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Volume 57, Number 6

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-Mathematical Reviews
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HARDCOVER (CORNERSTONES)
2010. 2ND ED. XX, 390 P. 18 ILLUS. \$69.95

ISBN 978-0-8176-4551-9.

Functions, Spaces, and Expansions

Mathematical Tools in Physics and Engineering
O. Christensen, Technical University of Denmark, Lyngby, Denmark
This graduate-level text is an exposition of mathematical tools in analysis for students, researchers, and practitioners across science and engineering. Every topic covered has been specifically chosen because it plays a key role outside the field of pure mathematics. Although the treatment of each topic is mathematical in nature, and concrete applications are not delineated, the principles presented are fundamental to exploring the computational aspects of physics and engineering. Readers should have a solid understanding of linear algebra, in Rn and in general vector spaces. Familiarity with the concepts of calculus and real analysis, including Riemann integrals and infinite series of real or complex numbers, is required.
HARDCOVER (APPLIED AND NUMERICAL HARMONIC ANALYSIS)
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A Problem-Based Approach
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This book is an introduction to Diophantine equations. The book introduces readers to the elementary methods necessary in solving Diophantine equations, presents classical Diophantine equations, and contains exercises with complete solutions. The book is intended for undergraduates, high school students and their teachers, mathematics contests participants and any person interested in essential mathematics. SOFTCOVER
2010. APPROX. 360 P. 20 ILLUS. APPROX. \$49.95 ISBN 978-0-8176-4548-9

## Homological Algebra of Semimodules and Semicontramodules

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This monograph examines semi-infinite homological algebra, concentrated mostly on the semi-infinite theory of associative algebraic structures, but also includes some material on the semi-infinite homology and cohomology of Lie algebras and topological groups. The main objects of study are the double-sided derived functors SemiExt and SemiTor, and the phenomenon of comodule-contramodule correspondence, connecting them with the more conventional, one-sided Ext and CtrTor. Contramodules also play a prominent role in this book.
HARDCOVER (MONOGRAFIE MATEMATYCZNE, VOLUME 70) 2010. 300 P. APPROX. \$89.95 ISBN 978-3-0346-0435-2

## Spinors in FourDimensional Spaces

G. F. Torres del Castillo, Universidad Autónoma de Puebla, México

Without using the customary Clifford algebras frequently studied in connection with the representations of orthogonal groups, this book gives an elementary introduction to the two-component spinor formalism for four-dimensional spaces with any signature. Some of the useful applications of four-dimensional spinors, are derived in detail using illustrative examples. The work is aimed at graduate students and researchers in mathematical and theoretical physics interested in the applications of the two-component spinor formalism in any four-dimensional vector space or Riemannian manifold with a definite or indefinite metric tensor.
HARDCOVER (PROGRESS IN MATHEMATICAL PHYSICS, VOLUME 59) 2010. VIII, 176 P. $\$ 99.00$
ISBN 978-0-8176-4983-8

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June/July 2010

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In the past thirty years or so, combinatorics and finite mathematics have become ever more prominent players in our discipline. Collaboration graphs, Google search algorithms, queueing theory, random graphs, Internet routing, and coloring problems are just some of the hot topics of which we are all aware. There are many more, and these are all showcased in the articles of the present issue of the Notices. M. C. Escher's art is well known to be mathematically inspired, and surely some of that inspiration is combinatorial. We believe that this material combines to make an inspiring and informative issue.
-Steven G. Krantz
Editor

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## Notices

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SUBSCRIPTION INFORMATION: Subscription prices for Volume 57 (2010) are US\$488 list; US\$390 institutional member; US\$293 individual member. (The subscription price for members is included in the annual dues.) A late charge of $10 \%$ of the subscription price will be imposed upon orders received from nonmembers after January 1 of the subscription year. Add for postage: Surface delivery outside the United States and India-US\$27; in India-US\$40; expedited delivery to destinations in North America-US\$35; elsewhere-US $\$ 120$. Subscriptions and orders for AMS publications should be addressed to the American Mathematical Society, P.O. Box 845904 , Boston, MA 02284-5904 USA. All orders must be prepaid.
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I thank Randi D. Ruden for her splendid editorial work, and for helping to assemble this issue. She is essential to everything that I do.
-Steven G. Krantz Editor
mathoverflow

The Internet is changing the way mathematics is done. Online journals, MathSciNet, and the arXiv have existed for some time. More recently a number of blogs have become home to rather sophisticated mathematical discussion [1]. There are also mathematical wikis combining exposition and research (see ncatlab.org/n1ab) and several massively collaborative online "Polymath" projects (see po7ymathprojects.org).

On the new site MathOverflow, users ask and answer focused mathematical questions. They also vote on questions and answers, making it easy to sort out the good stuff. The majority of active participants are faculty and graduate students, including many leading researchers. On a typical day, the site receives about thirty new questions and over 30,000 page views from around 2,500 different users. It's not just the new material each day that matters but the evolving searchable archive of mathematical knowledge.

MathOverflow is a natural tool for mathematicians, since doing mathematics is itself largely driven by asking questions. We often formulate a problem as a series of small focused questions and then try to answer them. Sometimes the answers are surprising and lead to more questions, and off you go on an adventure. Other times you get stuck, in which case you might try looking online. You're likely to find relevant material on MathOverflow, since it is heavily indexed by search engines. Many mathematicians first visit the site by following a search query, so the system is designed to make it easy to jump right in. You can start posting questions and answers without registering. Mathematical symbols, entered as LATEX between dollar signs, are automatically rendered.

The site has a personal feel to it. Helping someone with his or her problem is satisfying, and it's pleasant to find that somebody has taken the time to help with yours. It's like a global math department tea. You interact with excellent mathematicians of all ages, and you get to do some really fun mathematics. We know of cases where someone asked a question that arose in their research and got a satisfying answer within an hour, or even minutes [2]! The site launched in October of 2009, and questions have already led to research papers with the asker and answerer as coauthors.

When people first visit MathOverflow, they are often surprised by the sophistication of the mathematics. The software allows the community to moderate itself, keeping the site from becoming anarchic. For example, established members of the community have increased power to make decisions. This ability is measured by a point system, somewhat unfortunately called "reputation". Your reputation increases when others vote up
your posts. As you contribute more, you gain the ability to organize and improve the site. At first you can only ask and answer questions, but soon you can vote and create new tags. Eventually you can edit other people's posts and vote to close bad questions. Thus, moderation duties are efficiently shared among the most active users.

A "bad question" is one which is either irrelevant to mathematicians or one where it is unclear what constitutes an answer. Blogs often host discussions, and wikis compile expository material; MathOverflow fills a different niche. It is optimized for focused questions which (could) have clear answers. These optimizations make it awkward to use for broad or open-ended questions. You should use MathOverflow only when it is the right tool for the job. The best questions for the site include motivation and background and have a clear goal. Discussions, homework questions, and broad requests are politely directed to more suitable websites.

MathOverflow's underlying software is Stack Exchange, the engine behind the wildly successful programming site Stack Overflow. All contributed content is under the same Creative Commons license used by Wikipedia. Copies of the database are regularly made available for analysis, reproduction, and future proofing.

It is hard to properly explain MathOverflow without showing it to you, so please visithttp://mathoverf1ow.net and poke around! If you have a question someone probably knows how to answer, try asking it. If you find the main page overwhelming, go to http:// mathoverflow.net/tags and click on a tag corresponding to your specialty. Currently some areas of math are disproportionately represented but, just as with the arXiv, it is gradually filling out as it matures. We're still not sure what the future holds for MathOverflow; this will depend on how the mathematical community decides to use it. We'd love to have mathematicians from more areas actively involved. Already you're sure to find something you're interested in.

## References

[1] JOHN BAEZ, Math blogs, Notices of the AMS, March 2010.
[2] "Shtetl-optimized: Prove my lemma, get acknowledged in a paper!", http://scottaaronson.com/b1og/?p=432.
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## Epilogue on Legendre Portraits

After publication of my article "Changing faces: The mistaken portrait of Legendre" [Notices, December 2009, pp. 1440-1443], many websites have replaced the wrong portrait (that of Louis Legendre) with the newly discovered caricature portrait of Adrien-Marie Legendre. Details of recent developments can be found on Gérard Michon's website at
http://www.numericana.com/ answer/record.htm

Meanwhile, several readers have told me that the traditional portrait of A.-M. Legendre had long been suspected to be wrong. In particular, Bille Carlson reports that in 1971-72 he found a letter by M. Costamagna, dated 18 November 1959, in the Legendre file at the Archives of the Académie des Sciences in Paris, pointing out that the signature below the portrait did not match that of the mathematician and concluding that it was probably the portrait "d'un Legendre conventionnel, ancien boucher, régicide,..."; in other words, Louis Legendre. Finally, two readers have written to correct my translation of the letter from Legendre to Jacobi. When Legendre wrote "sans quoi vous auriez eu de mes nouvelles", he was simply saying "otherwise you would have heard from me" and was not suggesting that he had particular news to convey.
-Peter Duren
University of Michigan duren@umich.edu
(Received March 1, 2010)

## Honesty in Mathematical Writing

I would like to discuss an experience I had recently that exposes some problems in mathematics journal publishing.

I discovered an error in an AMS Memoir. The proof of a main result overlooks certain cases, whose inclusion requires that the proof
be extended and modified. One of the authors simply denied to the managing editor the claim I referred to (this claim, though not explicitly stated in the paper, enters into the proof). The managing editor, after consulting the authors, declined to consider my correction (and in particular to have it examined by an independent party), quoting formal reasons and my differences with the authors. If I wanted to have my correction not only published but also refereed, I was supposed to send it elsewhere. Asking in what form the Memoirs could publish a correction, I received no reply. After nearly three years I am not aware of any action taken by the editor or the authors to prepare or publish a correction, nor even whether my explanation was carefully read and understood. Both the result and the method have been used in subsequent publications.

Quoting standards and backlog and pointing to other journals are an editor's favorite methods for disposing of submissions. These methods are easy, because they do not require any careful argument, and very effective, because they prevent anyone from attempting dispute. But if one asks what scientific purpose these methods serve, particularly in the case of corrections, I cannot identify one. Corrections do not spoil the mathematics in a journal, nor do they occupy much of its space. And if every journal wants to have its errors treated somewhere else, I see nothing but disorder coming out of the process.

My case seems not to be isolated (see T. P. Hill's letter in the January 2010 issue of the Notices), and I see a general trend promoted in which authors care primarily about their CVs and journals about their impact factors. Who should then care about correctness? Nowadays, I see it as hardly practicable/realistic that everyone verify the proof of every result he likes or uses. When a referee is not thorough, some authors and journals apparently not only prefer to conveniently pile all errors onto the reader to handle but also view the
reader's concerns as acts of public disruption. So are proofs to be judged by popularity, and fixed through gossip?

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}
(Received March 19, 2010)


## Department Chairs Should Audit Business Courses

It seems to me that any professor who is a serious candidate for department head should already have knowledge of estimating, developing, and executing (tight) budgets for the benefit of departmental students, staff, and faculty. As departmental budgets continue to be cut, especially at state universities, I believe it would be beneficial to all concerned for department heads to be knowledgeable on the budget process. If this seems to violate "historical traditions" too much, then appoint an "associate department head for XYZ" (where $\mathrm{X}=$ budgets), who reports directly to the department head. If possible, your business schools should offer a special course in financial/business management for potential department heads/associate department heads for XYZ to audit. This approach has a hidden bonus, viz., it could easily lead to the invention of novel mathematics as a formal contribution to the mathematics of finance/ business.

> -Albert A. Mullin
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(Received March 30, 2010)

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[^0]
# The Mathematical Side of M. C. Escher 

Doris Schattschneider

While the mathematical side of Dutch graphic artist M. C. Escher (18981972) is often acknowledged, few of his admirers are aware of the mathematical depth of his work. Probably not since the Renaissance has an artist engaged in mathematics to the extent that Escher did, with the sole purpose of understanding mathematical ideas in order to employ them in his art. Escher consulted mathematical publications and interacted with mathematicians. He used mathematics (especially geometry) in creating many of his drawings and prints. Several of his prints celebrate mathematical forms. Many prints provide visual metaphors for abstract mathematical concepts; in particular, Escher was obsessed with the depiction of infinity. His work has sparked investigations by scientists and mathematicians. But most surprising of all, for several years Escher carried out his own mathematical research, some of which anticipated later discoveries by mathematicians.

And yet with all this, Escher steadfastly denied any ability to understand or do mathematics. His son George explains:

Father had difficulty comprehending that the working of his mind was akin to that of a mathematician. He greatly enjoyed the interest in his work by mathematicians and scientists, who readily understood him as he spoke, in his pictures, a common language. Unfortunately, the specialized language of mathematics hid from him the fact that mathematicians were struggling with the same concepts as he was.

[^1]Scientists, mathematicians and M. C. Escher approach some of their work in similar fashion. They select by intuition and experience a likely-looking set of rules which defines permissible events inside an abstract world. Then they proceed to explore in detail the consequences of applying these rules. If well chosen, the rules lead to exciting discoveries, theoretical developments and much rewarding work. [18, p. 4]

In Escher's mind, mathematics was what he encountered in schoolwork-symbols, formulas, and textbook problems to solve using prescribed techniques. It didn't occur to him that formulating his own questions and trying to answer them in his own way was doing mathematics.

## Until 1937

M. C. Escher grew up in Arnhem, Holland, the youngest in a family of five boys. His father was a civil engineer and his four older brothers all became scientists. The home atmosphere may have instilled in him some habits of scientific inquiry, including the patient, methodical approach that would characterize his later work. Also, the young boys were given regular lessons in woodworking techniques that would later become very useful to Escher in making woodcuts.

His school life may have been less useful than his home life. Recalling his school years, Escher once confessed "I was an extremely poor pupil in arithmetic and algebra, and I still have great difficulty with the abstractions of figures and letters. I was slightly better at solid geometry because it appealed to my imagination, but even in that subject I never excelled at school" [1, p. 15]. He did well in drawing, however, and his high school art teacher encouraged him to make linocuts.

In 1919 Escher entered the Haarlem School for Architecture and Decorative Arts intending to study architecture, but with the advice of his drawing and graphic arts teacher, Samuel Jessurun de Mesquita, and the consent of his parents, soon switched to a program in graphic arts. Among his prints executed while in Haarlem are three that show plane-filling; two of these are based on filling rhombuses, and one has a rectangle filled with eight different elegant heads, four upside down, each repeated four times [53, pp. 7-8]. Plane-filling would soon become an obsession.

Upon finishing his studies at the Haarlem School in 1922, he traveled for most of a year throughout Italy and Spain, filling a portfolio with sketches of landscapes and details of buildings, as well as meticulous drawings of plants and tiny creatures in nature. During this odyssey, he visited the Alhambra in Granada, Spain, and there marveled at the wealth of decoration in majolica tiles, sketching a section that especially attracted him "for its great complexity and geometric artistry" [1, pp. 24, 41]. This first encounter with the tilings in the Alhambra likely increased his interest in making his own tilings. In any case, during the mid-1920s, he produced a few periodic "mosaics" with a single shape, some of them hand-printed on silk [53, p. 11]. Unlike the Moorish tiles that always had geometric shapes, Escher's tile shapes (which he called "motifs") had to be recognizable (in outline) as creatures, even if only of the imagination. These early attempts show that he understood (intuitively, at least) how to utilize basic congruence-preserving transformationstranslations, half-turns ( $180^{\circ}$ rotations), reflections and glide-reflections-to produce his tilings.

Escher married in 1924, and the couple settled in Rome, where two sons were born. Until 1935 he continued to make frequent sketching trips, most in southern Italy, returning to his Rome studio to compose his sketches for woodcuts and lithographs. In 1935, with the growing rise of Fascism in Italy and his sons in ill health, Escher felt it best to move his family from Italy to Switzerland. In 1936 he undertook a long sea journey, and during the trip he spent three days at the Alhambra, joined by his wife Jetta. There they made careful color sketches of many of the majolica tilings. This second Alhambra visit, coupled with his move from the scenery of Italy, marked an enormous change in his work: landscapes would be replaced by "mindscapes". ${ }^{1}$ No longer would his sketches and prints be inspired by what he found in mountainous villages, nature, and architecture. Now the ideas would be found only in the recesses of his mind.

[^2]Escher later wrote that after this Alhambra visit, "I spent a large part of my time puzzling with animal shapes" [1, p. 55]. By carefully studying the Alhambra sketches and noting the geometric relationships of the tiles to one another, he was able to make a dozen new symmetry drawings of interlocked motifs. ${ }^{2}$ One of these showed interlocked Chinese boys. In spring 1937 he produced his first print that used a portion of a plane-filling to produce a metamorphosis of figures. In Metamorphosis $I,{ }^{3}$ the buildings of the coastal town of Atrani morph into cubes which in turn evolve into the Chinese boys [53, pp. 19, 286]. The print was a fantasy, linking his new interest in plane-fillings with his love of the Amalfi coast, but Escher never liked it because it didn't tell a story-how do you link Chinese boys to an Italian town?

In July 1937 the Escher family moved to a suburb of Brussels, where a third son was born. That October Escher showed his meager portfolio of symmetry drawings to his older half-brother Beer, a professor of geology, who immediately recognized that these periodic patterns would be of interest to crystallographers, since crystals were defined by their periodic molecular structure. He offered to send Escher a list of technical papers that might be helpful. There were ten articles in Beer's list, all from Zeitschrift für Kristallographie, published between 1911 and 1933, by F. Haag, G. Pólya, P. Niggli, F. Laves, and H. Heesch [53, pp. 24, 337]. Escher found only the articles by Haag and Pólya useful.

Haag's article [28] provided a clear definition for Escher of "regular" plane-fillings and also provided some illustrations. In one of his copybooks, Escher carefully wrote Haag's definition of "regular division of the plane" (here translated):

> Regular divisions of the plane consist of congruent convex polygons joined together; the arrangement by which the polygons adjoin each other is the same throughout.

In the same copybook, Escher also sketched several of Haag's polygon tilings. After studying them, he quickly discovered that the word "convex" in Haag's definition was superfluous, for by manipulating the tile's shape, he was able to sketch several examples of nonconvex polygonal tilings. It was probably at this point that he inserted parentheses around the word "convex" in Haag's definition. Of course he also readily discovered that the word "polygon" was far too restrictive for his purposes; it could easily

[^3]

Figure 1. A copy of the display in [43], signed by Pólya.
be replaced by "tile" or "shape". Haag's definition (with Escher's amendments) was adopted by Escher and would guide all of his symmetry investigations. He later carefully recorded the definition on the back of his symmetry drawing 25 (1939) of lizards (the drawing is depicted in Escher's lithograph Reptiles).

Pólya's article [43] would have a great influence on Escher. Escher carefully copied (by hand, in ink) the full text that outlined the four isometries of the plane and announced Pólya's classification of periodic planar tilings by their symmetry groups. Pólya was evidently unaware that this classification had been carried out by Fedorov more than thirty years earlier. Escher was already intuitively aware of the congruence-preserving transformations Pólya spoke of but probably didn't understand any of the discussion about symmetry groups. What struck him was Pólya's full-page chart that displayed an illustrative tiling for each of the seventeen plane symmetry groups (Figure 1). Escher carefully sketched each of these seventeen tilings in a copybook and studied them, map-coloring some of them [47]. Among these, there were tilings that displayed symmetries he had not recorded in the Alhambra; for example, tilings whose only symmetries other than translations were glide-reflections or fourfold $\left(90^{\circ}\right)$ and twofold $\left(180^{\circ}\right)$ rotations. Within one month of studying these, Escher had completed his first symmetry drawings displaying fourfold rotation symmetry: squirming lizards interlocked four at a time, pinwheeling where four feet met [53, p. 127]. He featured a portion of one of these drawings at the center of his woodcut Development I, completed in the same month.

Escher was so grateful for the help that Pólya's paper provided that he wrote to the mathematician to thank him. He sent Pólya the print Development I and asked the mathematician whether or not he had written a book on symmetry for "laymen" as his article indicated he had hoped to do. Although a writer once characterized Pólya's reply as polite but formal, indicating he hadn't written the hoped-for book [53, p. 22], Pólya wrote to me in 1977 that he and Escher had corresponded more than once and that he regretted losing the correspondence in his haste to come to America in 1940. A recent
discovery of a forgotten suitcase full of Pólya's notes and other collected letters and papers, now in the Pólya archives at Stanford University, shows that Pólya even sent Escher his own attempt at an Escher-like tiling. Among these papers is Pólya's drawing of a tiling by snakes, inscribed "sent to MCE", at the address where Escher resided from 1937 to 1940. Also, there is an outline of Pólya's never-completed book The Symmetry of Ornament and many sketches of tilings, both for the planned book and for the 1924 article that so influenced Escher [53, pp. 335-36].

## Escher as a Mathematical Researcher

From 1937 to 1941 Escher plunged into a methodical investigation that can only be termed mathematical research. Haag's article had given him a definition of "regular division of the plane", and Pólya's article showed him that there were many tile shapes that could produce these. He wanted to find more and characterize them. The questions he pursued, using his own techniques, were:
(1) What are the possible shapes for a tile that can produce a regular division of the plane, that is, a tile that can fill the plane with its congruent images such that every tile is surrounded in the same manner?
(2) Moreover, in what ways are the edges of such a tile related to each other by isometries?
The only isometries that Escher allowed in order to map a tile to an adjacent tile were translations, rotations, and glide-reflections-a reflection would require a tile's edge to be a straight segment, not a natural condition for his creature tiles. In 1941-1942 he recorded his many findings in a definitive Notebook that was to be his private encyclopedia about regular divisions of the plane and how to produce and color them [46], [48], [53]. The Notebook had two parts: its cover inscribed (here translated) "Regular divisions of the plane into asymmetric congruent polygons; I Quadrilateral systems MCE 1-1941 Ukkel; II Triangle systems X-1942, Baarn".

Escher's study of "quadrilateral systems" was extensive. He represented these tilings symbolically with a grid of congruent parallelograms in which each parallelogram represented a single tile. He shaded the grids checkerboard style, so that each parallelogram shared edges only with parallelograms of the opposite color. He was interested in asymmetric tiles (after all, his creature tiles were to be primarily asymmetric), and in order to indicate the asymmetry, placed a hook inside each parallelogram. The hook provided orientation, while small circles and squares on the tile's boundary indicated twofold and fourfold centers about which the tile could rotate into an adjacent tile. Escher was aware that certain symmetries required special parallelogram grids and so considered five different categories: arbitrary parallelogram,


Figure 2. A copybook page showing Escher's method of investigation of regular divisions of the plane. His symbolic notation is explained in our text.
rhombus, rectangle, square, and isosceles right triangle (a grid of squares in which the diagonals have been drawn). He labeled these A-E, respectively. As he sought to answer his two questions, he filled the pages of several school copybooks with his sketches of marked grids representing tilings, scratching out those that didn't work out or that duplicated an earlier discovery. Each time he discovered a marked grid that represented a regular division of the plane, he recorded it and made an example of a tiling with a "shaped tile", its vertices marked by letters.

To quickly record how each edge of a tile was related to another edge of the same tile or an adjacent tile, Escher devised his own notation: = meant "related by a translation" and || "related by a glide-reflection". An S on its side meant "related by a $180^{\circ}$ rotation" and L meant "related by a $90^{\circ}$ rotation". Figure 2 shows one copybook page with five different "rhombus systems" on the left and shaped tilings for two of these systems on the right. Note Escher's "voorbeeld maken!" at the bottom of the page-"make an example!" His results were recorded entirely visually, with no need for words. Ultimately he found ten different classes of these tilings and numbered the classes I - X. His Notebook charts giving both visual and descriptive versions of the classes are in [53, pp. 58-61].

To discover still other regular divisions, those for which three colors would be required for mapcoloring, Escher employed a technique that he called "transition". Figure 3 recreates one of his
examples. He would begin with a two-color regular division from one of his ten categories (Figure 3 begins with type $\mathrm{II}^{\text {A }}$ ). Each of these categories had four tiles meeting at every vertex and required only two colors. He would then choose a tile and a segment of the boundary that connected one of its vertices (say B) to another carefully chosen boundary point (say A) that was not a vertex of the tiling (Figure 3a). Using A as a pivot point, he would then pivot the boundary segment connecting A and B (stretching it if necessary) so that vertex B slid along the boundary of the tile, stopping at a new position (say C). Repeating this on the corresponding segments of the boundaries of all tiles produced a new tiling with vertices at which three tiles met, requiring three colors for map-coloring (Figure 3b). The process could be continued with the new segment AC , sliding C along the boundary until it reached a vertex $D$ of the original tiling. This produced a new tiling that again required only two colors (Figure 3c). At the intermediate (3-color) stage, the network of tile edges was certainly not homeomorphic to the original, but surprisingly, at the end (2-color) stage, the new network of tile edges might also not be homeomorphic to that of the original tiling. Escher thought of the intermediate (3-color) tiling as having components of both the beginning and ending 2 -color tilings, and so labeled it with both types. In Figure 3, his type $\mathrm{II}^{\mathrm{A}}$ system is transformed to $\mathrm{II}^{\mathrm{A}}-\mathrm{III}^{\mathrm{A}}$, and that is transformed to system $\mathrm{III}^{\mathrm{A}}$. In this instance, the tiles in the final tiling have three, not four, edges


Figure 3. Escher's transition process takes (a) a 2 -colored tiling with $180^{\circ}$ rotation centers at tile vertices and midpoints of edges, to (b) a 3-colored tiling, to (c) a 2 -colored tiling with $180^{\circ}$ rotation centers at midpoints of tile edges.
and meet six at a vertex; Escher noted that this was an exceptional case [53, p. 62].

Escher did not record these discoveries with words, but in his Notebook he displayed sixteen pages of carefully drawn illustrations of transitions that cover all of his ten categories [53, pp. 62-69]. In many cases, he discovered more than one distinct transition of the same tiling. Using today's terminology, he discovered how to produce tilings of different isohedral types beginning with a single isohedral tiling. And he also recorded in a chart (a digraph!) which of his ten categories led to others. This chart makes clear that his process of transition can change the topological and combinatorial properties of a tiling but not change its symmetry group [48], [53, p. 60].

The last section of Escher's "quadrilateral systems" study summarizes in ten pages his investigations of what he called "2-motif" tilings [53, pp. 70-76]. He would begin with a regular division, map-colored with two or three colors, then split each tile (in the same manner) into two distinct shapes, so that the resulting tiling could be colored with two colors. This investigation was spurred by his fascination with what he called "duality" - many of his prints play with the idea of interchanging the role of figure and ground, or juxtaposition of opposites. Sky and Water I and Circle Limit IV (Angels and Devils) are famous examples.

For example, in Sky and Water I (Figure 4) a horizontal row of interlocked flat tiles at the center alternates white fish and black birds, dividing the print into upper and lower halves (sky and water). The fish in this center row can serve as figure and the birds as ground, or vice versa. But as the eye moves upward from this row of tiles, the creatures separate and take on distinct roles. The black birds become three-dimensional as they rise, while the white fish melt to become sky. The fish become the background against which figures of birds fly.

As the eye moves downward from the center row of tiles, the opposite transformation takes place. Now the fish gain three-dimensional form and the black birds dissolve to become water in which the fish swim. In mathematics, the essence of dual objects is that each completely defines the other, such as a set and its complement, a statement and its negation. In addition to the figure/ground duality, there are other kinds of duality represented in this single print: black and white, sky and water. And opposites: bird and fish often denote opposites (think "neither fish nor fowl"), and in the print, each bird is placed exactly opposite a fish, with the invisible surface of the water acting as a compositional mirror.

Part II of Escher's Notebook is brief, devoted to what he called "triangle systems"-regular divisions having $120^{\circ}$ rotation centers (system A) or $60^{\circ}, 120^{\circ}$, and $180^{\circ}$ rotation centers (system B). After explaining the necessary placement of rotation centers, he records only twenty different tilings, several with two motifs, and all carefully map-colored to respect symmetry. Most require three colors. Unlike in his quadrilateral systems, some of his tiles have rotation symmetry, and from these he derives other tiles with one or two motifs [53, pp. 79-81].

In 1941, as he was nearing the end of these investigations, Escher and his family moved to Baarn, Holland, where he would spend all but the last two years of his life. In the years following, he produced more than 100 regular divisions of the plane, each final version numbered and carefully drawn on graph paper, its creature tiles outlined in ink, map-colored using watercolors, respecting the symmetries of the tiling. As his portfolio of symmetry drawings grew, he referred to it as his "storehouse". Fragments of these drawings would be featured in many prints, notecards, exhibit announcements, painted and tiled public works, and even carved on the surface of a ball. In all, there are 134 numbered symmetry drawings and many unnumbered sketches.

Escher carried out several other minor mathematical investigations in order to achieve certain effects in his art. Some of these results were recorded in a notebook entitled Regular Division of the Plane: Abstract Motifs, Geometric Problems, and others were gathered in small folios. He studied several Moorish-like tilings and investigated linked rings (seen in his last print, Snakes). He enumerated several tilings by congruent triangles while designing bank notes. He also recorded two theorems he evidently discovered but did not prove. One was about concurrent lines in a triangle, and the other about concurrent diagonals in a special tiling hexagon [53, pp. 82-93]. At my request, the first theorem was verified by A. Liu and M. Klamkin [35] and the second by J. F. Rigby [44].

An investigation in 1942 that was an amusement, shared with his children and grandchildren [18, pp. 9-11], [50], was combinatorial-to determine how many different patterns could be generated by following this algorithm:

> Decorate a square with an asymmetric motif and use four copies of the decorated square (independently chosen from any of four rotated aspects) to fill out a $2 \times 2$ larger square, then translate the larger square in the direction of its edges to fill the plane.

With a methodical search of the $4^{4}$ possible $2 \times 2$ filled squares, eliminating obvious duplications, and by sketching examples with a simple motif for the rest, he ultimately found twenty-three distinct patterns. That is, no two of these twenty-three patterns were identical, allowing rotations. He also asked the combinatorial question in two special cases in which reflected aspects of the decorated square were also allowed. In these cases, the choices of the four copies of the decorated square were restricted as follows:

Case (1)-two choices must be the same rotated aspect and independently, the other two choices the same reflected aspect. Case (2)-two choices must be different rotated aspects and independently, the other two choices different reflected aspects.

Escher's results were sketched in copybooks and later printed with inked carved wooden stamps using a motif that produced patterns resembling knitted or crocheted pieces. He also made a "ribbon" design, outlining crossing bands in a square, and carved four wooden stamps-one of the original design, one of its reflection, and two others that reversed the crossings in the first two stamps. He did not attempt to find the number of patterns produced by the $4^{16}$ possible $2 \times 2$ squares filled with aspects of these, but he did produce several patterns with them and colored them with a minimum number of colors so that continuous ribbon strands had the same color and no two bands of the same color ever crossed [53, pp. 44-52], [17, p. 41].

## Escher's Interactions with Mathematicians

Until 1954 few mathematicians outside of Holland knew of Escher's work. That year the International Congress of Mathematicians (ICM) was held in Amsterdam, and N. G. de Bruijn arranged for an exhibit of Escher's prints, symmetry drawings, and carved balls at the Stedelijk Museum [11]. He wrote in the catalog, "Probably mathematicians will not only be interested in the geometrical motifs; the same playfulness which constantly appears in mathematics in general and which, to a great


Figure 4. M. C. Escher's Sky and Water I, 1938. Woodcut, $435 \mathrm{~cm} \times 439 \mathrm{~cm}$.
many mathematicians is the peculiar charm of their subject, will be a more important element" [2].

When Roger Penrose visited the exhibit, he was amazed and intrigued. Escher's print Relativity especially caught his eye. It shows three prominent staircases in a triangular arrangement (and some smaller staircases), as seen from many different viewpoints, with several persons simultaneously climbing or


Figure 5. Penrose's tribar. descending them in an impossible manner, defying the law of gravity. Penrose was inspired to find a structure whose parts were individually consistent but, when joined, became "impossible". After returning to England he came up with the idea of the now-famous Penrose tribar in which three mutually perpendicular bars appear to join to form a triangle (Figure 5). Following that, his father devised an "endless staircase", another object that can be drawn on paper but is impossible to construct as it appears [41, pp. 149-50]. Penrose then closed the loop of discovery by sending the sketches of these impossible objects to Escher, who in turn used them in crafting the perpetual motion in his print Waterfall and the never-ending march of the monks in Ascending and Descending.

Penrose also visited Escher's home in 1962 and brought a gift of identical wooden puzzle pieces


Figure 6. Coxeter's Figure 7, with Escher's markings (here computerenhanced for visibility).
derived from a $60^{\circ}$ rhombus. Escher soon sent Penrose the puzzle's solution, enclosing a sketch of the unique way in which the pieces fitted together. Here, congruent tiles were surrounded in two distinct ways. In 1971 Escher produced his only tiling with one tile that was not a regular division (today it would be called 2-isohedral). It was the last of his numbered symmetry drawings, with a little ghost that filled the plane according to the rules of Penrose’s puzzle [41, pp. 144-45; 53, p. 229].
H. S. M. Coxeter also saw Escher's work for the first time during that ICM in 1954, and upon returning to Canada, he wrote Escher a letter to express his appreciation of the artist's work. Three years later, he wrote again to ask if he might use two of Escher's symmetry drawings to illustrate an article based on his presidential address to the Royal Society of Canada. The article discussed symmetry in the Euclidean plane and also in the Poincaré disk model of the hyperbolic plane and on a sphere surface [3]. Escher readily agreed, and when he later received a reprint of the article, he wrote to Coxeter, "some of the text-illustrations and especially figure 7, page 11, gave me quite a shock" [5, p. 19]. The figure's hyperbolic tiling, with triangular tiles diminishing in
size and repeating (theoretically) infinitely within the confines of a circle, was exactly what Escher had been looking for in order to capture infinity in a finite space.

Escher worked over the figure with compass and straightedge and circled important points (Figure 6). From this, he managed to discern enough of the geometry to produce his print Circle Limit I. But he wanted to know more, and sent a large diagram to Coxeter showing what he had figured out, namely, the location of centers of six of the circles (Figure 7). In his letter, he politely asked Coxeter for "a simple explanation how to construct the [remaining] circles whose centres approach [the bounding circle] from the outside till they reach the limit." He also asked, "Are there other systems besides this one to reach a circle limit?" [5, p. 19], [54, p. 263]. Coxeter replied with a minimal answer to Escher's first request:

The point that I have marked on your drawing (with a red o on the back of the page) lies on three of your circles with centres $1,4,5$. These centres therefore lie on a straight line (which I have drawn faintly in red) and the fourth circle through the red point must have its centre on this same red line. [54, p. 264]

From this, Escher was supposed to construct the complete scheme. By contrast, Coxeter answered the second question at length, beginning, "Yes, infinitely many! This particular pattern is denoted by $[4,6]$ " and then explained for which $p$ and $q$ patterns $[p, q]$ exist, referring to the text


Figure 7. Escher's diagram sent to Coxeter, exhibiting what the artist had figured out. The original drawing is faint, drawn in pencil on tracing paper. This is a reconstruction by the author, and shows Coxeter's red markings.

Generators and Relations, and enclosing a "spare copy of $\left\{\begin{array}{l}3 \\ 7\end{array}\right\}$ " [54, p. 264].

Escher was disappointed with this reply, yet it only increased his determination to figure things out. He wrote to his son George:
[Coxeter] encloses an example of using the values three and seven, of all things! However this odd seven is of no use to me at all; I long for two and four (or four and eight)... My great enthusiasm for this sort of picture and my tenacity in pursuing the study will perhaps lead to a satisfactory solution in the end. ... it seems to be very difficult for Coxeter to write intelligibly to a layman. Finally, no matter how difficult it is, I feel all the more satisfaction from solving a problem like this in my own bumbling fashion. [1, p. 92], [54, pp. 264-5]

Escher did successfully carry out his "Coxetering", as he called his work with hyperbolic tilings, and in 1959-1960 he produced three other Circle Limit prints. Upon earlier receiving Circle Limit I, Coxeter had praised Escher for his understanding of the conformal pattern, and in 1960, when he received the complex Circle Limit III, Coxeter wrote Escher a three-page letter sprinkled with symbols explaining the print's mathematical content, with references to several technical texts, and the implications for coloring seen in the "compound $\{3,8\}$ $[6\{8,8\}]\{8,3\}$ of six $\{8,8\}$ 's inscribed in a $\{3,8\}$ " $[54$, p. 265]. And Escher despaired to George, "Three pages of explanation of what I actually did... It is a pity that I understand nothing, absolutely nothing of it..." [1, pp. 100-01], [54, p. 265].

In 1960 Coxeter arranged for Escher to give two lectures at the University of Toronto about his work, and the Coxeters hosted the artist at their home. The Coxeter-Escher correspondence continued for several years, with two letters of note. In March 1964 Coxeter wrote "After looking again and again at your Circle Limit III on my study wall, I finally realized that my remark about its 'impossibility' was based on my own misunderstanding, as you will see in the enclosed," which was his review of Escher's book [20] for Mathematical Reviews. He added, "The more I look at your work, the more I admire it" [9]. That review [4] was the first time Coxeter revealed that the white arcs forming the backbones of fish in Escher's Circle Limit III were not, as he and others had assumed, badly rendered hyperbolic lines but rather were branches of equidistant curves. In 1979 and again in 1995 he published articles [5], [6] devoted to those white arcs, explaining, "they 'ought' to cut the circumference at the same angle, namely $80^{\circ}$ (which they do, with remarkable accuracy). Thus Escher's work, based on his intuition, without any computation, is perfect..." [5, pp. 19-20].


Figure 8. Escher's geometric grid for the print Square Limit.

In other articles Coxeter gave mathematical analyses of Escher's work and indicated that the artist had anticipated some of his own discoveries [54]. In May 1964 Escher sent Coxeter his print Square Limit and explained with a diagram its underlying geometric grid of self-similar triangles (see reconstruction in Figure 8). Escher's explanatory sketch was on graph paper, in red and blue colored pencil. It showed the first three rings surrounding the center square to indicate how the division process can continue forever. He had devised this fractal structure himself, and while a Euclidean construction with straight segments, it possessed the desired property of his Circle Lim-its-figures diminished as they approached the bounding square [17, pp. 104-05], [53, p. 315], [59, pp. 182-183]. A $90^{\circ}$ rotation about the center of the diagram is a color symmetry, sending red tiles to blue, blue to red, and white to white. In Escher's print, the triangles are replaced by fish.

Escher had only brief interactions with other mathematicians; none would influence his work as did Pólya, Penrose, and Coxeter. Edith Müller, who had been A. Speiser's Ph.D. student, wrote to me that Escher had learned of her dissertation (a symmetry analysis of the Alhambra tilings) and visited her in 1948 in Zurich to discuss her (and his) work. She told him about how Speiser had learned to make lace in order to better understand symmetry.

Heinrich Heesch, another student of Speiser, carried out extensive research on tilings in the mid-1930s but did not publish until the 1960s. He, too, defined "regular" tilings as plane-fillings with congruent tiles in which every tile was surrounded in the same manner. Also, like Escher, he was interested in characterizing the conditions on edges of


Figure 9. Escher's lecture slide about regular divisions of the plane, showing five "quadrilateral systems".
asymmetric tiles that could tile in this manner and for which the tiling had no reflection symmetries. He proved there were exactly twenty-eight types of these tiles and displayed a visual chart of them in his 1963 book with Otto Kienzle [29]. He assigned each edge of a tile a letter-T, $G$, or $C_{n}$-according to how it related to another edge by translation, glide-reflection, or $360^{\circ} / n$ rotation. He sent the book to Escher, who at that time was very ill; for Escher, this information came more than twenty years after his own discoveries of all but one of those twenty-eight types [53, pp. 324-26].

In the last two years of Escher's life, mathematics teacher Hans de Rijk (a.k.a. Bruno Ernst) collaborated with Escher to write a book that would interpret the body of the artist's work, with special attention to the mathematical underpinnings of many prints. Every Sunday without fail they would spend time together as the manuscript took shape. This book [17] and a definitive catalog of Escher's graphic work [37] were both published in 1976, four years after the artist's death, and were the first to show many of Escher's painstaking preliminary drawings for his prints, some of them geometric marvels. A shorter version of de Rijk's analysis of Escher's work is in [1], pp. 135-54.

## Escher's Work Used to Teach Mathematical Ideas

Escher enjoyed the role of teacher, giving lectures to diverse audiences-scientific gatherings, school students, museum audiences, even Rotary clubs. His lecture poster (Figure 9) shows in five different illustrative tilings how he explained the actions of translations (verschuiving), rotations (assen), and glide-reflections (glijspiegeling) that would carry a tile into an adjacent tile. Numbers identify the
various aspects of a tile, circles and squares identify twofold and fourfold rotation centers, respectively, and adjacent dashed lines act as rails along which a tile glides and then reflects (in a line equidistant from the rails). Escher used large brightly-colored cardboard cutouts in the shapes of these tiles, mounted on straightened wire hangers, to demonstrate the motions of the isometries.

When Escher's book [20] was published in Holland in 1960, it included a short essay in the introduction by crystallographer P. Terpstra, to teach about symmetry and the seventeen plane symmetry groups. When the British translation was published, the essay appeared as a separate pamphlet; it never appeared with the American edition. Evidently the publisher, like Escher, thought it too technical.

Caroline MacGillavry, a crystallographer at the University of Amsterdam, was the first scientist to see the possibility of using Escher's art as a teaching tool in a text. When she first visited his studio in the late 1950s, she marveled: "The notebook in which he wrote his 'layman's theory' has been a revelation to me. It contains practically all the 2-, 3-, and 6-colour rotational two-dimensional groups, with and without glide-reflection symmetry" [39, p. x]. That visit gave birth to her idea of collaborating with Escher to use his symmetry drawings in a text for beginning geology students, to teach the classification of colored periodic tilings according to their symmetries. The International Union of Crystallography agreed to sponsor the publication. In the book's introduction, she notes,

Escher's periodic drawings...make excellent material for teaching the principles of symmetry. These patterns are complicated enough to illustrate clearly the basic concepts of translation and other symmetry, which are so often obscured in the clumsy arrays of little circles, pretending to be atoms, drawn on blackboards by teachers of crystallography classes. On the other hand, most of the designs do not present too great difficulties for the beginner in the field. [39, p. ix]

In reviewing Escher's store of periodic drawings (by then, more than 100), she noted that one of the simplest symmetry groups, type $p 2$ with no color symmetries, was not represented. At her request, Escher produced a new symmetry drawing to fill the gap [39, plate 2], [53, p. 210]. He also produced another requested type [39, plate 34], [53, p. 211] and refreshed or redrew some others for the publication.

Coxeter may have been the first mathematician (outside of Holland) to use Escher's work to illustrate a mathematics text. His Introduction to

Geometry was unusual when it was published in 1961, with many nonstandard topics, including symmetry and planar tessellations, which he illustrated with Escher's symmetry drawings used earlier in [3]. Martin Gardner devoted a Mathematical Games column in Scientific American to a review of the book and republished the drawings, bringing Escher's symmetry work to the attention of the wider scientific world [21]. It was not long before scores of math texts (at all levels) and articles on teaching displayed Escher's periodic drawings and prints. While the elementary concepts of planar isometries, similarities, and symmetry are obvious ones for which Escher's symmetry drawings and prints provide wonderful illustrations, the drawings can also be used in teaching higher-level concepts of abstract algebra and group theory. In her article [57], Marjorie Senechal discusses how, by studying the color symmetry groups of Escher's periodic drawings, students can better understand the definition of a group, commutativity and noncommutativity, group action, orbits, generators, subgroups, cosets, conjugates, normal subgroups, stabilizers, permutations and permutation representations, and group extensions.

Teachers (and texts) of mathematics and science also use Escher's prints for artful depictions of mathematical objects (knots, Möbius bands, spirals, loxodromes, fractals, polyhedra, divisions of space) and to provide intriguing visual metaphors for abstract mathematical concepts (infinity, duality, reflection, relativity, self-reference, recursion, topological change) [49]. In his Pulitzer-Prizewinning book Gödel, Escher, Bach: An Eternal Golden Braid, Douglas Hofstadter uses Escher's work in essential ways to convey ideas of recursion and self-reference, and several authors have used Escher's prints to illustrate complex ideas of perception and illusion.

Often those who view art impose on it their reading of the artist's intention, and mathematicians' use of Escher's work to illustrate the idea of infinity and other mathematical concepts might be questioned. But it should be noted that Escher was intrigued by these concepts and set out to embody their essence in many of his prints. His fascination with infinity and how to capture it was a theme he returned to again and again. He spoke eloquently of this quest in his essay "Approaches to Infinity":

Man is incapable of imagining that time could ever stop. For us, even if the earth should cease turning on its axis and revolving around the sun, even if there were no longer days and nights, summers and winters, time would continue to flow on eternally.

Anyone who plunges into infinity, in both time and space, further and further without stopping, needs fixed
points, mileposts, for otherwise his movement is indistinguishable from standing still. There must be stars past which he shoots, beacons from which he can measure the distance he has traversed. He must divide his universe into distances of a given length, into compartments recurring in an endless sequence. Each time he passes a borderline between one compartment and the next, his clock ticks. ... [37, pp. 37-40]

For Escher, mathematical concepts, especially infinity and duality, were a constant source of artistic inspiration.

## Mathematical Research Related to or Inspired by Escher's Work

Several aspects of Escher's work anticipated by decades theoretical investigations by members of the scientific community. And some of his work has directly inspired mathematical investigations. We note here (necessarily briefly) many of these investigations.

Classification of "regular" tilings using edge relationships of tiles was Escher's method and also that of H . Heesch, but it was limited to asymmetric tiles and tilings with symmetry groups having no reflections. In the 1970s Branko Grünbaum and Geoffrey Shephard undertook a systematic classification of several kinds of tilings having transitivity properties with respect to the symmetry group of the tiling-isohedral (tile-transitive), isogonal (vertex-transitive), isotoxal (edge-transitive). Their method relied on using adjacency symbols and incidence symbols that recorded how (in the case of isohedral tilings) each tile was surrounded; the transitivity condition implied that every tile was surrounded in the same way. Their book [25] remains the fundamental reference on all aspects of tilings.

Two-color and 2-motif tilings were Escher's way of expressing duality. It is interesting to note that the first classification of two-color symmetry groups was carried out in 1936 (at almost the same time Escher was making his independent investigations) by H. J. Woods, who was interested in these black-white mosaics for textile designs [10], [62]. When a monohedral (one tile) tiling was colored in two colors, and a symmetry of the tiling interchanged the tiles and interchanged their colors, he called it "counterchange symmetry". (For example, in a checkerboard-colored tiling of the plane by squares, a reflection of the tiling in an edge of one column of squares would be a counterchange symmetry.) The scientific community and Escher were unaware of Woods's work. Later this kind of symmetry, so prevalent in Escher's work, was called "antisymmetry" by Russian crystallographers; that terminology is


Figure 10. The beginning of Horseman.
not used today. Escher noted that some crystallographers had trouble accepting the idea of antisymmetry; he said that he couldn't work without it [1, p. 94].

Escher's method of splitting tiles to produce 2-motif tilings has been shown to be a powerful one. Today, the term 2-isohedral is used to describe tilings in which the symmetry group of the tiling produces two orbits of tiles-there are two distinct congruence classes of tiles with respect to the symmetry group of the tiling. It has been proved that every 2 -isohedral tiling can be derived beginning with an isohedral tiling and applying the processes of splitting and gluing [13] and that this same process extends to produce $k$-isohedral tilings [32]. Andreas Dress [14] and I [55] have studied other aspects of these tilings.

Color symmetry was not a serious concern of crystallographers until the 1950s; even then, it was not easily embraced, and it took many years before color symmetry groups were studied systematically. When crystallographers and mathematicians did begin to investigate color symmetry groups, they (like Caroline MacGillavry) turned to Escher's work for illustrations and discoveries. Even today, there are competing notations for color symmetry groups [7], [25], [60], [61].

Metamorphosis, or topological change, was one of Escher's key devices in his prints. His interlocked creatures often began as parallelograms, squares, triangles, or hexagons, then seamlessly morphed into recognizable shapes, preserving an underlying lattice, as in his visual demonstration in Plate I in [19]. At other times the metamorphosis of creatures changed that lattice, as occurs in his Metamorphosis III. William Huff's design studio produced some intriguing examples of "parquet deformations" that preserve lattice structure [30], and, more recently, Craig Kaplan has investigated the varieties of deformation employed by Escher [34].

Covering surfaces with symmetric patterns was Escher's passion-the Euclidean plane, the hyperbolic plane, sphere surfaces, and cylinders-and always these coverings represented nontrivial symmetry groups of the patterned surface. Douglas Dunham has explored many families of Escher-like tilings of the hyperbolic plane and how to render them by computer [15], [52, pp. 286-296]. Others have studied how to cover different surfaces with periodic designs and sometimes asked, "What
symmetry groups do these coverings represent?" See [7], [45], [56].

Escher's algorithm to produce patterns with decorated squares has inspired mathematicians and computer scientists to use combinatorial techniques (Burnside counting) and computer techniques to check his work and to answer more questions. Escher's results of twenty-three patterns for his simplest case and ten for his case (1) are exactly right. The correct answer for his case (2) is thirty-nine; for this case, Escher missed three patterns and counted one pattern twice [50]. Other questions have been asked and answered: How many patterns are there with two stamps (the original and its reflection) if Escher's restrictions on choice are removed [12]? How many with two stamps and translation only in one direction [42]? How many with the four "ribbon pattern" stamps and the additional action of under-over interchange added to the group of symmetries [22]? Can Escher's algorithm be computer-automated [38], [40]? Can allowable coloring of the ribbon patterns be automated [23]?

Creating tile shapes was almost an obsession with Escher. He would begin with a simple tile (often a polygon) that he knew would produce a regular division, then painstakingly coax the boundary into a recognizable shape. Who but Escher could conjure the polygon in Figure 10 into a helmeted horseman? [53, pp. 110-11] He explained,

The border line between two adjacent shapes having a double function, the act of tracing such a line is a complicated business. On either side of it, simultaneously, a recognizability takes shape. But the human eye and mind cannot be busy with two things at the same moment and so there must be a quick and continual jumping from one side to the other. [39, p. vii]

Kevin Lee was the first to implement Escher's process with a computer program [36]. Craig Kaplan and David Salesin devised a computer program to address a complementary question-beginning with any shape, can it be gently deformed (still being recognizable) into a tile that will produce an isohedral tiling [33]?

Local vs. global definition of "regularity" was not Escher's concern; he followed the local rule that every tile be surrounded in the same way. But every one of Escher's "regular divisions" is an isohedral tiling; it satisfies the global regularity condition that the symmetry group is transitive on the tiles. An isohedral tiling necessarily has local regularity, but are the two definitions equivalent? In the Euclidean plane, yes, at least for asymmetric tiles and edge-to-edge tilings by polygons, but not
so in the hyperbolic plane or in higher dimensions [51]. P. Engel also addresses this question in [16].

Symmetry of a tile inducing symmetry of its tiling was encountered and noted by Escher. When he used a tile with reflection symmetry (such as a dragonfly), it always induced reflections as symmetries of the tiling. He would note the tile was symmetric, and add an asterisk * to his classification symbol. But in a couple of instances, he created a tiling in which the tile was almost symmetric (and with slight modification can be made symmetric), yet the reflection line for the tile is not a reflection line for any of its tilings. In [24], Branko Grünbaum calls such tiles "hypersymmetric" and asks if they can be characterized. This is an open question.

Orderliness not induced by symmetry groups occurs at least twice in Escher's work: in his fractal construction of squares of diminishing size (Figure 8) and in his combinatorially perfect but not color-symmetry perfect coloring of one of his most complex designs with butterflies [1, p. 76]. Branko Grünbaum and others have asked for serious studies of other kinds of "orderliness" in tilings and patterns, not only that defined by symmetry groups [26], [27].
"Completing" Escher's lithograph "Print Gallery" recently posed a mathematical challenge to H . Lenstra and B. de Smit-how they came to understand the underlying geometric grid, "unroll" it, complete missing bits of the unrolled print, and roll it up again is described in [58].

In 1960 Escher wrote, "Although I am absolutely innocent of training or knowledge in the exact sciences, I often seem to have more in common with mathematicians than with my fellow artists" [20, Introduction]. Although he struggled with mathematics as a school student, when he became a graphic artist he was driven to pursue mathematical research, learn new geometric ideas, depict mathematical concepts, and pose mathematical questions. He could not have imagined the scope of influence his work would have for the scientific community.

## Acknowledgments

The author thanks the M. C. Escher Company for permission to reproduce works by M. C. Escher. Bill Casselman took the photo of Coxeter's article used in Figure 6. Figures 3, 5, 6, 7, 8, and 10 were created by the author using The Geometer's Sketchpad.

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# Call for Nominations 

 (2) (90)
## Frank and Brennie Morgan

 AMS-MAA-SIAM Prize for Outstanding Research in Mathematics by an Undergraduate StudentThe prize is awarded each year to an undergraduate student (or students having submitted joint work) for outstanding research in mathematics. Any student who is an undergraduate in a college or university in the United States or its possessions, or Canada or Mexico, is eligible to be considered for this prize.

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# The Giant Component: The Golden Anniversary 

Joel Spencer

## Paul Erdős and Alfréd Rényi

Oftentimes the beginnings of a mathematical area are obscure or disputed. The subject of random graphs had, however, a clear beginning, and it occurred fifty years ago. Alfréd Rényi (1921-1970) was head of the Hungarian Mathematical Institute which today bears his name. Rényi had a great love of literature and philosophy, a true Renaissance man. His life, in the words of Paul Turán, was one of intense and creative involvement in the exchange of ideas and in public affairs. His mathematics centered on probability theory. Paul Erdős (1913-1996) was a central figure in twentiethcentury mathematics. This author recalls well the memorial conference organized by his long-time friend and collaborator Vera Sós in 1999, at which fifteen plenary lecturers discussed his contributions. What was so surprising to us all was the sheer breadth of his work, which spanned so many vital areas of mathematics. To this very prejudiced author, it is his work in discrete mathematics that is having the greatest lasting impact. In this area, Erdős tended toward asymptotic questions, a style very much relevant to today's world of $o, O, \Theta$, and $\Omega$. For both Erdős and Rényi, mathematics was a collaborative enterprise. They both had numerous coauthors, and they wrote thirty-two joint papers.

In 1960 they produced their masterwork, On the Evolution of Random Graphs [7]. Begin with $n$ vertices and no edges and add edges randomly (that is, uniformly from among the potential edges) one by one. Let $G[n, e]$ be the state when there are $e$ edges. Of course, $G[n, e]$ could be any graph with $n$ vertices and $e$ edges and technically is the uniform

[^4]probability distribution over all such graphs. Erdős and Rényi studied the typical behavior of $G[n, e]$ as $e$ "evolved" from 0 to $\binom{n}{2}$. When $e$ approaches and passes $\frac{n}{2}$ the random graph undergoes a phase transition. In a typical computer run on a million vertices, the size of the largest component is only 168 at $e=400000$ edges. By $e=600000$ it has exploded to size 309433.

In modern language we let $G(n, p)$ be the probability space over graphs on $n$ vertices where each pair is adjacent with independent probability $p$. Such graphs have very close to a proportion $p$ of the edges. The behaviors of $G[n, e]$ and $G(n, p)$ where $e=p\binom{n}{2}$ are asymptotically the same for all the topics we discuss here, and we shall use the modern language.

We shall parameterize $p=\frac{c}{n}$. The graph with $\frac{n}{2}$ edges then corresponds to $c=1$. In this range the graph will split into components. We let $C_{1}, C_{2}$ denote the largest and second largest components in the graph, with $\left|C_{i}\right|$ denoting their number of vertices. We define the complexity of a component with $V$ vertices and $E$ edges as $E-V+1$. Trees and unicyclic graphs have complexity 0 and 1 , respectively, and are called simple.

Theorem 1 (Erdős-Rényi). The behavior of $G(n, p)$ with $p=\frac{c}{n}$ can be broken into three parts.
Subcritical $c<1$ : All components are simple and very small, $\left|C_{1}\right|=O(\ln n)$.
Critical $c=1:\left|C_{1}\right|=\Theta\left(n^{2 / 3}\right) A$ delicate situation!
Supercritical $c>1:\left|C_{1}\right| \sim y n$ where $y=y(c)$ is the positive solution to the equation

$$
\begin{equation*}
e^{-c y}=1-y \tag{1}
\end{equation*}
$$

$C_{1}$ has high complexity. All other components are simple and very small, $\left|C_{2}\right|=O(\ln n)$.
Remark. The full statements here and below are rather unwieldy. Thus $\left|C_{1}\right| \sim y n$ really means that for all $c>1$ and all $\epsilon>0$ the limit as $n \rightarrow \infty$ of the probability that $(y-\epsilon) n<\left|C_{1}\right|<(y+\epsilon) n$ is 1 . We allow ourselves the more informal description.
Remark. The average degree of a vertex is $p(n-1) \sim c$. The critical behavior takes place when the average degree reaches 1 .

When $c>1$, Erdős and Rényi called $C_{1}$ the giant component. There are two salient features of the giant component: its existence and its uniqueness. Their system does not have a Jupiter and an also huge Saturn; it is more like Jupiter and Ceres. Several books ([3], [4], [13]) give very full discussions.

