Cartan as a Teacher

Pierre Cartier

"We Are All Your Students, Mr. Cartan"

At the end of June 1965, at the (then) new mathematics library of the École Normale Supérieure in Paris (ENS), a party was held in honor of Henri Cartan. He was leaving his position of "directeur des études mathématiques" at the ENS, after twenty-five years of service. Many mathematicians gathered on the occasion of this farewell. Cartan, in one of his customary understatements, commented: "I asked to extend the invitation to my former pupils, and I see so many people…". To which Vladimir Arnol'd, visiting France for the first time, and with his standard wit, answered: "But… Mr Cartan…in Moscow as well as in Paris, we are all your students." Cartan was extremely pleased.

Of the two great creations of the revolutionary Convention (1793), if École Polytechnique is, in the words of Arthur Wightman, a "peculiar combination of West Point and M.I.T.", École Normale is properly the highest in a network of teacher's colleges.

When I was admitted at ENS in 1950, the mathematics department consisted of a rich and spacious library, with two offices at the entrance gate, shared by the professor, his teaching assistant (a "caïman"[1] in our student's slang), and the librarian. The professor was Henri Cartan; his caïman was Jean Frenkel, a veteran from WWII;[2] the librarian was the wife of the head librarian of the school, named Madame Martin. There were two more classrooms, called E and F, with the standard joke that every event was announced to take place in Room E *or* F. Everything took place in these two small rooms: the Bourbaki Seminar, the standard lectures for the students, the entrance examination, and also the Cartan seminar (see later).

Mathematics students were allowed only three years for the standard curriculum, whereas our friends in the other departments were given four years. Obtaining a fellowship for a supplementary year (to begin a doctoral thesis) was quite hazardous.

Cartan was everywhere. He himself taught the three years. In the first year, he supplemented—and modernized—the Sorbonne course on calculus. In this first year, we were supposed to follow at the Sorbonne the main courses: calcul différentiel et intégral (calculus), general physics, and classical mechanics. These courses were quite old-fashioned.

In the second year, instead of writing a master's thesis, the mathematics students were required to follow an advanced course: algebra (or analysis, or "géométrie supérieure"). But the main feat was for us Cartan's course called "cours aux carrés".[3] Having married during the summer of 1951, to the great dismay of Cartan, rather conservative in social habits, I was less faithful a student, but I remember memorizing hastily Erdös-Selberg's proof of the prime number theorem[4] as part of "algèbre supérieure". Cartan impressed me very much with a course on potential theory, using the new tool of distributions and a simplified version of Sobolev spaces known as Dirichlet space.

For many years, the main academic obligation for the students of ENS was to submit to a national competition called "agrégation". Officially, this is a qualifying examination for the profession of high school teacher, in an extended sense, from junior

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[1] *=alligator*

[2] *And a Jew who left France in 1942 to join de Gaulle's Free French Forces.*

[3] *Second-year students*

[4] *A so-called "elementary proof", a tricky and uninformative extension of Čebičev reasoning!*

Élie Cartan, Henri Cartan's father, 1932

high school to the elite undergraduate program known as "classes préparatoires".

In our third year, still under the guidance of Cartan, we prepared for the "agrégation". He read very carefully our memoirs, twice a month, and trained us for the oral examination. I hated the completely old-fashioned curriculum and was impatient to return to more serious—and modern—matters. But preparing for the oral examination—a course in which one *teaches how to teach*—is a fruitful exercise, and I was later happy to teach such a course.

I remember a somewhat embarrassing episode. I was supposed to explain the standard result that a function, whose derivative exists everywhere and is identically zero, is a constant. I prepared carefully during the Christmas vacation, using a proof in Bourbaki allowing some exceptional points. When it was my turn, I explained to the class that I intended to "improve" Bourbaki's proof by allowing an exceptional set of measure 0. I remember Cartan commenting: "Now, listen carefully, Cartier is going to prove his first theorem!" I understood, by his tone, that something was wrong. I often taught Lebesgue's integral and never slipped again!

But the real event was Cartan's seminar. I vividly remember my first class at ENS, the second Monday of November 1950. According to what became the weekly routine, I attended in the morning Cartan's class for the "conscrits" (freshmen), then at 2:30 p.m. his seminar, and in the late afternoon, we were supposed to learn from the craftsmen the art of cutting and assembling metal, wood, glass, for a possible career in experimental physics. I was not very good at that and learned just enough to help my friends prepare fantastic illuminations for the school's night in the next spring.

Cartan had an indomitable curiosity and openness. He was also very tenacious, and his friends often called him "mosquito" for his insistence on biting. Though not a Huguenot, he was as rigorous. I recently learned that, during the summer, he was the organ player for a small Protestant community in Die, where the mourning took place after his death. He could be quite formal, but in a very British way, compatible with wit. I remember my first visit to his just reunited family (in their summer home). His daughter was wearing trousers for the first time, and in a very formal tone he asked his wife: "My dear friend, have you noticed the dress of your daughter?" Despite this formality, he had an open mind: always well dressed, he never blamed me for my sloppiness, intended to provoke my elders!

A glance at the table of contents of his seminar shows an unusual diversity of topics: algebraic topology, sheaf theory, several complex variables, automorphic functions, algebraic geometry, index theorem.… He himself gave many of the lectures, the other speakers spent long hours with him for preparation, and he wrote (and typed) a large part of the proceedings himself.

