of Mathematics* titled "On the permutability of self-adjoint operators". This led to a distinguished career in functional analysis, with numerous important papers on unbounded operators on Hilbert spaces and on a variety of related topics. His lectures were regarded by students and faculty as a model of mathematical clarity and precision. Both within the mathematics department and among his circle of friends and relatives, he was considered to be a kind, gentle, and compassionate man. Although he suffered a serious heart attack in 1979, he made a remarkable recovery, aided by an intensive rehabilitation and exercise program that led to his becoming an avid runner; indeed, he won a medal in a 1983 race. He retired from Washington University in 1995.

In 1989 Eddie and his wife visited his hometown of Rheydt in the Moenchen-Gladbach-Rheydt region and were very well received there; remarkably, he said he harbored no bitterness over his family's experiences. He is survived by Anne, his wife of fifty-two years, and their children Karl, who teaches film studies at Montclair State University and has produced a number of films, and Franziska, who works with photographers and serves as a stylist for advertising agencies in the St. Louis area.

—Guido Weiss and Edward Wilson, Washington University

Mathematics Opportunities

Summer Program for Women Undergraduates

The 2010 Summer Program for Women in Mathematics (SPWM2010) will take place at George Washington University in Washington, DC, from June 26 to July 31, 2010. This is a five-week intensive program for mathematically talented undergraduate women who are completing their junior years and may be contemplating graduate study in mathematical sciences. The goals of this program are to communicate an enthusiasm for mathematics, to develop research skills, to cultivate mathematical self-confidence and independence, and to promote success in graduate school.

Applicants must be U.S. citizens or permanent residents studying at a U.S. university or college who are completing their junior years or the equivalent and have mathematical experience beyond the typical first courses in calculus and linear algebra. Sixteen women will be selected. Each will receive a travel allowance, campus room and board, and a stipend of US$1,750. The deadline for applications is **March 1, 2010**. Early applications are encouraged. Applications are accepted only by mail. For further information, please contact the director, Murli M. Gupta, email: `mmg@gwu.edu`; telephone: 202-994-4857; or visit the program's website at `http://www.gwu.edu/~spwm/`. Application material is available on the website.

—From an SPWM announcement

NSF Support for Undergraduate Training in Biological and Mathematical Sciences

The National Science Foundation (NSF) offers opportunities for support through its Undergraduate Biology and Mathematics (UBM) program. The goal of the program is to enhance undergraduate education and training at the intersection of the biological and mathematical sciences and to better prepare undergraduate biology or mathematics students to pursue graduate study and careers in fields that integrate the mathematical and biological sciences.

The program will provide support for jointly conducted long-term research experiences for interdisciplinary teams of at least two undergraduates from departments in the biological and mathematical sciences. Projects should focus on research at the intersection of the mathematical and biological sciences and should provide students exposure to contemporary mathematics and biology addressed with modern research tools and methods. Projects must involve students from both areas in collaborative research experiences and include joint mentorship by faculty in both fields.

Between six and nine awards are expected to be made in 2010. The deadline for full proposals is **February 11, 2010**. For more information, see `http://www.nsf.gov/pubs/2008/nsf08510/nsf08510.htm#awd_info`. The UBM program is a joint effort of the Education and Human Resources (EHR), Biological Sciences (BIO), and Mathematical and Physical Sciences (MPS) directorates of the NSF.

—From an NSF announcement

Monroe H. Martin Prize

The Institute for Physical Science and Technology at the University of Maryland, College Park, is seeking applications and nominations for the eighth Monroe H. Martin Prize. The prize is awarded for an outstanding paper in applied mathematics (including numerical analysis) by a young researcher who is a resident of North America and who has not reached his or her thirty-sixth birthday by July 31, 2010. Submitted papers must be written by single authors and must have been published or accepted for publication. The work must not have been performed in connection with the completion of requirements for an academic degree. The candidate must neither be nor have been affiliated with the University of Maryland.

Applications and nominations should include a copy of the paper or contribution with a cover letter. The deadline for submissions is **July 31, 2010**. Submissions should be sent to R. Roy, Director, Institute for Physical Science and Technology, University of Maryland, College Park, Maryland 20742-2431. The award will be announced by November 1, 2010. The recipient will be asked to present his or her work at the Monroe H. Martin Lecture at the University of Maryland in December 2010 and will be awarded a prize of US$5,000 plus travel expenses.

The Monroe H. Martin Prize was established to commemorate the achievements of the late Monroe H. Martin, former director of the Institute for Fluid Dynamics and Applied Mathematics and chair of the Department of Mathematics at the University of Maryland. Previous prize winners are Neil Berger (1975), Marshall Slemrod (1980), Jonathan Goodman (1985), Marek Rychlik (1990), A. M. Stuart (1995), Z. Xia (1995), R. J. McCann (2000), Y. Grabovsky (2000), C. Sinan Gunturk (2005), and Jared Tanner (2005).

—Institute for Physical Science and Technology announcement

Clay Mathematics Institute 2010 Summer School

The 2010 Clay Mathematics Institute (CMI) Summer School on Probability and Statistical Physics in Two (and More) Dimensions will be held in Buzios, Brazil, from July 11 to August 7, 2010.

In the past ten to fifteen years, various areas of probability theory related to rigorous statistical mechanics, disordered systems, and combinatorics have enjoyed an intensive development. A number of these developments deal with two-dimensional random structures. The questions related to critical systems are twofold: understanding large-scale properties of lattice-based models (on a periodic deterministic lattice or in the case where the lattice is itself random) and, on the other hand, being able to construct and manipulate a continuous object that describes directly their scaling limits. In the case of a fixed planar lattice, a number of conjectures originating in the physics literature have now been proved, but many questions remain open. In the case of statistical physics on random planar graphs, sometimes referred to as quantum gravity, many results have been recently understood, and a relation between discrete and continuous structures is now emerging. The aim of the Summer School is to provide a complete picture of the current state of the art in these and related topics.

Organization: During the first two weeks of the school, three foundational courses will be combined with afternoon activities in thematic working groups and evening seminars. Weeks three and four will be dedicated to shorter advanced courses (the fourth week will run in parallel with the Fourteenth Brazilian School of Probability).

Foundational Courses: The foundational courses include "Large random planar maps and their scaling limits" (Jean-Francois LeGall and Gregory Miermont), "SLE and other conformally invariant objects" (Vincent Beffara), and "Noise sensitivity and percolation" (Jeffrey Steif and Christophe Garban).

Minicourses: These courses include "Random geometry and Gaussian free field" (Scott Sheffield), "Conformal invariance of lattice models" (Stanislav Smirnov), "Integrable combinatorics" (Philippe Di Francesco), "Fractal and multifractal properties of SLE" (Gregory Lawler), "The double dimer model" (Rick Kenyon), "Random polymers" (Frank den Hollander), and "Self-avoiding walks" (Gordon Slade). The latter two will be held jointly with the Brazilian School on Probability. Another possible minicourse, "Supersymmetric methods in disordered systems and two-dimensional critical behavior" (John Cardy), is awaiting confirmation. See the website `http://www.claymath.org/summerschool` for updated information.

Financial Support: Graduate students and postdoctoral fellows who are within five years of receipt of the Ph.D. degree can apply for financial support. Support is decided on a competitive basis and may include accommodations plus funds toward the cost of economy travel.

The scientific committee consists of David Alexandre Ellwood, Charles Newman, Vladas Sidoravicius, and Wendelin Werner.

The deadline for applications is **March 1, 2010**. More information and online application forms are available at `http://www.claymath.org/summerschool` or by sending email to `summerschool@claymath.org`.

—From a CMI announcement

News from CRM

The Centro di Ricerca Matematica Ennio De Giorgi (CRM) invites applications for four one-year junior visiting positions for the academic year 2010–11. Applicants should be new or recent Ph.D. recipients in mathematics and should have exceptional potential in research. The annual stipend is 25,000 euros (approximately US$36,800), along with a research allowance of 2,500 euros (approximately US$3,600) intended for support of other researchers invited to CRM by the junior visitors. Junior visitors will participate in a variety of scientific activities, including intensive research periods, workshops, and seminars, and will interact with prominent scientists who participate in the senior visiting program at CRM.

The application deadline is **January 28, 2010**, and junior visitors are expected to begin their research activities at CRM no later than October 2010. For further information and application details, see the website `http://www.crm.sns.it/hpp/grants.html?year=2010`.

The 2010 scientific program includes the following workshops:

January 25–February 5, 2010: Periodic Approximation in Dynamics

February 9–12, 2010: Linear and Nonlinear Hyperbolic Equations

March 8–14, 2010: Italy-India Conference on Diophantine and Analytic Number Theory

March 1–April 30, 2010: Euclidean Harmonic Analysis, Nilpotent Lie Groups and PDEs

May 2–June 30, 2010: Configuration Spaces: Geometry, Combinatorics and Topology

September 6–10, 2010: Geometric Evolutions and Minimal Surfaces in Lorentzian Manifolds

September 13–17, 2010: On the Contested Expanding Role of Applied Mathematics from the Renaissance to the Enlightenment

October 12–16, 2010: Optimal Transportation and Applications

For further information see the website `http://www.crm.sns.it`.

—From a CRM announcement

Everett Pitcher Lectures

The next series of Everett Pitcher Lectures will be held March 22–25, 2010, on the campus of Lehigh University in Bethlehem, Pennsylvania. The speaker will be Rick Durrett of Cornell University, who will speak on "Three Faces of Probability", with three lectures on improbabilities, life on a random graph, and cancer models. Durrett's research focuses on probability problems that come from ecology and genetics. He is a member of the National Academy of Sciences.

The lectures, which are free and open to the public, are held in honor of Everett Pitcher, who was secretary of the AMS from 1967 until 1988. Pitcher served in the mathematics department at Lehigh University from 1938 until 1978, when he retired as Distinguished Professor of Mathematics. He passed away in December 2006 at the age of ninety-four. For further information, contact the Everett Pitcher Lecture Series, Department of Mathematics, Lehigh University, Bethlehem, PA, 18015; telephone 610-758-3731; or see the website `http://www.lehigh.edu/~math/pitcher.html`.

—From a Lehigh University announcement

About the Cover

Differential geometry issue

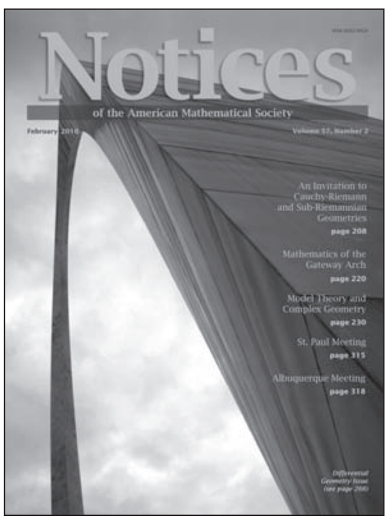
In 2009 geometer Robert Osserman visited St. Louis to observe and to lecture at Washington University on the mathematical properties of the Gateway Arch. An article based on Osserman's lecture is included in this issue. The cover photograph by Geir Arne Hjelle attests to the awesomeness of a marriage between mathematics and architecture on a grand scale.

The cover photograph was taken on November 23, 2008, from the south leg of the St. Louis Arch, looking north and slightly up. For this magnificent wide angle, unusual shot, the photographer used a Canon EOS 40D camera fitted with a Canon EF-S 17-85mm f4-5.6 IS USM lens.

"I was down at the arch," Geir Arne comments, "looking for a cover picture for my St. Louis photo book, and I guess this was one of the candidates." The photographer later favored a different shot. You can see the other shot at `http://www.flickr.com/photos/hjelle/3439055921/`.

The cover image was composed using GIMP (GNU Image Manipulation Program) freeware. Find it at `http://www.gimp.org/`.

—Steven G. Krantz



Inside the AMS

From the AMS Public Awareness Office

• **Mathematical Moment.** Recent topics include "Knowing *How* to Fold Them" (how math is involved in understanding the complex behavior of proteins) and "Resisting the Spread of Disease". Listen to a podcast interview with Mac Hyman talking about his research on H1N1 (the swine flu) and view more than seventy-five topics at `http://www.ams.org/mathmoments/`.

• **Feature Column.** Recent essays include "Marian Rejewski and the First Break into Enigma", by Bill Casselman; "A Non-commutative Marriage System in the South Pacific", by Tony Phillips; "School Choice", by Joe Malkevitch; and "We Recommend a Singular Value Decomposition", by David Austin. Read the current essay and explore the archive at `http://www.ams.org/featurecolumn`.

—Annette Emerson and Mike Breen
AMS Public Awareness Officers
`paoffice@ams.org`

AMS Email Support for Frequently Asked Questions

A number of email addresses have been established for contacting the AMS staff regarding frequently asked questions. The following is a list of those addresses together with a description of the types of inquiries that should be made through each address.

abs-info@ams.org for questions regarding a particular abstract.

abs-submit@ams.org for information on how to submit abstracts for AMS meetings and MAA sessions at January Joint Mathematics Meetings. Type HELP as the subject line.

acquisitions@ams.org to contact the AMS Acquisitions Department.

ams@ams.org to contact the Society's headquarters in Providence, Rhode Island.

amsdc@ams.org to contact the Society's office in Washington, DC.

amsmem@ams.org to request information about membership in the AMS and about dues payments or to ask any general membership questions; may also be used to submit address changes.

ams-survey@ams.org for information or questions about the Annual Survey of the Mathematical Sciences or to request reprints of survey reports.

bookstore@ams.org for inquiries related to the online AMS Bookstore.

classads@ams.org to submit classified advertising for the *Notices*.

cust-serv@ams.org for general information about AMS products (including electronic products), to send address changes, place credit card orders for AMS products, or conduct any general correspondence with the Society's Customer Services Department.

development@ams.org for information about giving to the AMS, including the Epsilon Fund.

eims-info@ams.org to request general information about Employment Information in the Mathematical Sciences (EIMS). To view rates and post an ad, go to `www.ams.org/eims`.

emp-info@ams.org for information regarding AMS employment and career services.

eprod-support@ams.org for technical questions regarding AMS electronic products and services.

mathcal@ams.org for queries about the "Mathematics Calendar" section of the *Notices*. To submit conference information, see the form at `http://www.ams.org/cgi-bin/mathcal/mathcal-submit.pl`.

mathrev@ams.org to submit reviews to *Mathematical Reviews* and to send correspondence related to reviews or other editorial questions.

meet@ams.org to request general information about Society meetings and conferences.

meetreg-request@ams.org to request meeting registration forms be emailed.

meetreg-submit@ams.org to submit completed registration forms.

mmsb@ams.org for information or questions about registration and housing for the Joint Mathematics Meetings (Mathematics Meetings Service Bureau).

msn-support@ams.org for technical questions regarding MathSciNet.

notices@ams.org (or `smf@ams.org`) to send correspondence to the managing editor of the *Notices*, including items for the news columns. The editor (`notices@math.wustl.edu`) is the person to whom to send articles and letters. Requests for permission to reprint from the *Notices* should be sent to `reprint-permission@ams.org` (see below).

notices-ads@ams.org to submit electronically paid display ads for the *Notices*.

notices-booklist@ams.org to submit suggestions for books to be included in the "Book List" in the *Notices*.

notices-letters@ams.org to submit letters and opinion pieces to the *Notices*.

notices-whatis@ams.org to comment on or send suggestions for topics for the "WHAT IS…?" column in the *Notices*.

paoffice@ams.org to contact the AMS Public Awareness Office.

president@ams.org to contact the president of the American Mathematical Society.

prof-serv@ams.org to send correspondence about AMS professional programs and services.

pub@ams.org to send correspondence to the AMS Publication Division.

pub-submit@ams.org to submit accepted electronic manuscripts to AMS publications (other than *Abstracts*). See `http://www.ams.org/submit-book-journal` to electronically submit accepted manuscripts to the AMS book and journal programs.

reprint-permission@ams.org to request permission to reprint material from Society publications.

sales@ams.org to inquire about reselling or distributing AMS publications or to send correspondence to the AMS Sales Department.

secretary@ams.org to contact the secretary of the Society.

statements@ams.org to correspond regarding a balance due shown on a monthly statement.

tech-support@ams.org to contact the Society's typesetting Technical Support Group.

textbooks@ams.org to request examination copies or inquire about using AMS publications as course texts.

webmaster@ams.org for general information or for assistance in accessing and using the AMS website.

Deaths of AMS Members

LINDA BARKLEY, Boeing Communications, died on August 15, 2009. Born on December 12, 1951, she was a member of the Society for 31 years.

ENRIQUE BAYO, professor, from San Juan, PR, died on May 28, 2002. Born on April 20, 1922, he was a member of the Society for 51 years.

LEONARD D. BERKOVITZ, professor emeritus, Purdue University, died on October 13, 2009. Born on January 24, 1924, he was a member of the Society for 59 years.

THOMAS DIETMAIR, ERGO Insurance Group, Germany, died on September 6, 2009. Born on January 22, 1961, he was a member of the Society for 17 years.

F. BROCK FULLER, professor emeritus, Caltech, died on November 6, 2009. Born on July 8, 1927, he was a member of the Society for 59 years.

PAUL GERMAIN, professor, Paris, France, died in February 2009. Born on August 28, 1920, he was a member of the Society for 55 years.

ALBERT E. HURD, professor, University of Victoria, Canada, died on October 28, 2009. Born on October 22, 1931, he was a member of the Society for 51 years.

EDWARD S. KENNEDY, professor emeritus, American University of Beirut, died on May 4, 2009. Born on January 3, 1912, he was a member of the Society for 72 years.

ARNAUD I. LAFONTE, from Rio de Janeiro, Brazil, died on October 28, 2009. Born on October 5, 1938, he was a member of the Society for 4 years.

GEORGE J. MALTESE, professor, Wesleyan University, died on October 27, 2009. Born on June 24, 1931, he was a member of the Society for 51 years.

DAVID B. MERONK, retired professor, Bowling Green State University, died on July 30, 2009. Born on August 10, 1934, he was a member of the Society for 17 years.

ALAN A. MEYERHOFF, professor, Rutgers University, New Brunswick, died on May 18, 2009. Born on March 20, 1926, he was a member of the Society for 35 years.

TETSURO MIYAKAWA, professor, Kanazawa University, died on February 11, 2009. Born on March 10, 1948, he was a member of the Society for 21 years.

CHARLES T. MOLLOY, Falls Church, VA, died on May 5, 2009. Born on November 22, 1914, he was a member of the Society for 58 years.

A. EDWARD NUSSBAUM, professor emeritus, Washington University at St. Louis, died on October 31, 2009. Born on January 10, 1925, he was a member of the Society for 57 years.

HOWARD L. PENN, professor, U.S. Naval Academy, died in November 2009. Born on November 9, 1946, he was a member of the Society for 37 years.

SAM PERLIS, professor emeritus, Purdue University, died on June 22, 2009. Born on April 18, 1913, he was a member of the Society for 71 years.

MARIAN POUR-EL, professor emeritus, School of Mathematics, University of Minnesota, died on June 10, 2009. Born on April 19, 1928, she was a member of the Society for 56 years.

Calvin R. Putnam, professor emeritus, Purdue University, died on April 24, 2008. Born on May 25, 1924, he was a member of the Society for 63 years.

Bettina Richmond, professor, Western Kentucky University, died on November 22, 2009. Born on January 30, 1958, she was a member of the Society for 30 years.

Pierre Samuel, professor emeritus, University of Paris-Sud, died on August 23, 2009. Born on September 12, 1921, he was a member of the Society for 63 years.

Alice T. Schafer, former AWM president, died on September 27, 2009. Born on June 18, 1915, she was a member of the Society for 68 years.

Stefan Schwabik, professor emeritus, Institute of Mathematics of the Academy of Sciences of the Czech Republic, died on November 4, 2009. Born on March 15, 1941, he was a member of the Society for 16 years.

Peter Seibert, from Mexico City, died on August 13, 2009. Born on April 13, 1927, he was a member of the Society for 49 years.

Nobuo Shimada, from Nagoya, Japan, died on December 17, 2007. Born on October 13, 1925, he was a member of the Society for 47 years.

Edsel F. Stiel, professor emeritus, California State University, Fullerton, died on January 18, 2008. Born on December 19, 1933, he was a member of the Society for 48 years.

Takayuki Tamura, professor emeritus, University of California, Davis, died on June 1, 2009. Born on March 12, 1919, he was a member of the Society for 51 years.

Lincoln H. Turner, from Lakewood, CO, died on November 25, 2007. Born on June 30, 1928, he was a member of the Society for 50 years.

John H. Ursell, professor emeritus, Queen's University, died on July 30, 2009. Born on June 9, 1938, he was a member of the Society for 24 years.

J. Richard VandeVelde, professor, Arrupe House, Loyola University of Chicago, died on August 11, 2009. Born on February 21, 1935, he was a member of the Society for 43 years.

Alfred G. Vassalotti, associate professor emeritus, Hofstra University, died on October 21, 2009. Born on January 11, 1929, he was a member of the Society for 45 years.

Theresa P. Vaughan, retired professor, University of North Carolina at Greensboro, died on June 13, 2009. Born on October 13, 1941, she was a member of the Society for 37 years.

Frederick N. Webb, from Littleton, MA, died on July 12, 2009. Born on July 22, 1944, he was a member of the Society for 8 years.

Alvin M. White, professor emeritus, Harvey Mudd College, died on June 2, 2009. Born on June 21, 1925, he was a member of the Society for 49 years.

Kathleen B. Whitehead, retired professor, Tufts University, died on April 18, 2009. Born on November 9, 1920, she was a member of the Society for 66 years.

Joseph Zelle, from Euclid, OH, died on July 11, 2009. Born on February 25, 1912, he was a member of the Society for 51 years.

Reference and Book List

*The **Reference** section of the* Notices *is intended to provide the reader with frequently sought information in an easily accessible manner. New information is printed as it becomes available and is referenced after the first printing. As soon as information is updated or otherwise changed, it will be noted in this section.*

Contacting the *Notices*

The preferred method for contacting the *Notices* is electronic mail. The editor is the person to whom to send articles and letters for consideration. Articles include feature articles, memorial articles, communications, opinion pieces, and book reviews. The editor is also the person to whom to send news of unusual interest about other people's mathematics research.

The managing editor is the person to whom to send items for "Mathematics People", "Mathematics Opportunities", "For Your Information", "Reference and Book List", and "Mathematics Calendar". Requests for permissions, as well as all other inquiries, go to the managing editor.

The electronic-mail addresses are `notices@math.wustl.edu` in the case of the editor and `notices@ams.org` in the case of the managing editor. The fax numbers are 314-935-6839 for the editor and 401-331-3842 for the managing editor. Postal addresses may be found in the masthead.

Upcoming Deadlines

January 15, 2010: Applications for AMS-AAAS Mass Media Summer Fellowships. See `http://www.aaas.org/programs/education/MassMedia/`, or contact Stacey Pasco, Manager, Mass Media Program, AAAS Mass Media Science and Engineering Fellows Program, 1200 New York Avenue, NW, Washington, DC 20005; telephone 202-326-6441; fax 202-371-9849; email: `spasco@aaas.org`. Also see `http://www.ams.org/government/massmediaann.html` or contact the AMS Washington Office, 1527 Eighteenth Street, NW, Washington, DC 20036; telephone 202-588-1100; fax 202-588-1853; email: `amsdc@ams.org`.

January 15, 2010: Applications for Jefferson Science Fellows Program. See `http://sites.nationalacademies.org/PGA/jefferson/` or email: `jsf@nas.edu` or telephone 202-334-2643.

January 28, 2010: Applications for four 1-year junior visiting positions at CRM for the academic year 2010–11. See `http://www.crm.sns.it`.

January 28, 2010: Proposals for National Science Foundation Scientific Computing Research Environments for the Mathematical Sciences (SCREMS). See `http://www.nsf.gov/pubs/2007/nsf07502/nsf07502.htm`.

Where to Find It

A brief index to information that appears in this and previous issues of the *Notices*.

February 1, 2010: Applications for February review for National Academies Postdoctoral and Senior Research Associateship Program. See http://sites.nationalacademies.org/PGA/RAP/PGA_050491 or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email: rap@nas.edu.

February 1, 2010: Applications for AWM Travel Grants and Mentoring Travel Grants. See http://www.awm-math.org/travelgrants.html; telephone: 703-934-0163; email: awm@awm-math.org; or contact Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030.

February 5, 2010: Applications for Math for America Fellowship Program. See http://www.mathforamerica.org/.

February 11, 2010: Full proposals for NSF Undergraduate Biology and Mathematics (UBM) program. See "Mathematics Opportunities" in this issue.

February 15, 2010: Applications for Enhancing Diversity in Graduate Education (EDGE) Summer Program. See http://www.edgeforwomen.org/?page_id=5.

February 15, 2010: Applications for Institute for Pure and Applied Mathematics (IPAM) Research in Industrial Projects for Students (RIPS) undergraduate summer research program. See http://www.ipam.ucla.edu.

February 15, 2010: Applications for AMS-AAAS Congressional Fellowship. See http://www.ams.org/government/congressfellowann.html or contact the AMS Washington Office, telephone: 202-588-1100; email: amsdc@ams.org.

February 27, 2010: Entries for AWM Essay Contest. See http://www.awm-math.org/biographies/contest.html.

March 1, 2010: Applications for Summer Program for Women in Mathematics (SPWM2010). See "Mathematics Opportunities" in this issue.

March 1, 2010: Applications for Clay Mathematics Institute (CMI) Summer School on Probability and Statistical Physics in Two (and More) Dimensions. See "Mathematics Opportunities" in this issue.

April 15, 2010: Applications for fall 2010 semester of Math in Moscow. See http://www.mccme.ru/mathinmoscow or write to: Math in Moscow, P.O. Box 524, Wynnewood, PA 19096; fax: +7095-291-65-01; email: mim@mccme.ru. For information on AMS scholarships see http://www.ams.org/outreach/mimoscow.html or write to: Math in Moscow Program, Membership and Programs Department, American Mathematical Society, 201 Charles Street, Providence RI 02904-2294; email: student-serv@ams.org.

May 1, 2010: Applications for May review for National Academies Postdoctoral and Senior Research Associateship Program. See http://sites.nationalacademies.org/PGA/RAP/PGA_050491 or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email: rap@nas.edu.

May 1, 2010: Applications for the fall 2010 program of the Christine Mirzayan Science and Technology Policy Graduate Fellowship program of the National Academies. See http://sites.nationalacademies.org/PGA/policyfellows/index.htm or contact The National Academies Christine Mirzayan Science and Technology Policy Graduate Fellowship Program, 500 Fifth Street, NW, Room 508, Washington, DC 20001; telephone: 202-334-2455; fax: 202-334-1667; email: policyfellows@nas.edu.

May 1, 2010: Applications for AWM Travel Grants. See http://www.awm-math.org/travelgrants.html; telephone: 703-934-0163; or email: awm@awm-math.org. The postal address is: Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030.

June 1, 2010: Applications for NSF's Enhancing the Mathematical Sciences Workforce in the Twenty-First Century (EMSW21) program. See http://www.nsf.gov/pubs/2005/nsf05595/nsf05595.htm.

July 31, 2010: Nominations and applications for the 2010 Monroe H. Martin Prize. See "Mathematics Opportunities" in this issue.

August 1, 2010: Applications for August review for National Academies Postdoctoral and Senior Research Associateship Program. See http://sites.nationalacademies.org/PGA/RAP/PGA_050491 or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email: rap@nas.edu.

October 1, 2010: Applications for AWM Travel Grants. See http://www.awm-math.org/travelgrants.html; telephone: 703-934-0163; email: awm@awm-math.org; or contact Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030.

November 1, 2010: Applications for November review for National Academies Postdoctoral and Senior Research Associateship Program. See http://sites.nationalacademies.org/PGA/RAP/PGA_050491 or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email: rap@nas.edu.

MPS Advisory Committee

Following are the names and affiliations of the members of the Advisory Committee for Mathematical and Physical Sciences (MPS) of the National Science Foundation. The date of the expiration of each member's term is given after his or her name. The website for the MPS directorate may be found at http://www.nsf.gov/home/mps/. The postal address is Directorate for the Mathematical and Physical Sciences, National Science Foundation, 4201 Wilson Boulevard, Arlington, VA 22230.

Hector D. Abruna (10/10)
Department of Chemistry and
Cornell University

Taft Armandroff (10/12)
W. M. Keck Observatory
Kamuela, Hawaii

James Berger (10/11)
Department of Statistical Science
Duke University

Daniela Bortoletto (10/11)
Department of Physics
Purdue University

Kevin Corlette (10/12)
Department of Mathematics
University of Chicago

Eric A. Cornell (10/10)
JILA
University of Colorado

Juan J. de Pablo (10/12)
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University of Wisconsin-Madison

Joseph M. DeSimone (10/12)
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Barbara J. Finlayson-Pitts (10/11)
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University of California, Irvine

Irene Fonseca (10/11)
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Carnegie Mellon University

Sharon C. Glotzer (10/12)
Department of Chemical Engineering
University of Michigan

Suzanne Hawley (10/11)
Astronomy Department
University of Washington

Iain M. Johnstone (chair) (10/10)
Department of Statistics
Stanford University

David E. Keyes (10/10)
Department of Applied Physics & Applied Mathematics
Columbia University

Jerzy Leszczynski (10/12)
Department of Chemistry and Biochemistry
Jackson State University

Theresa A. Maldonado (10/10)
(MPSAC/CEOSE Liaison
Department of Electrical and Computer Engineering
Texas A&M University

Dennis L. Matthews (10/10)
College of Engineering and School of Medicine
University of California Davis

James W. Mitchell (10/12)
Department of Chemical Engineering
Howard University

Ramesh Narayan (10/11)
Harvard University and Harvard-Smithsonian Center for Astrophysics

Sharon L. Neal (10/11)
Department of Chemistry and Biochemistry
University of Delaware

Luis Orozco (10/12)
Department of Physics
University of Maryland, College Park

John Peoples Jr. (10/11)
Fermilab
Batavia, IL

Elsa Reichmanis (10/11)
School of Chemical and Biomolecular Engineering
Georgia Institute of Technology

Fred S. Roberts (10/12)
DIMACS
Rutgers University

Joel E. Tohline (10/10)
Department of Physics and Astronomy
Louisiana State University

Geoffrey West (10/11)
Santa Fe Institute
Santa Fe, NM

Book List

The Book List highlights books that have mathematical themes and are aimed at a broad audience potentially including mathematicians, students, and the general public. When a book has been reviewed in the Notices, *a reference is given to the review. Generally the list will contain only books published within the last two years, though exceptions may be made in cases where current events (e.g., the death of a prominent mathematician, coverage of a certain piece of mathematics in the news) warrant drawing readers' attention to older books. Suggestions for books to include on the list may be sent to* `notices-booklist@ams.org`.

*Added to "Book List" since the list's last appearance.

The Archimedes Codex: How a Medieval Prayer Book Is Revealing the True Genius of Antiquity's Greatest Scientist, by Reviel Netz and William Noel. Da Capo Press, October 2007. ISBN 978-03068-1580-5. (Reviewed September 2008.)

The Best of All Possible Worlds: Mathematics and Destiny, by Ivar Ekeland. University of Chicago Press, October 2006. ISBN-13: 978-0-226-19994-8. (Reviewed March 2009.)

The Calculus of Friendship: What a Teacher and Student Learned About Life While Corresponding About Math, by Steven Strogatz. Princeton University Press, August 2009. ISBN-13: 978-06911-349-32.

The Calculus Wars: Newton, Leibniz, and the Greatest Mathematical Clash of All Time, by Jason Socrates Bardi. Thunder's Mouth Press, April 2007. ISBN-13: 978-15602-5992-3. (Reviewed May 2009.)

