2010 Gauss Prize Awarded

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Yves Meyer

On August 19, 2010, the 2010 Carl Friedrich Gauss Prize for Applications of Mathematics was awarded at the opening ceremonies of the International Congress of Mathematicians (ICM) in Hyderabad, India. The prizewinner is Yves Meyer of the École Normale Supérieure de Cachan, France. He was honored "for fundamental contributions to number theory, operator theory and harmonic analysis, and his pivotal role in the development of wavelets and multiresolution analysis."

The Carl Friedrich Gauss Prize for Applications of Mathematics was instituted in 2006 to recognize mathematical results that have opened new areas of practical applications. It is granted jointly by the International Mathematical Union and the German Mathematical Society and is awarded every four years at the ICM.

"Whenever you feel competent about a theory, just abandon it." This has been Meyer's principle in his more than four decades of outstanding mathematical research work. He believes that only researchers who are newborn to a theme can show imagination and make big contributions. Accordingly, Meyer has passed through four distinct phases of research activity corresponding to his explorations in four disparate areas—quasicrystals, the Calderón-Zygmund program, wavelets, and the Navier-Stokes equation. The varied subjects that he has worked on are indicative of his broad interests. In each one of them Meyer has made fundamental contributions. His extensive work in each would suggest that he does not leave a field of research that he has entered until he is convinced that the subject has been brought to its logical end.

The seeds for Meyer's highly original approach in every branch of mathematics that he has ventured into were perhaps sown early in his career. He started his research career after having been a high school teacher for three years following his university education. He completed his Ph.D. in 1966 in just three years. "I was my own supervisor when I wrote my Ph.D.," Meyer has said. This individualistic perspective to a problem has been his hallmark till this day.

In 1970 Meyer introduced some totally new ideas in harmonic analysis (a branch of mathematics that studies the representation of functions or signals as a superposition of some basic waves) that turned out to be useful not only in number theory but also in the theory of the so-called quasicrystals. There are certain algebraic numbers called the Pisot-Vijayaraghavan numbers and certain numbers known as Salem numbers. These have some remarkable properties that show up in harmonic analysis and Diophantine approximation (approximation of real numbers by rational numbers). For instance, the Golden Ratio is such a number. Yves Meyer studied these numbers and proved a remarkable result. Meyer's work in this area led to notions of Meyer and model sets which played an important role in the mathematical theory of quasicrystals.

Quasicrystals are space-filling structures that are ordered but lack translational symmetry and are aperiodic in general. Classical theory of crystals allows only two-, three-, four-, and sixfold rotational symmetries, but quasicrystals display fivefold symmetry and symmetry of other orders. Just like crystals, quasicrystals produce modified Bragg diffraction, but where crystals have a simple repeating structure, quasicrystals exhibit more complex structures, like aperiodic tilings. Penrose tilings are an example of such an aperiodic structure that displays fivefold symmetry. Meyer studied certain sets in the n-dimensional Euclidean space (now known as a Meyer set) that are characterized by a certain finiteness property of its set of distances. Meyer's idea was that the study of such sets includes the study of possible structures of quasicrystals. This formal basis has now become an important tool in the study of aperiodic structures in general.

In 1975 Meyer collaborated with Ronald Coifman on what are called Calderón-Zygmund operators. The important results that they obtained gave rise to several other works by others, which have led to applications in areas such as complex analysis, partial differential equations, ergodic theory, number theory, and geometric measure theory. This approach of Meyer and Coifman can be looked on as the interplay between two opposing paradigms: the classical complex-analytic approach and the more modern Calderón-Zygmund approach, which relies primarily on real-variable techniques. Nowadays, it is the latter approach that dominates, even for problems that actually belong to the area of complex analysis.

The Calderón-Zygmund approach was the result of the search for new techniques because the complex-analytic methods broke down in higher dimensions. This was done by S. Mihlin, A. Calderón, and A. Zygmund, who investigated and resolved the problem for a wide class of operators, which we now refer to as singular integral operators, or Calderón-Zygmund operators. These singular integral operators are much more flexible than the standard representation of an operator, according to Meyer. His collaborative work with Coifman on certain multilinear integral operators has proved to be of great importance to the subject. With Coifman and Alan MacIntosh he proved the boundedness and continuity of the Cauchy integral operator, which is the most famous example of a singular integral operator, on all Lipschitz curves. This had been a long-standing problem in analysis.