After a few weeks, during which I understood nothing, I had acquired an elder brother, Jean-Pierre Serre, and a benevolent uncle, Samuel Eilenberg, a Protestant and a Polish Jew, well in line with my roots in Mittel Europa. I made an acquaintance with all—or almost all—French mathematicians. At that time, everything in mathematics was in Paris, around Cartan, with two extensions in Nancy[5]—where Dieudonné, Delsarte, and Schwartz maintained Bourbaki's spirit—and in the University of Strasbourg with Ehresmann (for a while a Bourbaki accomplice). The glorious generation—Hadamard, Borel, Fréchet, Paul Lévy, Élie Cartan[6]—was very old or already gone, and the ambitious youth gathered around Cartan. By his central position at the ENS, Cartan was more or less the thesis advisor of everyone (at least formally).

From Eilenberg, in the academic year 1950–51, we heard a long series of lectures in which he developed an axiomatic theory of group homology as a model for his well-known axiomatization of algebraic topology in his book with Steenrod. This series was to be followed by a similar attempt toward the cohomology of sheaves—a much more difficult subject, whose completion was one of the first major achievements by Grothendieck.

Three times a year, the same group of people attended the Bourbaki seminar in its ascending

[5] *The other arm of Bourbaki was in Chicago; hence the famous series of Nancago mathematics books.*

[6] *Father of Henri Cartan!*

phase.[7] There we learned about Weil's proof of the Riemann hypothesis in the case of function fields, Zariski's work (as reported by P. Samuel), Koszul's thesis about the homology of Lie algebras, the work of Petrowsky in partial differential equations and of Gelfand in the theory of group representations…all the hot subjects in mathematics. Moreover, the supposedly secret drafts of forthcoming Bourbaki volumes were freely circulated by Serre, Cartan, and others. Among the closest friends of Cartan were André Weil, who visited France every summer and gave a series of lectures at the ENS in the winter of 1951 (about adeles and ideles), Armand Borel, who introduced Cartan to Leray's work on sheaves, and Claude Chevalley, who became a professor at La Sorbonne after 1954 (and one year in Japan).

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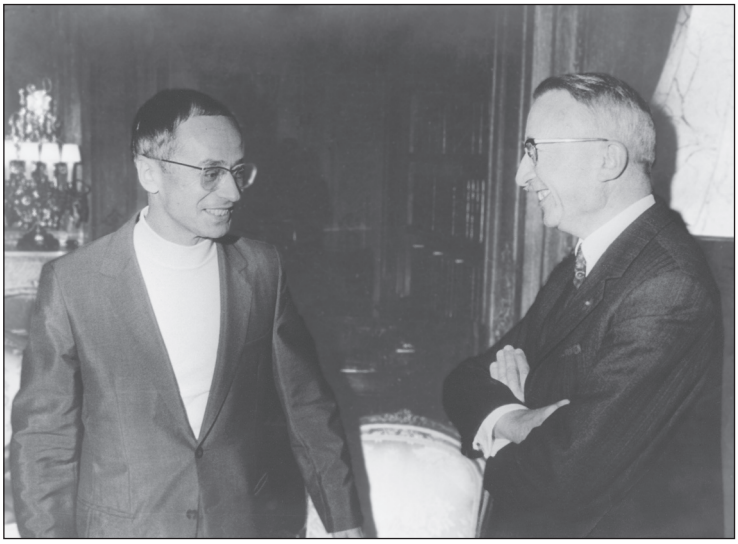
French mathematics was at a turning point. The undergraduate curriculum (even in its enhanced form for the "classes préparatoires") was a mix of coordinate geometry, synthetic geometry (based on the "theorem" that every one-to-one correspondence between the points of a projective line is given by a Möbius transformation),[8] differential calculus with applications to geometry and kinematics. The foundations were sometimes shaky, there was hardly any hint of groups of transformations (in geometry), and the use of matrices was ignored or not advised.[9] The good teachers dared to give the foundations of the real number system, but in the absence of set-theoretical terminology, the exposition was quite obscure. In the land of Lebesgue, hardly any mention was made of the Lebesgue integral, and we had to learn Lie groups from the thesis of Élie Cartan or in Pontrjagin!

By a sequence of well-planned steps, Cartan made General Bourbaki win! He managed to hire the ambitious youth at La Sorbonne: Schwartz, Choquet, Dixmier, Godement, and Chevalley. In 1957 the takeover was complete (except for André Weil, who was never forgiven for his refusal to be drafted in 1939, at the beginning of WWII!). The curriculum was deeply renovated, and the textbook of textbooks became Bourbaki (whose golden age extends from 1950 to 1975!). The forceful gesticulations of Dieudonné, as well as the power of persuasion of Choquet and Lichnerowicz (both not members of Bourbaki) convinced everyone to worship general topology, linear algebra, functional analysis, and group theory. Henri Cartan was always a moderate in this debate and never threw the baby out with the bath water. He made one major mistake—discarding classical mechanics with this comment: "The teaching of classical mechanics (in France) is very poor"—which was true!— "and for the physicist, only quantum mechanics matters." We know better, as no one can understand quantum mechanics without a thorough acquaintance with its Newtonian (or Hamiltonian) version. One of the reasons for this mistake was the almost complete ignorance of the challenges of mathematical physics by the members of the Bourbaki *coterie*. It is also to be said that the teaching of physics in France was even more backward than in mathematics. One example: the first serious course on quantum mechanics was given in 1964, not in Paris, but in the National Center for Nuclear Physics (at Saclay).

