Fields Medals Awarded

On August 19, 2010, four Fields Medals were awarded at the opening ceremonies of the International Congress of Mathematicians (ICM) in Hyderabad, India. The medalists are ELON LINDENSTRAUSS, NGÔ BAO CHÂU, STANISLAV SMIRNOV, and CÉDRIC VILLANI.

The Fields Medals are given every four years by the International Mathematical Union (IMU). Although there is no formal age limit for recipients, the medals have traditionally been presented to mathematicians under forty years of age as an encouragement to future achievement. The medal is named after the Canadian mathematician John Charles Fields (1863–1932), who organized the 1924 ICM in Toronto. At a 1931 meeting of the Committee of the International Congress, chaired by Fields, it was decided that funds left over from the Toronto ICM "should be set apart for two medals to be awarded in connection with successive International Mathematical Congresses." In outlining the rules for awarding the medals, Fields specified that the medals "should be of a character as purely international and impersonal as possible." During the 1960s, in light of the great expansion of mathematics research, the possible number of medals to be awarded was increased from two to four. Today the Fields Medal is recognized as the world's highest honor in mathematics.

Previous recipients of the Fields Medal are: Lars V. Ahlfors and Jesse Douglas (1936); Laurent Schwartz and Atle Selberg (1950); Kunihiko Kodaira and Jean-Pierre Serre (1954); Klaus Roth and René Thom (1958); Lars Hörmander and John W. Milnor (1962); Michael F. Atiyah, Paul J. Cohen, Alexander Grothendieck, and Stephen Smale (1966); Alan Baker, Heisuke Hironaka, Sergei Novikov, and John G. Thompson (1970); Enrico Bombieri and David Mumford (1974); Pierre R. Deligne, Charles Fefferman, Grigorii A. Margulis, and Daniel G. Quillen (1978); Alain Connes, William P. Thurston, and Shing-Tung Yau (1982); Simon K. Donaldson, Gerd Faltings, and Michael H. Freedman (1986); Vladimir Drinfel'd, Vaughan F. R. Jones, Shigefumi Mori, and Edward Witten (1990); Jean Bourgain, Pierre-Louis Lions, Jean-Christophe Yoccoz, and Efim Zelmanov (1994); Richard Borcherds, William Timothy Gowers, Maxim Kontsevich, and Curtis T. McMullen (1998); Laurent Lafforgue and Vladimir Voevodsky (2002); and Andrei Okounkov, Grigori Perelman, (medal declined), Terence Tao, and Wendelin Werner (2006).

Elon Lindenstrauss

Citation: "For his results on measure rigidity in ergodic theory and their applications to number theory."

Elon Lindenstrauss

Photographs courtesy of ICM/IMU.

Elon Lindenstrauss has developed extraordinarily powerful theoretical tools in ergodic theory, a field of mathematics initially developed to understand celestial mechanics. He then used them, together with his deep understanding of ergodic theory, to solve a series of striking problems in areas of mathematics that are seemingly far afield. His methods are expected to continue to yield rich insights throughout mathematics for decades to come.

Ergodic theory studies dynamical systems, which are simply mathematical rules that describe how a system changes over time. So, for example, a dynamical system might describe a billiard ball ricocheting around a frictionless, pocketless billiard table. The ball will travel in a straight line until it hits the side of the table, which it will bounce off of as if from a mirror. If the table is rectangular, this dynamical system is pretty simple and predictable, because a ball sent in any direction will end up bouncing off each of the four walls at a consistent angle. But suppose, on the other hand, that the billiard table has rounded ends like a stadium. In that case, a ball from almost any starting position headed in almost any direction will shoot all over the entire stadium at endlessly varying angles. Systems with this kind of complicated behavior are called “ergodic”.

The way that mathematicians pin down this notion that the trajectories spread out all over the space is through the notion of “measure invariance”. A measure can be thought of as a more flexible way to compute area, and having an invariant measure essentially ensures that if two regions of the space in some sense have equal areas, points will travel into them the same percentage of the time. By contrast, in the rectangular table (which of course is not ergodic), the center will get very little traffic in most directions.