Meyer credits the research phase on wavelets, which have had a tremendous impact on signal and image processing, with having given him a second scientific life. A wavelet is a brief wave-like oscillation with amplitude that starts out at zero, increases, and decreases back to zero, like what may be recorded by a seismograph or heart monitor. But in mathematics these are specially constructed to satisfy certain mathematical requirements and are used in representing data or other functions. As mathematical tools they are used to extract information from many kinds of data, including audio signals and images. Sets of wavelets are generally required to analyze the data. Wavelets can be combined with portions of an unknown signal by the technique of convolution to extract information from the unknown signal.

Representation of functions as a superposition of waves is not new. It has existed since the early 1800s, when Joseph Fourier discovered that he could represent other functions by superposing sines and cosines. Sine and cosine functions have well-defined frequencies but extend to infinity; that is, although they are localized in frequency, they are not localized in time. This means that, although we might be able to determine all the frequencies in a given signal, we do not know when they are present. For this reason a Fourier expansion cannot represent properly transient signals or signals with abrupt changes. For decades scientists have looked for more appropriate functions than these simple sine and cosine functions to approximate choppy signals.

To overcome this problem, several solutions have been developed in the past several decades to represent a signal in the time and the frequency domain at the same time. The effort in this direction began in the 1930s with the Wigner transform, a construction by Eugene Wigner, the famous mathematician-physicist. Basically wavelets are building blocks of function spaces that are more localized than Fourier series and integrals. The idea behind the joint time-frequency representations is to cut the signal of interest into several parts and analyze each part separately with a resolution matched to its scale. In wavelet analysis, appropriate approximating functions that are contained in finite domains thus become very suitable for analyzing data with sharp discontinuities.

The fundamental question that the wavelet approach tries to answer is how to cut the signal. The time-frequency domain representation itself has a limitation imposed by the Heisenberg uncertainty principle that both the time and frequency domains cannot be localized to arbitrary accuracy simultaneously. Therefore, unfolding a signal in the time-frequency plane is a difficult problem, which can be compared to writing the score and listening to the music simultaneously. So groups in diverse fields of research developed techniques to cut up signals localized in time according to resolution scales of their interest. These techniques were the precursors of the wavelet approach.

The wavelet analysis technique begins with choosing a wavelet prototype function, called the mother wavelet. Time-resolution analysis can be performed with a contracted, high-frequency version of the mother wavelet. Frequency-resolution analysis can be performed with a dilated, low-frequency version of the same wavelet. The wavelet transform or wavelet analysis is the most recent solution to overcome the limitations of the Fourier transform. In wavelet analysis the use of a fully scalable modulated window solves the signal-cutting problem mentioned earlier. The window is shifted along the signal, and for every position the spectrum (the transform) is calculated. Then this process is repeated several times with a slightly shorter (or longer) window for every new cycle. The result of this repetitive signal analysis is a collection of time-scale representations of the signal, each with different resolution; in short, multiscale resolution or multiresolution analysis. Simply put, the large scale is the big picture, whereas the small scale shows the details. It is like zooming in without loss of detail. That is, wavelet analysis sees both the forest and trees.

In geophysics and seismic exploration, one could find models to analyze waveforms propagating underground. Multiscale decompositions of images were used in computer vision because the scale depended on the depth of a scene. In audio processing, filter banks of constant octave bandwidth (dilated filters) applied to the analysis of sounds and speech and to handle the problem of Doppler shift multiscale analysis of radar signals were evolved. In physics, multiscale decompositions were used in quantum physics by Kenneth G. Wilson for the representation of coherent states and also to analyze the fractal properties of turbulence. In neurophysiology, dilation models had been introduced by the physicist G. Zweig to model the responses of simple cells in the visual cortex and in the auditory cochlea. Wavelet analysis would bring these disparate approaches together into a unifying framework. Meyer is widely acknowledged as one of the founders of wavelet theory.

In 1981 Jean Morlet, a geologist working on seismic signals, had developed what are known as Morlet wavelets, which performed much better than the Fourier transforms. Actually, Morlet and Alex Grossman, a physicist whom Morlet had approached to understand the mathematical basis of what he was doing, were the first to coin the term “wavelet” in 1984. Meyer heard about the work and was the first to realize the connection between Morlet’s wavelets and earlier mathematical constructs, such as the work of Littlewood and Paley, used for the construction of functional spaces and for the analysis of singular operators in the Calderón-Zygmund program.