## Francis Galton and Henry Watson

Let's switch gears. A Galton-Watson process begins with a single node "Eve". ${ }^{1}$ Eve has $Z$ children, where $Z$ has a Poisson distribution with mean $c$. (That is, Eve has $k$ children with probability $e^{-c} c^{k} / k$ !. The full study of Galton-Watson processes considers other distributions as well.) These children then have children independently with the same distribution, and the process continues through the generations. Let $T=T_{c}$ be the total population generated. There is a precise formula

$$
\begin{equation*}
\operatorname{Pr}\left[T_{c}=k\right]=\frac{e^{-c k}(c k)^{k-1}}{k!} \tag{2}
\end{equation*}
$$

However, it is also possible the $T$ is infinite.
Theorem 2. The Galton-Watson process has three regions.
Subcritical $c<1$ : $T$ is finite with probability 1 , and $E[T]=\sum_{k=0}^{\infty} c^{k}=\frac{1}{1-c}$.
Critical $c=1$ : $T$ is finite with probability 1 but has infinite expectation. ${ }^{2}$
Supercritical $c>1$. T is infinite with probability $y=y(c)$ given by (1).

With $c>1$ let $z=1-y$ be the probability Eve generates a finite tree. If Eve has $k$ children the full tree will be finite if and only if all of the children generate finite trees, which has probability $z^{k}$. Thus

$$
\begin{equation*}
z=\sum_{k=0}^{\infty} e^{-c} \frac{c^{k}}{k!} z^{k} \tag{3}
\end{equation*}
$$

and some manipulation gives that $y=1-z$ satisfies (1). One needs further argument to show that $z=1, y=0$ is a spurious solution.

[^5]
## Erdős Meets Galton

Fix a vertex $v$ in $G(n, p)$ with $p=\frac{c}{n}$ and perform a breadth first search (BFS) to find its components. It has binomial distribution $B[n-1, p]$ neighbors, asymptotically Poisson with mean $c$. Its neighbors then have Poisson $c$ new neighbors, and so on. The component of $v$ is approximated by the Galton-Watson process. For $c<1$ this approximation works well. But for $c>1$ the Galton-Watson process may go on forever while the component of $v$ can have at most $n$ vertices. An ecological limitation causes the processes to converge. BFS requires new vertices. After $\delta n$ vertices have been found, the new distribution is binomial $B[n(1-\delta), p]$, which is Poisson with mean $c(1-\delta)$. The success of BFS causes $\delta$ to rise, which makes it harder to find new vertices, leading the process to eventually die. The effect of the ecological limitation is only felt after a positive proportion $\delta n$ of vertices have been found. Consider BFS from each vertex $v$. With probability $1-y$ the process will die early, giving a small component. But for $\sim y n$ the process will not die early. All of these vertices have their components merge into the giant component.

## Jupiter Without Saturn

Why can we not have Jupiter and Saturn, two components both of size bigger than $\delta n$ ? This would be highly unstable. Each additional edge would have probability at least $(\delta n)^{2} /\binom{n}{2} \sim 2 \delta^{2}$ of merging them. High instability and nonexistence are not the same. Indeed, while there are many proofs of the uniqueness of the giant component, we do not know one that is both simple and rigorous.

## The Critical Window

Erdős and Rényi normally repressed their enthusiasm in their formal writings. But not now!

This double "jump" in the size of the largest component when $\frac{e}{n}$ passes the value $\frac{1}{2}$ is one of the most striking facts concerning random graphs. [7]
In the 1980 s , spearheaded by the work of Béla Bollobás and Tomasz Łuczak, the value $c=1$ was stretched out, and a critical window was found. The stretching was done by adding a second-order term. The correct parameterization is

$$
\begin{equation*}
p=\frac{1}{n}+\frac{\lambda}{n^{4 / 3}} . \tag{4}
\end{equation*}
$$

Now there are three regions:
Barely Subcritical $p n \sim 1$ and $\lambda \rightarrow-\infty$ : All components are simple. $\left|C_{1}\right| \sim\left|C_{2}\right|$, their sizes increasing with $\lambda$.
The Critical Window $p n \sim 1$ and $\lambda$ a constant: Here (and only here!) we have chaotic behavior,
distributions instead of almost sure behavior. Parameterizing $\left|C_{i}\right|=X_{i} n^{2 / 3}$, the $X_{i}(i=1,2$ and beyond) have a nontrivial distribution and a nontrivial joint distribution. The complexity $Y_{i}$ of $C_{i}$ also has a nontrivial distribution.
Barely Supercritical $p n \sim 1$ and $\lambda \rightarrow+\infty$ : $\left|C_{1}\right| \gg n^{2 / 3} \gg\left|C_{2}\right| . C_{1}$ is the dominant component, much bigger than $C_{2}$ but still small. $C_{1}$ has high complexity, but all other components are simple.

Stirling's formula applied to (2) with $c=1$ gives $\operatorname{Pr}\left[T_{1}=k\right] \sim(2 \pi)^{-1 / 2} k^{-3 / 2}$ and $\operatorname{Pr}\left[T_{1} \geq k\right] \sim$ $2(2 \pi)^{-1 / 2} k^{-1 / 2}$. Now consider $G(n, p)$ with $p n=1$ and let $C(v)$ be the component containing $v$. Call $C(v)$ large if its size is at least $K n^{2 / 3}$ and let $Z$ be the number of $v$ with $C(v)$ large. Estimating $|C(v)|$ by $T_{1}$ would give that $C(v)$ is large with probability $2(2 \pi)^{-1 / 2} K^{-1 / 2} n^{-1 / 3}$, and so $Z$ would have expectation $2(2 \pi)^{-1 / 2} K^{-1 / 2} n^{2 / 3}$. The actual value of $E[Z]$ is somewhat smaller due to the ecological limitation, but let us assume it as a heuristic. If any large component exists, every vertex of it would be in a large component, so that $Z$ would be at least $K n^{2 / 3}$. When $K$ is large, $E[Z]$ is much lower than $K n^{2 / 3}$, so that with high probability there would be no large component. Conversely, when $\epsilon$ is a small positive constant, we expect many components of size bigger than $\epsilon n^{2 / 3}$.

## A Strange Physics

Let $c_{i} n^{2 / 3}$ be the size of the $i$ th largest component at $p=n^{-1}+\lambda n^{-4 / 3}$. Let $\Delta \lambda$ be an "infinitesimal" and increase $\lambda$ by $\Delta \lambda$. There are $\left(c_{i} n^{2 / 3}\right)\left(c_{j} n^{2 / 3}\right)$ potential edges that would merge $C_{i}, C_{j}$, and each is added with probability $(\Delta \lambda) n^{-4 / 3}$. The $n$ factors cancel: $C_{i}, C_{j}$ merge to form a component of size $\left(c_{i}+c_{j}\right) n^{2 / 3}$ with probability $c_{i} c_{j}(\Delta \lambda)$. This gravitational attraction merges the large components and forms the dominant component. We can include the complexity in this model. When $C_{i}, C_{j}$ with complexities $r_{i}, r_{j}$ merge, the new component has complexity $r_{i}+r_{j}$. Further, each $C_{i}$ has $\sim \frac{1}{2} c_{i}^{2} n^{4 / 3}$ potential internal edges. In the infinitesimal time $\Delta \lambda$ with probability $c_{i}^{2}(\Delta \lambda) / 2$, such an edge is added, and the complexity of $C_{i}$ is incremented by 1 . Over time, the complexities get larger and larger. The limiting process, called the multiplicative coalescent process, has interesting connections to Brownian motion [2].

## A Computer Exercise

Computer experimentation vividly shows the rapid development in the critical window. In the run ${ }^{3}$ on the following page, we begin with $n=10^{5}$ vertices and no edges. At each step a random edge is added, and a Union-Find algorithm is used to keep

[^6]track of component sizes. We parameterize the number of edges as $e=\binom{n}{2}\left(n^{-1}+\lambda n^{-4 / 3}\right)$ and take "snapshots" at $\lambda=-4,-3, \ldots,+4$. The ten largest component sizes (listed $0, \ldots, 9$ here, and divided by $n^{2 / 3}$ ) are given for each $\lambda$. At $\lambda=+2$ there is a 1.16 Jupiter and 0.86 Saturn. The next digit, under $N$, gives the new ranking ( - if not in the top ten) for that component for the next $\lambda$. Components $0,1,2,3,4$ have $N=0$, meaning they have all merged by $\lambda=+3$. At $\lambda=3$ Jupiter has blown up to 4.21. (Smaller components have also joined Jupiter, explaining the discrepancy in the sum.) The size of the second largest component has decreased (it is the component formerly ranked 5) to a 0.22 Ceres.

## Inside the Critical Window

At $\lambda=-4$ there is a "jostling for position" among the top components, while by $\lambda=+4$ a dominant component has emerged. The last time the largest component loses that distinction occurs during the critical window [8]. At $\lambda=-4$ all components are simple, while by $\lambda=+4$ the dominant component has high complexity. Complexity at least 4 is necessary for nonplanarity. Planarity is lost in the critical window [16]. In a masterful work [12], the development of complex components is studied. One exceptionally striking result: the probability that the evolution ever simultaneously has two complex components is, asymptotically, $\frac{5}{18} \pi$.

## Classical Bond Percolation

Mathematical physicists examine $Z^{d}$ as a lattice; the pairs $\vec{v}, \vec{w}$ that are one apart are called bonds. They imagine that each bond is occupied with independent probability $p$. (For graph theorists, the occupied bonds form a random subgraph of $Z^{d}$.) The occupied components then form clusters, or components. There is a critical probability, denoted $p_{c}$ (dependent on $d$ ), so that:
Subcritical $p<p_{c}$ : All components are finite. Critical $p=p_{c}$ : A delicate situation!
Supercritical $p>p_{c}$ : There is precisely one infinite component.

There are natural analogies between this infinite model and the asymptotic Erdős-Rényi model. Infinite size corresponds to $\Omega(n)$, while finite size corresponds to $O(\ln n)$. There is particular interest in $p$ being very close to $p_{c}$. Let $f(p)$ be the probability that $\overrightarrow{0}$ (or, by symmetry, any particular $\vec{v}$ ) lies in an infinite component. The critical exponent $\beta$ is that ${ }^{4}$ real number such that $f\left(p_{c}+x\right) \sim x^{\beta+o(1)}$ as $x \rightarrow 0^{+}$. The probability $p_{c}+x$ corresponds to $p n=1+x$ and $f$ to the probability that a given vertex $v$ lies in the giant component or, equivalently, the proportion $y(1+x)$ of vertices in

[^7]| . | -4 | N | -3 | N | -2 | N | -1 | N | 0 | N | +1 | N | +2 | N | +3 | N | +4 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 0 | 0.14 | 1 | 0.18 | 0 | 0.24 | 1 | 0.28 | 0 | 0.37 | 0 | 0.82 | 0 | 1.16 | 0 | 4.21 | 0 | 5.88 |
| 1 | 0.10 | 2 | 0.16 | 1 | 0.19 | 2 | 0.26 | 1 | 0.36 | 1 | 0.39 | 1 | 0.86 | 0 | 0.22 | 0 | 0.24 |
| 2 | 0.10 | 3 | 0.13 | 3 | 0.13 | 4 | 0.19 | 3 | 0.26 | 0 | 0.28 | 3 | 0.49 | 0 | 0.13 | 0 | 0.12 |
| 3 | 0.09 | 4 | 0.12 | 4 | 0.13 | 0 | 0.16 | 2 | 0.21 | 2 | 0.23 | 4 | 0.46 | 0 | 0.12 | 0 | 0.10 |
| 4 | 0.07 | 0 | 0.09 | 5 | 0.13 | 3 | 0.14 | 5 | 0.16 | 0 | 0.20 | 2 | 0.32 | 0 | 0.11 | 2 | 0.10 |
| 5 | 0.07 | 0 | 0.08 | 7 | 0.09 | 5 | 0.12 | 4 | 0.15 | 4 | 0.14 | 5 | 0.16 | 1 | 0.10 | 1 | 0.10 |
| 6 | 0.07 | 5 | 0.06 | 6 | 0.09 | 7 | 0.10 | 8 | 0.12 | 5 | 0.12 | 2 | 0.12 | 3 | 0.09 | 1 | 0.10 |
| 7 | 0.06 | 8 | 0.06 | 2 | 0.08 | 6 | 0.09 | - | 0.12 | 3 | 0.10 | 1 | 0.11 | 4 | 0.09 | 4 | 0.10 |
| 8 | 0.05 | - | 0.06 | 8 | 0.06 | 9 | 0.09 | - | 0.10 | 3 | 0.10 | 7 | 0.09 | 2 | 0.09 | 0 | 0.08 |
| 9 | 0.05 | 9 | 0.05 | - | 0.06 | 0 | 0.07 | - | 0.10 | 9 | 0.10 | 0 | 0.08 | 5 | 0.08 | 6 | 0.07 |

the giant component. As $y(1+x) \sim 2 x=x^{1+o(1)}$ the $\beta$ value for the Erdős-Rényi model is considered 1. This is known to be the $\beta$ value in $Z^{d}$ for all sufficiently high dimensions $d$. Grimmett [10] gives many other critical exponents, and in all cases the analogous value for the Erdős-Rényi model matches the known value in high-dimensional space. Mathematical physicists loosely use the term mean field behavior to describe percolation phenomena in high dimensions, and the ErdősRényi model has this mean field behavior.

## Recent Results

Today it is recognized that percolation and the critical window appear in many guises. Here is a highly subjective description of recent work. We generally give simplified versions.

## Random 2-SAT

We generate $m$ random clauses $C_{1}, \ldots, C_{m}$ on Boolean variables $x_{1}, \ldots, x_{n}$. That is, each clause $C=y \vee z$ with $y, z$ drawn randomly from $\left\{x_{1}, \overline{X_{1}}, \ldots, x_{n}, \overline{X_{n}}\right\}$. We ask if all $C_{i}$ can be simultaneously satisfied. The answer changes from yes to no in the critical window $m=n+\lambda n^{2 / 3}$ [5].

## $d$-regular Graph

Let $G_{n}$ be a sequence of transitive $d$-regular graphs. Under reasonable conditions $p_{c}=\frac{1}{d-1}$ acts as the critical probability for a random subgraph of $G_{n}$. For $p<p_{c}$ the components are small, while for $p>p_{c}$ there is a giant component. More delicately, at $p=p_{c}$ the largest component has size $\Theta\left(n^{2 / 3}\right)$. The scaling $p d=1+\lambda n^{-1 / 3}$ acts as the critical window [18].

## An Improving Walk

Consider an infinite walk starting at $W_{0}=1$ with $W_{t}=W_{t-1}+X_{t}-1$ where $X_{t}$ is Poisson with mean $\frac{t}{n}$. When $W_{t}=0$ (crashes) it is reset to $W_{t}=1$. When $t=\frac{1-\epsilon}{n}$ the walk has negative drift and crashes repeatedly. When $t=\frac{1+\epsilon}{n}$ the walk has positive drift and goes to infinity. The walk will crash for the last time in the critical window $t=n+\lambda n^{2 / 3}$ [9].

## A First-Order Phase Transition

Modify the Erdôs-Rényi evolution as follows. In each round an edge is added to $G$, initially empty. Two random pairs $\{u, v\},\{w, x\}$ are given. Add that pair for which the product of the component sizes of the two vertices is smaller. This provides a powerful antigravity that deters large components from joining. Parameterizing $e=t \frac{n}{2}$ edges chosen (so that $t=1$ is the critical value in the ErdósRényi evolution) the giant component occurs at $t \sim 1.77$. More interesting, extensive computer simulation (but no mathematical proof!) indicates strongly that when the critical value is reached, there is a first-order phase transition. That is, let $t=t_{c}$ be the critical probability and let $f(t)$ be the proportion of vertices in the largest component at "time" $t$. Then the limit of $f(t)$ as $t$ approaches $t_{c}$ from above appears not to be zero but rather something like 0.6 [1].

## General Critical Points

Let $G$ be a graph on $n$ vertices. Set $d^{*}=$ $\left(\sum_{v} d_{v}^{2}\right)\left(\sum_{v} d_{v}\right)$, noting $d_{v}^{*}$ denotes the average degree of a vertex if you first select an edge uniformly and then one of its vertices uniformly. Let $G_{p}$ denote the random subgraph of $G$, accepting each edge with independent probability $p$. Then, under certain mild conditions on $G, p=\frac{1}{d^{*}}$ is the critical point in the evolution of $G_{p}$. When $p=\frac{1-\epsilon}{d^{*}}, G_{p}$ contains no giant component, while when $p=\frac{1+\epsilon}{d^{*}}, G_{p}$ does contain a unique giant component [6].

## Degree Sequence

For given $d_{1}, \ldots, d_{n}$ we consider the graph on $n$ vertices chosen uniformly among all with that degree sequence, that is, $v$ having precisely $d_{v}$ neighbors. Suppose for each $n$ we have a degree sequence, $\lambda_{i}(n) \sim \lambda_{i} n$ vertices having degree $i$. Then (with $d^{*}$ from above), $d^{*} \sim\left[\sum i^{2} \lambda_{i}\right] /\left[\sum i \lambda_{i}\right]$. Set $Q:=\sum_{i} i(i-2) \lambda_{i}$ so that $Q>0$ if and only if $d^{*}>2$. In analyzing BFS one thinks of an edge going to a vertex with the above distribution, which then is on an expected number $d^{*}-1$ of new edges. With $d^{*}<2$ the process will die, while
for $d^{*}>2$ it might continue. When $Q<0$ the random graph with this degree sequence has no giant component, while when $Q>0$ it does [17].

Set $Q_{n}:=\sum_{i} i(i-2) \lambda_{i}(n)$ and assume $Q_{n} \rightarrow$ 0 . Under moderate assumptions, $Q_{n}=\lambda n^{-1 / 3}$ provides a critical window. For $\lambda \rightarrow-\infty$ the random graph is subcritical, and all components have size $o\left(n^{2 / 3}\right)$. When $\lambda \rightarrow+\infty$ the random graph is supercritical, there is a dominant component of size $\gg n^{2 / 3}$ and all other components have size $o\left(n^{2 / 3}\right)$. In the power law random graphs, thought by many to model the Web graph and other phenomena, it is assumed that $\lambda_{i} \sim i^{-\gamma}$ for a constant $\gamma$. For certain $\gamma$ the above critical window does not work, and work in progress indicates that there is a critical window whose exponent depends on $\gamma$ [14] [11].

## A Potts Model

In the Potts model, the distribution of graphs is biased toward having more components. There are three parameters, $p \in[0,1], q \geq 1$, and the number of vertices $n$. A graph $G$ with $e$ edges, $s:=\binom{n}{2}-e$ nonedges, and $c$ components has probability $p^{e}(1-p)^{s} q^{c} / Z$, where $Z$ is a normalizing constant chosen so that the sum of the probabilities is 1 . For $q=2$ this is called the Ising model, for $q \geq 3$ and integral, this is the Potts model. For $2<q<3$ the critical value is $p n=c_{q}:=2 \frac{q-1}{q-2} \ln (q-1)$. At $p n=c_{q}+\epsilon$ there is a giant component, while at $p n=c_{q}-\epsilon$ the largest component has logarithmic size. The critical window has parameterization $p n=c_{q}+\frac{\lambda}{n}$. There the graph has two different personalities. Either it has a giant component or the largest component has logarithmic size. Both occur with positive limiting probability, and these limiting probabilities sum to 1 . At no $p$ is there a middle ground with the size of the largest component being bigger than logarithmic but sublinear [15].

## In Conclusion

The mathematical landscape at the time of Paul Erdős's birth, nearly one hundred years ago, was very different from what it is today. Discrete mathematics was disparaged as "the slums of topology". Probability was useful for gambling, but not proper work for a serious mathematician. Today both areas are thriving. It is the fecund intersection of discrete mathematics and probability that has seen the most spectacular growth. A wide variety of random processes on large discrete structures are studied. These processes, to use Erdős and Rényi's well-chosen word, undergo an evolution. At a critical moment they undergo a phase transition, from water to ice, from satisfiable to not satisfiable, from freeflow to gridlock, from small components to a giant component. To understand a process we need to understand
these critical moments. The Erdős-Rényi process provides a bedrock to which all other processes may be compared.

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## Coxeter and Escher

As Doris Schattschneider mentions briefly in her article on the mathematics of M. C. Escher in this issue, the response of H. S. M. Coxeter to some queries from Escher about hyperbolic tesselations was not entirely satisfactory. Coxeter had sent Escher a reprint of an article on symmetry that included as its Figure 7 the tesselation of the Poincaré disc associated to a $2-4-6$ reflection group (reproduced as Figure 6 in Schattschneider's article). Escher was much intrigued by this diagram, and wanted to know how to reproduce the figure in his own terms, presumably by compass and straightedge. Relatively elementary considerations based on Pythagoras' Theorem and symmetry enabled him to construct the non-Euclidean arcs bounding several larger triangles of the tesselation, but he did not see how to continue. Coxeter side-stepped the matter rather abruptly. Figure 7 of Schattschneider's article tells much of the story-Escher sent Coxeter a hand-drawn diagram of what he had been able to reproduce, along with the query, "...how are to be constructed further circles with radii shorter again and again ...??" Coxeter sent the diagram back to Escher with some added markings in red, on its reverse side, that indicated only how to draw a line that contained somewhere along it the center of one of the circles that Escher had not been able to find. He gave absolutely no indication of exactly where on that line the center was actually located. His comment that "This can be continued in the same manner" wasn't all that helpful. We are led to ask, why didn't he say more?

The cover image reproduces Coxeter's Figure 7 overlaid with a diagram that makes some attempt to indicate what is going on. Most of what's there is just another version of Schattschneider's figure, which is in turn a version of what Escher included in the drawing sent to Coxeter together with the red point and line that Coxeter added and sent back. There are two basic principles that seem to be required in drawing this tesselation. One is the basic duality between points $P$ inside the Poincaré disc and lines $\ell$ outside it: The non-Euclidean arcs through $P$ have their centers on $\ell$.


It is not clear whether Escher was aware of this principle or not. He does draw several lines compatible with this assertion, but we don't know exactly what he deduced from this. To tell the truth, we don't really know whether Coxeter's red additions told him anything he didn't already know. This principle did not alone give him sufficient information to add new circles, and he may have thought there was no point in drawing lines unless they really contributed. The second principle is what he needed: Suppose given a non-Euclidean arc with endpoints $P$ and $Q$ on the unit circle. The centers of all circles orthogonal to the arc lie on the straight line through $P$ and $Q$. This allows one to find centers by intersecting lines drawn according to either or both of these principles, as is shown for the red circle on the cover.


We do not know where this principle was first formulated in the literature, but it is likely that it was used to construct many diagrams to be found in Coxeter's own writings. For example, secant lines-i.e., lines intersecting the unit circle obliquely-apparently used for this purpose can be found in the first figure of his article on Escher's Circle Limit III in volume 12 of the journal Leonardo. It is unlikely that Coxeter drew those figures himself, however. Somewhere he makes a remark about secant lines in this and similar figures, but it is not clear from what he says that he knows what their purpose is. There is therefore a mystery-what exactly was the reason that Coxeter didn't tell Escher about this technique?

We wish to thank both Coxeter's daughter, Susan Thomas, and the Royal Society of Canada for permission to reproduce his Figure 7, which was originally published in volume 51 of the Transactions of the Royal Society of Canada. We also wish to thank Doris Schattschneider for taking part in correspondence involved in figuring out how Coxeter's diagrams were constructed.
-Bill Casselman Graphics Editor
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# Graph Theory in the Information Age 

Fan Chung

In the past decade, graph theory has gone through a remarkable shift and a profound transformation. The change is in large part due to the humongous amount of information that we are confronted with. A main way to sort through massive data sets is to build and examine the network formed by interrelations. For example, Google's successful Web search algorithms are based on the WWW graph, which contains all Web pages as vertices and hyperlinks as edges. There are all sorts of information networks, such as biological networks built from biological databases and social networks formed by email, phone calls, instant messaging, etc., as well as various types of physical networks. Of particular interest to mathematicians is the collaboration graph, which is based on the data from Mathematical Reviews. In the collaboration graph, every mathematician is a vertex, and two mathematicians who wrote a joint paper are connected by an edge.

Figure 1 illustrates a portion of the collaboration graph consisting of about 5,000 vertices, representing mathematicians with Erdős number 2 (i.e., mathematicians who wrote a paper with a coauthor of Paul Erdős).

Graph theory has two hundred years of history studying the basic mathematical structures called graphs. A graph $G$ consists of a collection $V$ of vertices and a collection $E$ of edges that connect pairs of vertices. In the past, graph theory has

[^8]

Figure 1. An induced subgraph of the collaboration graph.
been used in a wide range of areas. However, never before have we confronted graphs of not only such tremendous sizes but also extraordinary richness and complexity, both at a theoretical and a practical level. Numerous challenging problems have attracted the attention and imagination of researchers from physics, computer science, engineering, biology, social science, and mathematics. The new area of "network science" emerged, calling for a sound scientific foundation and rigorous analysis for which graph theory is ideally suited. In the other direction, examples of real-world graphs
lead to central questions and new directions for research in graph theory.

These real-world networks are massive and complex but illustrate amazing coherence. Empirically, most real-world graphs have the following properties:

- sparsity-The number of edges is within a constant multiple of the number of vertices.
- small world phenomenon-Any two vertices are connected by a short path. Two vertices having a common neighbor are more likely to be neighbors.
- power law degree distribution-The degree of a vertex is the number of its neighbors. The number of vertices with degree $j$ (or having $j$ neighbors) is proportional to $j^{-\beta}$ for some fixed constant $\beta$.
To deal with these information networks, many basic questions arise: What are basic structures of such large networks? How do they evolve? What are the underlying principles that dictate their behavior? How are subgraphs related to the large (and often incomplete) host graph? What are the main graph invariants that capture the myriad properties of such large graphs?

To answer these problems, we first delve into the wealth of knowledge from the past, although it is often not enough. In the past thirty years there has been a great deal of progress in combinatorial and probabilistic methods, as well as spectral methods. However, traditional probabilistic methods mostly consider the same probability distribution for all vertices or edges while real graphs are uneven and clustered. The classical algebraic and analytic methods are efficient in dealing with highly symmetric structures, whereas real-world graphs are quite the opposite. Guided by examples of real-world graphs, we are compelled to improvise, extend, and create new theory and methods. Here we will discuss the new developments in several topics in graph theory that are rapidly developing. The topics include a general random graph theory for any given degree distribution, percolation in general host graphs, PageRank for representing quantitative correlations among vertices, and the game aspects of graphs.

## Random Graph Theory for General Degree Distributions

The primary subject in the study of random graph theory is the classical random graph $G(n, p)$, introduced by Erdős and Rényi in 1959 [38, 39] (also independently by Gilbert [44]). In $G(n, p)$, every pair of a set of $n$ vertices is chosen to be an edge with probability $p$. In a series of papers, Erdős and Rényi gave an elegant and comprehensive analysis describing the evolution of $G(n, p)$ as $p$ increases. Note that a random
graph in $G(n, p)$ has the same expected degree at every vertex, and therefore $G(n, p)$ does not capture some of the main behaviors of real-world graphs. Nevertheless, the approaches and methods in classical random graph theory provide the foundation for the study of random graphs with general degree distributions.

Many random graph models have been proposed in the study of information network graphs, but there are basically two different approaches. The "online" model mimics the growth or decay of a dynamically changing network, and the "offline" model of random graphs consists of specified families of graphs as the probability spaces together with some specified probability distribution.

One online model is the so-called preferential attachment scheme, which can be described as "the rich get richer". The preferential attachment scheme has been receiving much attention in the recent study of complex networks [11, 57], but its history can be traced back to Vilfredo Pareto in 1896, among others. At each tick of the clock (so to speak), a new edge is added, with each of its endpoints chosen with probability proportional to their degrees. It can be proved $[15,31,57]$ that the preferential attachment scheme leads to a power law degree distribution. There are several other online models, including the duplication model (which seems to be more feasible for biological networks, see [35]), as well as many recent extensions, such as adding more parameters concerning the "talent" or "fitness" of each node [50].

There are two main offline graph models for graphs with general degree distribution-the configuration model and random graphs with expected degree sequences. A random graph in the configuration model with degree sequences $d_{1}, d_{2}, \ldots, d_{n}$ is defined by choosing a random matching on $\sum_{i} d_{i}$ "pseudo nodes", where the pseudo nodes are partitioned into parts of sizes $d_{i}$, for $i=1, \ldots, n$. Each part is associated with a vertex. By using results of Molloy and Reed [58, 59], it can be shown [2] that under some mild conditions, a random power law graph with exponent $\beta$ almost surely has no giant component if $\beta \geq \beta_{0}$ where $\beta_{0}$ is a solution to the equation involving the Riemann zeta function $\zeta(\beta-2)-2 \zeta(\beta-1)=0$.

The general random graph model $G(\mathbf{w})$ with expected degree sequence $\mathbf{w}=\left(w_{1}, w_{2}, \ldots, w_{n}\right)$ follows the spirit of the Erdôs-Rényi model. The probability of having an edge between the $i$ th and $j$ th vertices is defined to be $w_{i} w_{j} / \operatorname{Vol}(G)$, where Vol $(G)$ denotes $\sum_{i} w_{i}$. Furthermore, in $G(\mathbf{w})$ each edge is chosen independently of the others, and therefore the analysis can be carried out. It was proved in [28] that if the expected average degree is strictly greater than 1 in a random graph in $G(\mathbf{w})$, then there is a giant component (i.e., a connected component of volume a positive fraction of that of the whole graph). Furthermore,
the giant component almost surely has volume $\delta \operatorname{Vol}(G)+O\left(\sqrt{n} \log ^{3.5} n\right)$, where $\delta$ is the unique nonzero root of the following equation [29]:

$$
\begin{equation*}
\sum_{i=1}^{n} w_{i} e^{-w_{i} \delta}=(1-\delta) \sum_{i=1}^{n} w_{i} \tag{1}
\end{equation*}
$$

Because of the robustness of the $G(\mathbf{w})$ model, many properties can be derived. For example, a random graph in $G(\mathbf{w})$ has average distance almost surely equal to $(1+o(1)) \frac{\log n}{\log \tilde{w}}$, and the diameter is almost surely $\Theta\left(\frac{\log n}{\log \tilde{w}}\right)$, where $\tilde{w}=\sum_{i} w_{i}^{2} / \sum_{i} w_{i}$ provided some mild conditions on $\mathbf{w}$ are satisfied [27]. For the range $2<\beta<3$, where the power law exponents $\beta$ for numerous real networks reside, the power law graph can be roughly described as an "octopus" with a dense subgraph having small diameter $O(\log \log n)$ as the core, while the overall diameter is $O(\log n)$ and the average distance is $O(\log \log n)$ (see [31]).

For the spectra of power law graphs, there are basically two competing approaches. One is to prove analogues of Wigner's semicircle law (which is the case for $G(n, p)$ ), while the other predicts that the eigenvalues follow a power law distribution [40]. Although the semicircle law and the power law have very different descriptions, both assertions are essentially correct if the appropriate matrices associated with a graph are considered [33, 34]. For $\beta>2.5$, the largest eigenvalue of the adjacency matrix of a random power law graph is almost surely $(1+o(1)) \sqrt{m}$, where $m$ is the maximum degree. Moreover, the $k$ largest eigenvalues have power law distribution with exponent $2 \beta-1$ if the maximum degree is sufficiently large and $k$ is bounded above by a function depending on $\beta, m$ and $w$. When $2<\beta<2.5$, the largest eigenvalue is heavily concentrated at $\mathrm{cm}^{3-\beta}$ for some constant $c$ depending on $\beta$ and the average degree. Furthermore, the eigenvalues of the (normalized) Laplacian satisfy the semicircle law under the condition that the minimum expected degree is relatively large [34].

The online model is obviously much harder to analyze than the offline model. One possible approach is to couple the online model with the offline model of random graphs with a similar degree distribution. This means to find the appropriate conditions under which the online model can be sandwiched by two offline models within some error bounds. In such cases, we can apply the techniques from the offline model to predict the behavior of the online model (see [30]).

## Random Subgraphs in Given Host Graphs

Almost all information networks that we observe are subgraphs of some host graphs that often have sizes prohibitively large or with incomplete information. A natural question is to attempt to deduce the properties of a random subgraph from
the host graph and vice versa. It is of interest to understand the connections between a graph and its subgraphs. What invariants of the host graph can or cannot be translated to its subgraphs? Under what conditions can we predict the behavior of all or any subgraphs? Can a sparse subgraph have very different behavior from its host graph? Here we discuss some of the work in this direction.

Many information networks or social networks have very small diameters (in the range of $\log n$ ), as dictated by the so-called small world phenomenon. However, in a recent paper by Liben-Nowell and Kleinberg [52], it was observed that the tree-like subgraphs derived from some chain-letter data seem to have relatively large diameter. In the study of the Erdős-Rényi graph model $G(n, p)$, it was shown [60] that the diameter of a random spanning tree is of order $\sqrt{n}$, in contrast with the fact that the diameter of the host graph $K_{n}$ is 1 . Aldous [4] proved that in a regular graph $G$ with a certain spectral bound $\sigma$, the diameter of a random spanning tree $T$ of $G$, denoted by $\operatorname{diam}(T)$, has expected value satisfying

$$
\frac{c \sigma \sqrt{n}}{\log n} \leq \mathrm{E}(\operatorname{diam}(T)) \leq \frac{c \sqrt{n} \log n}{\sqrt{\sigma}}
$$

for some absolute constant c. In [32], it was shown that for a general host graph $G$, with high probability the diameter of a random spanning tree of $G$ is between $c \sqrt{n}$ and $c^{\prime} \sqrt{n} \log n$, where $c$ and $c^{\prime}$ depend on the spectral gap of $G$ and the ratio of the moments of the degree sequence.

One way to treat random subgraphs of a given graph $G$ is as a (bond) percolation problem. For a positive value $p \leq 1$, we consider $G_{p}$, which is formed by retaining each edge independently with probability $p$ and discarding the edge with probability $1-p$. A fundamental problem of interest is to determine the critical probability $p$ for which $G_{p}$ contains a giant connected component. In the applications of epidemics, we consider a general host graph being a contact graph, consisting of edges formed by pairs of people with possible contact. The question of determining the critical probability then corresponds to the problem of finding the epidemic threshold for the spreading of the disease.

Percolation problems have long been studied [45, 49] in theoretical physics, especially with the host graph being the lattice graph $\mathbb{Z}^{k}$. Percolation problems on lattices are known to be notoriously difficult even for low dimensions and have only been resolved very recently by bootstrap percolation [8, 9]. In the past, percolation problems have been examined for a number of special host graphs. Ajtai, Komlós, and Szemerédi considered the percolation on hypercubes [3]. Their work was further extended to Cayley graphs [16, 17, 18, 55] and regular graphs [42]. For expander graphs with degrees bounded by $d$, Alon, Benjamini, and Stacey
[5] proved that the percolation threshold is greater than or equal to $1 /(2 d)$. In the other direction, Bollobás, Borgs, Chayes, and Riordan [13] showed that for dense graphs (where the degrees are of order $\Theta(n)$ ), the giant component threshold is $1 / \rho$, where $\rho$ is the largest eigenvalue of the adjacency matrix. The special case of having the complete graph $K_{n}$ as the host graph concerns the ErdősRényi graph $G(n, p)$, which is known to have the critical probability at $1 / n$, as well as the "double jump" near the threshold.

For general host graphs, the answer has been elusive. One way to address such questions is to search for appropriate conditions on the host graph so that percolations can be controlled. Recently it has been shown [26] that if a given host graph $G$ satisfies some (mild) conditions depending on its spectral gap and higher moments of its degree sequence, for any $\epsilon>0$, if $p>$ $(1+\epsilon) / \tilde{d}$, then asymptotically almost surely the percolated subgraph $G_{p}$ has a giant component. In the other direction, if $p<(1-\epsilon) / \tilde{d}$, then almost surely the percolated subgraph $G_{p}$ contains no giant component. We note that the second order average degree $\tilde{d}$ is $\tilde{d}=\sum_{v} d_{v}^{2} /\left(\sum_{v} d_{v}\right)$, where $d_{v}$ denotes the degree of $v$.

In general, subgraphs can have spectral gaps very different from those of the host graph. However, if a graph $G$ has all its nontrivial eigenvalues of the (normalized) Laplacian lying in the range within $\sigma$ from the value 1 , then it can be shown [25] that almost surely a random subgraph $G_{p}$ has all its nontrivial eigenvalues in the same range (up to a lower-order term) if the degrees are not too small.

## PageRank and Local Partitioning

In graph theory there are many essential geometrical notions, such as distances (typically, the number of hops required to reach one vertex from another), cuts (i.e., subsets of vertices/edges that separate a part of the graph from the rest), flows (i.e., combinations of paths for routing between given vertices), and so on. However, real-world graphs exhibit the small world phenomenon, so any pair of vertices are connected through a very short path. Therefore the usual notion of graph distance is no longer very useful. Instead, we need a quantitative and precise formulation to differentiate among nodes that are "local" from "global" and "akin" from "dissimilar". This is exactly what PageRank is meant to achieve.

In 1998 Brin and Page [19] introduced the notion of PageRank for Google's Web search algorithm. Different from the usual methods in pattern matching previously used in data retrieval, the novelty of PageRank relies entirely on the underlying Web graph to determine the "importance" of a Web page. Although PageRank is originally designed
for the Web graph, the concept and definitions work well for any graph. Indeed, PageRank has become a valuable tool for examining the correlations of pairs of vertices (or pairs of subsets) in any given graph and hence leads to many applications in graph theory.

The starting point of the PageRank is a typical random walk on a graph $G$ with edge weights $w_{u v}$ for edge $u, v$. The probability transition matrix $P$ is defined by: $P(u, v)=\frac{w_{u v}}{d_{u}}$, where $d_{u}=\sum_{v} w_{u, v}$. For a preference vector $s$, and a jumping constant $\alpha>0$, the PageRank, denoted by $\operatorname{pr}(\alpha, s)$ as a row vector, can be expressed as a series of random walks as follows:

$$
\begin{equation*}
\mathrm{pr}_{\alpha, f}=\alpha \sum_{k=0}^{\infty}(1-\alpha)^{k} s P^{k} \tag{2}
\end{equation*}
$$

Equivalently, $\operatorname{pr}(\alpha, s)$ satisfies the following recurrence relation:
(3) $\operatorname{pr}(\alpha, s)=\alpha s+(1-\alpha) \operatorname{pr}(\alpha, s) P$.

In the original definition of Brin and Page [19], $s$ is taken to be the constant function with value $1 / n$ at every vertex motivated by modeling the behavior of a typical surfer who moves to a random page with probability $\alpha$ and clicks a linked page with probability $1-\alpha$.

Because of the close connection of PageRank with random walks, there are very efficient and robust algorithms for computing and approximating PageRank [6, 12, 47]. This leads to numerous applications, including the basic problem of finding a "good" cut in a graph. A quantitative measure for the "goodness" of a cut that separates a subset $S$ of vertices is the Cheeger ratio:

$$
h(S)=\frac{|E(S, \bar{S})|}{\operatorname{vol}(S)},
$$

where $E(S, \bar{S})$ denotes the set of edges leaving $S$ and $\operatorname{vol}(S)=\sum_{v \in S} d_{v}$. The Cheeger constant $h_{G}$ of a graph is the minimum Cheeger ratio over all subsets $S$ with vol $(S) \leq \operatorname{vol}(G) / 2$. The traditional divide-and-conquer strategy in algorithmic design relies on finding a cut with small Cheeger ratio. Since the problem of finding any cut that achieves the Cheeger constant of $G$ is NP-hard [43], one of the most widely used approximation algorithms was a spectral partitioning algorithm. By using eigenvectors to line up the vertices, the spectral partitioning algorithm reduces the number of cuts under consideration from an exponential number of possibilities to a linear number of choices. Nevertheless, there is still a performance guarantee provided by the Cheeger inequality:

$$
2 h_{G} \geq \lambda \geq \frac{h_{f}^{2}}{2} \geq \frac{h_{G}^{2}}{2}
$$

where $h_{f}$ is the minimum Cheeger ratio among subsets that are initial segments in the order
determined by the eigenvector $f$ associated with the spectral gap $\lambda$.

For large graphs with billions of nodes, it is not feasible to compute eigenvectors. In addition, it is of interest to have local cuts in the sense that for given seeds and the specified size for the parts to be separated, it is desirable to find a cut near the seeds separating a subset of the desired size. Furthermore, the cost/complexity of finding such a cut should be proportional to the specified size of the separated part but independent of the total size of the whole graph. Here, PageRank comes into play. Earlier, Spielman and Teng [63] introduced local partitioning algorithms by using random walks with the performance analysis using a mixing result of Lovász and Simonovitz [54] (also see [56]). As it turns out, by using PageRank instead of random walks, there is an improved partitioning algorithm [6] for which the performance is supported by a local Cheeger inequality for a subset $S$ of vertices in a graph $G$ :

$$
h_{S} \geq \lambda_{S} \geq \frac{h_{g}^{2}}{8 \log \operatorname{vol}(S)} \geq \frac{h_{S}^{2}}{8 \log \operatorname{vol}(S)},
$$

where $\lambda_{S}$ is the Dirichlet eigenvalue of the induced subgraph on $S, h_{S}$ is the local Cheeger constant of $S$ defined by $h_{S}=\min _{T \subseteq S} h(T)$, and $h_{g}$ is the minimum Cheeger ratio over all PageRank $g$ with the seed as a vertex in $S$ and $\alpha$ appropriately chosen depending only on the volume of $S$. This approximation partition algorithm can be further improved using the fact that the set of seeds for which the PageRank leads to the Cheeger ratio satisfying the above local Cheeger inequality is quite large (about half of the volume of $S$ ). We note that the local partitioning algorithm can also be used as a subroutine for finding balanced cuts for the whole graph.

Note that PageRank is expressed as a geometric sum of random walks in (2). Instead, we can consider an exponential sum of random walks, called heat kernel pagerank, which in turn satisfies the heat equation. The heat kernel pagerank leads to an improved local Cheeger inequality [23, 24] by removing the logarithmic factor in the lower bound. Numerous problems in graph theory can possibly take advantage of PageRank and its variations, and the full implications of these ideas remain to be explored.

## Network Games

In morning traffic, every commuter chooses his/her most convenient way to get to work without paying attention to the consequences of the decision to others. The Internet network can be viewed as a similar macrocosm that functions neither by the control of a central authority nor by coordinated rules. The basic motivation for each individual can only be deduced by greed and selfishness. Every player chooses the most convenient
route and uses strategies to maximize possible payoff. In other words, we face a combination of game theory and graph theory for dealing with large networks both in quantitative analysis and algorithm design. Many questions arise. Instead of just proving the existence of Nash equilibrium, we would like to design algorithms to effectively compute or approximate the Nash equilibrium. How rapidly can such algorithms converge? There has been a great deal of progress in the computational complexity of Nash equilibrium [22, 37].

The analysis of selfish routing comes naturally in network management. How much does uncoordinated routing affect the performance of the network, such as stability, congestion, and delay? What are the trade-offs for some limited regulation? The so-called price of anarchy refers to the worst-case analysis to evaluate the loss of collective welfare from selfish routing. There has been extensive research done on selfish routing [62]. The reader is referred to several surveys [41, 51] and some recent books on this topic [61].

Many classical problems in graph theory can be reexamined from the perspective of game theory. One popular topic on graphs is chromatic graph theory. For a given graph $G$, what is the minimum number of colors needed to color the vertices of $G$ so that adjacent vertices have different colors? In addition to theoretical interests, the graph coloring problem has numerous applications in the setting of conflict resolution. For example, each faculty member (as a vertex) wishes to schedule classes in a limited number of classrooms (as colors). Two faculty members who have classes with overlapping time are connected by an edge, and then the problem of classroom scheduling can be viewed as a graph coloring problem. Instead of having a central agency to make assignments, we can imagine a game-theoretic scenario that the faculty members coordinate among themselves to decide a nonconflicting assignment. Suppose there is a payoff of 1 unit for each player (vertex) if its color is different from all its neighbors. A proper coloring is then a Nash equilibrium, since no player has an incentive to change his/her strategy.

Kearns et al. [48] conducted an experimental study of several coloring games on specified networks. Many examples were given to illustrate the difficulties in analyzing the dynamics of large networks in which each node takes simple but selfish steps. This calls for rigorous analysis, especially along the line of the combinatorial probabilistic methods and generalized Martingale approaches that have been developed in the past ten years [20]. Some work in this direction has been done on a multiple round model of graph coloring games [20], but more work is needed.

## Summary

It is clear that we are at the beginning of a new journey in graph theory, emerging as a central part of the information revolution. It is a long way from the "seven bridges of Königsberg", a problem posed by Leonhard Euler in 1736. In contrast to its origin in recreational mathematics, graph theory today uses sophisticated combinatorial, probabilistic, and spectral methods with deep connections with a variety of areas in mathematics and computer science. In this article, some vibrant new directions in graph theory have been selected and described to illustrate the richness of the mathematics involved, as well as the utilization through major threads of current technology. The list of the sampled topics is by no means complete, since these areas of graph theory are still rapidly developing. Abundant opportunities in research, theoretical and applied, remain to be explored.

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# Hadwiger's Conjecture and Seagull Packing 

Maria Chudnovsky

The four-color theorem $[1,2]$ is one of the most well-known results in graph theory. Originating from the question of coloring a world map, posed in the middle of the nineteenth century, it has since fascinated hundreds of researchers and motivated a lot of beautiful mathematics. It states that every planar graph (that is, a graph that can be drawn in the plane without crossings) can be properly colored with four colors. Kuratowski's theorem [15] connects the planarity of a graph with the absence of certain topological structures in it. Thus, the four-color theorem tells us that if a graph lacks a certain topological obstruction, then it is four-colorable. In 1941 Hadwiger made a conjecture that generalized this idea, from four-colorability to general $t$-colorability. Since then, Hadwiger's conjecture has received a great deal of attention, but only a few special cases have been solved. In this article we survey some of the known results on the conjecture and discuss recent progress.

To make this more precise, let us say that a graph $G$ contains a $K_{t}$-minor, or a clique minor of size $t$, if there exist $t$ pairwise vertex-disjoint connected subgraphs $F_{1}, \ldots, F_{t}$ of $G$, such that for every distinct $i, j \in\{1, \ldots t\}$, there is an edge with one end in $V\left(F_{i}\right)$ and the other in $V\left(F_{j}\right)$. (For a graph $F$, we denote by $V(F)$ the vertex set of $F$, and by $E(F)$ the edge set of $F$.) For coloring problems, it is necessary to assume that graphs are loopless, that is, every edge has two distinct ends, and we adopt this convention here. We can now state Hadwiger's conjecture [10].

[^9]Conjecture 1. For every integer $t \geq 2$, every graph that is not $(t-1)$-colorable contains a $K_{t}$-minor.

Let us examine this conjecture for small values of $t$. For $t=2$, it states that if a graph is not one-colorable, then it contains a $K_{2}$-minor. Indeed, if $G$ is not one-colorable, then there is an edge in $G$ with ends $u, v$, say; now setting $V\left(F_{1}\right)=\{u\}, V\left(F_{2}\right)=\{v\}$, and $E\left(F_{1}\right)=E\left(F_{2}\right)=\varnothing$, we observe that $F_{1}, F_{2}$ satisfy the conditions of the definition of a $K_{2}$-minor. For $t=3$, it is not difficult to see that if a graph $G$ is not two-colorable, then it contains a cycle $C$ with at least three vertices. Let the vertices of $C$ be $c_{1}, \ldots, c_{k}$ in (cyclic) order. Set $V\left(F_{1}\right)=\left\{c_{1}\right\}, V\left(F_{2}\right)=\left\{c_{2}\right\}, E\left(F_{1}\right)=E\left(F_{2}\right)=\varnothing$ and make $F_{3}$ be the path $c_{3}-\ldots-c_{k}$. Again, $F_{1}, F_{2}, F_{3}$ satisfy the definition of a $K_{3}$ minor, and the conjecture holds. The situation for $t=4$ is a little more complicated, but still quite simple. Graphs with no $K_{4}$-minor are known as series-parallel graphs. Series-parallel graphs can be built starting from single vertices by applying a few simple operations. What is important for our purposes is that all series-parallel graphs have a vertex of degree at most two (after deleting parallel edges) and therefore can be colored with at most three colors. Thus Hadwiger's conjecture holds for $t=4$.

Things become more complicated when we turn to the case of $t=5$. In this case, Hadwiger's conjecture asserts that every graph with no $K_{5}$ minor is four-colorable. Together with Kuratowski's theorem [15], which states that a graph is planar if and only if it has no $K_{5}$-minor and no $K_{3,3}$-minor (it is not really important for this article what exactly a $K_{3,3}$-minor is), this implies the four-color theorem. Thus the case of $t=5$ is quite deep. Wagner [18] showed that Hadwiger's conjecture
for $t=5$ is equivalent to the four-color theorem, and so Hadwiger's conjecture is true for $t=5$.

Hadwiger's conjecture is also known to be true for $t=6$. This is a theorem of Robertson, Seymour, and Thomas [17], and the proof also uses the four-color theorem. More precisely, the authors show that every minimal counterexample to Hadwiger's conjecture with $t=6$ is obtained from a planar graph by adding one new vertex with arbitrary neighbors (these are called apex graphs). Since every planar graph is four-colorable, it follows that every apex graph is five-colorable, which contradicts the fact that we were looking at a counterexample to Hadwiger's conjecture with $t=6$. For $t \geq 7$, Hadwiger's conjecture is open.

Since the proofs of all the known cases of Hadwiger's conjecture rely on the fact that for small $t$ graphs with no $K_{t}$-minor are close to being planar, it is interesting to consider the opposite extreme. What about very dense graphs, with very large chromatic number, where we therefore should expect to find very large clique minors? Studying these cases may lead to new proof techniques, since such graphs are certainly not close to being planar. This may also be a good place to look for a counterexample.

Let us therefore turn to graphs with no stable set of size three (meaning that there do not exist three vertices, all pairwise nonadjacent). Let $G$ be such a graph, and let $|V(G)|=n$. Since in every proper coloring of $G$, all color classes have size at most two, it follows that the chromatic number of $G$ is at least $\left\lceil\frac{n}{2}\right\rceil$, and so, if true, Hadwiger's conjecture would tell us that $G$ has a clique minor of size at least $\left\lceil\frac{n}{2}\right\rceil$. This problem will be the focus of the rest of this article.

Conjecture 2. Let $G$ be a graph with $n$ vertices and no stable set of size three. Then $G$ contains a clique minor of size $\left\lceil\frac{n}{2}\right\rceil$.

At first, Conjecture 2 looks weaker than the case of Hadwiger's conjecture for graphs with no stable sets of size three. However, it was shown in [16] that Conjecture 2 implies that Hadwiger's conjecture holds for graphs with no stable set of size three.

First let us remark that Conjecture 2 becomes easy if we replace $\left\lceil\frac{n}{2}\right\rceil$ with $\left\lceil\frac{n}{3}\right\rceil$ :
Theorem 1. Let $G$ be a graph with $n$ vertices and no stable set of size three. Then $G$ contains a clique minor of size $\left\lceil\frac{n}{3}\right\rceil$.
Proof. Take a maximal collection, $F_{1}, \ldots, F_{k}$, of vertex-disjoint induced two-edge paths in $G$ pairwise with edges between them (that means that each $F_{i}$ has three vertices $a_{i}, b_{i}, c_{i}$, and two edges $a_{i} b_{i}$ and $b_{i} c_{i}$; and $a_{i}$ is nonadjacent to $c_{i}$ in $\left.G\right)$. Let $H$ be the graph obtained from $G$ by deleting the union of the vertex sets of $F_{1}, \ldots, F_{k}$. It is not difficult to see, using the maximality of the collection
$F_{1}, \ldots, F_{k}$, that $V(H)$ is the union of two cliques, say $H_{1}$ and $H_{2}$. We may assume that $\left|H_{1}\right| \geq\left|H_{2}\right|$. Let $F_{k+1}, \ldots, F_{k+\left|H_{1}\right|}$ be the one-vertex graphs consisting of the vertices of $H_{1}$. Then $F_{k+1}, \ldots, F_{k+\left|H_{1}\right|}$ pairwise have an edge between them. Moreover, since $G$ has no stable set of size three, and $a_{i}$ is nonadjacent to $c_{i}$ for every $i \in\{1, \ldots, k\}$, it follows that for every $i \in\{1, \ldots, k\}$, every vertex of $V(G) \backslash V\left(F_{i}\right)$ has a neighbor in $V\left(F_{i}\right)$. Therefore, $F_{1}, \ldots, F_{k+\left|H_{1}\right|}$ is a clique minor in $G$. Now an easy calculation shows that $k+\left|H_{1}\right| \geq\left\lceil\frac{n}{3}\right\rceil$. Thus we produced a clique minor of size at least $\left\lceil\frac{n}{3}\right\rceil$ in $G$, as required.

Notice that each of the subgraphs $F_{i}$ is either a single vertex or an induced two-edge path. Let us call an induced two-edge path in $G$ a seagull (in $G$ ). Seagulls seem to be a good tool for attacking Conjecture 2, and we will come back to them later in this text. Moreover, in $[12,13]$ a similar but more general technique was used, in a very elegant way, to obtain large clique minors in graphs with arbitrarily large stable sets.

If Conjecture 2 is true, then there exist $\left\lceil\frac{n}{2}\right\rceil$ disjoint connected subgraphs of $G$, pairwise with edges between them. It therefore follows that at least $\left\lceil\frac{n}{4}\right\rceil$ of these subgraphs have at most two vertices. Let us call a clique minor an edge clique minor if all the subgraphs $F_{1}, F_{2}, \ldots$ have at most two vertices. Seymour made the following stronger conjecture:

Conjecture 3. Let $G$ be a graph with $n$ vertices and no stable set of size three. Then $G$ has an edge clique minor of size $\left\lceil\frac{n}{2}\right\rceil$.

If Conjecture 3 is false, Hadwiger's conjecture may still be true, but, in order for Hadwiger's conjecture to be true, Conjecture 3 needs to hold with $\left\lceil\frac{n}{2}\right\rceil$ replaced by $\left\lceil\frac{n}{4}\right\rceil$. Unfortunately, no general results are known for $\left\lceil\frac{n}{2}\right\rceil$ replaced by $\lceil c n\rceil$ for any constant $c>0$. However, recently, Blasiak [4] was able to make progress on Seymour's conjecture by adding a simplifying assumption. He assumed that $V(G)$ is the union of three cliques, and then he could prove that $G$ contains an edge clique minor of size $\left\lceil\frac{n}{2}\right\rceil$. For even $n$ this gives Seymour's conjecture (in the case when $V(G)$ is the union of three cliques). The idea of Blasiak's proof is as follows. Let $V(G)=M_{1} \cup M_{2} \cup M_{3}$, where $M_{1}, M_{2}, M_{3}$ are cliques. Let us call an edge of $G$ big if its ends are in two different $M_{i}$ 's. Thus for every two big edges $e$ and $f$, there exists $i \in\{1,2,3\}$ such that both $e$ and $f$ have an end in $M_{i}$, and therefore there is an edge between $e$ and $f$. This means that any matching of big edges is an edge clique minor in $G$ (a matching is a collection of disjoint edges). Blasiak showed (using Tutte's theorem [3]) that, except in cases that had already been understood, $G$ has a perfect matching of big edges (or a matching of big edges covering all but one vertex, if $G$ has an
odd number of vertices), thus producing an edge clique minor of size $\left\lceil\frac{n}{2}\right\rceil$.

This is a very nice result, but it is really disappointing that the clique minor it produces is one too small, compared with Hadwiger's conjecture. It is thus very tempting to try to fix this shortcoming. Now, after a few years of frustration, here is a fix and a strengthening [5]:

Theorem 2. Let $G$ be a graph with n vertices and with no stable set of size three. Assume that some clique of $G$ has cardinality at least $\frac{n}{4}$, and at least $\frac{n+3}{4}$ if $n$ is odd. Then $G$ has a clique minor of size $\left\lceil\frac{4}{\frac{n}{2}}\right\rceil$.

Theorem 1 asserts that Hadwiger's conjecture is true for a larger class of graphs than that considered by Blasiak, but unlike Blasiak's theorem, it does not guarantee an edge clique minor. In fact, in all the novel cases that Theorem 1 deals with, the minor it produces uses seagulls. Here is an outline of the proof of Theorem 1. Let $p=\left\lceil\frac{n}{2}\right\rceil$. By a previous result [16], we may assume that $G$ is $p$-connected (in fact, it is not difficult to prove that if $G$ is not $p$-connected, then it contains an edge clique minor of size $p$ ). Now let $Z$ be a clique in $G$ of maximum size, and let $F_{1}, \ldots, F_{t}$ be a collection of pairwise vertex-disjoint seagulls in $G \backslash Z$ with $t$ as large as possible. Let $F_{t+1}, \ldots, F_{t+|Z|}$ be one-vertex graphs consisting of the vertices of $Z$. As before, $F_{1}, \ldots, F_{t+|Z|}$ is a clique minor in $G$. The main result of [5] is that $t+|Z| \geq p$, and therefore $G$ contains a clique minor of size at least $p$, as required. Let us remark that here, again (as in the proof that every graph with no stable set of size three has a clique minor of size $\left\lceil\frac{n}{3}\right\rceil$ ), the minor consists of one-vertex graphs and seagulls.