In many dynamical systems, there is more than one invariant measure, that is, more than one way of computing area for which almost all the trajectories will go into equal areas equally often. In fact, there are often infinitely many invariant measures. What Lindenstrauss showed, however, is that in certain circumstances, there can be only a very few invariant measures. This turns out to be an extremely powerful tool, a kind of hammer that can break hard problems open.

Lindenstrauss then adroitly wielded his hammer to crack some hard problems indeed. One example of this is in an area called “Diophantine approximations”, which is about finding rational numbers that are usefully close to irrational ones. Pi, for example, can be approximated pretty well as 22/7. The rational number 179/57 is a bit closer, but because its denominator is so much larger, it’s not as convenient an approximation. In the early nineteenth century, the German mathematician Johann Dirichlet proposed one possible standard for judging the quality of an approximation: The imprecision of a rational approximation p/q should be less than $1/(q^2)$. He then went on to show, in a not very difficult proof, that there are infinitely many approximations to any irrational number that meet this standard. (To put this in formula form, he showed that for any real number α, there are infinitely many integers p and q such that $|\alpha - p/q| < 1/q^2$.)

Eighty years ago the British mathematician John Edensor Littlewood proposed an analogue to Dirichlet’s statement to approximate two irrational numbers at once: It should be possible, he figured, to find approximations p/q to α and r/q to β so that the product of the imprecision of the two approximations would be as small as you please. (In formula form, the claim is that for any real numbers α and β and any tiny positive quantity ε you like, there will be approximations p/q to α and r/q to β so that $|\alpha - p/q| \times |\beta - r/q| < \varepsilon/q^3$.) He gave the problem to his graduate students, thinking it should not be that much harder than Dirichlet’s proof. But the Littlewood conjecture turned out to be extraordinarily difficult, and until recently, no substantial progress had been made on it.

Then Lindenstrauss brought his ergodic theory tools to the problem, in joint work with Manfred Einsiedler and Anatole Katok. Ergodic theory might seem an odd choice for a problem that does not involve dynamical systems or time, but such unlikely pairings are sometimes the most powerful. Here’s one way of reformulating Littlewood’s problem to see a connection: First imagine a unit square, and glue the top edge to the bottom edge to make a cylinder. Now glue the right edge to the left edge and you’ll get a shape called a torus that looks like a donut. You can roll up the entire coordinate plane to this same shape by gluing any point (x, y) to the point whose x-coordinate is the fractional part of x and whose y-coordinate is the fractional part of y. This torus is the space of our dynamical system. We can then define a transformation by taking any point (x, y) to another point $(x+\alpha, y+\beta)$. If α and β are irrational (or more precisely, not rationally related), this dynamical system will be ergodic. The Littlewood conjecture then becomes the claim that you can make these trajectories suitably close to the origin by applying the transformation enough times. The number of times you apply the transformation becomes the denominator of the fractions approximating α and β.

Using a reformulation of the Littlewood conjecture in terms of a more complex dynamical system, the team made a huge step of progress on the conjecture: They showed that if there are any pairs of numbers for which the conjecture is false, there are only a very few of them, a negligible portion of them all.

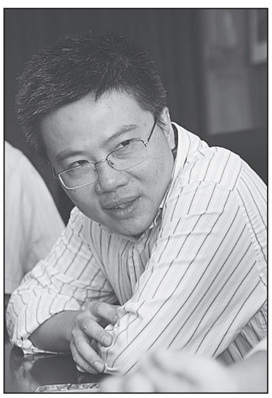
Another example of the power of Lindenstrauss’s work is his proof of the first nontrivial case of the arithmetic quantum unique ergodicity conjecture. Ergodic systems come up frequently in physics, because as soon as you have three bodies interacting, for example, the system starts to behave in a somewhat ergodic fashion. But if those interactions happen at the quantum scale, you cannot describe them with the ordinary tools of ergodic theory, because quantum theory does not allow for well-defined paths of points at well-defined positions; instead, you can only consider the probability that a point will exist in a particular position at a particular time. Analyzing such systems mathematically has proven extraordinarily difficult, and physicists have had to rely on numerical simulations alone, without a firm mathematical underpinning.