Meyer studied whether it was possible to construct an orthonormal basis with wavelets. (An orthonormal basis is like a coordinate system in the space of functions and, like the familiar coordinate axes, each base function is orthogonal to the other. With an orthonormal basis you can represent every function in the space in terms of the basis functions.) This led to his first fundamental result in the subject of wavelets in a Bourbaki seminar article that constructs a lot of orthonormal bases with Schwarz class functions (functions that have values only over a small region and decay rapidly outside). This article was a major breakthrough that enabled subsequent analysis by Meyer. “In this article,” says Stéphane Mallat, “the construction of Meyer had isolated the key structures in which I could recognize similarities with the tools used in computer vision for multiscale image analysis and in signal processing for filter banks.”

A Mallat-Meyer collaboration resulted in the construction of mathematical multiresolution analysis and a characterization of wavelet orthonormal bases with conjugate mirror filters that implement a first wavelet transform algorithm that performed faster than the fast Fourier transform (FFT) algorithm. Thanks to the Meyer-Mallat result, wavelets became much easier to use. One could now do a wavelet analysis without knowing the formula for the mother wavelet. The process was reduced to simple operations of averaging groups of pixels together and taking differences, over and over. The language of wavelets also became more comfortable to electrical engineers.

In the 1980s the digital revolution was all around, and efficient algorithms were critically needed in signal and image processing. The JPEG standard for image compression was developed at that time. In 1987 Ingrid Daubechies, a student of Grossman, while visiting the Courant Institute at New York University and later during her appointment at AT&T Bell Labs, discovered a particular class of compactly supported conjugate mirror filters, which were not only orthogonal (like Meyer’s) but were also stable and which could be implemented using simple digital filtering ideas. The new wavelets were simple to program, and they were smooth functions, unlike some of the earlier jumpy functions. Signal processors now had a dream tool: a way to break up digital data into contributions of various scales.

Combining Daubechies’s and Mallat’s ideas, one could do a simple orthogonal transform that could be rapidly computed in modern digital computers. Daubechies wavelets turn the theory into a practical tool that can be easily programmed and used by a scientist with a minimum of mathematical training. Meyer’s first Bourbaki paper actually laid the foundations for a proper mathematical framework for wavelets. That marked the beginning of modern wavelet theory. In recent years wavelets have begun to provide an interesting alternative to Fourier transform methods.

Interestingly, Meyer’s first reaction to the work of Grossman and Morlet was “So what! We harmonic analysts knew all this a long time ago!” But he looked at the work again and realized that Grossman and Morlet had done something different and interesting. He built on the difference to eventually formulate his basis construction. “Meyer’s construction of the orthonormal bases and his subsequent results in the area were the key discovery that opened the door to all further mathematical developments and applications. Meyer was at the core of the catalysis that brought together mathematicians, scientists, and engineers that built up the theory and resulting algorithms,” says Mallat.

Since the work of Daubechies and Mallat, applications that have been explored include multiresolution signal processing, image and data compression, telecommunications, fingerprint analysis, statistics, numerical analysis, and speech processing. The fast and stable algorithm of Daubechies was improved subsequently in a joint work between Daubechies and Albert Cohen, Meyer’s student, which is now being used in the new standard JPEG2000 for image compression and is now part of the standard toolkit for signal and image processing. Techniques for restoring satellite images have also been developed based on wavelet analysis.

More recently, he has found a surprising connection between his early work on the model sets used to construct quasicrystals—the “Meyer sets”—and “compressed sensing”, a technique used for acquiring and reconstructing a signal utilizing the prior knowledge that it is sparse or compressible. Based on this, he has developed a new algorithm for image processing. A version of such an algorithm has been installed in the space mission Herschel of the European Space Agency, which is aimed at providing images of the oldest and coldest stars in the universe.

"To my knowledge," says Wolfgang Dahmen, "Meyer has never worked directly on a concrete application problem." Thus Meyer's mathematics provide good examples of how the investigations of fundamental mathematical questions often yield surprising results that benefit humanity.

Yves Meyer was born in 1939. After graduating from Ecole Normale Supérieure, Paris, in 1960, he taught high school for three years, then took a teaching assistantship at Université de Strasbourg. He received his Ph.D. from Strasbourg in 1966. He has been a professor at École Polytechnique and Université Paris-Dauphine and has also held a full research position at Centre National de la Recherche Scientifique. He was professor at École Normale Supérieure de Cachan, France, from 1999 to 2009 and is currently professor emeritus. He is a foreign honorary member of the American Academy of Arts and Sciences. He has also been awarded a doctorate (Honoris causa) by Universidad Autonoma de Madrid.

