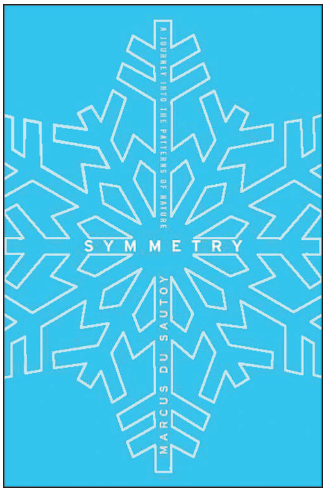

Symmetry: A Journey into the Patterns of Nature

Reviewed by Brian E. Blank

Symmetry: A Journey into the Patterns of Nature
Marcus du Sautoy
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In January 1975, Jacques Tits gave a lecture during which he wrote down the order, $2^{46} \cdot 3^{20} \cdot 5^9 \cdot 7^6 \cdot 11^2 \cdot 13^3 \cdot 17 \cdot 19 \cdot 23 \cdot 29 \cdot 31 \cdot 41 \cdot 47 \cdot 59 \cdot 71$, or about $8 \cdot 10^{53}$, of a sporadic simple group M that Bernd Fischer and, independently, Robert Griess had predicted in 1973. Andrew Ogg, an expert in modular functions who happened to be in the audience, was uniquely qualified to recognize something peculiar about this order: he had recently determined the primes that give rise to a certain family of genus 0 modular curves, and the primes he had discovered were precisely the fifteen prime divisors on the blackboard. It was an astonishing coincidence. At the time, to suggest anything more than that would have been moonshine.

Three years later, in 1978, Fischer, Donald Livingstone, and Michael Thorne calculated the character table of the Monster, as John Horton Conway had christened M, by assuming an additional prediction of Griess, the existence of an irreducible character of degree $\chi_1 = 196883$. Shortly thereafter, John McKay noticed that the coefficient c_1 of the elliptic modular function,

$$j(\tau) = \frac{1}{q} + 744 + 196884q + 21493760q^2 + 864299970q^3 + 20245856256q^4 + \cdots$$
$$= \frac{1}{q} + 744 + \sum_{k=1}^{\infty} c_k q^k \qquad (q = \exp(2\pi i\tau)),$$

is given by $c_1 = 1 + \chi_1$. The evidence of a link between M and the j-function became too overwhelming to dismiss when McKay and John Thompson discovered additional equations involving the characters of M and the Fourier coefficients of j, of which the first few are $c_2 = c_1 + \chi_2$, $c_3 = c_1 + c_2 + \chi_3$, and $c_4 = c_1 + c_3 + \chi_3 + \chi_4$. At this stage, Thompson cautiously referred to these curious equalities as "numerology", but, prompted by some of his suggestions, Conway and Simon Norton discovered further evidence of a deep relationship between the Monster and modular functions, a relationship they dubbed *monstrous moonshine*.

The whimsical term "moonshine" had a certain suitability. There was something humorous about so unexpected a connection between seemingly unrelated branches of mathematics. Writing in 1979, six months before Griess constructed M, Ogg remarked, "It is particularly amusing that new light should be shed on the function j, one of the most intensely studied in all of mathematics, by the most exotic group there is (or is not, as the case may be)." At the time, the light to which Ogg referred was not the bright, direct light that a theoretical connection would have shone. It was a dimmer, reflected light. As Conway put it, "The stuff we were getting… had the feeling of mysterious moonbeams lighting up dancing Irish leprechauns."

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Few writing tasks could seem more foolhardy than authoring a popular book about monstrous moonshine, but the publication of Marcus du Sautoy's *Finding Moonshine* in Great Britain gave the appearance that one had emerged. Concerned, perhaps, that *Finding Moonshine* might be mistaken for a revenuer's memoir, du Sautoy's American publisher issued it as *Symmetry: A Journey into the Patterns of Nature*.

It is difficult to categorize du Sautoy's book in a few sentences, but the mathematical content is centered around finite simple groups and their classification (CFSG). After setting the scene with some autobiography, du Sautoy uses a discussion of symmetry to introduce the finite group concept. The groups of rotations of regular prime-sided polygons become the first simple groups the reader encounters. After learning about the regular solids of the Pythagoreans and Theaetetus, du Sautoy's reader visits the Alhambra with the author, as he hunts for the seventeen symmetries of its decorations. After finding the last, du Sautoy asks, rhetorically, "How can I be sure there isn't an 18th one out there to be discovered?" It is a question to which he will return.

