Thinking about Technology in the Classroom

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Decisions regarding the use of technology in the mathematics classroom are subject to powerful market forces. Amazing products, exorbitant claims, and tools in search of a problem abound. Fortunately, the underlying market is also complex, with thoughtful educators, private and public granting agencies, and (most recently) freeware available on the Internet serving to shape this market and its bottom line. What seems to be needed is the development of principles that can help schools, teachers, and authors of standards navigate these stormy waters. Much as Deborah Ball has worked to define mathematical knowledge for teaching, so is there a need to develop a better understanding of the characteristics and uses of technology that make it effective as a teaching tool.

Here we might do well to begin with the writings of Norbert Wiener. In God and Golem, Inc, Wiener shares some ominous thoughts about a world in which technology plays the role of master rather than servant. He raises the specter of a form of natural selection between man and machine, one whose outcome remains to be determined. In the mathematics classroom, such musings may lead us to reflect on the practice of asking students to use computer technology for operations that they themselves are unable to perform. By way of example, it has become commonplace to ask students as young as middle school to turn to calculators for the correlation coefficients of hypothetical data sets. While the mathematics of correlation may be beyond both teacher and student, such exercises are defended on the grounds that they convey a qualitative understanding of numerical measures that are frequently alluded to. Fair enough—as long as we are aware of the fateful step this form of instruction can represent.

One way of addressing such concerns is to focus on the transparency of the technology under consideration. To what extent is the user aware of what the machine is doing vs. dealing with it as a black box? Here the common spreadsheet provides an attractive alternative to many of its glitzier counterparts. In the case of correlation, Excel has a built-in function that corresponds to the black box approach. However, it also provides a format in which, based only on the machine's ability to do basic arithmetic, the student can perform and visualize the underlying calculations. Were there a classroom version of Excel that gives the teacher control of when the built-in function CORREL is to be made accessible, we would have a teaching tool in which the machine plays the role of servant throughout.

Setting aside such issues of student-machine relationship, it becomes important to consider the questions confronting teachers. "How would a particular form of technology support me in the challenges I face?" is sure to be on the table. And since issues of motivation and student involvement are high on many teachers' lists, it would be surprising if these did not figure into decisions regarding the use of technology. While the importance of such considerations should not be minimized, teachers should also be encouraged to move beyond them with questions such as, "How will my students' interaction with this form

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of technology affect their mathematical development?" and, "Given the role that procedural skills and proof play in our standards for instruction, how can technology help me meet my obligations in these areas?"

Considering these in reverse order, technology is an unlikely tool for teaching proofs; also there exist deep concerns about the role that technology might play in *undermining* the acquisition of procedural skills. As such, it is important to backtrack to the issue of mathematical development which thoughtful teachers see as central to their enterprise. Separating "procedural skills" and "proof" is a vast chasm that might be termed "structural understanding". And here we make the following assertion: Properly used, technology can be a powerful tool in cultivating structural understanding; badly used, it becomes an impediment.

Such assertions about structural understanding are likely to lead to requests for a more precise definition. One approach is to sidestep such requests on the grounds that the concept is highly dependent on the topic and level at which it is being applied. Alternatively, one could assert that structural understanding is what will enable students to make the transition from procedural skill to proof. Or one can be honest and admit that, like "mathematical knowledge for teaching", it is a term in need of better understanding. In adopting the latter approach, it seems appropriate to begin with some examples that might help us arrive at a definition.

One of the services that technology provides is the calculation of remainders arising in whole number division problems. Excel's MOD function can free its users from this form of drudgery, enabling them to reflect on the structure of the problem that requires such calculations. In this context, spreadsheets can be used to reenact the discovery of base-10 positional notation by simulating the making of "stacks of ten" while also converting pennies to dimes, dimes to dollars, etc. In the quest for structural understanding, students can be provided with an Excel template which they are expected to complete as follows.

◇	A	B	C	D	E	F	G	H	I
1	This spreadsheet allows you to enter a wholle number M in cell B3. It then simulates the makeing of "stacks fo ten" to convert M into "pennies", "dimes", "dollars", ...								
2							dollars	dimes	pennies
3	**M=**	473				**0**	**4**	**7**	**3**
4									
5			*scratchwork*						
6						0	4	47	473
7						0	4	7	3

With such a spreadsheet in hand, it becomes natural to consider "stacks of 8", peasant multiplication, the Euclidean algorithm, etc.

Creative attempts to cultivate structural understanding abound in the book *The Enjoyment of Mathematics* by Rademacher and Toeplitz. Here Chapter 23 deals with "Periodic Decimal Fractions" and, as survivors of the math wars will recall, this topic figured prominently into debates about the place of long division in the curriculum. But in their remarkable exposition of Euler's theorem, these authors confront the problem of finding the decimal expansion of 3/41 in the following format:

$$\frac{3}{41} = \underset{3}{0}\,.\,\overline{\underset{30}{0}\,\underset{13}{7}\,\underset{7}{3}\,\underset{29}{1}\,\underset{3}{7}}\ldots$$

This technique (closely related to what was once called "short division") focuses attention on the central structural issue at hand, namely that of cycles in the sequence of remainders. Had spreadsheets existed in 1929, might these authors have employed the following?

◇	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	This spreadsheet allows you to enter a proper fraction in A3:A4. It then uses the MOD function to perform "short division" by determining the remainders to be generated in Row 4 and the fraction's decimal expansion in Row 3.														
2															
3	3	=	.	0	7	3	1	7	0	7	3	1	7	0	7
4	41		3	30	13	7	29	3	30	13	7	29	3	30	13

Absent until now has been any mention of geometry, where straightedge and compass constructions are a likely starting point for efforts to achieve structural understanding. Here there exist remarkable forms of dynamic geometry software that harness technology in the implementation of the underlying procedures and in the continuous deformation of the resulting figures. While such software has much to offer, there exist questions analogous to ones discussed above. In discovering that "the medians of a triangle seem to intersect at a single point," should the surprising role of their trisection points be arrived at synthetically or numerically? Should it be possible for the teacher to specify that it is synthetic geometry that is under study? Do the marketers of such software have a tendency to identify computational power with pedagogical effectiveness?

Such questions I am pleased to leave to teachers with greater experience in the classroom use of these products. In my own work with high school students enrolled in California's State Summer School for Mathematics and Science (`http://www.ucop.edu/cosmos/`) and in prerequisite-free freshman seminars at UC Davis, I have tended to emphasize spreadsheet templates such as those described above. For anyone interested in trying their hand at some templates whose completion is intended to cultivate "structural understanding", I would be pleased to respond with an email attachment. On such a basis, contributions towards a definition of structural understanding would be welcome as well.