Thus, trying to prove Theorem 1 gives rise to the following question: for an integer $k$, what are necessary and sufficient conditions for a graph $G$ to have $k$ pairwise vertex-disjoint seagulls? And, of course, there is the corresponding algorithmic question: given a graph $G$ and an integer $k$, is there a polynomial time algorithm to test if $G$ contains $k$ pairwise vertex-disjoint seagulls? For general graphs $G$, Dor and Tarsi showed that this problem is NP-complete [7]. But what about graphs $G$ with no stable set of size three? In joint work with Seymour [5], we were able to prove the following (the five-wheel is a graph consisting of an induced cycle $C$ on five vertices, and a vertex complete to $V(C))$ :

Theorem 3. Let $G$ be a graph with no stable set of size three, and let $k$ be an integer. Assume that if $k=2$, then $G$ is not the five-wheel. Then $G$ contains $k$ pairwise vertex-disjoint seagulls if and only if

- $|V(G)| \geq 3 k$, and
- $G$ is $k$-connected, and
- for every clique $C$ of $G$, if $D$ denotes the set of vertices in $V(G) \backslash C$ that have both a neighbor and a non-neighbor in $C$, then $|D|+|V(G) \backslash C| \geq 2 k$, and
- the complement graph of $G$ has a matching with $k$ edges.

Now the proof of Theorem 1 boils down to checking that the conditions of Theorem 2 hold for the appropriate graph $G$ and integer $k$. The proof of Theorem 2 is far too complex to be included here, but let us just mention that it uses many beautiful results from graph and matroid theory, such as Hall's theorem [11], the Tutte-Berge formula [3], and Edmonds' matroid union theorem [8].

And what about the algorithmic question? Recently, in joint work with Oum and Seymour [6], we were able to design an algorithm that checks (in polynomial time) whether each of the necessary and sufficient conditions of Theorem 2 is satisfied. The only step that was not previously known is checking the third condition (that is, an algorithm that, given an integer $k$ and a graph $G$ with no stable set of size three, tests whether for every clique $C$ of $G$, if $D$ denotes the set of vertices in $V(G) \backslash C$ that have both a neighbor and a non-neighbor in $C$, then $|D|+|V(G) \backslash C| \geq 2 k)$. We were able to reduce this step to the well-known question of finding a maximum matching in a bipartite graph.

However, even before the result of [6], Theorem 2 could be used in an indirect way to test if a given graph contains $k$ pairwise disjoint seagulls. The algorithm was not as simple conceptually, but it used some interesting tools, and so it is worthwhile mentioning. Let us consider the fractional version of the question. Let $G$ be a graph with no stable set of size three, and let $S$ be the set of all seagulls in $G$. A fractional packing of seagulls in $G$ is a function $f: S \rightarrow[0,1]$ such that for every vertex $v$ of $G$

$$
\sum_{S \in S} f(S) \leq 1
$$

The value of the packing $f$ is $\sum_{S \in S} f(S)$. Now the question is: when does $G$ have a fractional packing of seagulls of value at least $k$ ? This question can be easily presented as a linear programming problem, of size polynomial in $|V(G)|$, and thus it can be answered in polynomial time using the ellipsoid method [14].

Here is another theorem from [5]:
Theorem 4. Let $G$ be a graph with no stable set of size three, and let $k \geq 0$ be a real number. The following are equivalent:

- There is a fractional packing of seagulls of value at least $k$.
- $|V(G)| \geq 3 k$, and $G$ has connectivity at least $k$, and for every clique $C$, if $D$ denotes the set of vertices in $V(G) \backslash C$ that have both a neighbor and a non-neighbor in $C$, then $|D|+|V(G) \backslash C| \geq 2 k$.

Now we proceed as follows. Let $k$ be an integer and let $G$ be a graph with no stable set of size three. First we test if $G$ has a fractional packing of seagulls of value at least $k$. If not, then $G$ does not have $k$ pairwise vertex-disjoint seagulls, and we can stop. If yes, Theorem 2 and Theorem 3 imply that in order to check whether $G$ contains $k$ pairwise vertex-disjoint seagulls, it remains to check the following two conditions:

- If $k=2$, then $G$ is not the five-wheel.
- The complement graph of $G$ has a matching with $k$ edges.
Checking the first condition is straightforward, and there is a well-known polynomial-time algorithm for the second, due to Edmonds [9]. Thus we obtain the required polynomial-time algorithm.

To conclude, in this article we described some recent progress on Hadwiger's conjecture for graphs with no stable set of size three (Conjecture 2). In general, this conjecture is still open and may even be false. Proving it would be a great step toward solving Hadwiger's conjecture in general, and it is also likely to be a good place to look for a counterexample, if one exists.

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## W H A T I S...

# a Kinetic Solution for Degenerate ParabolicHyperbolic Equations? 

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Nonlinear degenerate parabolic-hyperbolic equations are one of the most important classes of nonlinear partial differential equations. Nonlinearity and degeneracy are two main features of these equations and yield several striking phenomena that require new mathematical ideas, approaches, and theories. On the other hand, because of the importance of these equations in applications, there is a large literature for the design and analysis of various numerical methods to calculate these solutions. In addition, a well-posedness theory (existence, uniqueness, and stability) is in great demand.

A nonlinear degenerate parabolic-hyperbolic equation typically takes the form:

$$
\partial_{t} u+\nabla \cdot \mathbf{f}(u)=\nabla \cdot(\mathbf{A}(u) \nabla u), \quad u \in \mathbb{R}
$$

Here $t \in \mathbb{R}_{+}:=[0, \infty) ; \mathbf{x}=\left(x_{1}, \ldots, x_{d}\right) \in \mathbb{R}^{d}$; $\nabla=\left(\partial_{x_{1}}, \ldots, \partial_{x_{d}}\right)$ is the gradient with respect to $\mathbf{x} ; \mathbf{f}: \mathbb{R} \rightarrow \mathbb{R}^{d}$ is differentiable and $\mathbf{f}^{\prime}(u)$ is locally bounded; and the $d \times d$ matrix $\mathbf{A}(u)$ is symmetric, nonnegative, and locally bounded. In physics, such an equation is also called a nonlinear advection-diffusion equation. The diffusion term

[^10]$\nabla \cdot(\mathbf{A}(u) \nabla u)$ reflects a transport of molecules, by random molecular motion within the fluid, from a region of higher concentration to one of lower concentration. The advection term $\nabla \cdot \mathbf{f}(u)$ describes a transport of the fluid flow.

One of the prototypes is the porous medium equation:

$$
\partial_{t} u=\Delta\left(u^{p}\right), \quad p>1
$$

where $\Delta=\sum_{j=1}^{d} \partial_{x_{j} x_{j}}$ is the Laplace operator. The equation is degenerate on the level set $\{u=0\}$; away from this set, the equation is strictly parabolic. Even though the nonlinear equation is parabolic, the solutions exhibit a certain hyperbolic feature, which results from the degeneracy. One striking family of solutions was found around 1950 in Moscow by Zel'dovich-Kompaneets and Barenblatt. The supports of these solutions propagate at finite speeds (cf. [5]), whereas a solution to a nondegenerate parabolic equation propagates at an infinite speed.

The simplest example for the isotropic case (i.e., $\mathbf{A}(u)$ is diagonal) with both phases is

$$
\partial_{t} u+\partial_{x}\left(\frac{u^{2}}{2}\right)=\partial_{x x}[u]_{+},
$$

where $[u]_{+}=\max \{u, 0\}$. The equation is hyperbolic when $u<0$ and parabolic when $u>0$, and the level set $\{u=0\}$ is a free boundary separating the hyperbolic phase from the parabolic phase. For any constant states $u^{ \pm}$with $u^{+}<0<u^{-}$and
$y_{*} \in \mathbb{R}$, there exists a unique nonincreasing profile $\phi=\phi(y)$ such that

$$
\begin{aligned}
& \lim _{y \rightarrow-\infty}\left(\phi(y), \phi^{\prime}(y)\right)=\left(u^{-}, 0\right), \\
& \phi(y) \equiv u^{+} \text {for } y>y_{*}, \\
& \lim _{y \rightarrow y_{*}-0}\left(\phi(y), \phi^{\prime}(y)\right)=(0,-\infty), \\
& \lim _{y \rightarrow y_{*}-0} \frac{d[\phi(y)]_{+}}{d y}=\frac{1}{2} u^{+} u^{-}
\end{aligned}
$$

so that $u(t, x)=\phi(x-s t), s=\frac{u^{+}+u^{-}}{2}$, is a discontinuous solution connecting $u^{+}$to $u^{-}$. Although $u(t, x)$ is discontinuous, $[u(t, x)]_{+}$is a continuous function even across the interface $\{u=0\}$. See Figure 1.


Figure 1. A discontinuous profile connecting the hyperbolic phase to the parabolic phase.

The well-posedness issue for the initial value problem is relatively well understood if-relying on the work of Lax, Oleinik, Volpert, and Kruzhkov, and more recently of Lions-Perthame-Tadmor [4]one removes the term $\nabla \cdot(\mathbf{A}(u) \nabla u)$. It is equally well understood when the set $\{u: \operatorname{rank}(\mathbf{A}(u))<$ $d\}$ consists only of isolated points with certain orders of degeneracy (cf. [3,5]). For the isotropic case, the well-posedness of entropy solutions was established by various people: solutions with bounded variation by Volpert-Hudjaev (1969), solutions in $L^{\infty}$ by Carrillo (1999), and unbounded solutions by Chen-DiBenedetto (2001). Here an entropy solution is a discontinuous solution that satisfies both the equation and an additional admissible inequality (called an entropy inequality, motivated by the Second Law of Thermodynamics) in the sense of distributions. However, the general case with solutions in $L^{1}$ remained open until 2003, when the kinetic approach was developed in ChenPerthame [1] in order to deal with both parabolic and hyperbolic phases. This unified approach is motivated by the macroscopic closure procedure of the Boltzmann equation in kinetic theory, the hydrodynamic limit of large particle systems in
statistical mechanics, and early works on kinetic schemes to calculate shock waves and on the theoretical kinetic formulation for the pure hyperbolic case (see $[1,4]$ ).

More precisely, consider the quasi-Maxwellian kinetic function $\chi$ on $\mathbb{R}^{2}$ :

$$
\chi(v ; u)=\left\{\begin{array}{cl}
+1 & \text { for } 0<v<u \\
-1 & \text { for } u<v<0 \\
0 & \text { otherwise }
\end{array}\right.
$$

If $u \in L^{\infty}\left(\mathbb{R}_{+} ; L^{1}\left(\mathbb{R}^{d}\right)\right)$, then $\chi(v ; u) \in L^{\infty}\left(\mathbb{R}_{+}\right.$; $\left.L^{1}\left(\mathbb{R}^{d+1}\right)\right)$.

A function $u(t, \mathbf{x}) \in L^{\infty}\left(\mathbb{R}_{+} ; L^{1}\left(\mathbb{R}^{d}\right)\right)$ is called a kinetic solution if $u(t, \mathbf{x})$ satisfies the following:
(i) The kinetic equation:

$$
\begin{aligned}
& \partial_{t} \chi(v ; u)+\mathbf{f}^{\prime}(v) \cdot \nabla \chi(v ; u) \\
& \quad-\nabla \cdot(\mathbf{A}(v) \nabla \chi(v ; u)) \\
& =\partial_{v}(m+n)(t, \mathbf{x} ; v)
\end{aligned}
$$

holds in the sense of distributions with the initial data $\left.\chi(v ; u)\right|_{t=0}=\chi\left(v ; u_{0}\right)$, for some nonnegative measures $m(t, \mathbf{x} ; v)$ and $n(t, \mathbf{x} ; v)$, where $n(t, \mathbf{x} ; v)$ is defined by

$$
\langle n(t, \mathbf{x} ; \cdot), \psi(\cdot)\rangle:=\sum_{k=1}^{d}\left(\nabla \boldsymbol{\beta}_{k}^{\psi}\right)^{2} \in L^{1}\left(\mathbb{R}_{+} \times \mathbb{R}^{d}\right)
$$

for any $\psi \in C_{0}^{\infty}(\mathbb{R})$ with $\psi \geq 0$ and $\boldsymbol{\beta}_{k}^{\psi}(u):=$ $\int^{u} \sigma_{k}(v) \sqrt{\psi(v)} d v$, where $\sigma_{k}(v)$ is the $k$ th column of the matrix $\boldsymbol{\sigma}(v)$ such that $\mathbf{A}(v)=\boldsymbol{\sigma}(v) \boldsymbol{\sigma}(v)^{\top}$;
(ii) There exists $\mu \in L^{\infty}(\mathbb{R})$ with $0 \leq \mu(v) \rightarrow 0$ as $|v| \rightarrow \infty$ such that

$$
\int_{0}^{\infty} \int_{\mathbb{R}^{d}}(m+n)(t, \mathbf{x} ; v) d t d \mathbf{x} \leq \mu(v)
$$

(iii) For any two nonnegative functions $\psi_{1}, \psi_{2} \in$ $C_{0}^{\infty}(\mathbb{R})$,

$$
\begin{gathered}
\sqrt{\psi_{1}(u(t, \mathbf{x}))} \sum_{i=1}^{d} \partial_{x_{i}} \beta_{i k}^{\psi_{2}}(u(t, \mathbf{x})) \\
=\sum_{i=1}^{d} \partial_{x_{i}} \beta_{i k}^{\psi_{1} \psi_{2}}(u(t, \mathbf{x}))
\end{gathered}
$$

for almost every ( $t, \mathbf{x}$ ).

## Well-posedness in $L^{1}$

The advantage of the notion of kinetic solutions is that the kinetic equation is well defined even when $\mathbf{f}(u(t, \mathbf{x}))$ and $\mathbf{A}(u(t, \mathbf{x}))$ are not locally integrable so that $L^{1}$ is a natural space in which the kinetic solutions are posed. This notion also covers the so-called renormalized solutions used in the context of scalar hyperbolic conservation laws by Bénilan-Carrillo-Wittbold (2000). Based on this notion, a new approach has been developed in [1] to establish a well-posedness theory for the initial value problem for kinetic solutions
in $L^{1}$. In particular, the $L^{1}$-contraction of kinetic solutions is established, which implies that the kinetic solution forms a semigroup with respect to $t>0$.

## Consistency

The uniqueness result implies that any kinetic solution in $L^{\infty}$ must be an entropy solution. On the other hand, any entropy solution is actually a kinetic solution. Therefore, the two notions are equivalent for solutions in $L^{\infty}$, although the notion of kinetic solutions is more general.

## Connection with the Classical Entropy Method

By the very construction of the kinetic approach, any results using the classical entropy method can easily be translated in terms of the old Kruzkov entropies by integrating in $v$. In the case of uniqueness for the general case, this was performed by Bendahmane-Karlsen (2004).

Condition (iii), which is a fundamental and natural property similar to a chain rule, automatically holds in the isotropic case. It is also the cornerstone for the uniqueness in the general case. Moreover, condition (ii) implies that $m+n$ has no support at $u=\infty$.

Furthermore, let $u \in L^{\infty}\left(\mathbb{R}_{+} \times \mathbb{R}^{d}\right)$ be the unique entropy solution with periodic initial data $u_{0} \in L^{\infty}$ for period $\mathbb{T}_{P}=\Pi_{i=1}^{d}\left[0, P_{i}\right]$. Assume that $\mathbf{f}(u)$ is in $C^{1}, \mathbf{A}(u)$ is continuous, and both satisfy the nonlinearity-diffusivity condition: For any $\tau \in \mathbb{R}$ and $\mathbf{y} \in \mathbb{R}^{d}$ with $|\mathbf{y}|=1$, the set

$$
\left\{v: \tau+\mathbf{f}^{\prime}(v) \cdot \mathbf{y}=0, \mathbf{y} \mathbf{A}(v) \mathbf{y}^{\top}=0\right\} \subset \mathbb{R}
$$

has Lebesgue measure zero. Then
$\left\|u(t, \cdot)-\frac{1}{\left|\mathbb{T}_{P}\right|} \int_{\mathbb{T}_{P}} u_{0}(\mathbf{x}) d \mathbf{x}\right\|_{L^{1}\left(\mathbb{T}_{P}\right)} \rightarrow 0 \quad$ as $t \rightarrow \infty$, where $\left|\mathbb{T}_{P}\right|$ is the volume of the period $\mathbb{T}_{P}$.

The nonlinearity-diffusivity condition implies that there is no interval of $v$ in which $\mathbf{f}(v)$ is affine and $\mathbf{A}(v)$ is degenerate. Unlike the pure hyperbolic case, the equation is no longer self-similar invariant. The argument for achieving this decay result is based on the kinetic approach developed in [1], involves a time-scaling and a monotonicity-in-time property of the entropy solution, and employs the advantages of the kinetic equation, in order to recognize the role of the nonlinearity-diffusivity of the equation (see [2]).

Follow-up results based on this approach include $L^{1}$-error estimates and continuous dependence of solutions both on $\mathbf{f}$ and $\mathbf{A}$, and more general degenerate parabolic-hyperbolic equations. Further regularity results of solutions have been established by Tadmor-Tao (2007).

## Further Reading

[1] G.-Q. Chen and B. Perthame, Well-posedness for nonisotropic degenerate parabolic-hyperbolic equations, Annales de l'Institut Henri Poincaré: Analyse non linéaire 20 (2003), 645-668.
[2] _, Large time behavior of periodic solutions to anisotropic degenerate parabolic-hyperbolic equations, Proc. Amer. Math. Soc. 137 (2009), 3003-3011.
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[4] P.-L. Lions, B. Perthame, and E. Tadmor, A kinetic formulation of multidimensional scalar conservation laws and related equations, J. Amer. Math. Soc. 7 (1994), 169-191.
[5] J. L. VÁzquez, The Porous Medium Equation: Mathematical Theory, Oxford University Press, Oxford, 2007.


## A Mathematical Medley <br> Fifty Easy Pieces on Mathematics

George G. Szpiro, Nene Zïrcher Zeitung, Zurich, Switzerland
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# The Mathematical Dramatist: Interview with Gioia De Cari 

Julie Rehmeyer

Gioia De Cari was a third of the way through her doctoral thesis in math at the Massachusetts Institute of Technology when she up and left. She became an actress and a playwright and hasn't thought about research mathematics again.

What happened? Why would a woman abandon a promising career, a love of mathematics, and years of labor in order to start from scratch in the risky world of art?

Those are the questions De Cari explores in her one-woman play Truth Values: One Girl's Romp through M.I.T.'s Male Math Maze. She depicts her experiences in the MIT math department in the late 1980s, showing the thrill and the grind of research, her sense of alienation, the supportiveness and remoteness of her professors, her struggle to connect with her fellow students, the ever-present sexism that ground her down, and how she found her true calling.

Truth Values premiered in August 2009 at the New York International Fringe Festival, winning the 2009 Fringe NYC Overall Excellence Award. It then traveled to Cambridge, MA, in September, where it sold out its entire three-week run. Its three performances in San Francisco during the 2010 Joint Mathematics Meetings sold out as well. It is continuing to play in both public and private performances around the country.

JR: What made you decide to write the play?
GC: In 2000 or so I did a solo show called The 9th Envelope that was like an Alice in Wonderland fantasy story, and I wove in some interludes about math. What really surprised me was how captivated audiences were by the math parts. People would come up to me afterward to talk about them. I thought, "Oh wow, my next show better be all about math!"

[^11]That was the genesis of Truth Values. But as I got into it, I found there were all kinds of things that were difficult about turning autobiographical material into a work of art. In particular, how do you find the right tone? My perspective was that everyone I had known in the math world was just doing the best they could, even if it wasn't as good as it needed to be. I didn't want to go in a negative direction with it, but the play also couldn't leave out the sexism, because that was a strong aspect of what happened to me. I was fighting with myself about it, thinking, "Look at how far MIT has come. I shouldn't bring this up now." In fact, I'd decided to shelve the project.

But then Larry Summers came along. [In 2005, while Summers was president of Harvard, he remarked in a public forum that he believed that differences in inherent aptitude were a bigger factor than sexual discrimination in the low numbers of women in the upper echelons of academia.] When he said that, that's when I thought, I've got to speak up here.

The most upsetting thing to me, even more than Summers's comments, was what happened to Nancy Hopkins in the wake of the comments. She was a biologist at MIT, and she was there when Summers made his remarks. She said afterwards that she left because otherwise she would have blacked out or thrown up. The press just ripped her to shreds over this. She got hate mail for a year.

As an artist, you have more license to say certain things than academics or scientists do. So at that point I felt like I had a responsibility to speak up, and I finished the play.

JR: Before you wrote the play, what did you say when people asked why you left MIT?

GC: At first I tried to keep my math background a secret. Honestly, what does it have to do with being an actor? Then, invariably, someone would want to know where I went to school and I'd have to
tell them and they'd get all interested, and they'd back me in a corner and say, No, no, no you have to tell me!

So, I would tell them some stories about being in math. I was always surprised that people actually found them interesting, and these stories eventually became the show.

Sexism was the thing I really didn't want to talk about. When I first thought about creating the show, I thought maybe I could leave it out entirely. But of course I couldn't. It was too important a part of the tapestry of my experiences in that world. When I was at MIT, several professors asked me, "You're married, so why are you here? Why aren't you having babies?" One of my professors asked me to deliver cookies to a seminar. I was driven out of my office by an overly amorous fellow student.

It's such a difficult thing, with sexism, to suss out exactly what's happening. All the time while these things were happening, the question was in my mind, Is this sexism, or is it something else? I'd think, oh, I'm making a mountain out of a molehill. It's just a plate of cookies! It's trivial, isn't it? Why is this bothering me? It's after a zillion little things that are no big deal that it sneaks up on you.

JR: What impact did it have on you to be asked, for example, if you really wouldn't rather stay home and have babies? Why was it a big deal?

GC: I was shocked. I never imagined that anyone would ask me that. I didn't even experience it as sexism. It just upset me and I didn't know why. I didn't understand it at all. It took me so many years and writing this play to understand how I felt about that.

I think the reason it was so hard is that it pressed my buttons. I had come from Berkeley, which was a very liberal and progressive place, but in point of fact, some of my friends and family tended to be pretty socially conservative in many respects, and there tended to be a strong emphasis on having a family. The fact that I was at graduate school, not having a family yet and deciding to put it off, was something that was not in line with a lot of people in my circle and their values. I always felt bad because I always wanted to please everyone in my circle, all my family and all my friends.

So I felt like, well, I can go and do the Ph.D. and I'll make my dad happy and proud, and then I can
go have a family and make everyone else happy and proud. Everything was about pleasing everyone else. It didn't occur to my young, naïve, immature self that these kinds of questions are also about pleasing yourself. So that question pressed my button and made me feel like, I'm not doing the thing I'm supposed to do.

Of course, it took me many years to think it through and think, Do you think anyone asked men this? Were these guys asking the male students, Gee, you're married, why aren't you having kids? You wonder. Maybe they were, who knows!

JR: Many women respond to these issues by desexualizing themselves: dressing just like the guys, picking up the masculine norms of the place, removing any hint of femininity from their speech or movement or actions. How did you handle it?

GC: In the play I show how I tried to do that and then started having nightmares, because that ran so deeply against the grain for me. The femme side of me is very strong and deeply important to me. When I tried to excise it, I had recurring nightmares that my breasts were chopped off, night after night, to the point where I gave up on hiding my femininity. I decided that it really deeply went against my nature, so I had to go back to dressing and behaving in a way that I felt more comfortable.

But then, my usual reasonably feminine look started to morph, without me quite intending it. I started acting out by wearing more and more outrageously feminine outfits, to the point where I started calling them "fashion experiments". I just sort of found myself doing it every day. I was guided by some wicked elf on my shoulder. I once found myself wearing something so outrageous that I got myself into quite a predicament.... But I won't spoil that. You'll have to see the play to find out.

JR: It seems like this illustrates how the gender issues go beyond simple sexism and into issues of style and culture.

GC: Absolutely. Also, I would say, there's an aspect that's not a man-woman thing necessarily. It might be a personality thing. It might be about people who are people-oriented versus dataoriented, concept-oriented.

I wonder if women are more encouraged to be people-oriented, and so it's brought out in them more. Maybe if men are not inclined that way, they're left to their own devices more. I wonder.


Postcard advertisement for Truth Values.

I felt for myself the personality issue was a big, big part of it. Humanity interests me far more than mathematics ever could. That's not necessarily the right sort of personality for mathematics.

JR: That's so interesting, and it connects with a moment in the play: As you are getting increasingly unhappy in math, you ask for advice from more senior women in mathematics, and one of them tells you that if you can do anything else other than mathematics, you should. That seems to be a common attitude in math, that you're only a real mathematician if you suck at everything else. By that standard, admitting that humanity interests you more than mathematics is tantamount to admitting that you're not cut out to be a mathematician. But why should it? Why can't you be extremely interested in humanity and also be a very fine mathematician?

GC: Yes, I've had this conversation with other people, too. After performances of my play, a lot of people talk to me about these sorts of things, especially people from other fields in science than math. They've suggested, "Wouldn't science be better off if we had more people in it who did approach it from a humanitarian point of view?" Maybe we would do better science. Maybe it would up the creativity level. If we're only going to select
people who are only really narrow-minded and their focus is really tiny on their work, what does that do to science?

JR: So do you think staying in mathematics could have been a good decision for you?

GC: No, I don't. I'm happy with my decision to leave math. I think it was the right decision. I mean it's crazy to be an artist. That's a crazy decision. But somehow I ended up an artist. I didn't leave math thinking that would happen, I just left math to leave it. Somehow it has worked out well for me, and I've never questioned the decision to leave one bit.

JR: What if the mathematical community had had less sexism? Do you think it could have worked for you?

GC: I would have had to carve out a way of working that suited me that included both math and art. For example, someone who has done that is Tom Lehrer. He carved out a really interesting career for himself that bridges math and art. If I'd have done something along those lines, maybe it would have worked out. Maybe. But I think the standard academic life, or teaching somewhere, I'm not sure I was suited.

JR: During the play, you comment, "There is a kind of exquisite artistry to a mathematical proof. It's a thought sculpture built from the poetry of pattern." That's such an eloquent description of what draws so many mathematicians to math. Does that beauty still appeal to you, even though you don't do math these days?

GC: No. I feel like that description is an artist's description of mathematics. I feel like math is a sculptural medium that I used to work in as an artist that I don't work with anymore, just like someone who once sculpted in marble now works in clay. Now I work in words and emotion and music and lyrics and body shapes and gesture and story. I think of acting as a body sculpture. That's my medium now.

JR: As you were writing, were you worried about how people you knew would react?

GC: I was worried about that, but I dealt with that by creating characters that are collages based on a lot of different elements, not only on math people I knew at the time. As an artist, it doesn't interest me to do portraits anyway. I love to create character collages, that's the way I like to work. Once I allowed myself to go fully in that direction, I wasn't so worried anymore, because there are no portraits of anyone in there.

JR: How has the mathematical community responded so far?

GC: I've had wonderful response to it. It caused quite a stir at MIT. MIT math had a departmental meeting in advance of it playing in Cambridge. Maybe they were a little worried about what I was going to say (laughing). But then a lot of people in the math department came to see it, and some
came more than once. They really were very, very positive about it. The response has been fantastic.

JR: What impact would you like the play to have, either on people in general or on the mathematical community?

GC: It does seem to have an impact. It stirs people up, it gets them talking and thinking about issues about women and math and science. That's lovely, and I'm so glad that that's happened and that I can be of service in that way.

But that wasn't my intention exactly. I was acting purely as an artist. I had something to say and I wanted to say it in the most artistically satisfying way possible. I didn't really have an agenda or a moral or something I wanted to impart. I just knew I had to say what I had to say.

When I was initially working on this play, I immersed myself in a whole bunch of the most well-received, greatest, most celebrated solo shows that I could. The thing about solos is that when one person morphs into all these characters, if it's done well, the audience comes away with a sense of
our shared humanity, how we might not be all that different after all. There's something very deeply moving about that.

I hope that people come away from my show with that experience. I also hope that they laugh and have a good time, that they are moved and touched by it, and especially that they might think a little differently about what it's like to be a woman in math and science than when they sat down at the beginning of the show. Sometimes, you have to have a vicarious experience of someone in a certain situation before you really get it. You can talk about it in academic colloquia all day long, but sometimes it's that visceral thing you get from storytelling and theater art that kind of opens your eyes. I think maybe Larry Summers needs that.

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## Book Review

# The Calculus of Friendship 

## Reviewed by Lawrence S. Braden

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Ostensibly, The Calculus of Friendship would at first appear to be a simple story of a high school student and his teacher, drawn together by their love of calculus, in which calculus was the bond that cemented their friendship over three decades. It is that, but so much more. Perhaps the book should be titled A Calculus of Friendship, using the tertiary definition of calculus as a recipe, or formula. But let Strogatz tell it in his own words: "Like calculus itself, this book is an exploration

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of change. It's about the transformation that takes place in a student's heart, as he and his teacher reverse roles, as they age, as they are buffeted by life itself. Through all these changes, they are bound together by a love of calculus. For them it is more than a science. It is a game they love playing togetherso often the basis of friendship between men-a constant while all around them is in flux" (from the Prologue, page xii).

Most readers would assume at first glance that the young math-geek Strogatz would be intellectually drawn to the subject by the older (by thirty years) Mr. Don Joffray, a physically imposing man, "a stronger version of Lee Marvin, whom I'd seen in lots of war movies." (Indeed, Joffray was at one
time the national whitewater kayaking champion). The truth is much more subtle and interesting. In fact Strogatz thought of Joffray as "soft", not giving his students the red meat of mathematics. While the tenth grader Strogatz was learning the epsilon-delta definition of "continuity" in precalculus from his MIT-trained teacher Mr. Johnson, he found out that Joffray was telling his students that "a function is continuous if you can draw it without lifting your pencil from the paper." (Lest we judge Mr. Joffray too harshly for this, none other than Creighton Buck once told me that in his freshman calculus classes at Wisconsin, he did not make much of an issue of epsilons and deltas either.) Don Joffray always wished that he knew more mathematics (who of us has not?). Thus Strogatz's delicious morsels were a balm unto his soul and the beginning of a beautiful friendship.

I am exactly fifteen years older than Strogatz and fifteen years younger than Joffray and have taught calculus to bright high school students for thirty-five years in private schools in Hawaii and New England. Philosophically, I am more of a Mr. Johnson than a Mr. Joffray, perhaps to my students' detriment, perhaps not. Who can say? But we all love calculus and all love sharing the wonderment of such a subject with others.

Joffray was probably learning as much about calculus from Strogatz and some of his other gifted students as they were learning from him. Joffray was the sort of teacher who would pose a problem to the class to which he did not know the answer. He took the greatest delight when one of his students would solve a problem such as this, especially when he was unable to solve it himself. So from the first the two of them were aware that the student was by far the more mathematically powerful of the pair and knew almost as much mathematics as the teacher.

But Joffray, a happy, enthusiastic man, challenged his students to a higher level than many more-traditional teachers. There was simply no telling what the "lesson of the day" would be. One day Joffray walked into the class and told of a goat he saw tethered to a tree that weekend. The goat, wishing to escape, went around and around the tree, keeping the rope taut. What was the equation of the goat's path? Young Strogatz could learn all the calculus from the book himself. But it was questions such as these that hooked him. Indeed, he says on page 5, "With the passage of time I see now that I was like the goat tethered to the tree-and Mr. Joffray was the tree. I pulled taut on the rope and tried to get away from him, but only ended up wrapping myself closer and closer to him, all these years. How did that happen? It wasn't because he taught me so much in the usual sense. No, his approach was so humble and unconventional, it confused me. It made me feel superior to him. I'm embarrassed to admit that, but it's true."

At the end of Strogatz's junior year, his school (Loomis) held its annual awards ceremony. Strogatz won the Rensselaer Prize for the top junior in math and science. Joffray made a speech, likening his student to a mountain climber, ascending mathematical peaks and then returning with what he had seen. He made Strogatz feel generous, and even heroic.

The bonding started much more slowly than one might have thought. It was the student (perhaps once every year or so) writing to his old teacher and telling him about interesting problems he had come across in his college studies. One of the first concerned the irrationality of the square root of 2 and the proof that every precalculus high school student is taught-a very neat and clever proof, but one rooted in number theory. At Princeton Strogatz was taught a geometrical proof by his teacher, Benedict Gross. It was much more complicated than the standard one, but purely geometrical and very clever. After sharing it with Joffray, he challenged Joffray to use the method to prove that the sides of the Golden Rectangle are also incommensurable.

The chapters of The Calculus of Friendship are sewn together with an underpinning of advanced mathematics. As the friendship slowly(!) blossoms, the author uses specific mathematical problems they shared as chapter headings. (Appropriately, the chapter containing the problem about the square root of 2 is entitled "Irrationality".) Some of the others, for a flavor of the book, are "Shifts", "The Monk and the Mountain", "Pursuit", "Randomness", "Infinity and Limits", "Chaos" (of course!), and "Celebration". Each of these is annotated, in chronological order, with the appropriate years when their relationship went through that phase. For instance, "The Monk and the Mountain" refers to the June 1961 column of Scientific American in which Martin Gardner posed the riddle of the monk who starts ascending a mountain at dawn, reaching the summit at dusk. He spends the night and, at dawn, starts down the mountain on the same spiraling path, reaching the bottom sometime before dusk. Every topologist knows that there is a point on the path where the monk must be at the same time each day. Perhaps this is the first "fixed point theorem" a student encounters, beautifully solved by imagining that there are two monks on the path the same day...one ascending and one descending. Surely they must meet. Strogatz likens this phase of their relationship (1989-1990) to the monk and the mountain. His own career as a teacher was about to take off at the Massachusetts Institute of Technology, and Joffray's, at age sixty-one or so, reached its apex and was headed for decline. It was at that time that Strogatz and Loomis classmate Ed Rak, a mathematical genius, were invited to return to Loomis to give a speech at a faculty banquet honoring their old mentor.

Joffray had won the 1990 Swan Award for Teaching, given by a consortium of schools "to a teacher whose achievement represents the highest standards of the profession." It was at this point that their correspondence really took off.

And thus it started, neither party imagining just where this correspondence would lead. Strogatz is not only a first-rate mathematician but a master teacher as well, as anyone who has seen him teach will testify. (I daresay that any person teaching high school mathematics should see his twenty-four lectures on chaos from The Teaching Company on DVD. They are masterpieces of clarity, and any inquisitive person, no matter what his or her background, would probably be enthralled with the subject.) Strogatz is the sort of person who not only enjoys learning his subject but enjoys sharing that learning with others, and who better than Mr. Joffray? The problems that Strogatz shared with Joffray, and his comments and solutions, are truly beautiful ones, most certainly more advanced than one usually is likely to run across in the Advanced Placement Calculus curriculum.

As an example, much emphasis is placed on convergence (absolute, conditional, or none) of infinite series in high school calculus. Few of these can actually be summed (ignoring specific ones obtained by "plugging in" to a Maclaurin series.) But what on earth is to be made of the infinite sum of $(\sin j) / j$, summing over the positive integers? Joffray asked Strogatz about this problem, which arose in his class. (Regrettably, the integral test, a standard tool, is invalid here.) What followed was a beautiful nine-page exposition into Fourier series, and the problem was solved. (Okay, if you are curious, it sums to $(\pi-1) / 2$.)

Strogatz was so jazzed (his word) by his back-and-forth communications concerning this with his old teacher that he shared it with pal Rennie Mirollo while waiting for their food in a Chinese restaurant. Mirollo, with the help of the place mat, solved it using not Fourier series but Taylor series in the complex plane. A totally different and elegant solution! Joffray was ecstatic (there is no other word for it) upon receiving the second proof. This book is chock full of elegant problems, elegant solutions, and the give-and-take between two master teachers.

But something was missing from all of this. After years of corresponding with one another, Strogatz's wife said "Well, after all this time, you two probably know quite a lot about each other." Then it hit him...he did not really know his old teacher at all, not as a person. "You don't!" she replied. "That is really such a guy thing!" Strogatz had, without ever realizing it, relegated Joffray to a category of "friendship" shared with his weekly basketball group-e.g., "how many children they had" was irrelevant. Strogatz came to realize, much to his shame, that it was he who had taken
refuge in the mathematics, shutting the door when Joffray talked about the loss of his twenty-seven-year-old son to cancer, or when Joffray mentioned Strogatz's impending marriage. Life may be continuous, but it is certainly bound to contain nondifferentiable points. On his side, Strogatz never mentioned that his marriage had failed or that his father had died. For a while Joffray's letters lay unopened on Strogatz's desk...he had stopped corresponding altogether. He at that stage of his life was emotionally exhausted. He didn't even respond when Joffray wrote to say that he had suffered a stroke and was partially blind. In April 2004 Strogatz's brother unexpectedly died. Joffray wrote to say that he had heard the news via the Loomis alumni magazine and sent condolences. It was this that that shook Strogatz out of his lethargy. He phoned his old mentor and said that he wanted to come up and, for the first time, really have a down-to-earth talk. With a tape recorder.

The rest is history. And a beautiful book, bound to become a classic in the mathematical literature, in this reviewer's opinion. Like Hardy's A Mathematician's Apology, you do not have to know any mathematics whatsoever to read this book. It is a candid and all-too-human story told with brutal honesty...warts and all, sharing with the reader the elation and sincere regrets bound up in the re-lationship-but in the end, the victories, too. With some beautiful mathematics throughout!

Strogatz's friend Alan Alda (yes, that Alan Alda) conducted a very perspicuous interview with the author. It may be accessed at http://press. princeton.edu/video/strogatz/a1da.htm7.

But you would be cheating yourself if you only watched the interview and did not read the book.
came more than once. They really were very, very positive about it. The response has been fantastic.

JR: What impact would you like the play to have, either on people in general or on the mathematical community?

GC: It does seem to have an impact. It stirs people up, it gets them talking and thinking about issues about women and math and science. That's lovely, and I'm so glad that that's happened and that I can be of service in that way.

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I am exactly fifteen years older than Strogatz and fifteen years younger than Joffray and have taught calculus to bright high school students for thirty-five years in private schools in Hawaii and New England. Philosophically, I am more of a Mr. Johnson than a Mr. Joffray, perhaps to my students' detriment, perhaps not. Who can say? But we all love calculus and all love sharing the wonderment of such a subject with others.

Joffray was probably learning as much about calculus from Strogatz and some of his other gifted students as they were learning from him. Joffray was the sort of teacher who would pose a problem to the class to which he did not know the answer. He took the greatest delight when one of his students would solve a problem such as this, especially when he was unable to solve it himself. So from the first the two of them were aware that the student was by far the more mathematically powerful of the pair and knew almost as much mathematics as the teacher.

But Joffray, a happy, enthusiastic man, challenged his students to a higher level than many more-traditional teachers. There was simply no telling what the "lesson of the day" would be. One day Joffray walked into the class and told of a goat he saw tethered to a tree that weekend. The goat, wishing to escape, went around and around the tree, keeping the rope taut. What was the equation of the goat's path? Young Strogatz could learn all the calculus from the book himself. But it was questions such as these that hooked him. Indeed, he says on page 5, "With the passage of time I see now that I was like the goat tethered to the tree-and Mr. Joffray was the tree. I pulled taut on the rope and tried to get away from him, but only ended up wrapping myself closer and closer to him, all these years. How did that happen? It wasn't because he taught me so much in the usual sense. No, his approach was so humble and unconventional, it confused me. It made me feel superior to him. I'm embarrassed to admit that, but it's true."

At the end of Strogatz's junior year, his school (Loomis) held its annual awards ceremony. Strogatz won the Rensselaer Prize for the top junior in math and science. Joffray made a speech, likening his student to a mountain climber, ascending mathematical peaks and then returning with what he had seen. He made Strogatz feel generous, and even heroic.

The bonding started much more slowly than one might have thought. It was the student (perhaps once every year or so) writing to his old teacher and telling him about interesting problems he had come across in his college studies. One of the first concerned the irrationality of the square root of 2 and the proof that every precalculus high school student is taught-a very neat and clever proof, but one rooted in number theory. At Princeton Strogatz was taught a geometrical proof by his teacher, Benedict Gross. It was much more complicated than the standard one, but purely geometrical and very clever. After sharing it with Joffray, he challenged Joffray to use the method to prove that the sides of the Golden Rectangle are also incommensurable.

The chapters of The Calculus of Friendship are sewn together with an underpinning of advanced mathematics. As the friendship slowly(!) blossoms, the author uses specific mathematical problems they shared as chapter headings. (Appropriately, the chapter containing the problem about the square root of 2 is entitled "Irrationality".) Some of the others, for a flavor of the book, are "Shifts", "The Monk and the Mountain", "Pursuit", "Randomness", "Infinity and Limits", "Chaos" (of course!), and "Celebration". Each of these is annotated, in chronological order, with the appropriate years when their relationship went through that phase. For instance, "The Monk and the Mountain" refers to the June 1961 column of Scientific American in which Martin Gardner posed the riddle of the monk who starts ascending a mountain at dawn, reaching the summit at dusk. He spends the night and, at dawn, starts down the mountain on the same spiraling path, reaching the bottom sometime before dusk. Every topologist knows that there is a point on the path where the monk must be at the same time each day. Perhaps this is the first "fixed point theorem" a student encounters, beautifully solved by imagining that there are two monks on the path the same day...one ascending and one descending. Surely they must meet. Strogatz likens this phase of their relationship (1989-1990) to the monk and the mountain. His own career as a teacher was about to take off at the Massachusetts Institute of Technology, and Joffray's, at age sixty-one or so, reached its apex and was headed for decline. It was at that time that Strogatz and Loomis classmate Ed Rak, a mathematical genius, were invited to return to Loomis to give a speech at a faculty banquet honoring their old mentor.

Joffray had won the 1990 Swan Award for Teaching, given by a consortium of schools "to a teacher whose achievement represents the highest standards of the profession." It was at this point that their correspondence really took off.

And thus it started, neither party imagining just where this correspondence would lead. Strogatz is not only a first-rate mathematician but a master teacher as well, as anyone who has seen him teach will testify. (I daresay that any person teaching high school mathematics should see his twenty-four lectures on chaos from The Teaching Company on DVD. They are masterpieces of clarity, and any inquisitive person, no matter what his or her background, would probably be enthralled with the subject.) Strogatz is the sort of person who not only enjoys learning his subject but enjoys sharing that learning with others, and who better than Mr. Joffray? The problems that Strogatz shared with Joffray, and his comments and solutions, are truly beautiful ones, most certainly more advanced than one usually is likely to run across in the Advanced Placement Calculus curriculum.

As an example, much emphasis is placed on convergence (absolute, conditional, or none) of infinite series in high school calculus. Few of these can actually be summed (ignoring specific ones obtained by "plugging in" to a Maclaurin series.) But what on earth is to be made of the infinite sum of $(\sin j) / j$, summing over the positive integers? Joffray asked Strogatz about this problem, which arose in his class. (Regrettably, the integral test, a standard tool, is invalid here.) What followed was a beautiful nine-page exposition into Fourier series, and the problem was solved. (Okay, if you are curious, it sums to $(\pi-1) / 2$.)

Strogatz was so jazzed (his word) by his back-and-forth communications concerning this with his old teacher that he shared it with pal Rennie Mirollo while waiting for their food in a Chinese restaurant. Mirollo, with the help of the place mat, solved it using not Fourier series but Taylor series in the complex plane. A totally different and elegant solution! Joffray was ecstatic (there is no other word for it) upon receiving the second proof. This book is chock full of elegant problems, elegant solutions, and the give-and-take between two master teachers.

But something was missing from all of this. After years of corresponding with one another, Strogatz's wife said "Well, after all this time, you two probably know quite a lot about each other." Then it hit him...he did not really know his old teacher at all, not as a person. "You don't!" she replied. "That is really such a guy thing!" Strogatz had, without ever realizing it, relegated Joffray to a category of "friendship" shared with his weekly basketball group-e.g., "how many children they had" was irrelevant. Strogatz came to realize, much to his shame, that it was he who had taken
refuge in the mathematics, shutting the door when Joffray talked about the loss of his twenty-seven-year-old son to cancer, or when Joffray mentioned Strogatz's impending marriage. Life may be continuous, but it is certainly bound to contain nondifferentiable points. On his side, Strogatz never mentioned that his marriage had failed or that his father had died. For a while Joffray's letters lay unopened on Strogatz's desk...he had stopped corresponding altogether. He at that stage of his life was emotionally exhausted. He didn't even respond when Joffray wrote to say that he had suffered a stroke and was partially blind. In April 2004 Strogatz's brother unexpectedly died. Joffray wrote to say that he had heard the news via the Loomis alumni magazine and sent condolences. It was this that that shook Strogatz out of his lethargy. He phoned his old mentor and said that he wanted to come up and, for the first time, really have a down-to-earth talk. With a tape recorder.

The rest is history. And a beautiful book, bound to become a classic in the mathematical literature, in this reviewer's opinion. Like Hardy's A Mathematician's Apology, you do not have to know any mathematics whatsoever to read this book. It is a candid and all-too-human story told with brutal honesty...warts and all, sharing with the reader the elation and sincere regrets bound up in the re-lationship-but in the end, the victories, too. With some beautiful mathematics throughout!

Strogatz's friend Alan Alda (yes, that Alan Alda) conducted a very perspicuous interview with the author. It may be accessed at http://press. princeton.edu/video/strogatz/alda.htm1.

But you would be cheating yourself if you only watched the interview and did not read the book.


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An Episodic History of Mathematics Mathematical Culture Through Problem Solving

Steven G. Krantz

Can be used as a text in the history of mathematics, and in teacher preparation and capstone courses.

An Episodic History of Mathematics will acquaint students and readers with mathematical language, thought, and mathematical life by means of historically important mathematical vignettes. It will also serve to help prospective teachers become more familiar with important ideas in the history of mathematicsboth classical and modern.

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This history of mathematics is also an opportunity to have some fun because the focus in this text is also on the practical-getting involved with the mathematics and solving problems. This book is unabashedly mathematical. In the course of reading Episodic History, the neophyte will become involved with mathematics by working on the same problems that, for instance, Zeno and Pythagoras and Descartes and Fermat and Riemann worked on.

## Combinatorics A Guided Tour

## David A. Mazur

Combinatorics: A Guided Tour maintains a conversational, reader-friendly tone and emphasizes the kinds of thinking that are characteristic of combinatorics: bijective and combinatorial proofs, recursive analysis, and counting problem classification. It is flexible enough to be used for undergraduate courses in combinatorics, second courses in discrete mathematics, introductory graduate courses in applied mathematics programs, as well as for independent study or reading courses.

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To order or to inquire about examination copies, call 1-800-331-1622.
These books are available at WWW.maa.org and through Amazon.com.

## Tate Receives 2010 Abel Prize

The Norwegian Academy of Science and Letters has awarded the Abel Prize for 2010 to John Torrence Tate, University of Texas at Austin, for "his vast and lasting impact on the theory of numbers." The Abel Prize recognizes contributions of extraordinary depth and influence to the mathematical sciences and has been awarded annually since 2003. It carries a cash award of 6,000,000 Norwegian kroner (approximately US\$1 million). John Tate received the Abel Prize from His Majesty King Harald at an award ceremony in Oslo, Norway, on May $25,2010$.

## Biographical Sketch

John Torrence Tate was born on March 13, 1925, in Minneapolis, Minnesota. He received his B.A. in mathematics from Harvard University in 1946 and his Ph.D. in 1950 from Princeton University under the direction of Emil Artin. He was affiliated with Princeton University from 1950 to 1953 and with Columbia University from 1953 to 1954. He joined the faculty of Harvard University in 1954 and remained there for thirty-six years before joining the University of Texas, Austin. He retired from Texas in 2009.

John Tate has received many distinguished international awards and honors. As early as 1956, he was awarded the AMS Cole Prize for outstanding contributions to number theory, and he received the AMS Steele Prize for Lifetime Achievement in 1995. With Mikio Sato he received the 2002/2003 Wolf Prize in Mathematics for "his creation of fundamental concepts in algebraic number theory." He was an invited speaker at the International Congress of Mathematicians in 1962 in Stockholm and again in 1970 in Nice. He was elected to the U.S. National Academy of Sciences in 1969, was named a foreign member of the French Académie des Sciences in 1992, and elected an honorary member of the London Mathematical Society in 1999.

## Citation

Beyond the simple arithmetic of $1,2,3, \ldots$ lies a complex and intricate world that has challenged some of the finest minds throughout history. This world stretches from the mysteries of the prime numbers to the way we store, transmit, and secure information in modern computers. It is called the theory of numbers. Over the past century it has grown into one of the most elaborate and sophisticated branches of mathematics, interacting profoundly with other areas such as algebraic geometry and the theory of automorphic forms.

John Tate is a prime architect of this development.

Tate's 1950 thesis on Fourier analysis in number fields paved the way for the modern theory of automorphic forms and their $L$-functions. He revolutionized global class field theory with Emil Artin, using novel techniques of group cohomology. With Jonathan Lubin, he recast local class field theory by the ingenious use of formal groups. Tate's invention of rigid analytic spaces spawned the whole field of rigid analytic geometry.


John Tate He found a $p$-adic analogue of Hodge theory, now called Hodge-Tate theory, which has blossomed into another central technique of modern algebraic number theory.

A wealth of further essential mathematical ideas and constructions were initiated by Tate, including Tate cohomology, the Tate duality theorem, Barsotti-Tate groups, the Tate motive, the Tate module, Tate's algorithm for elliptic curves, the Néron-Tate height on Mordell-Weil groups of abelian varieties, Mumford-Tate groups, the Tate isogeny theorem and the Honda-Tate theorem for abelian varieties over finite fields, Serre-Tate deformation theory, Tate-Shafarevich groups, and the Sato-Tate conjecture concerning families of elliptic curves. The list goes on and on.

Many of the major lines of research in algebraic number theory and arithmetic geometry are only possible because of the incisive contribution and illuminating insight of John Tate. He has truly left a conspicuous imprint on modern mathematics.

## About the Prize

The Niels Henrik Abel Memorial Fund was established in 2002 to award the Abel Prize for outstanding scientific work in the field of mathematics. The prize is awarded by the Norwegian Academy of Science and Letters, and the choice of Abel Laureate is based on the recommendation of the Abel Committee, which consists of five internationally recognized mathematicians.

Previous recipients of the Abel Prize are: JeanPierre Serre (2003), Michael Atiyah and I. M. Singer (2004), Peter Lax (2005), Lennart Carleson (2006), S. R. S. Varadhan (2007), John G. Thompson and Jacques Tits (2008), and Mikhail L. Gromov (2009).
-From announcements of the Norwegian
Academy of Science and Letters

# Yau and Sullivan Awarded 2010 Wolf Prize 

The 2010 Wolf Prize in Mathematics has been awarded to:

Shing-Tung Yau, Harvard University, "for his work in geometric analysis that has had a profound and dramatic impact on many areas of geometry and physics" and

Dennis Sullivan, Stony Brook University and City University of New York Graduate School and University Center, Stony Brook, New York, "for his innovative contributions to algebraic topology and conformal dynamics."

The US $\$ 100,000$ prize is divided equally between the prizewinners. The prize was presented by the president of the State of Israel in a ceremony at the Knesset (parliament) in Jerusalem on May 13, 2010. The list of previous recipients of the Wolf Prize in Mathematics is available on the website of the Wolf Foundation, http://www.wolffund.org.i1.

## Description of the Prizewinners' Work

The following description of the work of Yau and Sullivan was prepared by the Wolf Foundation.

Shing-Tung Yau has linked partial differential equations, geometry, and mathematical physics in a fundamentally new way, decisively shaping the field of geometric analysis. He has developed new analytical tools to solve several difficult nonlinear partial differential equations, particularly those of the Monge-Ampere type, critical to progress in Riemannian, Kähler, and algebraic geometry and in algebraic topology, that radically transformed these fields. The Calabi-Yau manifolds, as these are known, a particular class of Kähler manifolds, have become a cornerstone of string theory aimed at understanding how the action of physical forces in a high-dimensional space might ultimately lead to our four-dimensional world of space and time. Yau's work on T-duality is an important ingredient for mirror symmetry, a fundamental problem at the interface of string theory and algebraic and symplectic geometry. While settling the positive mass and energy conjectures in general relativity, he also created powerful analytical tools, which have broad applications in the investigation of the global geometry of space-time.

Yau's eigenvalue and heat kernel estimates on Riemannian manifolds count among the most


Shing-Tung Yau


Dennis Sullivan
profound achievements of analysis on manifolds. He studied minimal surfaces, solving several classical problems, and then used his results to create a novel approach to geometric topology. Yau has been exceptionally productive over several decades, with results radiating into many areas of pure and applied mathematics and theoretical physics. In addition to his diverse and fundamental mathematical achievements, which have inspired generations of mathematicians, Yau has also had an enormous impact worldwide on mathematical research through his training of an extraordinary number of graduate students and establishing several active mathematical research centers.

Dennis Sullivan has made fundamental contributions in many areas, especially in algebraic topology and dynamical systems. His early work helped lay the foundations for the surgery theory approach to the classification of higher dimensional manifolds, most particularly providing a complete classification of simply connected manifolds within a given homotopy type. He developed the notions of localization and completion in homotopy theory and used this theory to prove the Adams conjecture (also proved independently by Quillen). Sullivan and Quillen introduced the rational homotopy type of space.

Sullivan showed that it can be computed using a minimal model of an associated differential graded
algebra. Sullivan's ideas have had far-reaching influence and applications in algebraic topology. One of Sullivan's most important contributions was to forge the new mathematical techniques needed to rigorously establish the predictions of Feigenbaum's renormalization as an explanation of the phenomenon of universality in dynamical systems. Sullivan's "no wandering domains" theorem settled the classification of dynamics for iterated rational maps of the Riemann sphere, solving a sixty-year-old conjecture by Fatou and Julia. His work generated a surge of activity by introducing quasiconformal methods to the field and establishing an inspiring dictionary between rational maps and Kleinian groups of continuing interest. His rigidity theorem for Kleinian groups has important applications in Teichmüller theory and for Thurston's geometrization program for 3-manifolds. His recent work on topological field theories and the formalism of string theory can be viewed as a by-product of his quest for an ultimate understanding of the nature of space and how it can be encoded in strange algebraic structures.

Sullivan's work has been consistently innovative and inspirational. Beyond the solution of difficult outstanding problems, his work has generated important and active areas of research pursued by many mathematicians.

## Biographical Sketches

Shing-Tung Yau was born in 1949 in Shantou, Guangdong Province, China. He received his Ph.D. in 1971 from the University of California Berkeley under the direction of Shiing-Shen Chern. He has held positions at the Institute for Advanced Study, Princeton; the State University of New York at Stony Brook; and Stanford University. He has been at Harvard since 1987. He received the Fields Medal in 1982 and the National Medal of Science in 1997; he was also awarded the Veblen Prize in 1981 and the Crafoord Prize in 1994.

Dennis Sullivan was born in 1941 in Port Huron, Michigan. He received his Ph.D. in 1966 from Princeton University under the direction of William Browder. He has held positions at the Massachusetts Institute of Technology and the Institut des Hautes Études Scientifiques. His awards include the Veblen Prize in Geometry (1971), the Prix Élie Cartan of the French Academy of Sciences (1981), and the 1994 King Faisal International Prize for Science; he received the National Medal of Science in 2004 and the AMS Steele Prize in 2006.


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# On Buckets and Fires 

Herbert S. Wilf

I appreciate the opportunity to visit my prejudices about mathematics education upon the world.

For many years now I have watched while committees, or projects, or individuals have debated their ideas about curricula, for primary education, for secondary education, for colleges and universities, and so forth. I have more than once made a mental analogy of these developments with various diets. Would you like the Atkins curriculum or the South Beach? The low fat or the high protein? The curriculum du jour is a constantly shifting target, but everyone agrees that the situation is getting more and more dire. We say that our students are not learning mathematics. We worry that we are not following the model of China, or Korea, or some such, and we debate whether that's a good thing or a bad thing. We think we had better use calculators in the classrooms, or perhaps we should forbid calculators in the classrooms, but we should certainly think carefully about that subject.

Over the years I have become more and more certain that these curricular discussions are somewhere between irrelevant and marginal. Much more important than what students are being taught, at least until the years in which they have chosen a major subject, is by whom they are being taught and how. For education is a supremely human-tohuman process. You cannot just lay out the goodies on the table. That will not stimulate student appetites.

It has been said (perhaps first by Plutarch) that "A mind is a fire to be kindled, not a bucket to be filled." The job of the teacher is to light that

[^14]fire-never mind whether the student is learning the multiplication table or studying quadratic equations. The subject matter is only the fuel. The teacher lights it up.

The earliest years in the education of an individual are those in which the teacher's human qualities are the most important. As the individual finishes secondary education and enters college, it seems reasonable that the curriculum should become somewhat more important. So my remarks here apply more often to secondary education than to college. However, I have seen in my own office many undergraduates who enter their freshman years propelled like rockets toward particular careers-medicine, the law, computer science, whatever. My advice to them has always been the same: "Cool it. You are a student in one of the great universities of the world. You can study medicine later. For now, try archaeology, Sanskrit, art appreciation, mathematics, etc., all of which are available in abundance here. Make your choices based on who will be teaching the various courses. Sign up for the ones that have stellar teachers. Don't worry about what they're teaching. Just listen, think, and enjoy."