Cartan and Samuel Eilenberg at a Bourbaki congress, Pelvoux, 1951.



Henri Cartan and Jean-Pierre Serre, Paris, July 1970, on the occasion of the awarding of the Gaston Julia Prize to Serre.

[7] *To reach its peak of fame and attendance from 1970 to 1990, followed by a slow decline.*

[8] *Laurent Schwartz embarrassed his teacher with the counterexample of going to the complex conjugate!*

[9] *Ostensibly because there was supposedly no agreement about the product rule: lines by columns or columns by lines!*

So around 1960, Bourbaki had won, in main part due to the efforts of Cartan, helped by the political know-how of Schwartz. There followed the dubious episode of the so-called "modern math", or the failed attempt to use Bourbaki as a textbook for kindergarten! The fiercest proponents were ultra-zealous disciples of Bourbaki, not of the same mathematical caliber, helped by an attraction toward abstractness, an integral part of the then prevalent fashions (in art and elsewhere).

For about fifteen years French mathematics was ruled by two enlightened despots: Cartan and Schwartz, controlling between them most of the academic world. Nominally, every doctoral student in mathematics had one of them as thesis advisor, and they suggested research topics. Cartan's way was more open and deep, but everyone benefited from their valued advice. Committee work was reduced to a couple of days off for our masters, where everything was settled, and in most cases, in an optimal way.

Under this reign, what was the fate of the young French mathematician? Let me take myself as an example. At the end of my first year at ENS, I was at pain to choose my future; I was attracted by philosophy and (experimental) physics as well as by mathematics. The advice of Althusser, the professor of philosophy, was warm support, with a fatherly caution about the degrees to be earned. Yves Rocard, head of the physics department, whose son Michel was later a prime minister of France, wanted to recruit me to build with him the French atom bomb, to be exploded ten years later. Cartan, following the advice of Eilenberg, invited me to one of those secret meetings of Bourbaki! My fate was decided.

A few years later, Cartan turned me down for the Princeton fellowship (awarded later on to Douady) on the excuse that this didn't fit a married man[10] (he was not especially happy about my early marriage!). Then he turned me down for the position of "caïman" at the ENS, explaining to me later that I would have sacrificed my research work because of my involvement in teaching (he was perfectly right!).

A deeper interference came later in 1961. After completing my thesis, two years of postdoc at the Institute for Advanced Study in Princeton, and almost three years of military service at the time of the war of independence of Algeria, it was time for me to apply for a professorship (I was almost thirty!). For each position I applied to, I got the same answer: "Why should you come here, since Cartan says that you have been appointed at Strasbourg?" (where I didn't apply!). There was no way out. I tried to rebel and told Cartan I would stay only two years in Strasbourg—no more. He laughed, and I willingly spent ten years there—in the opinion of my wife, our best years! He later gave me an explanation. He was enormously interested in bringing back together French and German mathematicians, as witnessed by his long-lasting friendship with Behnke and Hirzebruch. In Cartan's opinion, I was the best fit in the young generation to work in this direction. He was right, and I immensely enjoyed working for the *Versöhnung*[11] with the help of my German friends Dold and Puppe. I was from that time on as convinced as he was of the necessity of a Federal Europe (still in the making), well in line with the thoughts of my mother, who explained to me, when I was five, that I should see the day of the United States of Europe.

Some years later, in 1974, I didn't yield to his pressure to join Orsay University, but it was already another time, in the aftermath of the 1968 student revolution… Grothendieck was gone already…

Thank you, Henri Cartan, for your fatherly influence on me! It made me grow up!

[10] *But not much later, I was invited for two years at the Institute for Advanced Study, where my wife was welcomed!*

Jacques Dixmier

Our Teacher

I was a student at the École Normale, from October 1942 to September 1945. This was a time of war, and Cartan was unable to return to his alma mater in Strasbourg, which was then under Nazi rule. In this way, we could benefit from an outstanding teacher.

In my first year in the school, I had to attend the calculus course at La Sorbonne, Cartan's course being officially a kind of tutorial to help us master the curriculum in calculus. But this curriculum was rather outmoded, and the aim of Cartan was to "modernize" the subject matter. The main point was to introduce us to so-called modern algebra (groups, rings, vector spaces…). Therefore, Cartan's lectures were combining a rather standard course on calculus but in the spirit of the not yet published volume of Bourbaki, entitled *Functions of a Real Variable (Elementary Theory)*. So, for instance, the Γ-function was described as in the famous booklet by Emil Artin. In the more "modern" or "advanced" part, we were told about integers mod p, Grassmann calculus, Fourier transforms, and fixed-point theorems.

Cartan knew how to involve us in the class, and we would have been more impressed if we had known his research work. We were seventeen in the class, to be reduced to nine during the second year, because of the hardships of the war and also

[11] *Mutual forgiveness, in German!*

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because in the second year he taught only the students who had chosen mathematics as a major. After consulting the class, he decided to teach a course on Lie groups. He developed all the important notions from a rather general point of view but always illustrated by elementary examples. His goal was to lead us up to Lie's third theorem (reconstructing a Lie group from its Lie algebra). Because we were lacking the relevant background in topology, he concentrated on the local theory. I would not claim it was an easy course, and he admitted it frankly.

In our third year, our curriculum was centered around the so-called "agrégation", a national competition for prospective teachers. The main emphasis of the course was—and still is—to prepare the students to deliver a lecture—in a sense *teaching how to teach*.