Chez les Weils (French), by Sylvie Weil. Buchet-Chastel, January 2009. ISBN-13: 978-22830-236-93.

Crossing the Equal Sign, by Marion D. Cohen. Plain View Press, January 2007. ISBN-13: 978-18913-866-95.

Crocheting Adventures with Hyperbolic Planes, by Daina Taimina. A K Peters, March 2009. ISBN-13: 978-15688-145-20.

Decoding the Heavens: A 2,000-Year-Old Computer—and the Century-Long Search to Discover Its Secrets, by Jo Marchant. Da Capo

Press, February 2009. ISBN-13: 978-03068-174-27.

The Drunkard's Walk: How Randomness Rules Our Lives, by Leonard Mlodinow. Pantheon, May 2008. ISBN-13: 978-03754-240-45.

Embracing the Wide Sky: A Tour Across the Horizons of the Human Mind, by Daniel Tammet. Free Press, January 2009. ISBN-13: 978-14165-696-95.

Ernst Zermelo: An Approach to His Life and Work, by Heinz-Dieter Ebbinghaus. Springer, April 2007. ISBN-13 978-3-540-49551-2. (Reviewed August 2009.)

Gaming the Vote (Why Elections Aren't Fair and What We Can Do About It), by William Poundstone. Hill and Wang, February 2009. ISBN-13: 978-08090-489-22.

The Housekeeper and the Professor, by Yoko Ogawa. Picador, February 2009. ISBN-13: 978-03124-278-01.

How Math Explains the World: A Guide to the Power of Numbers, from Car Repair to Modern Physics, by James D. Stein. Collins, April 2008. ISBN-13: 978-00612-417-65.

How to Think Like a Mathematician: A Companion to Undergraduate Mathematics, by Kevin Houston. Cambridge University Press, March 2009. ISBN-13: 978-05217-197-80.

Leonhard Euler and His Friends: Switzerland's Great Scientific Expatriate, by Luis-Gustave du Pasquier (translated by John S. D. Glaus). CreateSpace, July 2008. ISBN: 978-14348-332-73.

Lewis Carroll in Numberland: His Fantastical Mathematical Logical Life: An Agony in Eight Fits, by Robin Wilson. W. W. Norton & Company. ISBN-13: 978-03930-602-70.

Logic's Lost Genius: The Life of Gerhard Gentzen, by Eckart Menzler-Trott, Craig Smorynski (translator), Edward R. Griffor (translator). AMS-LMS, November 2007. ISBN-13: 978-0-8218-3550-0.

The Mathematical Mechanic: Using Physical Reason to Solve Problems, by Mark Levi. Princeton University Press. ISBN: 978-0691140209.

Mathematicans Fleeing from Nazi Germany: Individual Fates and Global Impact, by Reinhard Siegmund-Schultze. Princeton University Press, July 2009. ISBN 978-0-691-14041-4.

Mathematicians of the World, Unite!: The International Congress of Mathematicians: A Human Endeavor, by Guillermo P. Curbera. A K Peters, March 2009. ISBN-13: 978-15688-133-01.

Mathematics and Common Sense: A Case of Creative Tension, by Philip J. Davis. A K Peters, October 2006. ISBN 1-568-81270-1. (Reviewed June/July 2009.)

Mathematics Emerging: A Sourcebook 1540–1900, by Jacqueline Stedall. Oxford University Press, November 2008. ISBN-13: 978-01992-269-00.

Mathematics in Ancient Iraq: A Social History, by Eleanor Robson. Princeton University Press, August 2008. ISBN-13: 978-06910-918-22.

Mathematics in India, by Kim Plofker. Princeton University Press, January 2009. ISBN-13: 978-06911-206-76.

Mathematics in 10 Lessons: The Grand Tour, by Jerry P. King. Prometheus Books, May 2009. ISBN: 978-1-59102-686-0.

The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook, by Victor J. Katz et al. Princeton University Press, July 2007. ISBN-13: 978-0-6911-2745-3.

The Millennium Prize Problems, edited by James Carlson, Arthur Jaffe, and Andrew Wiles. AMS, June 2006. ISBN-13: 978-08218-3679-8. (Reviewed December 2009.)

More Mathematical Astronomy Morsels, by Jean Meeus. Willmann-Bell, 2002. ISBN 0-943396743.

The Numbers Behind NUMB3RS: Solving Crime with Mathematics, by Keith Devlin and Gary Lorden. Plume, August 2007. ISBN-13: 978-04522-8857-7. (Reviewed March 2009.)

The Numbers Game: The Commonsense Guide to Understanding Numbers in the News, in Politics, and in Life, by Michael Blastland and Andrew Dilnot. Gotham, December 2008. ISBN-13: 978-15924-042-30.

The Numerati, by Stephen Baker. Houghton Mifflin, August 2008. ISBN-13: 978-06187-846-08. (Reviewed October 2009.)

Our Days Are Numbered: How Mathematics Orders Our Lives, by Jason Brown. McClelland and Stewart, April 2009. ISBN-13: 978-07710-169-67.

Out of the Labyrinth: Setting Mathematics Free, by Robert Kaplan and Ellen Kaplan. Oxford University Press, January 2007. ISBN-13: 978-0-19514-744-5. (Reviewed June/July 2009.)

A Passion for Discovery, by Peter Freund. World Scientific, August 2007. ISBN-13: 978-9-8127-7214-5.

**Perfect Rigor: A Genius and the Mathematical Breakthrough of the Century,* by Masha Gessen. Houghton Mifflin Harcourt, November 2009. ISBN-13: 978-01510-140-64.

Picturing the Uncertain World: How to Understand, Communicate, and Control Uncertainty Through Graphical Display, by Howard Wainer, Princeton University Press, April 2009. ISBN-13: 978-06911-375-99.

Plato's Ghost: The Modernist Transformation of Mathematics, by Jeremy Gray. Princeton University Press, September 2008. ISBN-13: 978-06911-361-03. (Reviewed in this issue.)

The Princeton Companion to Mathematics, edited by Timothy Gowers (June Barrow-Green and Imre Leader, associate editors). Princeton University Press, November 2008. ISBN-13: 978-06911-188-02. (Reviewed November 2009.)

**Proofs from THE BOOK*, by Martin Aigner and Günter Ziegler. Expanded fourth edition, Springer, October 2009. ISBN-13: 978-3-642-00855-9

Pythagoras' Revenge: A Mathematical Mystery, by Arturo Sangalli. Princeton University Press, May 2009. ISBN-13: 978-06910-495-57.

Recountings: Conversations with MIT Mathematicians, edited by Joel Segel. A K Peters, January 2009. ISBN-13: 978-15688-144-90.

Rock, Paper, Scissors: Game Theory in Everyday Life, by Len Fisher. Basic Books, November 2008. ISBN-13: 978-04650-093-81.

**Roger Boscovich*, by Radoslav Dimitric (Serbian). Helios Publishing Company, September 2006. ISBN-13: 978-09788-256-21.

Sacred Mathematics: Japanese Temple Geometry, by Fukagawa Hidetoshi and Tony Rothman. Princeton University Press, July 2008. ISBN-13: 978-0-6911-2745-3.

**Solving Mathematical Problems: A Personal Perspective,* by Terence Tao. Oxford University Press, September

2006. ISBN-13: 978-0-199-20560-8. (Reviewed in this issue.)

Sphere Packing, Lewis Carroll, and Reversi, by Martin Gardner. Cambridge University Press, July 2009. ISBN: 978-0521756075.

Strange Attractors: Poems of Love and Mathematics, edited by Sarah Glaz and JoAnne Growney. A K Peters, November 2008. ISBN-13: 978-15688-134-17. (Reviewed September 2009.)

**The Strangest Man,* by Graham Farmelo. Basic Books, August 2009. ISBN-13: 978-04650-182-77.

Super Crunchers: Why Thinking-by-Numbers Is the New Way to Be Smart, by Ian Ayres. Bantam, August 2007. ISBN-13: 978-05538-054-06. (Reviewed April 2009.)

Tools of American Math Teaching, 1800–2000, by Peggy Aldrich Kidwell, Amy Ackerberg-Hastings, and David Lindsay Roberts. Johns Hopkins University Press, July 2008. ISBN-13: 978-0801888144. (Reviewed January 2010.)

The Unfinished Game: Pascal, Fermat, and the Seventeenth-Century Letter That Made the World Modern, by Keith Devlin. Basic Books, September 2008. ISBN-13: 978-0-4650-0910-7.

What Is a Number?: Mathematical Concepts and Their Origins, by Robert Tubbs. Johns Hopkins University Press, December 2008. ISBN-13: 978-08018-901-85.

What's Happening in the Mathematical Sciences, by Dana Mackenzie. AMS, 2009. ISBN-13: 978-08218-447-86.

Why Does E=mc²? (And Why Should We Care?), by Brian Cox and Jeff Forshaw. Da Capo Press, July 2009. ISBN-13: 978-03068-175-88.

Zeno's Paradox: Unraveling the Ancient Mystery Behind the Science of Space and Time, by Joseph Mazur. Plume, March 2008 (reprint edition). ISBN-13: 978-0-4522-8917-8.

Doctoral Degrees Conferred

2008–2009

ALABAMA

Auburn University (10)

Department of Mathematics and Statistics

Baker, Charla, The intersection problem for Latin squares with holes of size 2 and 3

Bobga, Benkam, Some necessary conditions for list colorability of graphs and a conjecture on completing partial Latin squares

Chidume, Chukwudi, Iteration methods for approximation of solutions of nonlinear equations in Banach spaces

Divavahi, Venkata, Graph decompositions

Gammon, Kevin, Factorwise rigidity involving hereditarily indecomposable spaces

Jin, Kang, On the lattice Boltzmann method: Implementation and applications

Newman, Nicolas, Enclosings of small cycle systems

Nudurupati, Sai, Robust nonparametric discriminant analysis procedures

Peng, Man, Palm measure invariance and exchangeability for marked point processes

Turkmen, Asuman, Robust partial least squares for regression and classification

University of Alabama-Birmingham (11)

Department of Biostatistics

Azuero, Andres, Comparisons of sequential testing approaches for detection of association between disease and haplotype blocks

Banerjee, Samprit, Bayesian genome-wide QTL mapping for multiple traits

Chen, Lang, Microarray data analysis for SNPs effects and inferring alternative splicing

Dube, Tina, Assessing and correcting the effects of measurement error among correlated covariates in a proportional hazards setting

Keith, Scott, Free-knot splines and bootstrapping for nonlinear modeling in complex samples

Li, Qing, Interim monitoring efficacy, safety and futility in Phase III clinical trials

Nair, Nitin, Adaptive procedures to detect treatment effects under unexpected covariate interactions

Prucka, William, Wavelet-based regression and classification for longitudinal diffusion tensor imaging data

You, Zhiying, Power and sample size of cluster randomized trials

Department of Mathematics

Kulkarni, Mandar, Multi-coefficient Dirichlet-Neumann type elliptic inverse problems with application to reflection seismology

Mavinga, Nsoki, Nonlinear second order parabolic and elliptic equations with nonlinear boundary conditions

University of Alabama-Huntsville (2)

Department of Mathematics

Bowie, Miranda, Liar's domination and the domination continuum

Puckett, Matthew, Minimum wave speed and uniqueness of monotone traveling waves

University of Alabama-Tuscaloosa (4)

Department of Mathematics

Fayoumi, Hiba, A study of binary systems on sets and applications to several classes of such systems

Kang, Hyuna, Stochastic and key rate duration for value at risk

Liu, Congxiao, Mathematical study for the switch-initiating field of $2d$ ferroanti-ferromagnet exchange coupled systems

Wu, Leina, Grid refinement method for partial differential equations

ARIZONA

Arizona State University (14)

School of Mathematical and Statistical Sciences

Alvarado, Alejandra, Arithmetic progressions on curves

Fortney, Jon Pierre, Dirac structures in pseudo-gradient systems with an emphasis on electrical networks

Kang, Yun, The dynamics of plant-herbivore interactions and their implications for spatial expansion

Kelkar, Ashwini, A study of the subgraphs and the conjecture of the middle two layers graph, using modular matchings

Kupresanin, Ana Maria, Topics in functional canonical correlation and regresson

La Marca, Michael Benedetto, Control of continuum models of production systems

Park, Russell, Optimal compression and numerical stability for Gegenbauer reconstructions with applications

Saxena, Rishu, High order methods for edge detection and applications

Spiriti, Steven Mark, Random search optimization for free-knot splines and P-splines

Stefan, Wolfgang, Total variation regularization for linear ill-posed inverse problems: Extensions and applications

Unver, Ali Kemal, Observation based PDE models for stochastic production systems

Wallington, Rachel, Number fields with solvable Galois groups and small Galois root discriminants

The above list contains the names and thesis titles of recipients of doctoral degrees in the mathematical sciences (July 1, 2008, to June 30, 2009) reported in the 2009 Annual Survey of the Mathematical Sciences by 223 departments in 158 universities in the United States. Each entry contains the name of the recipient and the thesis title. The number in parentheses following the name of the university is the number of degrees listed for that university. A supplementary list containing names received since compilation of this list will appear in a summer 2010 issue of the *Notices*.

Yang, Hongling, A study of additive coefficient models

Zhang, Guoyi, Smoothing splines using compactly supported, positive definite, radial basis functions

University of Arizona (9)

Department of Mathematics

Bhattacharya, Abhishek, Nonparametric statistics on manifolds with applications to shape spaces

Kalaycioglu, Selin, Computing the projective indecomposable modules of large finite groups

Program in Applied Mathematics

Arciero, Julia, Theoretical models of blood flow regulation

Beauregard, Matthew, Nonlinear dynamics of elastic filaments conveying a fluid and numerical applications to the static Kirchhoff equations

Rael, Rosalyn, Comparing theory and data on multi-species interactions using evolutionary game theory

Reich, Daniel, Stochastic networks: Tractable approaches for identifying strategic paths

Robertson, Suzanne, Spatial patterns in stage-structured populations with density-dependent dispersal

Stover, Joseph, A general theory of monotonicity for interaction map particle systems and a stochastic spatial model for biological invasions

Vandiver, Rebecca, Morphoelasticity: The mechanics and mathematics of elastic growth

CALIFORNIA

California Institute of Technology (8)

Department of Applied and Computational Mathematics

Lyon, Mark, High-order unconditionally-stable FC-AD PDE solvers for general domains

Othmer, Jonathan, Mapping nucleic acid free energy landscapes

Stern, Ari, Geometric discretization of Lagrangian mechanics and field theories

Department of Mathematics

Pragel, Daniel, Embeddings of one-factorizations of hypergraphs and decompositions of partitions

Saha, Abhishek, On critical values of L-functions for holormorphic forms on $GSp(4) \times GL(2)$

Vuletic, Mirjana, The Pfaffian Schur process

Wong, Manwah, Orthogonal polynomials, paraorthogonal polynomials, and point perturbation

Zhuang, Dongping, A geometric study of commutator subgroups

Claremont Graduate University (7)

School of Mathematical Sciences

Bazan, Carlos, PDE-based image and structure enhancement for electron tomography of mitochondria

Hui, Kwok, Risks analysis of software development using Bayesian belief network and non-linear programming methods

Limon, Alfonso, A multilevel framework for PDEs whose solution exhibits fast transitions

Muhi El-Ddin, Imad, Watermarking schemes robust against affine attacks: An application of digital image processing in information technology

Nguyen, James, A hardware implementation of the level set method for robotic path finding with multiple obstacle avoidance

Park, Jeho, An empirical approach to communication and performance modeling for message passing parallel applications on cluster systems

Zhu, Bing, Computational modeling and bifurcation analysis of bubbling fluidized processes

Naval Postgraduate School (1)

Department of Applied Mathematics

Dea, John, High-order non-reflecting boundary conditions for the linearized Euler equations

Stanford University (5)

Department of Statistics

Hoefling, Holger, Topics in statistical learning

Kapur, Karen, Modelling background and cross-hybridization effects of microarray probes

Khare, Kshitij, Constructions of Gaussian fields from Markov processes, and related topics

She, Yiyuan, Sparse regression with exact clustering

Zhou, Hua, Topics on Markov chains with polynomial eigenfunctions

University of California, Berkeley (41)

Department of Mathematics

Armstrong, Scott, Principal half-eigenvalues of fully nonlinear homogeneous elliptic operators

Baginski, Paul, Stable $\aleph_0$-categorical algebraic structures

Blasiak, Jonah, Cyclage, catabolism, and the affine Hecke algebra

Brown, Jeffrey, Gromov-Witten invariants of toric fibrations

Chen, Qingtao, Some mathematical aspects of quantum field theory

Cotton-Clay, Andrew, Symplectic Floer homology of area-preserving surface diffeomorphisms and sharp fixed point bounds

Dutter, Seth, Logarithmic jet spaces and intersection multiplicities

Flenner, Joseph, The relative structure of Henselian valued fields

Judson, Zachary, Connectivity in the Higson corona of the positive real access

Long, Yun, Mixing time of the Swendsen-Wang dynamics in complete graphs and trees

Mkrtchyan, Sevak, Scaling limits of random skew plane partitions

Peters, Emily, A planar algebra construction of the Haagerup subfactor

Prat-Waldron, Arturo, Pfaffian line bundles over loop spaces, spin structures and the index theory

Ramakrishnan, Janak, Type of o-minimal theories

Rothbach, Brian, Borel orbits of $X^2 = 0$ in Gl_n

Sain, Jeremy, Berezin quantization from ergodic actions of compact quantum groups and quantum Gromov-Hausdorff distance

Sammis, Ian, Implicit and fourth-order semi-Lagrangian contouring for geometric moving interface problems

Schommer-Pries, Christopher, The classification of two-dimensional extended topological field theories

Snyder, Noah, Quantum groups, tensor categories and knot invariants

Somersille, Stephanie, Biased tug of war, the biased infinity Laplacian and comparison with exponential cones

Stovall, Lindsay, L^p inequalities for certain generalized Radon transforms

Tohaneanu, Mihai, Local energy estimates and structure estimates on Schwarzschild and Kerr black hole backgrounds

Tolland, Andrew, Gromov-Witten gauge theory

Trokhimchouk, Maxim, Regularity for certain parabolic diffusion systems

Varilly-Alvarado, Anthony, Arithmetic of del Pezzo surfaces of degree 1

Wu, Guoliang, Impulse control of multidimensional diffusion and jump diffusion processes

Zhu, Xinwen, Gerbal representations of double loop groups

Department of Statistics

Ahn, Soyeon, Statistical topics in gene regulation

Bae, Chongsoon, Analyzing random forests

Hather, Greg, Statistical analysis of DNA sequence motifs and microarray data

Liang, Richard, Two continuum-sites stepping stone models in population genetics with delayed coalescence

Obozinski, Guillaume, Simultaneous variable selection and simultaneous subspace selection for multitask learning

Ralph, Peter, Most recent common ancestors, genetic inheritance, stochastic gene transcription and Brownian motion on disconnected sets: A probabilistic analysis of a few models

Rosenberg, David, Semi-supervised learning with multiple views

Sly, Allan, Spatial and temporal mixing of Gibbs measures

Vu, Vince, High dimensional data analysis: Entropy and regression

Xu, Na, Transcriptome detection by multiple RNA tiling array analysis and identifying functional conserved noncoding elements by statistical testing

Yan, Donghui, Some issues with dimensionality in statistical inference

Group in Biostatistics

Gilbert, Houston, Multiple hypothesis testing: Methodology, software implementation, and applications to genomics

Moore, Kelly, Targeted maximum likelihood estimation of treatment effects in randomized controlled trials and drug safety analysis

Rubin, Daniel, Applications of double robustness

University of California, Davis (20)

Department of Mathematics

Baba, Shinpei, Decomposition theorems for complex projective structures

Berg, Christopher, Combinatorics of $(l, 0)$-JM partitions, l-cores, the ladder crystal and the finite Hecke algebra

Hallowell, Karl, Higher spin approaches to quantum field theory and (pseudo)-Riemannian geometrics

Haws, David, Matroid polytopes: Algorithms, theory, and applications

Miller, Adam, How disturbance creates the storage effect and promotes maintenance of species diversity

Mulherkar, Jaideep, Some properties of the XXZ spin chain and their applications to quantum computing

Needell, Deanna, Topics in compressed sensing

Raz, Hillel, Lieb-Robinson bounds in the anharmonic lattice

Sershen, Cheryl, A dynamic model of DNA structure and function

Stevens, Alice, Knots in Heegaard surfaces

Strawbridge, Eva, Mechanics, dynamics, and structures of DNA

Department of Statistics

Barkauskas, Donald, Statistical analysis of matrix-assisted laser desorption/ionization Fourier transform ion cyclotron resonance mass spectrometry data with applications to cancer biomarker detection

Chen, Kun, Functional approaches for high dimensional and gene expression data

Chen, Ying, Statistical approaches for detection of relevant genes and pathway in analysis of gene expression data

Liu, Bitao, Functional data analysis for online auction data

Liu, Jingyi, A shrinkage-average method for fitting high dimensional linear models

Liu, Ziqi, Nonparametric bootstrap method on Stiefel manifold and GPAV algorithm for ASP fit

Norris, Ann (Michelle), Parametric and semiparametric joint modeling for longitudinal diagnostic outcomes

Weng, Qian, Modeling progression of neurodegenerative disease with longitudinal neuroimaging data

Xu, Qiuyan, Estimation of integrated covolatility for asynchronous assets in the presence of microstructure noise

University of California, Irvine (8)

Department of Mathematics

Bjork, Andrew-David, Criteria for rank-one transformations to be weakly mixing, and the generating property

Cao, Yanping, On the α-regularization of nonlinear partial differential equations-analysis and error estimates

Cox, Sean, Covering theorems for the core model, with applications

Hu, Zhengzheng, Phase-field modeling of thin film epitaxial growth

Kypriotakis, Kyriakos, Combinatorial principles in Jensen type extender models of the form $L[E]$

Le, Phong, Coherent decomposition of p-adic Newton polygons for L-functions of exponential sums

Luo, Songting, Numerical methods for static Hamilton-Jacobi equations

Matayoshi, Jeffrey, The zeros of random polynomials

University of California, Los Angeles (22)

Department of Mathematics

Blunk, Mark, Del Pezzo surfaces of degree 6 over an arbitrary field

Chen, Lin, Chern-Simons theory of knot invariants

Chifan, Ionut, Deformation spectral gap rigidity in von Neumann algebras

Citro, Craig, L-invariants of adjoint square Galois representations coming from modular forms

Conley, Clinton, Some applications of combinatorics in descriptive set theory

Dong, Bin, Applications of variational models and partial differential equations on medical image and surface processing

Gautam, Sushrut Zubin, Two geometric obstruction results in harmonic analysis

Herman Jr., Paul Edward, Applications of beyond endoscopy

Kim, Yunho, Variational methods: Theory and its applications to image deblurring and denosing problems

Lie, Victor Daniel, Relational (quadratic) time-frequency analysis

Liu, Jian, Controlling the dynamics of recurrent neural networks with synaptic learning rules

Lu, Steve, New constructions in pairing-based cryptography

Mathews, Bryant, Canonical dimension of projective homogeneous varieties of inner Type A and Type B

Nguyen, Dinh Huu, On P-generic splitting varieties for Milnor K-symbols mod P

Roy, Tristan, Global analysis of the defocusing cubic wave equation in dimension 3

Shao, Shuanglin, Restriction of estimates for paraboloids and cones in the cylindrically symmetric case

Shiber, Dan, Tracial and non-tracial random matrix models in free probability

Thompson, Alexander, A metamathematical condition equivalent to the existence of a complete left invariant metric for a Polish group

Vanvalkenburgh, Michael, Hermite/Laguerre-Gaussian modes and lower bounds for quasimodes of semiclassical operators

Virtanen, Jukka Tapio, Structure of elementary particles in non-Archimedean spacetime

Department of Statistics

Anderson, Ariana, Machine learning for classification and diagnosis of functional magnetic resonance image scans

Zhu, Long, Recursive composition in computer vision: Modeling, inference, and learning

University of California, Riverside (7)

Department of Statistics

Bi, Yingtao, Theoretical analysis of classification under CCC-noise and its application to semi-supervised text mining

Kim, Sungsu, Inverse circular regression with possibly asymmetric error distribution

Montes de Oca, Veronica, Nonparametric cusum algorithms with applications to network surveillance

Pal, Rupam Ranjan, Efficient designs for discriminating between two competing linear regression models

Shah, Payal, Sequential sampling methods using generalized linear models with applications to pest density estimation

Suarez Espinoza, Javier, Inference for the multiparameter skew normal distribution

Yu, Hua, Neutral zone classifiers within a decision-theoretic framework

University of California, San Diego (16)

Department of Mathematics

Armel, Jonathan, Holomorphic extension of solutions to homogeneous analytic partial differential equations

Averett, Maia Christine, On real Johnson-Wilson theories

Beeson, Amanda, Groups of special units

Comstock, Jana, A finite presentation of knotted trivalent graphs

Eldredge, Nathaniel, Holomorphic extension of solutions to homogeneous analytic partial differential equations

Fuller, Evan, Generating functions for composition/word statistics

Grice, Jon, Discrete quantum control

Gurvich, Michael, Some results on the topology of quasitoric manifolds and their equivalent mapping spaces

Horn, Paul, Random subgraphs of a given graph

Jehring, Kristin, Harmonic functions on Walsh's Brownian motion

Kinnally, Michael, Stational distributions for stochastic delay differential equations with non-negativity contraints

Lee, Nam Heon, A sufficient condition for stochastic stability of an Internet congestion control model in terms of fluid model stability

Niedermaier, Andrew, Statistics on wreath products

Raleigh, Sean, Ruled Legendrian knots and Lagrangian surfaces

Szypowski, Ryan, Least-squares finite elements and constrained evolution systems

Yacobi, Oded, An analysis of multiplicity spaces in classical symplectic branching

University of California, Santa Barbara (12)

Department of Mathematics

Dai, Dongyan, Convergence of the Yamabe flow on manifolds with boundary

Hendrata, Melisa, A dynamical system model for simulating myxobacteria life cycle

Kessenich, Paul, Global existence with small initial data for three-dimensional incompressible isotropic viscoelastic materials

Levitt, John, Embeddings of Mori dream spaces

Mohler, George, Efficient, non-stiff, and multiscale methods for complex fluids

Qi, Jiawei, Effective dynamics for ferrofluids

Taylor, Scott, Boring split links and unknots

Department of Statistics and Applied Probability

Eli, Kollman, Calibrating market and asset returns from stochastic volatility skew effects

Jang, Homin, Shrinkage estimators of variances and their applications to microarray data analysis

Marick, Sinay, Bayesian inference for linear and generalized linear models with flexible prior structure on the covariance matrix

Tung, David, Some contributions to inference based on spacing

Wignall, Brian, Structural modeling and top-down reduced-form modeling for multi-name credit derivatives

University of California, Santa Cruz (1)

Department of Applied Mathematics and Statistics

Schuresko, Michael, Controlling global network connectivity of robot swarms with local interactions

University of Southern California (12)

Department of Mathematics

Aydilek, Harun, Optimal decisions under recursive utility

Glatt-Holtz, Nathan, Stochastic equations in geophysical fluid dynamics

Guest, Simon, A solvable version of the Baer-Suzuki theorem and generalizations

Jedwab, Andrea, Representations of finite dimensional Hopf algebras

Oleksandr, Lytvak, Fully discrete approach for estimating local volatility in the generalized Black-Scholes setting

Piterbarg, Yuliya, Population modeling and Bayesian estimation for the deconvolution of blood alcohol concentration from transdermal biosensor data

Psiloyenis, Yiannis, Mixing conditions and long return times on Markov towers

Seliger, Philip, Optimal device design

Song, Qian, Optimal and exact control of deterministic, stochastic or parabolic differential equations

Yang, Yan, Mathematical modeling and analysis of representation error and system uncertainties for discretized distributed parameters systems

Zeng, Yu, Large deviations approach to the bi-stable systems

Zhou, Yuegang, Credit risk of a leveraged firm in a controlled optimal stopping framework

COLORADO

Colorado State University (13)

Department of Mathematics

Bikowski, Jutta, Electrical impedance tomography reconstructions in two and three dimensions; form Calderon to direct methods

Frederick, Christopher, Persistence homology of sequences of neighborhood complexes for graphs

Jamshidi, Arthur, Modeling spatio-temporal systems with skew radial basis functions: Theory, algorithms and applications

Jónsdóttir, Margrét, Automorphism towers of general linear groups

Ladd, Joshua, Large-scale computational analysis of national animal identification system mock data, including trace back and trace forward

Lee, Sheldon, An adaptive algorithm for an elliptic optimization problem, and stochastic-deterministic coupling: A mathematical framework

Department of Statistics

Delorey, Mark, Penalized estimation for sample surveys in the presence of auxiliary variables

E, Lidong, Application of generalized fiducial inference

Hancock, Stacey, Estimating the number of structural breaks in nonstationary time series

Huang, Wenying, Spatial processes with stochastic heteroscadesticity

Iverson, Todd, Kernel methods for a discrete state/continuous time process with auxiliary information

Wang, Ke, Spatial models with applications in computer experiment

Yang, Yu, Estimation for Lévy-driven CARMA processes

University of Colorado, Boulder (13)

Department of Applied Mathematics

Barker, Andrew, Parallel monolithic fluid-structure interaction algorithms with application to blood flow simulation

Haut, Terry, Analysis and applications of nonlocal spectral formulations of fluids with free interfaces and surfaces

Sanders, Geoffrey, Two extensions to adaptive smooth aggregation (aSA) multigrid: Eigensolver initialization and nonsymmetric problems

Simpson, David, Bifurcations in piecewise-smooth, continuous systems

Department of Mathematics

D'Andrea, Jonas, Wavelet frames on fractal spaces of Dutkay-Jorgensen type

Denoncourt, Hugh, Some combinatorial models for reduced expressions in Coxeter group

Ernst, Dana, A diagrammatic representation of an affine C Temperly-Lieb algebra

Formichella, Marc, Functional equations among Barnes' integrals and hypergeometric series

Kang, Sooran, The Yang-Mills functional and Laplace's equation on quantum Heisenberg manifolds

Mishev, Ilia, Coxeter group actions on supplementary pairs of Saalschützian ${}_4F_3(1)$ hypergeometric series

Pohlmann, Brent, Structural properties of acyclic heaps with applications to Kazhdan-Lusztig theory

Schumacher, Timothy, Removable boundary singularities for the equation $\Delta u = u^\alpha$ in non-smooth domains

Shaw, Jason, Commutator relations and the clones of finite groups

University of Colorado, Denver (5)

Department of Biostatistics and Informatics

Boyd, Adam, Censoring-robust treatment effect estimation in clinical trials with time-to-event outcomes

Department of Mathematical and Statistical Sciences

Beezley, Jonathan, High-dimensional data assimilation and morphing ensemble Kalman filters with applications in wildfire modeling

Flink, Stephen, Truncated quadrics and elliptic curves

Harris, Angela, Cycle structures in graphs

Zakharyan, Armen, Stochastic diffusion model of heterogeneous populations

University of Northern Colorado (2)

School of Mathematical Sciences

Phipps, Marnie, A phenomenological investigation on eighth graders' number sense of fractions

Toney, Allison, Women with advanced degrees in mathematics in doctoral programs in mathematics education

CONNECTICUT

University of Connecticut, Storrs (13)

Department of Mathematics

Alagic, Gorjan, Uncertainty principles for compact groups

Bella, Thomas, Topics in numerical linear algebra related to quasiseparable and other structured matrices