The quantum unique ergodicity conjecture says, roughly, that if you calculate area using the measure that is natural in classical dynamics, then as the energy of the system goes up, this probability distribution becomes more evenly distributed over the space. Furthermore, this measure is the only one for which that is true. Lindenstrauss was able to prove this in an arithmetic context for particular kinds of dynamical systems, creating one of the first major, rigorous advances in the theory of quantum chaos.

Elon Lindenstrauss was born in 1970 in Jerusalem. He received his Ph.D. in mathematics from the Hebrew University of Jerusalem in 1999. He is professor of mathematics at Hebrew University and at Princeton University. He has been a member of the Institute for Advanced Study, Princeton; a Clay Mathematics Institute Long-Term Prize Fellow; a visiting member of the Courant Institute of Mathematical Sciences at New York University; and an assistant professor at Stanford University. His distinctions include the Leonard M. and Eleanor B. Blumenthal Award for the Advancement of Research in Pure Mathematics (2001), the 2003 Salem Prize, the European Mathematical Society Prize (2004), and the Anna and Lajos Erdős Prize in Mathematics (2009).

Ngô Bao Châu

Citation: "For his proof of the fundamental lemma in the theory of automorphic forms through the introduction of new algebro-geometric methods."



Ngô Bao Châu

Ngô Bao Châu removed one of the great impediments to a grand, decades-long program to uncover hidden connections between seemingly disparate areas of mathematics. In doing so, he provided a solid foundation for a large body of theory and developed techniques that are likely to unleash a flood of new results.

The path to Ngô's achievement began in 1967, when the mathematician Robert Langlands had a mind-boggling bold vision of a sort of mathematical wormhole connecting fields that seemed to be light years apart. His proposal was so ambitious and unlikely that when he first wrote of it to the great number theorist André Weil, he began with this sheepish note: "If you are willing to read [my letter] as pure speculation I would appreciate that; if not—I am sure you have a wastebasket handy." Langlands then laid out a series of dazzling conjectures that have proven to be a road map for a large area of research ever since.

The great majority of those conjectures remain unproven and are expected to occupy mathematicians for generations to come. Even so, the progress on the program so far has been a powerful engine for new mathematical results, including Andrew Wiles's proof of Fermat's Last Theorem and Richard Taylor's proof of the Sato-Tate conjecture. The full realization of Langlands's program would unify many of the fields of modern mathematics, including number theory, group theory, representation theory, and algebraic geometry.

Langlands's vision was of a bridge across a division in mathematics dating all the way back to Euclid's time, that between magnitude and multitude. Magnitudes are the mathematical form of butter, a continuous smear of stuff that can be divided up into pieces as small as you please. Lines and curves, planes, the space we live in, and even higher-dimensional spaces are all magnitudes, and they are commonly studied with the tools of geometry and analysis. Multitudes, on the other hand, are like beans, discrete objects that can be put in piles but cannot be split without losing their essence. The whole numbers are the canonical example of multitudes, and they are studied with the tools of number theory. Langlands predicted that certain numbers that arise in analysis—specifically, the eigenvalues of certain operators on differential forms on particular Riemannian manifolds, called automorphic forms—were actually a code that, if unraveled, would classify fundamental objects in the arithmetic world.

One of the tools developed from the Langlands program is the "Arthur-Selberg trace formula", an equation that shows precisely how geometric information can calculate arithmetic information. That is valuable in itself and, furthermore, is a building block in proving Langlands's principle of functoriality, one of the great pillars of his program. But Langlands ran across an annoying stumbling block in trying to use the trace formula. He kept encountering complicated finite sums that clearly seemed to be equal, but he couldn't quite figure out how to show it. It seemed like a straightforward problem, one that could be solved with a bit of combinatorial fiddling, so he called it a "lemma"—the term for a minor but useful result—and assigned it to a graduate student.

When the graduate student could not prove it, he tried another. Then he worked on it himself. Then he consulted with other mathematicians. At the same time as everyone continued to fail to prove it, the critical need for the result became increasingly clear. So the problem came to have a slightly grander title: the "fundamental lemma".