Finding a seamless transition, du Sautoy directs his attention to the theory of equations. Sometime late in the 1400s or early in the 1500s, 125 years or so after the completion of the Alhambra, the Bolognese mathematician Scipione del Ferro discovered formulas that express the roots of cubic polynomials in terms of radicals. Another Bolognese mathematician, Lodovico Ferrari, solved the quartic in 1540. In du Sautoy's formula-free discussion of these advances, algebra takes a back seat to the antics of the cast of eccentric characters. The work of Niels Abel and Évariste Galois on higher degree equations follows, with biographies of these interesting but short-lived mathematicians. The breakthrough of Galois in 1832 brings us not only to the notion of the simple group but also to noncyclic examples ($\mathrm{PSL}(2,p)$ and A_n).

Camille Jordan, Felix Klein, Sophus Lie, Émile Mathieu, Arthur Cayley, and William Burnside are the remaining nineteenth-century group theorists to whom du Sautoy accords more than a few lines of attention. The contributions of Jordan and Lie were fundamental to the emerging theory of groups, but, more germane to the subject at hand, Lie introduced an important family of continuous groups, and Jordan enriched the supply of simple groups with several infinite families of finite analogues of Lie groups. In publications of 1861 and 1873, Mathieu complicated the developing pattern by discovering five finite simple groups that did not belong to the known families. Burnside inadvertently gave a name to such groups many years later when he remarked, "These apparently sporadic simple groups would probably repay a closer examination than they have yet received" [3, p. 504].

The inclusion of Cayley in *Symmetry* is welcome, if surprising. Although he did not play a direct role in the classification of simple groups, his indirect influence was instrumental, for it was Cayley who, in a series of papers published in 1854, introduced the abstract group concept. Cayley's dictum, "A group is defined by means of the laws of combination of its symbols," translated by du Sautoy as, "Forget the equation and its solutions, just look at the interaction of the permutations," provides an effective segue from the theory of equations to modern group theory.

The abstract approach did more than unify group-theoretic results that were arising in geometry, number theory, the theory of equations, and the theory of invariants. It also suggested questions that would otherwise have seemed pointless. Thus it was Cayley who first asked, What are all the possible groups of a given order? Among the mathematicians whom Cayley influenced, Otto Hölder is particularly noteworthy (but is overlooked by du Sautoy). In 1889, Hölder completed the basic theorem on composition series that Jordan began in 1870, and, in 1892, it was Hölder who kicked off the CFSG program when he stated, "It would be of the greatest interest to gain an overview of the entire collection of simple groups." In the same article, he proved that there is no simple group with order p_1p_2 or $p_1p_2p_3$, where the p_i's are primes, not necessarily distinct. He also determined all simple groups up to order 200. Later that year, Frank Nelson Cole, remarking that it was "desirable to extend this census as far as possible", stretched it to 500 (and to 660 the next year). The progress of subsequent censuses may be found in [4].

The decade of the 1890s saw several other developments related to the topics of *Symmetry*. In a coda to his chapter, *The Palace of Symmetry*, du Sautoy states that "The language of group theory gives us the means to prove that 17—and no more—different symmetry groups are possible on a two-dimensional wall." This imperceptive assertion, which refers to the independent determinations of the plane crystallographic groups by William Barlow, Yevgraf Stepanovich Fyodorov, and Arthur Schönflies between 1891 and 1894, comes as a letdown. In the Alhambra chapter, du Sautoy had promised, "As we shall see later in our story, [the proof that there cannot be an 18$^{\text{th}}$ pattern] depends on mastering group theory." *Language* is not *mastery*, and we do not see. (Although we know how many symmetries *could* be at the Alhambra, there is some controversy about how many there actually are [10]. I would rather gaze at a Mark Rothko canvas than say anything more about these busy ornamentations.)

Some other important steps taken in the 1890s are not mentioned. Examples of such omissions are the theory of group characters, introduced by Gerhard Frobenius in 1896, and the development of Lie theory and its finite analogues. The last of Wilhelm Killing's papers on the classification of the finite dimensional simple Lie algebras over $\mathbb{C}$ appeared in 1890. Later in the decade, beginning with his thesis in 1894, Élie Cartan completed Killing's project by constructing the exceptional simple Lie algebras. In 1896 Leonard Eugene Dickson received his doctorate, the first in mathematics awarded by the University of Chicago. His dissertation, supplemented by the more than thirty papers on group theory he wrote over the next few years, was the basis of his 1901 book, *Linear Groups with an Exposition of the Galois Field Theory*. In it, Dickson included all the classical projective groups over finite fields and listed fifty-three of the fifty-six noncyclic simple groups of order less than 1 000 000. Shortly thereafter, in 1905, Dickson constructed finite simple analogues of the exceptional Lie group G_2. Additionally, he introduced finite analogues of E_6. Dickson is cited but once in *Symmetry*, and that reference concerns a remark Dickson made decades after this work.