Corluy, Marc, Rates of conveyance in the central limit theorem for Markov chains

Jura, Matthew, Reverse mathematics and the coloring number of graphs

Levin, Oscar, Computability theory, reverse mathematics and ordered fields

Markkanen, Tyler, Separating the degree spectra of structures

Prime, Russell, Averaging quadratic L-functions over function fields

Rong, Zhou, Numerical simulation of cell movement

Sang, Hailin, Asymptotic properties of generalized kernel density estimators

Termine, Lisa, Existence of solutions to semilinear elliptic differential equations using computer verification

Wooster, Robert, Evolution systems of measure for stochastic differential equation with Levy noise

Department of Statistics

Xie, Wangang, Bayesian phylogenetic model selection and applications

Zhao, Yifang, Contributions to microarray data analysis

Wesleyan University (4)

Department of Mathematics and Computer Science

Gould, Franklin, On certain classes of minimally almost periodic groups

Hall, Becky, An improved method for computing group cohomology of congruence subgroups of $SL_3(\mathbb{Z})$

Pan, WeiWei, Group actions on categorified bundles

Shea, Steve, Finitary isomorphisms of r-processes

Yale University (4)

Department of Mathematics

Aryapoor, Masood, The Penrose transform and self-dual Yang-Mills equations in split signature

Le Donne, Enrico, Bi-Lipschitz homogeneous geodesic manifolds

Mohammadi, Amir, Unipotent flows in positive characteristics and applications

Warshall, Andrew, Depth and related properties of infinite finitely generated groups

DELAWARE

University of Delaware (8)

Department of Mathematical Science

Gutekunst, Todd, Subsets of finite groups exhibiting additive regularity

Heryudono, Alfa, Adaptive radial basis function methods for the numerical solution of partial differential equations with application to the simulation of the human tear film

Maki, Kara, Computational solutions of linear systems and models of the human tear film

Wang, Zeying, Skew Hadamard difference sets, strongly regular graphs and p-ary bent functions

Wei, Ang, Random-harmonic functions and multivariate Gaussian estimates

Wu, Junhua, Geometric structures and linear codes related to conics in classical projective planes of odd orders

Xu, Liwei, Computational methods for a class of problems in acoustic, elastic and water waves

Zeev, Noam, Direct and inverse scattering problems for thin dielectric and partially coated objects

DISTRICT OF COLUMBIA

George Washington University (4)

Department of Mathematics

Luse, Terry, Invariants of knots, graphs, and Feynman diagrams

Pingrey, Sarah, Strong degree spectra of relations

Department of Statistics

Hikawa, Hiroyuki, Local linear Peters-Belson regression and its applications to employment discrimination cases

Yan, Lihan, Group sequential robust designs in genetic studies

Howard University (2)

Department of Mathematics

Al-Islam, Najja, Pseudo almost periodic solutions to some systems of nonlinear hyperbolic second-order partial differential equations

Salaam, Lifoma, Combinatorial statistics on phylogenetic trees

FLORIDA

Florida Atlantic University (4)

Department of Mathematical Sciences

Bozovic, Vladimir, Algebraic and combinatorial aspects of group factorizations

Hopkins, Mary, Weakly integrally closed monoids and forbidden patterns

Kasprikova, Eva, Higher-order commutators in the method of orbits

Wess, Mark, Computing topological dynamics from time series

Florida Institute of Technology (1)

Department of Mathematical Sciences

Huang, Weijun, Sequential stochastic games

Florida State University (12)

Department of Mathematics

Lebedev, Yuri, Open math library for computing on Riemann surfaces

Lin, Haomin, An optimal control problem for a time dependent Ginzburg-Landau model of superconductivity

Mavroudis, Konstantinos, Constant proportions portfolio strategies in an evolutionary context under a dividend factor model

Salta, Emmanuel, Variance reduction techniques in pricing financial derivatives

Shah, Manan, Randomized quasi-Monte Carlo methods and the computation of endogenous mortgage rates

Tzigantcheva, Milena, Stochastic volatility extensions of the swap market model

Yoo, Eunjoo, Variance gamma pricing of American futures options

Department of Statistics

Balov, Nikolay, Covariance on manifolds

Ivanescu, Andrada, Revealing sparse signals in functional data

Lin, Lanjia, Association models for clustered data with binary and continuous responses

Liu, Yang, Transformation models for survival data analysis and applications

Simino, Jeanette, Discrimination and calibration of prognostic survival models

University of Central Florida (8)

Department of Mathematics

Boustique, Hatim, Lattice valued convergence: Quotient maps

Bryant, Donald, Analysis of Kolmogorov's superposition theorem and its implications with low and high dimensional data

Galiffa, Daniel J., The Sheffer *B*-type 1 orthogonal polynomial sequences

Khosravi, Mehrdad, Pseudoquotients: Construction, applications, and their Fourier transform

Konate, Souleymane, Efficient cone beam reconstruction for the distorted circle and line trajectory

Landon, Benjamin, Degree of approximation of Holder continuous functions

Lopez, Jerry, Optimal dual frames for erasures and discrete Gabor frames

Macon, Lisa Fischer, Almost regular graphs and edge face colorings of plane graphs

University of Florida (6)

Department of Mathematics

Dashti, Seyyed, Effective symbolic dynamics

Nguyen, Hung Ngoc, Representations of finite groups of Lie type

Department of Statistics

Li, Zhen, Bayesian methodologies for genomic data with missing covariates

Roy, Vivekananda, Theoretical and methodological developments for Markov chain Monte Carlo algorithms for Bayesian regression

Wu, Song, A robust approach for genetic mapping of complex traits

Yap, John, Nonparametric covariance estimation in functional mapping of complex dynamic traits

University of Miami (3)

Department of Mathematics

Coburn, Brian, Multi-species influenza models with recombination

Donzelli, Fabrizio, Algebraic density property of homogeneous spaces

Zivanovic, Sanja, Attractors in dynamics with choice

University of South Florida (1)

Department of Mathematics and Statistics

Miladinovic, Branko, Kernel density estimation of reliability with applications to extreme value distributions

GEORGIA

Emory University (11)

Department of Biostatistics

Kwee, Lydia, Multilocus methods for association mapping of complex traits

Lu, Chengxing, Statistical methods to adjust for misclassified repeated exposures in modeling disease-exposure associations

Price, Megan, Issues in causal inference and applications to public health

Wiener, Jeffrey, Evaluating agreement among observers or methods of measurement for quantitative data

Department of Mathematics and Computer Science

Avart, Christian, Colorful flowers

Castle, Mariangely, Polya's theorem with zeros

Chung, Julianne, Numerical approaches for large-scale ill-posed inverse problems

Dudek, Andrzej, Problems in extremal combinatorics

Malagon, Audrey, Killing forms of Lie algebras

Poerschke, Annika, On algorithmic hypergraph regularity

Razouk, Nader, Localization phenomena in matrix functions: Theory and algorithms

Georgia Institute of Technology (13)

School of Mathematics

Bilinski, Mark, Approximating the circumference of 3-connected claw-free graphs

Bird, Csaba, Problems and results in partially ordered sets, graphs and geometry

Greenberg, Samuel, Random sampling of lattice configurations using local Markov chains

Jiménez, David, Analysis of two problems in signal quantizations and A/D conversion

Li, Yongfeng, Nonlinear oscillation and control in the BZ chemical reaction

Litherland, Trevis, On the limiting shape of random young tableaux for Markovian words

Marcus, Adam, New combinatorial techniques for non-linear orders

Pearson, John, The noncommutative geometry of ultrametric Cantor sets

Pugliese, Alessandro, Theoretical and numerical aspects of coalescing of eigenvalues and singular values of parameter dependent matrices

Savinien, Jean, Cohomology and *K*-theory of aperiodic tilings

Xu, Hua, Aspects of random matrix concentration and subsequence problems

Young, Stephen, Random dot product graphs: A flexible model for complex networks

Yurchenko, Aleksey, Some problems in the theory of open dynamical systems and deterministic walks in random environments

University of Georgia (13)

Department of Mathematics

Bagci, Irfan, Cohomology and support varieties in Lie superalgebras

Hower, Jeremiah, On elliptic curves and arithmetical graphs

Zhuang, Chao, Approximation methods and applications in financial optimization problems

Department of Statistics

Breazel, Ellen Hepfer, Effects of common errors in microsatellite data on estimates of population differentiation and inferring genotypic structure of complex loci using genome-wide expression data

Chen, Jien, Applications of empirical likelihood to quantile estimation and longitudinal data

Le-Rademacher, Jennifer, Principal component analysis for interval-valued and histogram-valued data and likelihood functions and estimators for symbolic data

Liu, Shangbin, Statistical issues on mass spectrometry-based protein identification and quantitation

Qiu, Junshan, Variable selection methods with applications

Sun, Guoying, A genetical genomics to genome scan for complex traits

Tewari, Susanta, Construction of high-resolution likelihood-based integrated genetic and physical map of neurospora crassa

Wang, Qin, Sufficient dimension reduction and sufficient variable selection

Ye, Jun, Geostatistical methods for spatio-temporal analysis of fMRI data

Zhen, Xiaoa, Categorical time series

IDAHO

University of Idaho (3)

Department of Mathematics

Bauer, Ryan, Disjoint cycles and tree packings in graphs

Buzbas, Erkan Ozge, Likelihood methods for balancing selection under the *k*-allele models

Tran, Luong Dinh, Nonlinear extensions of the Erdős-Ginzburg-Ziv theorem

ILLINOIS

Illinois Institute of Technology (5)

Department of Applied Mathematics

Aijun, Du, Deviation, ergodicity and reduction for stochastic dynamical systems

Chen, Bao Hua, Stochastic modeling of water vapor in the climate system

Vidozzi, Frederico, Two essays on multivariate stochastic processes and applications to credit risk modeling

Vidozzi, Sebastiano, Selected applications of the theory of Markov processes to mathematical finance

Zeng, Xiaoyan, Integration and approximation using lattice designs

Illinois State University (2)

Department of Mathematics

Casey, Stephanie, Subject matter knowledge for teaching statistical association

Williams, Nicole, Interpreting scatterplots: The role of statistical knowledge and personal knowledge

Northern Illinois University (6)

Department of Mathematical Sciences

Bilgili, Devrim, Quantitative trait loci detection using an accelerated failure time cure model

Corley, Todd, Classification methods for primitive table algebras of dimension 4

Lund, Mark, Nevanlinna theory in annuli

Meshes, Jonathan, Functions in class *F* in the unit disc which have integrals not in *F*

Racovitan, Michael, Generalizations for the blocks of the symmetric group and Nakayama conjecture

Sokolov, Vadim, Quadratic inverse eigenvalue problems: Theory, methods and applications

Northwestern University (11)

Department of Engineering Sciences and Applied Mathematics

Conway, Jessica, Complex patterns in oscillatory systems

Hansen, David, Axisymmetric Stokes flow solutions and bubble microstreaming

McCallum, Matthew, Eigenfunctions of cyclic integral transforms

Menon, Vilas, Building better models of the CA1 pyramidal neuron: A combined computational and experimental approach

Merkey, Brian, Biofilm modeling for wastewater treatment: Multiple species and multiple components

Sample, Christine, Nonlinear dynamics of interacting interfaces in electrochemical and biological systems

Smith, Bryan, The extended finite element method for special problems with moving interfaces

Department of Mathematics

Nanes, Kalman, On the ergodicity of partially hyperbolic flows

Ouyang, Cheng, Asymptotics of implied volatility in local volatility models

Qian, Randy, Diffractive theorems for the wave equation with inverse square potential

Wang, Fang, Minimal measures for Lagrangian systems

Southern Illinois University, Carbondale (5)

Department of Mathematics

A, Min, Solvability of diffusion models of insect movement in complex landscapes

Ghebremicael, Aman, Latin squares and applications

Schwartz, Andrew, Decompositions of graphs and trees

Teweldemedhin, Amanuel, Nonparametric estimators of mean residual life function

Ye, Ping, Tests of symmetry with ordered alternatives in three-dimensional contingency tables

University of Chicago (15)

Department of Mathematics

Beceanu, Marius, A critical centre-stable manifold for the semilinear Schrödinger equation

Bennett, Hanna, Volume distortion in groups

Biringer, Ian, Algebra versus geometry in hyperbolic 3-manifolds

Broshi, Michael, Moduli of finite flat group schemes with *G*-structure

Conidis, Chris, Applications of computability theory

Cudney, Richard, A category of representations at the critical level

Fowler, James, Poincaré duality groups and homology manifolds

Ginensky, Adam, A generalization of the Clifford index and determinantal equations for curves and their secant varieties

Malestein, Justin, Topological properties of a surface and the lower central series

Patel, Deepam, De-Rham epsilon factors

Patterson, Eric, Singularities of configuration and graph hypersurfaces

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Smith Zbarsky, Emma, Families of *A*-infinity algebras and homotopy group actions

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University of Illinois at Chicago (15)

Division of Epidemiology and Biostatistics

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Joslin, Charlotte, The association between Acanthamoeba keratitis and domestic water exposure in the Chicago area

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Department of Mathematics, Statistics, and Computer Science

de los Angeles Ortiz Rodriguez, Rosario, A statistical look at day trading

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University of Illinois, Urbana-Champaign (25)

Department of Mathematics

Barrus, Michael, On induced subgraphs, degree sequences, and graph structure

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Duong, Han, Minimal volume lattice d-simplices

Gunaydin, Ayhan, Model theory of fields with multiplicative group

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Landquist, Eric, Infrastructure, arithmetic, and class number computations in purely cubic function fields of characteristics at least 5

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Lopes, Vincius Cifu, Grothendieck semirings and definable endofunctions

Marikova, Jana, O-minimal fields with standard past map

Masri, Nadia, Fourier coefficients of modular forms and their applications

Mizrak, Ozgur, Asymptotic stability of the ground states of the nonlinear Schrödinger equation

Popa, Ana-Maria, On completely bounded multipliers of the Fourier algebra $A(G)$

Rojas, Ricardo, Loops and finite affine planes

Sahin, Mehmet, Moduli space of sheaves on P^2 with Hilbert polynomial $4n + 1$

Santhanam, Rekha, Units of equivariant ring spectra

Sheikh, Naeem N., Extremal problems for partitions of edge-sets of graphs

Stan, Florin, Trace problems in algebraic number fields and applications to characters of finite groups

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Department of Statistics

Hong, Hyokyoung, Prediction of conditional quantiles in ordinal regression with applications to aging research

Huebner, Alan, Modeling correlated ordinal data: Marginal and conditional approaches

INDIANA

Indiana University, Bloomington (16)

Department of Mathematics

Alan, Muhammed Ali, Weighted pluripotential theory

Altok, Serdar, Reversibility of simple random walk on Galton-Watson trees

Bateman, Michael, Maximal averages over rectangles in the plane

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Crawford, Jesse, Interpretation of eigenvalues in multivariate statistical analysis

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Hackman, Michelle, A new family of screw-motion invariant minimal surfaces

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Nguyen, Toan, Asymptotic stability of noncharacteristic viscous boundary layers

Van Cott, Cornelia, Obstructions to slicing generalized doubles

Yarahmadian, Shantia, Pointwise Green function bounds and stability of noncharacteristic boundary layers

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Purdue University (37)

Department of Mathematics

Bakhtary, Parsa, On the cohomology of a simple normal crossings divisor and its dual complex, with applications to isolated singularities

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Fang, Qirong, Efficient and stable spectral methods for acoustic scattering over bounded obstacles

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Kim, Jaehong, Holomorphic Banach bundles over compact manifolds

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Kwan, Yuen Yick, Fast spectral methods for incompressible flows

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Yao, Song, Quadratic nonlinear expectations and risk measures

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Zhang, Songfu, Smoluchowski-Kramers approximation for stochastic equations with Levy-noise

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Department of Statistics

An, Lingling, Dynamic clustering of time series gene expression

Anderson, David, Multifractal fractional sum-difference models for internet traffic

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Jeng, Xinge Jessie, Covariance adaptation and regularization in large-scale hypothesis testing and high-dimensional variable selection

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Shu, Lei, Stepwise model building for mixed-effect models with random scale effects

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Zeng, Lingmin, Group variable selection methods in analyses of genomic data

University of Notre Dame (7)

Department of Mathematics

Broad, Steven, Index formulas for higher order Loewner vector fields

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Jensen, Raymond, Integral boundary invariants for conformally compact manifolds of constant curvature and measure-theoretic Einstein condition

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Department of Mathematics

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Dagli, Mehmet, Lie algebra decompositions with applications to quantum dynamics

Drignei, Mihaela Cristina, Inverse Sturm-Liouville problems using multiple spectra

Lee, Jangwoon (Leo), Analysis and finite element approximations of stochastic optimal control problems constrained by stochastic elliptic partial differential equations

Mikkelson, Rana, Minimum rank of graphs that allow loops

Pryporova, Olga, Types of convergence of matrices

Shin, Jaemin, Inverse scattering problems for first-order systems

Xie, Xiaoliang, Large time-stepping methods for higher order time-dependent evolution equations

Xu, Ying, Modeling and direct numerical simulation of particle-laden turbulent flows

Department of Statistics

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Lu, Lu, Statistical methods in a high school transcript survey

Ma, Haiming, New developments in planning accelerated life tests

Ramler, Ivan, Improved statistical methods for k-means clustering of noisy and directional data

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University of Iowa (11)

Department of Applied Mathematics and Computational Sciences

Gillispie, Brian, Instabilities of elastic objects in motion

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Department of Biostatistics

Bayman, Emine O., Bayesian hierarchical models for multi-center clinical trials: Power and subgroup analyses

Chen, Yiyi, Optimal adaptive group sequential design for Phase II clinical trials: A Bayesian decision theoretic approach

Department of Mathematics

Chun, Sangmin, The annihilator semigroup ring

Froelich, Jennifer, Universal deformation rings related to the symmetric group S5

Ljumanovich, Leonida, Horn theories

Stehnova, Jitka, Theta correspondence for unitary groups

KANSAS

Kansas State University (6)

Department of Mathematics

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Shklyarov, Dmytro, Hirzebruch-Riemann-Roch theorem for differential graded algebras

Department of Statistics

Jung, Yoonsung, Effects of unequal treatment variances on the analysis of crossover designs

Ndum, Edwin, Individual treatment effect heterogeneity in multiple time points trials

Tang, Zhongwen, Lack of fit of logistic GEE models and cost efficient Bayesian optimal designs for nonlinear combinations of parameters in nonlinear regression models

Zhang, Ke (Kurt), Inference of nonparametric hypothesis testing on high dimensional longitudinal data and its application in DNA copy number variation and microarray data analysis

University of Kansas (5)

Department of Mathematics

Kastens, Jude, Some new developments on two separate topics: Statistical cross validation and floodplain mapping

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Moen, Kabe, Linear and multilinear fractional operators: Weighted inequalities, sharp bounds, and other properties

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Wichita State University (1)

Department of Mathematics and Statistics

Vargas, German, Spectral methods solution of Navier-Stokes equations for steady viscous flows

KENTUCKY

University of Kentucky (13)

Department of Mathematics

Bouchat, Rachelle, Algebraic properties of edge ideals

Brown, Tricia, Rees products of posets and inequalities

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Chifman, Julia, Direct products and the intersection map of certain classes of finite groups

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Nicolas, Carlos, Structural and enumerative properties of k-triangulations of the n-gon

Schmidt, Jack, Rewriting systems for finite groups

Stokes, Erik, The h-vectors of matroids and the arithmetic degree of squarefree strongly stable ideals

Department of Statistics

Chirilla, Costel, Empirical processes and ROC curves with an application to linear combinations of diagnostic tests

Yu, Lei, Extended polychotomous logistic Markov model for longitudinal categorical responses

University of Louisville (3)

Department of Mathematics

Brauch, Timothy, Applications of the combinatorial nullstellensatz on bipartite graphs

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Louisiana State University, Baton Rouge (6)

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Alali, Bacim, Multiscale analysis of heterogeneous media for local and nonlocal continuum theories

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Meng, Chao, Stochastic and Copula models for credit derivatives

Morris, Karli, Trace forms of Abelian extensions of number fields

Ozer, Koray, Laplace transform inversion and time-discretization methods for evolution equations

Tugurlan, Maria Cristina, Fast marching methods—Parallel implementation and analysis

University of Louisiana at Lafayette (4)

Department of Mathematics

Marinov, Tchavdar, An inverse problem for the Euler-Bernoulli equation and a new scheme for solving a hierarchical size-structured model with nonlinear growth, mortality, and reproduction rates

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MARYLAND

Johns Hopkins University (22)

Department of Applied Mathematics and Statistics

Beer, Elizabeth, Random latent position graphs: Theory, inference, and applications

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Lin, Xue, Rank-based methods for statistical analysis of gene expression microarray data

Ye, Xugang, Random disambiguation paths: Models, algorithms, and analysis

Department of Biostatistics

Carvalho, Benilton, Statistical methods and software for high density oligonucleotide arrays

Chan, Kwun Chuen (Gary), Recurrent marker process before failure event: A backward process approach

Cheng, Yu-Jen, Statistical methods for failure time data with biased sampling and measurement errors

Di, Chongzhi, Likelihood ratio testing under nonidentifiability with applications to biomedical studies

Eckel, Sandrah, Quantifying individual and city level modification of the health effects of air pollution in older adults

Ho, Yen-Yi, Statistical methods for failure time data with biased sampling and measurement errors

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Shum, Kim Fai (Kenny), Estimating the mean medical cost in small samples

Department of Mathematics

Agarwala, Susama, The geometry underlying renormalization

Banerjee, Abhishek, Nearby cycles, Archimedean complex and periodicity in cyclic homology

Breiner, Christine, Embedded minimal surfaces of finite topology and one end: Conformal structure and asymptotic properties

Chen, Yifei, Strong rational connectedness of toric varieties

Hezari, Hamid, Eigenvalues and eigenfunctions of Schrödinger operators: Inverse spectral problems and zeros of eigenfunctions

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University of Maryland, Baltimore County (5)

Department of Mathematics and Statistics

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Belding, Juliana, Number theoretic algorithms for elliptic curves

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Delgado, Robert, Density properties of subsets of the Euler characteristic-2 surface group, $\mathrm{PSL}(2, R)$ character variety

Draper, Thomas, Non-linear complexity of Boolean permutations

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Galaz-Garcia, Fernando, Non-negatively curved fixed-point homogeneous manifolds in low dimensions

Groer, Christopher, Parallel and serial algorithms for the vehicle routing problem and several variations

Hoban, Ryan, Local rigidity of triangle groups in $\mathrm{Sp}(4, R)$

Johnson, Gregory, Abstract elementary classes with Löwenheim-Skolem number cofinal with ω

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Long, Jane, The cohomology of the affine group over $Fp2$ and of $\mathrm{PSL}(3, Fp)$

Lott, Paul Aaron, Fast solvers for models of fluid flow with spectral elements

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Olsson, James Scott, Combining evidence from unconstrained spoken term frequency estimation for improved speech retrieval

Pauletti, Miguel, Parametric AFEM for geometric evolution equations and coupled fluid-membrane interaction

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Sun, Weiran, Mathematical problems arising when connecting kinetic to fluid regions

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Boston University (7)

Department of Mathematics and Statistics

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Uminsky, David, The viscous N-vortex problem: A Helmholtz/Kirchhoff approach

Boston University School of Public Health (6)

Department of Biostatistics

Blood, Emily, Performance of mixed effects models in the analysis of mediated longitudinal data generated from a structural equation model

Chibnik, Lori, Development and evaluation of risk prediction models in the presence of correlated markers and non-linear associations between markers and outcome using logistic regression and net-benefit analysis

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Stedman, Margaret, A comparison of methods for physician randomized trials with binary and survival outcomes

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Brandeis University (3)

Department of Mathematics

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Department of Mathematics

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School of Engineering and Applied Science

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Massachusetts Institute of Technology (19)

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Davis, Christopher, The overconvergent de Rham-Witt complex

Eu, Ching-Hwa, Hochschild homology/cohomology of preprojective algebras of ADET quivers

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Yin, Jingbin, A q-analogue of spanning trees: Nilpotent transformations over finite fields

Northeastern University (5)

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Tufts University (4)

Department of Mathematics

Espanol, Malena Inez, Multilevel methods for discrete ill-posed problems: Application to deblurring

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University of Massachusetts, Amherst (7)

Department of Biostatistics, Public Health

Li, Xiaohong, Approaches to estimation of haplotype frequencies and haplotype-trait associations

Xu, Bo, Predictors of treatment means for a one factor completely randomized design

Department of Mathematics and Statistics

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Milburn, Brett, Dirac maps and generalized complex geometry on homogeneous spaces

Shieh, Meng-Shiou, Correction methods, approximate biases, and inference for misclassified data

Worcester Polytechnic Institute (1)

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Huang, Xueying (Shirley), In vivo MRI-based three-dimensional fluid-structure interaction models and mechanical image analysis for human carotid atherosclerotic plaques

MICHIGAN

Michigan State University (13)

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Knapp, Adam, Computations of Lagrangian Floer homology and gauge theoretic invariants for Montesinos twins

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Department of Statistics and Probability

Aggarwal, Deepa, On some inference problems for current status data

Zhu, Yongfang, Statistical models for assessing the individuality of fingerprints

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Coffield, Daniel, A model for single phase flow in horizontally fractured porous media using homogenization techniques

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University of Michigan (25)

Department of Mathematics

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Gentry, Sara, Mathematical modeling of mutation acquisition in hierarchical tissues: Quantification of the cancer stem cell hypothesis

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Iwen, Mark, Combinatorial compressive sampling with applications

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Lee, KyungYong, On realization of line arrangements as multiplier ideals

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Vavrichek, Diane, Accessibility and JSJ decompositions of groups

Yang, Bo, Application of perturbation methods to pricing credit and equity derivatives

Department of Statistics

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Linn, Matthew, Nonlinear filtering of random fields in the presence of long-memory noise and related problems in stochastic analysis

Sen, Bodhisattva, A study of bootstrap and likelihood based methods in non-standard problems

Singhal, Harsh, Statistical inverse problems on graphs with applications to flow volume estimation in computer networks

Sun, HoKeun, Hybrid bootstrap for mapping quantitative trait loci and change point problems

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Wayne State University (5)

Department of Mathematics

Holdai, Veera, Multichannel change point problems

Kan, Shaobai, System identification in stochastic hybrid systems

Li, Weiyuan, Fundamental solution and L^p estimates for higher order subelliptic Schrödinger operators on stratified groups

Truong, Bao, Variational analysis and applications to multiobjective optimization

Zhu, Huiqing, Discontinuous Galerkin methods for singularly perturbed problems

Western Michigan University (7)

Department of Mathematics

Madden, Sandra, High school mathematics teachers' evolving study of comparing distributions

Department of Statistics

Awosoga, Oluwagbohunmi, Meta analyses of multiple baseline time series design intervention models for dependent and independent series

Nantz, Eric, Testing equality of competing risks of patient discontinuation across multiple disease states

Nyirenda, Themba, Heteroscedastic linear model estimation based on ranks: An iterated reweighted least squares

Ponnathapura, Srinand, Statistical procedures for bioequivalence analysis

Rosales, Mathew, Robustness of confidence intervals for the mean of delta distribution

Tena, Jezania Kira, Test procedures for equality of two variances in delta distributions

MINNESOTA

University of Minnesota-Twin Cities (10)

Division of Biostatistics, School of Public Health

Li, Meijuan, Statistical models for a censored quantitative trait and candidate genes association mapping in structured populations with multilevel genetic relatedness

Wei, Peng, Network-based mixture models for genomic data

School of Statistics

Adragni, Kofi Placid, Principal fitted components on small samples

Deng, Qiqi, Developing a distribution-free change-point model for unknown-baseline data

Dong, Hua, Statistical properties of Gaussian reproducing kernel in univariate density estimation

Flegal, James, Monte Carlo standard errors for Markov chain Monte Carlo

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Rendahl, Aaron, Graphical methods of determining predictor importance and effect

Wang, Huixin, On large margin hierarchical classification

Wu, Seongho, Homotopy, nonconvexity and regularization

MISSISSIPPI

Mississippi State University (1)

Department of Mathematics and Statistics

Shahul-Hameed, Jaffar Ali, Multiple positive solutions for classes of elliptic systems with combined nonlinear effects

University of Mississippi (4)

Department of Mathematics

Chism, Lyrial, On independence polynomials and independence equivalence in graphs

Craddock, Michelle, Reflexivity and the Grothendieck property for positive tensor products of Banach lattices

Lai, Wei-Kai (Brian), The Radon-Nikodym property for positive tensor products of Banach lattices

Song, Lanzhen, Independence polynomials of k-tree related and well-covered graphs

MISSOURI

Missouri University of Science and Technology (3)

Department of Mathematics and Statistics

Marciniak, Malgorzata, Holomorphic extensions in toric varieties

Mukhopadhyay, Purna, A modified approach for obtaining sieve bootstrap prediction intervals for time series

Supapakorn, Thidaporn, Estimating bounds for nonidentifiable parameters using potential outcomes

St. Louis University (1)

Department of Mathematics and Computer Science

Manche, Jaime, Best representations and completions in weakened topologies for the integers

University of Missouri-Columbia (9)

Department of Mathematics

Bihun, Oksana, Approximate isometries and distortion energy functionals

Cowell, Simon, Asymptotic unconditionality in Banach spaces

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Shane, Christopher, Uniqueness theorems for non-symmetric convex bodies

Department of Statistics

Lin, Xiaoyan, Bayesian hierarchical models for the recognition-memory experiment

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Zhang, Jing, Bayesian spatial analysis with application to the Missouri Ozark Forest Ecosystem Project

Zhu, Liang, Semiparametric analysis of multivariate longitudinal data

Washington University in St. Louis (11)

Department of Electricial and System Engineering

Yang, Ruoting, Data-driven two degree of freedom control for robust intelligent life science automation

Department of Mathematics

Abel, Haley, The role of positive selection in molecular evolution: Alternative models for within-locus selective effects

Bohanon, Joseph, Groups in which the normalizer of every non-normal sugroup is maximal

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Lott, Tim, Dehn fillings of hyperbolic punctured-torus bundles

Ronshausen, Emily, The Liouville property in the discrete group-action setting

Standeven, Bennett, The role of first order logic in analysis of several complex variables

MONTANA

Montana State University (3)

Department of Mathematical Sciences

Alonso, Nicomedes, III, An alternating-direction Sinc-Galerkin method for elliptic problems on finite and infinite domains

Segal, Sara, Action research in mathematics education: A study of a master's program for teachers

Vallines Mira, Raquel, Teachers' beliefs regarding effective teaching strategies for American Indian students in mathematics

University of Montana, Missoula (2)

Department of Mathematical Sciences

Elias, Joran, Randomness of tree ensemble methods

Yates, Rebekah, Norm-preserving criteria for uniform algebra isomorphisms

NEBRASKA

University of Nebraska-Lincoln (6)

Department of Mathematics

Bockelman, Brian, Solving partial differential equations with Sinc methods

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University of New Hampshire (6)

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Das, Paramita, Planar algebras of families of group-type subfactors

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Paddack, Megan, The process of making meaning: The interplay between teachers' knowledge of mathematical proofs and their classroom practice

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Shubov, Mikhail, Improved estimation of Fourier coefficients for ill-posed inverse problems

Yuan, Wei, Kadison-Singer algebras

NEW JERSEY

New Jersey Institute of Technology (3)

Department of Mathematical Sciences

Espin, Leonardo, Self-similar flows in finite or infinite two-dimensional geometries

Joshi, Yogesh, Discrete dynamical population models: Higher dimensional pioneer-climax models

Matakuti, Kamyar, Numerical detection of complex singularities in two and three dimensions

Princeton University (17)

Department of Mathematics

Brooks, Shimon, Entropy bounds for quantum limits

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Greene, Joshua, Donaldson's theorem, Heegaard Floer homology, and results on knots