So, for each of many years, he offered to the second-year students a new basic course. These courses were enormously influential, and he offered them besides his many commitments: research work, doctoral students, academic infighting, family tragedies… He was a real master, and we cannot be too grateful to him.

Adrien Douady

Memories of the Cartan Seminar

On June 28, 2004, a celebration was held at the École Normale Supérieure in Paris in honor of Henri Cartan on the occasion of his 100th birthday. Among the speakers was Adrien Douady.[12] *Douady gave recollections of the Cartan seminars. Here are excerpts of his talk,*[13] *prepared by Régine Douady, his widow, from his handwritten notes, translated into English and slightly edited by P. Cartier and L. Illusie.*

Nowadays there are two kinds of seminars: the *colloquia*, in which a prestigious visitor is invited to talk on a topic of his or her choice, and *groupes de travail* (work groups) in which a small team decides to study some question, to fully understand an important article. The Cartan seminar was of this second type. Cartan would choose a theme or a recent result that he wanted to understand in detail. He would assemble around him a team of speakers and assign the talks to be given, reserving a good number of them for himself. The seminar took place in Room U of the ENS. In the audience you would sometimes find Serre, Weil, Dieudonné, Godement, Chevalley (who ran his own seminar right afterward), and, of course, Cartan. He did not tolerate the slightest inaccuracy, the slightest imprecision, and he criticized the speaker to the point of totally destabilizing him. You had to be well prepared. One month earlier Cartan would hand you a paper to read and digest. He would sometimes explain the plan. You were allowed, and even encouraged, to reconstruct the whole thing. I was of course happy to do that. I've never been able to read a paper (I go to sleep after page 3).

I remember my talks, and those of Bernard Morin as well. I wrote on the blackboard for him. Blind, he had an acute geometric vision. That was his revenge: seeing what others don't.

But what really mattered for Cartan were the notes of the exposés. Again he would tolerate no imprecision. It was out of the question to say that two groups were isomorphic without specifying an isomorphism between them, or to say that a diagram commuted up to sign: the sign had to be given. Above all he wanted the text to be perfectly clear. For this he asked you to revise your text as many times as necessary. *Vingt fois sur le métier remettez votre ouvrage* (*Redo your work twenty times*), said Boileau. With Cartan it was rather thirty times than twenty. His point was that a text should have at least thirty readers, otherwise there's no need to write it. Therefore, if you spend half an hour to spare the reader a minute of perplexity, then it's well worth it. He would return your manuscript covered with annotations made with a red pen in his small, curly handwriting. Then you would revise it and give him back a new version. There was no word processor at the time. I typed. I had a green typewriter, given to me by my father, which I still have. I photocopied my text, cut out shreds of 2 or 3 lines, pasted, photocopied again. I was allergic to the droplets falling from the ribbon of the typewriter when you type. You worked hard. A good reason for this is that when your text was eventually "Cartan acceptable", another person was to work on it: Denise Lardeux, the secretary of the IHP (Institut Henri Poincaré). She used stencils, whose smell impregnated her office and her blouse. You could make corrections—with a correcting red ink having an even worse smell—but only by replacing one letter by another one, or a word by another one of the same length. All that made you confident that your text was really in final form.

Through this hard training all Cartan's students acquired a solid mathematical style. Serre's style reached perfection: it's a model of clarity, saying all that has to be said without an unnecessary word. Unfortunately Serre had very few Ph.D. students.

We try to pass the torch to our students. They toil and sweat, but they are grateful to us. One of them recalled that after I had said to her, "This is not clear", she had replied, "But it's clear in my

Adrien Douady was professor of mathematics at the Université de Paris-Sud 11.

[12] *Adrien Douady died accidentally on November 2, 2006.*

[13] *A recording of his talk is available at the website of the Diffusion des savoirs de l'ENS.*

1923 ÉNS Sciences, Paris, 1926. Front row, left to right: | (1) René de Possel, (2) Henri Cartan, (3) Paul Dubreil, (5) Jean Coulomb.

head". Then I said: "What do you want to give to the reader: your text or your head?"

A few words, now, about the contents of the Cartan seminars. There were two main themes: algebraic topology and complex analytic geometry. In the 1940s one discovered the extraordinary power of the cohomological methods created for algebraic topology. These methods were later applied to algebraic geometry, complex analytic geometry, number theory, etc. Their development in each of these fields, as well as in their foundations (homological algebra), occupied a large (too large?) part of the activity of mathematicians up to the 1980s. Weil, Zariski, Cartan, Serre, Grothendieck, and Dieudonné were leaders in this vast movement, with the Cartan seminar as a prominent place for it.

Just after I had passed the "agrégation", I spent 1957–58 in Princeton. I attended the course of John Moore on algebraic topology and of Spencer in complex analytic geometry (I had tried to attend Kodaira's, but in vain, as he did not speak loudly, and I was already a little deaf). I had benefited from many conversations with Cartan and, above all, Andreotti. When I came back, in October 1958, Cartan immediately enrolled me in his seminar, starting with two exposés on spectral sequences. The goal was a construction of Adams to prove that the only spheres admitting a continuous composition law with neutral element are S^0, S^1, S^3, S^7 and are parallelizable. At the time K-theory didn't exist, and you used "secondary cohomological operations". They were horrible, ill-defined machines. A few years later, thanks to K-theory, a much simpler proof was found, and it was observed that all "secondary cohomological operations" are actually differentials in the Atiyah-Hirzebruch spectral sequence relating cohomology and K-theory.

