AMS Short Courses

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Two Short Course proposals have beeen selected for presentation just before the Joint Mathematics Meetings begin. These Short Courses will take place on January 4 and 5, 2011 (Tuesday and Wednesday).

The cost to participate is the same for both courses. Advance registration fees are: member of the AMS or MAA, US$100; nonmembers are US$134; students, unemployed, or emeritus are US$48. These fees are in effect until December 15. If you choose to register at the meeting, the fees are US$140 for members of the AMS or MAA, US$170 for nonmembers, and US$69 for students, unemployed, or emeritus.

Evolutionary Game Dynamics

Karl Sigmund, University of Vienna, organizer

Evolutionary game theory studies basic types of social interactions in populations of players. It is the ideal mathematical tool for methodological individualism, i.e., the reduction of social phenomena to the level of individual actions. Evolutionary game dynamics combines the strategic viewpoint of classical game theory (independent, rational players trying to outguess each other) with population dynamics (successful strategies increase their frequencies).

A substantial part of the appeal of evolutionary game theory comes from its highly diverse applications, such as social dilemmas, evolution of language, or mating behavior in animals. Moreover, its methods are becoming increasingly popular in computer science, engineering, algorithmic game theory, network analysis, machine learning, statistical procedures, and control theory. They help to design and control multi-agent systems, often with large numbers of agents (for instance, when routing drivers over highway networks, or data packets over the Internet). While traditionally these fields have used a top down approach, by directly controlling the behavior of each agent in the system, attention has recently turned to an indirect approach: allowing the agents to function independently, while providing incentives that lead them to behave in the desired way. Instead of the traditional assumption of equilibrium behavior, researchers opt increasingly for the evolutionary paradigm, and consider the dynamics of behavior in populations of agents employing simple, myopic decision rules.

The lectures offer a menu whose main course is based on deterministic and stochastic dynamics describing the evolution of frequencies of behavioral types, and whose side dishes consist of examples drawn from disciplines as diverse as microbiology, genetics, animal behavior, evolutionary psychology, route planning, e-auctions, common resources management, or micro-economics.

An introductory part of the course is devoted to a brief sketch of the origins of the field, and in particular to the examples that motivated evolutionary biologists to introduce a population dynamical viewpoint into game theory. This leads to some of the main concepts: evolutionary stability, replicator dynamics, invasion fitness, etc. Much of it can be explained by means of simple examples such as the Rock-Paper-Scissors game. It came as a surprise when childish games of that sort, intended only for the clarification of concepts, were found to actually lurk behind important classes of real-life social and biological interactions. The Ultimatum Game, the Prisoner's Dilemma, or the Stag-hunt Game have been used in hundreds of economic experiments, leading to fascinating insights into the role of fairness norms, moral emotions, unconscious motivations, and cultural differences. Behavioral economics promoted the design of efficient mechanisms for broadband auctions or globalized e-commerce, but also led to hot debates on the economic roles of punishment, inequity aversion, beliefs in supernatural agents, or concerns for reputation.

The transmission of successful strategies by genetic and cultural means results in a rich variety of stochastic processes and, in the limit of very large populations, deterministic adjustment dynamics including differential inclusions and reaction-diffusion equations. Some economists view these types of dynamics as basic tools for so-called equilibrium refinement and equilibrium selection concepts. (Indeed, most games have so many equilibria that it is hard to select the "right one"). However, evolutionary games have also permitted the move away from the equilibrium-centered viewpoint. Today, we understand that it is often premature to assume that behavior converges to an equilibrium. In particular, an evolutionarily stable strategy need not be reachable. Limit phenomena such as periodic or heteroclinic cycles, or chaotic attractors, may be considered, perhaps not as "solutions of the game", but as predictions of play. On the other hand, large classes of games leading to global convergence are presently much better understood.

The team for this AMS course consists of **Ross Cressman**, Wilfried Laurier University; **Josef Hofbauer**, University of Vienna; **Sabin Lessard**, Université de Montréal; **Bill Sandholm**, University of Wisconsin, **Karl Sigmund**, University of Vienna; and **Sylvain Sorin**, Université Pierre et Marie Curie, Paris. These speakers have substantially contributed to the field. They will provide a thoroughly up-to-date introduction to evolutionary games for mathematicians interested in the bottom-up analysis of social behavior.

Relevant Literature

CRESSMAN, R., *Evolutionary Dynamics and Extensive Form Games*, MIT Press, Cambridge, MA, 2003.

HOFBAUER, J. and SIGMUND, K., Evolutionary game dynamics, *Bull. AMS* **40** (2003), 479–519.

LESSARD, S. and LADRET, V., The probability of fixation of a single mutant in an exchangeable selection model, *Journal of Math Biology* **54** (2007), 721–744.