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Program in Applied and Computational Mathematics

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Department of Statistics and Biostatistics

Chen, Chuanwen, Admissible, consistent multiple testing with applications

Li, Jixin, Saddle point approximations

Mukherjee, Somnath, A statistical test spectrum—from robust to powerful

Sun, Qiankun, Statistical modeling and inference for multiple temporal or spatial cluster detection

Wang, Jue, Some properties of robust statistics under asymmetric models

Zhang, Juan, Higher order conditional inference using parallels with approximate Bayesian techniques

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Department of Mathematics and Computer Science

Puri, Karan, Factorization of isometries of hyperbolic 4-space and a discreteness condition

NEW MEXICO

New Mexico State University, Las Cruces (3)

Department of Mathematical Sciences

Eydenberg, Michael, Kernel methods on Ω-ultradistributions

Noussi, Hubert, Stabilization of competition models in the chemostat via feedback linearization

Tran, Hien, Statistics of course data with applications to financial risk analysis

University of New Mexico (13)

Department of Mathematics and Statistics

Collins, Dave, Nonparametric solution to flowgraph models

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Deshler, Jessica, Predicting student success in undergraduate mathematics courses

Grandini, Daniele, Quaternionic Kähler reductions of Wolf spaces

Guba, Oksana, The spectra of ordinary differential equations

Meng, Chen, Phylogenetic models for hybridization in a coalescent framework

Panek, Danusz, On sharp extrapolation theorems

Peterson, Kara, Modeling artic sea ice using the material-point method and an elastic-decohesive rheology

Ringler, Adam, Positivity problems in higher dimensional geometry

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Villagren-Hernandez, Alejandro, Monte Carlo strategies for calibration in climate models

Ying, Wenxia, Bayesian modeling of nonstationary extremal events

Zhu, Min, Functional transformed models: Frequentist on Bayesian approaches

NEW YORK

Clarkson University (2)

Department of Mathematics and Computer Science

Tang, Yuefeng, Interpolation algorithms in program verification

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Columbia University (18)

Department of Mathematics

Cohen, Dvora, The existence of asymptotically parallel spinors on manifolds asymptotic to hypersurfaces in Minkowski space

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Goia, Irina, Bessel and volatility-stabilized processes

Li, Zhi, On Λ-adic Saito-Kurokawa lifting and its application

Maharaj, Yogishwar, Line bundles, curves, and quasi-projective surfaces

Mapes, Sonja, Finite atomic lattices and their relationship to resolutions of monomial ideals

McFeron, Donovan, Remarks on some non-linear heat flows in Kahler geometry

Mosinca, Natalia, Probability on graphs and groups: Theory and applications

Pedersen, Helge, Splice diagrams, singularity links and universal Abelian covers

Ross, Joseph, The Hilbert-Chow morphism and the incidence divisor

Simring, Eric, Representations and combinatorics of Ocneanu's essential paths

Welji, Shaffiq, On the Heegaard Floer homology of singular knots and their unoriented resolutions

Zhang, Wei, Modularity of generating functions of special cycles on Shimura varieties

Zhou, Yun, A stochastic representation problem with applications in optimal singular control, Dynkin games and obstacle problems

Department of Statistics

Ichiba, Tomoyuki, Topics in multi-dimensional diffusion theory: Attainability, reflection, ergodicity and rankings

Jin, Man, Variable selection in canonical discriminant analysis

Paik, Jane, Contributions to the analysis of survival data subject nonstandard sampling schemes

Pospisil, Libor, Risk measures and maximum drawdown

Cornell University (12)

Center for Applied Mathematics

Diener, Nicolas, Mathematical models for swing options and subprime mortgage derivatives

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Owen, Megan, Distance computation in the space of phylogenetic trees

Restrepo, Mateo, Computational methods for static allocation and real-time redeployment of ambulances

Verdugo, Angel, Dynamics of gene networks with time delays

Department of Mathematics

Adams, Bryant, Representing processes as update automata and transducers

Horwitz, Noam, Free resolutions of monomial ideals

Koch, Sarah, A new link between Teichmüller theory and complex dynamics

Matucci, Francesco, Algorithms and classification in groups of piecewise-linear homeomorphisms

Minnes, Mor Mia, Computability and complexity properties of automatic structures and their applications

Murgescu, Radu, On the p-class groups of the pure number field $\mathbf{Q}(N^{1/p})$ and its Galois closure $\mathbf{Q}(N^{1/p}, \zeta_p)$

Workman, John, End-point estimates and multi-parameter paraproducts on higher dimensional tori

Graduate Center, City University of New York (7)

Ph.D. Program in Mathematics

Dean, Margaret, Finitely generated metabelian and solvable groups

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Orosz, Brooke, Problems in additive number theory

Tepper, Michael, Endomorphisms of n-dimensional projective space over function fields

New York University, Courant Institute (17)

Courant Institute of Mathematical Sciences

Albuts, Thomas, Dimension and measure of SLE on the boundary

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Hand, Paul, Homogenization in cardiac electrophysiology and blow-up in bacterial chemotaxis

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Le, Nam Quang, Analysis of several sharp-interface limits in variational problems

Lee, Darren, Two constructions of complete minimal surfaces

Mintchev, Stanislav, Self-organization phenomena in networks of pulse-coupled phase oscillators

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Render, Paris, Harmonic functions for random walks in random environments

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Schutzer, Emmanuel, Marking the $(1, 2)$ points of the Brownian web and applications

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Yarmola, Tatiana, Invariant measures for degenerate random perturbations of discrete-time dynamical systems

Yilmaz, Atilla, Large deviations for random walk in a random environment

Yin, Yong, Static Born-Infeld theory

Rensselaer Polytechnic Institute (9)

Department of Mathematical Sciences

Cristobal La Torre, Juan, Effective transplant properties of stochastics biophysical models

Ding, Xueru, Statistical equilibria of the coupled barotropic flow and shallow water flow on a rotating sphere

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Frenkel, Yevgeniy, Propagation of ultrashort electromagnetic pulses in a Maxwell-Duffing medium

Lin, Kui, Error estimation for the direct inversion model and numerical schemes for the full inversion model in elastography

Morales, Jose, Synthetic-aperture radar imaging and waveform design for dispersive media

Rasmussen, Kathryn, Optical pulse dynamics in active media with negative refractive index

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Wang, Chaoxiong, Power control for multiuser communication systems and computation of generalized Nash equilibria

State University of New York at Buffalo (5)

Department of Biostatistics

Cai, Xueya, Generalized linear model approach for estimating and testing equality of control correlations

Miller, Austin, Statistical methods for biodosimetry in the presence of both Berkson and classical measurement error

Department of Mathematics

Buettgens, Matthew, Property Tao and von Neumman algebras

Iovanov, Miodrag C., The representation theory of profinite algebras—interactions with category theory, algebraic topology and compact groups

Li, Bo, Regularity estimates for Berezin's operator calculus

State University of New York at Binghamton (4)

Department of Mathematics and Science

Junes, Leandro, Duality of higher order non-Euclidean property for oriented matroids

Pallekonda, Seshendra, Bounded category of an exact category

Sullivan, Margaret, DNA computing with cutting, pasting, filtering and washing

Tuncer, Nigar, Globalization theorems in topology

State University of New York at Stony Brook (15)

Department of Applied Mathematics and Statistics

Chang, Su-Wei, Growth mixture modeling as an exploratory analysis tool to identify longitudinal quantitative trait loci in genome-wide association studies

Chu, Adrienne, Classification of gastrointestinal bleeding data

Kifle, Hagos, Lattice Boltzmann simulation of immiscible two-phase flow in three dimensional porous media

Kim, Daesang, Network flow model for the simulation of CO2 sequestration process

Kim, Dongyung, Conservative data collections and comparison study for front-tracking

Lee, Jungyeon, Extending commingling analysis to exponential distributions and its genetic applications

Yongzhi, Chen, Task mapping on supercomputers with cellular networks

Department of Mathematics

Chandramouli, Vasu V. M. Sarma, Renormalization and non-rigidity

Clarke, Andrew, Rigidity of minimal submanifolds

Hao, Ning, D-bar spark theory and Deligne cohomology

Hazard, Peter, Hénon-like maps renormalisation

Ostrovsky, Stanislov, Weighted L^2 interpolation on non-uniformly separated sequences

Savelyev, Yakov, Quantum characteristic classes

Thind, Jaimal, Categorical combinatorial constructions of A, D, E root systems

Xu, Dezhen, Configurations of graphs and the master equation

Syracuse University (5)

Department of Mathematics

Graves, Christina, On the structure of reliability polynomials

Graves, Stephen, Growth of tessellations

Lu, Yao, Fast multiscale integral equation methods for image restoration

McCaffery, Adam, Structure of infinitely-ended, edge-transitive planar maps and their Petrie walks

Zhang, Haizhang, Sampling with reproducing kernels

The University of Albany, SUNY (3)

Department of Mathematics and Statistics

Clark, Timothy, Poset resolutions of monomial ideals

Savvopoulou, Anna, Almost everywhere convergence of convolution measures

Wedrychowicz, Christopher, Almost everywhere convergence of weighted averages

University of Rochester (7)

Department of Biostatistics and Computational Biology

Chen, Linlin, The correlation structure of microarray data and its statistical implications

Ma, Yan, Inference for instrument reliability and rater agreement within a multi-rater and longitudinal data setting

Pearson, Alexander, Subset selection for high-dimensional data, with applications to gene array data

Department of Mathematics

Huang, Haijun, Heat equation with noise restricted to fractal set

Li, Huibin, Invariant measure of Burger's system of equations

Palsu-Andriescu, Emanuel, Fourier integral operators on Colombeau spaces

Salch, Andrew, Moduli and cohomology

NORTH CAROLINA

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Department of Statistical Science

Bullard, Floyd, Exoplanet detection: A comparison of three statistics or how long should it take to find a small planet

Ghosh, Joyee, Efficient Bayesian computation and model search in linear hierarchical models

Mitra, Robin, Bayesian methods to impute missing covariates for causal inference and model selection

Niemi, Jarad, Bayesian analysis and computational methods for dynamic modeling

Ouyang, Zhi, Bayesian additive regression kernels

Puggioni, Gavino, Using data augmentation and stochastic differential equations in spatio temporal modeling

Scott, James, Bayesian adjustment for multiplicity

Vance, Eric Alan, Statistical methods for dynamic network data

Zhang, Liang, Statistical computation for model space exploration in high-dimensional problems

North Carolina State University (40)

Department of Mathematics

Andjelkovic, Ivan, Auxiliary signal design for fault detection for nonlinear systems: Direct approach

Benedict, Brandy, Axisymmetric poroelastic boundary element methods for biphasic mechanics of articular cartilage

Davis, Jimena, Uncertainty quantification in the estimation of probability distributions on parameters in size-structured population models

Dickson, Kelly, Condition estimates for numerical continuation with applications to atomic and molecular fluids

Ellwein, Laura, Cardiovascular and respiratory regulation, modeling and parameter estimation

Ernstberger, Jon, High speed parameter estimation for a homogenized energy model

Ernstberger, Stacey, Sensitivity methods for dynamical systems

Govan, Angela, Ranking theory with application to popular sports

Janovitz-Freireich, Itnuit, Computation of the exact and approximate radicals of ideals: Techniques based on matrices of traces, moment matrices and Bezoutians

Jiang, Qunlei, Analysis and computation for a fluid mixture model of tissue deformations

Kalhorn, Rebecca, On the solvability of nonlinear boundary value problems on time scales

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Olson, Sarah, Mathematical models for analysis of tissue regeneration in articular cartilage

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Siskind, Ryan, Model development for shape memory polymers

Wan, Wei, Dynamic game theoretic models in marketing and finance

Yang, Yipeng, Path dependent stochastic models and their applications in finance and communications

Yuhasz, George, Berlekamp/Massey algorithms for linearly generated matrix sequences

Department of Statistics

Anand, Suraj, Novel statistical approaches to assessing the risk of QT prolongation and sample size calculation in 'thorough QT/QTc studies'

Brinkley, Jason, Generalized estimators of the attributable benefit of an optimal treatment regime

Cho, Yoonjin, Comparing predictive values of two diagnostic tests

Choi, Jungsoon, Multivariate spatial-temporal modeling of environment-health processes

Curtis, Steven, Variable selection methods with applications to shape restricted regression

Demirhan, Eren, Variable selection for multivariate smoothing splines with correlated random errors

Krishna, Arun, Shrinkage-based variable selection for linear regression and mixed effects models

Liu, Song, Variable selection in generalized additive models and additive Cox models with nonnegative garrote

Ogorek, Benjamin, Orthology-based multilevel modeling of mouse and human gene pairs

Saha, Paramita, Robust inference with quantile regression in stochastic volatility models with application to value at risk calculation

Schumann, David, Robust variable selection

Wu, Yuefeng, Asymptotic behavior of some Bayesian nonparametric and semiparametric procedures

Zhang, Min, Asymptotic joint distributions of time to default with application to the pricing of credit derivatives

Zhang, Qianyi, Seasonal unit root test: A comparison

Zhang, Ying, Testing for unit roots in seasonal time series with long period

University of North Carolina at Chapel Hill (19)

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Abraham, Anita, Model selection methods for the linear mixed model for longitudinal data

Andraca Carrera, Eugenio, Approaches to parameter and variance estimation in generalized linear models

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Clement, Meagan, Analysis techniques for diffusion tensor imaging data

Lu, Tsui-Shan, Statistical inferences for outcome dependent sampling with continuous multivariate outcomes

Park, Ju-Hyun, Bayesian density regression and predictor-dependent clustering

Saville, Benjamin, Bayesian multilevel models and medical applications

Shi, Xiaoyan, Diagnostic measures for missing covariate data and semiparametric models for neuroimaging

Simpson, Sean, Linear models with generalized AR(1) covariance structure for longitudinal and spatial data

Department of Mathematics

Bouzarth, Elizabeth, Regularized singularities and spectral deferred correction methods: A mathematical study of numerically modeling Stokes fluid flow

Dodson, Benjamin, Caustics and the indefinite signature Schrödinger equation, linear and nonlinear

Newport, Jason, Solutions to the nonlinear Schrödinger equation with Dirac mass initial data

Olli, Jeanette, Tiling systems, division point measures, and endomorphisms

Department of Statistics and Operation Research

Lim, Changwon, Robust statistical theory and methodology for nonlinear models with applications to toxicology

Samarov, Daniel, Analysis and advanced extensions of canonical correlation analysis

Sen, Suman, Classification on manifolds

Wang, Fangfang, Statistical analysis of some financial time series

Wang, Zhaohui, Capacity investment strategies under operational flexibility

Yoon, Jungyeon, Contributions to stochastic volatility models and option pricing

University of North Carolina at Charlotte (4)

Department of Mathematics and Statistics

Huang, Hongwei, Statistical properties and applications of Turing's formula

Lee, Jimin, Some statistical methods for failure time data

Li, Jing, Risk minimizing portfolio optimization and hedging with conditional value-at-risk

Wang, Xian, Selection of mixed copulas and finite mixture models with applications in finance

NORTH DAKOTA

North Dakota State University, Fargo (2)

Department of Mathematics

Lambert, Joshua, Determining the biplanar crossing number of $C_k \times C_l \times C_m \times C_n$

Pile, Angela, The space of regular polygons with a small number of edges

OHIO

Bowling Green State University (3)

Department of Mathematics and Statistics

Attanayake, Champike, Finite elements and practical error—analysis of Huxley and EFK equations

Dong, Fanglong, Bayesian model checking in multivariate discrete regression problems

Rashid, Mamunur, Inference on logistic regression models

Case Western Reserve University (15)

Department of Epidemiology and Biostatistics

Chang, Shelley, Predicting methicillin-resistant S. aureus carriage and dissemination in a veterans affairs medical center

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Department of Mathematics

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Liu, Peng, Adaptive mixture estimation and subsampling PCA

Kent State University (6)

Department of Mathematical Sciences

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Dugan, Carrie, Solvable groups whose character degree graphs have diameter three

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Ignatyev, Oleksiy, The compact support property for hyperbolic SPDE: Two contrasting equations

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Ohio State University, Columbus (32)

Department of Mathematics

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Kar, Aditi, Discrete groups and CAT(0) asymptotic cones

Khoury, Jr., Michael, Multiplicity-one results and explicit formulas for quasi-split p-adic unitary groups

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Liu, Sheng-Chi, Mass equidistribution of Hecke eigenforms on the Hilbert modular varieties

McSweeney, John, Time to coalescence for a class of nonuniform allocation models

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Schnell, Christian, The boundary behavior of cohomology classes and singularities of normal function

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Department of Statistics

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Han, Gang, Modeling the output from computer experiments having quantitative and qualitative input variables and its applications

Kohlschmidt, Jessica, Ranked set sampling: A look at allocation issues and missing data complications

Kumar, Arun, Sequential calibration of computer models

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Markiewicz, Shannon, Nonparametric inference using order restricted randomized designs

Paul, Rajib, Theoretical and algorithmic developments in Markov chain Monte Carlo

Roy, Soma, Sequential-adaptive design of computer experiments for the estimation of percentiles

Sroka, Christopher, Extending ranked set sampling to survey methodology

Walters, Kimberly, The use of post-intervention data from waitlist controls to improve estimation of treatment effect in longitudinal randomized controlled trials

Wang, Zhen, Semi-parametric Bayesian models extending weighted least squares

Yao, Yonggang, Statistical applications of linear programming for feature selection via regularization methods

Zhang, Xiuyun, Efficient algorithms for fitting Bayesian mixture models

Ohio University, Athens (2)

Department of Mathematics

Blackwood, Brian, A study of partial orders on nonnegative matrices and von Neumann regular rings

Stover, Derrick, Continuous mappings and some new classes of spaces

University of Cincinnati (5)

Department of Mathematical Sciences

Baena, John, Fast signature schemes over odd characteristic

Clough, Crystal, Square: A new family of multivariate encryption schemes

Jin, Yan, Bayesian solution to the analysis of data with values below the limit of detection (LOD)

Sauer, Johnothon, Semigroups and their zero-divisor graphs

Teymuroglu, Zeynep, Continuum models for the spread of alcohol abuse

University of Cincinnati, Medical College (3)

Department of Environmental Health

Liu, Xiaolei, Sample size calculations in matched case-control studies and unmatched case-control studies with controls contaminated

Xi, Huifeng, Allele frequency distribution and its implication in association study

Yu, Tao, A self-building system for ranked set sampling

University of Toledo (2)

Department of Mathematics

Liu, Nanshan, Theory and applications of Legendre polynomials and wavelets

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OKLAHOMA

Oklahoma State University (4)

Department of Mathematics

Dollarhide, John, On the L-function of multiplicative character sums

Fisher, Brian, Students' conceptualizations of multivariable limits

Department of Statistics

Bryan, Jennifer, Rank transforms and tests of interaction for repeated measures experiments with various covariance structures

Shi, Lei, Homozygosity mapping with unknown common ancestors

University of Oklahoma (7)

Department of Mathematics

Cho, Sangbum, A finite presentation of the Goeritz group

Frye, Timothy, Graph reflection groups

Martin, Jason, Expert conceptualizations of the convergence of Taylor series, yesterday, today and tomorrow

Mecham, TaraLee, Largeness of graphs of Abelian groups

Mukherjee, Antara, Isoperimetric inequalities using Varopoulos transport

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OREGON

Oregon State University (6)

Department of Mathematics

Cook, Sam, Killing spinors and affine symmetry tensors in Gödel's universe

Pierce, Larry, Computing entropy for Z^2-actions

Schmurr, Jason, Triangular billiards surfaces and translation covers

Ultman, Shari, The cohomology rings of seven dimensional primitive cohomogeneity one manifolds

Wills, Dean, Connections between combinatorics of permutations and algorithms and geometry

Department of Statistics

Taylor, Aimee, Statistical enhancement of support vector machines

Portland State University (4)

Department of Mathematics and Statistics

Ahmed, Aden Omar, Quaternions, octonions, and quantizations of games

Godinez Vazquez, Humberto, Data assimilation, adaptive observations and applications

Khan, Faisal Shah, Quantum multiplexers, Parrando games, and proper quantization

Swinyard, Craig, Students' reasoning about the concept of limit in the context of reinventing the formal definition

University of Oregon (7)

Department of Mathematics

Brown, Jonathan Scott, Finite w-algebras of classical type

Collins, John Patrick, Gluing Bridgeland stability conditions and Z_2-equivariant sheaves on curves

Leeman, Aaron Christopher, Stabilization of chromatic functors

Phan, Christopher Lee, Koszul and generalized Koszul properties for noncommutative graded algebras

Vanderpool, Ruth, Non-existence of a stable homotopy category for P-complete abelian groups

Wade, Jeremy, Summability of Fourier orthogonal expansions and a discretized Fourier orthogonal expansion involving radon projections for functions on the cylinder

Walsh, Mark G., Metrics of positive scalar curvature and generalised Morse functions

PENNSYLVANIA

Carnegie Mellon University (14)

Department of Mathematical Science

Arama, Danut, On a variational approach for Stokes conjectures in water waves

Brunick, Gerard, A weak existence result with applications to the financial engineer's calibration problem

Chebolu, Prasad, Topics in random graphs

Melsted, Pall, Algorithms on random graphs

Obermeyer, Fritz, Automated equational reasoning in nondeterministic λ-calculi modulo theoreis H^*

Offner, David, Extremal problems on the hypercube

Picollelli, Michael, Extremal problems and random processes on graphs

Rand, Alexander, Delannay refinement algorithms for numerical methods

Sousa, Bernardo, Variational methods for phase transitions

Wimmer, Karl, Fourier methods and combinatorics in learning theory

Department of Statistics

Bertolet, Marianne, To weight or not to weight: Incorporating sampling designs into model-based analyses

Damouras, Sotirios, Nonparametric time series analysis using Gaussian processes

Luca, Diana, Genetic matching by ancestry in genome-wide association studies

Xie, Libo, Long-memory stochastic volatility models: A new method for inference and applications in options pricing

Lehigh University (2)

Department of Mathematics

Mohler, Joel, Residues of Weyl group multiple Dirichlet series

Qian, Huiyu, On data-driven chi square statistics

Pennsylvania State University (21)

Department of Mathematics

Endres, Erik, Shocks and wave interactions for compressible flow

Ethier, John, Strong forms of orthogonality for sets of hypercubes

George, Chris, The Mackey analogy for $SL(n,\mathbb{R})$

Kursungoz, Kagan, Parity considerations in Andrews-Gordon identities and the k-marked Durfee symbols

Li, Hengguang, Elliptic equations with singularities: A-priori analysis and numerical approaches

Otgonbayar, Uuye, The local index theorem in noncommutative geometry

Pan, Jia, Optimal intermediated investment in a liquidity-driven business cycle

Weaver, Bryce, Growth rate of periodic orbits for geodesic flows on surfaces with regions of positive curvature

Xue, Guangri, Mathematical modeling and computation of multiphysics fuel cells

Zhang, Lei, Phase field model for the nucleation in solid state phase transformations: Theories, algorithms and applications

Zheng, Bin, Finite element approximations of high order partial differential equations

Zhu, Yunrong, Robust preconditioners for H(grad), H(curl) and H(div) systems with strongly discontinuous coefficients

Department of Statistics

Benaglia, Tatiana, Nonparametric estimation in multivariate finite mixture models

Dong, Yuexiao, Dimension reduction for non-illiptically distributed predictors

Gaugler, Trent, Nonparametric estimation in multivariate finite mixture models

Kim, Daeyoung, Mixture inference at the edge of identifiability

Recta, Virginia, A model-based analysis of semicontinuous spatial data

Tyekucheva, Svitlana, Augmenting the bootstrap to analyze high-dimensional genomic data

Wang, Yu, Nonlinear dimension reduction in feature space

Zhang, Wei, A general class of agreement coefficients for categorical and continuous responses

Zhu, Junjia, Essays on monitoring profile data

Temple University (2)

Department of Mathematics

Du, Xiuhong, Additive Schwarz preconditioned GMRES, inexact Krylov subspace methods and applications of inexact CG

Wang, Linhong, Noetherian skew power series rings

University of Pennsylvania (18)

Department of Mathematics

Auel, Asher N., Cohomological invariants of line bundle-valued symmetric bilinear forms

Bressler, Andrew E., Quantum random walks on the integer lattice via generating functions

Favero, David, A study of the geometry of the derived category

Herreros, Pilar, Rigidity of magnetic and geodesic flows

Holschbach, Armin, A Chebotarev-like density theorem in algebraic geometry

Jankowski, Christopher, On type II_0 E_0-semigroups induced by q-pure maps on $M_n(\mathbb{C})$

Obus, Andrew, Ramification of primes in fields of moduli of three-point covers

Rowe, Paul, Policy compliance, confidentiality and complexity in collaborative systems

Shonkwiler, Clayton, Poincaré duality angles on Riemannian manifolds with boundary

Tsay, Joe-Kai, Formal analysis of the Kerberos authentication protocol

Vela-Vick, David Shea, Applications of Ozsvath-Szabo invariants to contact geometry

Yoo, Meesue, Combinatorial formulas connected to diagonal harmonics and Macdonald polynomials

Zhu, Tong, Nonlinear Polya urn models and self-organizing processes

Zong, Ying, On hyper-symmetric abelian varieties

Department of Statistics

Gupta, Abhishek, Statistical methods for high-dimensional data analysis

Lewin, David, Causal inference methods for randomized controlled trials with noncompliance

Nagaraja, Chaitra, Statistical methods for modeling house prices and indices

Wang, Lie, Variance function estimation in nonparametric regression

University of Pittsburgh (22)

Department of Biostatistics

Blakesley-Ball, Richard, Parametric control familywise error rates with dependent P-values

Cheng, Chunrong, Enhanced inter-study prediction and biomarker detection in microarray with application to cancer studies

Haile, Sarah, Inference on competing risks in breast cancer data

Jakobsdottir, Johanna, Genetics of age-related maculopathy and score statistics for X-linked quantitative trait loci

Li, Jia, Statistical issues in meta-analysis for identifying signature genes in the integration of multiple genomic studies

Mi, Zhibao, Robust cross-platform disease prediction using gene expression microarrays

Oh, Sunghee, Effects of missing value imputation on down-stream analyses in microarray data

Scott, John, Spatio-temporal mixed models for diffusion tensor magnetic resonance imaging

Tudorascu, Dana, Partial least squares on data with missing covariates: A comparison approach

Zhu, Fang, An index of local sensitivity to nonignorability and a penalized pseudolikelihood method for data with nonignorable nonresponse

Department of Mathematics

Balwe, Chetan, Geometric motivic integration on Artin n-stacks

Berry, Robert, Lipschitz estimates for geodesics in the Heisenberg group

Labovschii, Alexandr, Mathematical architecture for models of fluid flow phenomena

Manica, Evandro, Effects of coupling and heterogeneity in the pre-Bötzinger complex cells using first return maps

Poerio, Thomas, Topological algebraic structure in the density topology and on Souslin lines

Reynolds, Angela, Mathematical models of acute inflammation and a full lung model of gas exchange under inflammatory stress

Stanculescu, Iuliana, Turbulence modeling as an ill-posed problem

Department of Statistics

Ghebregiorgis, Ghideon, Modeling and analyzing multivariate longitudinal left-censored biomarker data

Ha, Wonho, Applications of the reflected Ornstein-Uhlenbeck process

Han, Sangdae, Comparing spectral densities in replicated time series by smoothing spline ANOVA

Nickleach, Scott, Numerical algorithms for stock option valuation

Wang, Jie, Bathtub failure rates in reliability and dependence in multiple testing

RHODE ISLAND

Brown University (11)

Center for Statistical Science

Dunsiger, Shira, Analysis of longitudinal binary data from behavioral medicine

Li, Hong, Statistical methods for monitoring disease recurrence

Division of Applied Mathematics

Carvalho, Luis, Bayesian centroid estimation

Grinberg, Leopold, Topics in ultrascale scientific computing with application in biomedical modeling

Luo, Xian, A spectral element/smoothed profile method for complex-geometry flows

Narayan, Akil, A generalization of the Wiener rational basis functions on infinite intervals

Papamichael, Giorgios, Event-by-event Monte Carlo simulation of radiation transport in vapor and liquid water

Pollard, Thomas, Comprehensive 3-d change detection

Schiemenz, Alan, Advances in the discontinuous Galerkin method: Hybrid schemes and applications to the reactive infiltration instability in an upwelling compacting mantle

Srinivasan, Ravi, Closure and complete integrability in Burgers turbulence

Zhang, Wei, Statistical inference and probabilistic modeling in compositional vision

University of Rhode Island (2)

Department of Mathematics

Basu, Sukanya, Global behavior of solutions to a class of second-order rational difference equations

Kostrov, Yevgeniy, Global behavior in rational difference equations

SOUTH CAROLINA

University of South Carolina (12)

Department of Epidemiology and Biostatistics

Ogbuanu, Ikechukwu, Do prenatal immunological exposures modify the effect of Interleukin-13 (IL13) gene on childhood atopy?

