Perelman Awarded Millennium Prize

In March 2010 the Clay Mathematics Institute (CMI) announced that Grigoriy Perelman of St. Petersburg, Russia, is the recipient of the Millennium Prize for resolution of the Poincaré conjecture.

The Poincaré conjecture is one of the seven Millennium Prize Problems established by CMI in 2000. The decision to award the prize to Perelman was made in accord with the governing rules for the prizes: recommendation first by a Special Advisory Committee (Simon Donaldson, David Gabai, Mikhail Gromov, Terence Tao, and Andrew Wiles), then by the CMI Scientific Advisory Board (James Carlson, Simon Donaldson, Gregory Margulis, Richard Melrose, Yum-Tong Siu, and Andrew Wiles), with final decision by the Board of Directors (Landon T. Clay, Lavinia D. Clay, and Thomas M. Clay). [Editor's Note: At the time of this writing in mid-May 2010, CMI president James Carlson said that Perelman had not yet made a decision about whether to accept the prize.]

Formulated in 1904, the Poincaré conjecture is fundamental to achieving an understanding of three-dimensional shapes (compact manifolds). The simplest of these shapes is the three-dimensional sphere. Since we cannot directly visualize objects in *n*-dimensional space, Poincaré asked whether there is a test for recognizing when a shape is the three-sphere by performing measurements and other operations inside the shape. The goal was to recognize all three-spheres even though they may be highly distorted. Poincaré found the right test, simple connectivity. However, no one before Perelman was able to show that the test guaranteed that the given shape was in fact a three-sphere.

In the last century there were many attempts to prove, and also to disprove, the Poincaré conjecture using the methods of topology. Around 1982, however, a new line of attack was opened. This was the Ricci flow method pioneered and developed by Richard Hamilton. It was based on a differential equation related to the one introduced by Joseph Fourier 160 years earlier to study the conduction of heat. With the Ricci flow equation, Hamilton obtained a series of spectacular results in geometry. However, progress in applying it to the conjecture eventually came to a standstill, largely because formation of singularities defied mathematical understanding.

Perelman's breakthrough proof of the Poincaré conjecture was made possible by a number of new elements. He achieved a complete understanding of singularity formation in Ricci flow, as well as the way parts of the shape collapse onto lower-dimensional spaces. He introduced a new quantity, the entropy, which, instead of measuring disorder at the atomic level as in the classical theory of heat exchange, measures disorder in the global geometry of the space. This new entropy, like the thermodynamic quantity, increases as time passes. Perelman also introduced a related local quantity, the L-functional, and he used the theories originated by Cheeger and Aleksandrov to understand limits of spaces changing under Ricci flow. He showed that the time between formation of singularities could not become smaller and smaller, with singularities becoming spaced so closely—infinitesimally close—that the Ricci flow method would no longer apply. Perelman deployed his new ideas and methods with great technical mastery and described the results he obtained with elegant brevity. Mathematics has been deeply enriched.

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