To improve the quality of mathematics education, we need to enhance the abilities of teachers to inspire students. We do not need to rearrange the deck chairs on our curricula, if you'll excuse a mixed metaphor.

How to do that?
Primum non nocere; first, do no harm. A great deal of harm is inflicted upon young students whose confidence in their mathematical abilities may be fragile. Teachers who themselves feel mathematically challenged sometimes resort to making students feel worse in order to make themselves feel better. Fixing this problem means, above all, making sure that those who teach mathematics know mathematics. As a rough rule of thumb, the
teacher should know enough to be able to field a question from a student, a follow-up question from another student, and a third question that goes still deeper. Depth of knowledge is the best security that the teacher can have, the best insurance against his/her trying to put down bright students, and the best route to lighting that fire. So, you want a good mathematics teacher? Make sure he/she knows a whole bunch of mathematics.

When people find out what I do for a living, a very frequent response is "oh, my worst subject". We discuss this for a while, and it develops that a teacher somewhere back in the lower grades "made me feel stupid", perhaps in public, and this flipped his/her mathematics switch permanently to the "Off" position. Public humiliation is a very difficult experience to get past.

Here are a few examples of answers that teachers might give to student questions that have the effect of deflating the self-esteem of a student and discouraging a student from asking more questions:
. "You just have to substitute ... into ..." [Just?]
. "Where were you when I told the class that ...?"
. "I covered that point last Wednesday. Here is what I said ..."
. "That fact is a trivial consequence of ..."
Well, you get the idea.
To inspire a student, a teacher must exemplify the kind of person that the student would like to be. Most of us would like to be happy in our work. An inspiring teacher must be happy in her work. In this case, since the subject is mathematics, the teacher must enjoy mathematics. If that's not the case, the students will see through her in one nanosecond and lose interest in the teacher and the subject. That leaves one small question-how can we get more teachers who enjoy mathematics? That's a tough one, but please let's work on it. It's very important.

Perhaps the attribute of being inspiring is itself teachable to some extent and not entirely genetically encoded.

Perhaps in the process of training teachers we might require them to study some branch of mathematics that is entirely new to them, just to instill in them a feeling for the difficulty of learning a new subject, so they'll have more empathy with their own struggling students.

Our quality of education will improve when we have more teachers who are knowledgeable in their subject and who enjoy doing it and talking about it. This has all been said before. I claim no originality here. But I do claim that these obvious truths have made a deep impression on me, and I very much hope that we can work together to create the inspirational educational system that our children deserve.

## Roads to Infinity

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# C|R|O|S|STO|R|D $1,1,2,3,5, \ldots$ 

Answers: Page 795

## by Ned White

## ACROSS

1 String along
7 Artist whose canvas of "The Sacrament of the Last Supper" is a 39-Across
11 $\qquad$ nut
14 Yom Kippur observer
15 Black, to Blake
16 Hit for U-2, Three Dog Night, Bee Gees, or Metallica
17 89, a * $\qquad$ .
18 Work by *, 23 years after 20-Across and 58-Across
19 Cross, for one
20 With 58-Across, work by * addressing the mathematics of rabbit population growth
21 Proverbial heir, with "the"
22 Dynamic leader?
23 Bygone Renault
25 Brand with the tag "True Canadian Taste"
27 Implies
30 Digital communication syst.?
31 Andean tuber
32 Nike rival
35 Macho type
39 What the middle nine rows of this puzzle closely represent
43 Greedily devour (with "down")
44 $\qquad$ 'acte
45 N.Y.C. travel choice
46 Tiebreakers, for short
48 Boxster feature
51 Assumption, for one, in Louisiana
54 Where "Aida" premiered
55 Polo competitor
56 Fan sounds
58 See 20-Across
62 Crawl space?
63 Where * resided
64 * $\qquad$ (and where * now resides, starting with the circle at 49-Down)
65 Ranch extension?
66 Con
67 Geronimo, for one
68 Hanoi holiday
69 Word to follow the start of 39-Across, as inspired by *
70 Quark-antiquark particles

DOWN
1 Frontier name, for short
2 Pin holder
3 Second name in lens-making
4 Release from a stroller, maybe
5 Many combines
6 Muff
7 Put off
8 Up to it
9 Little show-off's attention-getter
10 Electees
11 They may be high
12 Año opener
13 Arc lamp gas
21 Developed
22 "Zelig" portrayer
24 Crunch opener
26 Dept. of Labor arm
27 Harleys, slangily
28 Elvis or Bogart, for example
29 "The Lion King" lioness


33 Elvis and Bogart, for example
34 Represents
36 Corp. heads, collectively
37 Like Woody Herman's sax
38 Minimal tide
40 "Star Wars" extra
41 Red-spotted swimmers
42 "Star Trek: T.N.G." half-Betazoid
47 Graceland, for many
49 Shake, for some
50 Novelist Smollett or Wolff
51 Tube in a lab
52 Lapis lazuli
53 Bomb defuser, sometimes
54 The Rockies, e.g.
57 Film barker
59 Chevron competitor
60 "All The Way" lyricist Sammy
61 Mer interrupters
63 Spray in the kitchen
64 Frodo's friend

# AMS Homework Software Survey 

Elaine Kehoe

Can using homework software help improve passing rates in freshman mathematics classes? High failure rates in first-year mathematics courses are perceived by administrators as a failure of mathematics instruction, and so reducing failure rates is essential. An AMS Task Force believes that the use of software to help grade homework and tests has the potential to improve student learning in freshman mathematics courses with a comparatively modest investment and to lead to higher passing rates in these courses.

As a result of the work of the AMS Task Force on the First-Year Mathematics Experience, the AMS conducted an online survey of 1,230 U.S. mathematics and statistics departments asking for their experiences using homework software and for the concerns of departments that were considering such software. (The full report of the Task Force appeared in the AMS Notices 56, pp. 754-60.)

Of the 1,230 departments, 467 responded to the survey. The survey responses were broken down by type of degree granted at the institutions (Ph.D., M.S., B.S.) and by private-versus-public institution, as well as by users versus prospective users.

Almost half of the 162 doctoral mathematics departments surveyed reported using homework software. Only $30 \%$ of the 190 master's-degreegranting departments and $13 \%$ of the 848 bache-lor's-degree-granting departments reported using this software.

Overall, users were happy with homework software. Current users were more positive about the benefits of homework software and much less concerned about drawbacks than were prospective users. They considered the primary benefit to be better student learning and the primary drawback to be students' not showing their work.

[^15]
## Response Rates

Response rates, using college and university rankings taken from the 1995 NRC study, were as follows:

1. The top eighty doctoral mathematics departments: 71\% (57 responses)
2. The remaining eighty-two doctoral departments: 65\% (53 responses)
3. 190 comprehensive university (master's-degree-granting) departments: $45 \%$ ( 81 responses)
4. 848 college (bachelor's-degree-granting) departments: 30\% (257 responses)

## Types of Software

All the homework software programs discussed can accept both algebraic expressions and numerical answers. Students log in to a website to see their assignments and enter answers; each student gets an individualized assignment with different numbers. Students can try again after a wrong answer and may be given hints or directed to tutorials on the topic. Instructors can manage and download homework scores and can typically limit the number of tries, penalize too many tries, allow hints to be displayed, and so forth. Chat rooms and users' groups are often available. Commercial software has online prompts and phone-in help services. This software can also be used to produce quizzes that are usually given in supervised computer labs.

Publisher-supplied software is tailored to the course textbook. Students pay a fee to get an account at the website. MyMathLab is from Pearson. WebAssign was developed independently but is now affiliated with Cengage Brooks/Cole and supports texts of other publishers; it can include tutorials and virtual labs.

Among faculty-created software, WeBWorK, developed by University of Rochester mathematics faculty, has been the most successful. MapleTA is an outgrowth of eGrade, created by University of

Nebraska mathematics faculty, and can be hosted locally or on a Maplesoft server.

A learning system adapts instruction and questions as it learns about the strengths and weaknesses of a student. Forty-three mathematics departments use such systems. About six institutions mentioned using each of ALEKS and Hawkes Learning Systems, mainly for high school mathematics and remedial college mathematics. The best known locally developed mathematics learning system is Virginia Tech's Mathematics Emporium.

## Survey Questions

Following are the questions contained in the survey.

Question 1. Benefits. Respondents were asked whether the following benefits were a major issue, a minor issue, or not an issue in motivating the use or prospective use of homework software:
A. Can grade homework that previously was ungraded
B. Frees instructors/TAs from grading duties
C. Promotes better learning (e.g., immediate feedback, ability to try again)
D. Reduces student copying

The first three benefits were rated a major issue by a majority of respondents and a major or minor issue by almost all respondents, users or prospective users, and across all types of departments. However, "better learning" received a significantly higher overall rating than the first two; it was a major issue for $76 \%$ of respondents, averaged over current and prospective users. Benefits A and C were more likely to be rated as major issues by users than by prospective users, whereas benefit B received about the same ratings from both groups. Only 25\% rated benefit D a major issue.

Some respondents noted that homework software led to greater effort and persistence by students; that it was attractive to students; and that it enforced some consistency in homework assignments in multisection courses. Others praised its supplementary tutorials and its appropriateness for drilling in basic skills.

Question 2. Drawbacks. The following drawbacks were rated by respondents:
A. Students do not show work
B. Types of questions are restricted
C. Students get frustrated with format
D. Students get lazy on first try
E. Funds for student graders are lost
F. Students can easily get electronic help

The first three were typically a major or minor issue at more than $80 \%$ of departments in all categories and among users and prospective users. Only Drawback A was a major issue for a majority of all responders. Generally, prospective users rated these drawbacks major issues more frequently than did current users. Drawback D was a major issue more for prospective users than for
users at private Ph.D. and M.S. departments and all B.S. departments.

Some respondents explained how they worked around drawbacks; for example, in-class quizzes helped to mitigate the problem of students' getting homework help from the Internet. Other complaints were that available homework questions did not cover a topic in the textbook, that it was tedious to create needed additional questions, and that students "game" the system by entering nonsense answers so that some systems will give them enough hints to make the answer obvious.

Question 3. Status. This question asked whether the respondent was a current user, a prospective user, or not interested in becoming a user. The respective total percentages were 54,25 , and $22 \%$ aggregated across types of institutions.

At least 44\% of all doctoral departments use homework software. Doctoral departments probably have greater incentives for use, such as very large introductory courses for which there are inadequate resources for hand grading. Another advantage for doctoral departments is the availability of departmental technical support in getting started with the software. With lighter teaching loads, some doctoral faculty members may choose to spend time learning how to use such software, constructing new homework sets for this software, and instructing other faculty members in its use.

Question 4. Which Courses (for prospective users only). Prospective users were asked which courses their departments might be interested in using homework software for and whether they anticipated using it for all or part of the grading.

Few prospective users planned to use homework software for all grading in any of the listed courses. Typically, at least $75 \%$ of Ph.D. and M.S. prospective users were considering such software for grading some of the homework in precalculus, business calculus, general calculus, and calculus for scientists. For multivariable calculus, differential equations, and statistics, the percentages dropped to $60-70 \%$. In general, there was little difference between the plans of prospective users at private versus public institutions, although in general prospective users at public institutions were a little more likely than those at private institutions to plan to use such software for all grading.

Question 5. Which Courses. Users were asked in which courses software was used and whether it was used for all or part of the grading.

The highest percentage of homework software use was for college algebra and below (87\%), calculus for scientists ( $81 \%$ ), and precalculus ( $78 \%$ ). The percentages were about a third higher at Ph.D. departments (which used it in an average of 3.8 courses) than in M.S. and B.S. departments (which used it in an average of 2.7 courses). At
doctoral departments, users were substantially more likely than prospective users to use software for grading all homework in all the listed courses except differential equations. An example of the doctoral pattern is calculus for scientists: $35 \%$ of users employed software to grade all homework in this course, whereas only $8 \%$ of prospective users contemplated all-software grading. These users may lack the resources for hand grading, or they may have developed a comfort level based on using software for part of the homework.

For college algebra and precalculus, MyMathLab was used $50 \%$ of the time, followed by WebAssign with $25 \%$. General calculus, calculus for scientists, and multivariable calculus were split fairly evenly among MyMathLab, WebAssign, and WeBWorK (which was originally designed for calculus courses). Statistics usage was dominated by MyMathLab and statistics-specific software.

Question 6. Enrollments of Courses Using Homework Software. Responses reflected typical enrollments in introductory mathematics courses at the respondent's type of institution.

Question 7. Changing Level of Usage. There was negligible decline $(\leq 5 \%)$ in the percentage of sections using homework software in any course. Furthermore, 35 to $50 \%$ of respondents reported increasing the number of sections using homework software, depending on the course. A few respondents were unhappy with a particular software system; they tended to switch to another system rather than discontinue the use of any homework software.

Question 8. Which Software. The most popular system was MyMathLab, cited 110 times, followed by WebAssign, cited 80 times (this number will grow with WebAssign's new association with Cengage Brooks/Cole), and WeBWorK, cited 55 times. MapleTA was cited only 15 times. About seventy respondents listed other systems. Doctoral departments were more likely (almost 50\%) than M.S. and B.S. departments to use multiple systems. Textbooks were the most common determinant in the choice of software; the textbook publisher's software was often used. Top-eighty doctoral departments were twice as likely to use WeBWorK (almost 40\%) as M.S. or B.S. institutions. WeBWorK needs local technical support to be mounted and maintained (although small-size usage can be hosted by a WeBWorK server at the Mathematical Association of America).

Question 9: Software Enrollments. MyMathLab was used by 230,000 students; WeBWorK by a little over 100,000 students; and WebAssign by a little under 100,000 students. These numbers were affected by size of institution, enrollment in courses using software, and number of sections using software.

The following graphs show the distribution of the total number of students using homework software at different types (Ph.D., M.S., B.S.) of departments:


Ph.D. Departments


The histograms suggest that (1) Ph.D. departments generally used homework software coursewide; (2) M.S. departments had more partial-course adoptions; and (3) in many B.S. departments, just a few sections of a course used homework software (although another interpretation is that only one or two small-enrollment courses used homework software).

Question 10. Obstacles to Adopting Software. The following obstacles were rated by respondents:
A. Difficulties in installing software on local server
B. Inadequate technical support for start-up period
C. Inadequate financial support for acquiring software/hardware
D. Inadequate long-term technical support
E. Inadequate long-term financial support for software/hardware
F. Trouble for faculty members in learning to use the system and deal with its limitations
G. Resistance by faculty members to using new software

Most were rated minor issues. Over 75\% of top-eighty Ph.D. departments and of M.S. departments had technical start-up problems, major or minor, with all software systems. Also, $60 \%$ of these departments had problems with long-term technical support for WebAssign. However, the percentage of technical support problems for WebAssign at other Ph.D. departments and B.S. departments were comparable to those of other software, around $40 \%$. Only a modest number of technical support problems were reported by departments using extensive learning systems such as ALEKS or Hawkes Learning System. All nine Ph.D. departments using MapleTA had start-up and long-term technical support problems (it is not known whether these departments were maintaining MapleTA locally or at a Maplesoft server).

As noted above, WeBWorK is locally maintained, and so local technical and financial support loom larger for WeBWorK. Around 70\% of WeBWorK users at all types of departments reported major or minor installation problems. There were also significant start-up technical support problems, except at top-eighty Ph.D. departments.

Faculty had trouble learning to use homework software in 75 to $90 \%$ of departments. However, less than a quarter of responses rated this as a major factor for any of the systems. Faculty resistance was pervasive- 80 to $100 \%$ of respondentsand was rated as a major factor by upward of $50 \%$ of respondents.

Question 11. Attitudes. The following groups rated their attitudes toward homework software as "very favorable, generally favorable, mixed, generally negative, very negative, don't know":
A. Students
B. Tenured/tenure-track faculty
C. Lecturers and other nontenure-track faculty
D. Graduate teaching assistants
E. Faculty in client departments
F. Campus administrators

There were very few negative responses. However, for most systems, at least $50 \%$ of the tenured/ tenure-track faculty had mixed attitudes. Also, for all software, students and nontenure-track faculty had more favorable attitudes than did tenure-track faculty. At public doctoral departments MyMathLab was rated favorably by over $70 \%$ of students and nontenure-track faculty but by only $23 \%$ of tenured/tenure-track faculty. Also, WebAssign had particularly low favorable ratings, around $25 \%$, among tenured/tenure-track faculty at M.S. departments.

## General Comments

All software systems received praise. Several department chairs wanted more of their faculty members to become interested in using homework software. Some respondents noted that their faculty believed that more students were doing the homework after the switch to homework software. Several respondents pointed to noticeable improvement in the passing rates in college algebra and lower courses, but one noticed no change. One complained that managing course homework spreadsheets was hard with commercial homework software. One doctoral respondent complained that funds for student graders were cut when homework software was adopted.

## Procedural Issues

Several survey questions required subjective judgments of the whole department's experiences or of the general views of faculty. The views of the individual completing the survey may have biased some of these subjective responses.

Other subjective factors need to be considered. We suspect that, in departments not using homework software, a survey of individual faculty members would frequently find that some were receptive to considering its use and some were not. Because faculty typically have considerable autonomy in how they run their classes, most faculty members in a department might not be interested in homework software, even though it was being used by other faculty members. Note that question 9, asking for the number of students using homework software, provides good data about the extent of software use in a department.

Clearly, departments using homework software or seriously interested in using it are more likely to respond to this survey. This is particularly evident with the thirty community colleges contacted, in which eight of the eleven responders were users. On the other hand, it is not reasonable to assume
that almost all homework software users responded to this survey. One of the methods used in surveys such as this to estimate the fraction of nonresponders interested in the topic of the survey is to look at the straggler responses-the very last responses that come in a month or more after most responses.Typically these late responses tend to be representative of the nonresponders. In this survey, the late responses displayed the same percentage of users as the early responses. So it is reasonable that there are a number of users who did not respond to this survey.

The Project Advisory Committee consisted of: Alan Tucker (State University of New York-Stony Brook), Project Director; Ellen Maycock, AMS Associate Executive Director and Project Liaison; Dale Alspach (Oklahoma State University); Jack Bookman (Duke University); Donna Krawczyk (University of Arizona); P. Gavin LaRose (University of Michigan); James Sellers (Pennsylvania State University); Brett Sims (Borough of Manhattan Community College); and Jean Taylor (Rutgers University, emerita).

This survey project was supported by Grant No. DUE0837131 from the National Science Foundation to the American Mathematical Society. The full report of the survey can be found on the AMS website, www.ams.org/profession/leaders/ webassess.htm.

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## Mathematics People

## 2010-2011 AMS Centennial Fellowship Awarded

The AMS has awarded its Centennial Fellowship for 2010-2011 to JoËl BeLLAÏCHE of Brandeis University. The fellowship carries a stipend of US\$77,000, an expense allowance of US $\$ 7,700$, and a complimentary Society membership for one year.

Joël Bellaïche was born and raised in Paris. As an undergraduate and graduate student, he attended the École Normale Supérieure in Paris. He defended his thesis in 2002 at the University of Orsay, where his advisor was Laurent Clozel. After a short postdoctoral stay at the University of Padua in Italy and at the Institute for Advanced Study in Princeton, and after one year as a maître de confèrences at the University of Nice (France), he moved permanently to the United States and became a Ritt Assistant Professor at Columbia University in 2004. Since January 2008 he has been an associate professor at Brandeis University.

Bellaïche's main interest is in the Birch and Swinner-ton-Dyer conjecture and, more generally, the Bloch-Kato conjectures. Those conjectures, still wide open, relate some analytic invariants of a motive over a number field (more specifically, the values of its $L$-function or its $p$-adic $L$-function) with some arithmetic invariant of the motive (e.g., the Mordell-Weil group of an elliptic curve). Bellaïche's approach uses the theory of automorphic forms.

Next year he plans to work on higher-rank $p$-adic $L$ functions and their relations with the universal families of automorphic forms called eigenvarieties. He plans to visit several places, including Princeton, Montreal, Chicago, and Paris.

Please note: Information about the competition for the 2011-2012 AMS Centennial Fellowships will be published in the "Mathematics Opportunities" section of an upcoming issue of the Notices.

> -Allyn Jackson

## Khot Wins Waterman Award

Subhash Кнот of the Courant Institute of Mathematical Sciences, New York University, has been awarded the 2010 Alan T. Waterman Award of the National Science Foundation. He is a theoretical computer scientist who works in the field of computational complexity, which seeks to understand the power and limits of efficient computation.

A fundamental phenomenon in computer science is the existence of computational problems that cannot be quickly solved. These "computationally intractable" problems, as they are called, present far-reaching consequences. For instance, they limit our ability to use mathematics to tackle large-scale problems arising in science and engineering, such as the optimal design of protein folding. Conversely, they make computer security possible, as computational intractability thwarts hackers' attempts to access personal information stored in online databases. Understanding and addressing this phenomenon, therefore, has huge potential benefits for science and engineering. Khot has made significant inroads in identifying computational intractability. He has uncovered a problem about probabilistic games called "the Unique Games Problem". His work shows that it lies at the core of a variety of intractable computational problems.

Khot earned a bachelor's degree from the Indian Institute of Technology, Bombay, in 1999 and a doctorate in computer science from Princeton University in 2003. He has been the recipient of an NSF CAREER award, a Sloan Foundation Fellowship, and a Microsoft New Faculty Fellowship.

The Waterman Award was established in 1975 and is considered the NSF's most prestigious honorary award, given annually to an outstanding researcher under the age of thirty-six in any field of science and engineering supported by the NSF. The award includes a grant of US $\$ 500,000$ over three years for scientific research or advanced study in the recipient's field of science.
-From an NSF announcement

## Tropp Awarded Popov Prize

Joel A. Tropp of the California Institute of Technology has been awarded the sixth Vasil Popov Prize for outstanding research contributions to approximation theory and related areas. According to the prize citation, Tropp was honored "for his outstanding contributions to the development of sparse reconstruction methods in the context of approximation from redundant systems, greedy algorithms, and, most recently, compressed sensing. In particular, he has shown that greedy algorithms will with high probability exactly recover sparse vectors from random measurements (for example) based on Gaussian or Bernoulli distributions. This was a cornerstone result in showing the efficacy of greedy algorithms for decoding in compressed sensing.... Tropp's work has significantly advanced the understanding of greedy algorithms and
sublinear reconstruction algorithms in new, highly relevant application contexts."

The Popov Prize honors the memory of Vasil A. Popov (1942-1990), the Bulgarian analyst best known for his work in nonlinear approximation. It is awarded every three years and carries a cash award of US $\$ 2,000$. Previous winners of the Popov Prize are Albert Cohen, Arno Kuijlaars, Emmanuel Candes, Serguei Denissov, and Mauro Maggioni.
-Popov Prize Selection Committee

## Kreck Receives Cantor Medal

The Deutsche-Mathematiker-Vereinigung (German Mathematical Society, DMV) has awarded the 2010 Cantor Medal to Matthias Kreck, director of the Hausdorff Research Institute for Mathematics in Bonn.

Born in 1972, Kreck received his doctorate from the University of Bonn under the direction of Friedrich Hirzebruch. Kreck held positions in Wuppertal, Mainz, Heidelberg, and Bonn and also served for many years as director of the Mathematisches Forschungsinstitut Oberwolfach. His research focuses on manifolds and their classification.

The Cantor Medal is the highest honor given by the DMV and is presented at most every two years for outstanding research in mathematics. Previous recipients are Karl Stein (1990), Jürgen Moser (1992), Erhard Heinz (1994), Jacques Tits (1996), Volker Strassen (1999), Yuri Manin (2002), Friedrich Hirzebruch (2004), Hans Föllmer (2006), and Hans Grauert (2008).
-From the DMV Mitteilungen

## Mantegazza Awarded 2010 Balaguer Prize

The Ferran Sunyer i Balaguer Foundation has awarded the Ferran Sunyer i Balaguer Prize for 2010 to Carlo MANtegazza of Scuola Normale Superiore di Pisa for his monograph Lecture Notes on Mean Curvature Flow. According to the prize citation, the monograph is "an introductory text to the mean curvature flow at a high level. It is an excellent book for Ph.D. students and for researchers... [that] fills a gap in the literature" and organizes an important amount of material on the mean curvature flow. The monograph "is very well written, with a precise and clear style." It includes chapters on definition of the flow and small time existence, evolution of geometric quantities, monotonicity formula and type I singularities, and type II singularities. It features a series of appendices on quasilinear parabolic equations on manifolds, interior estimates of Ecker and Huisken, Hamilton's maximum principle for tensors, classification of homothetically shrinking closed curves, and a series of important results without proofs.

The Ferran Sunyer i Balaguer Foundation of the Institut d'Estudis Catalans (IEC) awards this international prize every year to honor the memory of Ferran Sunyer i Balaguer (1912-1967), a self-taught Catalan mathematician who
gained international recognition for his research in mathematical analysis despite the serious physical disabilities with which he was born. The prize carries a cash award of 15,000 euros (approximately US\$20,000); the winning monographs are published by Birkhäuser Verlag.

> -From a Ferran Sunyer i Balaguer Foundation
> announcement

## Hersh Awarded Michler Prize

Patricia Hersh of North Carolina State University has been awarded the fourth annual Ruth I. Michler Memorial Prize by the Association for Women in Mathematics (AWM) and Cornell University. She will spend the fall semester of 2010 doing research at Cornell, where she plans to study topology and combinatorics of stratified spaces from Schubert calculus, combinatorial representation theory, and total positivity theory with Allen Knutson. She will also collaborate with Irena Peeza on combinatorial commutative algebra and cellular resolutions and with Ed Swartz on rings of graph colorings. Hersh received her Ph.D. from the Massachusetts Institute of Technology, where she studied enumerative properties and decomposition in partially ordered sets. Her primary interests are in algebraic and topological combinatorics, particularly the interactions between combinatorics and such fields as topology, commutative algebra, representation theory, and theoretical computer science. She is particularly interested in combinatorial methods for studying topological structure.

The Michler Prize is awarded annually to a woman recently promoted to associate professor or an equivalent position in the mathematical sciences. It consists of a residential fellowship in the Cornell University mathematics department without teaching obligations. The amount of the award is US $\$ 45,000$, with an additional travel allowance provided by the Cornell University mathematics department.
-From an AWM announcement

## Sloan Fellowships Awarded

The Alfred P. Sloan Foundation has announced the names of the recipients of the 2010 Sloan Research Fellowships. Each year the foundation awards 118 fellowships in the fields of mathematics, chemistry, computational and evolutionary molecular biology, computer science, economics, neuroscience, and physics. Grants of US\$50,000 for a twoyear period are administered by each fellow's institution. Once chosen, fellows are free to pursue whatever lines of inquiry most interest them, and they are permitted to employ fellowship funds in a wide variety of ways to further their research aims.

Following are the names and institutions of the 2010 awardees in mathematics: Spyros Alexakis, University of Toronto; Omer Angel, University of British Columbia; Jason Behrstock, Lehman College, City University of New York; Janet Best, Ohio State University; Alexander

Bufetov, Rice University; MAtthew DeVos, Simon Fraser University; LARRY Guth, University of Toronto; RADU Laza, Stony Brook University; Tyler Lawson, University of Minnesota; Max Lieblich, University of Washington; Svitlana Mayboroda, Purdue University; Alexei ObLomkov, University of Massachusetts, Amherst; RaAnan Schul, Stony Brook University; Amit Singer, Princeton University; Balázs Szegedy, University of Toronto; Joel A. Tropp, California Institute of Technology; MARK Tygert, Courant Institute of Mathematical Sciences, New York University; Monica Visan, University of California Los Angeles; Maria G. Westdickenberg, Georgia Institute of Technology; and Xiaoyi Zhang, University of Iowa.
-From a Sloan Foundation announcement

## Prizes of the Mathematical Society of Japan

The Mathematical Society of Japan (MSJ) has awarded several prizes for 2009. Kenji Yajima of Gakushuin University was awarded the 2009 Autumn Prize for his fundamental work on the boundedness of wave operators. His work on the $L^{p}$ boundedness of wave operators for Schrödinger operators has had a tremendous impact on mathematical physics, operator theory, scattering theory, harmonic analysis, and nonlinear partial differential equations. The Autumn Prize is awarded to a member of the MSJ who has made exceptional contributions in his or her field of research.

Tatsuo Nishitani of Osaka University, Hiroaki AIkawa of Hokkaido University, and Takayoshi Ogawa of Tohoku University received Analysis Prizes. Nishitani was recognized for his contributions to the study of $C^{\infty}$ well-posedness for the noneffectively hyperbolic operators and, in particular, his recent development of the $C^{\infty}$ and of the Gevrey five-class well-posedness under the Levi condition and Gevrey three- or four-class well-posedness without it. Aikawa was honored for his contributions to the development of potential theory on nonsmooth domains and, in particular, for his work on characterizing geometric properties of domains in terms of behavior of harmonic functions and harmonic measures. Ogawa was recognized for his contributions to real analysis and its applications to nonlinear partial differential equations in critical function spaces and, in particular, for his end point estimates of the heat semigroup in the Hardy-Besov spaces.

Ko Honda of the University of Southern California and Yoshikata Kida of Kyoto University were awarded Geometry Prizes. Honda was recognized for his fundamental work on the topology of contact structures in dimension three, in particular on the relations between tightness of contact structures and the possible monodromies of adapted open book decompositions. Kida was honored for his outstanding research on the rigidity for actions of the mapping class group of a compact orientable surface from the viewpoint of measure equivalence theory or that of orbit equivalence theory.

The Takebe Katahiro Prizes are awarded to young members of the MSJ who have obtained outstanding results, and the Takebe Katahiro Prize for Encouragement of Young Researchers is awarded to young members of the MSJ who are deemed to have begun promising careers in research by obtaining significant results. The 2009 Takebe Katahiro Prizes were awarded to Ryoki Fukushima of Kyoto University for his work on analysis of Brownian motion in random obstacles, to AKIhiro Shimomura of Tokyo Metropolitan University for his work on the scattering problem for nonlinear dispersive equations, and to Hiroshi Iritani of Kyushu University for his research on Gromov-Witten invariants. The 2009 Takebe Katahiro Prizes for Encouragement of Young Researchers were awarded to: Yoshihiro SAWANO of Gakushuin University for his study of function spaces by real variable methods, Hironobu SaSaki of Chiba University for his study of inverse scattering for nonlinear dispersive equations with Yukawa type interaction, Kouichi Yasui of Kyoto University for his research on handle decompositions of four-manifolds, Nobu Kishimoto of Kyoto University for his research on unique solvability of nonlinear dispersive equations and function spaces, Hideyuki Miura of Osaka University for his work on mathematical analysis of nonlinear partial differential equations in fluid mechanics, and Kokoro Tanaka of Tokyo Gakugei University for his studies of surface knots and surface braids.
-From a Mathematical Society of Japan announcement

## CMS Prizes Awarded

The Canadian Mathematical Society (CMS) has announced the awarding of several major prizes.

Kai Behrend of the University of British Columbia has been named the recipient of the 2011 Jeffery-Williams Prize, which recognizes mathematicians who have made outstanding contributions to mathematical research. The award recognizes Behrend's contributions to the theory of algebraic stacks and the geometry of moduli spaces of stable maps, as well as his work on Gromov-Witten theory, Donaldson-Thomas theory, and the virtual fundamental class.

Rachel Kuske of the University of British Columbia has been awarded the 2011 Krieger-Nelson Prize, which recognizes outstanding research by a woman mathematician. According to the prize citation, she is "one of Canada's leading applied mathematicians and has also become an acknowledged expert and innovator in the field of mathematics education." The award recognizes her contributions to the study of ordinary, stochastic, and partial differential equation models for a wide range of applications, including neuroscience, mathematical biology, buckling under compression, mathematical finance, and hydraulic-fracture mechanics, as well as her work in the mathematics community as founder and co-chair of the mentor network of the Association for Women in

Mathematics and as a member of the editorial boards of several mathematical journals.

## Hertz Foundation Fellowships Awarded

The Fannie and John Hertz Foundation has awarded Hertz Fellowships to fifteen students in the applied sciences and engineering for support of up to five years of graduate studies. The fellows were selected from a pool of nearly six hundred applicants. Four students in the mathematical sciences were chosen as fellows. They are David Lecoanet, University of Wisconsin, Madison; Maria Monks, Massachusetts Institute of Technology; Justin Solomon, Stanford University; and Jeff Weber, Willamette University. The fellowships provide up to US $\$ 36,000$ per year for graduate studies with no strings attached, allowing exceptional applied scientists and engineers the freedom to innovate.
-From a Hertz Foundation announcement

## Prizes of the CRM

The Centre de Recherches Mathématiques (CRM) in Montreal, Canada, has awarded several prizes for 2010.

The 2009-2010 André Aisenstadt Prize has been awarded to OMER ANGEL of the University of British Columbia. The prize, consisting of C $\$ 3,000$ (approximately US $\$ 3,000$ ) and a medal, recognizes achievements in research by young Canadian mathematicians. According to the prize citation, "Angel, with Schramm, put the local study of random triangulations on firm ground by proving the existence of a limit object. Subsequently he showed how to sample such a uniform random triangulation, a tool which allowed him to determine the critical percolation probability and establish the widely held belief (among physicists) that a random metric on a manifold has Hausdorff dimension 4." More recently, he, along with Holroyd, Romik, and Virag, "gave a very precise description of some of the statistics of a random sorting network, allowing them to make the remarkable conjecture that a typical short (random) path between objects on a lattice stays close to a geodesic (under the standard embedding)."

The CRM and the Statistical Society of Canada (SSC) have awarded the 2010 CRM-SSC Prize in Statistics to Grace Y. Yi of the University of Waterloo. According to the prize citation, "she has contributed in a significant way to the development of statistical methods for longitudinal studies and for the analysis of time-to-event data, especially for the treatment of missing observations and measurement errors. Her work on the asymptotic behavior of parametric and semiparametric inference techniques has also been influential in statistics and biostatistics." The prize, which includes a cash award of $C \$ 3,000$ (approximately US $\$ 3,000$ ), is given to a Canadian citizen or a
permanent resident of Canada whose research was carried out primarily in Canada.
-From a CRM announcement

## Leitgeb Awarded Humboldt Professorship

Hannes Leitgeb of the University of Bristol, United Kingdom, has been awarded an Alexander von Humboldt Professorship for 2010 by the Alexander von Humboldt Foundation. According to the prize citation, Leitgeb is "one of the world's leading academics working at the interface of logic, mathematics, and cognitive science. His numerous innovative achievements include his work on artificial neural networks. This is where Leitgeb closes the interdisciplinary gap between research into artificial intelligence and brain research. In Munich Leitgeb will build up a new center of mathematical philosophy, which will cooperate closely with neurophilosophy and neuroscience."

The Alexander von Humboldt Professorship honors researchers from outside of Germany who are internationally recognized leaders in their fields and allows them to spend five years conducting research at German universities. The award is valued at up to five million euros (approximately US\$6.7 million) and is endowed by the Federal Ministry of Education and Research.
-From a Humboldt Foundation announcement

## Tygert Receives NAS Award

MARK Tygert of the Courant Institute of Mathematical Sciences, New York University, has received the 2010 NAS Award for Initiatives in Research of the National Academy of Sciences (NAS). He was honored for his "development of fast algorithms in mathematical physics, operator compression, and linear algebra, using deep, innovative ideas based on randomization and harmonic analysis." The award, which carries a cash prize of US $\$ 15,000$, recognizes innovative young scientists and encourages research likely to lead toward new capabilities for human benefit.
-From an NAS announcement

## Nesterov and Ye Awarded John von Neumann Theory Prize

The 2009 John von Neumann Theory Prize, the highest prize given in the field of operations research and management science, has been awarded to Yurii Nesterov of the Center for Operations Research and Econometrics, Université Catholique de Louvain, and Yinyu Ye of Stanford University. According to the prize citation, Nesterov
"is the world's leading authority on the efficiency of algorithms for continuous optimization." Ye "has been at the forefront of research on interior-point methods and applications of conic optimization for over twenty years." The award, which is presented by the Institute for Operations Research and the Management Sciences (INFORMS), carries a cash prize of US\$5,000.
-From an INFORMS announcement

## Rollo Davidson Prizes Awarded

The Rollo Davidson Trust has awarded the 2010 Rollo Davidson Prizes to Gady Kozma of the Weizmann Institute for his work in probability and Fourier analysis and to Sourav Chatterjee of the Courant Institute of Mathematical Sciences, New York University, for his work on Stein's method, spin glasses, and concentration of measure.

The Rollo Davidson Trust was founded in 1975 and awards an annual prize to young mathematicians working in the field of probability.
-Elaine Kehoe

## Putnam Prizes Awarded

The winners of the seventieth William Lowell Putnam Mathematical Competition have been announced. The Putnam Competition is administered by the Mathematical Association of America (MAA) and consists of an examination containing mathematical problems that are designed to test both originality and technical competence. Prizes are awarded to both individuals and teams.

The five highest-ranking individuals, listed in alphabetical order, were: William A. Johnson, University of Washington, Seattle; Xiaosheng Mu, Yale University; Qingchun Ren, Massachusetts Institute of Technology; Arnav Tripathy, Harvard University; and Yufei Zhao, Massachusetts Institute of Technology. Each received a cash award of US\$2,500.

Institutions with at least three registered participants obtain a team ranking in the competition based on the rankings of three designated individual participants. The five top-ranked teams (with team members listed in alphabetical order) were: Massachusetts Institute of Technology (Qingchun Ren, Bohua Zhan, Yufei Zhao); Harvard University (Iurie Boreico, Arnav Tripathy, Alex Zhai); California Institute of Technology (Jason C. Bland, Sam Elder, Gjergji Zaimi); Stanford University (Young Hun Jung, Seok Hyeong Lee, Jeffrey Wang); and Princeton University (Peter Z. Diao, Adam C. Hesterberg, John V. Pardon).

The first-place team receives an award of US\$25,000, and each member of the team receives US $\$ 1,000$. The awards for second place are US\$20,000 and US\$800; for third place, US $\$ 15,000$ and US $\$ 600$; for fourth place,

US\$10,000 and US\$400; and for fifth place, US\$5,000 and US\$200.
-From a Putnam announcement

## Guggenheim Fellowships Awarded

The John Simon Guggenheim Memorial Foundation has announced the names of 180 artists, scholars, and scientists from the United States, Canada, and the United Kingdom who were selected as Guggenheim Fellows for 2010. Guggenheim Fellows are appointed on the basis of distinguished achievement in the past and exceptional promise for future accomplishment. The mathematicians selected to receive the 2010 fellowships are InGRID Daubechies, Princeton University; Tomasz Mrowka, Massachusetts Institute of Technology; and JOHN Wettlaufer, Yale University.
-From a Guggenheim Foundation news release

## Intel Science Talent Search Winners Announced

Three students whose work involves the mathematical sciences have received scholarship awards in the 2010 Intel Science Talent Search. Akhil Mathew, an eighteen-year-old student from Madison, New Jersey, was awarded third place and a US $\$ 50,000$ scholarship for his project on Deligne categories, a setting for studying a wide range of algebraic structures with ties to theoretical physics. Katherine Rudolph, an eighteen-year-old student from Naperville, Illinois, won the eighth-place award and a US $\$ 20,000$ scholarship for her project that investigated dense packing of identical spheres, the results of which can be used in fields from chemistry to cryptology. Yale FAN, an eighteen-year-old student from Beaverton, Oregon, was awarded ninth place and a US $\$ 20,000$ scholarship for his research that demonstrated the advantages of quantum computing in performing difficult computations.

## NSF Graduate Research Fellowships Announced

The National Science Foundation (NSF) has awarded a number of Graduate Research Fellowships for fiscal year 2010. Further awards may be announced later in the year. This program supports students pursuing doctoral study in all areas of science and engineering and provides a stipend of US\$30,000 per year for a maximum of three years of full-time graduate study. Following are the names of the awardees in the mathematical sciences selected so
far in 2010, followed by their undergraduate institutions (in parentheses) and the institutions at which they plan to pursue graduate work.

Zachary Abel (Harvard University), Massachusetts Institute of Technology; William C. Abram (University of Chicago), University of Michigan, Ann Arbor; Timothy M. ADAMO (University of Oxford), University of Oxford; Nikhyl B. Aragam (University of California Los Angeles), University of California Los Angeles; Rebecca Bellovin (Stanford University), Stanford University; Nate Bottman (University of Washington), Harvard University; GeOrge A. Boxer (Princeton University), Harvard University; NichoLAS D. Brubaker (University of Delaware), University of Delaware; Alhaji Cherif (Cornell University), Arizona State University; Sarah Constantin (Princeton University), Yale University; IAIN J. CRUICKSHANK (United States Military Academy), University of Chicago; DAMEK S. DAVIS (University of California Irvine), University of California San Diego; Kevin A. Del Bene (Rensselaer Polytechnic Institute), Rensselaer Polytechnic Institute; Peter Z. Diao (Princeton University), Stanford University; Alexander P. Ellis (Columbia University), Columbia University; MAx Engelstein (Yale University), Stanford University; ClaUdia Falcon (University of North Carolina, Chapel Hill), University of North Carolina, Chapel Hill; Iordan V. GANEV (Miami University), University of Chicago; Evan S. GAWLIK (California Institute of Technology), Princeton University; Jesse T. Geneson (Harvard University), University of California Berkeley; SEan P. Howe (University of Arizona), Massachusetts Institute of Technology; NATHAN Kallus (University of California Berkeley), Massachusetts Institute of Technology; ADAM D. KAPELNER (University of Pennsylvania), University of Pennsylvania; RACHEL KARPman (Scripps College), University of Michigan, Ann Arbor; Erin M. Kiley (Worcester Polytechnic Institute), Worcester Polytechnic Institute; Jeffrey Kuan (California Institute of Technology), Princeton University; Jaclyn A. Lang (University of Cambridge, Churchill College), University of California Los Angeles; JASON D. LEE (Duke University), Stanford University; John D. Lesieutre (Massachusetts Institute of Technology), Massachusetts Institute of Technology; Yingkun Li (University of California Los Angeles), University of California Los Angeles; Tova Lindberg (Bethany Lutheran College), Washington University; DANIEL A. Litt (Harvard University), Princeton University; Miles E. Lopes (University of California Berkeley), University of California Berkeley; Zachary A. Maddock (Columbia University), Columbia University; SuSAN C. MASSEY (University of Washington), University of Washington; Talea L. MAYO (University of Texas, Austin), University of Texas, Austin; Shira A. Mitchell (Harvard University), Harvard University; Maria Monks (Massachusetts Institute of Technology), University of California Berkeley; RALPH E. Morrison (Williams College), University of Wisconsin, Madison; SAMUEL R. NOLEN (Vanderbilt University), Cornell University; Matthew Paff (Cornell University), Stanford University; Heather S. Palmeri (Rensselaer Polytechnic Institute), Rensselaer Polytechnic Institute; GInA-MARIA Pomann (North Carolina State University), North Carolina State University; AARON H. Potechin (Cambridge

University), Massachusetts Institute of Technology; ERIC RIEDL (University of Notre Dame), Harvard University; Douglas P. Rizzolo (University of California Berkeley), University of California Berkeley; Stephanie Sapp (University of California Berkeley), University of California Berkeley; Brandon M. Seward (University of Michigan), University of Michigan, Ann Arbor; Daniel Shapero (McGill University), New York University; John M. Silberholz (University of Maryland), Massachusetts Institute of Technology; LAURA P. STARKSTON (Harvard University), Columbia University; Nike Sun (Cambridge University), Stanford University; Yi Sun (Harvard University), Massachusetts Institute of Technology; Hiroaki Tanaka (Northwestern University), Northwestern University; Elizabeth K. Tuley (University of Maryland), Princeton University; MADELEINE R. Udell (Stanford University), Stanford University; Anil B. Venkatesh (Duke University), Duke University; Robin S. Walters (Harvard University), University of Chicago; John S. Wilmes (Reed College), University of Chicago; Jed C. YANG (University of California Los Angeles), University of California Los Angeles; Melissa L. Yeung (University of Chicago), Columbia University; and Joshua N. ZaHL (University of California Los Angeles), University of California Los Angeles.
-From an NSF announcement

## American Academy Elections

Fifteen mathematical scientists have been elected to membership in the American Academy of Arts and Sciences. They are: Andrea L. Bertozzi, University of California Los Angeles; Peter Constantin, University of Chicago; MARK L. Green, University of California Los Angeles; Joseph B. Kadane, Carnegie Mellon University; Robert Kottwitz, University of Chicago; ANDREW J. MAJDA, Courant Institute of Mathematical Sciences, New York University; Grigori Mints, Stanford University; Freydoon Shahidi, Purdue University; Jalal Shatah, Courant Institute of Mathematical Sciences, New York University; Madhu Sudan, Massachusetts Institute of Technology and Microsoft Research New England; Moshe Y. Vardi, Rice University; MARY F. Wheeler, University of Texas, Austin; and Maciej R. ZWORSKI, University of California Berkeley. Elected as foreign honorary members were DAVId Schmeidler, Tel Aviv University, and Ernest B. Vinberg, Moscow State University.

The American Academy of Arts and Sciences was founded in 1780 to foster the development of knowledge as a means of promoting the public interest and social progress. The membership of the academy is elected and represents distinction and achievement in a range of intellectual disciplines-mathematical and physical sciences, biological sciences, social arts and sciences, and humanities and fine arts.
-From an AAAS announcement

# Mathematics Opportunities 

## NSF CAREER Program Guidelines Available

The guidelines for the Faculty Early Career Development (CAREER) Program of the National Science Foundation (NSF) are now available on the World Wide Web. The program solicitation number is 08-557. Information is available at http://nsf.gov/pubs/2008/nsf08557/ nsf08557.htm. The deadline for submission of proposals is July 22, 2010.
-From an NSF announcement

## Call for Nominations for SASTRA Ramanujan Prize

The Shanmugha Arts, Science, Technology Research Academy (SASTRA) invites nominations for the 2010 SASTRA Ramanujan Prize. The prize carries a cash award of US $\$ 10,000$, and the winner will be invited to give a talk at the SASTRA conference in December 2010. The deadline for nominations is August 15, 2010. For more information, email sastraprize@math.uf1.edu or see the websitehttp://www.math.uf1.edu/sastra-prize/ nominations-2008.htm7.

## Call for Nominations for Heineman Prize

The American Physical Society (APS) and the American Institute of Physics (AIP) are seeking nominations for the 2011 Dannie Heineman Prize for Mathematical Physics. The prize recognizes outstanding publications in the field of mathematical physics. The prize carries a cash award of US\$10,000, an award certificate, and travel expenses to the meeting at which the prize is given. The deadline for nominations is July 1, 2010. For
more information, see the APS website at http://www. aps.org/programs/honors/prizes/heineman.cfm.
-From an APS announcement

## Call for Nominations for Parzen Prize

To promote the dissemination of statistical innovation, the Emanuel and Carol Parzen Prize for Statistical Innovation is awarded in even-numbered years to North American statisticians who have made outstanding and influential contributions by developing innovative statistical methods. Candidates must have received the Ph.D. degree more than twenty-five years before the nomination. The prize consists of an honorarium of US $\$ 1,000$ and travel expenses to College Station, Texas, to present a lecture at the prize ceremony. Nominations for the 2012 Parzen Prize should be submitted by October 1, 2011, to Thomas Wehrly, Department of Statistics, 3143 TAMU, Texas A\&M University, College Station, Texas 77843-3143.
-From a Texas A\&M announcement

## Call for Proposals for Research Programs at CRM

The Centre de Recerca Matemàtica (CRM) invites proposals for research programs for the academic year 2012-2013. CRM research programs cover between three months and nine months of intensive research in a given area of the mathematical sciences and their applications. Researchers from different institutions are brought together to work on open problems and to analyze the state and perspectives of their area. Guidelines and application instructions can be found at http://www.crm.es/. The deadline for submission of proposals is November 19, 2010.
-From a CRM announcement

## Inside the AMS

## AMS Announces Congressional Fellow

The American Mathematical Society (AMS) is pleased to announce that Hugh MacMillan has been chosen as its Con-


Hugh MacMillan gressional Fellow for 2010-2011.

The AMS will sponsor MacMillan's fellowship through the Congressional Fellowship program administered by the American Association for the Advancement of Science (AAAS). The fellowship is designed to provide a unique public policy learning experience, to demonstrate the value of science-government interaction, and to bring a technical background and external perspective to the decision-making process
in Congress.
Fellows spend a year working on the staff of a member of Congress or a congressional committee as a special legislative assistant in legislative and policy areas requiring scientific and technical input. The fellowship program includes an orientation on congressional and executive branch operations and a year-long seminar series on issues involving science, technology, and public policy.

Hugh MacMillan earned his Ph.D. in applied mathematics from the University of Colorado at Boulder and most recently worked as an assistant professor of mathematical sciences at Clemson University.

For more information on the AAAS Congressional Fellowship Program, visit the websitehttp://www.fe11owships. aaas.org.

## Erdős Memorial Lecture

The Erdős Memorial Lecture is an annual invited address named for the prolific mathematician Paul Erdôs (19131996). The lectures are supported by a fund created by Andrew Beal, a Dallas banker and mathematics enthusiast. The Beal Prize Fund, now US $\$ 100,000$, is being held by the AMS until it is awarded for a correct solution to the Beal Conjecture (see www.math.unt.edu/~mau1din/bea1. htm7. At Mr. Beal's request, the interest from the fund is used to support the Erdős Memorial Lecture.

The Erdős Memorial Lecturer for 2010 was Doron Zeilberger of Rutgers University. He delivered a lecture titled " $3 x+1$ " at the Spring Southeastern Section Meeting in Lexington, Kentucky, in March 2010.
-AMS announcement

## AMS Holds Workshop for Department Chairs

The AMS held its annual workshop for department chairs prior to the Joint Mathematics Meetings in San Francisco, California, in January 2010. This one-day session for mathematical sciences department chairs and leaders is organized in a workshop format to facilitate the sharing of ideas and experiences between peers and to create an environment in which attendees can address departmental challenges from new perspectives.

The 2010 workshop was led by Lawrence Gray, former head and director of undergraduate studies at the School of Mathematics, University of Minnesota; John Meakin, chair, Department of Mathematics, University of Nebraska, Lincoln; and Stephen Robinson, chair, Department of Mathematics, Wake Forest University.

Workshop sessions have included a range of issues facing departments, including planning and budgeting, personnel management, assessment, outreach, faculty development, communications, and departmental leadership. Workshop leaders used a case study approach this year to elicit discussion on these important topics.
-Anita Benjamin, AMS Washington Office

## From the AMS Public Awareness Office

In April the American Mathematical Society (AMS) organized and cosponsored Cinemath: A Festival of Math Films in Rhode Island. The first event was a screening at the Rhode Island School of Design of Between the Folds, the award-winning documentary exploring the science, art, creativity, and ingenuity of many of the world's best paper folders, including theoretical scientists who fuse mathematics and sculpture in the medium of origami. Following the screening, filmmaker Vanessa Gould (greenfusefilms.com) answered questions from the

audience. Rhode Island PBS aired several documentaries, including Between the Folds, Hunting the Hidden Dimension, It All Adds Up, and Hard Problems: The Road to the World's Toughest Math Contest. The Providence Public Library screened Stand and Deliver (based on the story of real-life math teacher Jaime Escalante, who passed away in March), Good Will Hunting, and A Beautiful Mind.

The Public Awareness Office conducted Who Wants to Be a Mathematician at the University of Michigan in Ann Arbor in March and at Providence College in April. Neil Gurram of Detroit Country Day School and Seth Neel of The Wheeler School were the big winners at the two events, each winning US\$3,000 and a TI-Nspire graphing calculator. Read about the contests at http://www.ams.org/programs/ students/wwtbam/ann-arbor-2010 and|http://www. ams.org/programs/students/wwtbam/ri-2010. Also see video of these games and others, including the national contest that took place at the Joint Mathematics Meetings in San Francisco, at http://www.youtube.com/user/ amermathsoc.

The Public Awareness Office distributed posters and materials to members of the National Council of Teachers of Mathematics (NCTM) in a spring mailing and also at the NCTM's annual meeting in a small display in the American Statistical Association's exhibit booth.
-Annette Emerson and Mike Breen AMS Public Awareness Officers paoffice@ams.org

## AMS Announces Mass Media Fellowship Award

The American Mathematical Society (AMS) is pleased to announce that Benjamin Pittman-Polletta has been awarded its 2010 Mass Media Fellowship. PittmanPolletta is a Ph.D. student in


Benjamin PittmanPolletta mathematics at the University of Arizona. He will work at the Oregonian for ten weeks over the summer under the sponsorship of the AMS.

The Mass Media Fellowship program is organized by the American Association for the Advancement of Science (AAAS). It is a highly competitive program designed to improve public understanding of science and technology by placing advanced science, mathematics, and engineering students in
newsrooms nationwide. Fellows work with media professionals to improve their communication skills and increase their understanding of the editorial process by which events and ideas become news.

The program is available to college or university students (in their senior years or on any graduate or postgraduate level) in the natural, physical, health, engineering, computer, or social sciences or mathematics who have outstanding written and oral communication skills and a strong interest in learning about the media. The program has supported more than 500 fellows in its thirty-six years.

For more information on the AAAS Mass Media Science \& Engineering Fellowship Program, visit the website http://www.aaas.org/programs/education/ MassMedia/.

> -AMS Washington Office

## Fan China Exchange Program Names Awardees

The Society's Fan China Exchange Program awards grants to support collaborations between Chinese and U.S. or Canadian researchers. Institutions in the United States or Canada apply for the funds to support a visitor from China or vice versa. This funding is made possible through a generous gift made to the AMS by Ky and Yu-Fen Fan in 1999. The awardees for 2010 follow.

North Carolina State University received a grant of US\$2,500 to support a visit by Zhengfeng Yang of East China Normal University.

The University of Texas at Arlington received a grant of US\$3,480 to support a visit by Pingzheng Zhang of the University of Jiangsu.

For information about the Fan China Exchange Program, visit the website http://www.ams.org/programs/ trave7-grants/china-exchange/china-exchange or contact the AMS Membership and Programs Department, email: chinaexchange@ams.org., telephone 401-4554170 (within the U.S. call 800-321-4267, ext. 4170).
-AMS Membership and Programs Department

## AMS Sponsors Exhibit at Capitol Hill Event

Susan Minkoff of the University of Maryland-Baltimore County (UMBC) represented the AMS at the annual Coalition for National Science Funding (CNSF) exhibition on Capitol Hill, held April 14, 2010. Minkoff presented her work on two different application problems, which are modeled at least in part by the wave equation. The two projects are both highly collaborative, involving faculty and students at UMBC and other institutions.

One project involved modeling and simulation of a next generation of trace gas sensors. These sensors will lead to new ways to detect diseases via breath biomarkers and


Left to right: Susan Minkoff with Rep. Vernon Ehlers (R-MI) and NSF Director Arden Bement.
more portable and robust technology for monitoring of atmospheric pollutants, chemical weapons, and so forth. The other project involved seismic imaging of the Earth's subsurface either to illuminate the near surface (environmental applications), the deep crust (earthquake modeling and prediction), or the region between these two regimes (possible petroleum targets).

The annual CNSF exhibition highlights research and education projects made possible through funding by the National Science Foundation. The 2010 exhibition included thirty-seven displays on a wide range of projects.

> - Anita Benjamin, AMS Washington Office

## Deaths of AMS Members

Dean C. Benson, from Kennewick, Washington, died on March 18, 2010. Born on October 25, 1918, he was a member of the Society for 59 years.

MARY L. BoAs, from Lake Forest Park, Washington, died on February 17, 2010. Born on March 10, 1917, she was a member of the Society for 70 years.

MICHAEL A. CASE, obtaining his doctorate in mathematics from Clemson University, died on April 3, 2010. Born on September 14, 1981, he was a member of the Society for 1 year.

Father Robert R. Dobbins, professor emeritus of mathematics at Iona College, died on November 13, 2009. Born on August 29, 1934, he was a member of the Society for 38 years.

Edwin Duda, from Miami, Florida, died on May 21, 2009. Born on October 15, 1928, he was a member of the Society for 50 years.

Ky FAN, professor emeritus, University of California, Santa Barbara, died on March 22, 2010. Born on September 19,1914 , he was a member of the Society for 64 years.
J. FANG, professor, from North, Virginia, died on February 16, 2010. Born on March 30, 1923, he was a member of the Society for 47 years.

Andrew J. Friedman died on October 26, 2009. Born on March 20, 1952, he was a member of the Society for 11 years.

Vladislav V. Goldberg, professor emeritus, New Jersey Institute of Technology, died on March 18, 2010.

Born on January 4, 1936, he was a member of the Society for 30 years.

George W. Heigho, from California, died on March 19, 2010. Born on July 10, 1933, he was a member of the Society for 47 years.
F. B. Hildebrand, from Wellesley, Massachusetts, died on November 29, 2002. He was a member of the Society for 62 years.

Raj K. Kaul, professor, State University of New York at Buffalo, died on April 9, 2010. Born on May 8, 1923, he was a member of the Society for 25 years.