SANDHOLM, W. H., *Population Games and Evolutionary Dynamics*, Harvard, MIT Press, 2010.

SIGMUND, K., *The Calculus of Selfishness*, Princeton University Press, 2010.

SORIN, S.; VIOSSAT, Y.; and HOFBAUER, J.: Time average replicator and best reply dynamics, *Math. Operations Research* **34** (2009), 263–269

Computational Topology

Afra Zomorodian, Dartmouth College, organizer

Introduction

The area of computational topology developed in response to topological impediments emerging from within geometric problems, such as reconstructing watertight surfaces in computer graphics. Topological problems, however, arise naturally in many areas of inquiry. In robotics, for instance, we need to capture the connectivity of the configuration space of a robot in order to plan optimal trajectories. In computational structural biology, optimal trajectories within the conformation space of a protein define its folding path. More recently, topological data analysis has emerged as a new paradigm for robust analysis of noisy, high-dimensional, heterogeneous, sampled data.

Like topology, computational topology is a large and diverse area. The aim of this short course is to introduce the audience to recent theoretical as well as practical developments of this field, starting with a theoretical grounding in algebraic topology and ending with analysis of real world data using fast software. Topics covered in the course may include:

1. Review of algebraic topology invariants, such as homotopy, homology, and the Euler characteristic, with an emphasis on algorithms and computation.

2. Theory for robust computation of features of spaces, such as persistent homology, multidimensional persistence

3. Data Structures for representing the underlying topology of sampled data, such as the witness complex.

4. Algorithms for computing robust invariants of spaces efficiently, such the persistence algorithm.

5. Software for analyzing sampled data, such as JPlex, mapper, and eucharis.

6. Applications of computational topology to real world data, such as in computer vision, biophysics, and sensor networks.

Lectures

We have the following confirmed speakers with preliminary titles and abstracts for their talks.

Topological Data Analysis

Gunnar Carlsson, Stanford University

Carlsson will talk about the general philosophy of topological data analysis, that one needs methods which are in some way robust to changes in metrics and to noise. He will describe various aspects of ordinary topology, specifically homology, diagrams, and mapping, and how they play out in the data analytics/statistical world. Carlsson will demonstrate software such as JPlex, mapper, and zigzags on real world data, such as image processing, protein folding, gene expression, and possibly climate data.

Planning with Uncertainty

Michael Erdmann, Carnegie Mellon University

Erdmann uses methods from combinatorial topology to create uncertainty-tolerant strategies for accomplishing robotic tasks. A key theorem shows that such a task has a guaranteed solution if and only if a certain simplicial complex associated with the task is homotopic to a sphere of a certain dimension. This special result leads to a general graph controllability theorem characterizing the ability of a system to achieve any goal despite control uncertainty, again in terms of the existence of a certain sphere. These theorems lead to algorithms, all of which have been implemented.

Optimal Generators

Jeff Erickson, University of Illinois at Urbana-Champaign

Many applications of computational topology arise from geometric problems, such as topological simplification of three-dimensional surface models in computer graphics. For these applications, we require geometric descriptions of topological features, based on various geometric measures, such as shortest homology cycles. Erickson will discuss recent results from computational geometry for computing such descriptions, including recent results on homology flows and cohomology cuts of surface-embedded graphs.

Euler Calculus and Data over Networks

Robert Ghrist, University of Pennsylvania
Ghrist will talk about Euler calculus: using sheaves and the Euler characteristic measure to aggregate data over networks. He will demonstrate and give a tutorial on eucharis, software for integration, and integral transforms over networks.

Cubical Homology and Dynamical Systems

Marian Mrozek, Jagiellonian University (Poland)
Mrozek discusses reduction algorithms for computing cubical homology, including classical free reductions as well as shaving, reductions via acyclic subspace, and coreductions, among other techniques. These techniques have been implemented in CHomP, a cubical homology software developed by Mrozek and colleagues. Mrozek will then describe applying homological invariants toward understanding chaotic dynamical systems.

Persistence: Theory & Practice

Afra Zomorodian, Dartmouth College
Zomorodian will introduce persistence, a notion for identifying features of data robustly. He will then discuss various computational theories of persistence for different classes of data, such as static and parameterized data. In each case, he will describe the theoretical structure, algorithms for computation of invariants, their implementation, and application toward real world examples, such as in biophysics and computer vision.

Schedule

It is planned that each speaker will give a ninety-minute talk. Currently, there is one software session for examining three software programs described by the first three speakers (possibly, persistence, mapper, and CHomP.) The idea is to have some interaction with software on the first day, so there is time for discussion on the second day. The panel discussion currently scheduled at the end of the second day may also be turned into a software session.