After three decades of work, only a few special cases had yielded to proof. The lack of a proof was such a roadblock to progress that many mathematicians had begun simply assuming it was true and developing results that depended upon it, creating a huge body of theory that would come crashing down if it turned out to be false.

Ngô Bao Châu was the one to finally break the problem open. The complicated identities in the fundamental lemma, he realized, could be seen as arising naturally out of sophisticated mathematical objects known as Hitchen fibrations. His approach was entirely novel and unexpected: Hitchen fibrations are purely geometric objects that are close to mathematical physics, nearly the last thing anyone expected to be relevant to this problem in the purest of pure math.

But it was instantly clear that he had made a profound connection. His approach turned the annoying, fiddly complexity of the fundamental lemma into a simple, natural statement about Hitchen fibrations. Even before he had managed to complete the proof, he had achieved something even more impressive: he had created genuine understanding.

Furthermore, by putting the problem in this much bigger framework, Ngô gave himself powerful new tools to assault it with. In 2004 he proved some important and difficult special cases working with his former thesis advisor, Gérard Laumon, and, in 2008, using his new methods, he cracked the problem in its full generality.

Ngô's methods are so novel that mathematicians expect them to break open a number of other problems as well. A prime target is another piece of Langlands's program, his "theory of endoscopy".

His techniques might even point the way toward a proof of the full principle of functoriality, which would be close to a full realization of Langlands's original vision. Langlands himself, who is now more than seventy years old and still hard at work, has developed a highly speculative but enticing approach to the problem. It is still far from clear that these ideas will lead to a proof, but if they do, they will have to rely on the kinds of geometric ideas that Ngô has introduced.

Ngô Bao Châu was born on June 28, 1972, in Hanoi, Vietnam. After secondary school in Vietnam, he moved to France and studied at the Université Paris 6, École Normale Supérieure de Paris. He completed his Ph.D. degree in Orsay under the supervision of Gérard Laumon. He is currently professor in the Faculté des Sciences at Orsay and a member of the Institute for Advanced Study in Princeton. In September 2010 he began his new appointment at the University of Chicago. Jointly with Laumon, Ngô was awarded the Clay Research Award in 2004. In 2007 he was awarded the Sophie Germain Prize and the Oberwolfach Prize.

Stanislav Smirnov

Citation: "For the proof of conformal invariance of percolation and the planar Ising model in statistical physics."

Stanislav Smirnov has put a firm mathematical foundation under a burgeoning area of mathematical physics. He gave elegant proofs of two long-standing, fundamental conjectures in statistical physics, finding surprising symmetries in mathematical models of physical phenomena.

Though Smirnov's work is highly theoretical, it relates to some surprisingly practical questions. For instance, when can water flow through soil and when is it blocked? For it to flow, small-scale pores in the soil must link up to provide a continuous channel from one place to another. This is a classic question in statistical physics, because the large-scale behavior of this system (whether the water can flow through a continuous channel of pores) is determined by its small-scale, probabilistic behavior (the chance that at any given spot in the soil, there will be a pore).

Stanislav Smirnov

It is also a natural question to model mathematically. Imagine each spot in the soil as lying on a grid or lattice, and color the spot blue if water can flow and yellow if it can't. Determine the color of each spot by the toss of a coin (heads for yellow, tails for blue), using a coin that might be weighted rather than fair. If a path of blue spots crosses from one side of a rectangle to the other, the water can pass from one side to the other.

Such "percolation models" behave in a remarkable way. For extreme values, the behavior is as you might expect: If the coin is heavily weighted against blue, the water almost certainly will not flow, and if it is heavily weighted toward blue, the water almost certainly will. But the probability of flow does not change evenly as the percentage of blue spots increases. Instead, the water is almost certainly going to be blocked until the percentage of blue spots reaches some threshold value, and once it does, the probability that the water will flow starts surging upward. This threshold is called the "critical point". Abrupt change of behavior like this is a bit like what happens to water as it heats: suddenly, at a critical temperature, the water boils. For that reason, this phenomenon is commonly called a phase transition.