MATTHEW KING, first-year graduate student, University of Virginia, died on April 19, 2010. Born on March 19, 1987, he was a member of the Society for 1 year.

RICHARD K. LASHOF, professor, from Alameda, California, died on February 4, 2010. Born on November 9, 1922, he was a member of the Society for 59 years.
E. L. Lehmann, professor, University of California, Berkeley, died on September 12, 2009. Born in 1917, he was a member of the Society for 37 years.

George F. Lenz, professor, University of New Orleans and Tulane University, died on February 11, 2010. Born on December 11, 1927, he was a member of the Society for 15 years.

WARREN S. LOUD, professor emeritus, from Minneapolis, Minnesota, died on January 15, 2010. Born on September 13, 1921, he was a member of the Society for 68 years.

Arik A. Melikyan, professor, Russian Academy of Sciences, died on April 6, 2009. Born on October 5, 1944, he was a member of the Society for 15 years.

James M. Ortega, professor, from Palmyra, Virginia, died on October 24, 2008. Born on June 15, 1932, he was a member of the Society for 47 years.

Edgar Reich, professor, University of Minnesota, died on July 6, 2009. Born on June 7, 1927, he was a member of the Society for 60 years.

Walter Rudin, professor, University of Wisconsin, died on May 20, 2010. Born on May 2, 1921, he was a member of the Society for 62 years.

JOHN SHAFER, professor, from Kensington, California, died on January 21, 2010. Born on July 9, 1924, he was a member of the Society for 43 years.

Gerald P. Shannon, professor, University College of North Wales, died on May 5, 2010. Born on March 20, 1944, he was a member of the Society for 18 years.

GARY L. VANCE, professor, from Michigan, died on June 8, 2008. Born on July 10, 1937, he was a member of the Society for 7 years.

Joseph S. Verret, professor, from Los Angeles, California, died on March 30, 2010. Born on December 11, 1945, he was a member of the Society for 19 years.

Bernard Vinograde, professor, from Santa Clara, Utah, died on March 29, 2010. Born on May 7, 1915, he was a member of the Society for 69 years.

Stephen Willard, professor, University of Alberta, Canada, died on August 7, 2009. Born on November 1, 1941, he was a member of the Society for 44 years.

Hongyou Wu, professor, Northern Illinois University, died on June 9, 2009. Born on October 15, 1961, he was a member of the Society for 21 years.

# Reference and Book List 

The Reference section of the Notices is intended to provide the reader with frequently sought information in an easily accessible manner. New information is printed as it becomes available and is referenced after the first printing. As soon as information is updated or otherwise changed, it will be noted in this section.

## Contacting the Notices

The preferred method for contacting the Notices is electronic mail. The editor is the person to whom to send articles and letters for consideration. Articles include feature articles, memorial articles, communications, opinion pieces, and book reviews. The editor is also the person to whom to send news of unusual interest about other people's mathematics research.

The managing editor is the person to whom to send items for "Mathematics People", "Mathematics Opportunities", "For Your Information", "Reference and Book List", and "Mathematics Calendar". Requests for permissions, as well as all other inquiries, go to the managing editor.

The electronic-mail addresses are notices@math.wust1.edu in the case of the editor and notices@ ams.org in the case of the managing editor. The fax numbers are 314-935-6839 for the editor and 401-331-3842 for the managing editor. Postal addresses may be found in the masthead.

## Information for Notices Authors

The Notices welcomes unsolicited articles for consideration for publication, as well as proposals for such articles. The following provides general
guidelines for writing Notices articles and preparing them for submission. Contact information for Notices editors and staff may be found on the Notices website, http://www.ams. org/notices.

Notices readership. The Notices publishes articles that have broad appeal for a diverse audience with many different types of readers: graduate students, academic mathematicians, industrial mathematicians, researchers in mathematically based fields, and amateur enthusiasts. The
paper edition of the Notices is sent to the approximately 33,000 members of the AMS, most of whom are professional mathematicians; about 25,000 of them reside in North America. Because the Notices is accessible for free over the Internet, the number of readers is much larger than the AMS membership. All readers may be assumed to be interested in mathematics research, but they are not all active researchers.

## Where to Find It

A brief index to information that appears in this and previous issues of the Notices.
AMS Bylaws-November 2009, p. 1320
AMS Email Addresses-February 2010, p. 268
AMS Ethical Guidelines-June/July 2006, p. 701
AMS Officers 2008 and 2009 Updates-May 2010, p. 670
AMS Officers and Committee Members-October 2009, p. 1133
Conference Board of the Mathematical Sciences—September 2009, p. 977

IMU Executive Committee-December 2009, p. 1465
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Mathematics Research Institutes Contact Information-August 2009, p. 854

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New Journals for 2008-June/July 2009, p. 751
NRC Board on Mathematical Sciences and Their Applications-March 2010, p. 423
NRC Mathematical Sciences Education Board—April 2010, p. 541
NSF Mathematical and Physical Sciences Advisory Committee-February 2010, p. 272
Program Officers for Federal Funding Agencies-October 2009, p. 1126 (DoD, DoE); December 2009, p. 1464 (NSF Mathematics Education)

Program Officers for NSF Division of Mathematical Sciences-
November 2009, p. 1313

## NoticesFeature Articles

Topics. The Notices seeks exceptional articles that report on major new developments in mathematics or that describe episodes from mathematics history that have connection to current research in the field. We also welcome articles discussing aspects of the mathematics profession, such as grant programs, the job market, professional opportunities for mathematicians, publishing, electronic communications, etc. We are also interested in articles about mathematics education at all levels. We publish reviews of books, films, plays, software, and mathematical tools.

Reaching the audience. Our goal is to educate the readership about new developments in mathematics and in the mathematics profession, as well as other matters of interest to the working mathematician. Each article is expected to have a large target audience of readers, perhaps 5,000 of the 33,000 subscribers. Authors must therefore write their articles for nonexperts rather than for experts or would-be experts. In particular, the mathematics articles in the Notices are expository. A Notices article should have an introduction that anyone can understand, and almost all readers should be able to understand the key points of the article.

Structure of articles. Most feature articles, including those on mathematics, are expected to be of long-term value and should be written as such. Ideally each article should put its topic in a context, providing some history and other orientation for the reader, and, as necessary, relating the subject matter to things that readers are likely to understand. In most cases, articles should progress to dealing with contemporary matters, not giving only historical material. The articles that are received the best by readers tend to relate different areas of mathematics to each other.

By design the Notices is partly magazine and partly journal, and authors' expository styles should take this into account. For example, many readers want to understand the mathematics articles without undue effort and without consulting other sources.

Format and length. Mathematics feature articles in the Notices are normally six to nine pages, sometimes a little longer. Shorter articles are more likely to be read fully than are longer articles. The first page is 400 or 500 words, and subsequent pages are about 800 words. From this one should subtract an allowance for figures, photos, and other illustrations and an appropriate allowance for any displayed equations and bibliography. The Notices is especially interested in the creative use of graphics and color and encourages illustrations. Articles on professional topics are typically 3 to 5 pages, as are book reviews.

Editorial process. The Notices aims to publish exceptionally wellwritten articles that appeal to a broad audience of mathematicians. Highly technical, specialized articles with a great deal of notation, insider jargon, and a long list of references are not suitable for the Notices. Some articles will be rejected by the editors without any external review. Other articles will be carefully refereed, and then a detailed editorial process will be used to bring the article up to the Notices standard. There will be considerable give and take between the author(s) and the editor, and it may take several drafts to get the article right.

## The "WHAT IS...?" Column

Nearly every issue of the Notices carries an installment of the "WHAT IS...?" column. The purpose of the column is to provide brief, nontechnical descriptions of mathematical objects in use in current research. The target audience for the columns is first-year graduate students.

Each "WHAT IS...?" column provides an expository description of a single mathematical object being used in contemporary research. Thus "WHAT IS $M$-Theory?" would be too broad, but "WHAT IS a Brane?" would be appropriate; ideally "WHAT IS a Brane?" would give a flavor of what $M$-theory is.

The writing should be nontechnical and informal. Narrative description conveying main ideas should be favored over notationheavy precision.

There is a strict limit of two Notices pages (1,400 words with no picture or 1,200 words with one picture). A list of "Further Reading" should contain no more than three references. Inquiries and comments about the "WHAT IS...?" column are welcome and may be sent to notices-whatis@ams. org.

## Upcoming Deadlines

June 15, 2010: Registration for International Mathematics Competition for University Students. See the website http://www.imc-math. org.uk/ or contact John Jayne, University College London, Gower Street, London WC1E 6BT, United Kingdom; telephone: +44-20-7679-7322, email: j.jayne@uc1.ac.uk.

July 1, 2010: Nominations for the 2011 Dannie Heineman Prize for Mathematical Physics. See "Mathematics Opportunities" in this issue.

July 22, 2010: Proposals for NSF CAREER Program. See "Mathematics Opportunities" in this issue.

July 31, 2010: Nominations and applications for the 2010 Monroe H. Martin Prize. Contact R. Roy, Director, Institute for Physical Science and Technology, University of Maryland, College Park, Maryland 20742-2431.

August 1, 2010: Applications for August review for National Academies Postdoctoral and Senior Research Associateship Program. See http:// sites.nationalacademies.org/ PGA/RAP/PGA_050491 or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone: 202-334-2760; fax: 202-334-2759; email: rap@nas.edu.

August 15, 2010: Nominations for SASTRA Ramanujan Prize. See "Mathematics Opportunities" in this issue.

September 30, 2010: Full proposals for NSF Integrative Graduate Education and Research Training (IGERT) program, by invitation only. See http://www.nsf.gov/ pubs/2010/nsf10523/nsf10523. htm?WT.mc_id=USNSF_25.

October 1, 2010: Applications for AWM Travel Grants. See http:// www. awm-math.org/travelgrants. htm1; telephone: 703-934-0163; email: awm@awm-math.org; or contact Association for Women in Mathematics,

11240 Waples Mill Road, Suite 200, Fairfax, VA 22030.

October 20, 2010: Proposals for NSF Postdoctoral Research Fellowships. See http://www.nsf.gov/ pubs/2008/nsf08582/nsf08582. htm.

November 1, 2010: Applications for November review for National Academies Postdoctoral and Senior Research Associateship Program. See http://sites. nationalacademies.org/PGA/RAP/ PGA_050491 or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone: 202-334-2760; fax: 202-334-2759; email: rap@nas.edu.

November 19, 2010: Proposals for research programs at CRM. See "Mathematics Opportunities" in this issue.

December 1, 2010: Letters of intent for proposals for thematic programs at the Bernoulli Center. See the website http://bernou11i.epf1. ch/new/index.php.

October 1, 2011: Nominations for the 2012 Emanuel and Carol Parzen Prize. See "Mathematics Opportunities" in this issue.

## Book List

The Book List highlights books that have mathematical themes and are aimed at a broad audience potentially including mathematicians, students, and the general public. When a book has been reviewed in the Notices, a reference is given to the review. Generally the list will contain only books published within the last two years, though exceptions may be made in cases where current events (e.g., the death of a prominent mathematician, coverage of a certain piece of mathematics in the news) warrant drawing readers' attention to older books. Suggestions for books to include on the list may be sent to notices-booklist@ ams.org.
*Added to "Book List" since the list's last appearance.

The Archimedes Codex: How a Medieval Prayer Book Is Revealing the True Genius of Antiquity's Greatest Scientist, by Reviel Netz and William Noel. Da Capo Press, October 2007.

ISBN 978-03068-1580-5. (Reviewed September 2008.)
*Bright Boys: The Making of Information Technology, by Tom Green. A K Peters, April 2010. ISBN: 978-1-56881-476-6.

The Calculus of Friendship: What a Teacher and Student Learned About Life While Corresponding About Math, by Steven Strogatz. Princeton University Press, August 2009. ISBN-13: 978-06911-349-32. (Reviewed in this issue.)

Chez les Weils (French), by Sylvie Weil. Buchet-Chastel, January 2009. ISBN-13: 978-22830-236-93.

Decoding the Heavens: A 2,000-Year-Old Computer-and the Cen-tury-Long Search to Discover Its Secrets, by Jo Marchant. Da Capo Press, February 2009. ISBN-13: 978-03068-174-27.

Ernst Zermelo: An Approach to His Life and Work, by Heinz-Dieter Ebbinghaus. Springer, April 2007. ISBN-13: 978-3-540-49551-2. (Reviewed August 2009.)
*Gaming the Vote (Why Elections Aren't Fair and What We Can Do About It), by William Poundstone. Hill and Wang, February 2009. ISBN-13: 978-08090-489-22.

The Housekeeper and the Professor, by Yoko Ogawa. Picador, February 2009.ISBN-13:978-03124-278-01. (Reviewed May 2010.)
*How to Read Historical Mathematics, by Benjamin Wardhaugh. Princeton University Press, March 2010. ISBN: 978-06911-401-48.
*IsaacNewton onMathematicalCertainty and Method, by Niccolò Guicciardini. MIT Press, October 2009. ISBN: 978-02620-131-78.

Logicomix: An Epic Search for Truth, by Apostolos Doxiadis and Christos Papadimitriou. Bloomsbury USA, September 2009. ISBN-13:978-15969-145-20.

Logic's Lost Genius: The Life of Gerhard Gentzen, by Eckart MenzlerTrott, Craig Smorynski (translator), Edward R. Griffor (translator). AMSLMS, November 2007. ISBN-13: 978-0-8218-3550-0.

The Mathematical Mechanic: Using Physical Reason to Solve Problems, by Mark Levi. Princeton University Press. ISBN: 978-0691140209.

Mathematicians: An Outer View of the Inner World, by Mariana Cook,

Princeton University Press, June 2009. ISBN13: 978-0-691-13951-7.

Mathematicans Fleeing from Nazi Germany: Individual Fates and Global Impact, by Reinhard SiegmundSchultze. Princeton University Press, July 2009. ISBN 978-0-691-14041-4.

Mathematics in Ancient Iraq: A Social History, by Eleanor Robson. Princeton University Press, August 2008. ISBN-13: 978-06910-918-22. (Reviewed March 2010.)

Mathematics in India, by Kim Plofker. Princeton University Press, January 2009. ISBN-13: 978-06911-206-76. (Reviewed March 2010.)

Mathematics in 10 Lessons: The Grand Tour, by Jerry P. King. Prometheus Books, May 2009. ISBN: 978-1-59102-686-0.

The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook, by Victor J. Katz et al. Princeton University Press, July 2007. ISBN-13: 978-0-6911-2745-3.

The Millennium Prize Problems, edited by James Carlson, Arthur Jaffe, and Andrew Wiles. AMS, June 2006. ISBN-13: 978-08218-3679-8. (Reviewed December 2009.)

More Mathematical Astronomy Morsels, by Jean Meeus. WillmannBell, 2002. ISBN 0-943396743.
*Mrs. Perkins's Electric Quilt: And Other Intriguing Stories of Mathematical Physics, Paul J. Nahin, Princeton University Press, August 2009. ISBN: 978-06911-354-03.
*Numbers Rule: The Vexing Mathematics of Democracy, from Plato to the Present, by George G. Szpiro. Princeton University Press, April 2010. ISBN: 978-06911-399-44.

The Numerati, by Stephen Baker. Houghton Mifflin, August 2008. ISBN13: 978-06187-846-08. (Reviewed October 2009.)

A Passion for Discovery, by Peter Freund. World Scientific, August 2007. ISBN-13: 978-9-8127-7214-5.

Perfect Rigor: A Genius and the Mathematical Breakthrough of the Century, by Masha Gessen. Houghton Mifflin Harcourt, November 2009. ISBN-13: 978-01510-140-64.

Picturing the Uncertain World: How to Understand, Communicate, and Control Uncertainty Through Graphical Display, by Howard Wainer,

Princeton University Press, April 2009. ISBN-13: 978-06911-375-99.

Pioneering Women in American Mathematics: The Pre-1940 Ph.D.'s, by Judy Green and Jeanne LaDuke. AMS, December 2008. ISBN-13:978-08218-437-65.

Plato's Ghost: The Modernist Transformation of Mathematics, by Jeremy Gray. Princeton University Press, September 2008. ISBN-13: 978-06911-361-03. (Reviewed February 2010.)

The Princeton Companion to Mathematics, edited by Timothy Gowers (June Barrow-Green and Imre Leader, associate editors). Princeton University Press, November 2008. ISBN-13: 978-06911-188-02. (Reviewed November 2009.)

Proofs from THE BOOK, by Martin Aigner and Günter Ziegler. Expanded fourth edition, Springer, October 2009. ISBN-13: 978-3-642-00855-9

Pythagoras' Revenge: A Mathematical Mystery, by Arturo Sangalli. Princeton University Press, May 2009. ISBN-13: 978-06910-495-57. (Reviewed May 2010.)

Recountings: Conversations with MIT Mathematicians, edited by Joel Segel. A K Peters, January 2009. ISBN13: 978-15688-144-90.

Roger Boscovich, by Radoslav Dimitric (Serbian). Helios Publishing Company, September 2006. ISBN-13: 978-09788-256-21.

Sacred Mathematics: Japanese Temple Geometry, by Fukagawa Hidetoshi and Tony Rothman. PrincetonUniversity Press, July 2008. ISBN-13: 978-0-6911-2745-3.
*The Solitude of Prime Numbers, by Paolo Giordano. Pamela Dorman Books, March 2010. ISBN: 978-06700-214-82.

Solving Mathematical Problems: A Personal Perspective, by Terence Tao. Oxford University Press, September 2006. ISBN-13: 978-0-199-20560-8. (Reviewed February 2010.)

Sphere Packing, Lewis Carroll, and Reversi, by Martin Gardner. Cambridge University Press, July 2009. ISBN: 9780521756075.

Strange Attractors: Poems of Love and Mathematics, edited by Sarah Glaz and JoAnne Growney. A K Peters, November 2008. ISBN-13: 978-15688-134-17. (Reviewed September 2009.)

The Strangest Man, by Graham Farmelo. Basic Books, August 2009. ISBN-13: 978-04650-182-77.
*Street-Fighting Mathematics: The Art of Educated Guessing and Opportunistic Problem Solving, by Sanjoy Mahajan. MIT Press, March 2010. ISBN: 978-0-262-51429-3.

Symmetry in Chaos: A Search for Pattern in Mathematics, Art, and Nature, by Michael Field and Martin Golubitsky. Society for Industrial and Applied Mathematics, second revised edition, May 2009. ISBN-13: 978-08987-167-26.

Teaching Statistics Using Baseball, by James Albert. Mathematical Association of America, July 2003. ISBN13: 978-08838-572-74. (Reviewed April 2010.)

Tools of American Math Teaching, 1800-2000, by Peggy Aldrich Kidwell, Amy Ackerberg-Hastings, and David Lindsay Roberts. Johns Hopkins University Press, July 2008. ISBN-13: 978-0801888144. (Reviewed January 2010.)

The Unfinished Game: Pascal, Fermat, and the Seventeenth-Century Letter That Made the World Modern, by Keith Devlin. Basic Books, September 2008. ISBN-13: 978-0-4650-0910-7.

Why Does E=mc? (And Why Should We Care?), by Brian Cox and Jeff Forshaw. Da Capo Press, July 2009. ISBN-13: 978-03068-175-88.

Zeno's Paradox: Unraveling the Ancient Mystery Behind the Science of Space and Time, by Joseph Mazur. Plume, March 2008 (reprint edition). ISBN-13: 978-0-4522-8917-8.


MATHEMATICS EVERYWHERE


## Mathematics Everywhere

Martin Aigner and Ehrhard Behrends, Freie Universitüt Berlin, Germany, Editors

Translated by Philip G. Spain
This series of lectures from renowned mathematicians demonstrates the prominent role of mathematics in our daily life, through science, technology and culture. The common theme throughout is mathematics' unique position as both the art of pure thought and universally applicable science. The book also includes a leisurely treatment of recent hot topics, including the solution of the Poincaré conjecture.

2010; 330 pages; Softcover; ISBN: 978-0-82 I8-4349-9; List US\$49; AMS members US\$39.20; Order code MBK/72

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www.ams.org/bookstore

## American Mathematical Society

## AMS EXEMPLARY PROGRAM PRIZE



At its meeting in January 2004, the AMS Council approved the establishment of a new award called the AMS Award for an Exemplary Program or Achievement in a Mathematics Department. It is to be presented annually to a department that has distinguished itself by undertaking an unusual or particularly effective program of value to the mathematics community, internally or in relation to the rest of society. Examples might include a department that runs a notable minority outreach program, a department that has instituted an unusually effective industrial mathematics internship program, a department that has promoted mathematics so successfully that a large fraction of its university's undergraduate population majors in mathematics, or a department that has made some form of innovation in its research support to faculty and/or graduate students, or which has created a special and innovative environment for some aspect of mathematics research.

The prize amount is $\$ \mathbf{5 , 0 0 0}$. All departments in North America that offer at least a bachelor's degree in the mathematical sciences are eligible.

The Prize Selection Committee requests nominations for this award, which will be announced in Spring 2011. Letters of nomination may be submitted by one or more individuals. Nomination of the writer's own institution is permitted. The letter should describe the specific program(s) for which the department is being nominated as well as the achievements that make the program(s) an outstanding success, and may include any ancillary documents which support the success of the program(s). The letter should not exceed two pages, with supporting documentation not to exceed an additional three pages.

All nominations should be submitted to the AMS Secretary, Robert J. Daverman, American Mathematical Society, 302C Aconda Court, University of Tennessee, Knoxville TN 37996-0614. Include a short description of the work that is the basis of the nomination, with complete bibliographic citations when appropriate. The nominations will be forwarded by the Secretary to the Prize Selection Committee, which will make the final decision on the award.

Deadline for nominations is September 15, 2010.

# Mathematics Calendar 

Please submit conference information for the Mathematics Calendar through the Mathematics Calendar submission form at http://www.ams.org/cgi-bin/mathcal-submit.pl. The most comprehensive and up-to-date Mathematics Calendar information is available on the AMS website at http://www.ams.org/mathcal/.

1-4 3rd International Conference (Chaos2010) on Chaotic Modeling, Simulation and Applications, Chania, Crete, Greece. (Mar. 2010, p. 430)

* 1-4 XI International Conference "Stability and Oscillations of Nonlinear Control Systems" (Pyatnitskiy conference) STAB10, Institute of Control Sciences of Russian Academy of Sciences, Moscow, Russia. Topics of the Conference: 1 . General Problems of the Stability Theory and Stabilization. 2. General Problems and Methods of Nonlinear Oscillations Theory (in particular, a special session "Analytical-Numerical Methods of the Oscillations Investigation" is being organised). 3. Lyapunov Function Methods for Nonlinear Control Systems and Hamilton-Jacobi- Lyapunov-Bellmann Methods in the Theory of Optimal Control and Game Problems of Control. 4. Smooth and Nonsmooth Dynamics (in particular, minisymposium "Dynamics of Nonsmooth Systems" is being organized). 5. Controllability and Observability Problems in Control Theory. 6. Robust Control Problems. 7. Control of Mechanical Systems. 8. Stability and Control of Hybrid and Switched Systems. 9. Applied Control Problems and ComputerOriented Methods.
Information: http://www.stab10.ru.

1-5 IMA Workshop: Natural Locomotion in Fluids and on Surfaces: Swimming, Flying, and Sliding, Institute for Mathematics and its Applications (IMA), University of Minnesota, Minneapolis, Minnesota. (Apr. 2009, p. 526)

* 1-6 AAA80 Workshop on General Algebra in Connection with the Workshop on Non-Classical Algebraic Structures, Mathematical Research and Conference Center (a part of the Stefan Banach International Mathematical Center) in Bedlewo, near Poznan, Poland.
Description: The main topics of this conference are: non-classical algebraic structures, universal algebra and lattice theory, applications of algebra in logic and classical algebraic structures.
Invited speakers includes: Joel Berman, Igor Dolinka, Ales Drapal, Heinz-Peter Gumm, Pawel Idziak, Jaroslav Jezek, Achim Jung, Andrzej Kisielewicz, Jimmie Lawson, Franco Montagna, Gabor Nagy, Jonathan D. H. Smith, Yde Venema.

Information: http://www.mini.pw.edu.pl/aaa80/.
2-4 Klein conference, Centro Internacional de Encuentros Matemáticos (CIEM), Castro Urdiales, Spain. (Mar. 2010, p. 430)
2-5 Number Theory and Representation Theory-A conference in honor of Dick Gross' 60th birthday, Science Center, Harvard University, Cambridge, Massachusetts. (Jun./Jul. 2009, p. 771)

This section contains announcements of meetings and conferences of interest to some segment of the mathematical public, including ad hoc, local, or regional meetings, and meetings and symposia devoted to specialized topics, as well as announcements of regularly scheduled meetings of national or international mathematical organizations. A complete list of meetings of the Society can be found on the last page of each issue.
An announcement will be published in the Notices if it contains a call for papers and specifies the place, date, subject (when applicable), and the speakers; a second announcement will be published only if there are changes or necessary additional information. Once an announcement has appeared, the event will be briefly noted in every third issue until it has been held and a reference will be given in parentheses to the month, year, and page of the issue in which the complete information appeared. Asterisks (*) mark those announcements containing new or revised information.
In general, announcements of meetings and conferences carry only the date, title of meeting, place of meeting, names of speakers (or sometimes a general statement on the program), deadlines for abstracts or contributed papers, and source of further information. If there is any application deadline with respect to participation in the meeting, this fact should be noted. All communications on meetings and conferences
in the mathematical sciences should be sent to the Editor of the Notices in care of the American Mathematical Society in Providence or electronically to notices@ams.org or mathcal@ams.org.
In order to allow participants to arrange their travel plans, organizers of meetings are urged to submit information for these listings early enough to allow them to appear in more than one issue of the Notices prior to the meeting in question. To achieve this, listings should be received in Providence eight months prior to the scheduled date of the meeting. The complete listing of the Mathematics Calendar will be published only in the September issue of the Notices. The March, June/July, and December issues will include, along with new announcements, references to any previously announced meetings and conferences occurring within the twelve-month period following the month of those issues. New information about meetings and conferences that will occur later than the twelve-month period will be announced once in full and will not be repeated until the date of the conference or meeting falls within the twelve-month period.
The Mathematics Calendar, as well as Meetings and Conferences of the AMS, is now available electronically through the AMS website on the World Wide Web. To access the AMS website, use the URL: http : // www. ams.org/.

3-5 12th Chico Topology Conference, Chico, California. (Jan. 2010, p. 74)

* 4-6 2010 CMS Summer Meeting, University of New Brunswick, Fredericton, Canada.
Description: The Canadian Mathematical Society (CMS) and the University of New Brunswick invite the mathematical community to the 2010 CMS Summer Meeting.
Plenary, prize and public lectures: Jason Brown (Dalhousie), H. E. A. Eddy Campbell (UNB), Gerda de Vries (Alberta), Jennifer Hyndman (UNBC), Mikhail Lyubich (Stony Brook), Henri Moscovici (Ohio State), Idun Reiten (Norwegian Univ. of Science and Technology), Kristin Schleich (UBC), Gunther Uhlmann (Washington), and Bálint Virág (Toronto). Furthermore, 19 scientific sessions are scheduled, including Mathematics Education, Contributed Papers, and Graduate Student Posters.
Information: http://www.cms.math.ca/Events/summer10/.
4-6 Topology and Geometry in Dimension Three: Triangulations, Invariants, and Geometric Structures, Oklahoma State University, Stillwater, Oklahoma. (Apr. 2010, p. 547)
4-9 XIIth International Conference on Geometry, Integrability and Quantization, Sts. Constantine and Elena resort, Varna, Bulgaria. (May 2010, p. 672)
* 5-7 National Conference On Nonlinear Analysis and Applications, Department of Mathematics, H.N.B. Garhwal University, Campus Pauri, Pauri Garhwal, Uttarakhand,India.
Description: To make a path for new researchers in nonlinear analysis and its applications to other disciplines. This conference will surely provide an opportunity for interaction among the young researchers and renowned academicians of the country.
Topics: It aims at covering all areas of nonlinear analysis with emphasis on the following topics: Approximation Theory, Fixed Point Theory, Fractal analysis, Operator Theory, Optimization Theory, Setvalued analysis.
Advisory Committee: S. R. Ansari, Srinagar; S. G. Dani, Mumbai; R. Dangwal, Pauri; R. C. Dimri, Srinagar; M. C. Joshi, Mumbai; M. C. Joshi, Nainital; D. S. Negi, Srinagar; R. S. Pathak, Varanasi; A. K. Singh, Tehri; Dinesh Singh, Delhi; S. L. Singh, Rishikesh; S. P. Singh, Bilaspur, S. D. Tripathi, Allahabad.
Information: http://www.hnbgu.ac.in.
5-10 36th International Conference "Applications of Mathematics in Engineering and Economics" (AMEE'10), Technical University Leisure House, Sozopol, Bulgaria. (Apr. 2010, p. 547)
* 7-8 UPC International Integrability Seminar 2010-UIIS, Universitat Politècnica de Catalunya, Barcelona, Spain.
Description: It consists of a series of selected one hour long lectures on research topics in integrability theory. The main topic is the discussion of the existence of conservation laws for dynamical systems. However, talks on related topics, like integrable systems, dimension reduction, or Lie symmetries are welcome.
Call for Scientific Communications: A session Scientific Communications will be included in the program. Short research talks in integrability of dynamical systems and related topics are welcome. Please send your proposal to email: uiis .ma1@upc . edu by May 10th, 2010. Registration: Please register by sending an e-mail to: uiis.ma1@ upc. edu, with your name and affiliation data.
Organizers: P. Acosta-Humanez, primi@ima.usergioarboleda. edu.co; D. Blázquez Sanz, david@ima.usergioarboleda.edu. co; T. Lázaro, jose.tomas.lazaro@upc.edu.
Scientific Committee: Yagasaki Kazuyuki, Niigata University; Carles Simó, Universitat de Barcelona.
Information: http://ima.usa.edu.co/uiis.htm.
* 7-9 Critical Issues in Mathematics Education: Reasoning and SenseMaking in the Math Curriculum, Mathematical Sciences Research Institute, 17 Gauss Way, Berkeley, California.

Organizers: Dave Auckly, Scott Baldridge, Deborah Loewenberg Ball, Aaron Bertram, Wade Ellis, Deborah Hughes Hallett, Gary Martin, and William McCallum (Chair).
Information: http://www.msri.org/calendar/workshops/ WorkshopInfo/569/show_workshop.

7-10 Istanbul Contact Geometry and Topology Workshop, Istanbul Center for Mathematical Sciences at Bogazici University, Istanbul, Turkey. (May 2010, p. 672)
7-11 AIM Workshop: Low dimensional structures in dynamical systems with variable time lags, American Institute of Mathematics, Palo Alto,CA. (Nov. 2009, p. 1360)
7-11 2010 Annual Canadian Operator Algebra and Operator Theory Symposium, University of New Brunswick, Fredericton, New Brunswick, Canada. (Apr. 2010, p. 548)
7-11 Conference Géométrie Algébrique en Liberté (GAeL) XVIII, Coimbra, Portugal. (Apr. 2010, p. 548)

7-11 International Conference on Applied Mathematics (with the first William Benter Prize in Applied Mathematics), City University of Hong Kong, Hong Kong. (Feb. 2010, p. 305)
7-11 International Functional Analysis Meeting in Valencia on the Occasion of the 80th Birthday of Professor Manuel Valdivia, University of Valencia, Valencia, Spain. (Feb. 2010, p. 305)
7-11 7th Annual Conference on Theory and Applications of Models of Computation- TAMC 2010, Charles University, Prague, Czech Republic. (Feb. 2010, p. 305)
7-18 IMA New Directions Short Course: New Mathematical Models in Economics and Finance, Institute for Mathematics and its Applications (IMA), University of Minnesota, Minneapolis, Minnesota. (Apr. 2010, p. 548)
7-25 IMA PI Summer Program for Graduate Students: Computational Wave Propagation, Michigan State University, East Lansing, Michigan. (Apr. 2010, p. 548)
8-9 2010 Clay Research Conference, Institut Henri Poincaré, Paris, France. (Dec. 2009, p. 1480)
8-11 Nonlinear evolution equations, Hotel "Conchiglia d'oro", Mondello (Palermo), Italy. (Mar. 2010, p. 430)
*9-11 SMTI 2010 National Conference, Hyatt Regency Cincinnati, Cincinnati, Ohio.
Description: The Science and Mathematics Teacher Imperative (SMTI) 2010 National Conference will explore the role that public research universities have in STEM teacher preparation, spanning the boundary between Colleges of Arts and Sciences and Colleges of Education, and building state-wide partnerships in tight fiscal times. SMTI is an initiative of the Association of Public and Land-grant Universities (APLU).
Information: http://www.aplu.org/NetCommunity/ Page. aspx?pid=1569.
10-12 Geometric and Probabilistic Aspects of General Relativity, University of Strasbourg, Strasbourg, France. (Oct. 2009, p. 1148)
10-14 Symmetry Plus Integrability 2010-The First International Conference on Integrable Systems and Nonlinear Waves on the Gulf of Mexico, In Honor of Yuji Kodama's 60th Birthday, South Padre Travelodge, South Padre Island, Texas. (May 2010, p. 672)

13-18 48th International Symposium on Functional Equations, Batz-sur-Mer, France. (Nov. 2009, p. 1360)

13-18 GNAMPA - ERC Summer School, Ischia, Italy. (Mar. 2010, p. 430)

* 14-16 Applied Geometric Algebras in Computer Science and Engineering 2010, University of Amsterdam, Amsterdam, The Netherlands.
Description: The 4th conference on Applied Geometric Algebras in Computer Science and Engineering will be held in the Doelenzaal,
which is part of the University of Amsterdam Library and situated in central Amsterdam.
Conference Aims: AGACSE 2010 is the 4th in this series of conferences and will bring together researchers from different disciplines within the Mathematics, Physics, Engineering and Computer Science communities. The conference will explore algebraic techniques for geometrical problems, study the relationships between them, and establish new links. Proceedings consisting of selected papers will probably be published by Springer.
Conference Chairs: Leo Dorst (University of Amsterdam, The Netherlands) and Joan Lasenby (University of Cambridge, England).
Information: http://staff.science.uva.nl/~leo/ agacse2010/index.html.

14-16 IMA Special Program: Interdisciplinary Research Experience for Undergraduates, Institute for Mathematics and its Applications (IMA), University of Minnesota, Minneapolis, Minnesota. (May 2010, p. 672)

14-17 SIAM Conference on Discrete Mathematics (DM10), Austin, Texas. (Dec. 2009, p. 1480)

14-18 Group Actions in Topology and Geometric Group Theory, The Third Group Action Forum Conference, Adam Mickiewicz University, Poznan, Poland. (Feb. 2010, p. 305)
14-18 Harmonic Analysis and Applications-A Conference in honor of the 70th birthday of Richard Wheeden, Mathematical Research Institute of the University of Seville (IMUS), University of Seville, Seville, Spain. (Apr. 2010, p. 548)
14-18 Summer school "p-adic cohomologies and arithmetic applications", Mathematical Research Institute, University of Sevilla (IMUS), Sevilla, Spain. (May 2010, p. 672)
14-18 Vector Bundles on Algebraic Curves (VBAC 2010)—New Invariants and Stability Conditions, Mathematics Department, Inst. Superior Tecnico, Lisbon, Portugal. (Apr. 2010, p. 548)

* 15-18 MMDS 2010: Workshop on Algorithms for Modern Massive Data Sets, Stanford University, Stanford, California.
Description: The goals of these meetings are to explore novel techniques for modeling and analyzing massive, high-dimensional, and nonlinearly structured scientific and internet data sets; and to bring together computer scientists, statisticians, mathematicians, and data analysis practitioners to promote cross-fertilization of ideas.
Talks/Organizers: Talks will address fundamental questions underlying recent work on algorithmic, statistical and computational aspects of large-scale data set analysis and provide a wide range modern applications. MMDS 2010 follows two tremendously successful meetings in 2006 and 2008 and is organized by Michael Mahoney (Stanford), Petros Drineas (RPI), Alexander Shkolnik (Stanford), Lek-Heng Lim (Berkeley) and Gunnar Carlsson (Stanford).
Information: http://mmds.stanford.ed. Contact: email: mmds-organizers@math.stanford.edu.
15-19 The Thirteenth International Conference on "Hyperbolic Problems: Theory, Numerics and Applications", Beijing, People's Republic of China. (Mar. 2010, p. 431)
16-20 Fourth International Conference on Recent Advances in Applied Dynamical Systems, Zhejiang Normal University, Jinhua, Zhejiang, China. (May 2010, p. 672)
16-21 XI International Conference "Current Geometry", Vietri sul Mare Salerno, Italy. (Feb. 2010, p. 305)
16-23 Budapest Semesters in Mathematics 25th Anniversary Reunion and Conference, Budapest, Hungary. (Nov. 2009, p. 1360)
17-19 Coimbra Meeting on 0-1 Matrix Theory and Related Topics, Department of Mathematics, University of Coimbra, Portugal. (Jun./ Jul. 2009, p. 771)

17-19 [FG60] Computational and Geometric Topology, Bertinoro, Forli, Italy. (Feb. 2010, p. 305)
18-August 15 Geometry, Topology, and Dynamics of Character Varieties, Institute for Mathematical Sciences, National University of Singapore, Singapore. (Aug. 2009, p. 864)
19-24 Automorphism Groups of Topological Structures, The Eilat Campus of Ben Gurion University of the Negev, Israel. (May 2010, p. 672)

19-24 International Algebraic Conference dedicated to the 70th birthday of A. V. Yakovlev, St. Petersburg, Russia. (Feb. 2010, p. 305)
Second Mathematical Chemistry Workshop of the Americas, Universidad de los Andes, Bogota, Colombia. (May 2010, p. 672)
19-26 XIVth International Summer Conference on Probability and Statistics (ISCPS-2010), Seminar of Statistical Data Analysis (SDA2010), Workshop on Branching Processes and Applications (WBPA2010), Sozopol, Bulgaria. (May 2010, p. 673)

20-25 The Twelfth National Conference in Algebra (China), Northwest Normal University, Lanzhou, Gansu, China. (Feb. 2010, p. 305)

* 20-26 International Category Theory Conference, DIMA-DISI, Genoa, Italy.
Description: Contributions in all aspects of Category Theory are welcome and we especially encourage experts to submit work on its applications as well as on its foundations. If you would like to give a talk, please submit a title and a one-page abstract by email to: Robert Rosebrugh; email: rrosebrugh@mta. ca before May 10, 2010.
Scientific Committee: Clemens Berger, Marta Bunge, Maria M. Clementino, Robin Cockett, Martin Hyland, Bill Lawvere, Bob Rosebrugh (chair), Ross Street, Enrico Vitale, Bob Walters.
Invited Speakers: Olivia Caramello, Cambridge-Pisa; Denis-Charles Cisinski, Paris; George Janelidze, Cape Town; Andréa Joyal, Montreal; Matias Menni, La Plata; Mark Weber, Paris.
Registration: Is open at the website: http://ct2010.disi. unige.it.
Information: http://ct2010.disi.unige.it.
21-23 Conference on Special Functions \& their Applications-CSFA 2010, School of Mathematics and Allied Sciences (SOMAAS) Jiwaji University, Gwalior (MP), India. (May 2010, p. 673)
* 21-23 Seventh Advanced Course in Operator Theory and Complex Analysis, El Puerto de Santa Maria, Spain.
Invited Speakers: Anton D. Baranov (St. Petersburg State University, Russia), Alexei Poltoratski (Texas A\&M University, USA), Serguei Shimorin (KTH Royal Institute of Technology, Sweden), Alexander Volberg (Michigan State University, USA).
Information: You can find further information about the courses and the possibility to contribute with a talk at: http://congreso. us.es/ceacyto/2010
21-25 AIM Workshop: Control and optimization of open quantum systems for information processing, American Institute of Mathematics, Palo Alto, California. (Apr. 2010, p. 548)
*21-25 Conference on Differential and Difference Equations and Applications, Rajecke Teplice (close to Zilina), Slovak Republic.
Description: Department of Mathematics, Faculty of Science University of Zilina organizes traditional Conference on differential and difference equations and their applications. The following sections are planned: Ordinary differential equations, Functional differential equations, Difference equations and dynamic equations on time scales, Partial differential equations, Numerical methods in differential and difference equations. The programme will consist of invited lectures, contributing talks and poster session.
Information: http://fpv.uniza.sk/cddea2010/.

21-25 Functions and Operators, The Faculty of Mathematics and Computer Science of the Jagiellonian University, Lojasiewicza 6, Krakow, Poland. (Apr. 2010, p. 548)
21-25 Harmonic Analysis and Related Topics, Instituto Superior Técnico, IST, Lisbon, Portugal. (Feb. 2010, p. 305)
21-25 International Conference on Algebras and Lattices, Prague, Czech Republic. (Mar. 2010, p. 431)

* 21-25 Mini-courses in Mathematical Analysis 2010, University of Padova, Padova, Italy.
Description: Following a long standing tradition, the Departments of Mathematics of the University of Padova are organizing the meeting "Mini-courses in Mathematical Analysis 2010". The meeting will take place at "Torre Archimede", a new building of the University of Padova in the city center. The program consists of four lecture courses delivered by invited speakers and a limited number of short communications. The meeting aims at introducing the participants to important current research fields in Mathematical Analysis. The meeting is particularly indicated not only to graduate students, postdocs and young researchers but also to well-established experts in Mathematical Analysis.
Information: http://minicourses.dmsa.unipd.it/.
21-26 "Alexandru Myller" Mathematical Seminar Centennial Conference, "Al. I. Cuza" University of Iaşi, Romania. (Jun./Jul. 2009, p. 771)

21-26 2nd International Conference for Promoting the Application of Mathematics in Technical and Natural Sciences (AMiTaNS'10), Sozopol, Bulgaria. (Mar. 2010, p. 431)

21-July 2 Summer School on Mathematical String Theory 2010, Virginia Tech, Blacksburg, Virginia. (May 2010, p. 673)
22-24 The 6th East Asia SIAM Conference in conjunction with the Applied Mathematics International Conference AMIC2010, Kuala Lumpur, Malaysia. (May 2010, p. 673)
*22-24 International Workshop on Mathematical Analysis 1 (IWOMA1), Institute for Mathematical Research, Universiti Putra Malaysia, Malaysia.
Description: The main goal of this workshop is to bring together experts and young talented scientists from Turkey (Fatih University) and Malaysia (Universiti Putra Malaysia) to discuss the modern aspects of the mathematical analysis and to ensure exchange of ideas in various applications of Mathematics in Engineering, Physics, Economics, Biology, etc. The invited talks will be published after peer reviewing by the Institute for Mathematical Research (INSPEM) of Universiti Putra Malaysia as Lecture series. This workshop will encourage international collaboration and interactive activities on the modern problems of mathematical analysis and provide an opportunity for young researchers from Malaysia to learn the current state of the researches in this field of the mathematics. Researchers from both theoretical and applied mathematics are very welcome to attend the workshop and to use this excellent possibility to exchange ideas with leading scientists from different research fields.
Information: http://einspem.upm. edu.my/IWOMA1.
22-25 Group Representation Theory and Related Topics, EPFL, Centre Interfacultaire Bernoulli, Lausanne, Switzerland. (Feb. 2010, p. 306)

22-July 2 RMMC 2010: Conservation Laws and Applications, University of Wyoming, Laramie, Wyoming. (Jan. 2010, p. 75)

* 24-26 Advances in superprocesses and nonlinear PDE's, Department of Mathematics, University of Colorado at Boulder, Colorado. Description: On the occasion of Professor Kuznetsov's 60th birthday, an international conference is being organized at Boulder. Note that there will be a few more speakers added on the web site soon. Topics: measure-valued processes, PDE's, SPDE's and more. Organized by J. Englander and B. Rider.

Information: http://euclid.colorado.edu/~wakefien/Conf/ conference.html.

24-27 ACA 2010: Applications of Computer Algebra, Vlora, Albania. (Jan. 2010, p. 75)

25-29 Conference "Algebraic Geometry, Algebraic K-theory, and Motives", dedicated to Andrei Suslin's 60th birthday, Steklov Mathematical Institute, St. Petersburg, Russia. (Mar. 2010, p. 431)
26-30 2010 International Conference on Topology and its Applications, Nafpaktos, Greece. (Jun./Jul. 2009, p. 771)
28-30 FAN 2010: Fluid dynamics, Analysis, and Numerics 2010. A conference in honor of J. Thomas Beale, Duke University, Durham, North Carolina. (Apr. 2010, p. 548)

28-30 International Symposium on Voronoi Diagrams in Science and Engineering (ISVD2010), Laval University, Quebec City, Canada. (May 2010, p. 673)
28-July 2 Fifth Pacific Rim Conference on Mathematics, Stanford University, Stanford, California. (May 2010, p. 673)

28-July 2 The Józef Marcinkiewicz Centenary Conference (JM 100), A. Mickiewicz University, Faculty of Mathematics and Computer Science, Poznańoznań, Poland. (Aug. 2009, p. 864)
28-July 3 Teichmüller Theory and its Interactions in Mathematics and Physics, Centre de Recerca Matemàtica, Bellaterra, Spain. (Dec. 2009, p. 1480)

28-July 4 The Second International School-seminar "Nonlinear Analysis And Extremal Problems", Institute for System Dynamics and Control Theory SB RAS, Irkutsk, Russia. (Mar. 2010, p. 431)
28-July 9 Operads and Universal Algebra, Chern Institute of Mathematics, Nankai University, Tianjin, China. (Apr. 2010, p. 548)
28-July 16 RTG Summer School on Inverse Problems \& Partial Differential Equations, University of Washington, Seattle, Washington. (Mar. 2010, p. 431)

29-July 4 23nd International Conference on Operator Theory, West University of Timisoara, Timisoara, Romania. (Feb. 2010, p. 306)

30-July 2 The 2010 International Conference of Applied and Engineering Mathematics, Imperial College, London, U.K. (Dec. 2009, p. 1480)

* 30-July 2 Symbolic Computation and its Applications, Center for Applied Mathematics and Theoretical Physics (CAMTP), Maribor, Slovenia.
Description: The conference is organized by the Center for Applied Mathematics and Theoretical Physics (CAMTP) of University of Maribor. It will be held at hotel Piramida in Maribor from June 30, 2010 to July 2, 2010. The conference will be devoted to presentation and discussion of new developments in symbolic computation and their applications to various theoretical mathematical and physical problems and to studying mathematical models arising in the real world applications. Main topics: Applications of symbolic computation to differential equations and dynamical systems; Applications of computer algebra to studying biological problems; Computational commutative algebra and algebraic geometry; D-modules; Cryptanalysis Contributions presenting applications of symbolic computation to other fields are welcome as well.
Information: http://www. camtp.uni-mb.si/camtp/SCA/.
July 2010
4-17 40th Probability Summer School, Saint-Flour, France. (Feb. 2010, p. 306)

5-9 11 th International Conference on p-adic Functional Analysis, Universite Blaise Pascal, Les Cezeaux, Aubiere, France. (Oct. 2009, p. 1148)

5-9 Modular Conference: Arithmetic of Modular Forms and Modularity Results, Centre de Recerca Matemàtica Apartat 50 E-08193, Bellaterra, Spain. (Jan. 2010, p. 75)

6-8 Conference on Industrial and Applied Mathematics, Bandung Institute of Technology, Bandung, Indonesia (Nov. 2009, p. 1360)
6-9 17th Workshop on Logic, Language, Information and Computation (WoLLIC 2010), University of Brasília, Brasília, Brazil. (Mar. 2010, p. 431)

7-9 Jornadas de Matem • tica Discreta y Algorotmica, Centro Internacional de Encuentros Matemáticos (CIEM), Castro Urdiales, Spain. (Mar. 2010, p. 432)

10-14 International Conference: The Sixth Dynamical Systems and Applications-2010, Sea Life Hotel, Antalya, Turkey. (Mar. 2010, p. 432)
11-14 24th European Conference on Operational Research (EURO XXIV), FCUL - University of Lisbon, Lisbon, Portugal. (Jan. 2010, p. 75)

11-16 Path Integrals 2010, Howard University's Blackburn Center, Washington, DC (USA). (May 2010, p. 673)
12-14 2010 International Conference on Theoretical and Mathematical Foundations of Computer Science (TMFCS-10), Orlando, Florida. (Feb. 2010, p. 306)

12-15 SIAM Conference on the Life Sciences (LS10), The David L. Lawrence Convention Center, Pittsburgh, Pennsylvania. (Apr. 2009, p. 526)

* 12-16 Fields Institute/University of Ottawa Extended Workshop on Groups and Group Actions in Operator Algebra Theory, University of Ottawa, Ontario, Canada.
Description: Operator algebras, including their links with groups and group actions, constitute one of only a few major research strengths common to both Canadian and Brazilian mathematics, and the impetus to the extended workshop was provided by a recent collaboration agreement signed by the Universidade Federal de Santa Catarina (FlorianÛpolis, Brazil) and the University of Ottawa. However, the workshop is open to everyone, and everybody is welcome. There will be two mini-courses of three one-hour lectures each, given by two main speakers: Ruy Exel (Universidade Federal de Santa Catarina) and Vadim Kaimanovich (University of Ottawa). Also, there will be one-hour lectures and 40-minute lectures given by invited speakers. Financial support: Is available for graduate students and postdoctoral fellows (funds permitting).
Deadline: For funding applications made on the workshop web page: April 15, with funding decisions made shortly thereafter.
Information: http://www.fields.utoronto.ca/programs/ scientific/10-11/groups/index.html.
12-16 2010 SIAM Annual Meeting (AN10), The David L. Lawrence Convention Center, Pittsburgh, Pennsylvania. (Apr. 2009, p. 526)
12-16 Soria Summer School on Computational Mathematics Algebraic Geometric Modelling in Information Theory (AGMINT), Soria, Spain. (Apr. 2010, p. 549)
12-22 Regulators III, Barcelona, Spain. (May 2010, p. 673)
12-August 6 Statistical Challenges Arising from Genome Resequencing, Isaac Newton Institute for Mathematical Sciences, Cambridge, United Kingdom. (Sept. 2009, p. 1030)
12-August 13 A minisemester on evolution of interfaces, Hokkaido University, Sapporo, Japan. (May 2010, p. 673)
13-17 5th International Conference on Origami in Science, Mathematics and Education (5OSME), Singapore Management University, Singapore, Singapore. (Jan. 2010, p. 75)

15-30 XIII Summer Diffiety School, Santo Stefano del Sole (Avellino), Italy. (Jan. 2010, p. 75)

16-23 International Conference on Spectral Geometry, Preceded by Pre-conference Minicourses, Dartmouth College, Hanover, New Hampshire. (May 2010, p. 674)
19-23 AIM Workshop: Components of Hilbert Schemes, American Institute of Mathematics, Palo Alto, California. (Apr. 2010, p. 549)
19-23 16th International Conference on Difference Equations and Applications ICDEA2010, Faculty of Physics and Mathematics of University of Latvia, Riga, Latvia. (May 2010, p. 674)

19-23 XIX Oporto Meeting on Geometry, Topology and Physics, Faro, Portugal. (Apr. 2010, p. 549)
19-August 13 Gyrokinetics in Laboratory and Astrophysical Plasmas, Isaac Newton Institute for Mathematical Sciences, Cambridge, United Kingdom. (Sept. 2009, p. 1031)

25-30 25th Summer Conference on Topology and its Applications, Jan Kochanowski University in Kielce, Poland. (May 2010, p. 674)
25-31 The XXI School of Algebra, Brasília, Brazil. (Dec. 2009, p. 1481)
26-29 Mixed Integer Programming 2010, Georgia Institute of Technology, Atlanta, Georgia. (May 2010, p. 674)
26-30 Analysis on Graphs and its Applications, Isaac Newton Institute for Mathematical Sciences, Cambridge, United Kingdom. (Dec. 2009, p. 1481)

26-30 6th International Conference on Lévy Processes: Theory and Applications, Technical University of Dresden, Dresden, Germany. (Apr. 2009, p. 526)

* 26-30 Poisson 2010 - Poisson Geometry in Mathematics and Physics, Instituto de Matematica Pura e Aplicada (IMPA), Rio de Janeiro, Brazil.
Description: The five-day conference will feature around twenty-five plenary speakers and will be preceded by a four day school (from July 20-23) aimed at preparing students and young researchers for the conference. Poisson 2010 is the seventh in a series of international conferences on Poisson geometry and related topics, held every two years. School lecturers include: A. Cattaneo, G. Cavalcanti and I. Moerdijk. Conference speakers include: A. Alekseev, D. Calaque, M. Crainic, E. Getzler, M. Gualtieri, S. Gukov, R. Heluani, D. Holm, Y. KosmannSchwarzbach, E. Meinrenken, S. Merkulov, E. Miranda, T. Ratiu, T. Schedler, P. Severa, X. Tang, B. Uribe, I. Vaisman, A. Weinstein, P. Xu, and N.-T. Zung.
Information: http://www.impa.br/opencms/pt/eventos/ store/evento_1001.
26-August 6 Winter School on Topics in Noncommutative Geometry, Departamento de Matematica, Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires, Argentina. (Apr. 2009, p. 526)
27-31 LinStat'2010 - International Conference on Trends and Perspectives in Linear Statistical Inference, Polytechnic Institute of Tomar, Portugal. (Jan. 2010, p. 75)
28-30 The Mathematics of Klee \& Grunbaum: 100 Years in Seattle, University of Washington, Seattle, Washington. (Feb. 2010, p. 306)
29-30 ICDEM 2010 Second International Conference on Data Engineering and Management 2010, Bishop Heber College, Tiruchirappalli, Tamil Nadu, India. (Apr. 2010, p. 549)


## August 2010

1-7 Perspectives in High Dimensions, Case Western Reserve University, Cleveland, Ohio 44106. (Mar. 2010, p. 432)

2-5 IMA Hot Topics Workshop: Integral Equation Methods, Fast Algorithms and Applications, Institute for Mathematics and its Applications (IMA), University of Minnesota, Minneapolis, MN 55455 (May 2010, p. 674)

2-6 AIM Workshop: Differentiable structures on finite sets, American Institute of Mathematics, Palo Alto, California. (Dec. 2009, p. 1481)

2-6 Formal Power Series and Algebraic Combinatorics 2010, San Francisco State University, San Francisco, CA, USA. (Nov. 2009, p. 1360)

2-7 Geometry, Topology, and Dynamics in Negative Curvature, Raman Research Institute, Bangalore, India. (May 2010, p. 674)
2-11 Mathematical Modeling in Industry XIV - A Workshop for Graduate Students, Center for Mathematical Research (CIMAT) in Guanajuato, Gto., México. (May 2010, p. 674)
2-13 Third Hands-On Research in Complex Systems School, University of Buea, Buea, Cameroon. (Dec. 2009, p. 1481)

3-5 International Conference on Mathematical Applications in Engineering: (ICMAE, 2010), Kuala Lumpur, Malaysia. (May 2010, p. 674)
4-7 Jairo Charris Seminar 2010: Algebraic Aspects of Darboux Transformations, Quantum Integrable Systems and Supersymmetric Quantum Mechanics, Universidad Sergio Arboleda, Sede Rodrigo Noguera Laborde, Santa Marta, Colombia. (Dec. 2009, p. 1481)
*5-12 Operator Theory, Analysis and Mathematical PhysicsOTAMP2010, The Mathematical Research and Conference Center, Bldlewo, Poland.
Description: This is the fifth conference of the OTAMP series. The previous ones were organized in 2002 (Bldlewo), 2004 (Bldlewo), 2006 (Lund), 2008 (Bldlewo). The aim of the conference is (similarly as for the previous ones) to bring together specialists working in operator theory, mathematical physics, and related areas of analysis. Surveys of the state-of-art in the above fields are planned. Recent scientific achievements of speakers and diversity of participants are considered as the highlights of the Conference.
Information: Visit: http://www.impan.pl/BC/Program/ conferences/100TAMP/.
8-11 Functional Analysis and Operator theory, Indian Statistical Institute, Bangalore, India. (Aug. 2009, p. 864)
9-13 AIM Workshop: Mahler's conjecture and duality in convex geometry, American Institute of Mathematics, Palo Alto, California. (Apr. 2010, p. 549)
9-13 Permutation Patterns 2010, Dartmouth College, Hanover, New Hampshire. (Feb. 2010, p. 306)
9-13 Workshop on Fluid Motion Driven by Immersed Structures, Fields Institute, Toronto, Ontario, Canada. (Apr. 2010, p. 549)

* 9-14 West-coast algebraic topology summer school, on the homotopy theory of moduli spaces, University of Oregon, Eugene, Oregon. Description: The summer school is aimed at graduate students and post-docs, though all are welcome. The topic of the summer school will be the homotopy theoretic approach to moduli spaces. Particular focus will be on approaches to Mumford's conjecture (Madsen-Weiss' theorem), the study of cobordism categories, and to other related results and recent developments.
Information:http://www.pims.math.ca/scientific-event/ 100808-wcatss.
11-14 The Fourth International Conference on Neural, Parallel \& Scientific Computations, ICNPSC-4, Department of Mathematics, Morehouse College, Atlanta, Georgia. (Dec. 2009, p. 1481)
11-December 22 Mathematical and Statistical Approaches to Climate Modelling and Prediction, Isaac Newton Institute for Mathematical Sciences, Cambridge, United Kingdom. (Sept. 2009, p. 1031)
12-15 International Conference on Recent Trends in Graph Theory and Combinatorics, ICRTGC-2010, Cochin, India. (Jun./Jul. 2009, p. 771)
* 12-16 ICM satellite conference on Algebraic and Combinatorial Approaches to Representation Theory, Indian Institute of Science, Bangalore, India.
Description: The algebraic and combinatorial analysis of the representation theory of Kac-Moody Lie algebras, Vertex algebras, Quantum
groups and Hecke algebras is an active area of current research due to its applications in many areas of mathematics and physics. This conference will bring together leading international experts working on various aspects of this theme. There will be expository survey talks as well as invited talks on current research developments.
Information: http://www.icts.res.in/program/repthy.
* 13-17 18th International Conference on Finite or Infinite Dimensional Complex Analysis and Applications (18th ICFIDCAA), University of Macau, Macau, China.
Description: The main themes of the conference include, not exclusively, Clifford and Harmonic Analysis, Geometric Analysis of Complex Manifolds, Complex Analysis In Relation to Functional Equations, Analytic Boundary Value Problems, Conformal and Quasi-conformal Mappings and PDEs, Special Functions and Related Topics, applications of complex analysis methods in, for instance signal and image processing. As one of the main roles, the 18th ICFIDCAA is to promote the creation and maintenance of contacts with scientists from diverse countries and cultures. In connecting with the conference two publications are being considered by the Committee of which one is a conference proceeding published by local publisher, and the other is a special issue with the SCI journal "Mathematical Methods in Applied Sciences" (see Publication Information). The sources for the publications consist of talks given in the conference and invited contributions
Information: http://www.fst.umac.mo/math/18thicfidcaa/ index.html.
13-17 ICM Satellite Conference on Probability and Stochastic Processes, Indian Statistical Institute, Bangalore, India. (Dec. 2009, p. 1481)

14-17 Satellite Conference of ICM 2010 on Mathematics in Science and Technology, India Habitat Centre, Lodhi Road, New-Delhi, India. (Nov. 2009, p. 1360)
15-19 Geometric, Asymptotic, Combinatorial Group Theory with Applications (GAGTA), Centre de recherches mathématiques, Université de Montréal, Pavillon André-Aisenstadt, 2920, Chemin de la tour, room 5357, Montréal (Québec), H3T 1J4 Canada. (Jan. 2010, p. 75)

* 15-28 Marie Curie Summer School-Mathematical Modelling of Cancer Growth and Treatment, University of Dundee, Scotland.
Description: Summer school August 15-25. Workshop August 2628. Last in a series of four previous summer schools funded by the European Union Marie Curie Conferences and Training Course Programme. Details of how to apply for attendance and participation will be made available shortly on the summer school website: http:// www.maths.dundee.ac.uk/summershcool2010. See there the lists of confirmed speakers, lecturers, instructors.
Information: Visit http://www.maths.dundee.ac.uk/ summerschool2010.
* 16-18 Measurement, Design, and Analysis Methods for Health Outcomes Research, Harvard School of Public Health, FXB Building, 651 Huntington Ave., Boston, Massachusetts.
Description: Participants will learn to design, implement, and analyze health outcomes studies, and critically review and use outcomes research data for clinical decision making, health care planning, and technology development. This program is geared towards introductory to intermediate outcomes research professionals. The program provides participants with an overview of several topics in the newly funded field of health outcomes research, equipping newcomers with knowledge of the language and concepts. Measurement, Design, and Analysis will help you understand the role of epidemiology, health economics, psychometrics, and biostatistics in conducting outcomes research, interpret the meaning of statistical measurement concepts, such as reliability, validity, responsiveness, sensitivity, and power, and compare the effectiveness of different treatments for the same illness. Be sure to mention your reference code: MDA10-CAL12.