Parker, David, HIV risk behaviors and knowledge: Associations with HIV prevalence in Estonia

Rasathurai, Sumithran, Bayesian identification methods

Wagner, Sara, Spatial assessment of groundwater uranium and cancer in South Carolina

Zhen, Huiling, Multiple imputation of missing data based on prediction of conditional quantiles

Zhou, Li, Quantile regression with ordinal and discrete data

Department of Mathematics

Sircar, Sarthok, Dynamics and rheology of biaxial liquid crystal polymers

Department of Statistics

Chen, Peng, Topics in binary regression models with group testing data

McLain, Alex, Accelerated testing with recurrent events and fundamental issues in marginal models

Pritchard, Nick, Geometric group testing

Taylor, Laura, Competing risks in a recurrent event setting

Wu, Meng, Based hypothesis test for unidimensionality

TENNESSEE

University of Memphis (5)

Department of Mathematical Sciences

Clarke, Teddy J., Asymptotic analysis of the abstract telegraph equation

Garland, Sarah Elizabeth, Factorial cross-over designs with randomization restrictions and balanced cross-over design from terraces

Naheed, Naima, Mathematical contributions to spin-polarized Thomas-Fermi theory

Parrish, Anca, On the geometric structure of Lorentz and Marcinkiewicz spaces

Parrish, Andrew, Pointwise convergence of ergodic averages in Orlicz spaces

University of Tennessee, Knoxville (4)

Department of Mathematics

Clayton, Timothy, Optimal control of epidemic models involving rabies and West Nile viruses

Khader, Maisa, The dissipative nonlinear wave equations with space-time dependent potentials

Labuz, Brendon, Generalized uniform covering maps characterized as inverse limits of uniform covering maps

Neilan, Michael, Numerical methods for fully nonlinear second order partial differential equations

Vanderbilt University (2)

Department of Mathematics

Acosta Reyes, Ernesto, Optimal non-linear signal models and stability of sampling-reconstruction

Kozakova, Iva, Percolation and Ising model on tree-like graphs

TEXAS

Baylor University (7)

Department of Statistical Sciences

Crixell, Jo Anna, Logistic regression with covariate measurement error in an adaptive design: A Bayesian approach

Greer, Brandi, Bayesian and likelihood interval estimation for comparing two Poisson rate parameters using under-reported data

Hetzer, Joel, Statistical considerations for the analysis of multivariate Phase II testing

Miyamoto, Kazutoshi, Bayesian and maximum likelihood methods for some two-segment generalized linear models

Powers, Stephanie, Bayesian approaches to inference and variable selection for misclassified or under-reported response models

Wang, Jie, Sample size determination for EMAX model, equivalence/non-inferiority test and drug combination in fixed dose trials

Zhang, Lin, Semiparametric AUC regression for testing treatment effect in clinical trials

Rice University (16)

Department of Computational and Applied Mathematics

Gonzalez del Cueto, Fernando, Filtering random layering effects for imaging and velocity estimation

Gonzalez, Edward, Efficient alternating gradient-type algorithms for the approximate non-negative matrix factorization problem

Papakonstantinou, Joanna, Historical development of the BFGS secant method and its characterization properties

Sun, Kai, Domain decomposition and model reduction for parabolic optimal control problems

Department of Mathematics

Douglas, Casey, Perturbed genus one Scherk surfaces and their limits

Dunning, Ryan, Asymptotics under self-intersection for minimizers of self-avoiding energies

Hardway, Heather, Pattern formation in systems of reaction diffusion equations modeling genetic networks

Horn, Peter, Higher-order analogues of genus and slice genus of classical knots

McLelland, Matthew, Deformation of symmetric Scherk type minimal surfaces

Department of Statistics

Ahern, Charlotte, Statistical modeling in the optimization of breast cancer screening schedules

Chatman, Jamie, Computing diversity in undergraduate admissions

Christian, James, Incorporating annotation data in quantitative trait loci mapping with mRNA transcripts

Fofanov, Viacheslav, Statistical models in protein structural alignments

Mathaes, Matthias, Statistical tests of neutrality based on SNP and Alu repeat data

Neeley, Eleanor Shannon, Models for the preprocessing of reverse phase protein arrays

Zhu, Hongxiao, Functional data classification and covariance estimation

Southern Methodist University (2)

Department of Mathematics

Haque, Mohammed Ziaul, An adaptive finite element method for systems of second-order hyperbolic partial differential equations in one space dimension

Taylor, Michael, Epidemic models for partial-temporary immunity with delay

Texas A&M University (18)

Department of Mathematics

Chen, Xianjin, Analysis and computation of multiple unstable solutions to nonlinear elliptic systems

Fulkerson, Michael, Radial limits of holomorphic functions on the ball

Jiang, Lijian, Multiscale finite element methods using limited global information and applications

Kimball, James, Bounds on codes from smooth toric threefolds with $\mathrm{Pic}(X) = 2$

Muntyan, Yevgen, Automata groups

Nam, Dukjin, Multiscale numerical methods for some types of parabolic equations

Nguyen, Nga, Surgery on frames

Oeding, Luke, G-varieties and the principal minors of symmetric matrices

Sendova, Tsvetanka, A new approach to the modeling and analysis of fracture through an extension of continuum mechanics to the nanoscale

Sethuraman, Swaminathan, Volumes of certain cones of multi homogeneous polynomials

Tucci, Gabriel, Some results in the hyperinvariant subspace problem and free probability

Vega, Maria, Hypergeometric functions over finite fields and their relations to algebraic curves

Yang, Haibo, RO(G)-graded equivariant Bredon cohomology and sheaves

Department of Statistics

Chen, Min, Modeling covariance structure in unbalanced longitudinal data

George, Nysia, Mixture modeling and outlier detection in microarray data analysis

Jeong, Jae Sik, Some applications of wavelets to time series data

Lee, Seokho, Principal components analysis for binary data

Maity, Arnab, Efficient inference in general semiparametric regression models

University of Houston (11)

Department of Mathematics

Branton, Sheena, Sub-actions for Young towers

Dulock, Matthew, Uniqueness of holomorphic curves encountering divisors

Filipliski, Natasha, Periodic solutions in systems with finite abelian symmetries

Kashyap, Upasana, Morita equivalence of dual operator algebras

Kouan, Cedvic, Pricing spread options under stochastic volatility

Larson, Craig, Graph theoretic independence and critical independent sets

Pons, Victoria, Numerical methods for non-smooth problems from calculus of variations: Applications

Raghupathi, Mrinal, Constrained Nevanlinna-Pick interpolation

Wang, Tong, Numerical methods for free boundary and particulate flow problems: Applications to biomechanics

Xie, Weiwei, Valuation of spread options by bivariate Edgeworth expansion

Yavich, Nikolay, Multilevel preconditioners for strongly anisotropic problems

University of North Texas (3)

Department of Mathematics

Kazemi, Parimah, A constructive method for finding critical points of the Ginzburg-Landau energy functional

Shao, Chuang, Urysohn ultrametric spaces and isometry groups

Snyder, Jason, The global structure of iterated function systems

University of Texas at Arlington (1)

Department of Mathematics

Saha, Snehanshu, A study on the b family of shallow water wave equations

University of Texas at Austin (20)

Department of Mathematics

Alonso, Ricardo, The Boltzmann equation: Sharp Povzner inequalities applied to regularity theory

Anthropelos, Michail, Agents' agreement and partial equilibrium pricing in incomplete markets

Callahan, Jason, The arithmetic and geometry of two-generator Kleinian groups

Calleja, Renato, Existence and persistence of invariant objects in dynamical systems and mathematical physics

Carneiro, Emanuel, Extremality, symmetry and regularity issues in harmonic analysis

Charters, Philippa, Generalizing binary quadratic residue codes to higher power residues over larger fields

DiTanna, Anthony, The optimal control of a Lévy process

Kahle, Alexander, Superconnections and index theory

Mallmann, Katja, The discriminant algebra in cohomology

Ortiz, Michael, Differential equivariant K-theory

Salerno, Adriana, Hypergeometric functions in arithmetic geometry

Schwab, Russell, Random and periodic homogenization for some nonlinear partial differential equations

Stirling, Spencer, Abelian Chern-Simons theory with toral gauge group, modular tensor categories, and group categories

Swenson, Michelle, Phylogenetic supertree methods

Zhou, Ti, Essays on pricing and portfolio choice in incomplete markets

Institute for Computational Engineering and Sciences

De Basabe, Jonas, High-order finite element methods for seismic wave propagation

Iglesias-Hernandez, Marco Antonio, An iterative representer-based scheme for data inversion in reservoir modeling

Santillana, Mauricio, Analysis and numerical simulation of the diffusive wave approximation of the shallow water equations

Stogner, Roy, Parallel adaptive C1 macro-elements for nonlinear thin film and non-Newtonian flow problems

Tharkabhushanam, Sri Harsha, A conservative deterministic spectral method for rarefied gas flows

UTAH

Brigham Young University (2)

Department of Mathematics

Lian, Zeng, Lyapunov exponents and invariant manifold for random dynamical systems in a Banach space

Rimmasch, Gretchen, Complete tropical Bezout's theorem and its consequences

University of Utah (6)

Department of Mathematics

Centeleghe, Tommaso, A conjectural mass formula for Galois representations mod$p$

Copene, Elizabeth, Ephaptic coupling of cardiac cells

Crofts, Scott, Duality of Spin$(m+1, n)$

Preszler, Jason, Nilpotent orbits of symplectic p-adic Lie algebras and quadratic forms

Simeonova, Lyubima, Wave propagation in composite materials: Effective properties and optimization

Thompson, Joshua, Grafting real complex projective structures

Utah State University (3)

Department of Mathematics and Statistics

Maslova, Inga, Testing and estimation for functional data with applications to magnetometer records

Morphet, William, Simulation, kringing, and visualization of circular-spatial data

Yurk, Brian, Modeling the evolution of insect phenology with particular reference to mountain pine beetle

VIRGINIA

George Mason University (1)

Department of Statistics

Markaryan, Tigran, Exact distributional properties of Efron's biased coin design with application to clinical trials

Old Dominion University (3)

Department of Mathematics and Statistics

Fernando, Anne, DGM—A finite difference scheme based on the discontinuous Galerkin method

Grant, Terri, Improved constrained global optimization for estimating molecular structure from atomic distance

Touron, Charles, An adaptive method for calculating blow-up solution

University of Virginia (7)

Department of Mathematics

Drupieski, Christopher, Cohomology of Frobenius-Lusztig kernels of quantized enveloping algebras

Kapp, Brian, On the structure of certain groups associated in division algebras over fields of power series

Khongsap, Totrakool, Spin Hecke algebras

Pitsillides, Achilleas, Structure of unitary groups over number fields

Schmitz, Rebecca, Toeplitz-composition C^*-algebras with piecewise continuous symbols

Tysse, Jill, The centers of spin symmetric and spin hyperoctahedral group algebras

Wang, Yao, On a stochastic wave equation modeling heat flow

WASHINGTON

University of Washington (17)

Department of Mathematics

Ballard, Matthew, Derived categories of quasi-projective schemes

Koo, Tak-Lun, Change of Selmer group under isogeny, Iwasawa invariants of Λ-adic representation, and coefficients of newforms

Kunkel, Christopher, Quaternionic contact pseudohermitian normal coordinates

Moody, Dustin, The Diffie-Hellman problem and generalization of Verheul's theorem

Sekar, Anusha, Earthquake source inversion for tsunami runup prediction

Vance, Stephanie, Lattices and sphere packings in Euclidean space

Whitcher, Ursula, Families of polarized K3 hypersurfaces

Department of Statistics

Alkema, Leontine, Uncertainty assessments of demographic estimates and projections

Di, Yanming, Conditional tests for localizing trait genes

Fu, Quiyan, Models and inference of transmission of DNA methylation patterns in mammalian somatic cells

Gile, Krista, Inference from partially-observed network data

Li, Qunhua, Statistical methods for peptide and protein identification using mass spectometry

Sloughter, James McLean, Probabilistic weather forecasting using Bayesian model averaging

Stanberry, Larissa, Statistical solutions to some problems in medical imaging

White, Toby, Extensions of latent class transition models with application to chronic disability survey data

Youn, Ahrim, Learning transcriptional regulatory networks from the integraton of heterogeneous high-throughout data

Zhang, Shengyu, Statistical analysis of portfolio risk and performance measures: The influence function approach

Washington State University (2)

Department of Mathematics

Morris, De Anne, Combinatorial properties of nonnegative and eventually nonnegative matrices

Yielding, Amy, Spectrally arbitrary zero-nonzero patterns

WEST VIRGINIA

West Virginia University (1)

Department of Mathematics

Zhou, Ju, Cycles in graph theory and matroids

WISCONSIN

Marquette University (1)

Department of Mathematics, Statistics and Computer Science

Ratnakumar, Shivani, Protocol for estimation of target registration error in registration of acquired CT to live x-ray left atrial images

Medical College of Wisconsin (2)

Division of Biostatistics

Fan, Xiaolin, Bayesian nonparametric inference for competing risks data

Pajewski, Nicholas, Bayesian semiparametric hierarchical models for genetic association studies in the presence of population structure and multiplicities

University of Wisconsin, Madison (21)

Department of Mathematics

Akers, Benjamin, On model equations or gravity-capillary waves

Andrejko, Erik, Between O and Ostaszewski

Darnall, Matthew, On the discrepancy of random (t, m, s)-nets

Galindo, Diego, Self similar solutions of cold ion plasma equations

Hua, Zheng, Derived categories of toric stacks and Calabi-Yau varieties

Kang, Hye-Won, Multiple scaling methods in chemical reaction networks

Kim, Yeon Hyang, Representations of almost periodic functions using L_2-frames

McGinn, Daniel, Quantifier elimination in intuitionistic JRS theories

Milovich, David, Order-theoretic invariants in set-theoretic topology

Nam, Sangnam, Construction and analysis of local wavelet-like pyramidal representations in several dimensions

Otto, Benjamin, Supercharacters of algebra groups

Owen, Robert, Outer model theory and the definability of forcing

Petro, Matthew, Moduli spaces of Riemann surfaces

Rault, Patrick, On uniform bounds for rational points on rational curves and thin sets

Rhoades, Robert, The interplay between harmonic weak Maass forms and classical modular forms

Sengun, Mehmet Haluk, Serre's conjecture over imaginary quadratic fields

Shi, Yingzhe, Numerical methods for coupling of multispecies kinetic and hydrodynamic equations

Tang, Yudong, Geodesic rays and test configurations

Thorne, Frank, Extensions of results on the distribution of primes

Tonejc, Jernej, Formal normal forms for almost complex structures

Yang, Xu, Numerical methods for multiscale kinetic transport and high frequency waves

University of Wisconsin, Milwaukee (8)

DEPARTMENT OF MATHEMATICAL SCIENCES

Chiappetta, Shawn, Non-overlapping domain decomposition parallel algorithms for convection-diffusion equations

Fuhrman, Kseniya, Mathematical modeling and analysis of virus-host interaction in aquatic systems

He, Hao, Utility maximization of a portfolio that includes an illiquid asset

Michael, Martin, Local and global solutions to quasilinear wave equations via Nash-Moser algorithm

Rus, George, Finite element methods for control of singular stochastic processes

Sebert, Florian, Algebraic and geometric properties of generalized wavelet matrices

Shomberg, Joseph, Explicit construction of a robust family of compact inertial manifolds

Zaglauer, Katharina, Fair pricing of participating life insurance contracts in a regime-switching market environment

WYOMING

University of Wyoming (2)

DEPARTMENT OF MATHEMATICS

Rahunanthan, Arunasalam, A study of spatial and time discretizations for discontinuous Galerkin methods

DEPARTMENT OF STATISTICS

Li, Yumei, Hidden Markov modeling of earthquake swarms

2009 Election Results

In the elections of 2009 the Society elected a president elect, a vice president, a trustee, five members at large of the Council, three members of the Nominating Committee, and two members of the Editorial Boards Committee.

President Elect

Elected as the president elect is **Eric M. Friedlander** from the University of Southern California. Term is one year as president elect (1 February 2010–31 January 2011), two years as president (1 February 2011–31 January 2013), and one year as immediate past president (1 February 2013–31 January 2014).

Vice President

Elected as the new vice president is **Sylvain Cappell** from Courant Institute of Mathematical Sciences, New York University. Term is three years (1 February 2010–31 January 2013).

Trustee

Elected as new trustee is **Mark L. Green** from the University of California, Los Angeles. Term is five years (1 February 2010–31 January 2015).

Members at Large of the Council

Elected as new members at large of the Council are

Alejandro Adem from the University of British Columbia

Richard Hain from Duke University

Jennifer Schultens from the University of California, Davis

Janet Talvacchia from Swarthmore College

Christoph Thiele from the University of California, Los Angeles

Terms are three years (1 February 2010–31 January 2013).

Nominating Committee

Elected as new members of the Nominating Committee are

William Beckner from University of Texas at Austin

Richard T. Durrett from Cornell University

Carla D. Savage from North Carolina University

Terms are three years (1 January 2010–31 December 2012).

Editorial Boards Committee

Elected as new members of the Editorial Boards Committee are

Anatoly Libgober from the University of Illinois at Chicago

Simon Tavener from Colorado State University

Terms are three years (1 February 2010–31 January 2013).

2010 AMS Election

Nominations by Petition

Vice President or Member at Large

One position of vice president and member of the Council *ex officio* for a term of three years is to be filled in the election of 2010. The Council intends to nominate at least two candidates, among whom may be candidates nominated by petition as described in the rules and procedures.

Five positions of member at large of the Council for a term of three years are to be filled in the same election. The Council intends to nominate at least ten candidates, among whom may be candidates nominated by petition in the manner described in the rules and procedures.

Petitions are presented to the Council, which, according to Section 2 of Article VII of the bylaws, makes the nominations. The Council of 23 January 1979 stated the intent of the Council of nominating all persons on whose behalf there were valid petitions.

Prior to presentation to the Council, petitions in support of a candidate for the position of vice president or of member at large of the Council must have at least fifty valid signatures and must conform to several rules and operational considerations, which are described below.

Editorial Boards Committee

Two places on the Editorial Boards Committee will be filled by election. There will be four continuing members of the Editorial Boards Committee.

The President will name at least four candidates for these two places, among whom may be candidates nominated by petition in the manner described in the rules and procedures.

The candidate's assent and petitions bearing at least 100 valid signatures are required for a name to be placed on the ballot. In addition, several other rules and operational considerations, described below, should be followed.

Nominating Committee

Three places on the Nominating Committee will be filled by election. There will be six continuing members of the Nominating Committee.

The President will name at least six candidates for these three places, among whom may be candidates nominated by petition in the manner described in the rules and procedures.

The candidate's assent and petitions bearing at least 100 valid signatures are required for a name to be placed on the ballot. In addition, several other rules and operational considerations, described below, should be followed.

Rules and Procedures

Use separate copies of the form for each candidate for vice president, member at large, or member of the Nominating and Editorial Boards Committees.

1. To be considered, petitions must be addressed to Robert J. Daverman, Secretary, American Mathematical Society, Department of Mathematics, 302C Aconda Court, University of Tennessee, 1534 Cumberland Avenue, Knoxville, TN 37996-0614, USA, and must arrive by 24 February 2010.
2. The name of the candidate must be given as it appears in the *Combined Membership List* (`www.ams.org/cml`). If the name does not appear in the list, as in the case of a new member or by error, it must be as it appears in the mailing lists, for example on the mailing label of the *Notices*. If the name does not identify the candidate uniquely, append the member code, which may be obtained from the candidate's mailing label or by the candidate contacting the AMS headquarters in Providence (amsmem@ams.org).
3. The petition for a single candidate may consist of several sheets each bearing the statement of the petition, including the name of the position, and signatures. The name of the candidate must be exactly the same on all sheets.
4. On the next page is a sample form for petitions. Petitioners may make and use photocopies or reasonable facsimiles.
5. A signature is valid when it is clearly that of the member whose name and address is given in the left-hand column.
6. The signature may be in the style chosen by the signer. However, the printed name and address will be checked against the *Combined Membership List* and the mailing lists. No attempt will be made to match variants of names with the form of name in the *CML*. A name neither in the *CML* nor on the mailing lists is not that of a member. (Example: The name Robert J. Daverman is that of a member. The name R. Daverman appears not to be.)
7. When a petition meeting these various requirements appears, the secretary will ask the candidate to indicate willingness to be included on the ballot. Petitioners can facilitate the procedure by accompanying the petitions with a signed statement from the candidate giving consent.

Nomination Petition

for 2010 Election

The undersigned members of the American Mathematical Society propose the name of

____

as a candidate for the position of (check one):

- ☐ **Vice President**
- ☐ **Member at Large of the Council**
- ☐ **Member of the Nominating Committee**
- ☐ **Member of the Editorial Boards Committee**

of the American Mathematical Society for a term beginning 1 February, 2011

Return petitions by 24 February 2010 to:
Secretary, AMS, 302C Aconda Court, University of Tennessee, Knoxville, TN 37996-0614 USA

Name and address (printed or typed)	
	Signature
	Signature
	Signature
	Signature
	Signature
	Signature

Mathematics Calendar

Please submit conference information for the Mathematics Calendar through the Mathematics Calendar submission form at http://www.ams.org/cgi-bin/mathcal-submit.pl. The most comprehensive and up-to-date Mathematics Calendar information is available on the AMS website at http://www.ams.org/mathcal/.

February 2010

* 1–March 6 **Math-Info 2010: Towards new interactions between mathematics and computer science**, International Center for Mathematical Meetings (C.I.R.M.), Marseille, France.

Description: This thematic month will be split into five weeks: * from Feb. 1–5: Lattice Reduction. * from Feb. 8–12: Dynamics and Computation. * from Feb. 15–19: Multi-dimensional Subshifts and Tilings. * from Feb. 22–26: Sage Days. * from March 3–5: Topological Methods for the Study of Discrete Structures. The mornings are devoted to lectures (around three lectures of four hours each for each week), given by renowned researchers. The specificity of this conference is that participants can propose topics and names for the afternoon working sessions in order to speak about their results or discuss some problems with other participants. **You can propose such working sessions** by sending us an email or directly on the webpage: http://www.lirmm.fr/arith/wiki/MathInfo2010/CallForWorkshopProposals. If you intend to participate, **you need to register** as soon as possible on the website: http://www.lirmm.fr/arith/wiki/MathInfo2010/Pre-registration.

Information: http://www.lirmm.fr/MathInfo2010; http://www.lirmm.fr/arith/wiki/MathInfo2010/Location.

* 1–12 **X Winter Diffiety School**, Academic Gymnasium, St. Petersburg State University, St. Petersburg, Russia.

Description: The aim of this permanent School is to introduce undergraduate and Ph.D. students in Mathematics and Physics as well as post-doctoral researchers in a recently emerged area of Mathematics and Theoretical Physics: SECONDARY CALCULUS. A "diffiety" is a new geometrical object that properly formalizes the concept of the solution space of a given system of (nonlinear) PDEs, much as an algebraic variety does with respect to solutions of a given system of algebraic equations. SECONDARY CALCULUS is a natural diffiety analogue of the standard Calculus on smooth manifolds, and as such leads to a very rich general theory of nonlinear PDEs. It appears that it is this the only natural language of quantum physics, just as the standard Calculus is for classical physics.

Information: http://www.levi-civita.org/Activities/DiffietySchools/XIWDS.

* 14–19 **Young Set Theory Workshop 2010**, Seminarzentrum Raach, near Vienna, Austria.

Description: The third annual Young Set Theory Workshop will take place between 15-19 February 2010 at Seminarzentrum Raach (http://www.szr.at) located one hour south of Vienna in Raach am Hochgebirge. The aim of this conference is to bring together Ph.D. students and postdocs in Set Theory in order to learn from leading researchers in the field, hear about the latest research, and to discuss research issues in a co-operative environment. The conference format will be similar to previous years, including tutorials, postdoc research talks, and discussion sessions.

Information: http://www.math.uni-bonn.de/people/logic/events/young-set-theory-2010/.

This section contains announcements of meetings and conferences of interest to some segment of the mathematical public, including ad hoc, local, or regional meetings, and meetings and symposia devoted to specialized topics, as well as announcements of regularly scheduled meetings of national or international mathematical organizations. A complete list of meetings of the Society can be found on the last page of each issue.

An announcement will be published in the *Notices* if it contains a call for papers and specifies the place, date, subject (when applicable), and the speakers; a second announcement will be published only if there are changes or necessary additional information. Once an announcement has appeared, the event will be briefly noted in every third issue until it has been held and a reference will be given in parentheses to the month, year, and page of the issue in which the complete information appeared. Asterisks (*) mark those announcements containing new or revised information.

In general, announcements of meetings and conferences carry only the date, title of meeting, place of meeting, names of speakers (or sometimes a general statement on the program), deadlines for abstracts or contributed papers, and source of further information. If there is any application deadline with respect to participation in the meeting, this fact should be noted. All communications on meetings and conferences in the mathematical sciences should be sent to the Editor of the *Notices* in care of the American Mathematical Society in Providence or electronically to notices@ams.org or mathcal@ams.org.

In order to allow participants to arrange their travel plans, organizers of meetings are urged to submit information for these listings early enough to allow them to appear in more than one issue of the *Notices* prior to the meeting in question. To achieve this, listings should be received in Providence **eight months** prior to the scheduled date of the meeting.

The complete listing of the Mathematics Calendar will be published only in the September issue of the *Notices*. The March, June/July, and December issues will include, along with new announcements, references to any previously announced meetings and conferences occurring within the twelve-month period following the month of those issues. New information about meetings and conferences that will occur later than the twelve-month period will be announced once in full and will not be repeated until the date of the conference or meeting falls within the twelve-month period.

The Mathematics Calendar, as well as Meetings and Conferences of the AMS, is now available electronically through the AMS website on the World Wide Web. To access the AMS website, use the URL: http://www.ams.org/.

* 27–28 **2nd International Conference on Wireless Information Networks & Business Information System (WINBIS2010)**, Kathmandu, Nepal.
Description: The aims of WINBIS'10 is bring together researchers, engineers, and practitioners interested on wireless information Networks & Business information systems to make international forum WINBIS'10 will not only present the papers, also give the ideas on how to analyze & approach problems by combining wireless information system & Business information system and it's applications.
Information: http://www.win-bis.com.

March 2010

* 14–17 **2010 Interpore Conference and Annual Meeting, Texas A&M, Mar 2010**, Texas A&M University, College Station, Texas.
Description: The goal of the conference is to provide a forum for discussing the academic and industrial challenges of porous media, as well as to foster collaboration among theoreticians, modelers and experimentalists working in porous media research. The *International Society for Porous Media* (InterPore) is a non-profit-making independent scientific organization established in 2008. The general aim of the Society is to advance and disseminate knowledge for the understanding, description, and modeling of natural and industrial porous media systems (see http://www.interpore.org/ for more information). The conference will include keynote lectures, invited oral presentations, and poster sessions. No parallel sessions are planned, and oral presentations are by invitation only.
Deadline: The abstracts for poster session presentations are to be sent to: interpore2010@gmail.com.
Information: See http://isc.tamu.edu/news-and-events/2010-interpore-conference-and-annual-meeting.html.

* 15–19 **Arizona School of Analysis with Applications**, University of Arizona, Tucson, Arizona.
Description: This school is primarily organized for graduate students and postdocs, but everybody is welcome. The core program consists of four minilectures by Rafael Benguria (Isoperimetric Inequalities for Eigenvalues of the Laplacian), Laszlo Erdos (Universality of Wigner Random Matrices), Michael Loss (Kinetic Theory and Kac's Master Equation), and Gunter Stolz (Localization in Disordered Media). Thanks to support by the University of Arizona and the NSF, we can contribute to the travel costs of many participants.
Information: http://www.mathphys.org/AZschool/.

* 22–26 **Conference "Recent Advances in Function Related Operator Theory"**, Hotel "Rincon of the Seas", Rincon, Puerto Rico.
Topics: Operator theory, Function theory in spaces of analytic and harmonic functions, Approximation theory, Applications of operator theory.
Information: More information through the conference web page: http://www.albany.edu/rafrot; or email: rafrot@albany.edu.

May 2010

* 13–15 **International Conference Devoted to the Memory of Academician M. Kravchuk (1892–1942)**, National Technical University of Ukraine, Kyiv, Ukraine.
Description: The Programme of the Conference includes the following 4 sections: 1. Differential and integral equations, its applications. 2. Algebra, geometry. Mathematical and numerical analysis. 3. Theory of probability and mathematical statistics. 4. History, methods of teaching of mathematics.
Opening ceremony: May 13, at 2:30 pm.
Registration fee: US $50.00 (you can pay, when you arrive to Kyiv). All registration fees include a book of abstracts, daily coffee breaks and excursion.
Information: kravchukconf@yandex.ru.

* 14–15 **A Celebration of Mathematics and the 40th Anniversary of Jeffery Hall**, Queen's University, Kingston, Ontario, Canada.
Description: In the post-Sputnik era, Canada and the United States faced an immense challenge to bring the scientific research levels of both nations to a higher standard. In the mathematical field, this challenge was met in Canada by several mathematicians, the most notable being Professor R. L. Jeffery of Queen's University. By strengthening the Canadian Mathematical Society and by organizing the summer research seminars held at Queen's University during the 1950s and 1960s, Professor Jeffery was instrumental in raising the level of mathematical research in Canada. Jeffery Hall, which presently houses the Department of Mathematics and Statistics at Queen's University, is named after him. This building was 40 years old in October, 2009. The world now faces a similar challenge to the one Professor Jeffery faced in the 1950s. To keep the importance of mathematical research in the forefront of our scientific consciousness, as well as to commemorate the 40th anniversary of Jeffery Hall and honour Professor Jeffery, the Department of Mathematics and Statistics at Queen's University is planning a two-day celebration from May 14-15, 2010. Several presentations are planned. Professor Sir David Cox of the University of Oxford, Professor Hale Trotter of Princeton University and Professor Gerhard Frey of the University of Duisburg, Essen, have already agreed to speak. A lecture surveying the development of research in mathematics in Canada, as well as R. L. Jeffery's role in the Canadian Mathematical Society, is also planned. It is anticipated that this celebration will galvanize and reinforce the mathematical talent in Canada to meet the present scientific and technological challenges. It is expected that many alumni and others will attend.
Information: If you would like to be put on the mailing list to receive more information, please write by mail to: Dr. A. M. Herzberg FRSC, Department of Mathematics, Queen's University, Kingston, Ontario K7L 3N6; by email: Miss A. Burns; burnsa@queensu.ca.

* 16–22 **ESF Mathematics Conference in partnership with EMS and ERCOM Algebraic Methods in Dynamical Systems**, The Institute of Mathematics Conference Centre, Bedlewo, Poland.
Description: The conference is intended to give an account of the new results concerning algebraic methods in dynamical systems. Morales-Ramis theorem establishing the quasi-abelianity of the differential Galois group of the variational equation along a particular solution of a meromorphically completely integrable Hamiltonian system have led during the recent years to a big number of new results on non-integrability of Hamiltonian systems. More recently, the application of differential Galois theory to the study of Hamiltonian systems is no longer limited to Picard-Vessiot theory but is extended to the more general setting based on recent results of Malgrange and Umemura.
Information: http://www.esf.org/conferences/10320.

* 17–20 **The Seventh International Conference on Computational Physics**, Fragrant Hill Hotel, Beijing, China.
Focus: The conference will focus on key aspects of computational physics, including theory, numerical methods, and applications. Twenty-two invited speakers have confirmed their participation in ICCP7. Papers submitted to ICCP7 will have an opportunity to be published in one of two journals (*Communications in Computational Physics*, *Chinese Journal of Computational Physics*). The Scientific Committee will give recommendation for publication of quality papers. For any enquiry you are welcome to send an email to: iccp7@iapcm.ac.cn.
Organizer: ICCP7 is organized by the Institute of Applied Physics and Computational Mathematics.
Information: For further information on the conference venue, please visit the website: http://www.xsfd.com/; http://www.iapcm.ac.cn/iccp7/.

* 23–28 **Conformal maps: from probability to physics**, Monte Verita, Ascona, Ticino, Switzerland.
Description: The conference centers on random structures in the context of complex analysis, inspired by interactions between math-

ematics and physics. The central example is perhaps the Stochastic Loewner Evolution, introduced by Oded Schramm. SLE arises as the scaling limit of interfaces in 2D lattice models at criticality (percolation, Ising model, self avoiding polymers), and its elegant combination of probability and complex analysis has led to the proofs of many conjectures originating in physics. Other topics include Diffusion Limited Aggregation (a model for electrodeposition and other phenomena), Hele-Shaw flow (describing interfaces between fluids of different viscosities, e.g. oil and water), 2D Quantum Gravity and Random Maps.
Organizers: K. Astala, S. Rohde, S. Smirnov.
Information: http://www.unige.ch/~hongler/ascona/.

* 26–29 **Workshop on Combinatorial and Additive Number Theory (CANT 2010)**, CUNY Graduate Center, New York, New York.
Description: This is the eighth in a series of annual spring workshops sponsored by the New York Number Theory Seminar on problems in combinatorial and additive number theory and related parts of mathematics. In addition to long and short talks, each day of the conference will include a problem and discussion session. Previous CANT conferences have attracted many graduate students as well as research mathematicians, and students are encouraged to attend. A list of lecturers and other information will be posted on the conference website. Mathematicians who wish to speak at the meeting are encouraged to submit a title and abstract to: melvyn.nathanson@lehman.cuny.edu.
Information: http://www.theoryofnumbers.com.

June 2010

* 7–11 **International Conference on Applied Mathematics (with the first William Benter Prize in Applied Mathematics)**, City University of Hong Kong, Hong Kong.
Description: In recent years, huge advances have been achieved through the application of mathematical ideas and techniques to a wide variety of fields. The International Conference on Applied Mathematics will cover a wide range of topics including all aspects of applied mathematics. The objective of this conference is to provide a forum for the exchange of expertise, experience, and insights by mathematical scientists, physicists, and young researchers who are active in the area of applied mathematics, and to encourage leading scientists from abroad to further strengthen their cooperation with local scientists. The first William Benter Prize in Applied Mathematics will be awarded and the winner will be presented with the prize (http://www6.cityu.edu.hk/rcms/WBP/the_prize.html) during the conference. The aim of the prize is to recognize outstanding mathematical contributions that have had a direct and fundamental impact on scientific, business, finance, or engineering applications.
Information: http://www6.cityu.edu.hk/rcms/WBP/int_conf.html.

