2010 Veblen Prize

TOBIAS H. COLDING and WILLIAM P. MINICOZZI and PAUL SEIDEL received the 2010 Oswald Veblen Prize in Geometry at the 116th Annual Meeting of the AMS in San Francisco in January 2010.

Citation

The 2010 Veblen Prize in Geometry is awarded to Tobias H. Colding and William P. Minicozzi II for their profound work on minimal surfaces. In a series of papers, they have developed a structure theory for minimal surfaces with bounded genus in 3-manifolds, which yields a remarkable global picture for an arbitrary minimal surface of bounded genus. This contribution led to the resolution of long-standing conjectures and initiated a wave of new results. Specifically, they are cited for the following joint papers, of which the first four form a series establishing the structure theory for embedded surfaces in 3-manifolds:

"The space of embedded minimal surfaces of fixed genus in a 3-manifold. I. Estimates off the axis for disks", *Ann. of Math.* (2) **160** (2004), no. 1, 27–68.

"The space of embedded minimal surfaces of fixed genus in a 3-manifold. II. Multi- valued graphs in disks", *Ann. of Math.* (2) **160** (2004), no. 1, 69–92.

"The space of embedded minimal surfaces of fixed genus in a 3-manifold. III. Planar domains", *Ann. of Math.* (2) **160** (2004), no. 2, 523–572.

"The space of embedded minimal surfaces of fixed genus in a 3-manifold. IV. Locally simply connected", *Ann. of Math.* (2) **160** (2004), no. 2, 573–615.

"The Calabi-Yau conjectures for embedded surfaces", *Ann. of Math.* (2) **167** (2008), no. 1, 211–243.

In the final paper cited here, the authors show that a complete embedded minimal surface of finite genus is properly embedded, proving the embedded version of the Calabi-Yau conjectures.

Biographical Sketch

Tobias Holck Colding was born in Copenhagen, Denmark, and got his Ph.D. in 1992 at the University of Pennsylvania under Chris Croke. He was on the faculty at the Courant Institute of New York University in various positions from 1992 to 2008 and since 2005 has been a professor at MIT. He was also a visiting professor at MIT from 2000 to 2001 and at Princeton University from 2001 to 2002 and a postdoctoral fellow at MSRI (1993–94). He is the recipient of a Sloan fellowship and has given a 45-minute invited address to the ICM in 1998 in Berlin. He gave an AMS invited address in Philadelphia in 1998 and the 2000 John H. Barrett Memorial Lectures at University of Tennessee. He also gave an invited address at the first AMS-Scandinavian International Meeting in Odense, Denmark, in 2000, and an invited address at the Germany Mathematics Meeting in 2003 in Rostock. He gave the 2008 Mordell Lecture at the University of Cambridge and will give the 2010 Cantrell Lectures at the University of Georgia. Since 2008 he has been a Fellow of the American Academy of Arts and Sciences, since 2006 a foreign member of the Royal Danish Academy of Sciences and Letters, and also since 2006 an honorary professor of the University of Copenhagen.

William P. Minicozzi II was born in Bryn Mawr, Pennsylvania, in 1967. He graduated from Princeton University in 1990 and received his Ph.D. from Stanford University in 1994 under the direction of Richard M. Schoen. After graduating, he spent a year at the Courant Institute of New York University as a visiting member. In 1995 he went to the Johns Hopkins University, where he was the J. J. Sylvester Professor of Mathematics from 2002 until 2007 and is currently a Krieger-Eisenhower Professor in the School of Arts and Sciences.

Minicozzi received a National Science Foundation postdoctoral fellowship in 1995 and an Alfred

Tobias Holck Colding

William P. Minicozzi

Paul Seidel

P. Sloan Foundation Research Fellowship in 1998. He gave an invited address at the 2006 International Congress of Mathematicians in Madrid and a London Mathematical Society Spitalfields Lecture in 2007, and he will give the thirty-fifth University of Arkansas Spring Lecture Series in 2010 and an AMS invited address in Syracuse in 2010. He currently lives in Maryland with his wife, Colleen, and three children, Tim, Nina, and Jason.

Response

From Tobias Holck Colding: I am greatly honored to be named along with Bill and Paul as a recipient of the 2010 Veblen Prize. I am particularly indebted to Bill, who has been an absolute delight to work with on a number of different topics. I would also like to take the opportunity to thank my other collaborators and, in particular, Jeff Cheeger and the Courant Institute and my many friends there where much of the work mentioned here was done.

From William P. Minicozzi: I am greatly honored to be named, along with Toby Colding and Paul Seidel, as a recipient of the Oswald Veblen Prize. Working with Toby has been a great experience, beginning in 1994 at Courant with our work on harmonic functions, continuing on to the work on minimal surfaces cited here, our work on finite time extinction in Ricci flow, and a number of other projects. It is very satisfying to see this work recognized.

I have been lucky to have the support of family and colleagues over the years. I would especially like to thank my wife, Colleen, for her love and support. I am deeply indebted to Toby for his vision for what's important, his contagious enthusiasm for geometry, and his boundless optimism. Finally, I would like to thank my thesis advisor, Rick Schoen, and my Johns Hopkins colleagues, Bernie Shiffman, Chris Sogge, Joel Spruck, and Steve Zelditch, for their support early in my career.

Citation

The 2010 Veblen Prize in Geometry is awarded to Paul Seidel of MIT for his fundamental contributions to symplectic geometry and, in particular, for his development of advanced algebraic methods for computation of symplectic invariants.