But of course, real soil does not come with neat, evenly spaced horizontal or vertical pores. So to apply this model to the real world, a couple of troublesome questions arise. First, how fine should the lattice grid be? Physicists are most interested in understanding processes at the molecular scale, in which case the grid should be very small indeed. Mathematicians then ask about the relationship between models with ever-smaller grids. Their hope is that as the grids get finer, the models will get closer and closer to one single model that effectively has an infinitely fine grid, called a "scaling limit".

To see why it is not obvious that the scaling limit will exist, imagine choosing a particular percentage of blue spots for a lattice and calculating the probability that the pores will line up to form a crossing. Then make the grid size smaller and calculate it again. As the grids get finer, the crossing probabilities may get closer and closer to some number, the way the numbers 1.9, 1.99, 1.999, 1.9999…get closer and closer to 2. In that case, this number will be the crossing probability for the scaling limit. But it is imaginable that the crossing probabilities will jump around and never converge toward a limit, like the sequence of numbers 2, 4, 2, 4, 2, 4… In that case, should the crossing probability for the scaling limit be 2 or 4? There is no good answer, so we have to say that the scaling limit does not exist.

Another potentially problematic question is what shape lattice to use. Even if we restrict ourselves to two dimensions, there are many choices: square lattices, triangular lattices, rhombic lattices, and so forth. Ideally, the model would be "universal", so that the choice of lattice shape does not matter, but that is not obviously true.

Physicists are pretty sure that neither of these potential problems is so bad. Using physical intuition, they have argued convincingly that the model will indeed approach a well-defined scaling limit as the grid gets finer. Furthermore, though the choice of lattice shape does affect the critical point, physicists have persuaded themselves that it will not affect many of the other properties they are interested in.

Physicists have figured out even more about two-dimensional lattices, including finding evidence for a surprising and beautiful symmetry. Imagine taking a lattice of any shape and stretching it or squinching it but leaving the angles all the same. The Mercator projection of the globe is an example of this: Greenland is huge, since distances are changed, but latitude and longitude lines nevertheless stay at right angles. Physicists have convinced themselves that if you transform two-dimensional percolation models in this way, it will not change their scaling limits (as long as you are near the critical points). Or, to use the technical term, they are persuaded that scaling limits are "conformally invariant".

In 1992 John Cardy, a physicist at the University of Oxford, used this insight to achieve one of percolation theory's big goals: a precise formula that calculates the crossing probabilities of the scaling limits of two-dimensional lattices near the critical point. The only problem was that, although his physical arguments were persuasive, neither he nor anyone else could turn that physical intuition into a mathematical proof.

In 2001 Smirnov put all this physical theory on a firm mathematical foundation. He proved that scaling limits are conformally invariant, though only for the triangular lattice (the shape that pennies, for example, fall into naturally when laid flat on a table and packed tightly together). In the process, he also proved the correctness of Cardy's formula for triangular lattices. His proof used an approach independent of ones used earlier by physicists that provided fundamental new insights. It also provided a critical missing step in the theory of Schramm-Loewner evolution, an important, recently developed method in statistical physics.

In another major achievement, Smirnov used similar methods to understand the Ising model, which describes such phenomena as magnetism, gas movement, image processing, and ecology. Just as with percolation, the large-scale behaviors of these phenomena are determined by their probabilistic, small-scale behavior. Consider, for example, magnetism. The atoms in a piece of iron behave like miniature magnets, with the electrons moving around the nucleus, creating a miniature magnetic field. The atoms try to pull their neighbors into the same alignment as their own. When enough atoms have their north poles pointing the same direction, the iron as a whole becomes magnetic. Mathematicians model this by visualizing the atoms as lying on the nodes of a lattice, with statistical rules that determine whether they are aligned with their north poles pointing up or down.

Like the percolation model, the Ising model undergoes a phase transition: as the iron is heated, the atoms vibrate more quickly, and if it is heated above a certain point, the vibrations are so strong that neighboring atoms suddenly no longer hold one another in alignment and the piece as a whole begins to lose its magnetism.