Information: http://secure.sph.harvard.edu/ccpe/ programs.cfm?CSID=MDA0810\&pg=cluster\&CLID=1 \&utm_ source=AMS\&utm_medium=Calendar\&utm_campaign=MDA
16-19 SIAM Conference on Nonlinear Waves and Coherent Structures (NW1 0), Sheraton Society Hill Hotel, Philadelphia, Pennsylvania. (Nov. 2009, p. 1361)

* 16-20 AIM Workshop: The mathematics of ranking, American Institute of Mathematics, Palo Alto, California.
Description: This workshop, sponsored by AIM and the NSF, will bring together researchers from a broad spectrum of both fundamental and applied areas to share their perspectives on ranking problems, with a focus on the underlying mathematics.
Information: http://aimath.org/ARCC/workshops/ mathofranking.html.
* 16-20 International Conference "Geometry, Topology, Algebra and Number Theory, Applications" dedicated to the 120th anniversary of Boris Delone, Moscow State University \& Steklov Institute of Mathematics, Moscow, Russia.
Confirmed plenary speakers: Eiichi Bannai (Kyushu University, Japan), Frederic Cohen (Rochester University, USA), Alexander Gaifullin (Moscow State University, Russia), Peter Gruber (Vienna University of Technology, Austria), Deok-Soo Kim (Hanyang University, Korea) Mikiya Masuda (Osaka City University, Japan), Yuri Nesterenko (Moscow State University, Russia), Alexey Parshin (Steklov Mathematical Institute of RAS, Russia).
Information: http://delone120.mi.ras.ru/.
16-21 Workshop on Operator Algebras and Conformal Field Theory, University of Oregon, Eugene, Oregon. (May 2010, p. 674)
16-December 17 MSRI Future Scientific Programs: Inverse Problems and Applications, Mathematical Sciences Research Institute, Berkeley, California. (Aug. 2009, p. 864)

16-December 17 MSRI Future Scientific Programs: Random Matrix Theory, Interacting Particle Systems and Integrable Systems, Mathematical Sciences Research Institute, Berkeley, California. (Aug. 2009, p. 864)

16-December 22 Partial Differential Equations in Kinetic Theories, Isaac Newton Institute for Mathematical Sciences, Cambridge, United Kingdom. (Sept. 2009, p. 1031)

* 18-21 International Congress in Honour of Professor H. M. Srivastava on his 70th Birth Anniversary, Uludag University, Bursa, Turkey. Description: Following the tradition of one of the previous conferences, the International Symposium on Analytic Function Theory, Fractional Calculus and Their Applications (which was held in honour of Professor H. M. Srivastava on his 65th birth anniversity at the University of Victoria, Victoria, British Columbia, Canada on August 22-27, 2005), Uludag University at Bursa (Turkey) is pleased to announce an international congress in the mathematical sciences in honour of Professor H. M. Srivastava on the occasion of his 70th birth anniversary. This intenational congress will be sponsored and supported by several academic institutions and organizations in and out of Turkey. Information: http://homepage.uludag.edu.tr/~srivastava.
19-20 MSRI-Connections for Women: Inverse Problems and Applications, Mathematical Sciences Research Institute, Berkeley, California. (Dec. 2009, p. 1482)

20-25 Third International Conference on Boundary Value Problems, Integral Equations and Related Problems, Beijing and Baoding, Hebei, China. (Aug. 2009, p. 864)
23-27 International Workshop on Geodesics, Chern Institute of Mathematics, Nankai University, Tianjin, China. (Jun./Jul. 2009, p. 771)
23-27 MSRI-Introductory Workshop on Inverse Problems and Applications, Mathematical Sciences Research Institute, Berkeley, California. (Dec. 2009, p. 1482)

23-27 Topics in Algorithmic and Geometric Group and Semigroup Theory, Centre de recherches mathèmatiques, Université de Montréal, Pavillon André-Aisenstadt, 2920, Chemin de la tour, room 5357, Montréal (Québec) H3T 1J4 Canada. (Jan. 2010, p. 76)
27-31 Differential Geometry and its Applications, Masaryk University, Faculty of Science, Brno, Czech Republic, Europe. (Feb. 2010, p. 306)
*29-31 LaCIM 2010, Université du Québec à Montréal, Montreal, Quebec, Canada.
Description: We commemorate this year a milestone in the existence of our laboratory. After 20 years of existence, the members of LaCIM are still actively pursuing research in combinatorial aspects of algebra, computer science, bioinformatics, physics and number theory. For this occasion we have invited distinguished speakers to present recent advances in areas of research in which the LaCIM members have contributed over the last decades. Contributions are also welcome, and for that purpose we invite authors to submit an extended abstract of up to 10 pages describing their work as explained below. Information: http://lacim.uqam.ca/lacim2010.

* 29-September 2 The International Satellite Conference on Harmonic Analysis (SATEHA-ICM2010), School of Mathematical Sciences, National Institute of Science Education and Research, Institute of Physics Campus, PO: Sainik School, Bhubaneswar, Orissa- 751005.
Description: The International Satellite Conference on Harmonic Analysis (SATEHA-ICM2010), is going to be organized by NISER, Bhubaneswar. It forms part of scientifc programs of the International Congress of Mathematicians (ICM) 2010, Hyderabad. The twin objectives of the conference are to create an occasion to the delegates who generally work in harmonic analysis to have a long lasting collaboration in future and to generate an enthusiasm among the aspiring young Indian researchers towards harmonic analysis.
Information: http://niser.ac.in/~sateha/.
29-Sepember 12 Summer School "Modern Problems of Wavelet Analysis and its Applications", Kursk State University, Kursk, Russia. (May 2010, p. 674)
* 30 Workshop on High-performance computing applied to Finance (HPCF 2010), Ischia, Naples, Italy.
Description: The focus of the workshop is on the computational issues in the evaluation of financial instruments on advanced architectures. The workshop is intended to bring together academics from finance, statistics, numerical analysis, and computer science; decisionmakers and strategists from the financial industries, and regulators from supervisory authorities in order to discuss recent challenges and results in using high-performance technologies for the evaluation of financial instruments.
Information: http://www.statmat.uniparthenope.it/ hpcf2010/scope.html.
* 30-September 1 MBI Workshop for Young Researchers in Mathematical Biology, Mathematical Biosciences Institute, The Ohio State University, Columbus, Ohio.
Description: This workshop is intended to broaden the scientific perspective of young researchers in mathematical biology and to encourage interactions with other scientists. Workshop activities include plenary talks, short talks, poster sessions, and discussion panels. We cordially invite young researchers to apply for participation in the workshop! All invitees will be expected to present a poster, and a select number will be invited to give short talks. The MBI will plan to cover local expenses for all invitees, but travel expenses may only be available on a competitive basis.
Accepted Plenary Speakers: Orly Alter (Biomedical Engineering, University of Texas at Austin), Peter Bates (Mathematics, Michigan State University), Anette Hosoi (MIT), James Keener (Mathematics, University of Utah), Reinhard Laubenbacher (Virginia Bioinformatics

Institute, Virginia Tech), John Lowengrub (Mathematics, University of California, Irvine), Van Savage (Biomathematics, UCLA Medical Center). Information: http://www.mbi.osu.edu/wyrmb/wyrmb2010. html.

30-September 3 Complexity and Group-based Cryptography, Centre de recherches mathématiques, Université de Montréal, Pavillon AndréAisenstadt, 2920, Chemin de la tour, room 5357, Montréal (Québec) H3T 1J4 Canada. (Jan. 2010, p. 76)

## September 2010

2-4 Moduli spaces, Institut de Recherche Mathématique Avancée, University of Strasbourg, France. (Jan. 2010, p. 76)
*2-4 Random Generation of Combinatorial Structures, GASCom 2010, Laboratoire de combinatoire et d'informatique mathématique (LaCIM), Université du Québec à Montréal, Montréal, Quebec, Canada. Description: This is the seventh conference in a series initiated in 1994 in Bordeaux (France). The conference topic is the random and exhaustive generation of combinatorial objects and bijective combinatorics with focus on theoretical approaches. In particular, the combinatorial, algebraic and algorithmic aspects are emphasized. Relations with other parts of mathematics, combinatorics, computer algebra, computer science, physics and biology are considered. The topics of the conference are the following: Random and Exhaustive Generation of Combinatorial Objects Enumerative, Algebraic and Bijective Combinatorics Tilings. This three days event will be preceded, from August 29 to August 31, by LaCIM 2010. This anniversary conference celebrates the 20 years of existence of the LaCIM. Both events will take place at the Université du Quèbec at Montrèal, and participants willing to attend the second event are invited to visit the LaCIM 2010 conference website.
Information: http://lacim.uqam.ca/gascom2010.
*2-4 The 6th William Rowan Hamilton Geometry and Topology Workshop. Knots, Surfaces and Three-Manifolds, The Hamilton Mathematics Institute, Trinity College Dublin, Ireland.
Description: The 6th Hamilton Geometry and Topology Workshop will consist of talks and discussions related to knots, surfaces and 3-manifolds.
Speakers: Brian Bowditch* (Warwick); Stefan Friedl (Warwick); Cameron Gordon (UT Austin); Eli Grigsby (Boston College); Brendan Guilfoyle (Tralee); Andras Juhasz (Cambridge); Jeremy Kahn (StonyBrook); Graham Niblo (Southampton); Brendan Owens (Glasgow); Alan Reid (UT Austin); Sucharit Sarkar (Columbia); Jennifer Schultens (Davis) * to be confirmed.
Co-sponsors: Boston College, the HMI and the NSF.
Funding: A limited amount of funding is available for both junior and senior researchers wishing to attend. Junior researchers (graduate students and early career faculty) are encouraged to apply.
Information: http://www.hamilton.tcd.ie/events/gt/ gt2010.htm.

* 4-5 Symposium on Biomathematics and Ecology: Education and Research (BEER-2010), Illinois State University, Normal, Illinois.
Description: The main goal of this symposium is to bring together researchers and educators of mathematics, biology and statistics to discuss the latest research trends in biomathematics and ecology, to share educational issues in Biomathematics. The symposium will have invited and contributed talks, workshops and short courses in both disciplines.
Information: http://www.biomath.ilstu.edu/beer.
6-9 XIX International Fall Workshop on Geometry and Physics, Oporto, Portugal. (Apr. 2010, p. 549)

6-10 Dynamics Days Europe 2010, University of Bristol, Bristol, United Kingdom. (May 2010, p. 675)

* 6-10 QMath 11 - Mathematical Results in Quantum Physics, University of Hradec Kralove, Hradec Kralove, Czech Republic.

Description: This conference is the 11th issue in the series of QMath meetings. Their purpose is to present the state of art in the "quantum part" of mathematical physics and bring together researchers from this field for scientific exchanges. It is also a great opportunity to initiate new collaborations.
Information: http: //qmath11.uhk.cz/.
7-10 First International Workshop on Differential and Integral Equations with Applications in Biology and Medicine, Aegean University, Karlovassi, Samos island, Greece. (Oct. 2009, p. 1148)

* 7-10 Workshop on Computer Graphics, Computer Vision and Mathematics 2010 (GraVisMa 2010), Brno University of Technology, Faculty of Information Technology, Brno, Czech Republic.
Main topics of workshops (but not limited to): 1) Computer Science fields: Computer Graphics (modelling, rendering, 3D imaging etc.), Computer Vision (3D reconstruction, non-linear image transformations etc.), Algorithms and Data Structures, Human Computer Interaction, 3DTV, Scientific and Medical Visualization, Scientific Computing, Applications. 2) Mathematical fields: Numerical Computation, Projective Geometry, Geometric Algebra, Conformal Algebra, Grassmann Algebra, Clifford Algebra, Other Mathematical Aspects. 3) Interesting applications: Influence Mathematics to Computer Science related fields, Computer Science inspiration to Mathematics, Educational Aspects, Programming Tools and Systems.
Other related topics: GraVisMa.
Deadlines: Paper registration Abstract and title: (recommended only) April 15, 2010. Paper upload: April 25, 2010, 23:59 GMT (London time). Acceptance notification: June 20, 2010. Final version upload: July 15, 2010 23:59 GMT (London time).
Information: http://gravisma.zcu.cz/GraVisMa-2010/ GraVisMa-2010.htm.
7-11 International Conference "Modern Stochastics: Theory and Applications II", Kyiv National Taras Shevchenko University, Kyiv, Ukraine. (Feb. 2010, p. 306)

7-11 Logic, Algebra and Truth Degrees 2010, Prague, Czech Republic. (Feb. 2010, p. 307)

7-12 Geometry, Dynamics, Integrable Systems 2010, Mathematical Institute SANU, Belgrade, Serbia. (Feb. 2010, p. 307)

9-12 The Second Asian Conference on Nonlinear Analysis and Optimization (NAO-Asia2010), Royal Paradise Hotel \& Spa, Patong Beach, Phuket, Thailand. (Apr. 2010, p. 549)

11-17 NAFSA 9-The 9th International School on Nonlinear Analysis, Function Spaces and Applications, Trest Castle, Czech Republic. (Oct. 2009, p. 1148)
12-17 ESF Mathematics Conference in Partnership with EMS and ERCOM/INI: Highly Oscillatory Problems: From Theory to Applications, The Isaac Newton Institute, Cambridge, United Kingdom. (Feb. 2010, p. 307)
13-15 IMA Hot Topics Workshop: Medical Device-Biological Interactions at the Material-Tissue Interface, Institute for Mathematics and its Applications (IMA), University of Minnesota, Minneapolis, Minnesota. (May 2010, p. 675)

13-15 Optimal Discrete Structures and Algorithms (ODSA 2010), University of Rostock, Rostock, Germany. (Apr. 2010, p. 549)

* 13-16 Courant Colloquium 2010, Georg-August-Universitaet Goettingen, Goettingen, Germany.
Description: Biannual conference of the Courant Research Center "Higher order structures in Mathematics", Göttingen.
Speakers include: Huzihiro Araki (Kyoto), Jean-Benoit Bost (Paris), Alberto Cattaneo* (Zürich), Klaus Fredenhagen (Hamburg), Edward Frenkel* (Berkeley), Eric Leichtnam (Paris), Johannes Huebschmann (Lille), Martin Markl (Prague), Grigory Mikhalkin (Toronto), Ulrike Tillmann (Oxford) Katrin Wendland (Augsburg).

Organizers: Dorothea Bahns, Karl-Henning Rehren, Thomas Schick (Göttingen).
Information: http://www. uni-math.gwdg.de/crc10/.
13-17 AIM Workshop: Emerging applications of complexity for CR mappings, American Institute of Mathematics, Palo Alto, California. (Apr. 2010, p. 550)

13-17 Third International Congress on Mathematical Software [ICMS'2010-developers meeting], Department of Mathematics, Kobe University, Kobe, Japan. (Feb. 2010, p. 307)
13-17 Random Matrix Theory and Its Applications I, Mathematical Sciences Research Institute, Berkeley, California. (Dec. 2009, p. 1482)

* 13-17 Summer School on Analysis: Spectral Theory and PDEs, Leibniz University, Hannover, Germany.
Description: The aim of the Summer School "Analysis: Spectral Theory and PDEs" is to offer young scholars the possibility to get an introduction to recent developments in spectral theory and partial differential equations. The school addresses students working towards a Master degree or a Ph.D. Young PostDocs are also welcome. 30 scholarships are available which cover travel expenses and accommodation.
Speakers: Adrian Constantin (Vienna), Eduard Feireisl (Prague), Raffe Mazzeo (Stanford), Werner Mueller (Bonn).
Organizers: J. Escher, B. Kroetz, E. Schrohe, Ch. Walker.
Information: For application details, please see the website: http: // www.math-conf.uni-hannover.de/ana10/.
13-December 17 Modern Trends in Optimization and Its Application, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, CA. (Nov. 2009, p. 1361)

15-18 Algebra, Geometry, and Mathematical Physics, Technical University of Crete, Chania, Crete, Greece. (Apr. 2010, p. 550)
15-18 Conference in Numerical Analysis (NumAn 2010): Recent Approaches to Numerical Analysis: Theory, Methods and Applications, Great Arsenale (Old Venetian Harbor) Chania, Island of Crete, Greece. (Mar. 2010, p. 433)
17-19 $S^{4}$ Conference on Symmetry, Separation, Superintegrability and Special Functions, School of Mathematics, University of Minnesota, Minneapolis, Minnesota. (Feb. 2010, p. 307)

19-22 Applied Statistics 2010 (AS2010), Ribno (Bled), Slovenia. (May 2010, p. 675)
*19-25 13th International Workshop for Young Mathematicians "Logic and Foundations of Mathematics", Jagiellonian University, Krakow, Poland.
Description: International Workshop for Young Mathematicians is a mathematical conference organized by the Zaremba Association of Mathematicians - Students of the Jagiellonian University. It is held mainly for students and PhD students of mathematics but other students interested in the topic are also welcome.
Information: http://omega.im.uj.edu.pl/workshop/.

* 19-25 ICNAAM 2010 minisymposium: Semigroups of Linear Operators and Applications, Hotel Rodos Palace, Rhodes, Greece.
Description: First Symposium on Semigroups of Linear Operators and Applications within ICNAAM 2010, under the auspices of European Society of Computational Methods in Sciences and Engineering (ESCMCE), bring together researchers from all the world to present new results in the theory of semigroups of linear operators and its applications to diffusion processes, PDE, etc. Besides scheduling talks from established mathematicians, we will give opportunity to junior researchers to present their works. The topics covered by Symposium include (but not limited to) Groups and semigroups of linear operators, Markov semigroups and applications to diffusion processes, Schroedinger and Feynmann-Kac semigroups, Applications of stochastic analysis to PDE - Diffusion processes
Information: http://www.fih.upt.ro/v3/personale/ lemledan/ICNAAM_2010.html.

20-21 MSRI-Connections for Women: An Introduction to Random Matrices, Mathematical Sciences Research Institute, Berkeley, California. (Dec. 2009, p. 1482)
20-24 10th International Conference on Parametric Optimization and Related Topics (paraoptX), Karlsruhe Institute of Technology, Karlsruhe, Germany. (Dec. 2009, p. 1482)

20-25 DYSES2010 (V Meeting of Dynamics of Socio-economic Systems Society), University of Sannio, Faculty of Economic and business sciences, Benevento, Italy. (Apr. 2010, p. 550)
20-25 XVI Geometrical Seminar, Hotel Breza, Vrnjacka banja, Serbia. (Mar. 2010, p. 433)
20-October 1 Berlin Mathematical School Summer School 2010 on Discretization in Geometry and Dynamics, Technische Universität Berlin, Germany. (Oct. 2009, p. 1148)

21-24 The 3rd International Conference on Nonlinear Dynamics, National Technical University "Kharkov Polytechnical Institute", Kharkov, Ukraine. (Mar. 2010, p. 433)

* 22-25 Seminal Interactions between Mathematics and Physics, Accademia Nazionale dei Lincei, Roma, Italy.
Description: The conference is the opening event of the newly founded Center for Mathematics and Theoretical Physics in Rome. The aim of the conference is to gather together leading international experts in the two sciences to promote cross-fertilization of Mathematics and Theoretical Physics at the highest level, and to provide the scientific community, and especially young researchers, with motivation and insights. The conference also aims at popularizing the field for a wider audience with two public lectures given by outstanding scientists, well-known even to the general public.
Organizers: A. De Sole, A. Giuliani, D. Guido, R. Longo, G. Morsella. Information: http://cmtp.uniroma2.it/10SIMP/.

23-26 Second International Conference on Numerical Analysis and Approximation Theory: NAAT 2010, Department of Applied Mathematics of the Faculty of Mathematics and Computer Science, Babes- Bolyai University, Cluj-Napoca, Romania. (Mar. 2010, p. 434)

* 24-26 Info-Metrics: Theory and Applications, American University, Washington, DC.
Description: Discuss and study the latest developments of Info-Metrics across the sciences. Generally speaking, info-metrics promotes the study of information processing and optimal decision rules based on efficient use of information. Empirically, it deals with developing a unified class of improved information processing methods for understanding very small or large noisy data that may be ill-behaved or sampled from a poorly understood process. In those cases, traditional estimation methods may not be proper as the problem is (almost always) inherently under-defined, or under-determined, unless one imposes unrealistic structure on the problem.
Conference topics: Include theory and methods and applications across the sciences. Examples include (i) Economics/Econometrics (Theory and Applications), (ii) Finance and Risk Management, (iii) Philosophy of Science, (iv) Predictive Games, (v) Natural Sciences and (vi) Social Sciences.

Information: http://www. american.edu/cas/economics/ info-metrics/conference/index.cfm.
27-30 54th Annual Meeting of the Australian Mathematical Society, The University of Queensland, Brisbane, Queensland, Australia. (Apr. 2010, p. 550)

28-October 1 Convex Optimization and Algebraic Geometry, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California. (Apr. 2010, p. 550)

## October 2010

2-3 AMS Eastern Section Meeting, Syracuse University, Syracuse, New York. (Sept. 2009, p. 1032)
4-8 AIM Workshop: Parameter identification in graphical models,
American Institute of Mathematics, Palo Alto, California. (Apr. 2010, p. 550)

4-9 Group Actions and Dynamics, Centre de recherches mathématiques, Université de Montréal, Pavillon André-Aisenstadt, 2920, Chemin de la tour, room 5357, Montréal (Québec) H3T 1J4 Canada. (Jan. 2010, p. 76)
7-10 International Conference on Algebra in honor of the 70th Birthday of Professor Shum Kar-Ping, Gajah-Mada University, Yogjakarta, Indonesia. (Apr. 2010, p. 550)

8-10 Yamabe Symposium on "Geometry and Low-Dimensional Topology", University of Minnesota, Minneapolis, Minnesota. (May 2010, p. 675)

9-10 AMS Western Section Meeting, University of California, Los Angeles, California. (Sept. 2009, p. 1032)
10-15 International Conference in Systems Biology (ICSB), Edinburgh International Conference Centre, The Exchange, Edinburgh, EH3 8EE, Scotland. (Jan. 2010, p. 76)
*11-15 Conference on Singularities, Geometry and Topology. In honour of the 60th Anniversary of Sabir Gusein-Zade, Residencia San Jose, EL Escorial, Madrid, Spain.
Description: This conference will present and discuss the most recent progress and the future research directions emerging from the relations between geometry, topology and singularity theory through various contemporary research fields. The rich corpus of works of Professor Sabir Gusein-Zade contributes significantly to the interactions between all these research topics.
Information: http://www.singacom.uva.es/oldsite/ seminarios/sabir/index.html.

11-15 Equations and First-order Properties in Groups, Centre de recherches mathématiques, Université de Montrèal, Pavillon AndréAisenstadt, 2920, Chemin de la tour, room 5357 Montréal (Québec) H3T 1J41 Canada. (Jan. 2010, p. 76)
11-15 Numerical Methods for Continuous Optimization, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California. (Apr. 2010, p. 550)

* 18-20 2010 International Multiconference on Computer Science and Information Technology (IMCSIT), Hotel Golebiewski, Wisla, Poland.
Organizers: IMCSIT is organized by the Polish Information Processing Society in cooperation with the IEEE Computer Society (Poland Chapter), Council of European Professional Informatics Societies (CEPIS), the Systems Research Institute Polish Academy of Sciences, and the Institute of Computer Science Polish Academy of Sciences.
Organizing Committee: Maria Ganzha (Chair), SRI PAS, Warsaw and University of Gdansk, Gdansk, Poland; Marcin Paprzycki, SRI PAS and WSM, Warsaw, Poland.
Important Deadlines: Paper submission: May 31, 2010. Notification of acceptance/rejection: July 12, 2010. Camera-ready paper due: August 23, 2010. Conference: October 18-20, 2010.
Contact: In case of questions, please contacts us at: http://www. contacts.imcsit.org.
Information: http://www.imcsit.org/ (Facebook group: http://tinyurl.com/yg6xtdz).
18-22 AIM Workshop: Algebraic systems with only real solutions, American Institute of Mathematics, Palo Alto, California. (May 2010, p. 675)

18-22 IMA Workshop: Computing with Uncertainty: Mathematical Modeling, Numerical Approximation and Large Scale

Optimization of Complex Systems with Uncertainty, Institute for Mathematics and its Applications (IMA), University of Minnesota, Minneapolis, Minnesota. (Apr. 2010, p. 550)
20-22 International Conference in Modeling Health Advances 2010, San Francisco, California. (May 2010, p. 434)
*21-24 Compact Moduli and Vector Bundles, University of Georgia, Athens, Georgia.
Description: The main topic of the conference will be the compact moduli spaces of stable varieties and pairs. The second major topic is vector bundles on compact moduli spaces, and in particular the conformal block bundles. We also plan to have a few talks on the current state of the art of the birational geometry of the moduli space of curves.
Information: http://www.math.uga.edu/~davids/conf10. html.
25-29 AIM Workshop: The geometry of the outer automorphism group of a free group, American Institute of Mathematics, Palo Alto, California. (May 2010, p. 675)

* 25-29 Hot Topics: Kervaire invariant, Mathematical Sciences Research Institute, Berkeley, California.
Organizers: Mike Hill (University of Virginia), Michael Hopkins (Harvard University), and Douglas C. Ravanel* (University of Rochester). Information: http://www.msri.org/calendar/workshops/ WorkshopInfo/584/show_workshop.
* 25-31 Algebra, Geometry, Mathematical Physics, The Sven Lovén Centre for Marine Sciences at the University of Gothenburg, Sweden. Description: The AGMP Newwork organizes scientific meetings in Algebra, Geometry, and Mathematical Physics. The main emphasis is on the interface between them. The AGMP ' 10 at Tjärnö also supports interdisciplinary graduate studies in the Baltic-Nordic region. Main topics include, but are not limited to: Algebra, applications in physics; Geometry, applications in physics; Lie theory, applications in physics; Representation theory, applications in physics; Deformation theory, applications in physics; Integrable systems, applications in physics; Quantum groups, Hopf algebras, applications in physics; Mathematical methods of high energy physics.
Information: http://www.agmf.ee/gb10/.
26-29 Discrete Optimization, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California. (Apr. 2010, p. 551)

26-29 SIAM Conference on Applied Linear Algebra (LA09), Embassy Suites Hotel, Monterey Bay-Seaside, California. (Sept. 2009, p. 1032)
27-30 Fourth International Workshop on Differential Algebra and Related Topics (DART-IV), Key Laboratory of Mathematics Mechanization, Chinese Academy of Sciences, Beijing, China. (May 2010, p. 675)
29-30 International Conference on Software Engineering, Management \& Applications (ICSEMA'10 ), College of Open Learning, Kathmandu, Nepal.
Description: The International Conference on Software Engineering, Management and Applications (ICSEMA 2010) brings together researchers, scientists, engineers, practitioners, academic organizations, other organizations which are related to software development and advanced graduate students to exchange and share their experiences, new ideas, and research results and research-in-progress about all aspects (theory, applications and tools) of Software Engineering Research, Management and Applications, and discuss the practical challenges encountered and the solutions adopted. Our conference is intended to foster the dissemination of state-of-the-art research in all software engineering areas, including their models, services, and novel applications associated with their utilization.
Information: http://www.icsema. com.
29-31 AMS Central Section Meeting, Notre Dame University, Notre Dame, Indiana. (Sept. 2009, p. 1032)

## November 2010

1-5 IMA Workshop: Numerical Solutions of Partial Differential Equations: Novel Discretization Techniques, Institute for Mathematics and its Applications (IMA), University of Minnesota, Minneapolis, Minnesota. (Apr. 2010, p. 551)

* 1-5 K-theory, $C^{*}$-algebras and index theory, Georg-August-Universität Göttingen, Goettingen, Germany.
Description: $K$-theory, $C^{*}$-algebras and index theory have been a driving force of research in geometry and analysis for decades, with many applications beyond these areas. A German-Russian cooperation program with this title has been funded by DFG-RFBR for the last years. The conference is the culmination of the work of this research program. We aim to bring together the members of the program and further researchers in the field to discuss recent developments and future prospects.
Information: http://www.uni-math.gwdg.de/kcit/.
* 1-December 19 Hyperbolic Conservation Laws and Kinetic Equations: Theory, Computation, and Applications, Institute for Mathematical Sciences, National University of Singapore, Singapore, Singapore.
Description: This program will provide a forum for people working in hyperbolic conservation laws, kinetic equations, mathematical physics, scientific computation, and engineering to jointly promote the research on the kinetic equations for the rarefied gas dynamics in NUS. This program will offer a series of comprehensive tutorial lectures by senior scientists from rarefied gas theory, semi-conductor industry, and nonlinear hyperbolic PDE to train PhD students and people who are interesting in those topics. For enquiries on scientific aspects of the program, please email Shih-Hsien Yu at: email: matysh(AT)nus.edu.sg.
Information: http://www2.ims.nus.edu.sg/Programs/ 010hyperbolic/index.php.
* 3-6 3rd International Conference for Young Mathematicians on Differential Equations and Applications dedicated to Ya. Lopatynski, The Ivan Franko National University of L'viv, L'viv, Ukraine.
Description: The conference is dedicated to the outstanding mathematician, talented pedagogue Yaroslav Lopatynsky, who was a founder of the Departments of Differential Equations in both L'viv University and Donetsk University. The aim of the conference is to bring together young mathematicians and world renowned scientists working in the fields of partial differential equations, ordinary differential equations, and operator theory. The meeting will be devoted to significant aspects of contemporary mathematical research on differential equations and the operator theory including the applications of these fields in other mathematical areas and the applications to real-world problems. Information: http://icl2010.franko.lviv.ua.

5-6 IMA Special Event: Finite Element Circus Featuring a Scientific Celebration of Falk, Pasciak, and Wahlbin, Institute for Mathematics and its Applications (IMA), University of Minnesota, Minneapolis, Minnesota (Apr. 2010, p. 551)

6-7 AMS Southeastern Section Meeting, University of Richmond, Richmond, Virginia. (Sept. 2009, p. 1032)

8-10 2010 IEEE International Conference on Technologies for Homeland Security, Westin Hotel, Waltham, Massachusetts. (Feb. 2010, p. 307)
8-12 MSRI—Inverse Problems: Theory and Applications, Mathematical Sciences Research Institute, Berkeley, California. (Dec. 2009, p. 1482)
*8-12 School of Applied Mathematics and Innovation 2010: Nonsmooth systems and applications, Universidad Sergio Arboleda, Sede Rodrigo Noguera Laborde, Santa Marta, Colombia.

Description: Lectures in SAMI 2010 will focus on the general 'nonsmooth' framework and specially on bifurcations. A nice set of application case studies will also be shown.
Lectures: Mario di Bernardo, Bifurcations of non-smooth systems; Enric Fossas, Non-smooth systems and control strategies: several examples in engineering; Enrique Ponce, Bifurcations in piecewise linear systems: case studies.
Organizer: IMA (Instituto de Matemáticas y sus Aplicaciones), Universidad Sergio Arboleda. LIIM (Laboratorio de InvestigaciÛ́n en Ingeniería Matemática), Universidad Nacional de Colombia, sede Manizales. People in charge: Gerard Olivar, LIIM- Universidad Nacional de Colombia, sede Manizales; Carlos Arturo Peña Rincón, IMA-Universidad Sergio Arboleda; Primitivo Acosta-Humánez, IMA-Universidad Sergio Arboleda.
Information: http://ima.usergioarboleda.edu.co/SAMI/ SAMI2010.htm.

16-19 Modern Trends in Optimization and Its Application, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California. (Apr. 2010, p. 551)

* 17-19 SIAM/MSRI Workshop on Hybrid Methodologies for Sym-bolic-Numeric Computation, Mathematical Sciences Research Institute, Berkeley, California.
Organizers: Mark Giesbrecht (University of Waterloo), Erich Kaltofen (North Carolina State University), Daniel Lichtblau (Wolfram Research), Seth Sullivant (North Carolina State University), and Lihong Zhi (Chinese Academy of Sciences, Beijing).
Information: http://www.msri.org/calendar/workshops/ WorkshopInfo/575/show_workshop.
* 23-27 International Conference on Mathematical Sciences, Turkey'10, Abant Izzet Baysal University, Izzet Baysal Kampüsü, Bolu, Turkey.
Description: This International Conference on Mathematical Sciences (ICMS Turkey '10) is dedicated to Professor Hari M. Srivastava on his 70th birth anniversary. The aim of this conference is to present new research activities (theoretical and practical) on mathematics and the recent developments to other scientists. High quality research submissions are invited for technical papers describing original unpublished results of theoretical, empirical, conceptual, and experimental work. Information: http://www.conference.math.sci.ibu.edu. tr/.
29-December 3 IMA Workshop: Numerical Solutions of Partial Differential Equations: Fast Solution Techniques, Institute for Mathematics and its Applications (IMA), University of Minnesota, Minneapolis, Minnesota. (Apr. 2010, p. 551)
* 30-December 3 Applications of Optimization in Science and Engineering, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California.
Description: Recent advances in optimization have made an important impact in a variety of disciplines, including control, communications, signal processing, image processing, machine learning, and computer vision. Conversely, applications in engineering and science have motivated new research directions in optimization. Featured speakers will discuss the role that optimization has played in their fields, opportunities for new applications, and the future challenges they see for optimization theory, algorithms, and software.
Organizing Committee: Stephen Boyd (Stanford, Engineering), Yonina Eldar (Technion-Israel Institute of Technology, Electrical Engineering), Tom Luo (University of Minnesota, Twin Cities), Bernhard Scholkopf (Max-Planck-Institute for Biological Cybernetics), and Lieven Vandenberghe (UCLA, Electrical Engineering).
Registration: Please apply or register online. Applications received by October 4, 2010 will receive fullest consideration.
Information: http://www.ipam.ucla.edu/programs/opws5/.


## December 2010

2-4 4th Global Conference on Power Control and Optimization (PCO’2010), Damai Puri Resort, Kucing, Sarawak, Malaysia. (May 2010, p. 675)

* 4-6 2010 CMS Winter Meeting, Coast Hotel and Suites, Vancouver (BC), Canada.
Description: The Canadian Mathematical Society (CMS) and the University of British Columbia invite the mathematical community to the 2010 CMS Winter Meeting.
Plenary, Prize and Public Lecturers: David Aldous (UC-Berkeley), Lia Bronsard (McMaster), David Donoho (Stanford), Ron Graham (UC-San Diego), Sujatha Ramdorai (Tata Institute; UBC), Peter Sarnak (Princeton), and Tamar Ziegler (Technion).
Information: http://www.cms.math.ca/Events/winter10/.
* 5-10 66th Annual Deming Conference on Applied Statistics, Atlantic City, New Jersey.
Description: The conference's aim is to provide a learning experience on recent developments in statistical methodologies. It is followed by two parallel short courses. The conference is composed of 12 3-hour tutorials on current statistical topics of interest. Recognized experts in the field of applied statistics have been invited to give the lectures and short courses based on their recently published books. Speaker's books are sold at a $40 \%$ discount. Attendees will receive bound proceedings.
Information: http://www.demingconference.com.
6-10 AIM Workshop: Waves and multiscale processes in the tropics, American Institute of Mathematics, Palo Alto, California. (May 2010, p. 675)
6-10 MSRI-Random Matrix Theory and its Applications II, Mathematical Sciences Research Institute, Berkeley, California. (Dec. 2009, p. 1482)

13-17 AIM Workshop: Random matrices, American Institute of Mathematics, Palo Alto, California. (May 2010, p. 676)

* 13-17 Conference in Geometry and Global Analysis Celebrating P. Gilkey's 65th Birthday, Universidade de Santiago de Compostela/ Universidade da Coruña Santiago de Compostela, A Coruña, Spain.
Description: The main purpose of this conference is to celebrate the contributions made by Prof. Peter B. Gilkey in Differential Geometry and Global Analysis. The conference will consist of a series of invited talks by reputable researchers on Differential Geometry and Global Analysis whose research is related to the work of Peter Gilkey. There will also be poster sessions where participants are welcome to present their contributions. The conference will be comprised of two parts. A scientific session will be held at the Faculty of Mathematics of the University of Santiago de Compostela from Monday, December 13th to Thursday, December 16, 2010. In addition, there will be another session taking place at the University of A Coruna on December 17th, where Peter Gilkey's career and fascinating mathematical achievements will be reviewed. Transport from Santiago de Compostela to A Coruna will be provided by the organization.
Information: http://xtsunxet.usc.es/gilkey2010/.
* 13-17 Conference "Quantization of Singular Spaces", University of Aarhus, Denmark.
Description: During the conference we will bring together mathematicians and physicists from diverse areas such as gauge theory, algebraic geometry, deformation quantization, Berezin-Toeplitz quantization and stochastic analysis in order to communicate recent research related to the problem of quantization of singular phase spaces.
Information: Visit: http://www.qgm.au.dk/events/ dec-2010-conference-quantization-of-singularspaces/.

15-16 The International Conference on Mathemtics Education Research, Malacca, Malaysia. (May 2010, p. 676)

17-21 The 15th Asian Technology Conference in Mathematics (ATCM 2010), University of Malaya, Kuala Lumpur, Malaysia. (Apr. 2010, p. 551)

19-21 "Mathematical Sciences for Advancement of Science and Technology" (MSAST 2010), IMBIC Hall, Salt Lake, Kolkata (Calcutta), India. (Apr. 2010, p. 551)
25-27 International Conference on Current trends in Mathematics, Allahabad, Uttar Pradesh, India. (May 2009, p. 659)
*27-30 The 76th Annual Conference of the Indian Mathematical Society, S. V. National Institute of Technology, SURAT -395007, Gujarat, India.
Description: The Indian Mathematical Society (IMS) (Estd. 1907) is the oldest and the largest Mathematical Society of INDIA. http:// www. indianmathsociety.org.in. The object of the Society is the promotion of Mathematical Study and Research. Its central activity is to inspire and encourage researchers, educationists, students and all the mathematics loving persons. The Annual Conferences of the Society are full of very enriching programmes such as Plenary and Memorial Award Lectures by eminent mathematicians, several invited Lectures and Symposia on current areas of research as also presentation of large number of research papers by researchers from all over the Country including the Paper Presentation Competition.
Information: http://www.svnit.ac.in.
29-31 ICCAM 2010: "International Conference on Computational and Applied Mathematics" Symposium Partial Differential Equations:Modeling, Analysis and Numerical Methods, First Hotel Bangkong 2 Soi Somprasong 1, Petchaburi Road, Tanonphayathai, Rajthavee, Bangkok 10400 Thailand. (Nov. 2009, p. 1361)

## January 2011

3-5 ICMS 2011: International Conference on Mathematical Sciences in Honour of Professor A. M. Mathai, St. Thomas College Pala, Kottayam 686574, Kerala, India. (May 2010, p. 676)

* 10-14 Algorithmic Game Theory, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California.
Description: The new research area of Algorithmic Game Theory provides new conceptual perspectives at a very fundamental level. This workshop will gather scientists and researchers from mathematics, computer science, economics, game theory, information theory, from academia as well as industry to provide a joint platform to discuss fundamental issues in this emerging interdisciplinary field.
Organizing Committee: Gunes Ercal-Ozkaya (University of Kansas), Tali Kaufman (MIT), Allon Percus (Claremont Graduate University), Vwani Roychowdhury (UCLA Electrical Engineering), and Sudhir Singh (UCLA Electrical Engineering). Please apply or register online.
Registration: Applications received by November 15, 2010 will receive fullest consideration.
Information: http://www.ipam.ucla.edu/programs/ agt2011/.
10-14 IMA Workshop: High Performance Computing and Emerging Architectures, Institute for Mathematics and its Applications (IMA), University of Minnesota, Minneapolis, Minnesota. (Apr. 2010, p. 552)

10-May 20 MSRI Future Scientific Programs: Arithmetic Statistics, Mathematical Sciences Research Institute, Berkeley, California. (Aug. 2009, p. 864)
10-May 20 MSRI Future Scientific Programs: Free Boundary Problems, Theory and Applications, Mathematical Sciences Research Institute, Berkeley, California. (Aug. 2009, p. 864)

* 13-14 Connections for Women: Free Boundary Problems, Theory and Applications, Mathematical Sciences Research Institute, Berkeley, California.

Organizers: Catherine Bandle (University of Basel), Claudia Lederman (University of Buenos Aires), Noemi Wolanski (University of Buenos Aires).
Information: http://www.msri.org/calendar/workshops/ WorkshopInfo/562/show_workshop.

17-21 AIM Workshop: Deformation theory, patching, quadratic forms, and the Brauer group, American Institute of Mathematics, Palo Alto, California. (May 2010, p. 676)

* 18-21 Efficiency of the Simplex Method; Quo Vadis Hirsch Conjecture?, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California.
Description: This workshop is devoted to the simplex method and the Hirsch conjecture, bringing together researchers with various contemporary approaches, including the smoothed analysis of the simplex method, analogies with interior point methods, explicit constructions and the systematic search for counterexamples through computational tools, and the investigation of combinatorial-topological abstractions of polyhedra.
Organizing Committee: Margaret Bayer (University of Kansas, Mathematics), Jesus De Loera (UC Davis, Mathematics), Antoine Deza (McMaster University), Gil Kalai (Hebrew University, Mathematics), and Shanghua Teng (USC).
Registration: Please apply or register online. Applications received by December 13, 2010, will receive fullest consideration.
Information: http://www.ipam.ucla.edu/programs/sm2011/.
* 18-21 Introductory Workshop: Free Boundary Problems, Theory and Applications, Mathematical Sciences Research Institute, Berkeley, California.
Organizer: Tatiana Toro (University of Washington).
Information: http://www.msri.org/calendar/workshops/ WorkshopInfo/563/show_workshop.
23-25 ACM-SIAM Symposium on Discrete Algorithms (SODA11), Holiday Inn, San Francisco Golden Gateway, San Francisco, California. (Mar. 2010, p. 434)
* 27-28 Connections for Women: Arithmetic Statistics, Mathematical Sciences Research Institute, Berkeley, California.
Organizers: Chantal David (Concordia University) and Nina Snaith* (University of Bristol).
Information: http://www.msri.org/calendar/workshops/ WorkshopInfo/565/show_workshop.
* 31-February 4 Introductory Workshop: Arithmetic Statistics, Mathematical Sciences Research Institute, Berkeley, California.
Information: http://www.msri.org/calendar/workshops/ WorkshopInfo/566/show_workshop.


## February 2011

*6-8 3rd International Conference on Wireless Information Networks \& Business Information System (WINBIS'11), Open Learning Society, Kathmandu, Nepal.
Description: The WINBIS conference is by now a well respected event joining international researchers to discuss the wide range of the development, implementation, application and improvement of wireless information networks \& business applications and systems. It is addressed to the scientific community, people involved in the development of wireless information networks \& business computer applications, consultants helping to properly implement computer technology and applications in the industry.
Information: http://www.win-bis.com.

* 16-19 International Conference on Operator Theory, Monastir, Tunisia.
Description: We have the pleasure to welcome you to the International Conference on Operator Theory ICOT-2011. Operator Theory and particularly Spectral Theory are at the root of several branches in Mathematics and offer a wide range of challenging research prob-
lems. The purpose of this conference is to highlight some of the major theoretical advances in the fields of Operator Theory, Spectral Theory and Functional Analysis. The wide scope of the Conference ICOT 2011 should provide an excellent opportunity for sharing ideas and problems among specialists.
Information: http://www.lpm-fss.org/icot2011.
* 28-March 4 SIAM Conference on Computational Science and Engineering (CSE11), Grand Sierra Resort and Casino, Reno, Nevada.
Description: The SIAM CS\&E conference seeks to enable in-depth technical discussions on a wide variety of major computational efforts on large problems in science and engineering, foster the interdisciplinary culture required to meet these large-scale challenges, and promote the training of the next generation of computational scientists.
Information: http://www.siam.org/meetings/cse11/.


## March 2011

* 7-11 Free Boundary Problems, Theory and Applications Workshop, Mathematical Sciences Research Institute, Berkeley, California.
Organizers: John King (University of Nottingham), Arshak Petrosyan (Purdue University), Henrik Shahgholian* (Royal Institute of Technology), and Georg Weiss (University of Tokyo).
Information: http://www.msri.org/calendar/workshops/ WorkshopInfo/564/show_workshop.

7-11 (NEW DATE) IMA Workshop: Computing in Image Processing, Computer Graphics, Virtual Surgery, and Sports, Institute for Mathematics and its Applications (IMA), University of Minnesota, Minneapolis, Minnesota. (Apr. 2010, p. 552)
14-June 17 Navigating Chemical Compound Space for Materials and BioDesign, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California. (Jan. 2010, p. 76)

* 28-April 1 International Conference on Homotopy and Non-Commutative Geometry, Batumi State University, Batumi, Republic of Georgia.
Description: The Conference is organized by Tbilisi Centre for Mathematical Sciences with the Georgian-German Noncommutative Partnership (topology, geometry, algebra) and is sponsored by Volkswagen Foundation.
Information: http://tcms.org.ge/Conferences/conf_2011.


## April 2011

* 11-15 Arithmetic Statistics, Mathematical Sciences Research Institute, Berkeley, California.
Information: http://www.msri.org/calendar/workshops/ WorkshopInfo/567/show_workshop.

11-15 IMA Workshop: Societally Relevant Computing, Institute for Mathematics and its Applications (IMA), University of Minnesota, Minneapolis, Minnesota. (Apr. 2010, p. 552)

May 2011
1-August 31 MITACS International Focus Period on Advances in Network Analysis and its Applications, Locations throughout Canada. (Apr. 2010, p. 552)
22-26 SIAM Conference on Applications of Dynamical Systems (DS1 1), Snowbird Ski and Summer Resort, Snowbird, Utah. (Mar. 2010, p. 434)

The following new announcements will not be repeated until the criteria in the next to the last paragraph at the bottom of the first page of this section are met.

June 2011

* 5-7 National Conference On Nonlinear Analysis and Applications, Department of Mathematics, H.N.B. Garhwal University, Campus Pauri, Pauri Garhwal, Uttarakhand.
Description: To make a path for new researchers in Nonlinear Analysis and its Applications to other diciplines. This conference will surely provide an opportunity for interaction among the young researchers and renowned academicians of the country.
Topics: Aims at covering all areas of nonlinear analysis with emphasis on the following topics: Approximation Theory, Fixed Point Theory, Fractal analysis, Operator Theory, Optimization Theory, Setvalued analysis.'
Advisory Committee: S. R. Ansari, Srinagar; S. G. Dani, Mumbai; R. Dangwal, Pauri; R. C. Dimri, Srinagar; M. C. Joshi, Mumbai; M. C. Joshi, Nainital; D. S. Negi, Srinagar; R. S. Pathak, Varanasi; A. K. Singh, Tehri; Dinesh Singh, Delhi; S. L. Singh, Rishikesh; S. P. Singh, Bilaspur; S. D. Tripathi, Allahabad.
Information: http://hnbgu.ac.in.


## July 2011

* 11-15 The 10th International Conference on Finite Fields and their Applications, Ghent, Belgium.
Topics: Include, but are not limited to: Field-theoretic aspects of finite fields: structure of finite fields, normal bases, polynomials, primitive elements, number-theoretic aspects of finite fields, exponential sums, arithmetic theory of function fields. Computational aspects: algorithms and complexity, polynomial factorization, decomposition and irreducibility testing, sequences and functions. Applications: algebraic coding theory, cryptography, affine and projective geometry over finite fields, algebraic geometry over finite fields, finite incidence geometry, designs and configurations, difference sets, combinatorics, generalisations of finite fields.
Information: http://cage.ugent.be/fq10/.
* 18-22 7th International Congress on Industrial and Applied Mathematics - ICIAM 2011 , Vancouver, BC, Canada.
Description: ICIAM 2011 will highlight the most recent advances in the discipline and demonstrate their applicability to science, engineering and industry. In addition to the traditional, strong focus on applied mathematics, the congress will emphasize industrial applications and computational science. Industrial mathematics will be featured prominently through an integrated program which will highlight the many outstanding contributions of applied mathematics. ICIAM 2011 will also be an opportunity to demonstrate to young researchers and graduate students the vast potential of mathematics.
Information: http://www.iciam2011.com/.
*26-29 Conference in Harmonic Analysis and Partial Differential Equations in honour of Eric Sawyer, Fields Institute, Toronto, Canada.
Description: The meeting will concentrate on Analysis and Partial differential Equations, specifically in relation to the diverse set of topics Dr. Eric T. Sawyer has worked on and influenced during his rich and prolific career: Weighted norm inequalities, Several complex variables, Unique continuation, Singular integrals, fractional integrals, Elliptic equations and systems, Degenerate equations, Monge-Ampère equations, Number theory and Combinatorics.
Information: http://www.fields.utoronto.ca/programs/ scientific/11-12/PDE/.


## August 2011

*1-5 Categories, Geometry and Physics, Santa Marta, Colombia. Description: Located on the beautiful Caribbean coastline, the aim of this one-week conference is to gather a group of people interested in
the applications of category theory to physically inspired mathematics, and provide a nice environment for a fruitful dialogue among the different pathways and alternative approaches to the subject.
Information: http://ima.usergioarboleda.edu.co/SJCH/ JCHS2011.htm.

## 1-5 Conference in Honour of Søren Asmussen-New Frontiers in

 Applied Probability, Sandbjerg Estate, Sønderborg, Denmark.Description: The conference honours one of the leading researchers in applied probability, Søren Asmussen, on the occasion of his 65th birthday. Professor Asmussen's research cuts across many of the major themes of modern applied probability, including queueing theory, insurance and financial mathematics, stochastic simulation, and stochastic control. His nearly 150 publications include major contributions to the theory of heavy tails, rare-event asymptotics and related simulation methods, matrix-geometric modelling, Markov processes, and regeneration. He has also written four major books: Branching Processes (co-authored with H. Hering), Applied Probability and Queues; Ruin Probabilities; and Stochastic Simulation: Algorithms and Analysis (co-authored with P. Glynn). His outstanding research contributions have been recognized through the Marcel F. Neuts Applied Probability Prize (1999), and the INFORMS Outstanding Publication Prizes in Simulation in 2002 and 2008.
Information: http://www.thiele.au.dk/asmussen.
April 2013

* 4-6 38th Arkansas Spring Lecture Series in the Mathematical Sciences: "Extension and Interpolation of Functions", University of Arkansas, Fayetteville, Arkansas.
Short Description: The principal speaker will be Charles Fefferman (Princeton University), on the topic of "Extension and Interpolation of Functions." There will also be talks by invited speakers. Applications for contributed talks by junior mathematicians are strongly encouraged. For further information, please contact Phil Harrington (email: psharrin@uark. edu) or Loredana Lanzani (email: lanzani@uark.edu).
Information: http://math.uark.edu/.


# New Publications Offered by the AMS 

# Algebra and Algebraic Geometry 



## Quantum Bounded Symmetric Domains

Leonid L. Vaksman<br>Translated by Olga Bershtein and Sergey D. Sinel'shchikov

This book provides exposition of the basic theory of quantum bounded symmetric domains. The area became active in the late 1990s at a junction of noncommutative complex analysis and extensively developing theory of quantum groups. It is well known that the classical bounded symmetric domains involve a large number of nice constructions and results of the theory of $C^{*}$-algebras, theory of functions and functional analysis, representation theory of real reductive Lie groups, harmonic analysis, and special functions. In a surprising advance of the theory of quantum bounded symmetric domains, it turned out that many classical problems admit elegant quantum analogs. Some of those are expounded in the book. Anyone with an interest in the subject will welcome this unique treatment of quantum groups.
The book is written by a leading expert in a very clear, careful, and stimulating way. I strongly recommend it to graduate students and research mathematicians interested in noncommutative geometry, quantum groups, $C^{*}$-algebras, or operator theory.

## - Vladimir Drinfeld, University of Chicago

Contents: Quantum disc; Basic quantum theory of bounded symmetric domains; Conclusion; Bibliography; Index.
Translations of Mathematical Monographs, Volume 238
July 2010, approximately 266 pages, Hardcover, ISBN: 978-0-8218-4909-5, 2000 Mathematics Subject Classification: 17B37, 20G42, 81R50; 22E47, 33D45, 43A85, 46L52, AMS members US\$84, List US\$105, Order code MMONO/238

## Analysis



# Differentiable Measures and the Malliavin Calculus 

Vladimir I. Bogachev, Moscow State University, Russia

This book provides the reader with the principal concepts and results related to differential properties of measures on infinite dimensional spaces. In the finite dimensional case such properties are described in terms of densities of measures with respect to Lebesgue measure. In the infinite dimensional case new phenomena arise. For the first time a detailed account is given of the theory of differentiable measures, initiated by S. V. Fomin in the 1960s; since then the method has found many various important applications. Differentiable properties are described for diverse concrete classes of measures arising in applications, for example, Gaussian, convex, stable, Gibbsian, and for distributions of random processes. Sobolev classes for measures on finite and infinite dimensional spaces are discussed in detail. Finally, the author presents the main ideas and results of the Malliavin calculus-a powerful method to study smoothness properties of the distributions of nonlinear functionals on infinite dimensional spaces with measures.

The target readership includes mathematicians and physicists whose research is related to measures on infinite dimensional spaces, distributions of random processes, and differential equations in infinite dimensional spaces. The book includes an extensive bibliography on the subject.

Contents: Background material; Sobolev spaces on $\mathbb{R}^{n}$; Differentiable measures on linear spaces; Some classes of differentiable measures; Subspaces of differentiability of measures; Integration by parts and logarithmic derivatives; Logarithmic gradients; Sobolev classes on infinite dimensional spaces; The Malliavin calculus; Infinite dimensional transformations; Measures on manifolds; Applications; References; Subject index.