Seidel's work also greatly influenced developments in nearby subjects, such as gauge theory and low-dimensional topology. Specifically, the Veblen Prize is awarded for the following work of Seidel:

The paper "A long exact sequence for symplectic Floer cohomology", *Topology* **42** (2003), no. 5, 1003–1063, in which Seidel studied the effect of a symplectic Dehn twist (which he himself had previously defined) on Floer homology.

The book *Fukaya Categories and Picard-Lefschetz Theory*, European Math. Soc. (EMS), Zurich, 2008. In this research monograph Seidel developed new explicit tools for computing the Fukaya category of a symplectic manifold in terms of Picard-Lefschetz theory. These techniques allowed him (in a separate paper) to verify Kontsevich's homological mirror symmetry conjecture in the case of a $K3$ surface, thus providing the first really substantial evidence for this important conjecture.

The paper (joint with I. Smith) "The symplectic topology of Ramanujan's surface", *Comment. Math. Helv.* **80** (2005), no. 4, 859–881. In this paper, the authors give the first examples of exotic symplectic structures on Euclidean space which are convex at infinity.

The paper (joint with K. Fukaya and I. Smith) "Exact Lagrangian submanifolds in simply-connected cotangent bundles", *Invent. Math.* **172** (2008), no. 1, 1–27. In this paper the authors proved a homological version of Arnold's conjecture about the topology of exact Lagrangian submanifolds of a cotangent bundle. (Similar results have been obtained independently by Nadler.)

Biographical Sketch

Paul Seidel was born in Florence, Italy, in 1970. He did his undergraduate studies at the University of Heidelberg with Albrecht Dold and his graduate studies at Oxford University with John Roe and Simon Donaldson. He has held visiting positions at the Institute for Advanced Study, the Max Planck Institut in Bonn, and ETH Zurich. For three years he was *chargé de recherche* at CNRS, affiliated with École Polytechnique. He held faculty positions at Imperial College London and at the University of Chicago. His current position is professor of mathematics at MIT. He has received a European Mathematical Society Prize (2000). In 2002 he was selected as a speaker for the International Congress of Mathematicians. He is married to

another mathematician, Ju-Lee Kim, and they have one daughter (Ilaria).

Response

It's an honor to be selected as one of the recipients of the Veblen Prize. I'd like to interpret this more broadly as a sign of appreciation for the part of mathematics that I've been working in, which is the study of symplectic topology using cohomological methods. This approach is possible thanks to breakthroughs made in the 1980s and early 1990s. I have no firsthand experience, but I think the pioneers who made those breakthroughs must have had a very hard time of it. We remain indebted to them for creating such a wonderful intellectual space for us to work and play in. I won't try to list them since the boundaries of the area are somewhat fuzzy and perceived influences can be very subjective. Personally, I learned many ideas from listening to Donaldson and Kontsevich, from collaborators (Khovanov, Thomas, Smith, Fukaya, Abouzaid, and Maydanskiy), or simply from people who walked into my office some day. By the way, if you've ever tried to explain an interesting piece of mathematics to me and found that I wasn't receptive to it, I hereby apologize to you! I know I have unfortunately missed some real opportunities in that way.

Looking ahead, I think the field is in good shape, as the tools provided to us by pseudoholomorphic curve theory are being systematically explored. Here are some current developments which I find encouraging (without necessarily being directly involved in them). First, the long-standing idea of using handle decompositions to understand the symplectic topology of Stein manifolds is becoming increasingly effective. In another direction, for closed symplectic manifolds which contain large families of pseudo-holomorphic spheres such as toric varieties, we are gradually getting a picture of how their symplectic geometry decomposes into simpler pieces. Next, there is a historically close connection with low-dimensional topology, which continues to pay off. Finally, our techniques have matured to the point where they can fruitfully interact with more distant areas. Through such interactions (and guided by ideas from mirror symmetry, in many cases), concepts from algebraic geometry and representation theory have become applicable in new ways. Still, right now there remains a hard nucleus of questions about symplectic structures on closed higher-dimensional manifolds, which has only barely been touched. I look forward to the moment when further development of the existing methods, or a completely new idea, will allow us to penetrate more deeply.

About the Prize

The Veblen Prize is awarded every three years for a notable research memoir in geometry or topology that has appeared during the previous five years in a recognized North American journal (until 2001 the prize was usually awarded every five years). Established in 1964, the prize honors the memory of Oswald Veblen (1880–1960), who served as president of the AMS during 1923–1924. It carries a cash award of US$5,000.

The Veblen Prize is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2010 prize, the members of the selection committee were: Yakov Eliashberg, Bruce A. Kleiner, and Peter S. Ozsvath.

Previous recipients of the Veblen Prize are: Christos D. Papakyriakopoulos (1964); Raoul H. Bott (1964); Stephen Smale (1966); Morton Brown and Barry Mazur (1966); Robion C. Kirby (1971); Dennis P. Sullivan (1971); William P. Thurston (1976); James Simons (1976); Mikhael Gromov (1981); Shing-Tung Yau (1981); Michael H. Freedman (1986); Andrew Casson (1991); Clifford H. Taubes (1991); Richard Hamilton (1996); Gang Tian (1996); Jeff Cheeger (2001); Yakov Eliashberg (2001); Michael J. Hopkins (2001); David Gabai (2004); Peter Kronheimer, Tomasz Mrowka, Peter Ozsváth, Zoltán Szabó (2007).