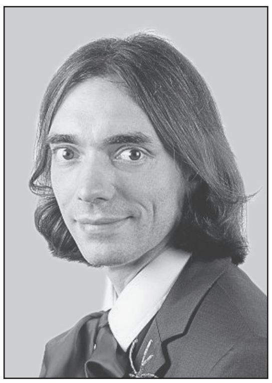
The same questions that mathematicians and physicists worry about in percolation also apply to the Ising model. The grid should be extremely small, since it is operating on the atomic level. So as the grid mesh gets finer and finer, does the model converge toward some infinitely fine version, a scaling limit? Furthermore, how does the lattice shape affect the critical point and other properties? And what happens if one stretches or squishes the lattice without changing the angles; does the scaling limit change?

For this model, too, Smirnov was able to show that the models do indeed converge toward a scaling limit as the grid mesh gets finer and that they are unaffected by stretches and squishes—that is, that they are conformally invariant. Later, with Dmitry Chelkak, he established universality, extending the results to a wide range of different lattices. He has also done significant work in analysis and dynamical systems. His work will continue to enrich both mathematics and physics in the future.

Stanislav Smirnov was born in 1970 in St. Petersburg, Russia. He received his Ph.D. from the California Institute of Technology in 1996 under the direction of Nikolai Makarov. He held a Gibbs Instructorship at Yale University and short-term positions at the Institute for Advanced Study, Princeton, and the Max Planck Institute for Mathematics (MPIM), Bonn. He moved to Sweden in 1998 and became professor at the Royal Institute of Technology and researcher at the Swedish Royal Academy of Sciences in 2001. Since 2003 he has been a professor at the University of Geneva, Switzerland. His distinctions include the St. Petersburg Mathematical Society Prize (1997), the Clay Research Award (2001), the Salem Prize (2001), the Gran Gustafsson Research Prize (2001), the Rollo Davidson Prize (2002), and the European Mathematical Society Prize (2004).

Cédric Villani

Citation: "For his proofs of nonlinear Landau damping and convergence to equilibrium for the Boltzmann equation."



Cédric Villani

Cédric Villani has provided a deep mathematical understanding of a variety of physical phenomena. At the center of much of his work is his profound mathematical interpretation of the physical concept of entropy, which he has applied to solving major problems inspired by physics. Furthermore, his results have fed back into mathematics, enriching both fields through the connection.

Villani began his mathematical career by reexamining one of the most shocking and controversial theories of nineteenth-century physics. In 1872 Ludwig Boltzmann studied what happens when the stopper is removed on a gas-filled beaker and the gas spreads around the room. Boltzmann explained the process by calculating the probability that a molecule of gas would be in a particular spot with a particular velocity at any particular moment—before the atomic theory of matter was widely accepted. Even more shockingly, though, his equation created an arrow of time.

The issue was this: when molecules bounce off each other, their interactions are regulated by Newton's laws, which are all perfectly time-reversible; that is, in principle, we could stop time, send all the molecules back in the direction they had come from, and they would zip right back into the beaker. But Boltzmann's equation is not time-reversible. The molecules almost always go from a state of greater order (e.g., enclosed in the beaker) to less order (e.g., spread around the room). Or, more technically, entropy increases.

Over the next decades, physicists reconciled themselves to entropy's emergence from time-reversible laws, and, indeed, entropy became a key tool in physics, probability theory, and information theory. A key question remained unanswered, though: How quickly does entropy increase? Experiments and numerical simulations could provide rough estimates, but no deep understanding of the process existed.

Villani, together with his collaborators Giuseppe Toscani and Laurent Desvillettes, developed the mathematical underpinnings needed to get a rigorous answer, even when the gas starts from a highly ordered state that has a long way to go to reach its disordered, equilibrium state. His discovery had a completely unexpected implication: though entropy always increases, sometimes it does so faster and sometimes more slowly. Furthermore, his work revealed connections between entropy and apparently unrelated areas of mathematics, such as Korn's inequality from elasticity theory.

After this accomplishment, Villani brought his deep understanding of entropy to another formerly controversial theory. In 1946 the Soviet physicist Lev Davidovich Landau made a mind-bending claim: that, in certain circumstances, a phenomenon can approach equilibrium without increasing entropy.