—from an IMU news release

2010 Nevanlinna Prize Awarded

On August 19, 2010, the 2010 Rolf Nevanlinna Prize was awarded at the opening ceremonies of the International Congress of Mathematicians (ICM) in Hyderabad, India. The prizewinner is DANIEL SPIELMAN of Yale University.

In 1982 the University of Helsinki granted funds to award the Nevanlinna Prize, which honors the work of a young mathematician (less than forty years of age) in the mathematical aspects of information science. The prize is presented every four years by the International Mathematical Union (IMU). Previous recipients of the Nevanlinna Prize are: Robert Tarjan (1982), Leslie Valiant (1986), Alexander Razborov (1990), Avi Wigderson (1994), Peter Shor (1998), Madhu Sudan (2002), and Jon Kleinberg (2006).

Spielman was honored "for smoothed analysis of linear programming, algorithms for graph-based codes and applications of graph theory for numerical computing." Linear programming (LP) is one of the most useful tools in applied mathematics. It is basically a technique for the optimization of an objective function subject to linear equality and inequality constraints. Perhaps the oldest algorithm for LP (an algorithm is a finite sequence of instructions for solving a computational problem; it is like a computer program) is what is known as the simplex method. The simplex algorithm was devised by George Dantzig way back in 1947 and is extremely popular for numerically solving linear programming problems even today. In geometric terms, the constraints define a convex polyhedron in a high-dimensional space, and the simplex algorithm reaches the optimum by moving from one vertex to a neighboring vertex of the polyhedron. The method works very well in practice, even though in the worst case (in artificially constructed instances) the algorithm takes exponential time. Thus understanding the complexity of LP and that of the simplex algorithm have been major problems in computer science. Mathematicians have been puzzled by the general efficacy of the simplex method in practice and have long tried to establish this as a mathematical theorem. Although worst-case analysis is an effective tool to study the difficulty of a problem, it is not an effective tool to study the practical performance of an algorithm. An alternative is the average case analysis, in which the performance is averaged over all instances of a given size, but real instances of the problem arising in practice may not be the same as average case instances. So average case analysis is not always appropriate.

Photograph courtesy ICM/IMU.

Daniel Spielman

In 1979 L. G. Kachian proved that LP is in P; that is, there is a polynomial time algorithm for linear programming, and this led to the discovery of polynomial algorithms for many optimization problems. The general belief then was that there was a genuine tradeoff between being good in theory and being good in practice; that these two may not coexist for LP. However, Narendra Karmarkar's interior point method in 1984 and subsequent variants thereof, whose complexity is polynomial, shattered this belief. Interior point methods construct a sequence of feasible points, which lie in the interior of the polyhedron but never on its boundary (as against the vertex-to-vertex path of the simplex algorithm), that converge to the solution. Karmarkar's algorithm and the later theoretical and practical discoveries are competitive with, and occasionally superior to, the simplex method.

But in spite of these developments, the simplex method remains the most popular method for solving LP problems, and there was no satisfactory explanation for its excellent performance till the beautiful concept of smoothed analysis introduced by Spielman and Shang-Hua Teng enabled them to prove a mathematical theorem.

Smoothed analysis gives a more realistic analysis of the practical performance of the algorithm than using worst-case or average-case scenarios. It provides "a means", according to Spielman, "of explaining the practical success of algorithms and heuristics that have poor worst-case behavior". Smoothed analysis is a hybrid of worst-case and average-case analysis that inherits the advantages of both. "Spielman and Teng's proof is really a tour de force," says Gil Kalai, a professor of mathematics at the Hebrew University in Jerusalem and an adjunct professor of mathematics and computer science at Yale.

The "smoothed complexity" of an algorithm is given by the performance of the algorithm under slight random perturbations of the worst-case inputs. If the smoothed complexity is low, then the simplex algorithm, for example, should perform well in practical cases in which input data are subject to noise and perturbations. That is, although there may be pathological examples in which the method fails, slight modifications of any pathological example yield a "smooth" problem on which the simplex method works very well. "Through smoothed analysis, theorists may find ways to appreciate heuristics they may have previously rejected. Moreover, we hope that smoothed analysis may inspire the design of successful algorithms that might have been rejected for having poor worst-case complexity," Spielman has said.