Christian Houzel

Henri Cartan and the École Normale Supérieure

When I was admitted at the École Normale Supérieure in 1956, I had not yet made up my mind between mathematics and physics. During the first week, I attended Cartan's lectures to the first-year students. They were devoted to the characterization of the additive group of real numbers as a totally ordered group which is a complete lattice. As the multiplicative group of positive real numbers has the same properties, this gives an elegant construction of the exponential function. I immediately understood that I was in the right place, and I chose mathematics as a major.

It was a great privilege to be in Paris at this time, in the late 1950s. Our professors were Henri Cartan, Gustave Choquet, Laurent Schwartz, Roger Godement, Jean-Pierre Serre, Jacques Dixmier, Claude Chevalley… This generation of mathematicians accomplished the renovation of French mathematics after the war, and Henri Cartan played a central role in it, due to his strong personality and his position at the École Normale Supérieure.

There was a sharp contrast between the mathematics I had learned in the *classes préparatoires*, which had a definite archaic taste, and the mathematics taught by Cartan. In each of the three years at the École, he gave us lectures on different topics. I remember a course on measure and integration in the first year, directly inspired by Bourbaki, and a luminous course on Lie groups in the second year. The third year was dedicated to the preparation of the *agrégation*, a special competitive examination to become a teacher. We had to give lessons as if in front of a class in a secondary school, and Cartan commented on our choices and our way of teaching. His remarks were always right and useful. In the other years, Cartan gave lectures on the calculus of variations, on analytic functions of several complex variables, on homology, and more.

To assist him in teaching, there was an assistant professor called a caïman. When I was in the third year, the caïman was Adrien Douady, from whom we learned much, but we were conscious that the task was heavy, and we asked Cartan whether it was possible to get a second assistant professor. Cartan managed to obtain the creation of a new position and at the beginning of 1961, he offered me that position, which I occupied for almost three years. So I had the privilege of collaborating with Cartan and Douady and meeting younger generations. These were very happy years.

Cartan's seminar, held every Monday afternoon at the École, was a must for the Parisian

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mathematicians. It existed for sixteen years, up to 1964, and it was not replaced afterward. Each year, Cartan devoted his seminar to the proof of a recent result. He started from scratch so that a student with a general knowledge in mathematics was able to understand. He developed the necessary tools, gave complete proofs, and, at the end of the year, he obtained a proof of the chosen result, very often better than the original one. Each year, the seminar was written down, and it remained a classical reference.

I had the opportunity to attend this seminar for six years. In the first one, I was very young and ignorant. The subject was automorphic functions, and I was astonished by such things, completely new for me. In the second and third year I learned some algebraic topology. The seminar was dedicated to Adams's works on Hopf spaces, with the theory of secondary cohomological operations, then to the periodicity of stable homotopy groups. In the next year, Cartan returned to functions of several complex variables and analytical spaces. The topic was suggested by Kodaira and Spencer's works on families of complex analytical manifolds and by the new results of Ahlfors and Bers on the moduli of compact Riemann surfaces. The seminar began with talks by Douady on Kodaira and Spencer's works. Then Cartan asked me to explain Ahlfors and Bers's work. At this time, Grothendieck, who attended the seminar, said that he had a way of constructing the moduli space of compact Riemann surfaces of given genus. His method was connected with his work in algebraic geometry. He proposed to Cartan to explain it, and his explanation occupied eleven sessions of the seminar: Cartan's seminar became Grothendieck's seminar! Cartan generously welcomed the new state of affairs. At the end of the year, Grothendieck asked me to explain local properties of analytical spaces. In his talks, he had used some of these properties without proof, and this complement was necessary. For me, this was hard work, and I still remember the care with which Cartan read my text.

For people of my generation, Henri Cartan was a master and a model. We were impressed by the power of his mathematical work but also by his moral qualities and by his sense of humor. He was engaged in the defense of persecuted mathematicians in dictatorial countries and in the construction of a political Europe. He was a good pianist and a fine musician. Through the reading of his correspondence with André Weil and Jean Dieudonné just after the war, I got a better knowledge of him. I hope I can publish these letters someday.

Jean-Pierre Kahane

Cartan at Orsay

Henri Cartan spent the last years of his academic career, the years from 1969 to 1975, at Orsay. Although it was a short period of time, Cartan played a decisive role in the development of Orsay, and he remains a glorious figure in the brief history of Université Paris-Sud. There are no archives of this interesting time, and I didn't try to collect testimonials; this article relies on my personal recollections.

Before 1968

The scientific center of Orsay was created around 1955, when the laboratories of nuclear physics moved from Paris to Orsay, under the supervision of Irène Joliot and Frédéric Joliot-Curie. Shortly afterward other important physics laboratories also moved from Paris or were created directly at Orsay. Until 1965 all these laboratories were part of the Faculté des Sciences de l'Université de Paris. This Faculté des Sciences, together with the Faculté des Lettres, were both located at the Sorbonne, but the building of the Sorbonne was too small to contain everything. A new location in Paris was necessary; it was built at the end of the 1960s on the former site of the wine market, La Halle aux Vins, and became the Jussieu campus. In the meantime, the mathematicians gathered at Institut Henri Poincaré, where the mathematics department of the Faculté des Sciences was located.

In 1958 it was decided to organize part of the teaching in all scientific subjects at Orsay. The first professors of mathematics who had their teaching duties at Orsay were successively Delange, Deny, Lesieur, Malgrange, myself, Malliavin, Néron, Cerf, and Poitou. We were professors at the Faculté des Sciences of Paris, and Orsay was merely a part of this Faculté, "annexe de la Faculté des sciences".