* 7–11 **International Functional Analysis Meeting in Valencia on the Occasion of the 80th Birthday of Professor Manuel Valdivia**, University of Valencia, Valencia, Spain.
Description: This conference will emphasize different areas of functional analysis. It will provide a venue for established scholars to interact with each other and with junior scholars. Partial support for a small number of participants is expected to be available. Recent recipients of doctoral degrees and pre-doc students are encouraged to apply. The meeting will take place in Valencia as a joint venture of the University of Valencia and the University Politécnica of Valencia. It will take place in the Mathematics building (and surrounding large lecture theaters) in Burjassot (Valencia) from June 7–11, 2010 (both days included). The structure of the conference is of plenary and long invited lectures in the morning, and twelve parallel sessions in the afternoon.
Information: http://www.adeit.uv.es/fav2010/.

* 7–11 **7th Annual Conference on Theory and Applications of Models of Computation- TAMC 2010**, Charles University, Prague, Czech Republic.
Description: TAMC is an international conference series with an interdisciplinary character, bringing together researchers working in computer science, mathematics and the physical sciences. The crossdisciplinary character, together with its focus on algorithms, complexity and computability theory, gives the conference a special flavor and distinction. The TAMC conference series arose naturally in response to important scientific developments affecting how we compute in the twenty-first century. Originally aimed to become a significant contributor to the scientific resurgence seen in East Asia, TAMC is now playing an important international role in contemporary applications of theoretical computer science. After six successful conferences held in China (Bejing 2004 and 2006, Kunming 2005, Shanghai 2007, Xi'an 2008, ChangSha 2009) TAMC is leaving Asia for the first time.
Information: http://www.tamc2010.cz.

* 15–19 **The Thirteenth International Conference on "Hyperbolic Problems: Theory, Numerics and Applications"**, Beijing, People's Republic of China.
Description: The objective of this conference is to bring together researchers, practitioners, and students with interest in theoretical analysis, numerical simulations, and applications of hyperbolic PDEs and related mathematical models appearing in the area of applied sciences. The conference will keep the traditional balance of the HYP series, blending theory, numerics and applications. In addition, additional new attracting points are also taken into granted.
Information: For more information about the conference list of the Scientific Committee and Invited Speakers, registration fee and the related things including possible support for young mathematicians from developing countries, please visit the HYP2010 conference website at: http://www.amt.ac.cn/hyp2010/.

* 16–21 **XI International Conference "Current Geometry"**, Vietri sul Mare Salerno, Italy.
Description: The power of synthesis of Geometry, which led in the past to the formulation of "grand unification theories", has got an essential role nowadays, especially because of the growing fragmentation of knowledge due to scientific progress. In order to avoid too big a dispersion, geometers need a constant dialogue. Therefore, a stable experience of personal meetings, apart from telematic interchanges, cannot be renounced. Current Geometry was born to allow a periodic update about actual progresses in Geometry (and its applications) on the international scene.
Information: http://www.levi-civita.org/Activities/Conferences/xicurrentgeometry.

* 17–19 **[FG60] Computational and Geometric Topology**, Bertinoro, Forli, Italy.
Description: The research groups of Geometric Topology and Computational Topology of the Universities of Bologna and Modena-Reggio Emilia, Departments of Mathematics, will be organizing a conference in order to celebrate the 60th birthdays of Massimo Ferri and Carlo Gagliardi.
Organizers: P. Bandieri, M. R. Casali, A. Cattabriga (secretary), P. Cristofori, P. Frosini, L. Grasselli, C. Landi, M. Mulazzani (chair).
Information: All the information on the conference and the form to register and/or submit a talk can be found on the conference's web page http://fg60.dm.unibo.it/.

* 19–24 **International Algebraic Conference dedicated to the 70th birthday of A. V. Yakovlev**, St. Petersburg, Russia.
Organizing committee: A. Generalov (chair), N. Gordeev, I. Fesenko, G. Leonov, A. Merkuriev, E. Novikova, I. Panin, A. Semenov, N. Vavilov, S. Vostokov, I. Zhukov.
Information: http://www.pdmi.ras.ru/EIMI/2010/iac/.

* 20–25 **The Twelfth National Conference in Algebra (China)**, Northwest Normal University, Lanzhou, Gansu, China.

Description: The National Conference in Algebra is the largest national conference in China. It is held every two years. All areas of algebra and their applications will be covered. At the last conference, there were several hundred presentations and over 500 participants. It will be a good opportunity for both the experts and beginning researchers to learn the most recent developments in algebra in China. The conference encourages participants from within and outside of China.
Information: http://www.nwnu.edu.cn/algebra.

* 21–25 **Harmonic Analysis and Related Topics**, Instituto Superior Técnico, IST, Lisbon, Portugal.
Description: The Summer School and Workshop "Harmonic Analysis and Related Topics" is intended both for Ph.D. students and young researchers and also experts in various topics related to Harmonic Analysis and Function Spaces. Summer School includes three short courses: "Sobolev, capacitary and isocapacitary inequalities" (Vladimir Maz'ya), "Weighted problems for operators of Harmonic Analysis in some Banach Function Spaces" (Vakhtang Kokilashvili), "Variable Lebesgue Spaces" (David Cruz-Uribe). The workshop is supposed to cover a number of topics in Harmonic Analysis, Function spaces, and related areas.
Information: http://www.math.ist.utl.pt/~hart2010/.

* 21–26 **2nd International Conference for Promoting the Application of Mathematics in Technical and Natural Sciences (AMiTaNS'10)**, Sozopol, Bulgaria.
Description: The conference will be scheduled in plenary and keynote lectures followed by special and contributed sessions. The accents of the conference will be on Difference and Spectral methods; Applied Analysis, Biomathematics, Continuum Mathematics, which can be complemented by new topics. You are welcomed to announce and organize special sessions that should be within the general topic of the conference. Everybody, who is interested in attending AMiTaNS'10 please let him/her prepare a short abstract within 300 words clearly stating the goal, tools and results and submit it at: conference@eac4amitans.org and fill out the online Application form. In case of problems please send its text version as attachment to e-mail: conference@eac4amitans.org.
Deadline: For both submissions is March 31, 2010.
Information: http://2010.eac4amitans.org/.

* 22–25 **Group Representation Theory and Related Topics**, EPFL, Centre Interfacultaire Bernoulli, Lausanne, Switzerland.
Description: This is an international conference that will take place in Lausanne, Switzerland. Group representation theory is a very active area of mathematics which has connections and overlaps with areas such as algebraic topology, K-theory, algebraic geometry and commutative ring theory. Research in the area has been driven by several conjectures and open problems which have pushed the development of new methodology and broader interactions with other areas of mathematics. The aim of the conference is to stimulate activity in and enhance interaction between representation theory and other areas of mathematics, like those presented above. In addition to the lectures, there will be a conference dinner in honor of Professor Jacques Thevenaz.
Information: http://grt.epfl.ch.

* 29–July 4 **23nd International Conference on Operator Theory**, West University of Timisoara, Timisoara, Romania.
Description: The conference is devoted to operator theory, operator algebras and their applications (differential operators, complex functions, mathematical physics, matrix analysis, system theory, etc.).
Information: http://www.imar.ro/~ot/.

July 2010

* 4–17 **40th Probability Summer School**, Saint-Flour, France.
Description: Founded in 1971, this school is intended for Ph.D. students, teachers and researchers who are interested in probability theory, statistics, and in applications of these techniques. The three 12-hour courses of this year will be given by Franco Flandoli, Giambattista Giacomin, and Takashi Kumagai.
Information: http://math.univ-bpclermont.fr/stflour/.

* 12–14 **2010 International Conference on Theoretical and Mathematical Foundations of Computer Science (TMFCS-10)**, Orlando, Florida.
Description: TMFCS is an important event in the theoretical, mathematical and logical areas of computer science. The conference will be held at the same time and location where several other major international conferences will be taking place.
Information: For other conferences associated with MULTICONF-10, please visit: http://www.promoteresearch.org.

* 28–30 **The Mathematics of Klee & Grunbaum: 100 Years in Seattle**, University of Washington, Seattle, Washington.
Description: A celebration of the long and illustrious careers of Victor Klee and Branko Grunbaum and their fundamental contributions to discrete mathematics and geometry. There will be 18 invited talks, a poster session and an open problems session. Details available at the above website.
Information: http://sites.google.com/a/alaska.edu/kleegrunbaum/.

August 2010

* 9–13 **Permutation Patterns 2010**, Dartmouth College, Hanover, New Hampshire.
Topics: The conference topics include (but are not limited to) enumeration questions, forbidden pattern questions, study of the permutation pattern order, algorithms for computing with permutation patterns, applications and generalizations of permutation patterns, and others.
Plenary Lectures: There will be two invited plenary lectures by Nik Ruškuc and Richard P. Stanley, as well as contributed talks.
Information: http://math.dartmouth.edu/~pp2010/.

* 27–31 **Differential Geometry and its Applications**, Masaryk University, Faculty of Science, Brno, Czech Republic, Europe.
Description: The DGA conferences take place regularly at one of the Czech universities every three years; the tenth conference in the series took place in Olomouc in 2007.
Invited plenary speakers: Shun-ichi Amari (Japan), Robert Bryant (MSRI, USA), Andreas Cap (Vienna, Austria), Anna Fino (Torino, Italy), Joseph M. Landsberg (Texas, USA), Franz Pedit (Amsherst & Tuebingen), Lorenz Schwachhoefer (Dortmund, Germany), Zhongmin Shen (Indianapolis, USA), Jian Song (Rutgers, USA), Vladimir Soucek (Prague, Czech Republic), Gudlaudur Thorbergsson (Koeln, Germany). The Editorial Board meeting of the journal of the same name, cf. http://www.elsevier.com/locate/difgeo will take place during the conference.
Information: http://dga.math.muni.cz/dga2010.

September 2010

* 7–11 **International Conference "Modern Stochastics: Theory and Applications II"**, Kyiv National Taras Shevchenko University, Kyiv, Ukraine.
Description: Conference is dedicated to the anniversaries of prominent Ukrainian scientists: A. Skorokhod, V. Korolyuk, I. Kovalenko.
Sessions: Diffusion Processes, Fractal Analysis, Gaussian and Related Processes and Fields, Generalized Renewal Processes, Information Security, Interacting Particle Systems, Limit Theorems, Markov and Semi-Markov Processes, Mathematics of Finance, Queuing Systems, Risk Processes and Actuarial Mathematics, Statistics of Stochastic Processes, Stochastic Analysis, Stochastic Differential Equations, Stochastic Models of Evolution Systems.
Keynote speakers: Yu. Belyaev, Sweden; V. Buldygin, Ukraine; A. Bulinski, Russia; J. M. Corcuera, Spain; P. Doukhan, France; M. Dozzi, France; A. Dudin, Belarus; A. Gushchin, Russia; V. Konakov, Russia;

Yu. Kondratiev, Germany; K. Kubilius, Lithuania; N. Leonenko, UK; N. Limnios, France; E. Merzbach, Israel; I. Molchanov, Switzerland; E. Orsingher, Italy; G. Peccati, France; D. Silvestrov, Sweden; E. Valkeila, Finland; V. Vatutin, Russia; A. Veretennikov, UK.
Information: http://probability.univ.kiev.ua/msta2conf.

* 7–11 **Logic, Algebra and Truth Degrees 2010**, Prague, Czech Republic.
Description: Mathematical Fuzzy Logic is a subdiscipline of Mathematical Logic which studies the notion of comparative truth. The assumption that "truth comes in degrees" has proved to be very useful in many, both theoretical and applied, areas of Mathematics, Computer Science, and Philosophy.
Goal: The main goal of this meeting is to foster collaboration between researchers in the area of Mathematical Fuzzy Logic, and to promote communication and cooperation with members of neighbouring fields.
Invited speakers: Arnon Avron, Félix Bou, Agata Ciabattoni, Roberto Cignoli, Ioana Leustean, Franco Montagna, James G. Raftery, and Hiroakira Ono.
Tutorials by: George Metcalfe and Vilém Novák.
Program committee: Petr Hájek (Chair), Antonio Di Nola, Christian Fermüller, Siegfried Gottwald, Daniele Mundici, and Carles Noguera.
Deadline for submissions: March 20, 2010.
Information: http://www.mathfuzzlog.org/latd2010/.

* 7–12 **Geometry, Dynamics, Integrable Systems 2010**, Mathematical Institute SANU, Belgrade, Serbia.
Main Topics: Integrable Systems, Classical Mechanics, Nonholonomic Mechanics, Rigid Body Dynamics, Lie Algebras and Lax Representation, Separation of Variables.
Scientific Committee: Alain Albouy, Sergio Benenti, Alexander Bobenko, Alexey Bolsinov, Alexey Borisov, Victor Buchstaber, Pantelis Damianou, Vladimir Dragovic, Boris Dubrovin, Valery Kozlov, Igor Krichever, Sergey Novikov (Chairman), Anatol Odzijewicz, Tudor Ratiu, Vered Rom-Kedar, Andrey Tsiganov, Marcelo Viana, Jean-Claude Zambrini, Rade Zivaljevic.
Information: Contact: gdis2010@mi.sanu.ac.rs; http://www.mi.sanu.ac.rs/~gdis2010/.

* 12–17 **ESF Mathematics Conference in Partnership with EMS and ERCOM/INI: Highly Oscillatory Problems: From Theory to Applications**, The Isaac Newton Institute, Cambridge, United Kingdom.
Description: Highly oscillatory phenomena occur in a wide range of mathematical applications: from fluid to solid mechanics, electromagnetics, acoustics, combustion, computerised tomography and imaging, molecular dynamics, quantum chemistry, plasma transport and electrical engineering. Such phenomena have attracted a great deal of mathematical attention, mainly in harmonic analysis, asymptotic analysis, homogenisation, differential geometry, theory of Hamiltonian systems and theory of integrable systems. They have an oft-undeserved reputation of being hopelessly difficult to analyse and to compute: the truth of the matter is that, once they have been understood from the mathematical standpoint, effective computational algorithms are bound to follow.
Information: http://www.esf.org/conferences/10340.

* 13–17 **Third International Congress on Mathematical Software [ICMS'2010—developers meeting]**, Department of Mathematics, Kobe University, Kobe, Japan.
Description: This congress is the third in the series, where the first meeting was held in Beijing in 2002 and the previous one in Castro Urdiales, Spain in 2006; see http://www.icms2006.unican.es/. We will welcome developers of mathematical software systems as well as researchers in algorithms and mathematicians who are interested in the development of mathematical software and systems. This is an almost unique chance to meet people in different disciplines in mathematics and computer science and exchange ideas on developments on mathematical software and systems. While the main audience of this meeting is assumed to be developers of mathematical software and software systems, we welcome the participation of mathematicians and scientists who are interested in using mathematical software for their research. The proceedings of the congress is planned and all accepted papers and short communications will be published as Springer Lecture Notes in Computer Science (LNCS).
Information: http://www.mathsoftware.org/.

* 17–19 S^4 **Conference on Symmetry, Separation, Superintegrability and Special Functions**, School of Mathematics, University of Minnesota, Minneapolis, Minnesota.
Description: This conference is in honor of Willard Miller Jr., who will be retiring from the University of Minnesota. Topics will include symmetry methods for differential equations, higher symmetries, separation and multi-separation of variables, special functions and orthogonal polynomials, Hamiltonian systems, both classical and quantum, super-integrability and its connections with separability and quasi-exact solvability, and applications to physical systems arising in classical and quantum mechanics and relativity.
Information: http://www.math.umn.edu/conferences/s4/.

November 2010

* 8–10 **2010 IEEE International Conference on Technologies for Homeland Security**, Westin Hotel, Waltham, Massachusetts.
Description: The tenth annual IEEE Conference on Technologies for Homeland Security (IEEE HST '10) will focus on innovative technologies for deterring and preventing attacks, protecting critical infrastructure and individuals, and mitigating damage and expediting recovery. Submissions are desired in the broad areas of critical infrastructure and key resources protection (CIKR), border protection and monitoring, and disaster recovery and response, with application within about five years.
Information: http://www.ieee-hst.org.

The following new announcements will not be repeated until the criteria in the next to the last paragraph at the bottom of the first page of this section are met.

September 2011

* 10–16 **Turning Dreams into Reality: Transformations and Paradigm Shifts in Mathematics Education**, Rhodes University, Grahamstown, South Africa.
Description: International Conference of The Mathematics Education into the 21^{st} Century Project.
Preliminary Announcement and Call for Papers: The Mathematics Education into the 21^{st} Century Project has just completed its tenth successful international conference in Dresden, Germany, following conferences in Egypt, Jordan, Poland, Australia, Sicily, Czech Republic, Malaysia and the USA. Our project was founded in 1986 and is dedicated to the planning, writing and disseminating of innovative ideas and materials in Mathematics, Statistics, Science and Computer Education.
Organizing Committee: The chairman is Professor Marc Schafer of Rhodes University.
Information: For further conference details please email Alan Rogerson, Chairman of the International Programme Committee: alan@rogerson.pol.pl.

New Publications Offered by the AMS

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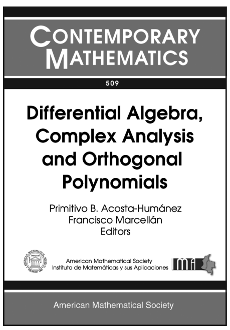
Analysis



Differential Algebra, Complex Analysis and Orthogonal Polynomials

Primitivo B. Acosta-Humánez, *Universidad Sergio Arboleda, Bogotá, Colombia*, and **Francisco Marcellán**, *Universidad Carlos III de Madrid, Leganés, Spain*, Editors

This volume represents the 2007–2008 Jairo Charris Seminar in Algebra and Analysis on Differential Algebra, Complex Analysis and Orthogonal Polynomials, which was held at the Universidad Sergio Arboleda in Bogotá, Colombia.

It provides the state of the art in the theory of Integrable Dynamical Systems based on such approaches as Differential Galois Theory and Lie Groups as well as some recent developments in the theory of multivariable and q-orthogonal polynomials, weak Hilbert's 16th Problem, Singularity Theory, Tournaments in flag manifolds, and spaces of bounded analytic functions on the unit circle.

The reader will also find survey presentations, an account of recent developments, and the exposition of new trends in the areas of Differential Galois Theory, Integrable Dynamical Systems, Orthogonal Polynomials and Special Functions, and Bloch–Bergman classes of analytic functions from a theoretical and an applied perspective.

The contributions present new results and methods, as well as applications and open problems, to foster interest in research in these areas.

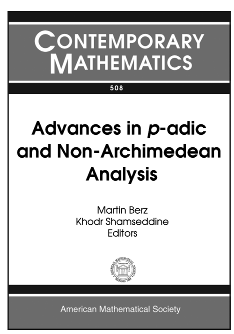
This item will also be of interest to those working in algebra and algebraic geometry.

A co-publication of the AMS and Instituto de Matemáticas y sus Aplicaciones (IMA).

Contents: D. Blázquez-Sanz and **J. J. Morales-Ruiz**, Differential Galois theory of algebraic Lie-Vessiot systems; **L. Fernández**, **F. Marcellán**, **T. E. Pérez**, and **M. A. Piñar**, Recent trends on two variable orthogonal polynomials; **C. A. Gomez S.**, On the integrability of the Riccati equation; **M. E. H. Ismail**, Two discrete systems of q-orthogonal polynomials; **J. Ławrynowicz**, **L. F. Reséndis O.**, and **L. M. Tovar S.**, Like-hyperbolic Bloch-Bergman classes; **J. T. Lázaro**, Some words about the application of Tchebycheff systems to weak Hilbert's 16th problem; **D. Mond**, From the index of a differential operator to the Milnor number of a singularity; **J. J. Morales-Ruiz** and **J.-P. Ramis**, Integrability of dynamical systems through differential Galois theory: a practical guide; **M. Paredes** and **S. Pinzón**, Tournaments and parabolic almost complex structures on flag manifolds.

Contemporary Mathematics, Volume 509

April 2010, approximately 235 pages, Softcover, ISBN: 978-0-8218-4886-9, 2000 *Mathematics Subject Classification:* 05C20, 12H05, 14E20, 14L99, 14M15, 20C20, 30C45, 33C50, 33D45, 34A26, 34C07, 34C08, 34M15, 35C05, 41A60, 42C05, 46E25, 53C15, 54C40, **AMS members US$63**, List US$79, Order code CONM/509



Advances in p-adic and Non-Archimedean Analysis

Martin Berz, *Michigan State University, East Lansing, MI*, and **Khodr Shamseddine**, *University of Manitoba, Winnipeg, Manitoba, Canada*, Editors

This volume contains the proceedings of the Tenth International Conference on p-adic and Non-Archimedean Analysis, held at Michigan State University in East Lansing, Michigan, on June 30–July 3, 2008.

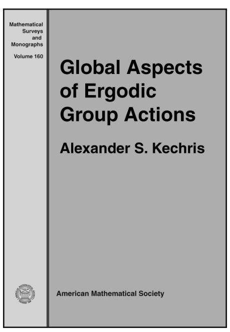
This volume contains a kaleidoscope of papers based on several of the more important talks presented at the meeting. It provides a cutting-edge connection to some of the most important recent developments in the field. Through a combination of survey papers, research articles, and extensive references to earlier work, this volume allows the reader to quickly gain an overview of current activity in the field and become acquainted with many of the recent sub-branches of its development.

Contents: J. Aguayo, **S. Navarro**, and **M. Nova**, Strict topologies on spaces of vector-valued continuous functions over non-Archimedean field; **B. Diarra**, Some subalgebras of the algebra of bounded linear operators of the one variable Tate algebra; **A. Escassut** and **N. Maïnetti**, The ultrametric corona problem;

A. K. Katsaras, Vector-valued p-adic measures; **H. A. Keller** and **H. Ochsenius**, On the Clifford algebra of orthomodular spaces over Krull valued fields; **K.-O. Lindahl**, Divergence and convergence of conjugacies in non-Archimedean dynamics; **H. M. Moreno**, A criterion for the invertibility of Lipschitz operators on type separating spaces; **M. Nilsson** and **R. Nyqvist**, On monomial dynamical systems on the p-adic n-torus; **H. Ochsenius** and **E. Olivos**, On the value group and norms of a form Hilbert space; **H. Ochsenius** and **W. H. Schikhof**, Compact perturbations of Fredholm operators on Norm Hilbert spaces over Krull valued fields; **J. Ojeda**, Applications of the p-adic Nevanlinna theory to problems of uniqueness; **C. Pérez-García** and **W. M. Schikhof**, Tensor products of p-adic locally convex spaces having the strongest locally convex topology; **C. G. Petalas** and **A. K. Katsaras**, Tensor products of p-adic measures; **A. Rodionov** and **S. Volkov**, p-adic arithmetic coding; **K. Shamseddine** and **M. Berz**, Analysis on the Levi-Civita field, a brief overview; **P.-A. Svensson**, Criteria for non-repelling fixed points; **F. Tangara**, A p-adic q-deformation of the Weyl algebra, for q a p^N-th root of unity.

Contemporary Mathematics, Volume 508

March 2010, 269 pages, Softcover, ISBN: 978-0-8218-4740-4, LC 2009042367, 2000 *Mathematics Subject Classification:* 46S10, 11S80, 12J25, 16W30, 46G10, 32P05, 11D88, 30G06, 47B37, **AMS members US$71**, List US$89, Order code CONM/508



Global Aspects of Ergodic Group Actions

Alexander S. Kechris, *California Institute of Technology, Pasadena, CA*

The subject of this book is the study of ergodic, measure preserving actions of countable discrete groups on standard probability spaces. It explores a direction that emphasizes a global point of view, concentrating on the structure of the space of measure preserving actions of a given group and its associated cocycle spaces. These are equipped with canonical topological actions that give rise to the usual concepts of conjugacy of actions and cohomology of cocycles. Structural properties of discrete groups such as amenability, Kazhdan's property (T) and the Haagerup Approximation Property play a significant role in this theory as they have important connections to the global structure of these spaces. One of the main topics discussed in this book is the analysis of the complexity of the classification problems of conjugacy and orbit equivalence of actions, as well as of cohomology of cocycles. This involves ideas from topological dynamics, descriptive set theory, harmonic analysis, and the theory of unitary group representations. Also included is a study of properties of the automorphism group of a standard probability space and some of its important subgroups, such as the full and automorphism groups of measure preserving equivalence relations and connections with the theory of costs.

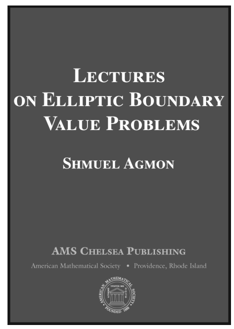
The book contains nine appendices that present necessary background material in functional analysis, measure theory, and group representations, thus making the book accessible to a wider audience.

Contents: Measure preserving automorphisms; The space of actions; Cocycles and cohomology; Realifications and complexifications; Tensor products of Hilbert spaces; Gaussian probability spaces; Wiener chaos decomposition; Extending representations to actions; Unitary representations of abelian groups; Induced representations and actions; The space of unitary representations; Semidirect products of groups; Bibliography; Index.

Mathematical Surveys and Monographs, Volume 160

February 2010, 237 pages, Hardcover, ISBN: 978-0-8218-4894-4, LC 2009042253, 2000 *Mathematics Subject Classification:* 37A15, 37A20, 03E15, **AMS members US$62**, List US$77, Order code SURV/160

Differential Equations



Lectures on Elliptic Boundary Value Problems

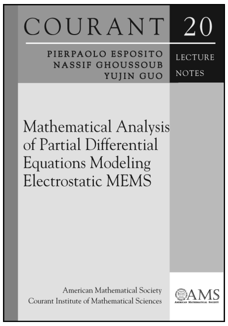
Shmuel Agmon, *Hebrew University of Jerusalem, Israel*

This book, which is a new edition of a book originally published in 1965, presents an introduction to the theory of higher-order elliptic boundary value problems. The book contains a detailed study of basic problems of the theory, such as the problem of existence and regularity of solutions of higher-order elliptic boundary value problems. It also contains a study of spectral properties of operators associated with elliptic boundary value problems. Weyl's law on the asymptotic distribution of eigenvalues is studied in great generality.

Contents: Notations and conventions; Calculus of L^2 derivatives—Local properties; Calculus of L^2 derivatives—Global properties; Some inequalities; Elliptic operators; Local existence theory; Local regularity of solutions of elliptic systems; Gårding's inequality; Global existence; Global regularity of solutions of strongly elliptic equations; Coerciveness; Coerciveness results of Aronszajn and Smith; Some results on linear transformations on a Hilbert space; Spectral theory of abstract operators; Eigenvalue problems for elliptic equations; The self-adjoint case; Non-self-adjoint eigenvalue problems; Completeness of the eigenfunctions; Bibliography; Notation index; Index.

AMS Chelsea Publishing, Volume 369

March 2010, 210 pages, Hardcover, ISBN: 978-0-8218-4910-1, LC 2009047651, 2000 *Mathematics Subject Classification:* 35J40; 35P10, **AMS members US$36**, List US$40, Order code CHEL/369.H



Mathematical Analysis of Partial Differential Equations Modeling Electrostatic MEMS

Pierpaolo Esposito, *Università degli Studi Roma Tre, Rome, Italy*, **Nassif Ghoussoub**, *University of British Columbia, Vancouver, BC, Canada*, and **Yujin Guo**, *University of Minnesota, Minneapolis, MN*

Micro- and nanoelectromechanical systems (MEMS and NEMS), which combine electronics with miniature-size mechanical devices, are essential components of modern technology. It is the mathematical model describing "electrostatically actuated" MEMS that is addressed in this monograph. Even the simplified models that the authors deal with still lead to very interesting second- and fourth-order nonlinear elliptic equations (in the stationary case) and to nonlinear parabolic equations (in the dynamic case). While nonlinear eigenvalue problems—where the stationary MEMS models fit—are a well-developed field of PDEs, the type of inverse square nonlinearity that appears here helps shed a new light on the class of singular supercritical problems and their specific challenges.

Besides the practical considerations, the model is a rich source of interesting mathematical phenomena. Numerics, formal asymptotic analysis, and ODE methods give lots of information and point to many conjectures. However, even in the simplest idealized versions of electrostatic MEMS, one essentially needs the full available arsenal of modern PDE techniques to do the required rigorous mathematical analysis, which is the main objective of this volume. This monograph could therefore be used as an advanced graduate text for a motivational introduction to many recent methods of nonlinear analysis and PDEs through the analysis of a set of equations that have enormous practical significance.

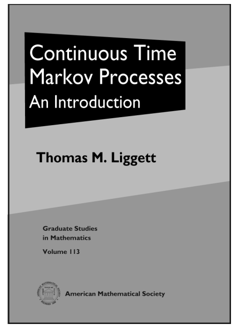
Titles in this series are co-published with the Courant Institute of Mathematical Sciences at New York University.

Contents: Introduction; *Part 1. Second-order equations modeling stationary MEMS:* Estimates for the pull-in voltage; The branch of stable solutions; Estimates for the pull-in distance; The first branch of unstable solutions; Description of the global set of solutions; Power-law profiles on symmetric domains; *Part 2. Parabolic equations modeling MEMS dynamic deflections:* Different modes of dynamic deflection; Estimates on quenching times; Refined profile of solutions at quenching time; *Part 3. Fourth-order equations modeling nonelastic MEMS:* A fourth-order model with a clamped boundary on a ball; A fourth-order model with a pinned boundary on convex domains; Appendix A. Hardy–Rellich inequalities; Bibliography; Index.

Courant Lecture Notes, Volume 20

March 2010, 318 pages, Softcover, ISBN: 978-0-8218-4957-6, LC 2009045518, 2000 *Mathematics Subject Classification:* 35J60, 35B45, 35B35, 35B40, 35P30, 74K15, 74F15, 35J20, 58E07, 74M05, **AMS members US$40**, List US$50, Order code CLN/20

Probability



Continuous Time Markov Processes

An Introduction

Thomas M. Liggett, *University of California, Los Angeles, CA*

Markov processes are among the most important stochastic processes for both theory and applications. This book develops the general theory of these processes, and applies this theory to various special examples. The initial chapter is devoted to the most important classical example—one-dimensional Brownian motion. This, together with a chapter on continuous time Markov chains, provides the motivation for the general setup based on semigroups and generators. Chapters on stochastic calculus and probabilistic potential theory give an introduction to some of the key areas of application of Brownian motion and its relatives. A chapter on interacting particle systems treats a more recently developed class of Markov processes that have as their origin problems in physics and biology.

This is a textbook for a graduate course that can follow one that covers basic probabilistic limit theorems and discrete time processes.

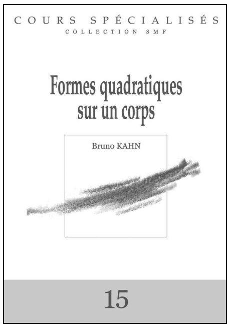
Contents: One-dimensional Brownian motion; Continuous time Markov chains; Feller processes; Interacting particle systems; Stochastic integration; Multidimensional Brownian motion and the Dirichlet problem; Appendix; Bibliography; Index.

