

About the Cover

Random Young diagrams

The cover was suggested (albeit loosely) by the article in this issue on free cumulants, written by Jonathan Novak and Pyotr Śniady. Free cumulants are a tool in the theory of non-abelian probability, and the distribution of Young diagrams was one of the earliest applications of that theory.

The images on the cover were produced by a kind of random walk that generates Young diagrams by adding one box randomly in each step, being sure that the result is again a Young diagram. Such a path of diagrams is in effect a numbering of the boxes in the diagram or, in other words, a Young tableau. Thus, in this process a Young diagram of size n is assigned a probability proportional to the number of Young tableaux giving rise to that diagram. It was proven by S. V. Kerov and A. M. Vershik on the one hand, and B. F. Logan and L. A. Shepp on the other, that as n goes to infinity the random diagrams, scaled by $1/\sqrt{n}$, tend to cluster around a fixed shape (indicated in the cover images by a grey curve). This is a non-abelian analogue of tossing coins and getting heads close to half the time. Later work analyzed in more detail the non-abelian Gaussian distribution involved.

This early work has been elaborated extensively, notably in continuing and remarkable work of Kerov and Philippe Biane. It is in Biane's work that free cumulants have proven to be especially valuable. Each Young diagram of size n corresponds to an irreducible representation of \mathfrak{S}_n , and the distribution of Young diagrams defined above is associated to the asymptotic behavior of the regular representation of \mathfrak{S}_n . Biane has shown how other weightings may be associated to the asymptotic limit of other families of representations of the symmetric group. Kerov and others have explored the asymptotic distribution associated to other infinite series of groups, such as $GL_n(\mathbb{F}_q)$ as $n \rightarrow \infty$, but much remains to be discovered.

I have found the written version of Biane's ICM talk in Beijing a good guide to the literature. Along with all the proceedings of the International Congresses, it may be found at

<http://www.mathunion.org/ICM/>

Kerov died at a relatively young age, in 2000. A short memorial article by Vershik can be found in volume 121 of the *Journal of Mathematical Sciences*. The preface of Kerov's book *Asymptotic Representation Theory of the Symmetric Group and Its Applications in Analysis* presents an introduction to his work. Many of his papers can be found at

<http://www.pdmi.ras.ru/~kerov/textfiles/index.html>

I was a little surprised to see that there is no entry on Kerov in Wikipedia, but then, according to Vershik's memorial, this seems to continue a trajectory along which Kerov deserved more recognition than he got.

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