That situation changed in 1965, when Orsay became the second Faculté des Sciences de l'Université de Paris, with a dean having the same status as the Parisian dean. We then became professors at the Faculté des Sciences d'Orsay. However, we kept only one mathematics department, common to the two Facultés. During those years the president of the department was Henri Cartan (except in 1967 when he spent one year in Princeton and asked me to replace him). In this position he became a key actor in the development of mathematics at Orsay, treating his younger colleagues as equal members of the department (for example, though it was not so convenient at the time, he decided that one or two plenary meetings of the department should take place at Orsay instead of Paris). We organized

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together the advanced courses and the thesis defenses; the *commission des thèses* in mathematics had no formal power, but the simple procedure of registering and discussing every planned defense proved very efficient in ensuring a good standard for the theses accepted by the department. Scientific life was developing at Orsay, with advanced courses and seminars, and a mathematical library that soon became quite good.

1968–1969

In 1968 the Faculté des Sciences de Paris moved to its new location, the quadrangle of Jussieu. Meanwhile Orsay developed all branches of science. The cooperation-competition was supervised by the two deans, Marc Zamansky and Georges Poitou, both mathematicians.

Occasionally there were common meetings of all professors of both Facultés. The joint mathematics department met regularly at Institut Henri Poincaré.

The student uprising of May 1968 was an explosion in Paris and a happening at Orsay. As a sign of solidarity with the students, Laurent Schwartz and Henri Cartan initiated a dangerous action, namely individual resignations from their academic positions. This act was emulated by a number of professors and became a kind of dramatic collective resignation during a joint Paris-Orsay meeting chaired by the dean, Zamansky. Zamansky disapproved and left the meeting; Poitou took the chair and collected all papers and signatures and did what had to be done, that is, nothing. The action had taken place, and the danger, having the resignations accepted, was over.

Poitou had a special style as dean and grand views about the future of universities. The same can be said about Zamansky, except that the style and the views were mutually opposed. It was agreed by everybody that the old universities, as empty shells, were dead. The dead body of the enormous University of Paris had to be replaced by new and living universities. For Zamansky there was no doubt: the Faculté des Sciences de Paris had to become the Scientific University of Paris. For Poitou all new universities should be multidisciplinary, and he acted in order to include Orsay in a much larger thematic and geographic domain that became the Université de Paris-Sud (now Université Paris-Sud). Zamansky failed, and the Faculté des Sciences de Paris split into two parts, which became the scientific parts of Université Paris 6 (now Université Pierre et Marie Curie) and Université Paris 7 (now Université Denis Diderot). The splitting involved different factors, including political and personal antagonisms. Mathematicians suffered from strong and partly artificial oppositions (now forgotten). Chevalley chose a new university, Paris 8-Vincennes, and Cartan chose Orsay.

Of course Cartan was more than welcome at Orsay and also at the new Université de Paris-Sud. A constitutive council of the university was created, elected by the different sections of the new university; that is, medicine at Kremlin-Bicêtre and other important hospitals and research laboratories; pharmacology at Châtenay-Malabry, with the greatest number of pharmacology students in France; law in Sceaux; institutes of technology in different branches at different places: Sceaux, Cachan and Orsay, with Orsay becoming the scientific center of the new university. Cartan was elected a member of this constitutive council. Then the first meeting of the council elected him as president. Though he didn't keep this position very long, his academic experience and the unique way he had of chairing a meeting were highly appreciated: he was always exquisitely polite and perfectly clear. Among the members of the council was the director of the Institute of Technology of Sceaux, Mrs. Alice Saulnier-Seïté, a trade-unionist at the time, who became minister of the universities a few years later. There were strong personalities, opposing views, bright speakers, and challengers, and Cartan proved a perfect helmsman of this moving boat.

1969-1975

As soon as he joined Orsay, Cartan took part in all aspects of the life of the mathematics department. By the way, the math building at Orsay, bâtiment 425, is named Mathématique, without an "s", as in Bourbaki "Éléments de mathématique". Cartan could feel at home.

The scientific environment was, and still is, quite favorable: Institut des Hautes Études Scientifiques (IHÉS) at Bures-sur-Yvette, a large campus of CNRS (National Center of Scientific Research) in Gif-sur-Yvette, several engineering schools in the immediate neighborhood (École Supérieure d'Électricité, École Supérieure d'Optique, and, in the mid-1970s, École Polytechnique). The IHÉS is within walking distance of the math department, and Cartan kept a strong relationship with permanent and visiting members of this institute.

We had several foreign members in the math department. They were invited on a temporary basis, and many of them wanted to settle in France and become French citizens. The same was true for some of the research people working at IHÉS. This was never easy and sometimes very difficult. Cartan got involved in the more difficult or urgent cases; he made the necessary steps and proved efficient in a sometimes incredible way.

The scientific life in the math department was organized around the library. From the very beginning we had decided to use all the funds we had for buying books and collections. We had obtained substantial help from the CNRS and a large endowment from the State through an occasional

resource called 5e Plan (the French policy included at that time some large programs, called Plans).

We had used this fund very carefully, but it came to its end as Cartan arrived at Orsay. We had an ordinary allowance for mathematics from the university, but we needed more. Cartan was a regular user of the library, and he appreciated the policy we had in this matter. Together with Gustave Choquet, who later moved from Paris to Orsay, he collected the information about the holdings and needs of the library, which he presented to the newly elected president of the University, Bernard Picinbono. Picinbono was convinced, all relevant committees and councils were equally convinced, and the library was saved.