Graduate Studies in Mathematics, Volume 113

April 2010, approximately 278 pages, Hardcover, ISBN: 978-0-8218-4949-1, 2000 *Mathematics Subject Classification:* 60J25, 60J27, 60J65; 35J05, 60J35, 60K35, **AMS members US$44**, List US$55, Order code GSM/113

New AMS-Distributed Publications

Number Theory



Formes Quadratiques sur Un Corps

Bruno Kahn, *Institut de Mathématiques de Jussieu, Paris, France*

This book presents the theory of quadratic forms over a field, focusing on the Pfister-Arason-Knebusch technique of extensions to function fields of quadrics.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Contents: La théorie de Witt; La théorie de Pfister; Corps de fonctions de quadriques; La théorie de Knebusch; Formes devenant isotropes sur le corps des fonctions d'une quadrique; Invariants élémentaires; Le théorème de réduction d'indice et ses applications; Formes de basse dimension; Invariants supérieurs; Descente; Rappels sur le groupe de Brauer; Rappels de cohomolgie galoisienne; Courbes algébriques; Un aperçu sur les formes quadratiques en caractéristique 2; Formes quadratiques et cycles algébriques; Solutions de certains exercises; Bibliographie; Glossaire; Index.

Cours Spécialisés—Collection SMF, Number 15

November 2009, 303 pages, Hardcover, ISBN: 978-2-85629-261-7, 2000 *Mathematics Subject Classification:* 11E04, 11E81, **Individual member US$74**, List US$82, Order code COSP/15

Classified Advertisements

Positions available, items for sale, services available, and more

KANSAS

KANSAS STATE UNIVERSITY
Department of Mathematics

Applications are invited for two Visiting Assistant Professorships commencing August 8, 2010. These will be annual appointments with the possibility of two subsequent one-year appointments depending on performance, funding, and need of services. A Ph.D. in mathematics or a Ph.D. dissertation—accepted with only formalities to be completed—is required by the time of appointment. The department seeks candidates whose research interests mesh well with current faculty. The department has research groups in algebra, analysis, differential equations, geometry/topology, and number theory. Successful candidates are expected to participate in the department's programs integrating undergraduate and graduate research, including mentoring undergraduate students during summer programs. The successful candidate should have strong research credentials as well as strong accomplishments or promise in teaching, and should value working with colleagues and students from diverse backgrounds.

Applicants must submit the following: a letter of application; curriculum vita; outline of teaching philosophy; a statement of research objectives; and four letters of reference, at least one of which addresses the applicant's teaching ability or potential. All application materials must be submitted electronically via `http:wwwmathjobs.org`. Screening of applications begins December 1, 2009, and continues until positions are closed.

Kansas State University is an Equal Opportunity Employer and actively seeks diversity among its employees and encourages applications from women and minorities. A background check is required.

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NEW JERSEY

RAMAPO COLLEGE OF NEW JERSEY
Assistant Professor of Mathematics
Tenure-Track–Fall 2010

JOB DESCRIPTION: Responsibilities encompass a wide range of undergraduate mathematics courses, and the teaching and development of General Education mathematics courses.

REQUIREMENTS: Ph.D. in mathematics by September 1, 2010, is required. College teaching experience preferred. Faculty members are expected to maintain active participation in research, scholarship, college governance, service, academic advisement, and professional development activities.

All applications must be completed online at `http:www.ramapojobs.com`. Attach resume, cover letter, statement of teaching philosophy, research interests, and a list of three references to your completed application. Since its beginning, Ramapo College has had an intercultural/international mission. Please tell us how your background, interest, and experience can contribute to this mission, as well as to the specific position for which you are applying.

Review of applications will begin immediately and continue until the position is filled. Position offers excellent state benefits. To request accommodations, call (201) 684-7718. Additional supportive materials in nonelectronic format may be sent to Dr. Maxim Goldberg-Rugalev, Search Committee chair, Ramapo College of New Jersey, 505 Ramapo Valley Road, Mahwah, NJ 07430.

Ramapo College is a member of the Council of Public Liberal Arts Colleges (COPLAC), a national alliance of leading liberal arts colleges in the public sector.

EEO/AFFIRMATIVE ACTION.

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BRAZIL

INSTITUTE FOR PURE AND APPLIED MATHEMATICS (IMPA)
Rio de Janeiro, Brazil

The Institute for Pure and Applied Mathematics (IMPA), invites applications for a two-year postdoctoral position in any field of mathematics, with a monthly take-home salary of R$6.000 (about US$3,500 at the current exchange rate). Candidates must have obtained their Ph.D. degrees after March 31, 2006. IMPA, located in Rio de Janeiro, Brazil, is widely recognized as one of the leading mathematical research centers worldwide. Its main goal is the generation of high-level mathematical

Suggested uses for classified advertising are positions available, books or lecture notes for sale, books being sought, exchange or rental of houses, and typing services.

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There are no member discounts for classified ads. Dictation over the telephone will not be accepted for classified ads.

Upcoming deadlines for classified advertising are as follows: March 2010 issue–December 28, 2009; April 2010 issue–January 28, 2010; May 2010 issue–February 26, 2010; June/July 2010 issue–April 28, 2010; August 2010 issue–May 28, 2010; September 2010–June 28, 2010.

U.S. laws prohibit discrimination in employment on the basis of color, age, sex, race, religion, or national origin. "Positions Available" advertisements from institutions outside the U.S. cannot be published unless they are accompanied by a statement that the institution does not discriminate on these grounds whether or not it is subject to U.S. laws. Details and specific wording may be found on page 536 (vol. 56).

Situations wanted advertisements from involuntarily unemployed mathematicians are accepted under certain conditions for free publication. Call toll-free 800-321-4AMS (321-4267) in the U.S. and Canada or 401-455-4084 worldwide for further information.

Submission: Promotions Department, AMS, P.O. Box 6248, Providence, Rhode Island 02940; or via fax: 401-331-3842; or send email to `classads@ams.org`. AMS location for express delivery packages is 201 Charles Street, Providence, Rhode Island 20904. Advertisers will be billed upon publication.

research. It offers also graduate level programs at the Ph.D. and MSc level. Currently, its faculty includes specialists in Real and Complex Dynamical Systems, Analysis, Algebra, Geometry, Probability, Fluid Dynamics, Optimization, Mathematical Economics, and Computer Graphics. Applications should be sent to: `opening@impa.br` until March 31, 2010. Further inquiries should be addressed to the same email address. For information on application submissions, see: `http://www.impa.br/opencms/en/pesquisa/postdoc/opening.html`.

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KOREA

POHANG UNIVERSITY OF SCIENCE AND TECHNOLOGY (POSTECH)
Department of Mathematics
P-Assistant Professorship Positions

The department of mathematics at Pohang University of Science and Technology (POSTECH) solicits applications for non-tenure-track P-Assistant Professorship. All areas of pure and applied mathematics are considered. Applicants are expected to have demonstrated exceptional research potential, including major contributions beyond or through the doctoral dissertation. The applicants should be within the first 4 years of their research activities since gaining their Ph.D. degrees. No linguistic ability in Korean is required for successful candidates who are fluent in English.

The P-Assistant Professorship is a two-year position and is non-renewable. Exceptional research performance during the two years may warrant considerations for possible tenure-track appointment. The annual salary is up to 60,000,000 KRW. An on-campus apartment unit may be provided at a low cost.

This position will remain open until it is filled.

Applicants should submit;

a. A signed cover letter
b. A curriculum vitae
c. A description of research
d. Three letters of recommendation sent directly by recommenders
e. Copies of publications including the Ph.D. dissertation

Please have your applications sent to:

Chair, Department of Mathematics
Pohang University of Science and Technology
Pohang, 790-784
The Republic of Korea;
fax: +82-54-279-2799;
phone: +82-54-279-2302;

Inquiries can be made to:

Yong-kyoo Lee (Mr.)
Department Administrative Manager;
email: `ykyoo@postech.ac.kr`.

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POHANG UNIVERSITY OF SCIENCE AND TECHNOLOGY (POSTECH)
Department of Mathematics
Tenure-track Faculty Positions Available

The Department of Mathematics at Pohang University of Science and Technology (POSTECH) invites applications for several tenure-track faculty positions in any areas of Pure and Applied Mathematics including Statistics. The positions are primarily at the Assistant Professor level, although senior candidates with strong credentials will also be considered. The positions are open to mathematicians of any nationality who are fluent in English, and applications are accepted throughout the year. To be considered for Spring & Fall, they must be received by early January & early July, respectively. New faculty members are provided with a competitive benefits package and free use of faculty apartments for ten years.

POSTECH was founded in 1986 by the world-leading steel company, POSCO, and is an internationally renowned academic institution dedicated to education and research in science and engineering. Successful candidates are expected to establish strong research programs and to excel in teaching at both the undergraduate and graduate levels.

1. Qualification: Applicants must hold a Ph.D. in Mathematics or a related discipline with an outstanding research record and should be able to teach in English.

2. Applicants should submit

a. A signed cover letter
b. A curriculum vitae
c. A description of research
d. Three letters of recommendation sent directly by recommenders
e. Copies of publications including the Ph.D. dissertation

For more information, please visit our webpage: POSTECH Homepage: `http://www.postech.ac.kr`.

Mathematics Department Homepage: `http://math.postech.ac.kr`.

Please have your package sent by email, fax, and regular mail, to

Hyungju Park, Chair
Department of Mathematics
POSTECH
Pohang, 790-784 Korea;
tel:+82-54-279-2302;
fax:+82-54-279-2799;
email: `alanpark@postech.ac.kr`.

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Meetings & Conferences of the AMS

IMPORTANT INFORMATION REGARDING MEETINGS PROGRAMS: AMS Sectional Meeting programs do not appear in the print version of the *Notices*. However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on the AMS website. See `http://www.ams.org/meetings/`. Final programs for Sectional Meetings will be archived on the AMS website accessible from the stated URL and in an electronic issue of the *Notices* as noted below for each meeting.

Lexington, Kentucky

University of Kentucky

March 27–28, 2010

Saturday – Sunday

Meeting #1057

Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: January 2010
Program first available on AMS website: February 11, 2010
Program issue of electronic *Notices*: March 2010
Issue of *Abstracts*: Volume 31, Issue 2

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: January 26, 2010

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Percy A. Deift, Courant Institute–New York University, *Open problems in integrable systems and random matrix theory.*

Irina Mitrea, Worcester Polytechnic Institute, *Recent progress in the area of elliptic boundary value problems on rough domains.*

Bruce Reznick, University of Illinois at Urbana Champaign, *The secret lives of polynomial identities.*

Bernd Ulrich, Purdue University, *Title to be announced.*

Doron Zeilberger, Rutgers University, *3x+1* (Erdős Memorial Lecture).

Special Sessions

Advances in Algebraic Coding Theory (Code: SS 6A), **Heide Gluesing-Luerssen**, University of Kentucky, and **Jon-Lark Kim**, University of Louisville.

Advances in Algebraic Statistics (Code: SS 2A), **Sonja Petrović**, University of Illinois, Chicago, and **Ruriko Yoshida**, University of Kentucky.

Advances in Algebraic and Geometric Combinatorics (Code: SS 14A), **Richard Ehrenborg** and **Margaret A. Readdy**, University of Kentucky.

Analysis and Control of Dispersive Partial Differential Equations (Code: SS 25A), **Michael J. Goldberg** and **Bingyu Zhang**, University of Cincinnati.

Combinatorial Algebra (Code: SS 7A), **Juan C. Migliore**, University of Notre Dame, and **Uwe Nagel**, University of Kentucky.

Commutative Algebra (Code: SS 1A), **Alberto Corso**, University of Kentucky, **Claudia Polini**, University of Notre Dame, and **Bernd Ulrich**, Purdue University.

Complex Analysis and Potential Theory (Code: SS 4A), **James E. Brennan** and **Vladimir Eiderman**, University of Kentucky.

Financial Mathematics and Statistics (Code: SS 22A), **Kiseop Lee**, University of Louisville, and **Jose Figueroa-Lopez**, Department of Statistics, Purdue University.

Function Theory, Harmonic Analysis, and Partial Differential Equations (Code: SS 5A), **Joel Kilty**, Centre College, **Irina Mitrea**, Worcester Polytechnic Institute, and **Katharine Ott**, University of Kentucky.

Geometric Function Theory and Analysis on Metric Spaces (Code: SS 3A), **John L. Lewis**, University of Kentucky, and **Nageswari Shanmugalingam**, University of Cincinnati.

Homotopy Theory and Geometric Aspects of Algebraic Topology (Code: SS 16A), **Serge Ochanine**, University of Kentucky, and **Marian F. Anton**, Centre College.

Interactions between Logic, Topology, and Complex Analysis (Code: SS 23A), **Matt Insall**, Missouri University of Science and Technology, and **Malgorzata Marciniak**, University of Toledo.

Inverse Problems, Riemann-Hilbert Problems, and Nonlinear Dispersive Equations (Code: SS 10A), **Peter A. Perry**, University of Kentucky, and **Peter Topalov**, Northeastern University.

Large Scale Matrix Computation (Code: SS 19A), **Qiang Ye**, University of Kentucky, and **Lothar Reichel**, Kent State University.

Mathematical Economics (Code: SS 21A), **Adib Bagh** and **Robert E. Molzon**, University of Kentucky.

Mathematical Problems in Mechanics and Materials Science (Code: SS 20A), **Michel E. Jabbour** and **Chi-Sing Man**, University of Kentucky, and **Kazumi Tanuma**, Gunma University.

Mathematical String Theory (Code: SS 18A), **Al Shapere**, Department of Physics and Astronomy, University of Kentucky, **Eric Sharpe**, Physics Department, Virginia Polytechnic Institute and State University, and **Mark A. Stern**, Duke University.

Mathematics Outreach (Code: SS 26A), **Carl W. Lee** and **David C. Royster**, University of Kentucky.

Matroid Theory (Code: SS 9A), **Jakayla Robbins**, University of Kentucky, and **Xiangqian Zhou**, Wright State University.

Multivariate and Banach Space Polynomials (Code: SS 15A), **Richard A. Aron**, Kent State University, and **Lawrence A. Harris**, University of Kentucky.

Noncommutative Algebraic Geometry (Code: SS 24A), **Dennis S. Keeler** and **Kimberly Retert**, Miami University.

Partial Differential Equations in Geometry and Variational Problems (Code: SS 8A), **Luca Capogna**, University of Arkansas, and **Changyou Wang**, University of Kentucky.

Recent Progress in Numerical Methods for Partial Differential Equations (Code: SS 12A), **Alan Demlow**, University of Kentucky, and **Xiaobing H. Feng**, University of Tennessee at Knoxville.

Relative Homological Algebra (Code: SS 11A), **Edgar E. Enochs**, University of Kentucky, and **Alina C. Iacob**, Georgia Southern University.

Sharp Spectral Estimates in Analysis, Geometry, and Probability (Code: SS 17A), **Richard S. Laugesen** and **Bartlomiej Siudeja**, University of Illinois.

Spectral and Transport Properties of Schrödinger Operators (Code: SS 13A), **Peter D. Hislop**, University of Kentucky, and **Jeffrey H. Schenker**, Michigan State University.

St. Paul, Minnesota

Macalester College

April 10–11, 2010

Saturday – Sunday

Meeting #1058

Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: February 2010
Program first available on AMS website: February 25, 2010
Program issue of electronic *Notices*: April 2010
Issue of *Abstracts*: Volume 31, Issue 2

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: February 16, 2010

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Charles Doering, University of Michigan, *Title to be announced*.

Matthew James Emerton, Northwestern University, *Title to be announced*.

Vladimir Touraev, Indiana University, *Title to be announced*.

Peter Webb, University of Minnesota, *Title to be announced*.

Special Sessions

Applications of a Geometric Approach to Chaotic Dynamics (Code: SS 16A), **Evelyn Sander**, George Mason University, **Judy Kennedy**, Lamar University, and **James Yorke**, University of Maryland.

Cohomology and Representation Theory of Algebraic Groups and Related Structures (Code: SS 7A), **Christopher Bendel**, University of Wisconsin-Stout, **Bobbe Cooper**, University of Minnesota, and **Terrell Hodge**, Western Michigan University.

Combinatorial Representation Theory (Code: SS 3A), **Tom Halverson**, Macalester College, and **Victor Reiner**, University of Minnesota.

Commutative Ring Theory (Code: SS 5A), **Michael Axtell**, University of St. Thomas, and **Joe Stickles**, Millikin University.

Differential Equations and Applications (Code: SS 15A), **Nicolai Tarfulea**, Purdue University Calumet, and **Catalin Turc**, Case Western Reserve University.

Fractals, Convolution Measures, and Frames (Code: SS 13A), **Keri Kornelson**, University of Oklahoma, and **Karen Shuman**, Grinnell College.

Geometric Flows, Moving Frames and Integrable Systems (Code: SS 8A), **Gloria Mari-Beffa**, University of Wisconsin-Madison, and **Peter Olver**, University of Minnesota.

Hecke Algebras and Deformations in Geometry and Topology (Code: SS 11A), **Matthew Douglass** and **Anne Shepler**, University of North Texas.

Mathematical Developments in Cell and Systems Biology (Code: SS 14A), **Anastasios Matzavinos**, Iowa State University, and **Nicoleta Eugenia Tarfulea**, Purdue University Calumet.

Matrices and Graphs (Code: SS 9A), **Luz M. DeAlba**, Drake University, **Adam Berliner**, St. Olaf College, **Leslie Hogben**, Iowa State University, and **In-Jae Kim**, Minnesota State University.

Partition Theory and the Combinatorics of Symmetric Functions (Code: SS 6A), **Eric S. Egge**, Carleton College, and **Kristina Garrett**, St. Olaf College.

Pattern Formation in Biological Systems (Code: SS 12A), **Magdalena Skolarska**, University of St. Thomas, and **Chad Topaz**, Macalester College.

Physical Knotting and Linking and its Applications (Code: SS 10A), **Eric Rawden**, University of St. Thomas, **Yuanan Diao**, University of North Carolina at Charlotte, and **Claus Ernst**, Western Kentucky University.

Probabilistic and Extremal Combinatorics (Code: SS 2A), **Ryan Martin** and **Maria Axenovich**, Iowa State University.

Quantum Invariants of 3-manifolds and Modular Categories (Code: SS 1A), **Thang Le**, Georgia Institute of Technology, **Eric Rowell**, Texas A&M University, and **Vladimir Touraev**, Indiana University.

Universal Algebra and Order (Code: SS 4A), **Jeffrey Olson**, Norwich University, **Jeremy Alm**, Illinois College, **Kristi Meyer**, Wisconsin Lutheran College, and **Japheth Wood**, Bard College.

Accommodations

Participants should make their own arrangements directly with the hotel. When making a reservation identify yourself as being with the AMS-Macalester College meeting. The AMS is not responsible for rate changes or for the quality of the accommodations. **Hotels have varying cancellation or early checkout penalties; be sure to ask for details when making your reservation**.

A special hotel rate has been negotiated at the **Crowne Plaza Saint Paul-Riverfront Hotel**. The room rate is US$109 per night, plus occupancy tax. Please be advised to make your reservations as soon as possible. **This special rate is valid while rooms are available up to and including March 15, 2010**. Participants can reserve their room through the direct link below, or by calling 651-292-1900 or 866-422-3185 and mentioning that they will be attending the AMS-Macalester College meeting. A complimentary continental breakfast is included with this room rate.

Link to hotel reservations at the group rate: `https://secure.ichotelsgroup.com/h/d/cp/1/en/cwshome/DPRD-7X8K3X/MSPSP?secure=true&secure=true&justLoggedIn=true&_requestid=638587`.

The Crowne Plaza is located in downtown Saint Paul, an area with many restaurants and museums. The hotel is 4 miles from Macalester College, where the meeting will take place. The meeting will provide free bus transportation to Macalester College on Saturday and Sunday morning and back to the hotel on Saturday evening. There are also public buses between the hotel and Macalester which run every half hour on Saturday and every hour on Sunday. For those with a car, parking is available at the hotel for US$10 per day, and it is an easy 12-minute drive to Macalester along Saint Paul's Grand Avenue.

Food Service

The Campus Center Dining Hall will be open for lunch and dinner on Saturday and Sunday. All-you-can-eat-meals cost approximately US$8. Macalester is located on Saint Paul's well-known Grand Avenue, which offers many varied dining opportunities. A list of nearby restaurants is available at `http://www.macalester.edu/admissions/campusvisit/restaurants.html`. A list of local restaurants will be available at the registration desk.

Local Information and Campus Map

For further information please consult the following sites:

Campus Map: `http://www.macalester.edu/about/mapbynumber.html`

Macalester College: `http://www.macalester.edu`

The Macalester College Math Department: `http://www.macalester.edu/mathcs`

Grand Avenue: `http://www.grandave.com/`

Visit Saint Paul: `http://www.visitsaintpaul.com/`

Other Activities

AMS Book Sale: Stop by the on-site AMS Bookstore—review the newest titles from the AMS, enter the FREE book drawing, enjoy up to 25% off all titles or even take home the new AMS T-shirt! Complimentary coffee will be served courtesy of AMS Membership Services.

AMS Editorial Activity: An acquisitions editor from the AMS Book program will be present to speak with prospective authors. If you have a book project that you would like to discuss with the AMS, please stop by the book exhibit.

Parking

Parking is readily available on campus and is free. We recommend that you park in the lot just south of the Leonard Center accessible via Snelling Avenue or in the lots just west of Olin-Rice or west of the Janet Wallace Fine Arts Center, both of which are accessed via Macalester Drive. See the campus map: `http://www.macalester.edu/about/mapbynumber.html`.

Social Event

The Department of Mathematics will host a reception for all participants. The reception will be held from 6:15 p.m. to 7:45 p.m. in the Smail Gallery of Olin-Rice Hall. Hearty appetizers, beer, and wine will be served. The AMS thanks the department for its hospitality.

Registration and Meeting Information

Registration will take place in Olin-Rice Hall, from 7:30 a.m. to 4:00 p.m. on Saturday, April 10, and 8:00 a.m. to noon on Sunday, April 11. The Invited addresses will take place in the J. B. Davis Lecture Hall of the Campus Center. Most Special Sessions will take place in Olin-Rice Hall and the others will take place in the Humanities Building, Carnegie Hall, and Old Main.

Registration fees: (payable on-site only) US$40/AMS members; US$60/nonmembers; US$5/emeritus members, students, or unemployed mathematicians. Fees are payable by cash, check, VISA, Mastercard, Discover, or American Express.

Travel

Macalester College is located on Saint Paul's Grand Avenue in a pleasant neighborhood which is 4 miles west of downtown Saint Paul and 6 miles east of downtown Minneapolis.

By Air:

All major airlines service the Minneapolis-Saint Paul International Airport (MSP). The airport is 6.3 miles from Macalester College and 8.5 miles to the Crowne Plaza Hotel in downtown St. Paul.

Cab fares from the airport to Macalester cost about US$20 and from the airport to the Crowne Plaza cost about US$25.

Super Shuttle (`http://www.supershuttle.com`) costs about US$14 each way between the airport and the Crowne Plaza.

Metro Transit (`http://metrotransit.org`) provides public bus options. The 54 bus from the airport to the Crowne Plaza takes about 25 minutes. Buses from the airport to Macalester take 35–45 minutes and require at least one transfer.

If you rent a car, it is easy to drive to the Crowne Plaza: Follow the signs for St. Paul/Highway 5. Hwy 5 turns into 7th Street in Saint Paul. Travel along Hwy 5 (7th Street) for 6.7 miles until you reach Kellogg Blvd. Turn right on Kellogg and travel for .5 miles until you reach the hotel at 11 E. Kellogg Blvd.

Driving from the airport to Macalester: Follow the signs for St. Paul/Highway 5. After you cross the river, exit right onto Edgecumbe Road (keep left as you exit) and follow for 1 mile; stay to the left and take Fairview Avenue. Follow Fairview Avenue for 1.5 miles. Turn right (east) on Saint Clair Avenue and go .5 miles to Snelling Avenue. Turn left (north) on Snelling, travel .25 miles to the Leonard Center parking lot.

By Car

Driving from the East or the West: Take interstate I94 to Exit 238 at Snelling Avenue. Travel 1 mile south on Snelling Avenue to the corner of Snelling and Grand (you are now at the corner of campus). Continue .25 miles further south along Snelling to reach the Leonard Center parking lot, just before the football stadium.

Driving from the South: Take interstate I35 to I35E. Take Exit 104A at Randolph Avenue. Turn left (west) and travel 1 mile along Randolph Avenue until you reach Snelling Avenue. Turn right (north) on Snelling and travel .8 miles. The Leonard Center parking lot is accessible from Snelling Avenue just after the football stadium.

Car Rental

Avis is the official car rental company for the sectional meeting in Albuquerque. All rates include unlimited free mileage. Weekend daily rates are available from noon Thursday to Monday at 11:59 p.m. Rates do not include any state or local surcharges, tax, optional coverages, or gas refueling charges. Renters must meet Avis's age, driver, and credit requirements. For the best available rate and to make a reservation please call Avis at 800-331-1600 or go online at `http://www.avis.com`. Please use the AMS meeting **Avis Discount Number J098887**.

Weather

April weather in Saint Paul is unpredictable. The average high for the month is 58 degrees F and the average low is 37 degrees F. It has been known to snow in mid April and it has been known to be warm, bright, and sunny. For up-to-the-minute weather please visit `http://www.weather.com/outlook/driving/interstate/local/55105`.

Information for International Participants

Visa regulations are continually changing for travel to the United States. Visa applications may take from three to four months to process and require a personal interview, as well as specific personal information. International participants should view the important information about traveling to the U.S. found at `http://www7.nationalacademies.org/visas/Traveling_to_US.html` and `http://travel.state.gov/visa/index.html`. If you need a preliminary conference invitation in order to secure a visa, please send your request to `wsd@ams.org`.

If you discover you do need a visa, the National Academies website (see above) provides these tips for successful visa applications:

* Visa applicants are expected to provide evidence that they are intending to return to their country of residence. Therefore, applicants should provide proof of "binding" or sufficient ties to their home country or permanent residence abroad. This may include documentation of the following:

– family ties in home country or country of legal permanent residence

– property ownership

– bank accounts

– employment contract or statement from employer stating that the position will continue when the employee returns;

* Visa applications are more likely to be successful if done in a visitor's home country than in a third country;

* Applicants should present their entire trip itinerary, including travel to any countries other than the United States, at the time of their visa application;

* Include a letter of invitation from the meeting organizer or the U.S. host, specifying the subject, location, and dates of the activity, and how travel and local expenses will be covered;

* If travel plans will depend on early approval of the visa application, specify this at the time of the application;

* Provide proof of professional scientific and/or educational status (students should provide a university transcript).

This list is not to be considered complete. Please visit the websites above for the most up-to-date information.

Albuquerque, New Mexico

University of New Mexico

April 17–18, 2010

Saturday – Sunday

Meeting #1059

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: February 2010
Program first available on AMS website: March 4, 2010
Program issue of electronic *Notices*: April 2010
Issue of *Abstracts*: Volume 31, Issue 3

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: February 23, 2010

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Kenneth Bromberg, University of Utah, *Title to be announced.*

Danny Calegari, California Institute of Technology, *Title to be announced.*

Ioana Dumitriu, University of Washington, *Title to be announced.*

Steffen Rohde, University of Washington, *Title to be announced.*

Special Sessions

Dyadic and Non-Dyadic Harmonic Analysis (Code: SS 2A), **M. Cristina Pereyra**, University of New Mexico, and **Stephanie A. Salomone**, University of Portland.

Financial Mathematics: The Mathematics of Financial Markets and Structures (Code: SS 4A), **Maria Cristina Mariani**, University of Texas at El Paso, **Ionut Florescu**, Stevens Institute of Technology, and **Maria P. Beccar-Varela**, University of Texas at El Paso.

Function Spaces, PDEs and Nonlinear Analysis (Code: SS 10A), **Osvaldo Mendez**, **Behzad Rouhani**, and **Mohamed Amine Khamsi**, University of Texas at El Paso.

Geometric Combinatorics (Code: SS 6A), **Art M. Duval**, University of Texas at El Paso, and **Jeremy Martin**, University of Kansas.

Geometric Function Theory (Code: SS 14A), **Lukas Geyer**, Montana State University, and **Donald Marshall** and **Steffen Rohde**, University of Washington.

Geometric Structures and PDEs (Code: SS 8A), **Charles Boyer** and **Dimiter Vassilev**, University of New Mexico.

Harmonic Analysis and Partial Differential Equations (Code: SS 5A), **Matthew Blair**, University of New Mexico, and **Hart Smith**, University of Washington.

Kleinian Groups and Teichmueller Theory (Code: SS 15A), **Kasra Rafi**, University of Oklahoma, **Hossein Namaze**, University of Texas, and **Kenneth Bromberg**, University of Utah.

Positivity in Noncommutative Settings (Code: SS 12A), **Roger Roybal**, California State University Channel Islands, and **Terry Loring**, University of New Mexico.

Random Matrix Theory and Applications (Code: SS 13A), **Ioana Dumitriu**, University of Washington, and **Raj Rao**, University of Michigan.

Selected Topics in Analysis and Numerics for PDEs (Code: SS 11A), **Thomas Hagstrom**, Southern Methodist University, and **Stephen Lau** and **Jens Lorenz**, University of New Mexico.

Strongly-nonlinear Phenomena: Theory and Applications to Nonlinear Optics, Hydrodynamics, Bose–Einstein Condensation and Biology (Code: SS 9A), **Alejandro Aceves**, Southern Methodist University, and **Alexander Korotkevich** and **Pavel Lushnikov**, University of New Mexico.

Subjects in between Pure and Applied Mathematics (Code: SS 7A), **Hanna Makaruk** and **Robert Owczarek**, Los Alamos National Laboratory.

Topics in Geometric Group Theory (Code: SS 1A), **Matthew Day**, California Institute of Technology, **Daniel Peter Groves**, University of Illinois at Chicago, **Jason Manning**, SUNY at Buffalo, and **Henry Wilton**, California Institute of Technology.

Trends in Commutative Algebra (Code: SS 3A), **Louiza Fouli**, New Mexico State University, and **Janet Vassilev**, University New Mexico.

Accommodations

Participants should make their own arrangements directly with the hotel. When making a reservation identify yourself as being with the UNM Math and Stat group attending the AMS Meeting. The AMS is not responsible for rate changes or for the quality of the accommodations. **Hotels have varying cancellation or early checkout penalties; be sure to ask for details when making your reservation.**

Plaza Inn Downtown Albuquerque, 900 Medical Arts Ave NE, Albuquerque, NM 87102; phone 505-243-5693 or 866-925-7881. Rates start at US$56.99 + tax. **Deadline for reservations is March 26, 2010**. Be sure to check the cancellation policy. Amenities include: Shuttle to/from airport and UNM from 6:30 a.m. to 11 p.m., complimentary continental breakfast, complimentary high-speed wireless Internet access in public areas and guest rooms, hot tub, pool, restaurant. The hotel is approximately one mile to campus. For more information please visit http://www.plazainnabq.com/.

Albuquerque Airport Fairfield Inn by Marriott, 2300 Centre Ave. SE, Albuquerque, NM 87106, phone: 505-247-1621. Price: US$59 + tax. **Deadline for reservations is March 19, 2010**. Be sure to check the cancellation policy. Amenities include: shuttle to/from airport and UNM, continental breakfast, coffee/tea in room, outdoor pool & spa, complimentary breakfast, wired high-speed Internet in guest rooms. The hotel is approximately 1.5 miles to campus.

Food Service

A list of local restaurants will be available at the registration desk.

Local Information and Campus Map

For further information please consult the website maintained by the Department of Mathematics at the University of New Mexico: `http://www.math.unm.edu`. To view a campus map please visit: `http://www.unm.edu/campusmap.html`. Dane Smith Hall is building 48 at F-G-3. For travel information please visit: `http://www.math.unm.edu/about/index.php`.

Other Activities

AMS Book Sale: Stop by the on-site AMS Bookstore—review the newest titles from the AMS, enter the FREE book drawing, enjoy up to 25% off all titles or even take home the new AMS T-shirt! Complimentary coffee will be served courtesy of AMS Membership Services.