In a gas, the two phenomena always go together. Gas approaches equilibrium by spreading around a room, losing any order it initially had and increasing entropy as much as possible. But Landau argued that plasma, a gas-like form of matter that contains so much energy that the electrons get ripped away from the atoms, was a different story. In plasma, the free-floating charged particles create an electrical field that in turn drives their motion. This means that unlike particles in a gas, which affect the motion only of other particles they happen to smash against, plasma particles influence the motion of faraway particles that they never touch as well. That means that Boltzmann's equation for gases does not apply—and the Vlasov-Poisson equation that does is time-reversible, and hence does not involve an increase in entropy. Nevertheless, plasma, like gas, spreads out and approaches an equilibrium state. It was believed that this happened only because of the collisions between atoms. But Landau argued that even if there were no collisions, the plasma would move toward equilibrium because of a decay in the electric field. He proved it—but only for a simplified linear approximation of the Vlasov-Poisson equation.

Despite a huge amount of study over the next six decades, little progress was made in understanding how this equilibrium state comes about or in proving Landau's claim for the full Vlasov-Poisson equation. Last year, Villani, in collaboration with Clément Mouhot, finally came to a deep understanding of the process and proved Landau right.

A third major area of Villani's work initially seemed to have nothing to do with entropy, until Villani found deep connections and transformed the field. He became involved in optimal transport theory, which grew out of one of the most practical of questions: Suppose you have a bunch of mines and a bunch of factories, in different locations, with varying costs for moving the ore from each particular mine to each particular factory. What is the cheapest way to transport the ore?

This problem was first studied by the French mathematician Gaspard Monge in 1781 and rediscovered by the Russian mathematician Leonid Kantorovich in 1938. Kantorovich's work on this problem blossomed into an entire field (linear programming), won him the Nobel Prize in economics in 1975, and spread into a remarkable array of areas, including meteorology, dynamical systems, fluid mechanics, irrigation networks, image reconstruction, cosmology, the placement of reflector antennas—and, in the last couple of decades, mathematics.

Villani and Felix Otto made one of the critical connections when they realized that gas diffusion could be understood in the framework of optimal transport. An initial configuration of gas particles can be seen as the mines, and a later configuration can be seen as the factories. (More precisely, it is the probability distribution of the particles in each case.) The farther the gas particles have to move to go from one configuration to the other, the higher the cost.

One can then imagine each of these possible configurations as corresponding to a point in an abstract mountainous landscape. The distance between two points is defined as the optimal transport cost, and the height of each point is defined by the entropy (with low points having high entropy). This gives a beautiful way of understanding what happens as gas spreads out in a room: it is as though the gas rolls down the slopes of this abstract terrain, its configurations changing as specified by the points on the downward path.

Now suppose that a fan is blowing when you open the beaker of gas, so that the gas does not spread uniformly as it diffuses. Mathematically, this can be modeled by considering the space in which the gas is spreading to be distorted or curved. Villani and Otto realized that the curvature of the space in which the gas spreads would translate into the topography of the abstract landscape. This connection allowed them to apply the rich mathematical understanding of curvature (in particular, Ricci curvature, which was critical in the recent solution of the Poincaré conjecture) to answer questions about optimal transport.

Furthermore, Villani and John Lott were able to take advantage of these links with optimal transport to further develop the theory of curvature. For example, mathematicians had not had a way of defining Ricci curvature at all in some situations, such as at a sharp corner. Villani and Lott (and simultaneously, using complementary tools, Karl-Theodor Sturm) were able to use the connection with optimal transport to offer a definition and push the mathematical understanding of curvature to new, deeper levels. This depth of understanding and development of novel connections between different areas is typical of Villani's work.

Cédric Villani was born in 1973 in France. After studying mathematics at École Normale Supérieure in Paris from 1992 to 1996, he was appointed assistant professor there. He received his Ph.D. in 1998. Since 2000 he has been a full professor at École Normale Supérieure de Lyon. He has held semester-long visiting positions in Atlanta, Berkeley, and Princeton. His distinctions include the Jacques Herbrand Prize of the French Academy of Science (2007), the Prize of the European Mathematical Society (2008), the Henri Poincaré Prize of the International Association for Mathematical Physics, and the Fermat Prize (2009). In 2009 he was appointed director of the Institut Henri Poincaré in Paris and part-time visitor at the Institut des Hautes Études Scientifiques.

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