Actually the needs of the library were also the first reason to organize the scientific life in a new way. We had a strong but informal link with CNRS, and the policy of CNRS (a very good policy) was to favor formal associations between CNRS and university laboratories or research teams. From the university point of view the math department was a laboratory; for purely financial reasons it appeared that it would be advantageous to have associated research teams. We formed research teams in harmonic analysis, topology, number theory, numerical analysis, and probability. Cartan was a member of the topology team, directed by Jean Cerf. Not only did he play an important role in the recruitment of topologists, but he perfectly understood the need to favor the fledgling teams in applied mathematics, numerical analysis, and probability and statistics. This enlargement of the mathematical field proved essential to maintaining and improving the scientific level of the department.

Cartan was a teacher. He had been the teacher of generations of mathematicians before moving to Orsay. In Orsay he had ordinary university students, and he taught classical subjects. The ordinary students of this time were actually quite decent students, interested in mathematics, interested in their studies, and very demanding of their teachers. Cartan impressed them and made them work. He, too, was very demanding of students and collaborators, checking everything with his personal style, blending rigor and irony. On the other hand he was extremely kind and thoughtful toward students and collaborators when they had personal difficulties. As a mathematician his figure was severe, but he was sensitive to other aspects of life and more than friendly as a human being.

His course was called "Algebra and Geometry". It included hyperbolic geometry and some important theorems in number theory (the Dirichlet theorem on prime numbers, for example). There was a mimeographed version of the course, but it was never published. His style of exposition was not what could be expected from a founder of Bourbaki. He wanted things to be understood in the most intuitive way. Examples (recollected by one of his assistants): an affine space is nothing but a linear space that lost its origin; or, a homographic transformation in the plane is simply a dilation/rotation (or a translation), if you place the fixed points at a right place.

Photograph © Mark Marian Schmidt.

Nicole and Henri Cartan, Paris, 1982.

When Cartan moved to Orsay, he was president of IMU, the International Mathematical Union. He chaired the Fields Medals Committee for the Fields Medals awards of 1970 (Baker, Hironaka, Novikov, Thomson). Just before retiring he was elected a full member of the French Academy of Sciences (he had been a corresponding member for nine years).

After the Retirement

Henri Cartan retired on October 1, 1975.

A symposium in his honor, on analysis and topology, was organized in the main amphitheater of Orsay on June 18, 19, and 20, 1975. I had been elected president of the university one month earlier, and it was the first scientific event that I attended in this capacity. A special booklet was issued by the university with the general nontechnical speeches, and the scientific contributions were published by *Astérisque*.

Cartan stayed on as emeritus in the department; he only had to share his office, and his name remained on the door until he died. He continued to use the library. For some time he was the only mathematician at Orsay who was a member of the Academy of Sciences. He was very active as an academician. The Academy of Sciences still applies an unwritten rule, *la règle de Cartan*, for electing new members: the number of people to be considered should be strictly larger than twice the number of available positions. He took part in a most important decision of the Academy: to decrease the average age of the members of the Academy, a number of new members are required to be younger than fifty-five at the time of

Cartan with a granddaughter, Dolomieu, August 1977.

election. He took part in every meeting as far as he could, and even after he was 100 he never failed to send a proxy when important votes were going to occur. I had the privilege of being called to the telephone on these occasions and hearing his firm, clear voice.

He was awarded the Médaille d'Or du CNRS in 1976 and the Wolf Prize in 1980, as well as being a member of a number of foreign academies. A celebration of his 100th anniversary took place in Paris. The most recent celebration was organized at Orsay, when the main amphitheater of the university was named Amphithéâtre Henri Cartan.

Of course, Cartan didn't need this in order to be remembered in Orsay. He had been one of the founders of the university and a kind of father of the mathematics department. We are proud to be part of his heritage.

Max Karoubi

Some Souvenirs of Henri Cartan

The first time I met Henri Cartan was at the École Normale Supérieure in Paris, when I was admitted in 1959. Despite excellent results in previous mathematics studies, the contact with the École Normale was a shock to me, as for most students of my generation. Actually, according to French tradition, we spent a lot of time at the École Normale, trying to understand Bourbaki's books, although we had no research experience…

Personally, I was not very successful at this exercise and largely preferred to attend Cartan's lectures, either at the École Normale or at the University of Paris. Indeed, Cartan was the best professor I ever knew. He had a unique way of captivating the audience from his first words, by the depth of his knowledge, of course, but also by his eloquence and perfect French. Nothing was left in the dark, no proof was omitted. It was clear that everything was thought out, not only in the details, never boring, but more importantly in the research on the most elegant way to prove theorems. At the end of his lectures, we all felt more clever!

Cartan was also supervising carefully the math studies of everyone, and we often had the opportunity to meet him during our four years at the École Normale. This was particularly important at the end of this period, when we started a research project. Then I naturally applied to a position at the CNRS (Centre National de la Recherche Scientifique), the analog of a "predoc" position today. Unfortunately, the application was rejected, probably because my research records were too thin… Cartan was surprised by this bureaucratic decision, not only for me, but also for other rejections he had heard about. He had the conviction that young students should not miss a chance to start scientific research if they were willing to. What happened then gives an idea of his prestige and influence: the French administration shifted gears over the following days, after some vigorous phone calls from Cartan to high rank officials in the government!