One additional event of the 1890s that was auspicious for CFSG was Burnside's switch in 1893, at the age of forty-one, from applied mathematics to group theory. His first result was to show that the alternating group A_5 is the only simple group with order $p_1p_2p_3p_4$, where the p_i's are primes, not necessarily distinct. (At more or less the same time, Frobenius proved that a group of order $p_1p_2\cdots p_n$ cannot be simple if $n \geq 2$ and the p_i's *are* distinct primes.) In 1904, Burnside proved the p^aq^b theorem, which states that a group whose order is divisible by fewer than three distinct primes is solvable. As a result, such a group is not simple unless it is cyclic of prime order.

In his history of CFSG, Solomon describes Burnside's p^aq^b theorem as the final triumph of the first era of investigation [13]. In the second edition of his book, Burnside made room for more promising techniques, such as group representations, by jettisoning antiquated material. The verification of the list of simple groups to order 660, which is found in the last chapter of the first edition, was among the discards. Surveying Dickson's greatly extended list, Burnside noted, "An examination of the orders of known non-cyclical simple groups brings out the remarkable fact that all of them are divisible by 12" [3, p. 330]. He did not elevate that observation to a conjecture, but, having pondered the divisor 2 for fifteen years, he was ready to assert, "The contrast. . .between groups of odd and even order suggests inevitably that [noncyclic] simple groups of odd order do not exist" [3, p. 503]. Burnside's restraint about the divisor 3 proved to be well judged—in 1960, Michio Suzuki discovered a simple group of order 29120, the first addition to Dickson's sixty-year-old list.

With its first ten chapters, which comprise about 300 pages, *Symmetry* outlines the development of group theory from its origins through the first twelve years of CFSG. The next half-century did not see continuous progress, but it did span important advances, such as the beautiful characterization of finite solvable groups by Philip Hall, du Sautoy's mathematical great-great-grandfather. Because these results are too technical for a book aimed at the lay person, *Symmetry* skips over them. Its final two chapters, which total some 50 pages, are concerned with the concluding forty-five years of CFSG, as well as monstrous moonshine. When du Sautoy picks up the trail, he describes the Odd Order Theorem of Walter Feit and John Thompson, which states that all odd order groups are solvable, a result that affirms Burnside's conjecture. Du Sautoy does his best to convey the difficulty of the theorem, but there is no way for his reader to gauge the sea change in methodology that has occurred since the Burnside epoch.

Ironically, the Feit-Thompson Theorem, "the single result that, more than any other, opened up the field" [7, p. 1], is a dead end in *Symmetry*. We are told that it "inspired a whole generation of young mathematicians", but, for du Sautoy's reader, it is a largely anonymous generation pursuing undisclosed paths. The true completion of CFSG came, we are told, when Michael Aschbacher and Stephen Smith plugged a "gap" in a "missing step" of a "16-point plan". That is the extent of what we learn about the classification machinery. After Feit-Thompson, even the *statements* of the theorems are too technical.

As good luck would have it, a long-dormant byway of CFSG was about to awaken. In 1965 Zvonimir Janko announced the discovery of a new sporadic simple group. This sixth sporadic group, the first to be found in more than ninety years (and, with order 175 560, the second addition to Dickson's list), was the start of an enormously successful eleven-year treasure hunt, which ended in 1975 with Janko's discovery of his fourth new sporadic group, the twenty-sixth and, as it turned out, last of these exceptions. To fully appreciate the feverish activity involved, read du Sautoy's account of Conway's twelve-hour-and-twenty-minute determination of the 4 157 776 806 543 360 000 element sporadic group Co_1. Or how Donald Higman and Charles Sims took a stroll around a quadrangle while dinner plates were being removed, and, by the time they sat down to dessert, had the 44 352 000 element sporadic group HS in their possession (modulo some paper and pencil calculations that went on into the early hours of the next morning). Further details, as told by Sims himself, may be found in a survey of the mathematics of Higman [2].

