2010 Conant Prize

BRYNA KRA received the 2010 AMS Levi L. Conant Prize at the 116th Annual Meeting of the AMS in San Francisco in January 2010.

Citation

The Levi L. Conant Prize for 2010 is awarded to Bryna Kra for her article, "The Green-Tao Theorem on arithmetic progressions in the primes: An ergodic point of view" (*Bull. Amer. Math. Soc. (N.S.)* **43** (2006), no. 1, 3–23).

The search for patterns in the prime numbers has fascinated both professional mathematicians and mathematical amateurs at least since the days of Euler, Goldbach, Lagrange, and Waring. Although the Prime Number Theorem provides asymptotic estimates on the distribution of primes, it does not yield information about regular patterns. The modern history of the subject began with a conjecture of Hardy and Littlewood in 1923 that, given k-tuples a_i and b_i of nonnegative integers, then, with obvious exceptions, there are infinitely many integers n such that the sets $a_{i_n}+b_i : 1 \leq i \leq k$ consist only of primes. In 1939 van der Corput proved that the primes contain infinitely many triples in arithmetic progression. Computational methods by Moran, Pritchard, and Thyssen found a progression of length 22 in 1995; a record that was finally broken in 2004, when Frind, Jobling, and Underwood found a progression of length 23 starting with the prime 56211383760397 and with common difference 44546738095860. That very same year Ben Green and Terence Tao achieved their striking breakthrough with a proof that the set of prime numbers contains arithmetic progressions of length k for every natural number k.

Kra's article is an engaging exposition of the many mathematical strands woven into the fabric of the proof—number theory, ergodic theory, harmonic analysis, discrete geometry, and combinatorics. The paper is written in a relaxed and readable style, while conveying a wealth of insight. Kra describes how a conjecture of Erdös and Turán sparked the imaginations of a succession of brilliant mathematicians—Szemerédi, Furstenberg, Gowers, Green, and Tao—all of whom contributed significant ideas from combinatorics, ergodic theory, and harmonic analysis. Although Szemerédi's Theorem itself is too weak to yield the Green-Tao Theorem directly, the contemplation of this theorem from many vantage points yielded enough insight to permit Green and Tao to prove their celebrated result.

Bryna Kra

Kra's narration captures the fascinating history of and conveys the key mathematical concepts behind the Green-Tao result. The article provides an instructive comparison of the proofs of Szemerédi's Theorem by Furstenberg, Gowers, and Tao, revealing the similarity lurking beneath the apparent differences in approach. It is an excellent and well-told lesson in the value of thinking and rethinking about important mathematical results.

Biographical Sketch

Bryna Kra earned her undergraduate degree from Harvard University in 1988 and her Ph.D. from Stanford University in 1995 under the direction of Yitzhak Katznelson. Before her appointment to Northwestern University in 2004, she held postdoctoral positions at the Hebrew University of Jerusalem, the University of Michigan, the Institut des Hautes Etudes Scientifiques, and Ohio State University, and was an assistant professor at Pennsylvania State University. Kra works in dynamical systems and ergodic theory with a focus on problems related to combinatorics and number theory, frequently in collaboration with Bernard Host. She was an invited speaker at the 2006 International Congress of Mathematicians and was awarded a Centennial Fellowship, also in 2006. Kra organizes a mentoring program for women in mathematics at Northwestern, runs a math enrichment program for children at a local elementary school, and is currently chair of the Northwestern math department.

Response

It is an honor and a pleasure to be awarded the Conant Prize. It is especially gratifying for me because this project is linked in my memory to the birth of my second son. The invitation to give a "Current Events" talk on Green and Tao's proof arrived while I was still in the hospital. As I sleepily rocked a newborn, their proof occupied my mind.

I would not be standing here without the support of many people. My parents have always been my strongest proponents, and I was pleased to finally write something that my mathematician father was happy to read! This article was only made coherent with the help of many colleagues who took the time to read and improve preliminary versions. And I especially thank my husband and children for their patience and support throughout.

About the Prize

The Conant Prize is awarded annually to recognize an outstanding expository paper published in either the *Notices of the AMS* or the *Bulletin of the AMS* in the preceding five years. Established in 2001, the prize honors the memory of Levi L. Conant (1857–1916), who was a mathematician at Worcester Polytechnic University. The prize carries a cash award of US$1,000.

The Conant Prize is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2010 prize, the members of the selection committee were: Georgia Benkart, Stephen J. Greenfield, and Ronald M. Solomon.

Previous recipients of the Conant Prize are: Carl Pomerance (2001); Elliott Lieb and Jakob Yngvason (2002); Nicholas Katz and Peter Sarnak (2003); Noam D. Elkies (2004); Allen Knutson and Terence Tao (2005); Ronald M. Solomon (2006); Jeffrey Weeks (2007); J. Brian Conrey, Shlomo Hoory, Nathan Linial, Avi Wigderson (2008), and John W. Morgan (2009).

About the Cover

Preventing boundary layer separation

The cover for this issue was suggested by Doug Arnold's article, also in this issue.

The game of golf began over four centuries ago, and golf balls were, for a long time, if not the fastest projectiles launched by man, at least the fastest visible ones. Given this, it ought not to be too surprising that basic and perhaps paradoxical principles of aerodynamics were first observed in their behavior. Early ones were made by stuffing a nearly incredible quantity of feathers into a sewn leather pocket (this labor-intensive task making golf a game for only the wealthy), and they were somewhat rough on the outside. In the nineteenth century smoother balls were made of gutta-percha, but it was soon observed that, although superior in many ways, they didn't travel as far as the earlier ones. Thus began a long development of golf balls with artificially created rough surfaces, including "brambles" (covered with bumps and resembling blackberries), "meshes" (with square indentations marked out in a kind of grid), hammered curves, and a primitive kind of dimple. It was only with the invention of piloted aircraft that aerodynamics developed systematically, and one by-product was that the effect of rough surfaces on the trajectories of golf balls was experimentally documented and at least partially understood.

It is a very complex phenomenon. The basic idea is that the dimples cause turbulence in the boundary layer of air flow around the golf ball, and this in turn retards the separation of the boundary layer from the surface. The drag on the ball is therefore decreased. This can be easily demonstrated in wind tunnel experiments. But why does the turbulence prevent separation? Although many books on fluid mechanics discuss drag on bluff objects (for example, the classic text *Essentials of Fluid Dynamics* by Ludwig Prandtl), the clearest nontechnical discussion we have found is that in the book *Shape and Flow* by the late Ascher Shapiro (especially on pp. 161–166). In brief: in a nonviscous atmosphere there would be no drag—the pressure at the back of a sphere would equal that at the front, and there would be no other cause of drag. But viscosity dissipates energy in the boundary layer flow, and a laminar boundary layer escapes as it approaches the high-pressure region at the back. For a dimpled ball, flow in the boundary layer is turbulent. This brings about mixing in the boundary layer, which means in particular that flow in the boundary layer is sped up by contact with the exterior flow. This in turn gives the boundary layer enough energy to
(continued on page 525)