Mathematical Surveys and Monographs, Volume 164
August 2010, approximately 501 pages, Hardcover, ISBN: 978-0-8218-4993-4, LC 2010005829, 2000 Mathematics Subject Classification: 28Cxx, 46Gxx, 58Bxx, 60Bxx, 60Hxx, AMS members US\$90, List US\$113, Order code SURV/164


## Linear Functional Analysis

Joan Cerdà, Universitat de Barcelona, Spain

Functional analysis studies the algebraic, geometric, and topological structures of spaces and operators that underlie many classical problems. Individual functions satisfying specific equations are replaced by classes of functions and transforms that are determined by the particular problems at hand.
This book presents the basic facts of linear functional analysis as related to fundamental aspects of mathematical analysis and their applications. The exposition avoids unnecessary terminology and generality and focuses on showing how the knowledge of these structures clarifies what is essential in analytic problems.
The material in the first part of the book can be used for an introductory course on functional analysis, with an emphasis on the role of duality. The second part introduces distributions and Sobolev spaces and their applications. Convolution and the Fourier transform are shown to be useful tools for the study of partial differential equations. Fundamental solutions and Green's functions are considered and the theory is illustrated with several applications. In the last chapters, the Gelfand transform for Banach algebras is used to present the spectral theory of bounded and unbounded operators, which is then used in an introduction to the basic axioms of quantum mechanics.

The presentation is intended to be accessible to readers whose backgrounds include basic linear algebra, integration theory, and general topology. Almost 240 exercises will help the reader in better understanding the concepts employed.

A co-publication of the AMS and Real Sociedad Matemática Española (RSME).

Contents: Introduction; Normed spaces and operators; Fréchet spaces and Banach theorems; Duality; Weak topologies; Distributions; Fourier transform and Sobolev spaces; Banach algebras; Unbounded operators in a Hilbert space; Hints to exercises; Bibliography; Index.
Graduate Studies in Mathematics, Volume 116
July 2010, 330 pages, Hardcover, ISBN: 978-0-8218-5115-9, LC 2010006449, 2000 Mathematics Subject Classification: 46-01; 46Axx, 46Bxx, 46Exx, 46Fxx, 46Jxx, 47B15, AMS members US\$50, List US\$62, Order code GSM/116


# Lyapunov Exponents and Invariant Manifolds for Random Dynamical Systems in a Banach Space 

Zeng Lian, New York University, Courant Institute of Mathematical Sciences, NY, and Kening Lu, Brigham Young University, Provo, UT

Contents: Introduction; Random dynamical systems and measures of noncompactness; Main results; Volume function in Banach spaces; Gap and distance between closed linear subspaces; Lyapunov exponents and oseledets spaces; Measurable random invariant complementary subspaces; Proof of multiplicative ergodic theorem; Stable and unstable manifolds; Appendix A. Subadditive ergodic theorem; Appendix B. Non-ergodic case; Bibliography.
Memoirs of the American Mathematical Society, Volume 206, Number 967
June 2010, 106 pages, Softcover, ISBN: 978-0-8218-4656-8, LC 2010012476, 2000 Mathematics Subject Classification: 37H15, 37L55; 37A30, 47A35, 37D10, 37D25, Individual member US\$41, List US\$68, Institutional member US\$54, Order code MEMO/206/967

> $C^{*}$-Algebras of Homoclinic and Heteroclinic Structure in Expansive Dynamics

Klaus Thomsen, IMF, Aarhus, Denmark

Contents: The Ruelle algebra of a relatively expansive system; On the functoriality of the Ruelle algebra; The homoclinic algebra of expansive actions; The heteroclinic algebra; One-dimensional generalized solenoids; The heteroclinic algebra of a group automorphism; A dimension group for certain countable state Markov shifts; Appendix A. Étale equivalence relations from abelian $C^{*}$-subalgebras with the extension property; Appendix B. On certain crossed product $C^{*}$-algebras; Appendix C. On an example of Bratteli, Jorgensen, Kim and Roush; Bibliography.
Memoirs of the American Mathematical Society, Volume 206, Number 970

June 2010, 122 pages, Softcover, ISBN: 978-0-8218-4692-6, LC 2010012520, 2000 Mathematics Subject Classification: 46L35, 37D20, Individual member US\$41, List US\$69, Institutional member US\$55, Order code MEMO/206/970

## Applications



## Mathematics in Finance

Santiago Carrillo Menéndez and José Luis Fernández Pérez, Universidad Autónoma de Madrid, Spain, Editors

This volume contains survey papers on mathematical finance based on some courses given at the "Lluís Santaló" Summer School of the Real Sociedad Matemática Española, held in July 2007 at the Universidad Internacional Menéndez Pelayo, Santander (Spain). The primary topics are pathwise approximations of stochastic differential equations, Hedge funds, and credit derivatives.
The paper by L. Seco and F. Chen provides a systematic survey of hedge funds from a rigorous mathematical point of view. The related paper by M. Escobar, S. Krämer, F. Scheibl, L. Seco and R. Zagst introduces a new theoretical framework for the pricing of hedge funds' equity, inspired by the framework of Black and Cox for the valuation of company equity as a call option.

A general framework for deriving high order, stable and tractable path-wise approximations of Stratonovich stochastic differential equations as applied to finance is the subject of the paper of L. G. Gyurkó and T. Lyons.
The paper by R. Zagst and M. Scherer is a short course on the different approaches used for pricing, hedging and risk management of credit derivatives.

Researchers and practitioners in mathematical finance will find in this book a collection of excellent, up-to-date and mathematically rigorous presentations of some of the most advanced techniques for pricing and risk management.

A co-publication of the AMS and Real Sociedad Matemática Española (RSME).

Contents: M. Escobar, S. Krämer, F. Scheibl, L.A. Seco, and R. Zagst, Hedge funds as knock-out options; L. G. Gyurkó and T. Lyons, Rough paths based numerical algorithms in computational finance; L. A. Seco and F. Chen, Hedge funds; R. Zagst and M. Scherer, Modeling and pricing credit derivatives.

Contemporary Mathematics, Volume 515
July 2010, 146 pages, Softcover, ISBN: 978-0-8218-4673-5, LC 2010003412, 2000 Mathematics Subject Classification: 91G20, 91G40, 91G60, 91G80, 65C30, 60H10, AMS members US\$47, List US\$59, Order code CONM/515

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# Small Modifications of Quadrature Domains 

Makoto Sakai, Kawasaki, Japan

Contents: Introduction and main results; Quadrature domains; Construction of measures for localization; Generalizations of the reflection theorem; Continuous reflection property and smooth boundary points; Proofs of (1) and (3) in Theorem 1.1; Corners with right angles; Properly open cusps; Microlocalization and the local-reflection theorem; Modifications of measures in $R^{+}$; Modifications of measures in $R^{-}$; Sufficient conditions for a cusp to be a laminar-flow point; Turbulent-flow points; The set of stationary points; Open questions; Bibliography; Symbol index; Index.

Memoirs of the American Mathematical Society, Volume 206, Number 969

June 2010, 269 pages, Softcover, ISBN: 978-0-8218-4810-4, LC 2010012474, 2000 Mathematics Subject Classification: 31Axx; 31A15, 76D27, Individual member US\$56, List US\$93, Institutional member US\$74, Order code MEMO/206/969


## Topological Quantum Computation

## Zhenghan Wang, Microsoft, Santa Barbara, CA

Topological quantum computation is a computational paradigm based on topological phases of matter, which are governed by topological quantum field theories. In this approach, information is stored in the lowest energy states of many-anyon systems and processed by braiding non-abelian anyons. The computational answer is accessed by bringing anyons together and observing the result. Besides its theoretical esthetic appeal, the practical merit of the topological approach lies in its error-minimizing hypothetical hardware: topological phases of matter are fault-avoiding or deaf to most local noises, and unitary gates are implemented with exponential accuracy. Experimental realizations are pursued in systems such as fractional quantum Hall liquids and topological insulators.
This book expands on the author's CBMS lectures on knots and topological quantum computing and is intended as a primer for mathematically inclined graduate students. With an emphasis on introducing basic notions and current research, this book gives the first coherent account of the field, covering a wide range of topics: Temperley-Lieb-Jones theory, the quantum circuit model, ribbon fusion category theory, topological quantum field theory, anyon theory, additive approximation of the Jones polynomial, anyonic quantum computing models, and mathematical models of topological phases of matter.
This item will also be of interest to those working in mathematical physics.
A co-publication of the AMS and CBMS.
Contents: Temperley-Lieb-Jones theories; Quantum circuit model; Approximation of the Jones polynomial; Ribbon fusion categories; (2+1)-TQFTs; TQFTs in nature; Topological quantum computers;

Topological phases of matter; Outlook and open problems; Bibliography.
CBMS Regional Conference Series in Mathematics, Number 112
April 2010, 115 pages, Softcover, ISBN: 978-0-8218-4930-9, LC 2010005661, 2000 Mathematics Subject Classification: 57M27, 81P68, 68Q12, 18D10; 57-02, 81-02, 68-02, 18-02, AMS members US\$27, All Individuals US\$27, List US\$34, Order code CBMS/112

# General and Interdisciplinary 



## Mathematics Everywhere

Martin Aigner and Ehrhard Behrends, Freie Universität Berlin, Germany, Editors Translated by Philip G. Spain

Mathematics is all around us. Often we do not realize it, though. Mathematics Everywhere is a collection of presentations on the role of mathematics in everyday life, through science, technology, and culture. The common theme is the unique position of mathematics as the art of pure thought and at the same time as a universally applicable science. The authors are renowned mathematicians; their presentations cover a wide range of topics. From compact discs to the stock exchange, from computer tomography to traffic routing, from electronic money to climate change, they make the "math inside" understandable and enjoyable. An additional attractive feature is the leisurely treatment of some hot topics that have gained prominence in recent years, such as Fermat's Theorem, Kepler's packing problem, and the solution of the Poincaré Conjecture. Or maybe you have heard about the Nash equilibrium (of "A Beautiful Mind" fame), or the strange future of quantum computers, and want to know what it is all about? Well, open the book and take an up-to-date trip into the fascinating world of the mathematics all around us.
Contents: Prologue: G. von Randow, Math becomes a cult-description of a hope; Case studies: J. H. van Lint, The mathematics of the compact disc; H.-O. Peitgen, C. Evertsz, B. Preim, D. Selle, T. Schindewolf, and W. Spindler, Image processing and imaging for operation planning in liver surgery;
R. Borndörfer, M. Grötschel, and A. Löbel, The quickest path to the goal; B. Fiedler, Romeo and Juliet, spontaneous pattern formation, and Turing's instability; S. Müller, Mathematics and intelligent materials; P. Gritzmann, Discrete tomography: from battleship to nanotechnology; J. Richter-Gebert, Reflections on reflections; Current topics: W. Schachermayer, The role of mathematics in the financial markets; A. Beutelspacher, Electronic money: an impossibility or already a reality?; M. Henk and G. M. Ziegler, Spheres in the computer-the Kepler conjecture; E. Behrends, How do quanta compute? The new world of the quantum computer; J. Kramer, Fermat's Last Theorem-the solution of a 300-year-old problem; K. Sigmund, A short history of the Nash equilibrium; R. Klein, Mathematics in the climate of global change; The central theme: M. Aigner, Prime numbers, secret codes and the boundaries of computability; E. Vogt, The mathematics of knots; D. Ferus, On soap bubbles; K. Ecker, Heat diffusion, the structure of space and the Poincaré Conjecture; E. Behrends, Chance and mathematics: a
late love; Epilogue: P. J. Davis, The prospects for mathematics in a multi-media civilization.

July 2010, 330 pages, Softcover, ISBN: 978-0-8218-4349-9, 2000
Mathematics Subject Classification: 00A05, 00A06, 00A08, AMS members US\$39, List US\$49, Order code MBK/72

## Logic and Foundations



# Structural Ramsey Theory of Metric Spaces and Topological Dynamics of Isometry Groups 

L. Nguyen Van Thé, University of Calgary, AB, Canada

Contents: Introduction; Fraïssé classes of finite metric spaces and Urysohn spaces; Ramsey calculus, Ramsey degrees and universal minimal flows; Big Ramsey degrees, indivisibility and oscillation stability; Appendix A. Amalgamation classes $\mathcal{M}_{S}$ when $|S| \leqslant 4$; Appendix B. Indivisibility of $U_{S}$ when $|S| \leqslant 4$; Appendix C . On the universal Urysohn space $\mathbf{U}$; Bibliography; Index.
Memoirs of the American Mathematical Society, Volume 206, Number 968
June 2010, 140 pages, Softcover, ISBN: 978-0-8218-4711-4, LC 2010012473, 2000 Mathematics Subject Classification: 03E02; 05C55, 05D10, 22A05, Individual member US\$43, List US\$72, Institutional member US\$58, Order code MEMO/206/968

## Probability



# Algebraic Methods in Statistics and Probability II 

Marlos A.G. Viana, University of Illinois, Chicago, IL, and Henry P. Wynn, London School of Economics, United Kingdom, Editors

This volume is based on lectures presented at the AMS Special Session on Algebraic Methods in Statistics and Probability-held March 27-29, 2009, at the University of Illinois at Urbana-Champaign-and on contributed articles solicited for this volume.

A decade after the publication of Contemporary Mathematics Vol. 287, the present volume demonstrates the consolidation of important areas, such as algebraic statistics, computational commutative algebra, and deeper aspects of graphical models.

In statistics, this volume includes, among others, new results and applications in cubic regression models for mixture experiments, multidimensional Fourier regression experiments, polynomial characterizations of weakly invariant designs, toric and mixture models for the diagonal-effect in two-way contingency tables, topological methods for multivariate statistics, structural results for the Dirichlet distributions, inequalities for partial regression coefficients, graphical models for binary random variables, conditional independence and its relation to sub-determinants covariance matrices, connectivity of binary tables, kernel smoothing methods for partially ranked data, Fourier analysis over the dihedral groups, properties of square non-symmetric matrices, and Wishart distributions over symmetric cones.
In probability, this volume includes new results related to discrete-time semi Markov processes, weak convergence of convolution products in semigroups, Markov bases for directed random graph models, functional analysis in Hardy spaces, and the Hewitt-Savage zero-one law.

This item will also be of interest to those working in algebra and algebraic geometry.

Contents: S. A. Andersson and T. Klein, Kiefer-complete classes of designs for cubic mixture models; V. S. Barbu and N. Limnios, Some algebraic methods in semi-Markov chains; R. A. Bates, H. Maruri-Aguilar, E. Riccomagno, R. Schwabe, and H. P. Wynn, Self-avoiding generating sequences for Fourier lattice designs; F. Bertrand, Weakly invariant designs, rotatable designs and polynomial designs; C. Bocci, E. Carlini, and F. Rapallo, Geometry of diagonal-effect models for contingency tables; P. Bubenik, G. Carlsson, P. T. Kim, and Z.-M. Luo, Statistical topology via Morse theory persistence and nonparametric estimation; G. Budzban and G. Högnäs, Convolution products of probability measures on a compact semigroup with applications to random measures; S. Chakraborty and A. Mukherjea, Completely simple semigroups of real $d \times d$ matrices and recurrent random walks; W.-Y. Chang, R. D. Gupta, and D. S. P. Richards, Structural properties of the generalized Dirichlet distributions; S. Chaudhuri and G. L. Tan, On qualitative comparison of partial regression coefficients for Gaussian graphical Markov models; M. A. Cueto, J. Morton, and B. Sturmfels, Geometry of the restricted Boltzmann machine; M. Drton and H. Xiao, Smoothness of Gaussian conditional independence models; W. Ehm, Projections on invariant subspaces; S. M. Evans, A zero-one law for linear transformations of Lévy noise; H. Hara and A. Takemura, Connecting tables with zero-one entries by a subset of a Markov basis; K. Khare and B. Rajaratnam, Covariance trees and Wishart distributions on cones; P. Kidwell and G. Lebanon, A kernel smoothing approach to censored preference data; M. S. Massa and S. L. Lauritzen, Combining statistical models; S. Petrović, A. Rinaldo, and S. E. Fienberg, Algebraic statistics for a directed random graph model with reciprocation; G. Pistone and M. P. Rogantin, Regular fractions and indicator polynomials; M. A. G. Viana, Dihedral Fourier analysis; T. von Rosen and D. Von Rosen, On a class of singular nonsymmetric matrices with nonnegative integer spectra; A. S. Yasamin, Some hypothesis tests for Wishart models on symmetric cones.

## Contemporary Mathematics, Volume 516

July 2010, 345 pages, Softcover, ISBN: 978-0-8218-4891-3, LC 2010006241, 2000 Mathematics Subject Classification: 20M20, 20C30, 28D $15,42 \mathrm{~A} 65,43 \mathrm{~A} 30,60 \mathrm{~F} 20,60 \mathrm{~B} 15,60 \mathrm{~K} 15,62 \mathrm{Kxx}, 62 \mathrm{G} 08$, 62H17, 62H05, 62H20, 62N01, AMS members US\$84, List US\$105, Order code CONM/516

# New AMS-Distributed Publications 

## Algebra and Algebraic Geometry



## Deformation Spaces

Hossein Abbaspour, Université de Nantes, France, Matilde Marcolli, California Institute of Technology, Pasadena, CA, and Thomas Tradler, New York City College of Technology (CUNY), $N Y$, Editors

The first instances of deformation theory were given by Kodaira and Spencer for complex structures and by Gerstenhaber for associative algebras. Since then, deformation theory has been applied as a useful tool in the study of many other mathematical structures, and even today it plays an important role in many developments of modern mathematics.
This volume collects a few self-contained and peer-reviewed papers by experts which present up-to-date research topics in algebraic and motivic topology, quantum field theory, algebraic geometry, noncommutative geometry and the deformation theory of Poisson algebras.

The papers originate from activities at the Max-Planck-Institute for Mathematics and the Hausdorff Center for Mathematics in Bonn.

A publication of Vieweg Verlag. The AMS is exclusive distributor in North America. Vieweg Verlag Publications are available worldwide from the AMS outside of Germany, Switzerland, Austria, and Japan.
Contents: Contributions by Grégory Ginot, Thomas M. Fiore and Igor Kriz, Toshiro Hiranouchi and Satoshi Mochizuki, Paulo Carrillo Rouse, Donatella Iacono and Marco Manetti, John Terilla, and Anne Pichereau.
Vieweg Aspects of Mathematics, Volume 39
March 2010, 181 pages, Hardcover, ISBN: 978-3-8348-1271-1, 2000 Mathematics Subject Classification: 58H15, 53D55, 18D50, 14F42, 13D10, 32G15, 14D23, 32G05, AMS members US\$54, List US\$60, Order code VWAM/39


# Cohomology of Groups and Algebraic K-Theory 

Lizhen Ji, University of Michigan, Ann Arbor, MI, Kefeng Liu, University of California, Los Angeles, CA, and Shing-Tung Yau, Harvard University, Cambridge, MA, Editors

Cohomology of groups is a fundamental tool in many subjects of modern mathematics. One important generalized cohomology theory is the algebraic $K$-theory. Indeed, algebraic $K$-groups of rings are important invariants of the rings and have played important roles in algebra, topology, number theory, etc. This volume consists of expanded lecture notes from a 2007 seminar at Zhejiang University in China, at which several leading experts presented introductions to and surveys of many aspects of cohomology of groups and algebraic $K$-theory, along with their broad applications. Two foundational papers on algebraic $K$-theory by Daniel Quillen are also included.

A publication of International Press. Distributed worldwide by the American Mathematical Society.

Contents: A. Bartels and W. Lück, On crossed product rings with twisted involutions, their module categories and $L$-theory; O. Baues, Deformation spaces for affine crystallographic groups; K. S. Brown, Lectures on the cohomology of groups; D. R. Grayson, A brief introduction to algebraic $K$-theory; D. Juan-Pineda and S. Millan-López, The braid groups of RP2 satisfy the fibered isomorphism conjecture; M. Karoubi, $K$-theory, an elementary introduction; M. Karoubi, Lectures on K-theory; W. Lück, On the Farrell-Jones and related conjectures; S. Prassidis, Introduction to controlled topology and its applications; H. Qin, Lecture notes on $K$-theory; D. Quillen, Higher algebraic $K$-theory: I; D. Quillen, Finite generation of the groups $K_{i}$ of rings of algebraic integers; D. Rosenthal, A user's guide to continuously controlled algebra; C. Soulé, Higher $K$-theory of algebraic integers and the cohomology of arithmetic groups (Notes by M. Varisco).

## International Press

March 2010, 517 pages, Softcover, ISBN: 978-1-57146-144-5, 2000 Mathematics Subject Classification: 18Fxx, 19Dxx, 20Jxx, AMS members US\$52, List US\$65, Order code INPR/88


## Automorphic Forms and the Langlands Program

Lizhen Ji, University of Michigan, Ann Arbor, MI, Kefeng Liu, University of California, Los Angeles, CA, Shing-Tung Yau, Harvard University, Cambridge, MA, and Zhu-Jun Zheng, Henan University, China, Editors

Classical modular forms on the upper half plane, with respect to the modular group $\operatorname{SL}(2, \mathbb{Z})$ and its congruence subgroups, have arisen naturally in number theory, complex analysis, topology, mathematical physics, and many other subjects. The closely related automorphic representations are basic notions in the celebrated Langlands program, which was proposed by Langlands in the late 1960s and has since revolutionized the fields of number theory, arithmetic algebraic geometry, and representation theory. This volume consists of expanded lecture notes from a 2007 international conference in Guangzhou, China, at which several leading experts in number theory presented introductions to, and surveys of, many aspects of automorphic forms and the Langlands program.
A publication of International Press. Distributed worldwide by the American Mathematical Society.
Contents: A. W. Knapp, Prerequisites for the Langlands program; A. W. Knapp, First steps with the Langlands program; S. S. Gelbart, Class field theory, the Langlands program and its application to number theory; W. T. Gan, Automorphic forms and automorphic representations; E. M. Lapid, Introductory notes on the trace formula; S. Friedberg, Euler products and twisted Euler products; X. Li, Arithmetic trace formulas and Kloostermania; D. Shelstad, Tempered endoscopy for real groups II: Spectral transfer factors; L. Guo, Algebraic Birkhoff decomposition and its applications.

## International Press

March 2010, 319 pages, Softcover, ISBN: 978-1-57146-141-4, 2000 Mathematics Subject Classification: 11Fxx, AMS members US\$44, List US\$55, Order code INPR/85


Harmonic Analysis Second Edition

Henry Helson，University of California，Berkeley，CA
．．．this volume can be an excellent way to get one＇s feet wet and see whether it is worthwhile to take the plunge．The book has helpful and challenging exercises for the reader．It should be emphasized that the great strength of the book lies in the large number of interesting special results it contains．Many of the prettiest facts of harmonic analysis are on display here in a very attractive setting．
－Bulletin of the American Mathematical Society
This second edition has been enlarged and considerably rewritten． Among the new topics are infinite product spaces with applications to probability，disintegration of measures on product spaces， positive definite functions on the line，and additional information about Weyl＇s theorems on equidistribution．Topics that have continued from the first edition include Minkowski＇s theorem， measures with bounded powers，idempotent measures，spectral sets of bounded functions and a theorem of Szegö，and the Wiener Tauberian theorem．
Readers of the book should have studied the Lebesgue integral，the elementary theory of analytic and harmonic functions，and the basic theory of Banach spaces．The treatment is classical and as simple as possible．This is an instructional book，not a treatise． Mathematics students interested in analysis will find here what they need to know about Fourier analysis．Physicists and others can use the book as a reference for more advanced topics．
A publication of Hindustan Book Agency．Distributed on an exclusive basis by the AMS in North America．Online bookstore rights worldwide．
Contents：Fourier series and integrals；The Fourier integral； Discrete and compact groups；Hardy spaces；Conjugate functions； Translation；Distribution．

## Hindustan Book Agency

April 2010， 236 pages，Softcover，ISBN：978－93－80250－05－2， 2000 Mathematics Subject Classification：42－01；43－01，AMS members US\＄24，List US\＄30，Order code HIN／47


# Recent Advances in Geometric Analysis 

 University，Taipei，Taiwan， Chang－Shou Lin，National Chung Cheng University，Chia－Yi， Taiwan，and Mao－Pei Tsui， University of Toledo，OH，EditorsThis volume presents an account of recent advances in geometric analysis and related topics，including Ricci flow，affine normal flow，geometric analysis on pseudo－convex hypersurfaces，Alexandrov space，manifolds with special holonomy， and the singular plateau problem．These papers，many by leading experts in the field，are drawn from lectures presented at the 2007 International Conference in Geometric Analysis，held at Taiwan University．The present volume is intended for both researchers and graduate students studying geometric analysis and related areas．
A publication of International Press．Distributed worldwide by the American Mathematical Society．

Contents：H．－D．Cao，Recent progress on Ricci solitons；J．Cao， B．Dai，and J．Mei，An optimal extension of Perelman＇s comparison theorem for quadrangles and its applications；D．－C．Chang and S．S．－T．Yau，Geometric analysis on a family of pseudoconvex hypersurfaces；S．Ji，A new proof for Faran＇s theorem on maps between B2 and B3；S．Karigiannis，Some notes on G2 and Spin（7）geometry；L．Ni，Closed type I ancient solutions to Ricci flow；J．Loftin and M．－P．Tsui，Limits of solutions to a parabolic Monge－Ampère equation；C．－S．Lin and C．－L．Wang，A function theoretic view of the mean field equations on tori；S．－C．Lau and N．C．Leung，Conformal geometry and special holonomy；L．－F．Tarn， Exhaustion functions on complete manifolds；S．Yamada，On singular plateau problem．

## International Press

March 2010， 229 pages，Softcover，ISBN：978－1－57146－143－8， 2000 Mathematics Subject Classification：53Dxx，AMS members US\＄40， List US\＄50，Order code INPR／87

## Differential Equations



# Trends in Partial Differential Equations 

Baojun Bian，Tongji University， Shanghai，China，Shenghong Li， Zhejiang University，Hangzhou， China，and Xu－Jia Wang，The Australian National University， Canberra，Australia，Editors

In a career of nearly sixty years of mathematical research，Guangchang Dong＇s influence on the development of partial differential equations in China has been immense，at both the teaching and research levels．To celebrate his eightieth birthday，an international conference called Elliptic and Parabolic Equations and Applications was held in August 2008 at Zhejiang University in Hangzhou，China．This volume presents
fifteen papers, some drawn from lectures given at the conference and others by his friends and former students.

A publication of International Press. Distributed worldwide by the American Mathematical Society.

Contents: B. Bian and Y. Wang, Viscosity solutions of Bellman equations arising in pricing passport options; K.-S. Chou and S.-Z. Du, On global partial regularity for borderline solutions of semilinear parabolic problems; B. Guan, The Dirichlet problem for complex Monge-Ampère equations and applications; Y. Han and G. Lu, Some recent works on multiparameter Hardy space theory and discrete Littlewood-Paley analysis; M.-C. Hong, Some analytic aspects of liquid crystal configurations; D.-X. Kong, Extremal sub-manifolds in Minkowski space-time; J. Ling and Z. Lu, Bounds of eigenvalues on Riemannian manifolds; Y. Lu, Wave breaking phenomena and stability of peakons for the Degasperis-Procesi equation; X.-N. Ma and Q. Ou, The convexity of level sets for solutions to partial differential equations; X.-B. Pan, Nucleation of instability of Meissner state of superconductors and related mathematical problems; Y.-B. Shen, Calculus of some geometric variations in Finsler geometry; W. Sheng and X.-J. Wang, Regularity and singularity in the mean curvature flow; L. Shen and G. Lv, Study of finite point method; J. Yin, P. Lei, and C. Wang, Nonlinear diffusion equations with different kind of degeneracy; X. Zhu, Canonical metrics on toric manifolds; List of publications by Guangchang Dong.

## International Press

March 2010, 527 pages, Softcover, ISBN: 978-1-57146-142-1, 2000 Mathematics Subject Classification: 35Rxx, AMS members US\$52, List US\$65, Order code INPR/86

## General and Interdisciplinary



Studies in the History of Indian Mathematics

C. S. Seshadri, Chennai Mathematical Institute, Tamil Nadu, India, Editor

This volume is the outcome of a seminar on the history of mathematics held at the Chennai Mathematical Institute during January-February 2008 and contains articles based on the talks of distinguished scholars both from the West and from India.
The topics covered include: (1) geometry in the øulvasåtras; (2) the origins of zero (which can be traced to ideas of lopa in Pàõini's grammar); (3) combinatorial methods in Indian music (which were developed in the context of prosody and subsequently applied to the study of tonal and rhythmic patterns in music); (4) a cross-cultural view of the development of negative numbers (from Brahmagupta (c. 628 CE) to John Wallis ( 1685 CE); (5) Kuññaka, Bhàvanà and Cakravàla (the techniques developed by Indian mathematicians for the solution of indeterminate equations); (6) the development of calculus in India (covering the millennium-long history of discoveries culminating in the work of the Kerala school giving a complete analysis of the basic calculus of polynomial and trigonometrical functions); (7) recursive methods in Indian mathematics (going back to Pàõini's grammar and culminating in
the recursive proofs found in the Malayalam text Yuktibhàùà (1530 CE)); and (8) planetary and lunar models developed by the Kerala School of Astronomy. The articles in this volume cover a substantial portion of the history of Indian mathematics and astronomy.
This book will serve the dual purpose of bringing to the international community a better perspective of the mathematical heritage of India and conveying the message that much work remains to be done, namely the study of many unexplored manuscripts still available in libraries in India and abroad.
A publication of Hindustan Book Agency. Distributed on an exclusive basis by the AMS in North America. Online bookstore rights worldwide.
Contents: K. Plofker, Biographical sketch of David Pingree; M. S. Sriram, Biographical sketch of K. V. Sarma; S. G. Dani, Geometry in the Sulvasutras; F. Staal, On the origins of zero; R. Sridharan and M. D. Srinivas, Combinatorial methods in Indian music: Pratyayas in Sangıtaratnakara of Sarngadeva; D. Mumford, What's so baffling about negative numbers?-A crosscultural comparison; A. K. Dutta, Kuttaka, Bhavana and Cakravala; K. Ramasubramanian and M. D. Srinivas, Development of calculus in India; P. P. Divakaran, Notes on Yuktibhasa: Recursive methods in Indian mathematics; M. S. Sriram, Planetary and lunar models in the Kerala works, Tantrasangraha (c. 1500) and Ganita Yuktibhasa (c. 1530).

## Hindustan Book Agency

June 2010, 402 pages, Hardcover, ISBN: 978-93-80250-06-9, 2000 Mathematics Subject Classification: 01-02, AMS members US\$52, List US\$65, Order code HIN/46

## Geometry and Topology



## Probabilistic Approach to Geometry

Motoko Kotani, Tohoku University, Sendai, Japan, and Masanori Hino and Takashi Kumagai, Kyoto University, Japan, Editors

The first Seasonal Institute of the Mathematical Society of Japan (MSJ-SI) -"Probabilistic Approach to Geometry" - was held at Kyoto University, Japan, on July 28-August 8,2008 . The conference aimed to make interactions between geometry and probability theory and to pursue new directions in these research areas. This volume contains the proceedings, selected research articles based on the talks, including survey articles on random groups, rough paths, and heat kernels by the survey lecturers at the conference. Readers will benefit from exploring this developing research area.
This item will also be of interest to those working in probability.
Published for the Mathematical Society of Japan by Kinokuniya, Tokyo, and distributed worldwide, except in Japan, by the AMS.

Contents: S. Aida, Rough path analysis: An introduction; M. Arnaudon and A. Thalmaier, Li-Yau type gradient estimates and Harnack inequalities by stochastic analysis; A. Atsuji, Estimates on the number of the omitted values by meromorphic
functions; K. D. Elworthy, Stochastic flows and geometric analysis on path spaces; R. Friedrich, The global geometry of stochastic Lœewner evolutions; R. Fukushima, A. Tanida, and K. Yano, Non-Markov property of certain eigenvalue processes analogous to Dyson's model; K. Hara, Rough path conditions for smooth paths; T. Hattori and A. Kasue, Functions of finite Dirichlet sums and compactifications of infinite graphs; Y. Inahama, A note on rough differential equations with unbounded coefficients; H. Izeki, A fixed-point property of finitely generated groups and an energy of equivariant maps; N. Juillet, On a method to disprove generalized Brunn-Minkowski inequalities; V. A. Kaimanovich and F. Sobieczky, Stochastic homogenization of horospheric tree products; N. Kajino, Remarks on non-diagonality conditions for Sierpinski carpets; T. Kato, Pattern formation from projectively dynamical systems and iterations by families of maps; T. Kondo, Fixed-point theorems for random groups; K. Kuwada, Couplings of the Brownian motion via discrete approximation under lower Ricci curvature bounds; K. Kuwae and T. Shioya, Infinitesimal Bishop-Gromov condition for Alexandrov spaces; H. Matsuzoe, Statistical manifolds and affine differential geometry; S. Ohta, Optimal transport and Ricci curvature in Finsler geometry;
Y. Ollivier, A survey of Ricci curvature for metric spaces and Markov chains; V. Pestov, Concentration of measure and whirly actions of Polish groups; L. Saloff-Coste, The heat kernel and its estimates; I. Shigekawa, Non-symmetric diffusions on a Riemannian manifold; A. Takatsu, On Wasserstein geometry of Gaussian measures; S. Watanabe, A limit theorem in singular regression problem; K. Yano, Two kinds of conditionings for stable Lévy processes; T. Yokota, A gap theorem for ancient solutions to the Ricciflow.

Advanced Studies in Pure Mathematics, Volume 57
March 2010, 514 pages, Hardcover, ISBN: 978-4-931469-58-7, 2000
Mathematics Subject Classification: 51-06; 34F05, 51F99, 58J65, AMS members US\$78, List US\$98, Order code ASPM/57

## Crossword Puzzle

$1,1,2,3,5, \ldots$ (page 752)

Answers


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The Department of Mathematics and Statistics of the University of Cyprus, invites applications for one position in the field of Probability-Statistics at the rank of Lecturer or Assistant Professor. The official languages of the University are Greek and/ or Turkish. For the above position knowledge of Greek is necessary. The deadline for applications is July 16th, 2010. For more information, see: http://www.ucy. ac.cy/goto/mathstatistics/e1-GR/ Vacancies.aspx.

000028

Suggested uses for classified advertising are positions available, books or lecture notes for sale, books being sought, exchange or rental of houses, and typing services.
The 2010 rate is $\$ 3.25$ per word. No discounts for multiple ads or the same ad in consecutive issues. For an additional \$10 charge, announcements can be placed anonymously. Correspondence will be forwarded.
Advertisements in the "Positions Available" classified section will be set with a minimum one-line headline, consisting of the institution name above body copy, unless additional headline copy is specified by the advertiser. Headlines will be centered in boldface at no extra charge. Ads will appear in the language in which they are submitted.
There are no member discounts for classified ads. Dictation over the telephone will not be accepted for classified ads.
Upcoming deadlines for classified advertising are as follows: August 2010 issue-May 28, 2010; September 2010 issue-June 28, 2010;October 2010 issue-July 29, 2010; November 2010 issue-August 30, 2010;

September 2010 issue-June 28, 2010; October 2010 issue-July 29, 2010. U.S. laws prohibit discrimination in employment on the basis of color, age, sex, race, religion, or national origin. "Positions Available" advertisements from institutions outside the U.S. cannot be published unless they are accompanied by a statement that the institution does not discriminate on these grounds whether or not it is subject to U.S. laws. Details and specific wording may be found on page 667 (vol. 56).
Situations wanted advertisements from involuntarily unemployed mathematicians are accepted under certain conditions for free publication. Call toll-free 800-321-4AMS (321-4267) in the U.S. and Canada or 401-455-4084 worldwide for further information.
Submission: Promotions Department, AMS, P.O. Box 6248, Providence, Rhode Island 02940; or via fax: 401-331-3842; or send email to classads@ams.org. AMS location for express delivery packages is 201 Charles Street, Providence, Rhode Island 20904. Advertisers will be billed upon publication.

# Meetings \& Conferences of the AMS 

IMPORTANT INFORMATION REGARDINGMEETINGS PROGRAMS: AMS Sectional Meeting programs do not appear in the print version of the Notices. However, comprehensive and continually updated meeting and programinformation with links to the abstract for each talk can be found on the AMS website. See http://www.ams.org/meetings/. Final programs for Sectional Meetings will be archived on the AMS website accessible from the stated URL and in an electronic issue of the Notices as noted below for each meeting.

## Berkeley, California

## University of California Berkeley

June 2-4, 2010
Wednesday - Friday

## Meeting \#1061

Eighth Joint International Meeting of the AMS and the Sociedad Matemática Mexicana.
Associate secretary: Susan J. Friedlander Announcement issue of Notices: April 2010
Program first available on AMS website: April 22, 2010
Program issue of electronic Notices: June 2010
Issue of Abstracts: Volume 31, Issue 3

## Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: Expired
The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/ internmtgs.htm7.

## Invited Addresses

Alejandro Adem, University of British Columbia and PIMS, Homotopy theory and spaces of representations.

Peter W.-K. Li, University of California, Irvine, What do we know about open manifolds?

Ernesto Lupercio, CINVESTAV, Orbifolds and quantum field theories.

Alberto Verjovsky, IM-UNAM, A smooth codimensionone foliation of the 5 -sphere by symplectic leaves.

Maciej Zworski, University of California Berkeley, Random perturbations in discrete quantization.

## Special Sessions

Algebraic Topology and Related Topics, Alejandro Adem, University of British Columbia, Gunnar E. Carlsson and Ralph L. Cohen, Stanford University, and Ernesto Lupercio, CINVESTAV.

Analytic Aspects of Differential Geometry, Nelia Charalambous, ITAM, Lizhen Ji, University of Michigan, and Jiaping Wang, University of Minnesota.

Commutative Algebra and Representation Theory, David Eisenbud and Daniel M. Erman, University of California, Berkeley, Jose Antonio de la Pena, UNAM, and Rafael Villareal, Cinvestav-IPN.

Complex Analysis and Operator Theory, Maribel Loaiza, Enrique Ramirez de Arellano, and Nikolai Vasilevski, CINVESTAV, Ilya M. Spitkovsky, College of William \& Mary, and Kehe Zhu, State University of New York at Albany.

Dynamical Systems, Alberto Verjovsky, IM-UNAM, and Rodrigo Perez, Indiana University-Purdue University, Indianapolis.

Graph Theory and Combinatorics with Emphasis on Geometric and Topological Aspects, Gelasio Salazar, Instituto de Fisica, Universidad Autonoma de San Luis Potosi, and Dan S. Archdeacon, University of Vermont.

Harmonic Analysis, Microlocal Analysis, and Partial Differential Equations, Gunther Uhlmann, University of Washington, and Salvador Perez Esteva, UNAM.

Low-Dimensional Topology, Kenneth L. Baker, University of Miami, and Enrique Ramirez Losada, CIMAT.

Singularity Theory and Algebraic Geometry, David Eisenbud, University of California, Berkeley, Anatoly S. Libgober, University of Illinois at Chicago, Jose Seade, UNAM, and Xavier Gomez-Mont, CIMAT.

Töplitz Operators and Discrete Quantum Models, Alejandro Uribe, University of Michigan, and Maciej Zworski, University of California, Berkeley.

## Syracuse, New York

Syracuse University

October 2-3, 2010
Saturday - Sunday
Meeting \#1062
Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of Notices: June/July 2010
Program first available on AMS website: August 19, 2010
Program issue of electronic Notices: October
Issue of Abstracts: Volume 31, Issue 4

## Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: June 15, 2010
For abstracts: August 10, 2010
The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/ sectional.htm1.

## Invited Addresses

Alan Frieze, Carnegie-Mellon University, Hamilton cycles in random graphs.

Yan Guo, Brown University, Asymptotic stability in some fluid problems.

William Minicozzi, Johns Hopkins University, Generic singularities of mean curvature flow.

Andrei Zelevinsky, Northeastern University, Title to be announced.

## Special Sessions

Advances in Theory and Applications of Evolution Equations (Code: SS 14A), Tokia Diagana, Howard University, and Gaston Ní Guerekata, Alexander Pankov, Xuming Xie, and Guoping Zhang, Morgan State University.

Analysis, Probability and Mathematical Physics on Fractals (Code: SS 9A), Luke Rogers, University of Connecticut, Robert Strichartz, Cornell University, and Alexander Teplyaev, University of Connecticut.

Analytic Combinatorics (Code: SS 13A), Miklos Bona, University of Florida, and Alex Iosevich, University of Rochester.

Commutative Algebra and Algebraic Geometry (Code: SS 8A), Anthony Geramita, Queen's University, Graham Leuschke and Claudia Miller, Syracuse University, and Michael Stillman, Cornell University.

Difference Equations and Applications (Code: SS 2A), Michael Radin, Rochester Institute of Technology.

Geometric Analysis and Flows (Code: SS 12A), William P. Minicozzi II, Johns Hopkins University, Xiaodong Cao, Cornell University, and Junfang Li, University of Alabama at Birmingham.

Graphs Embedded in Surfaces, and Their Symmetries (Code: SS 4A), Jack E. Graver and Mark E. Watkins, Syracuse University.

Harmonic Analysis (Code: SS 10A), Dmitriy Bilyk, University of South Carolina, and Svitlana Mayboroda, Purdue University.

Lie Algebras and Representation Theory (Code: SS 15A), David Hemmer, State University of New York at Buffalo, and Emilie Wiesner, Ithaca College.

Mathematical Image Processing (Code: SS 5A), Lixin Shen and Yuesheng Xu, Syracuse University.

Nonlinear Analysis and Geometry (Code: SS 1A), Tadeusz Iwaniec, Leonid V. Kovalev, and Jani Onninen, Syracuse University.

Quasiconformal Mappings, Riemann Surfaces, and Teichmüller Spaces (in honor of Clifford J. Earle) (Code: SS 7A), Yunping Jiang, Queens College and The Graduate Center, City University of New York, and Sudeb Mitra, Queens College, City University of New York.

Representations of Algebras (Code: SS 6A), Ed Green, Virginia Polytechnic Institute, Mark Kleiner and Dan Zacharia, Syracuse University, and Andrei Zelevinsky, Northeastern University.

Several Complex Variables (Code: SS 3A), Dan F. Coman and Evgeny A. Poletsky, Syracuse University.

Topology and Combinatorics (Code: SS 11A), Laura Anderson, SUNY Binghamton, and Patricia Hersh, North Carolina State University.

## Accommodations

This meeting was scheduled when there were no local conflicting events. Since then, Syracuse University has scheduled a football game Saturday and Parents' Weekend may also be scheduled during the same dates. There is likely to be a very high demand for hotel rooms and meeting participants are strongly advised to book hotel rooms as soon as possible.

Participants should make their own arrangements directly with the properties listed below. Special rates for the meeting are available at the properties shown below for the period of October 1-3, 2010. Rates quoted do not include sales tax. The AMS is not responsible for rate changes or for the quality of the accommodations. When making reservations participants should state that they are with the American Mathematical Society or AMS group. Hotels have varying cancellation or early checkout penalties; be sure to ask for details when making your reservation. To guarantee a room reservation, we advise making your travel plans early.

The United Inn, 1308 Buckley Road, North Syracuse, NY 13212; (315) 451-1212; http://www. unitedinnsyracuse.com/. The hotel is located approximately six miles from the university. Rates start at US\$49 per night. Rates quoted do not include the sales and occupancy tax of $12 \%$. Be sure to check cancellation and early checkout policies. Amenities include microfridge units in many rooms, phones with data ports, free high-speed wireless Internet, free continental breakfast, and 24 -hour complimentary coffee/tea; Restaurant and
lounge on property and complimentary parking. Deadline for reservations is September 24, 2010.

The Maplewood Inn, 400 7th North Street, Liverpool, NY 13088, (315) 451-1511; http://www. themaplewoodinn.com/. The hotel is located approximately five miles from the university. Rates start at US $\$ 95$ per night. Rates quoted do not include the sales and occupancy tax of $12 \%$. Be sure to check cancellation and early checkout policies. Amenities include complimentary high-speed Internet with wireless capabilities, complimentary airport shuttle, free continental breakfast, and complimentary parking. Deadline for reservations is September 1, 2010.

Holiday Inn Express Hotel Syracuse Airport, 5418 South Bay Road, Syracuse, NY 13212, (315) 454-0999; http://www.hiexpress.com/hote1s/us/en/syrss/ hote1detai1?hpIataNumber=99616580\&cm_ $\mathrm{mmC}=\mathrm{mdpr}$-_-goog1emaps-_-ex-_-syrss. The hotel is located approximately seven miles from the University. Rates start at US\$119 per night. Rates quoted do not include the sales and occupancy tax of $12 \%$. Be sure to check cancellation and early checkout policies. Amenities include complimentary high-speed Internet with wireless capabilities, free hot breakfast, and complimentary parking. Deadline for reservations is September 1, 2010.

Genesee Grande Hotel, 1060 E. Genesee St., Syracuse, NY 13210, (315) 476-4212; http://www. geneseegrande. com/. The hotel is located approximately five blocks from the university. Rates start at US\$129 per night. Rates quoted do not include the sales and occupancy tax of $12 \%$. Be sure to check cancellation and early checkout policies. Amenities include complimentary wireless Internet throughout the hotel, complimentary on-site parking, restaurant on-site, and complimentary airport transportation. Deadline for reservations is September 1, 2010.

Parkview Hotel, 713 E. Genesee St., Syracuse, NY 13210, (315) 701-2600; http: //www. theparkviewhote1. com/. The hotel is located approximately two miles from the university. Rates start at US\$109 per night. Rates quoted do not include the sales and occupancy tax of $12 \%$. Be sure to check cancellation and early checkout policies. Amenities include complimentary high-speed Internet with wireless capabilities, café on-site, and complimentary parking. Deadline for reservations is September 1, 2010.

## Food Service

The Schine Dining Complex in the Schine Student Center is cafeteria style and is open 11:00 a.m.-5:00 p.m. on Saturday and Sunday. Marshal Street and the portion of South Crouse Avenue adjacent to Marshal Street (found on the campus map: http://www.syr.edu/about/pdf/ NorthSouthCampus2009.pdf) have numerous privately owned restaurants with various hours.

## Other Activities

Book Sales: Stop by the on-site AMS bookstore and review the newest titles from the AMS, enjoy up to $25 \%$ off all AMS publications, or take home an AMS t-shirt!

Complimentary coffee will be served courtesy of AMS Membership Services.

AMS Editorial Activity: An acquisitions editor from the AMS book program will be present to speak with prospective authors. If you have a book project that you would like to discuss with the AMS, please stop by the book exhibit.

## Parking

Free parking will be available at lots Q3 and Q4 which are conveniently close to the site of the meeting. Please refer to the campus map for specific location at http://www. syr.edu/about/pdf/NorthSouthCampus2009.pdf.

## Registration and Meeting Information

Registration and AMS Book Exhibit will be held in the Carnegie Library Reading Room. Invited Addresses and all other sessions will be held in Carnegie Library, Physics Building, and the Hall of Languages. Please refer to the campus map for specific location at http://www. syr.edu/about/pdf/NorthSouthCampus2009.pdf. The registration desk will be open Saturday, October 2nd, 7:30 a.m.-4:00 p.m., and Sunday, October 3rd, 8:00 a.m.noon. Fees are US $\$ 50$ for AMS members, US $\$ 70$ for nonmembers; and US\$5 for students, unemployed mathematicians, and emeritus members. Fees are payable on-site via cash, check or credit card.

## Travel

By Plane: Hancock International Airport is only a few miles north of downtown Syracuse and car rental from all major agencies is available there. Taxi service from the airport to campus is available and costs approximately US $\$ 20$ one way.

By bus or train: Amtrak train and Greyhound bus services are located at the Regional Transportation Center, just north of downtown Syracuse. Taxi service is available to the university from all mass transit locations.

By car: When driving, Syracuse University is most easily reached from Interstate 81. New York State Thruway travelers should exit at Interstate 81 (Thruway Exit 36) and proceed south. Both northbound and southbound travelers on Interstate 81 should exit at Adams Street (Exit 18). Continue up the Adams Street hill to the third traffic light. Cross University Avenue and turn left into the University Avenue garage. The entrance to main campus is one block up from the University Avenue garage.

## Car Rental

Avis Rent A Car is the official car rental company for the meeting. Depending on variables such as location, length of rental, and size of vehicle, Avis will offer participants the best available rate which can range from 5\%-25\% discount off regular rates. Participants must use the assigned Meeting Avis Discount Number (J098887) and meet Avis rate requirements to receive the discount. (Rate discounts are available at all corporate and participating licensee locations.) Reservations can be made by calling 800-3311600 or online at www. avis.com.

All car rentals include unlimited free mileage and are available to renters 25 years and older. Renters must also
meet Avis's driver and credit requirements. Return to the same rental location or additional surcharges may apply. Rates do not include any state or local surcharges, tax, optional coverages, or gas refueling charges.

## Weather

Average high temperature in Upstate New York for early October is $65^{\circ} \mathrm{F}$, the average low temperature is $50^{\circ} \mathrm{F}$.

## Information for International Participants

Visa regulations are continually changing for travel to the United States. Visa applications may take from three to four months to process and require a personal interview, as well as specific personal information. International participants should view the important information about traveling to the U.S. found at http: // www7.nationalacademies.org/visas/Traveling_to_ US.htm1 and http://trave1.state.gov/visa/index. htm 1 . If you need a preliminary conference invitation in order to secure a visa, please send your request to d1s@ ams.org.

If you discover you do need a visa, the National Academies website (see above) provides these tips for successful visa applications:

* Visa applicants are expected to provide evidence that they are intending to return to their country of residence. Therefore, applicants should provide proof of "binding" or sufficient ties to their home country or permanent residence abroad. This may include documentation of the following:
- family ties in home country or country of legal permanent residence
- property ownership
- bank accounts
- employment contract or statement from employer stating that the position will continue when the employee returns; * Visa applications are more likely to be successful if done in a visitor's home country than in a third country;
* Applicants should present their entire trip itinerary, including travel to any countries other than the United States, at the time of their visa application;
* Include a letter of invitation from the meeting organizer or the U.S. host, specifying the subject, location and dates of the activity, and how travel and local expenses will be covered;
* If travel plans will depend on early approval of the visa application, specify this at the time of the application;
* Provide proof of professional scientific and/or educational status (students should provide a university transcript).

This list is not to be considered complete. Please visit the websites above for the most up-to-date information.

## Los Angeles, California

## University of California Los Angeles

October 9-10,2010
Saturday - Sunday

## Meeting \#1063

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of Notices: August 2010
Program first available on AMS website: August 26, 2010
Program issue of electronic Notices: October 2010
Issue of Abstracts: Volume 31, Issue 4

## Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: June 22, 2010
For abstracts: August 17, 2010
The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/ sectiona1.htm1.

## Invited Addresses

Greg Kuperberg, University of California Davis, Title to be announced.

Cris Moore, University of New Mexico, Title to be announced.

Stanley Osher, University of California Los Angeles, Title to be announced.

Terence Tao, University of California Los Angeles, The cosmic distance ladder (Einstein Public Lecture in Mathematics).

Melanie Wood, Princeton University, Title to be announced.

## Special Sessions

Algebraic Structures in Knot Theory (Code: SS 8A), Sam Nelson, Claremont McKenna College, and Carmen Caprau, California State University Fresno.

Applications of Nonlinear PDE (Code: SS 5A), Susan J. Friedlander and Igor Kukavica, University of Southern California.

Automorphic Forms and Number Theory (Code: SS 12A), William Duke, University of California Los Angeles, Ozlem Imamoglu, ETH Zurich, and Kimberly Hopkins, University of California Los Angeles.

Combinatorics and Probability on Groups (Code: SS 3A), Jason Fulman and Robert Guralnick, University of Southern California, and Igor Pak, University of California Los Angeles.

Continuous and Discrete Dynamical Systems (Code: SS 11A), Mario Martelli, Claremont Graduate University, and Robert Sacker, University of Southern California.

Extremal and Probabilistic Combinatorics (Code: SS 4A), Benny Sudakov, University of California Los Angeles, and Jacques Verstraete, University of California San Diego.

Free Probability and Subfactors (Code: SS 17A), Edward Effros and Dimitri Shlyakhtenko, University of California Los Angeles, and Dan-Virgil Voiculescu, University of California Berkeley.

Global Geometric Analysis (Code: SS 13A), William Wylie, University of Pennsylvania, Joseph E. Borzellino, California State University San Luis Obispo, and Peter Petersen, University of California Los Angeles.

Harmonic Analysis (Code: SS 9A), Christoph Thiele, University of California Los Angeles, and Ignacio UriarteTuero and Alexander Volberg, Michigan State University.

Homotopy Theory and K-theory (Code: SS 10A), Julie Bergner, University of California Riverside, and Christian Haesemeyer, University of California Los Angeles.

Large Cardinals and the Continuum (Code: SS 2A), Matthew Foreman, University of California Irvine, Alekos Kechris, California Institute for Technology, Itay Neeman, University of California Los Angeles, and Martin Zeman, University of California Irvine.

Mathematical Models of Random Phenomena (Code: SS 7A), Mark Burgin, University of California Los Angeles, and Alan C. Krinik, California State Polytechnic University Pomona.

Mathematics of Criminality (Code: SS 14A), Andrea Bertozzi, Martin Short, and George Mohler, University of California Los Angeles.

Metric and Riemannian Methods in Shape Analysis (Code: SS 16A), Andrea Bertozzi and Mario Micheli, University of California Los Angeles.

Nonlinear Phenomena-Applications of PDEs to Fluid Flows (Code: SS 15A), Andrea Bertozzi, Nebojsa Murisic, and David Uminsky, University of California Los Angeles.

Recent Trends in Probability and Related Fields (Code: SS 6A), Marek Biskup, University of California Los Angeles, Yuval Peres, Microsoft Research, and Sebastien Roch, University of California Los Angeles.

Rigidity in von Neumann Algebras and Ergodic Theory (Code: SS 18A), Adrian Ioana, University of California Los Angeles, Narutaka Ozawa, Tokyo University, and Sorin Popa and Yehudah Shalom, University of California Los Angeles.

Topology and Symplectic Geometry (Code: SS 1A), Robert Brown and Ciprian Manolescu, University of California Los Angeles, and Stefano Vidussi, University of California Riverside.

## Notre Dame, Indiana

Notre Dame University

November 5-7, 2010
Friday - Sunday

## Meeting \#1064

Central Section
Associate secretary: Georgia Benkart
Announcement issue of Notices: September 2010
Program first available on AMS website: September 23, 2010
Program issue of electronic Notices: November 2010
Issue of Abstracts: Volume 31, Issue 4

## Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: July 27, 2010
For abstracts: September 14, 2010
The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/ sectional. htm1.

## Invited Addresses

Laura DeMarco, University of Illinois at Chicago, Title to be announced.

Jordan Ellenberg, University of Wisconsin, Title to be announced.

David Fisher, Indiana University, Title to be announced.
Jared Wunsch, Northwestern University, Title to be announced.

## Special Sessions

Algebraic Group Actions on Affine Varieties (Code: SS 25A), Harm Derksen, University of Michigan, and Gene Freudenburg, University of Western Michigan.

Algebraic and Topological Combinatorics (Code: SS 9A), John Shareshian, Washington University, and Bridget Tenner, DePaul University.

Applications of Stochastic Processes in Cell Biology (Code: SS 20A), Peter Thomas, Case Western University.

Arithmetic, Groups and Geometry (Code: SS 23A), Jordan Ellenberg, University of Wisconsin, and Michael Larsen, Indiana University.

Commutative Algebra and Its Interactions with Algebraic Geometry (Code: SS 2A), Claudia Polini, University of Notre Dame, Alberto Corso, University of Kentucky, and Bernd Ulrich, Purdue University.

Complex Analysis and Dynamical Systems (Code: SS 15A), Laura DeMarco, University of Illinois at Chicago, and Jeffrey Diller, University of Notre Dame.

Computability and Its Applications (Code: SS 11A), Peter Cholak, Peter Gerdes, and Karen Lange, University of Notre Dame.

Computation, Analysis, Modeling in PDE and their Applications (Code: SS 17A), Bei Hu and Yongtao Zhang, University of Notre Dame.

Computational Electromagnetics and Acoustics (Code: SS 18A), David Peter Nicholls, University of Illinois at Chicago.

Differential Geometry and its Applications (Code: SS 16A), Jianguo Cao and Brian Smyth, University of Notre Dame.

Geometry and Lie Theory (Code: SS 10A), John Caine and Samuel Evens, University of Notre Dame.

Graphs and Hypergraphs (Code: SS 19A), David Galvin, University of Notre Dame, and Hemanshu Kaul, Illinois Institute of Technology.

Groups, Representations, and Characters (Code: SS 4A), James P. Cossey, University of Akron, and Mark Lewis, Kent State University.

Hilbert Functions in Commutative Algebra and Algebraic Combinatorics (Code: SS 3A), Fabrizio Zanello, Michigan Technological University, Juan Migliore, University of Notre Dame, and Uwe Nagel, University of Kentucky.

Interdisciplinary Session on Deterministic and Stochastic Partial Differential Equations (Code: SS 5A), Nathan GlattHoltz, Indiana University, and Vlad Vicol, University of Southern California.