AMS Editorial Activity: An acquisitions editor from the AMS Book program will be present to speak with prospective authors. If you have a book project that you would like to discuss with the AMS, please stop by the book exhibit.

Parking

Visitors can park anywhere on campus on weekends (6:00 p.m. Friday until 8:00 a.m. Monday) without a permit. City parking rules still apply. For further information please visit `http://pats.unm.edu/visitors.cfm`.

Registration and Meeting Information

Registration will take place in the atrium of Dane Smith Hall located on Las Lomas Blvd., across from University House on Yale Avenue, from 7:30 a.m. to 4:00 p.m. on Saturday, April 17, and 8:00 a.m. to noon on Sunday, April 18.

Registration fees: (payable on-site only) US$40/AMS members; US$60/nonmembers; US$5/emeritus members, students, or unemployed mathematicians. Fees are payable by cash, check, VISA, Mastercard, Discover, or American Express.

Travel

By Air:

Albuquerque Sunport International Airport: Albuquerque is served by many of the major commercial carriers and several commuter airlines. The Sunport is located two (2) miles south of UNM.

Traveling to Albuquerque by car: Albuquerque is served by two major interstates, I25 (North-South) and I40 (East-West). The Martin Luther King exit 1 mile south of the I25-I40 interchange allows access to the **Plaza Inn Albuquerque** (take the service road north from Martin Luther King, on the east side of I25) and UNM (go east to University Blvd., enter campus, turn right on Redondo Rd. and follow it to visitor parking or residence halls). The **Albuquerque Airport Fairfield Inn by Marriott** is located off of Yale (east of Yale) between the airport and UNM.

Transportation from/to the Sunport: Shuttle and taxi service available after all flights outside at baggage claim level. Albuquerque Cab (505-883-4888, typical charge US$18.00). Sunport provides a shuttle to Rental Car Center. Call 505-247-1621 for the free shuttle to **Albuquerque Airport Fairfield Inn by Marriott**, and 505-243-5693 for the free shuttle to **Plaza Inn Albuquerque**.

Getting around Albuquerque is easiest by car, but the city has regular bus service along Central Ave. from downtown to UNM for participants who wish to stay downtown, see `http://www.cabq.gov/transit/routes-and-schedules`. Taxi service is available but best arranged beforehand.

Car Rental

Avis is the official car rental company for the sectional meeting in Albuquerque. All rates include unlimited free mileage. Weekend daily rates are available from noon Thursday to Monday at 11:59 p.m. Rates do not include any state or local surcharges, tax, optional coverages, or gas refueling charges. Renters must meet Avis's age, driver, and credit requirements. For the best available rate and to make a reservation please call Avis at 800-331-1600 or go online at `http://www.avis.com`. Please use the AMS meeting **Avis Discount Number J098887**.

Weather

April weather is generally pleasant with daytime temperatures in the 60 degree F. range, and nighttime temperatures in the 30–45 degree F. range. For up-to-the-minute weather please visit `http://www.weather.com/outlook/driving/local/USNM0004`.

Information for International Participants

Visa regulations are continually changing for travel to the United States. Visa applications may take from three to four months to process and require a personal interview, as well as specific personal information. International participants should view the important information about traveling to the U.S. found at `http://www7.nationalacademies.org/visas/Traveling_to_US.html` and `http://travel.state.gov/visa/index.html`. If you need a preliminary conference invitation in order to secure a visa, please send your request to `dls@ams.org`.

If you discover you do need a visa, the National Academies website (see above) provides these tips for successful visa applications:

* Visa applicants are expected to provide evidence that they are intending to return to their country of residence. Therefore, applicants should provide proof of "binding" or sufficient ties to their home country or permanent

residence abroad. This may include documentation of the following:

– family ties in home country or country of legal permanent residence

– property ownership

– bank accounts

– employment contract or statement from employer stating that the position will continue when the employee returns;

* Visa applications are more likely to be successful if done in a visitor's home country than in a third country;

* Applicants should present their entire trip itinerary, including travel to any countries other than the United States, at the time of their visa application;

* Include a letter of invitation from the meeting organizer or the U.S. host, specifying the subject, location, and dates of the activity, and how travel and local expenses will be covered;

* If travel plans will depend on early approval of the visa application, specify this at the time of the application;

* Provide proof of professional scientific and/or educational status (students should provide a university transcript).

This list is not to be considered complete. Please visit the websites above for the most up- to-date information.

Newark, New Jersey

New Jersey Institute of Technology

May 22–23, 2010

Saturday – Sunday

Meeting #1060

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: March 2020
Program first available on AMS website: April 8, 2010
Program issue of electronic *Notices*: May 2020
Issue of *Abstracts*: Volume 31, Issue 3

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: February 2, 2010
For abstracts: March 30, 2010

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Simon Brendle, Stanford University, *Hamilton's Ricci flow and the sphere theorem in geometry*.

Konstantin M. Mischaikow, Rutgers University, *Computational topology applied to the global dynamics of nonlinear systems*.

Ricardo H. Nochetto, University of Maryland, *Curvature driven flows in deformable domains*.

Richard E. Schwartz, Brown University, *Polygonal outer billiards*.

Special Sessions

Automorphic Forms, L-functions, and Applications (Code: SS 6A), **Ameya Pitale**, American Institute of Mathematics, and **Anantharam Raghuram**, Oklahoma State University.

Biomembranes: Modeling, Analysis, and Computation (Code: SS 16A), **Ricardo H. Nochetto** and **Dionisios Margetis**, University of Maryland.

Elliptic and Parabolic Problems in Geometry (Code: SS 12A), **Simon Brendle**, Stanford University, and **Mu-Tao Wang**, Columbia University.

Expandable Computations, Algorithms, Methodologies and Experiments for Engineering Interpretation (Code: SS 1A), **Mustapha S. Fofana**, Worcester Polytechnic Institute, **Marie D. Dahleh**, Harvard School of Engineering and Applied Sciences, Harvard University, and **Kenji Kawashima**, Precision and Intelligence Laboratory, Tokyo Institute of Technology.

Financial Mathematics (Code: SS 9A), **Tim S.T. Leung**, Johns Hopkins University.

Graph Theory (Code: SS 10A), **Nathan W. Kahl**, Seton Hall University, **Michael J. Ferrara**, University of Colorado at Denver, and **Arthur H. Busch**, University of Dayton.

Groups, Computations, and Applications (Code: SS 2A), **Delaram Kahrobaei**, City University of New York.

Homology Theories for Knots and Skein Modules (Code: SS 3A), **Mikhail Khovanov**, Columbia University, and **Jozef H. Przytycki** and **Radmila Sazdanovic**, George Washington University.

Invariants of Knots, Links, and 3-Manifolds (Code: SS 4A), **Abhijit Champanerkar** and **Ilya S. Kofman**, College of Staten Island, CUNY, and **Philip J. P. Ording**, Medgar Evers College, CUNY.

Lie Algebras and Representation Theory (Code: SS 8A), **Gautam Chinta**, City College, City University of New York, **Andrew Douglas**, New York City College of Technology, City University of New York, and **Bart Van Steirteghem**, Medgar Evers College, City University of New York.

Logic and Groups (Code: SS 17A), **Peggy Dean**, **Claire Wladis**, and **Marcos Zyman**, Borough of Manhattan Community College, City University of New York.

Mathematical Neuroscience: Modeling, Analysis, and Simulations (Code: SS 14A), **Horacio G. Rotstein**, New Jersey Institute of Technology.

Mathematics and Computations of Fluid Dynamics (Code: SS 15A), **Yuan N. Young**, New Jersey Institute of Technology.

Mathematics of Optics and Matter Waves (Code: SS 13A), **Roy Goodman**, New Jersey Institute of Technology.

Nonlinear Waves (Code: SS 19A), **A. David Trubatch**, Montclair State University.

Recent Trends in Cayley Graphs to Model Interconnection Networks (Code: SS 18A), **Daniela Ferrero**, Texas State University, and **Beth Novick**, Clemson University.

Teichmüller Theory, Hyperbolic Geometry, and Complex Dynamics (Code: SS 5A), **Zheng Huang**, College of Staten Island, CUNY, and **Ren Guo**, University of Minnesota.

Topological and Computational Dynamics (Code: SS 7A), **Jean-Philippe Lessard**, Institute for Advanced Study and Rutgers University, and **Konstantin M. Mischaikow**, Rutgers University.

Vortex Dynamics: Theory and Applications (Code: SS 11A), **Denis Blackmore**, New Jersey Institute of Technology, **Morten Brøns**, Technical University of Denmark, and **Chjan Lim**, Rochester Polytechnic Institute.

Berkeley, California

University of California Berkeley

June 2–5, 2010

Wednesday – Saturday

Meeting #1061

Eighth Joint International Meeting of the AMS and the Sociedad Matemática Mexicana.
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: March 2010
Program first available on AMS website: April 22, 2010
Program issue of electronic *Notices*: June 2010
Issue of *Abstracts*: Volume 31, Issue 3

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: February 16, 2010
For abstracts: April 13, 2010

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/internmtgs.html.

Invited Addresses

Alejandro Adem, University of British Columbia and PIMS, *Title to be announced.*

Peter W-K Li, University of California Irvine, *Title to be announced.*

Ernesto Lupercio, CINVESTAV, *Title to be announced.*

Victor Perez Abreu, CIMAT, *Title to be announced.*

Alberto Verjovsky, IM-UNAM, *Title to be announced.*

Maciej Zworski, University of California Berkeley, *Title to be announced.*

Special Sessions

Algebraic Topology and Related Topics (Code: SS 3A), **Alejandro Adem**, University of British Columbia, **Gunnar E. Carlsson** and **Ralph L. Cohen**, Stanford University, and **Ernesto Lupercio**, CINVESTAV.

Analytic Aspects of Differential Geometry (Code: SS 2A), **Nelia Charalambous**, ITAM, **Lizhen Ji**, University of Michigan, and **Jiaping Wang**, University of Minnesota.

Commutative Algebra and Representation Theory (Code: SS 7A), **David Eisenbud** and **Daniel M. Erman**, University of California, Berkeley, **Jose Antonio de la Pena**, UNAM, and **Rafael Villareal**, Cinvestav-IPN.

Complex Analysis and Operator Theory (Code: SS 10A), **Maribel Loaiza**, **Enrique Ramirez de Arellano**, and **Nikolai Vasilevski**, CINVESTAV, **Ilya M. Spitkovsky**, College of William & Mary, and **Kehe Zhu**, State University of New York at Albany.

Dynamical Systems (Code: SS 4A), **Alberto Verjovsky**, IM-UNAM, and **Rodrigo Perez**, Indiana University-Purdue University, Indianapolis.

Graph Theory and Combinatorics with Emphasis on Geometric and Topological Aspects (Code: SS 9A), **Gelasio Salazar**, Instituto de Fisica, Universidad Autonoma de San Luis Potosi, and **Dan S. Archdeacon**, University of Vermont.

Harmonic Analysis, Microlocal Analysis, and Partial Differential Equations (Code: SS 1A), **Gunther Uhlmann**, University of Washington, and **Salvador Perez Esteva**, UNAM.

Low-Dimensional Topology (Code: SS 8A), **Kenneth L. Baker**, University of Miami, and **Enrique Ramirez Losada**, CIMAT.

Singularity Theory and Algebraic Geometry (Code: SS 6A), **David Eisenbud**, University of California, Berkeley, **Anatoly S. Libgober**, University of Illinois at Chicago, **Jose Seade**, UNAM, and **Xavier Gomez-Mont**, CIMAT.

Toeplitz Operators and Discrete Quantum Models (Code: SS 5A), **Alejandro Uribe**, University of Michigan, and **Maciej Zworski**, University of California, Berkeley.

Syracuse, New York

Syracuse University

October 2–3, 2010

Saturday – Sunday

Meeting #1062

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: August 19, 2010
Program issue of electronic *Notices*: October
Issue of *Abstracts*: Volume 31, Issue 4

Deadlines

For organizers: March 2, 2010
For consideration of contributed papers in Special Sessions: June 15, 2010
For abstracts: August 10, 2010

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Alan Frieze, Carnegie-Mellon University, *Title to be announced.*

Yan Guo, Brown University, *Title to be announced.*

William Minicozzi, Johns Hopkins University, *Title to be announced.*

Andrei Zelevinsky, Northeastern University, *Title to be announced.*

Special Sessions

Difference Equations and Applications (Code: SS 2A), **Michael Radin**, Rochester Institute of Technology.

Graphs Embedded in Surfaces, and Their Symmetries (Code: SS 4A), **Jack E. Graver** and **Mark E. Watkins**, Syracuse University.

Mathematical Image Processing, **Lixin Shen** and **Yuesheng Xu**, *Syracuse University.*

Nonlinear Analysis and Geometry (Code: SS 1A), **Tadeusz Iwaniec**, **Leonid V. Kovalev**, and **Jani Onninen**, Syracuse University.

Several Complex Variables (Code: SS 3A), **Dan F. Coman** and **Evgeny A. Poletsky**, Syracuse University.

Los Angeles, California

University of California Los Angeles

October 9–10, 2010

Saturday – Sunday

Meeting #1063

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: August 2010
Program first available on AMS website: August 26, 2010
Program issue of electronic *Notices*: October 2010
Issue of *Abstracts*: Volume 31, Issue 4

Deadlines

For organizers: March 10, 2010
For consideration of contributed papers in Special Sessions: June 22, 2010
For abstracts: August 17, 2010

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Greg Kuperberg, University of California Davis, *Title to be announced.*

Cris Moore, University of New Mexico, *Title to be announced.*

Stanley Osher, University of California Los Angeles, *Title to be announced.*

Terence Tao, University of California Los Angeles, *Title to be announced* (Einstein Public Lecture in Mathematics).

Melanie Wood, Princeton University, *Title to be announced.*

Special Sessions

Applications of Nonlinear PDE (Code: SS 5A), **Susan J. Friedlander** and **Igor Kukavica**, University of Southern California.

Combinatorics and Probability on Groups (Code: SS 3A), **Jason Fulman** and **Robert Guralnick**, University of Southern California, and **Igor Pak**, University of California Los Angeles.

Extremal and Probabilistic Combinatorics (Code: SS 4A), **Benny Sudakov**, University of California Los Angeles, and **Jacques Verstraete**, University of California San Diego.

Large Cardinals and the Continuum (Code: SS 2A), **Matthew Foreman**, University of California Irvine, **Alekos Kechris**, California Institute for Technology, **Itay Neeman**, University of California Los Angeles, and **Martin Zeman**, University of California Irvine.

Recent Trends in Probability and Related Fields (Code: SS 6A), **Marek Biskup**, University of California Los Angeles, **Yuval Peres**, Microsoft Research, and **Sebastien Roch**, University of California Los Angeles.

Topology and Symplectic Geometry (Code: SS 1A), **Robert Brown** and **Ciprian Manolescu**, University of California Los Angeles, and **Stefano Vidussi**, University of California Riverside.

Notre Dame, Indiana

Notre Dame University

October 29–31, 2010

Friday – Sunday

Meeting #1064

Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: August 2010
Program first available on AMS website: September 16, 2010
Program issue of electronic *Notices*: October 2010
Issue of *Abstracts*: Volume 31, Issue 4

Deadlines

For organizers: February 19, 2010
For consideration of contributed papers in Special Sessions: July 20, 2010
For abstracts: September 7, 2010

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Laura DeMarco, University of Illinois at Chicago, *Title to be announced.*

Jordan Ellenberg, University of Wisconsin, *Title to be announced.*

David Fisher, Indiana University, *Title to be announced.*

Jared Wunsch, Northwestern University, *Title to be announced.*

Special Sessions

Commutative Algebra and Its Interactions with Algebraic Geometry (Code: SS 2A), **Claudia Polini**, University of Notre Dame, **Alberto Corso**, University of Kentucky, and **Bernd Ulrich**, Purdue University.

Groups, Representations, and Characters (Code: SS 4A), **James P. Cossey**, University of Akron, and **Mark Lewis**, Kent State University.

Hilbert Functions in Commutative Algebra and Algebraic Combinatorics (Code: SS 3A), **Fabrizio Zanello**, Michigan Technological University, **Juan Migliore**, University of Notre Dame, and **Uwe Nagel**, University of Kentucky.

Interdisciplinary Session on Deterministic and Stochastic Partial Differential Equations (Code: SS 5A), **Nathan Glatt-Holtz**, Indiana University, and **Vlad Vicol**, University of Southern California.

Quasigroups, Loops, and Nonassociative Division Algebras (Code: SS 6A), **Clifton E. Ealy**, Western Michigan University, **Stephen Gagola**, University of Arizona, **Julia Knight**, University of Notre Dame, **J. D. Phillips**, Northern Michigan University, and **Petr Vojtechovsky**, University of Denver.

Singularities in Algebraic Geometry (Code: SS 1A), **Nero Budur**, University of Notre Dame, and **Lawrence Ein**, University of Illinois at Chicago.

Richmond, Virginia

University of Richmond

November 6–7, 2010

Saturday – Sunday

Meeting #1065

Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: September
Program first available on AMS website: September 23, 2010
Program issue of electronic *Notices*: November
Issue of *Abstracts*: Volume 31, Issue 4

Deadlines

For organizers: March 8, 2010
For consideration of contributed papers in Special Sessions: July 27, 2010
For abstracts: September 14, 2010

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Matthew H. Baker, Georgia Institute of Technology, *Title to be announced.*

Michael J. Field, University of Houston, *Title to be announced.*

Sharon R. Lubkin, North Carolina State University, *Title to be announced.*

Stefan Richter, University of Tennessee, Knoxville, *Title to be announced.*

Special Sessions

Operator Theory (Code: SS 2A), **Stefan Richter**, University of Tennessee, and **William T. Ross**, University of Richmond.

Pucon, Chile

December 15–18, 2010

Wednesday – Saturday

Meeting #1066

First Joint International Meeting between the AMS and the Sociedad de Matematica de Chile.
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: June 2010
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 15, 2010
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/internmtgs.html.

AMS Invited Addresses

Ricardo Baeza, Universidad de Talca, Chile, *Title to be announced.*

Igor Dolgachev, University of Michigan, *Title to be announced.*

Andres Navas, Universidad de Santiago de Chile, *Title to be announced.*

Rodolfo Rodriguez, Universidad de Concepcion, *Title to be announced.*

Gunther Uhlmann, University of Washington, *Title to be announced.*

S. R. Srinivasa Varadhan, New York University, *Title to be announced.*

AMS Special Sessions

Arithmetic of Quadratic Forms and Integral Lattices. (Code: SS 6A), **Maria Ines Icaza**, Universidad de Talca, Chile, **Wai Kiu Chan**, Wesleyan University, and **Ricardo Baeza**, Universidad de Talca, Chile.

Automorphic Forms and Dirichlet Series (Code: SS 5A), **Yves Martin**, Universidad de Chile, Chile, and **Solomon Friedberg**, Boston College.

Complex Algebraic Geometry (Code: SS 1A), **Giancarlo Urzua** and **Eduardo Cattani**, University of Massachusetts.

Foliations and Dynamics (Code: SS 4A), **Andrés Navas**, Universidad de Santiago de Chile, and **Rostislav Grigorchuk**, University of Texas.

Group Actions: Probability and Dynamics (Code: SS 3A), **Andrés Navas**, Universidad de Santiago de Chile, and **Rostislav Grigorchuk**, University of Texas.

Non-Associative Algebras (Code: SS 2A), **Alicia Labra**, Universidad de Chile, and **Kevin McCrimmon**, University of Virginia.

New Orleans, Louisiana

New Orleans Marriott and Sheraton New Orleans Hotel

January 5–8, 2011

Wednesday – Saturday

Joint Mathematics Meetings, including the 117th Annual Meeting of the AMS, 94th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: October 2010
Program first available on AMS website: November 1, 2010
Program issue of electronic *Notices*: January 2011
Issue of *Abstracts*: Volume 32, Issue 1

Deadlines

For organizers: April 1, 2010
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Statesboro, Georgia

Georgia Southern University

March 12–13, 2011

Saturday – Sunday

Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: August 12, 2010
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Iowa City, Iowa

University of Iowa

March 18–20, 2011

Friday – Sunday

Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: July 16, 2010
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Worcester, Massachusetts

College of the Holy Cross

April 9–10, 2011

Saturday – Sunday

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 9, 2010
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Las Vegas, Nevada

University of Nevada

April 30 – May 1, 2011

Saturday – Sunday

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Special Sessions

Advances in Modeling, Numerical Analysis and Computations of Fluid Flow Problems (Code: SS 2A), **Monika Neda**, University of Nevada Las Vegas.

Geometric PDEs (Code: SS 1A), **Matthew Gursky**, Notre Dame University, and **Emmanuel Hebey**, Universite de Cergy-Pontoise.

Lincoln, Nebraska

University of Nebraska-Lincoln

October 14–16, 2011

Friday – Sunday
Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: August 2011
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: October 2011
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Salt Lake City, Utah

University of Utah

October 22–23, 2011

Saturday – Sunday
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Boston, Massachusetts

John B. Hynes Veterans Memorial Convention Center, Boston Marriott Hotel, and Boston Sheraton Hotel

January 4–7, 2012

Wednesday – Saturday
Joint Mathematics Meetings, including the 118th Annual Meeting of the AMS, 95th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: October 2011
Program first available on AMS website: November 1, 2011
Program issue of electronic *Notices*: January 2012
Issue of *Abstracts*: Volume 33, Issue 1

Deadlines

For organizers: April 1, 2011
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

San Diego, California

San Diego Convention Center and San Diego Marriott Hotel and Marina

January 9–12, 2013

Wednesday – Saturday
Joint Mathematics Meetings, including the 119th Annual Meeting of the AMS, 96th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: October 2012
Program first available on AMS website: November 1, 2012
Program issue of electronic *Notices*: January 2012
Issue of *Abstracts*: Volume 34, Issue 1

Deadlines

For organizers: April 1, 2012
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Baltimore, Maryland

Baltimore Convention Center, Baltimore Hilton, and Marriott Inner Harbor

January 15–18, 2014

Wednesday – Saturday

Joint Mathematics Meetings, including the 120th Annual Meeting of the AMS, 97th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Matthew Miller
Announcement issue of *Notices*: October 2013
Program first available on AMS website: November 1, 2013
Program issue of electronic *Notices*: January 2013
Issue of *Abstracts*: Volume 35, Issue 1

Deadlines

For organizers: April 1, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

San Antonio, Texas

Henry B. Gonzalez Convention Center and Grand Hyatt San Antonio

January 10–13, 2015

Saturday – Tuesday

Joint Mathematics Meetings, including the 121st Annual Meeting of the AMS, 98th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: October 2014
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2015
Issue of *Abstracts*: Volume 36, Issue 1

Deadlines

For organizers: April 1, 2014
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Seattle, Washington

Washington State Convention & Trade Center and the Sheraton Seattle Hotel

January 6–9, 2016

Wednesday – Saturday

Joint Mathematics Meetings, including the 122nd Annual Meeting of the AMS, 99th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: October 2015
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2016
Issue of *Abstracts*: Volume 37, Issue 1

Deadlines

For organizers: April 1, 2015
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Atlanta, Georgia

Hyatt Regency Atlanta and Marriott Atlanta Marquis

January 4–7, 2017

Wednesday – Saturday

Joint Mathematics Meetings, including the 123rd Annual Meeting of the AMS, 100th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Georgia Benkart
Announcement issue of *Notices*: October 2016
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2017
Issue of *Abstracts*: Volume 38, Issue 1

Deadlines

For organizers: April 1, 2016
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Meetings and Conferences of the AMS

Associate Secretaries of the AMS

Western Section: Michel L. Lapidus, Department of Mathematics, University of California, Surge Bldg., Riverside, CA 92521-0135; e-mail: lapidus@math.ucr.edu; telephone: 951-827-5910.

Central Section: Georgia Benkart (after January 31, 2010), University of Wisconsin-Madison, Department of Mathematics, 480 Lincoln Drive, Madison, WI 53706-1388; e-mail: benkart@math.wisc.edu; telephone: 608-263-4283.

Eastern Section: Steven H. Weintraub, Department of Mathematics, Lehigh University, Bethlehem, PA 18105-3174; e-mail: steve.weintraub@lehigh.edu; telephone: 610-758-3717.

Southeastern Section: Matthew Miller, Department of Mathematics, University of South Carolina, Columbia, SC 29208-0001, e-mail: miller@math.sc.edu; telephone: 803-777-3690.

The Meetings and Conferences section of the *Notices* gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited in the table of contents on this page for more detailed information on each event. Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. **Information in this issue may be dated. Up-to-date meeting and conference information can be found at** www.ams.org/meetings/.

Meetings:

Important Information Regarding AMS Meetings

Potential organizers, speakers, and hosts should refer to page 92 in the January 2010 issue of the *Notices* for general information regarding participation in AMS meetings and conferences.

Abstracts

Speakers should submit abstracts on the easy-to-use interactive Web form. No knowledge of LaTeX is necessary to submit an electronic form, although those who use LaTeX may submit abstracts with such coding, and all math displays and similarily coded material (such as accent marks in text) must be typeset in LaTeX. Visit http://www.ams.org/cgi-bin/abstracts/abstract.pl. Questions about abstracts may be sent to abs-info@ams.org. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

Conferences: (see http://www.ams.org/meetings/ for the most up-to-date information on these conferences.)

Co-sponsored conferences:

February 18–22, 2010: AAAS Meeting in San Diego, CA (please see www.aaas.org/meetings for more information).

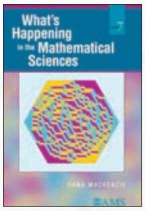
March 18-21, 2010: First International Conference on Mathematics and Statistics, AUS-ICMS '10, American University of Sharjah, Sharjah, United Arab Emirates (please see http://www.aus.edu/conferences/icms10/ for more information).

May 24-29, 2010: From Carthage to the World, the Isoperimetric Problem of Queen Dido and its Mathematics Ramifications, Carthage, Tunisia (for more information please see http://math.arizona.edu/~dido/welcome.html).

June 17-19, 2010: Coimbra Meeting on 0-1 Matrix Theory and Related Topics, University of Coimbra, Portugal (for more information please see http://www.mat.uc.pt/~cmf/01MatrixTheory).

American Mathematical Society

A SELECTION OF THE BEST-SELLING AMS TITLES IN 2009



What's Happening in the Mathematical Sciences

Dana Mackenzie

What's Happening in the Mathematical Sciences, Volume 7; 2009; 127 pages; Softcover; ISBN: 978-0-8218-4478-6; List US$19.95; AMS members US$15.95; Order code HAPPENING/7

Introduction to Analysis

Fifth Edition

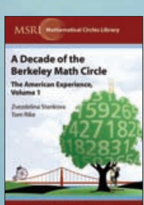
Edward D. Gaughan, *New Mexico State University, Las Cruces, NM*

Pure and Applied Undergraduate Texts, Volume 1; 1998; 240 pages; Hardcover; ISBN: 978-0-8218-4787-9; List US$62; AMS members US$50; Order code AMSTEXT/1

Lectures on Quantum Mechanics for Mathematics Students

L. D. Faddeev, *Steklov Mathematical Institute, St. Petersburg, Russia*, and **O. A. Yakubovskiĭ**, *St. Petersburg University, Russia*

Student Mathematical Library, Volume 47; 2009; 234 pages; Softcover; ISBN: 978-0-8218-4699-5; List US$39; AMS members US$31; Order code STML/47



A Decade of the Berkeley Math Circle

The American Experience, Volume I

Zvezdelina Stankova, *Mills College, Oakland, CA, and University of California, Berkeley, CA*, and **Tom Rike**, *Oakland High School, CA*, Editors

Titles in this series are co-published with the Mathematical Sciences Research Institute (MSRI).

MSRI Mathematical Circles Library, Volume 1; 2008; 326 pages; Softcover; ISBN: 978-0-8218-4683-4; List US$49; AMS members US$39; Order code MCL/1

Markov Chains and Mixing Times

David A. Levin, *University of Oregon, Eugene, OR*, **Yuval Peres**, *Microsoft Research, Redmond, WA, and University of California, Berkeley, CA*, and **Elizabeth L. Wilmer**, *Oberlin College, OH*

2009; 371 pages; Hardcover; ISBN: 978-0-8218-4739-8; List US$65; AMS members US$52; Order code MBK/58

Quantum Mechanics for Mathematicians

Leon A. Takhtajan, *Stony Brook University, NY*

Graduate Studies in Mathematics, Volume 95; 2008; 387 pages; Hardcover; ISBN: 978-0-8218-4630-8; List US$69; AMS members US$55; Order code GSM/95

Tools of the Trade

Introduction to Advanced Mathematics

Paul J. Sally, Jr., *University of Chicago, IL*

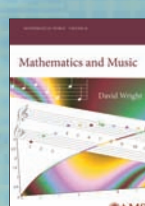
2008; 193 pages; Hardcover; ISBN: 978-0-8218-4634-6; List US$49; AMS members US$39; Order code MBK/55

Discrete Differential Geometry

Integrable Structure

Alexander I. Bobenko, *Technische Universität Berlin, Germany*, and **Yuri B. Suris**, *Technische Universität München, Garching bei München, Germany*

Graduate Studies in Mathematics, Volume 98; 2008; 404 pages; Hardcover; ISBN: 978-0-8218-4700-8; List US$69; AMS members US$55; Order code GSM/98



Mathematics and Music

David Wright, *Washington University, St. Louis, MO*

Mathematical World, Volume 28; 2009; 161 pages; Softcover; ISBN: 978-0-8218-4873-9; List US$35; AMS members US$28; Order code MAWRLD/28

The Geometry of Heisenberg Groups

With Applications in Signal Theory, Optics, Quantization, and Field Quantization

Ernst Binz and **Sonja Pods**, *University of Mannheim, Germany*

Mathematical Surveys and Monographs, Volume 151; 2008; 299 pages; Hardcover; ISBN: 978-0-8218-4495-3; List US$85; AMS members US$68; Order code SURV/151

Pioneering Women in American Mathematics

The Pre-1940 PhD's

Judy Green, *Marymount University, Arlington, VA*, and **Jeanne LaDuke**, *DePaul University, Chicago, IL*

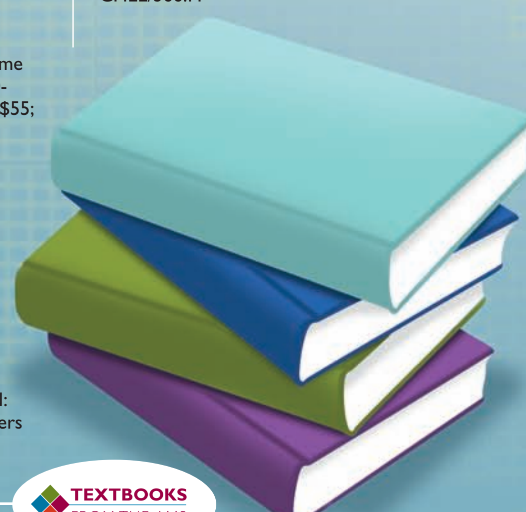
Co-published with the London Mathematical Society beginning with Volume 4. Members of the LMS may order directly from the AMS at the AMS member price. The LMS is registered with the Charity Commissioners.

History of Mathematics, Volume 34; 2009; 345 pages; Hardcover; ISBN: 978-0-8218-4376-5; List US$79; AMS members US$63; Order code HMATH/34

Class Field Theory

Emil Artin, and **John Tate**, *University of Texas at Austin, TX*

AMS Chelsea Publishing, Volume 366; 2008; 192 pages; Hardcover; ISBN: 978-0-8218-4426-7; List US$35; AMS members US$32; Order code CHEL/366.H



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