After this event, I had no other choice than starting to work seriously on a Ph.D. As a matter of fact, Cartan did not suggest any research project; his point of view (shared by some university professors) was simple. A student really interested in math should be able to raise problems according to his own taste, solve them, and, if sufficiently clever, write his Ph.D. after their solution… The role of Cartan was to check the interest of the questions involved and the correctness of the proofs.

This situation was at the same time stressful and challenging. Therefore, I started to look at many books; among them was Marston Morse's *Calculus of Variations in the Large*, a basic reference for the subject I was interested in. Reading this book was for me like swimming across the Atlantic Ocean, since I was not mature enough to understand the deep connections between differential geometry and algebraic topology.

Being Cartan's student saved me again. Indeed, I attended the 1963/1964 Cartan/Schwartz

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seminar about the Atiyah-Singer index theorem [2]. Its purpose was not only to understand the proof of this remarkable achievement but also to offer an opportunity for the students to approach exciting new research. The seminar was organized so that each student had the responsibility of some "exposés" which Cartan wanted written the "right" way. For instance, I was in charge of the exposés 4 and 5, and this was all right. However, Cartan was not satisfied with the exposé 16 I was in charge of, too. He decided to rewrite it completely (under my name). Actually, Cartan himself wrote many exposés during the years his famous seminar was running.

This Cartan/Schwartz seminar was the starting point of my research. After this experience, I did exactly what Cartan asked us to do: raised my own questions and tried to solve them. The new field of *K*-theory was a fascinating one (and still is), after the fundamental work of Grothendieck, Bott, Atiyah, Singer, Hirzebruch, and others. Many questions were left over, like the relation between Clifford algebras and Bott periodicity, the possibility of deriving the *K*-functor in order to get cohomology theories on a wide class of objects… During this period (1964–65), I had the chance to meet Grothendieck at the IHÉS. Although I was not formally his student, he helped me a lot, together with Cartan, to get my Ph.D. finally written. This was not finished, however: Cartan was asking for a "second thesis", which meant a different subject I should be able to master! Serre was kind enough to recommend to me a book about modular forms, a course he was giving at the Collège de France.

The positions at the CNRS were not supposed to be permanent (this has now changed). Therefore, after my thesis in 1967, I applied for a position at the University of Strasbourg, where Cartan was professor before the war and a little bit after. At that time, and also in 1972, when I applied for another position at the University of Paris, Cartan wrote a letter of recommendation. Of course, I don't know the contents of these letters, but from the decisions of the committees, I believe that the recommendations were positive in both cases. This is leading me to the special filial relationship between Cartan and his former students. He was ready to help when he had an opportunity, such as giving lectures about our work, so that we could get some recognition. For instance, when in Princeton during the years 1966–67, Cartan gave a lecture about my thesis before I visited the United States myself. He also gave a talk at Bourbaki's seminar about my work, as he did previously for others: Koszul, Douady, and so on. After this period, he was always inquiring about my research advances, reading my notes in the *Comptes Rendus*, asking about my own students, and this continued until the last years of his life.

Cartan influenced my mathematics directly much later than my thesis, not just in the field of *K*-theory but also in another aspect of my research linked with classical algebraic topology. The story started from a lecture Grothendieck gave at the IHÉS about a possible generalization to the integral case of the Quillen-Sullivan theory of rational homotopy types. Cartan wrote a short paper in *Inventiones* (in honor of Serre) about this subject [1]. In particular he wrote this sentence: "L'auteur expose ici ce qu'il croit avoir compris lors d'une conférence de Grothendieck á l'IHÉS le 12 décembre 1975."[14] Despite this modesty, I found Cartan's paper very inspiring and going beyond what Grothendieck explained in his lecture. Although the problem of algebraically determining integral homotopy types is still open and difficult, the way Cartan presented it was the origin of many of the papers I wrote the following years, for instance, the recent reference [4].

It is important to underline the deep European convictions of Cartan, after the two wars that were a devastation in Europe and caused so many deaths (including that of Cartan's own brother). I had the opportunity to work with him in more favorable circumstances when the first European Congress of Mathematics was launched in Paris during the year 1992. It had a difficult birth, due to the skepticism of many influential French mathematicians. I deeply think that, without Cartan's support, this Congress, and the ones which followed it,[15] would have had no chance to take place.

As a conclusion, I would like to add some words about the affability of Cartan and his wife, unfortunately deceased six months after him. When invited a few times to their homes, in Paris or Dolomieu, and also on many other occasions, I was greeted so warmly that I had the feeling of being part of the family circle.

We have lost a great mathematician and a man with a great sense of human values.

References

[1] H. CARTAN, Théories cohomologiques, *Invent. Math.* **35** (1976), 261–271.

[2] H. CARTAN and L. SCHWARTZ, *Séminaire 1963/64*, Société Mathématique de France.

[3] M. KAROUBI, Algèbres de Clifford et *K*-théorie, *Ann. Sci. Ec. Norm. Supérieure,* **1**(2) (1968), 161–270.

[4] M. KAROUBI, Cochaînes quasi-commutatives en topologie algébrique, *Pure and Applied Math. Quarterly* (Volume in honor of Serre), **5**(1), 2009.

[14] *The author explains here what he thought he has understood from a lecture of Grothendieck at the IHÉS on December 12, 1975.*

[15] *In Budapest, Barcelona, Stockholm, Amsterdam…*