The Association for Computing Machinery and the European Association for Theoretical Computer Science awarded the 2008 Gödel Prize to Spielman and Teng for developing the tool. Spielman and Teng's paper "Smoothed analysis of algorithms: Why the simplex algorithm usually takes polynomial time", in the *Journal of the ACM*, was also one of the three winners of the 2009 Fulkerson Prize awarded jointly by the American Mathematical Society (AMS) and the Mathematical Programming Society (MPS).

Since its introduction in 2001, smoothed analysis has been used as a basis for considerable research on problems ranging over mathematical programming, numerical analysis, machine learning, and data mining. "However," writes Spielman, "we still do not know if smoothed analysis, the most ambitious and theoretical of my analyses, will lead to improvements in practice."

A second major contribution by Spielman is in the area of coding. Much of the present-day communication uses coding, either for preserving secrecy or for ensuring error-free communication. Error-correcting codes (ECCs) are the means by which interference in communication is compensated. ECCs play a significant role in modern technology, from satellite communications to computer memories. In ECC, one can recover the correct message at the receiving end, even if a number of bits of the message were corrupted, provided the number is below a threshold. Spielman's work has aimed at developing codes that are quickly encodable and decodable and that allow communication at rates approaching the capacity of the communication channel.

In information theory, low-density parity-check code (LDPC) is a linear ECC, which corrects on a block-by-block basis and enables message transmission over a noisy channel. LDPC codes are constructed using a certain class of sparse graphs (in which the number of edges is much less than the possible number of edges). They are capacity-approaching codes, which means that practical constructions exist that allow the noise threshold to be set very close to the limit of what is theoretically possible. Using certain iterative techniques, LDPC codes can be decoded in time linear to their block length.

An important technique to make both coding and decoding efficient is based on extremely well connected but sparse graphs called expanders. It is this counterintuitive and apparently contradictory feature of expanders—that they are sparse and yet highly connected—that makes expanders very useful, points out Peter Sarnak, a professor of mathematics at Princeton University. Spielman and his coauthors have done fundamental work using such graph theoretic methods and have designed very efficient methods for coding and decoding. The most famous of Spielman's papers in this field was "Improved low-density parity check codes using irregular graphs", which shared the 2002 Information Theory Society Paper Award. This paper demonstrated that irregular LDPCs could perform much better than the more common ones that use regular graphs on the additive white Gaussian noise (AWGN) channel.

This was an extension of the earlier work on efficient erasure-correcting codes by Spielman and others that introduced irregular LDPC codes and proved that they could approach the capacity of the so-called binary erasure channel. So these codes provide an efficient solution to problems such as packet-loss over the Internet and are particularly useful in multicast communications. They also provide one of the best-known coding techniques for minimizing power consumption required to achieve reliable communication in the presence of white Gaussian noise. Irregular LDPC codes have had many other applications, including the recent DVB-S2 (Digital Video Broadcasting-Satellite v2) standard.

Spielman has applied for five patents for ECCs that he has invented, and four of them have already been granted by the U.S. Patent Office.

Combinatorial scientific computing is the name given to the interdisciplinary field in which one applies graph theory and combinatorial algorithms to problems in computational science and engineering. Spielman has recently turned his attention to one of the most fundamental problems in computing: the problem of solving a system of linear equations, which is central to scientific and engineering applications, mathematical programming, and machine learning. He has found remarkable linear time algorithms based on graph partitioning for several important classes of linear systems. These have led to considerable theoretical advances, as well as to practically good algorithms.

"The beautiful interface between theory and practice, be it in mathematical programming, error-correcting codes, the search for sparsifiers meeting the so-called Ramanujan bound, analysis of algorithms, computational complexity theory or numerical analysis, is characteristic of Dan Spielman's work," says Kalai.

Daniel Spielman was born in 1970 in Philadelphia, Pennsylvania. He received his B.A. in mathematics and computer science from Yale University in 1992 and his Ph.D. in applied mathematics from the Massachusetts Institute of Technology in 1995. He spent a year as a National Science Foundation (NSF) postdoctoral fellow in the Computer Science Department at University of California, Berkeley, and then taught at the Applied Mathematics Department of MIT until 2005. Since 2006 he has been Professor of Applied Mathematics and Computer Science at Yale University.

—from an IMU news release