Mathematical Modeling and Computation with Applications in Biology (Code: SS 22A), Mark Alber and Zhiliang Xu, University of Notre Dame.

Nonlinear Evolution Equations (Code: SS 7A), Alex Himonas and Gerard Misiolek, University of Notre Dame.

Number Theory and Physics (Code: SS 8A), Adrian Clingher, University of Missouri St. Louis, Charles Doran, University of Alberta, Shabnam N. Kadir, Wilhelm Leibniz Universitat, and Rolf Schimmrigk, Indiana University.

Numerical Algebraic Geometry (Code: SS 13A), Daniel J. Bates, Colorado State University, Jonathan D. Hauenstein, Texas A\&M University, Andrew J. Sommese, University of Notre Dame, and Charles W. Wampler, General Motors.

Quasigroups, Loops, and Nonassociative Division Algebras (Code: SS 6A), Clifton E. Ealy, Western Michigan University, Stephen Gagola, University of Arizona, Julia Knight, University of Notre Dame, J. D. Phillips, Northern Michigan University, and Petr Vojtechovsky, University of Denver.

Rigidity (Code: SS 24A), David Fisher, Indiana University, and Ralf Spatzier, University of Michigan.

Singularities in Algebraic Geometry (Code: SS 1A), Nero Budur, University of Notre Dame, and Lawrence Ein, University of Illinois at Chicago.

The Geometry of Submanifolds (Code: SS 21A), Yun Myung Oh, Andrews University, Mihaela Vajiac, Chapman University, and Ivko Dimitric, Pennsylvania State University.

Topology, Geometry and Physics (Code: SS 14A), Ralph Kaufmann, Purdue University, and Stephan Stolz, University of Notre Dame.

Undergraduate Mathematics Education: A Vision for the 21 st Century (Code: SS 12A), Steven Broad, St. Mary's College, Nahid Erfan and Alex Himonas, University of Notre

Dame, and Morteza Shafii-Mousavi, Indiana University South Bend.

## Richmond, Virginia

## University of Richmond

November 6-7, 2010
Saturday - Sunday

## Meeting \#1065

Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of Notices: September 2010
Program first available on AMS website: September 23, 2010
Program issue of electronic Notices: November 2010
Issue of Abstracts: Volume 31, Issue 4

## Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: July 27, 2010
For abstracts: September 14, 2010
The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/ sectional.htm1.

## Invited Addresses

Matthew H. Baker, Georgia Institute of Technology, Preperiodic points and unlikely intersections.

Michael J. Field, University of Houston, Title to be announced.

Sharon R. Lubkin, North Carolina State University, Model perspectives on self-organizing tissues.

Stefan Richter, University of Tennessee, Knoxville, Title to be announced.

## Special Sessions

Applications of Non-Archimedean Geometry (Code: SS 6A), Matthew H. Baker, Georgia Institute of Technology.

Codes and Designs (Code: SS 13A), James A. Davis, University of Richmond, and Qing Xiang, University of Delaware.

Computational and Applied Mathematics (Code: SS 11A), Ludwig Kohaupt, Beuth University, and Mohammad Siddique, Fayetteville State University.

Convexity and Combinatorics (Code: SS 9A), Valeriu Soltan and James F. Lawrence, George Mason University.

Differential Equations and Applications to Physics and Biology (Code: SS 8A), Junping Shi, College of William and Mary, and Zhifu Xie, Virginia State University.

Geometry of Banach Spaces and Connections with Other Areas (Code: SS 12A), Frank Sanacory, College at Old Westbury, and Kevin Beanland, Virginia Commonwealth University.

Kac-Moody Algebras, Vertex (Operator) Algebras, and Applications (Code: SS 7A), William J. Cook, Appalachian

State University, and Kailash C. Misra, North Carolina State University.

Mathematical Models in Biology and Medicine (Code: SS 10A), Lester Caudill, University of Richmond.

Mathematics and the Arts (Code: SS 5A), Michael J. Field, University of Houston, Gary R. Greenfield, University of Richmond, and Reza Sarhangi, Towson University.

Minimum Rank Problems (Code: SS 3A), Lon H. Mitchell, Virginia Commonwealth University, and Sivaram K. Narayan, Central Michigan University.

Numerical Methods for Solving Partial Differential Equations in Practice (Code: SS 15A), Kathryn Trapp, University of Richmond, and Katie Gurski, Howard University.

Operator Theory (Code: SS 2A), Stefan Richter, University of Tennessee, and William T. Ross, University of Richmond.

Statistical Properties of Dynamical Systems (Code: SS 4A), Michael J. Field and Matthew J. Nicol, University of Houston.

Topics in Graph Theory (Code: SS 1A), Daniel W. Cranston, Virginia Commonwealth University, and Gexin Yu, College of William \& Mary.

The History of Mathematics: A Transnational Discourse (Code: SS 14A), Della Fenster, University of Richmond, and Frédéric Brechenmacher, University of Lille-North of France-Université d'Artois.

## Pucon, Chile

## December 15-18, 2010

Wednesday - Saturday

## Meeting \#1 066

First Joint International Meeting between the AMS and the Sociedad de Matematica de Chile.
Associate secretary: Steven H. Weintraub
Announcement issue of Notices: June 2010
Program first available on AMS website: Not applicable
Program issue of electronic Notices: Not applicable
Issue of Abstracts: Not applicable

## Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced
The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/
internmtgs.htm7.

## AMS Invited Addresses

Ricardo Baeza, Universidad de Talca, Chile, $p$-cohomological dimension of fields of characteristic $p$.

Igor Dolgachev, University of Michigan, Cremona groups and their subgroups.

Andres Navas, Universidad de Santiago de Chile, Probabilistic, dynamical and topological aspects of orderable groups.

Rodolfo Rodriguez, Universidad de Concepcion, Numerical solution of time-domain electromagnetic problems arising from some metallurgical processes.

Gunther Uhlmann, University of Washington, Insideout: Inverse problems.
S. R. Srinivasa Varadhan, New York University, Large deviations.

## AMS Special Sessions

Algebra and Model Theory, Thomas Scanlon, University of California, Berkeley, and Xavier Vidaux, Universidad de Concepcion.

Algebraic Modeling of Knotted Objects, Vaughan F. R. Jones, University of California, Berkeley, Jesús Juyumaya, Universidad de Valparaíso, Louis H. Kauffman, University of Illinois at Chicago, and Sofia Lambropoulou, National Technical University of Athens.

Applications of Differential and Difference Equations in Biology and Ecology, J. Robert Buchanan, Millersville University, Fernando Córdova, Universidad Católica de Maule, and Jorge Velasco Hernandez, Institute Nacional de Petroleo.

Arithmetic of Quadratic Forms and Integral Lattices, Maria Ines Icaza, Universidad de Talca, Chile, Wai Kiu Chan, Wesleyan University, and Ricardo Baeza, Universidad de Talca, Chile.

Automorphic Forms and Dirichlet Series, Yves Martin, Universidad de Chile, Chile, and Solomon Friedberg, Boston College.

Complex Algebraic Geometry, Giancarlo Urzua and Eduardo Cattani, University of Massachusetts.

Foliations and Dynamics, Andrés Navas, Universidad de Santiago de Chile, and Steve Hurder, University of Illinois at Chicago.

Group Actions: Probability and Dynamics, Andres Navas, Universidad de Santiago de Chile, and Rostislav Grigorchuk, University of Texas.

Inverse Problems and PDE Control, Matias Courdurier, Pontificia Universidad Católica de Chile, Axel Osses, Universidad de Chile, and Gunther Uhlmann, University of Washington.

Non-Associative Algebras, Alicia Labra, Universidad de Chile, and Kevin McCrimmon, University of Virginia.

Probability and Mathematical Physics, Hui-Hsiung Kuo, Louisiana State University, and Rolando Rebolledo, Pontificia Universidad Catolica de Chile.

Representation Theory, Jorge Soto Andrade, Universidad de Chile, and Philip Kutzko, University of Iowa.

Spectral Theory and Mathematical Physics, Bruno Nachtergaele, University of California, Davis, and Rafael Tiedra, Pontificia Universidad Católica de Chile.

## New Orleans, Louisiana

## New Orleans Marriott and Sheraton New Orleans Hotel <br> January 6-9, 2011 <br> Thursday - Sunday <br> Meeting \#1067 <br> Note: Change in Dates!!

Joint Mathematics Meetings, including the 117th Annual Meeting of the AMS, 94th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Steven H. Weintraub
Announcement issue of Notices: October 2010
Program first available on AMS website: November 1, 2010
Program issue of electronic Notices: January 2011
Issue of Abstracts: Volume 32, Issue 1

## Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: July 28, 2010
For abstracts: September 22, 2010
The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/ national.htm1.

## Joint Invited Addresses

Kannan Soundararajan, Stanford University, To be announced (AMS-MAA Invited Address).

Chuu-Lian Terng, University of California Irvine, Title to be announced (AMS-MAA Invited Address).

## AMS Invited Addresses

Denis Auroux, University of California Berkeley, Title to be announced.

Andrea L. Bertozzi, University of California Los Angeles, Title to be announced.

Alexander Lubotzky, Hebrew University of Jerusalem, Expander graphs in pure and applied mathematics (AMS Colloquium Lectures).

George Papanicolaou, Stanford University, Title to be announced (AMS Josiah Willard Gibbs Lecture).

Scott Sheffield, Massachusetts Institute of Technology, Title to be announced.

Tatiana Toro, University of Washington, Title to be announced.

Akshay Venkatesh, Stanford University, Title to be announced.

## Call for MAA Contributed Papers

The MAA Committee on Contributed Paper Sessions solicits contributed papers pertinent to the sessions and their organizers listed below. Contributed Paper Session presentations are limited to fifteen minutes, except in the general session, where they are limited to ten minutes Each session room is equipped with a computer projector, an overhead projector, and a screen. Please note that the dates and times scheduled for these sessions remain tentative.

Alternative Approaches to Traditional Introductory Statistics Courses, Brian T. Gill, Seattle Pacific University; Nancy J. Boynton, SUNY Fredonia; and Michael A. Posner, Villanova University; Sunday afternoon. Do you teach a nontraditional selection of topics or use different methods in your introductory statistics course? Do you teach topics in a different order from the standard descriptives, probability, basic inference? What have you let go of from the traditional course? Tell us about your course-especially what makes it successful. We encourage contributions from specialized statistics courses such as those for business majors, biostats, etc. Also of interest are different methods of delivery, such as hybrid or online courses.

Successful teaching in statistics and the GAISE guidelines promote conceptual understanding, and encourage active participation. We invite submissions that provide details about how different approaches have proven successful in teaching introductory statistics courses. They may be organized to attract the attention and interest of students or to serve students with particular needs.

This session is sponsored by the SIGMAA on Statistics Education. Presenters will be considered for the Dex Whittinghill Award for Best Contributed Paper.

Cool Calculus: Lessons Learned Through Innovative and Effective Supplemental Projects, Activities, and Strategies for Teaching Calculus, Jessica M. Deshler, West Virginia University; Friday morning.

It is estimated that hundreds of thousands of students take (and fail) calculus every year at colleges and universities across the United States. Calculus frequently ranks near the top of "killer courses" lists created by students and administrators alike. Instructors are continuously searching for new and innovative ways to increase student understanding of the material.

This session invites papers that focus on the use of supplemental activities, projects, and innovative methods of instruction in the undergraduate calculus sequence. Proposals may include descriptions of development or implementation of calculus activities (including technology based applets, group work activities, etc.), methods of instruction (including successful implementation of group, supplemental instruction sessions, etc.), and evidence of impact on student learning, success, or attitudes. Proposals for both successful supplemental strategies as well as lessons learned from less successful attempts are invited. Presentations must be scholarly in nature; evidence of pedagogical effectiveness (or noneffectiveness) should be more than anecdotal, supported by quantitative or qualitative research.

Cryptology for Undergraduates, Robert Edward Lewand, Goucher College, and Chris Christensen, Northern Kentucky University; Thursday afternoon. In increasing numbers cryptology courses are being developed to address the interest and to serve the needs of undergraduate mathematics and computer science majors. Typical courses may include modules dealing with counting problems, probability, number theory, and matrices. Cryptology is also appearing as a topic in mathematics courses for nonmajors, as it has been recognized as a hook to interest these students in mathematics. As experience in offering these courses grows, innovative and interesting approaches to this topic are being developed. This session solicits presentations that address topics appropriate for undergraduate cryptology courses for mathematics or computer science majors, or presentations of cryptological topics or projects that could interest and motivate nonmathematics majors.

Developmental Mathematics Education: Helping UnderPrepared Students Transition to College-Level Mathematics, Kimberly J. Presser and J. Winston Crawley, Shippensburg University; Saturday afternoon. Many students are arriving at college today under-prepared for college-level mathematics courses. In order to help these students to be successful, we need to undertake new strategies for support services, courses offered, and perhaps even in our programs themselves. This session invites papers on all aspects of developmental mathematics education. In particular, what classroom practices are effective with such students and how does research in student learning inform these practices? For students interested in mathintensive majors such as the sciences, how can we best prepare these students for several subsequent mathematics courses? How can we best coordinate support services with the courses offered in our mathematics departments?

Effective Teaching of Upper Level Mathematics to Secondary Education Mathematics Majors, Joyati Debnath, Winona State University; Sunday morning. Secondary mathematics education prepares students for careers as secondary school mathematics teachers and provides opportunities to learn the processes of teaching and learning mathematics by providing both a strong foundation in mathematics content and hands-on experience in the classroom, and stressing teaching philosophies and standards and principles of the National Council of the Teachers of Mathematics (NCTM). Future teachers need to fully understand the mathematics they present. Teaching methods include utilizing different assessment techniques, appropriately using technology to enhance students' mathematical thinking, and featuring the cultural, historical, and scientific evolution of mathematics.

Fostering, Supporting and Propagating Math Circles for Students and Teachers, Tatiana Shubin, San Jose State University; Elgin H. Johnston, Iowa State University; and James Tanton, St. Mark's Institute of Mathematics; Saturday morning. A math circle is broadly defined as a semi-formal, sustained enrichment experience that brings mathematics professionals in direct contact with precollege students and/or their teachers. Circles foster passion and excitement for deep mathematics.

The SIGMAA for Math Circles for Students and Teachers (SIGMAA MCST) supports MAA members who share an interest in developing, supporting, and running math circles. It works to facilitate vertical integration of elementary, middle, and high school students, their teachers, undergraduate and graduate students, and faculty up through high-level research mathematicians.

SIGMAA MCST invites speakers to present reports on Math Circle activities and their effectiveness in achieving articulated goals. Presentations are expected to be scholarly in nature and serve to offer clear guidance on fostering circle activity. This may be achieved via examination of the structure of the circle, matters related to instigating and supporting the circle, effective topics of activity, and/or presentation of innovative student product, for instance.

This session invites presentations that describe successful course content, teaching methods, and projects or group work that are integrated into students' learning, effectively incorporating basic mathematical insights and leading to increased student enthusiasm for mathematics. This session may showcase curricular initiatives to improve learning and enhance the understanding of mathematical content for future secondary mathematics teachers. Presentations will also iterate the effect of methods on students. The session welcomes a range of mathematics courses, varying from calculus and modern algebra to discrete mathematics and modern analysis. Presenters are encouraged from four-year institutions, liberal arts colleges, and universities of all sizes. Abstracts can be accepted from individuals or teams of mathematicians.

Getting Students Involved in Writing Proofs, Aliza Steurer, Dominican University; Jennifer Franko-Vasquez, University of Scranton; and Rachel Schwell, Central Connecticut State University; Thursday afternoon. When confronted with the difficult task of involving students in the writing of a proof, many professors are unsure about how to proceed, yet they know (perhaps even from their own experience as students) how crucial student input is for learning. Should the students form groups to write the proof? Should one student present it at the board? How does one deal with the variety of learning styles students have? How does one effectively guide students so that they can write proofs on their own in homework assignments?

We seek novel ideas or approaches with evidence of their success in the classroom. Evidence to support the success of the idea or approach might include quantitative or qualitative measures (i.e., student responses, test scores, survey results, etc.). The members of our intended audience range from those new to teaching proof-based courses to those who have tried various methods in such classes.

Harnessing Mobile Communication Devices and Online Communication Tools for Mathematics Education, Michael B. Scott, California State University Monterey Bay, and Jason A. Aubrey, University of Missouri; Thursday morning. The nature of communication has changed substantially in the last twenty years. In particular the proliferation of mobile communication devices (cell phones, smart phones, laptops, etc.) and online communication
tools (Twitter, Facebook, virtual worlds, etc.) has had a profound effect on the way people communicate. Many instructors view this proliferation as a challenge, for example, text messaging in class. This evolution of communication can also present new learning opportunities for our students. This session gives instructors who are using these communication systems in an innovative manner an opportunity to share their experiences using these new systems to enhance student learning and report on their effectiveness.

Mobile communication devices can include cell phones, smart phones, iTouches, networked calculators, or any other personal device having the ability to communicate wirelessly. Online communication tools can include Twitter, Facebook, or other social networking sites, and can also include other sites, such as Second Life, where communication takes place in a nontraditional manner. The focus of the reports should be on how the use of these communication devices/tools improve student learning of mathematics inside or outside the classroom. This session is sponsored by the Committee on Technologies in Mathematics Education (CTiME) and WEB SIGMAA.

Humanistic Mathematics, Gizem Karaali, Pomona College; Mark Huber, Claremont McKenna College, Dagan Karp, Harvey Mudd College; Saturday afternoon. Humanistic mathematics is an approach to mathematics as a human endeavor. The phrase itself is an umbrella term for various threads of inquiry that deal with aesthetic, historical, literary, pedagogical, philosophical, psychological, and sociological aspects of doing, learning, and teaching mathematics. This session will bring together an eclectic collection of scholarly work that focuses on the people of mathematics, whether they be learners, teachers, or practitioners.

Submissions on all humanistic aspects of mathematics are invited. We are especially looking for work that brings together more than one strand of humanistic mathematics and encourage submissions that will stimulate discussion and further inquiry. Appropriate for this session are papers that discuss how a particular philosophical approach to mathematics can impact classroom pedagogy, how aesthetic ideas influenced the history of mathematics, or how one can use fiction in a mathematics classroom, as well as reflections upon large movements like calculus reform within their historical context, or critical discussions of the mathematical profession; other themes are also welcome as long as they fit in with the humanistic focus of the session. Submissions should be aimed at a broad mathematical audience.

This session is sponsored by the Journal of Humanistic Mathematics.

Influences of the Calculus Reform Movement on the Teaching of Mathematics, Steven R. Benson, Lesley University; Marilyn Carlson, Arizona State University; Ellen E. Kirkman, Wake Forest University; and Joe Yanik, Emporia State University; Sunday morning. In this session, speakers will address ways in which various aspects of the "calculus reform movement" have affected their own approach to teaching mathematics. Calculus reform, now over twentyfive years old, has influenced the teaching of undergraduate
mathematics in many ways and at all levels (including both pre- and post-calculus courses)-so much that the old (artificial) lines between "traditional" and "reform" have become blurred. From changing the amount of time spent on conceptual or computational topics to incorporation of appropriate technology to modification of "delivery" to the use of projects, to name but a few possibilities, various aspects of the vision of the reform movement have inspired teachers to make changes in the ways they approach their courses. Discussions of both pedagogy/ teaching strategies and course/curriculum design will be considered and foci may range from the individual course/ instructor to the department/institution level.

Presentations need not be on published research, but scholarly talks that provide evidence and reflection on that evidence are preferred. Most importantly this session is not a forum for pro- or anti-reform rhetoric, rather, it is an opportunity to share ideas and results with interested colleagues. Sponsored by the Committee on the Teaching of Undergraduate Mathematics.

Innovations in Service-Learning at All Levels, KarlDieter S. Crisman, Gordon College; Rachelle Ankney, North Park University; and Robert V. Perlis, Louisiana State University; Thursday afternoon. Service-learning is a growing concern on college campuses, but the mathematical sciences are sometimes seen as more challenging to bring into this valuable development (where the hyphen emphasizes the importance of connecting learning with service). Hence, this session will feature innovative and successful service-learning ideas in the mathematical sciences, at all levels, and in all topics. This is a timely discussion for many mathematics departments, as some institutions now mandate a service component as a graduation requirement, and others have valuable partnerships with organizations such as Campus Compact or with local communities.

Talks concerning the scholarship of learning involving service are welcome, and should address how the service connects to learning the mathematical content of the course. We encourage anyone with documented success using service-learning in math courses to submit abstracts, particularly those involving non-major courses or non"applied" major courses.

Innovative and Effective Ways to Teach Linear Algebra, David M. Strong, Pepperdine University; Gilbert Strang, Massachusetts Institute of Technology; and David C. Lay, University of Maryland; Friday morning. Linear algebra is one of the most interesting and useful areas of mathematics, because of its beautiful and multifaceted theory, as well as the enormous importance it plays in understanding and solving many real world problems. Consequently many valuable and creative ways to teach its rich theory and its many applications are continually being developed and refined. This session will serve as a forum in which to share and discuss new or improved teaching ideas and approaches.

These innovative and effective ways to teach linear algebra include, but are not necessarily limited to, handson, in-class demos; effective use of technology, such as Matlab, Maple, Mathematica, Java applets or Flash;
interesting and enlightening connections between ideas that arise in linear algebra and ideas in other mathematical branches; interesting and compelling examples and problems involving particular ideas being taught; comparing and contrasting visual (geometric) and more abstract (algebraic) explanations of specific ideas; and other novel and useful approaches or pedagogical tools.

Journals and Portfolios: Tools in Learning Mathematics?, Sarah L. Mabrouk, Framingham State College; Friday afternoon. Journals and portfolios are used in courses from college algebra to the calculus sequence and upper division courses as well as capstone courses and senior seminars. In order for these to be effective tools in learning mathematics, journals should allow for reflection on, response to, and synthesis of course material and portfolios should provide insight into the student's thinking, understanding, and problem solving and/or proof skills, demonstrating her/his progress in the study of mathematics. This session invites presentations discussing the effective use of journals/portfolios as tools in reflection on learning mathematics concepts and methods as well as their use in the development and improvement of problem solving and/or proof-writing skills. Of particular interest are the use of journals and portfolios in courses for mathematics majors and preservice teachers as well as in courses in which proof skills are developed and expanded such as geometry, number theory, abstract algebra, real and/or complex analysis, and capstone courses and seminars. Presentations should address prompts used for journaling, the outline for materials and commentary to be included in portfolios, the assessment of journals/portfolios, and the effectiveness of the use of journals/portfolios in learning mathematics, which should be demonstrated by more than anecdotal means.

The Mathematical Foundations for the Quantitative Disciplines, Yajun Yang, Farmingdale State College of SUNY; Laurette Foster, Prairie View A\&M University; Ray E. Collings, Georgia Perimeter College; and K. L. D. Gunawardena, University of Wisconsin-Oshkosh; Sunday afternoon.

We seek to address all of the college level courses below calculus, with particular emphasis on offerings in college algebra and precalculus that focus on conceptual understanding, the use of real-world data, and mathematical modeling to support the needs of the partner disciplines. One of several interrelated national initiatives currently being conducted by the MAA committee on Curriculum Renewal Across the First Two Years (CRAFTY), with the assistance of several other MAA committees, is to change the focus in the courses below calculus to better serve the majority of students taking these courses. The goal of this initiative, as expressed in CRAFTY's College Algebra Guidelines, is to encourage courses that place much greater emphasis on conceptual understanding and realistic applications via mathematical modeling compared to traditional courses where the primary emphasis is on developing algebraic skills that may be needed for mainstream calculus. The second initiative is the next round of the Curriculum Foundations project in which leading educators from various quantitative disciplines are brought together to discuss and develop recommendations to the
mathematics community on the current mathematical needs of their students.

For this session, we specifically seek presentations that present new visions for such courses; discuss experiences teaching such courses, particularly collaborations with other disciplines; discuss implementation issues (such as faculty training, placement testing, introduction of alternative tracks for different groups of students, transferability issues, etc.) related to offering such courses; present results of studies on student performance and tracking data in both traditional and new versions of these courses and in follow-up courses; discuss what the other disciplines and the workplace need from courses at this level; and discuss connections to the changing high school curricula and implications for teacher education. Cosponsored by CRAFTY and the MAA Committee on Two-Year Colleges.

Mathematics Experiences in Business, Industry and Government, Carla D. Martin, James Madison University; Philip E. Gustafson, Mesa State College; and Michael Monticino, University of North Texas; Saturday morning. The MAA Business, Industry and Government Special Interest Group (BIG SIGMAA) provides resources and a forum for mathematicians working in Business, Industry and Government (BIG) to help advance the mathematics profession by making connections, building partnerships, and sharing ideas. BIG SIGMAA consists of mathematicians in BIG as well as faculty and students in academia who are working on BIG problems.

Mathematicians, including those in academia, with BIG experience are invited to present papers or discuss projects involving the application of mathematics to BIG problems. The goal of this session sponsored by BIG SIGMAA is to provide a venue for mathematicians with experience in business, industry, and government to share projects and mathematical ideas in this regard. Anyone interested in learning more about BIG practitioners, projects, and issues, will find this session of interest.

The Mathematics of Games and Puzzles, Laura Taalman, James Madison University, and Robin L. Blankenship, Morehead State University; Thursday afternoon. Games and puzzles such as Sudoku, Nim, origami, SET, Mancala, Slitherlink, magic squares, flexagons, Chomp, Rubik's cubes, Farkle, knights tours, and many more provide a fertile ground for open and accessible problems for both faculty and undergraduate research projects. Investigations of such games and puzzles span a surprisingly wide range of mathematical topics, including number theory, probability, integer programming, game theory, graph theory, algorithms, combinatorics, algebra, and even topology.

This session is for talks about faculty research, upperlevel and lower-level classroom activities, and possible undergraduate research projects that relate to the mathematical structure of games and puzzles. We invite papers for any type of game or puzzle in any field of mathematics. Talks should be entertaining and accessible to an audience of both faculty and students but must also contain significant mathematical content. Speakers are encouraged to bring handouts of puzzles or games, involve the audience
in a game or puzzle, and/or discuss open problems in their topic, as appropriate.

The Mathematics of Sustainability, Elton Graves, Rose-Hulman Institute of Technology, and Peter T. Otto, Willamette University; Friday afternoon. Topics such as sustainable harvesting of food and natural resources, development of sustainable energy sources, conservation and recycling, greenhouse gas emissions, global warming, new types of "green" buildings, etc. are ideas which have now become global issues. This session is intended to encourage papers from colleagues who have used sustainability models or discussion in their undergraduate mathematics classroom.

Papers for this session should describe how mathematical sustainability models/discussions have been used in the undergraduate mathematics classroom. Models/ discussion may include but are not limited to global warming; green house gas models; sustainable use of resources including food, water, minerals; power generation; conservation, and sustainable structures.

Faculty members who have participated in interdisciplinary programs, classes, projects, or assignments are encouraged to present. Papers from all undergraduate mathematical courses or interdisciplinary courses with a mathematics component are welcome and encouraged.

Modeling in the ODE Driver's Seat, Kurt Bryan, RoseHulman Institute of Technology, and Brian J.Winkel, United States Military Academy; Friday morning. Like many topics in applied mathematics, a course in differential equations is at its best when driven up front by compelling applications. For a typical introductory course in ordinary differential equations this means physical situations and associated modeling that lead naturally to first and second order equations, systems of differential equations, numerical methods, and matrix algebra. When applications and modeling drive the subsequent analysis and solution techniques, students have a better sense of not only where they are going but why they are going there. We believe this, rather than an "analysis first, applications later", approach keeps students more engaged in the material.

To this end we seek presenters who will share with the audience lessons and activities that instructors can use to introduce students to elementary ODE topics through a modeling-first approach. We are particularly interested in projects that encourage students to develop mathematical models from verbal descriptions or data, or involve handson "laboratory experiments" suitable for classroom use, computational experiments, or other data collection and analysis. The activities should drive students to master new analytical techniques, make predictions, and assess the reasonableness of their predictions. We especially welcome fresh multidisciplinary projects involving modern applications.

New and Continuing Connections between Math and the Arts, Douglas E. Norton, Villanova University; Saturday morning. Connections between math and the arts are as old as the earliest visual representations of basic mathematical concepts and as new as computer representations of deep mathematical results-and are certainly not
restricted to the visual arts! They are manifest in cultures east, west, north, and south. This session is open for exploration of these connections in any of their varied forms but will include a particular thread of math-art connections in cultural contexts. This could mean, for example, from an ethnomathematics perspective, in a particular historical context, or in a particular contemporary subculture such as middle school English students or practicing topologists! The session is sponsored by SIGMAA-ARTS.

Philosophy of Mathematics in Teaching and Learning, Dan Sloughter, Furman University, and Martin E. Flashman, Humboldt State University; Saturday afternoon. Mathematicians usually ignore philosophical issues while teaching yet we frequently make ontological and epistemological commitments in much of what we do in the classroom. Every time we use a proof by induction or contradiction, discuss the existence or non-existence of a mathematical object, or refer to the discovery or creation of some piece of mathematics, we are endorsing some philosophical view of our subject.

This session will focus on the recognition and use of the philosophy of mathematics in the teaching and learning of mathematics. Can we understand mathematics without a philosophical context? Papers are encouraged to address questions such as: What philosophical issues (such as the nature of mathematical objects, the method of mathematical proof, and the nature of mathematical knowledge) belong in a mathematics course? How? In which course(s)? In what ways does the consideration of philosophical issues enhance a mathematics, or mathematics related, course? What does a learner gain by contact with issues from the philosophy of mathematics?

Research on the Teaching and Learning of Undergraduate Mathematics, Sean Larsen, Portland State University; Natasha M. Speer, University of Maine; and Stacy Brown, Pitzer College; Friday morning. This session sponsored by the SIGMAA on RUME (Special Interest Group of the MAA on Research in Undergraduate Mathematics Education) presents research that addresses issues concerning the teaching and learning of undergraduate mathematics by employing established quantitative and qualitative methodologies. The presented research builds on the existing literature in mathematics education and is informed by cognitive and socio-cultural theories of learning.

Proposals for reports of research on undergraduate mathematics education are invited. The research should build on the existing research literature and use established methodologies to investigate important issues in undergraduate mathematics teaching and learning.

The goals of the session are to share high-quality research on undergraduate mathematics education with the broader mathematics community. The session will feature research in a number of mathematical areas including linear algebra, abstract algebra, and mathematical proof.

The Scholarship of Teaching and Learning in Collegiate Mathematics, Jacqueline M. Dewar, Loyola Marymount University; Thomas F. Banchoff, Brown University; Pam Crawford, Jacksonville University; and Edwin P. Herman and Nathan Wodarz, University of Wisconsin-Stevens Point; Thursday morning. The scholarship of teaching
and learning is a growing field in which faculty bring disciplinary knowledge to bear on questions of teaching and learning and use student-based evidence to support their conclusions. Work in this area emphasizes pedagogical techniques and questions. The scope of the research can range from small, relatively informal investigations about teaching innovations in the classroom to larger or more formal investigations of student learning.

Reports that address issues concerning the teaching and learning of postsecondary mathematics are invited. Appropriate for this session are reports of classroombased investigations of teaching methods, student learning difficulties, or curricular assessment. Papers must discuss more than anecdotal evidence. For example, papers might reference the following types of evidence: student work, pre/post tests, interviews, surveys, think-alouds, etc.

The goals of this session are to feature scholarly work focused on teaching of postsecondary mathematics, provide a venue for mathematicians to make public their scholarly work on teaching, and highlight evidence-based arguments for the value of teaching innovations.

Trends in Undergraduate Mathematical Biology Education, Timothy D. Comar, Benedictine University; Raina Robeva, Sweet Briar College; and Mike Martin, Johnson County Community College; Sunday morning. This session will highlight successful implementations of biomathematics courses and content in undergraduate curriculum, entire biomathematics curricula, efforts to recruit students into biomathematics courses, involvement of undergraduate students in biomathematics research, preparation for graduate work in biomathematics and computational biology or for medical careers, and assessment of how these courses and activities impact the students.

Several reports emphasize that aspects of biological research are becoming more quantitative and that life science students should be introduced to a greater array of mathematical and computational techniques and to the integration of mathematics and biological content at the undergraduate level. Most recently the 2009 document, "Scientific Foundations for Future Physicians" copublished by the Association of American Medical Colleges and the Howard Hughes Medical Institute, recommends that future physicians need increased quantitative training.

Topics may include scholarly work addressing the issues related to the design of effective biomathematics courses and curricula, how best to gear content toward pre-med students, integration of biology into existing mathematics courses, collaborations between mathematicians and biologists that have led to new courses, course modules, undergraduate research projects, effective use of appropriate technology in biomathematics courses, and assessment issues. Sponsored by BIO SIGMAA.

Treasures from the Past: Using Primary Sources in the Classroom, Amy E. Shell-Gellasch, Beloit College; Daniel E. Otero, Xavier University; and David J. Pengelley, New Mexico State University; Friday afternoon. The use of primary sources in teaching is a growing trend in collegiate and even secondary education. It consists in presenting the writings of researchers from the historical past, either
in their original language or in translation, directly to students. In reading, deciphering and analyzing the original documents, students gain a rich understanding of not only the mathematics, but of its development and of how mathematics is practiced, both currently and historically.

This session promotes the use of original sources in the mathematical sciences in teaching. Submissions may address how specific mathematical texts of historical significance, or even secondary sources in the history of mathematics, have been used effectively in the classroom. Speakers may also present general ideas on how to implement the use of original sources in the teaching of mathematics.

Using Program Assessment to Improve Student Learning, Bonnie Gold, Monmouth University; William A. Marion, Valparaiso University; and Jay A. Malmstrom, Oklahoma City Community College; Sunday afternoon. This session invites papers from faculty whose departments not only have an assessment plan in place but also have completed at least one assessment cycle, including using the results to improve student learning. We ask you to address some of the following questions. How has your assessment process led to improved learning by your students? What problems did you find, either with the assessment plan itself or with your program? What changes did you make in your program? Have these changes led to the improvement you had hoped for? How have you documented that improvement? Where are you going next? Sponsored by the MAA Committee on Assessment.

Wavelets in Undergraduate Education, Caroline Haddad, SUNY Geneseo; Catherine A. Beneteau, University of South Florida; David K. Ruch, Metropolitan State College of Denver; Patrick J. Van Fleet, University of St. Thomas; Thursday morning. Wavelets are functions that satisfy certain mathematical properties and are used to represent data or other functions. They work extremely well in analyzing data with finite domains having different scales or resolutions. Interesting applications include digital image processing, FBI fingerprint compression, signal processing of audio files, de-noising noisy data, earthquake prediction, and solving partial differential equations. Wavelets have typically been studied at the graduate level but are making their way into the undergraduate curriculum. We are interested in presentations that effectively incorporate wavelets in an innovative way at the undergraduate level. This may include an undergraduate course in wavelets; a topic on wavelets in some other course using, but not limited to, hands-on demonstrations, projects, labs that utilize technology such as Matlab, Mathematica, Maple, Java applets, etc.; or research opportunities for undergraduates.

General Contributed Paper Session, Kristen Meyer, Wisconsin Lutheran College, and Thomas R. Hagedorn, The College of New Jersey; Thursday, Friday, Saturday, and Sunday mornings and afternoons. Papers may be presented on any mathematical topics. Papers that fit into one of the other sessions should be sent to that session, not to the general session.

## Submission Procedures for MAA Contributed Paper Abstracts

Abstracts must be submitted electronically at http:// www.ams.org/cgi-bin/abstracts/abstract.p1. Simply select the New Orleans meeting, fill in the number of authors, and then follow the step-by-step instructions. The deadline for abstracts is Wednesday, September 22, 2010.

Participants may submit at most two abstracts for MAA contributed paper sessions at any one meeting. If your paper cannot be accommodated in the session in which it is submitted, it will automatically be considered for the general session. Speakers in the general session are limited to one talk.

The organizer(s) of your session will automatically receive a copy of the abstract, so it is not necessary for you to send it directly to the organizer. All accepted abstracts are published in a book that is available to registered participants at the meeting. Questions concerning the submission of abstracts should be addressed to abscoord@ams.org.

## Statesboro, Georgia

## Georgia Southern University

March 12-13,2011
Saturday - Sunday
Meeting \#1068
Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of Notices: January 2011
Program first available on AMS website: January 27, 2011
Program issue of electronic Notices: March 2011
Issue of Abstracts: To be announced

## Deadlines

For organizers: August 12, 2010
For consideration of contributed papers in Special Sessions: November 23, 2010
For abstracts: January 20, 2011
The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/ sectional.htm1.

## Invited Addresses

Jason A. Behrstock, Lehman College (CUNY), Title to be announced.

Gordana Matic, University of Georgia, Title to be announced.

Jeremy T. Tyson, University of Illinois at UrbanaChampaign, Title to be announced.

Brett D. Wick, Georgia Institute of Technology, Title to be announced.

## Special Sessions

Advances in Biomedical Mathematics (Code: SS 4A), Yangbo Ye, University of Iowa, and Jiehua Zhu, Georgia Southern University.

Applied Combinatorics (Code: SS 2A), Hua Wang, Georgia Southern University, Miklos Bona, University of Florida, and Laszlo Szekely, University of South Carolina.

Fractals and Tilings (Code: SS 3A), Ka-Sing Lau, The Chinese University of Hong Kong, Sze-Man Ngai, Georgia Southern University, and Yang Wang, Michigan State University.

Harmonic Analysis and Partial Differential Equations (Code: SS 1A), Paul A. Hagelstein, Baylor University, Alexander Stokolos, Georgia Southern University, Xiaoyi Zhang, IAS Princeton and University of Iowa, and Shijun Zheng, Georgia Southern University.

## Iowa City, Iowa

## University of Iowa

March 18-20, 2011
Friday - Sunday

## Meeting \#1069

Central Section
Associate secretary: Georgia Benkart
Announcement issue of Notices: January 2011
Program first available on AMS website: February 5, 2011
Program issue of electronic Notices: March 2011
Issue of Abstracts: To be announced

## Deadlines

For organizers: August 18, 2010
For consideration of contributed papers in Special Sessions: November 30, 2010
For abstracts: January 25, 2011

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/ sectiona1.htm7.

## Special Sessions

Global and P-adic Representation Theory (Code: SS 3A), Muthukrishnan Krishnamurthy, Philip Kutzco, and Yangbo Ye, University of Iowa.

Modelling, Analysis and Simulation in Contact Mechanics (Code: SS 1A), Weimin Han, University of Iowa, and Mircea Sofonea, University of Perpignan.

Representations of Algebras (Code: SS 2A), Frauke Bleher, University of Iowa, and Calin Chindris, University of Missouri.

## Worcester, Massachusetts

## College of the Holy Cross

April 9-10, 2011
Saturday - Sunday

## Meeting \#1070

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of Notices: February 2011
Program first available on AMS website: March 3, 2011
Program issue of electronic Notices: April 2011
Issue of Abstracts: To be announced

## Deadlines

For organizers: September 9, 2010
For consideration of contributed papers in Special Sessions: December 21, 2010
For abstracts: February 15, 2011

## Las Vegas, Nevada

University of Nevada
April 30-May 1, 2011
Saturday - Sunday

## Meeting \#1071

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of Notices: February 2021
Program first available on AMS website: March 17, 2011
Program issue of electronic Notices: April 2011
Issue of Abstracts: To be announced

## Deadlines

For organizers: September 30, 2010
For consideration of contributed papers in Special Sessions: January 1, 2011
For abstracts: March 8, 2011
The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/ sectiona1.htm7.

## Invited Addresses

Elizabeth Allman, University of Alaska, Title to be announced.

Danny Calegari, University of California Santa Barbara, Title to be announced.

Hector Ceniceros, Stanford University, Title to be announced.

Tai-Ping Liu, Stanford University, Title to be announced.

## Special Sessions

Advances in Modeling, Numerical Analysis and Computations of Fluid Flow Problems (Code: SS 2A), Monika Neda, University of Nevada Las Vegas.

Geometric PDEs (Code: SS 1A), Matthew Gursky, Notre Dame University, and Emmanuel Hebey, Université de Cergy-Pontoise.

## Ithaca, New York

 Cornell University
## September 10-11, 2011

Saturday - Sunday

## Meeting \#1 072

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of Notices: June/July 2011
Program first available on AMS website: July 28, 2011
Program issue of electronic Notices: To be announced
Issue of Abstracts: To be announced

## Deadlines

For organizers: February 10, 2011
For consideration of contributed papers in Special Sessions: May 24, 2011
For abstracts: July 19, 2011

## Winston-Salem, North Carolina

## Wake Forest University

September 24-25, 2011
Saturday - Sunday

## Meeting \#1 073

Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of Notices: June 2011
Program first available on AMS website: August 11, 2011
Program issue of electronic Notices: September 2011
Issue of Abstracts: To be announced

## Deadlines

For organizers: February 24, 2011
For consideration of contributed papers in Special Sessions: June 7, 2011
For abstracts: August 2, 2011
The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/ sectional.htm7.

## Invited Addresses

Benjamin B. Brubaker, Massachusetts Institute of Technology, Title to be announced.

Shelly Harvey, Rice University, Title to be announced.
Allen Knutson, Cornell University, Title to be announced.

Seth M. Sullivant, North Carolina State University, Title to be announced.

## Lincoln, Nebraska <br> University of Nebraska-Lincoln <br> October 14-16, 2011 <br> Friday - Sunday

## Meeting \#1074

Central Section
Associate secretary: Georgia Benkart
Announcement issue of Notices: August 2011
Program first available on AMS website: September 1, 2011
Program issue of electronic Notices: October 2011
Issue of Abstracts: To be announced

## Deadlines

For organizers: March 14, 2011
For consideration of contributed papers in Special Sessions: June 28, 2011
For abstracts: August 23, 2011

## Salt Lake City, Utah

## University of Utah

October 22-23, 2011
Saturday - Sunday

## Meeting \#1075

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of Notices: August 2011
Program first available on AMS website: September 8, 2011
Program issue of electronic Notices: October 2011
Issue of Abstracts: To be announced

## Deadlines

For organizers: March 22, 2011
For consideration of contributed papers in Special Sessions: July 5, 2011
For abstracts: August 30, 2011
The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/ sectional.htm7.

## Invited Addresses

Graeme Milton, University of Utah, Title to be announced.

Lei Ni, University of California San Diego, Title to be announced.

Igor Pak, University of California Los Angeles, Title to be announced.

Monica Visan, University of California Los Angeles, Title to be announced.

## Port Elizabeth, Republic of South Africa

## Nelson Mandela Metropolitan University

November 29 - December 3, 2011
Tuesday - Saturday
Associate secretary: Matthew Miller
Announcement issue of Notices: To be announced Program first available on AMS website: Not applicable Program issue of electronic Notices: Not applicable Issue of Abstracts: Not applicable

## Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

## Boston, Massachusetts

## John B. Hynes Veterans Memorial Convention Center, Boston Marriott Hotel, and <br> Boston Sheraton Hotel

January 4-7, 2012
Wednesday - Saturday
Joint Mathematics Meetings, including the 118th Annual Meeting of the AMS, 95th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Michel L. Lapidus
Announcement issue of Notices: October 2011
Program first available on AMS website: November 1, 2011
Program issue of electronic Notices: January 2012
Issue of Abstracts: Volume 33, Issue 1

## Deadlines

For organizers: April 1, 2011
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

## Honolulu, Hawaii

## University of Hawaii

March 3-4, 2012
Saturday - Sunday
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of Notices: March 2012
Program first available on AMS website: To be announced
Program issue of electronic Notices: To be announced
Issue of Abstracts: To be announced

## Deadlines

For organizers: August 3, 2011
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

## Tampa, Florida

## University of South Florida

March 10-11,2012
Saturday - Sunday
Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of Notices: To be announced
Program first available on AMS website: To be announced
Program issue of electronic Notices: March 2012
Issue of Abstracts: To be announced

## Deadlines

For organizers: August 10, 2011
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

## Lawrence, Kansas

University of Kansas

March 30 - April 1, 2012
Friday - Sunday
Central Section
Associate secretary: Georgia Benkart
Announcement issue of Notices: To be announced Program first available on AMS website: To be announced Program issue of electronic Notices: To be announced Issue of Abstracts: To be announced

## Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

## San Diego, California

## San Diego Convention Center and San <br> Diego Marriott Hotel and Marina

January 9-12, 2013
Wednesday - Saturday
Joint Mathematics Meetings, including the 119th Annual Meeting of the AMS, 96th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Georgia Benkart
Announcement issue of Notices: October 2012
Program first available on AMS website: November 1, 2012
Program issue of electronic Notices: January 2012
Issue of Abstracts: Volume 34, Issue 1

## Deadlines

For organizers: April 1, 2012
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

## Baltimore, Maryland

## Baltimore Convention Center, Baltimore Hilton, and Marriott Inner Harbor

January 15-18, 2014
Wednesday - Saturday
Joint Mathematics Meetings, including the 120th Annual Meeting of the AMS, 97th Annual Meeting of the Mathematical Association of America, annual meetings of the

Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Matthew Miller
Announcement issue of Notices: October 2013
Program first available on AMS website: November 1, 2013
Program issue of electronic Notices: January 2013
Issue of Abstracts: Volume 35, Issue 1

## Deadlines

For organizers: April 1, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

## San Antonio, Texas

## Henry B. Gonzalez Convention Center and Grand Hyatt San Antonio

## January 10-13, 2015

Saturday - Tuesday
Joint Mathematics Meetings, including the 121st Annual Meeting of the AMS, 98th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Steven H. Weintraub
Announcement issue of Notices: October 2014
Program first available on AMS website: To be announced
Program issue of electronic Notices: January 2015
Issue of Abstracts: Volume 36, Issue 1

## Deadlines

For organizers: April 1, 2014
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

## Seattle, Washington

## Washington State Convention \& Trade <br> Center and the Sheraton Seattle Hotel

## January 6-9, 2016

Wednesday - Saturday
Joint Mathematics Meetings, including the 122nd Annual Meeting of the AMS, 99th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with
sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Michel L. Lapidus
Announcement issue of Notices: October 2015
Program first available on AMS website: To be announced
Program issue of electronic Notices: January 2016
Issue of Abstracts: Volume 37, Issue 1

## Deadlines

For organizers: April 1, 2015
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

## Atlanta, Georgia

## Hyatt Regency Atlanta and Marriott Atlanta Marquis

## January 4-7, 2017

Wednesday - Saturday
Joint Mathematics Meetings, including the 123rd Annual Meeting of the AMS, 100th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Georgia Benkart
Announcement issue of Notices: October 2016
Program first available on AMS website: To be announced Program issue of electronic Notices: January 2017
Issue of Abstracts: Volume 38, Issue 1

## Deadlines

For organizers: April 1, 2016
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

## Meetings and Conferences of the AMS

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The Meetings and Conferences section of the Notices gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited in the table of contents on this page for more detailed information on each event. Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. Information in this issue may be dated. Up-todate meeting and conference information can be found at www. ams.org/meetings/.

## Meetings:

2010
June 2-4
October 2-3
October 9-10
November 5-7
November 6-7
December 15-18

## 2011

January 5-8
March 12-13
March 18-20
April 9-10
April 30-May 1
September 10-11
September 24-25
October 14-16
October 22-23
November 29-
December 3

Berkeley, California
Syracuse, New York
Los Angeles, California
Notre Dame, Indiana
Richmond, Virginia
Pucon, Chile

New Orleans, Louisiana
Annual Meeting
Statesboro, Georgia p. 810
Iowa City, Iowa
Worcester, Massachusetts
Las Vegas, Nevada
Ithaca, New York
Winston-Salem, North
Carolina
Lincoln, Nebraska
Salt Lake City, Utah
Port Elizabeth, Republic
of South Africa
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2012

| January 4-7 | Boston, Massachusetts <br> Annual Meeting | p. 812 |
| :--- | :--- | :--- |
| March 3-4 | Honolulu, Hawaii | p. 813 |
| March 10-11 | Tampa, Florida | p. 813 |
| March 30-April 1 | Lawrence, Kansas | p. 813 |

2013
January 9-12
2014
January 15-18
2015
January 10-13

2016
January 6-9
2017
January 4-7
San Diego, California
p. 813 Annual Meeting

Baltimore, Maryland
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Annual Meeting
San Antonio, Texas
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Annual Meeting
Seattle, Washington
p. 814

Annual Meeting

Atlanta, Georgia
p. 814

## Important Information Regarding AMS Meetings

Potential organizers, speakers, and hosts should refer to page 92 in the January 2010 issue of the Notices for general information regarding participation in AMS meetings and conferences.


#### Abstract

s Speakers should submit abstracts on the easy-to-use interactive Web form. No knowledge of LATEX is necessary to submit an electronic form, although those who use LateX may submit abstracts with such coding, and all math displays and similarily coded material (such as accent marks in text) must be typeset in LATEX. Visit http://www.ams.org/cgi-bin/ abstracts/abstract.p1. Questions about abstracts may be sent to abs-info@ams.org. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.


Conferences: (seehttp://www.ams.org/meetings/for the most up-to-date information on these conferences.)
Co-sponsored conferences:
June 12-July 2, 2010: Mathematics Research Communities, Snowbird, VT (for more information please see http://www. ams.org/amsmtgs/mrc.htm7).
June 17-19, 2010: Coimbra Meeting on 0-1 Matrix Theory and Related Topics, University of Coimbra, Portugal (for more information please seehttp://www.mat.uc.pt/~cmf/01MatrixTheory).

## CAMBRIDGE

## See what Cambridge is releasing this Summer!

Causality,
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Theory and the
Differentiable
Structure of
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R. N. SEN
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## Geometric Analysis

 of Hyperbolic Differential Equations: An Introduction
S. Alinhac

London Mathematical Society Lecture Note Series
550.00: Pb: 978-0.-521-12822-3: 128 pp.

## Logic Colloquium 2007

Edited by
Françolise Delon,
Ulifich Kohlenbach, Penelope Maddy,
Frank Stephan
Lecture Notes in Logic

\$90.00: Hb: 978-0-521-76065-2: 288 pp.


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"One of the most original and insightful introductions to
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## A Window

 into Zeta and Modular Physics
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Klaus Kirsten
Floyd L. Williams
Mathematical Sciences
Research Institute Publications \$90.00: Hb: 978-0-521-19930-8: 360 pp.

## Networks, Crowds, and Markets

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Jon Kleingerg
"The elegant explanations in this book allow readers to rapidly gain a deep understanding of how networks work. Without resorting to either advanced math or even a bit of hand-waving, Easley and Kleinberg take us through the essential concepts and intriguing real-world applications."
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Computer Science,
and Engineering
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PETER L. HAMMER
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-Rafe Mazzeo, Stanford University
\$110.00: Hb: 978-0-521-89671-9: 330 pp.
\$48.00: Pb: 978-0-521-72149-3


Random Walk:
A Modern Introduction
Gregory F. Lawler Vlada Limic
Cambridge Studies in Advanced Mathematics
\$75.00: Hb: 978-0-521-51918-2:
376 pp.


Prices subject to change.

## American Mathematical Society

## For the Moid Reader...

## Mathematics Everywhere

Martin Aigner and Ehrhard Behrends, Freie Universitüt Berlin, Germany, Editors
Translated by Philip G. Spain
This series of lectures from renowned mathematicians demonstrates the prominent role of mathematics in our daily life, through science, technology and culture. The common theme throughout is mathematics' unique position as both the art of pure thought and universally applicable science. The book also includes a leisurely treatment of recent hot topics, including the solution of the Poincaré conjecture.

2010; 330 pages; Softcover; ISBN: 978-0-82 I8-4349-9; List US\$49;
AMS members US\$39.20; Order code MBK/72

## The Scientific Legacy of Poincaré

Éric Charpentier, Université Bordeaux 1, Talence, France, Étienne Ghys, École Normale Supérieure de Lyon, France, and Annick Lesne, Université Pierre et Marie Curie, Paris, France, Editors
Translated by Joshua Bowman
This book features presentations from 20 individual experts who each take one theme and present the approach and achievements of Henri Poincaré in that area. As a group, these presentations reflect the modernity of Poincare's work and illustrate how he remains a constant source of inspiration for researchers. The book focuses on main ideas over detailed proofs, making it relevant to a variety of readers with an interest in mathematics.
Co-published with the London Mathematical Society beginning with Volume 4. Members of the LMS may order directly from the AMS at the AMS member price. The LMS is registered with the Charity Commissioners.

History of Mathematics,Volume 36; 2010; 39 I pages; Hardcover; ISBN: 978-0-82 I8-4718-3; List US\$89;AMS members US\$71.20; Order code HMATH/36

## Riot at the Calc Exam and Other Mathematically Bent Stories

Colin Adams, Williams College, Willianstown, MA
This collection of humorous mathematics stories gives the reader a sense of what it would be like to participate in a mathematical world that is far from dreary. Several of the ideas for the stories were derived from lunchtime discussions at the humorous and friendly mathematics department at Williams College. The stories resonate with both mathematicians and laypeople with an interest in mathematics.
2009; 27 I pages; Softcover; ISBN: 978-0-82 I8-48I7-3; List US\$32;AMS members US\$25.60; Order code MBK/62


## Mathematical Connections

 A Capstone CourseJohn B. Conway, George Washington University, DC
This book illustrates connections between various courses taken by undergraduate mathematics majors. As such it can be used as a text for a capstone course. The chapters are essentially independent, and the instructor can choose the topics that will form the course and thus tailor the syllabus to suit the backgrounds and abilities of the students. At the end of such a course the graduating seniors should glimpse mathematics not as a series of independent courses but as something more like an integrated body of knowledge. The book has numerous exercises and examples so that the student has many opportunities to see the material illustrated and fleshed out.
2010; approximately 248 pages; Softcover; ISBN: 978-0-82 I8-4979-8; List US\$55;AMS members US\$44; Order code MBK/75

OTHER BOOKS OF
GENERAL INTEREST
OFFERED BY THE AMS

Famous Puzzles of Great Mathematicians Miodrag S. Petković University of Nis, Serbia

Pioneering Women in American Mathematics The Pre-1940 PhD's
Judy Green, Marymount University, Arlington, VA, and Jeanne LaDuke, DePaul University, Chicago, IL
The Life and Times of the Central Limit Theorem
Second Edition
William J. Adams, Pace University, New York, NY
The Knot Book
An Elementary Introduction to the Mathematical Theory of Knots
Colin C. Adams, Williams College, Williamstown, MA

## Mathematics and Music

David Wright, Washington University, St. Louis, MO
An Introduction to the History of Algebra
Solving Equations from Mesopotamian Times to the Renaissance
Jacques Sesiano, Swiss Federal Institute of Technology, Lausanne, Switzerland

The Game's Afoot! Game Theory in Myth and Paradox
Alexander Mehlmann, Vienna University of Technology, Austria

- A View from the Top

Analysis, Combinatorics and Number Theory
Alex Iosevich, University of Missouri, Columbia, MO

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2010. 390 p. 50 illus. in color. (Interdisciplinary Applied Mathematics, Volume 35) Hardcover ISBN 978-0-387-87707-5

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Andrew Pressley
-

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> semomataion
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179 illus., (Springer Undergraduate Mathematics Series) Softcover ISBN 978-1-84882-890-2

- \$49.95


## Visit Springer at the following conferences

## 2010 MathFest Meeting

Springer will be attending the upcoming MathFest meeting in Pittsburgh, PA from August 5th-August 7th. Stop by the Springer booth and browse over 1,000 books on screen! This new technology lets you browse titles with a single touch!

## 2010 ICM Conference

Springer will be attending the upcoming ICM Conference in Hyderabad, India from August 19th-August 27th. Please stop by one of our booths and browse our Titles on Display, sign up for SpringerAlerts, or speak with one of our publishing editors.



[^0]:    wouw,northwestern.edn/provost/awards/nemmers

[^1]:    Doris Schattschneider is professor emerita of mathematics at Moravian College in Bethlehem, Pennsylvania. Her email address is schattdo@moravian.edu.

[^2]:    ${ }^{1}$ The title of a 1995 exhibit of Escher's work at the National Gallery of Canada in Ottawa was titled "M. C. Escher: Landscapes to Mindscapes".

[^3]:    ${ }^{2}$ Escher's colored plane-fillings have been called tessellations, periodic drawings, tilings, and symmetry drawings. I prefer to use the last term.
    ${ }^{3}$ All of Escher's prints that are named in this essay can be found in the catalogs [1] and [37], and many of them can also be found in [20], [59], and on the official website www.mcescher.com.

[^4]:    Joel Spencer is professor of computer science and mathematics at the Courant Institute, New York University. His email address is spencer@cims.nyu.edu.

[^5]:    ${ }^{1}$ In the original work the nodes were all male!
    ${ }^{2}$ This author recalls in undergraduate days first seeing a finite random variable with infinite expectation and thinking it was a very funny and totally anomalous creation. Wrong! Such variables occur frequently at critical points in percolation processes.

[^6]:    ${ }^{3}$ Thanks to Juliana Freire.

[^7]:    ${ }^{4} \beta$ might not exist, but all mathematical physicists assume it does.

[^8]:    Fan Chung is professor of mathematics at the University of California, San Diego. Her email address is fan@ucsd. edu.
    *This article is based on the Noether Lecture given at the AMS-MAA-SIAM Annual Meeting, January 2009, Washington $D . C$.

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