In his last chapter, du Sautoy describes the constructions of M and the final sporadic group, J_4. His account highlights quite a contrast: the team assembled by Norton relied on Richard Parker's computer program to answer *Yea* or *Nay*, whereas Griess accomplished his construction of M entirely by bare-handed calculations. Du Sautoy also devotes eight pages of his last chapter to moonshine, a topic he introduced near the beginning of his book. In that earlier discussion, he describes how he learned of moonshine directly from Conway and Norton during a visit to Cambridge in 1985 as a prospective graduate student. At the end of Chapter 1, moonshine is left a tantalizing mystery. It remains a mystery, still tantalizing, one hopes, at the end of Chapter 12. At least readers will know that the experts have figured out quite a bit in the years since 1985. For those who want to learn more, Chapter 0 of [5] makes an excellent continuation.

I have come to the end of du Sautoy's book but have touched on less than half its content. That is because the author has managed to effectively embed selected episodes from the history of group theory into a narrative that sometimes resembles a personal journal and sometimes a travelogue. In the mix, we find many extended, nonmathematical discussions of symmetry as it is manifested in a broad panorama of guises. Facts about icosahedral Chinese incense burners from the first millennium CE and dodecahedral Roman dice from the fifth century BCE stay within our comfort zone. When du Sautoy strays from such topics, the results are dicier. He tells us that "Studies indicate that the more symmetrical among us are more likely to start having sex at an earlier age." Ovulating women, du Sautoy continues, can sniff out symmetry from the sweaty tee-shirts men have worn, but men, apparently, do not pick up the scent of a symmetrical woman. Enquiring minds that want to know more are out of luck: *Symmetry* does not come with any notes. One discussion that has no need for footnotes is an experiment devised by psychologists to pit symmetry against logic (page 278). You can try it out on your own, as I did when teaching truth tables in a transitions course. The results amused the students and enlightened the instructor.

We have now met du Sautoy the biographer, the historian, and the spokesman for symmetry. He is also an anthropologist who reports on the rituals of the strange tribe in the midst of which he lives. He does not have to go native—as a prominent group theorist, he *is* a native. Readers of the *Notices* will find this aspect of his book highly entertaining. You may not have encountered protesters outside your department bearing "No more group theory" placards, but I guarantee you will flash on scenes from your lives. When the author admits, "Like most mathematicians, I am naturally quite shy. I'm not someone...who likes to introduce myself to people. I hate parties, and I'm terrified of the telephone," I recognize many colleagues around the world, and I recognize myself. From among *Symmetry*'s readers, perhaps a parent or partner of a mathematician will empathize with the mother who complained, "There is something wrong with my son. He sits in his room all day studying mathematics" (page 337). Perhaps the general reader will come away understanding Lie's firsthand observation that, "A mathematician is comparatively well suited to be in prison" (page 228).

Du Sautoy examines the entire mathematical process—the inspiration, the serendipity, the hard labor, the setbacks, the frustration, the elation, the disillusionment, and the competition. "Big theorems are like jigsaw puzzles," he advises. "Who wouldn't enjoy being the person to put in the last piece?" He stresses our compulsion to classify, our obsession with pattern hunting, our reluctance to admit defeat. His book is structured as a year-long diary, and he updates us on the progress he is making, or the ground he is losing, with respect to a research problem concerning the enumeration of finite groups. As he realizes that the number theory of elliptic curves is shedding light on his work, we realize that *Finding Moonshine* was a sly choice of title.

On several occasions du Sautoy emphasizes that mathematics provides "a haven for the weird", a niche in which social oddness is tolerated. However, he does not sound all that tolerant when he describes a colleague as looking "like a tramp", or "a slightly mad clown", or "Neanderthal", or "frothing slightly at the mouth". One mathematician is said to have "distinctly inferior social skills". Another "will avoid eye contact with you at all costs". There is one mathematician who appears to be an outgoing communicator, but, "With him," du Sautoy divulges, "it's one-way traffic.... He doesn't seem remotely interested in what anyone else has to say. It's almost as if he is compelled to...forestall any possibility of normal two-way interaction." And then there is the group theorist who suffers a manic episode brought on by contemplating the ramifications of a new sporadic group. These are not anonymous members of the tribe. In every case, du Sautoy names names.

I have already mentioned the only major problem with *Symmetry*, an absence of footnotes. I do have a few minor quibbles. The term "shuffle" should probably not be used as a synonym for permutation: it suggests the special permutations employed, for example, in the definition of the wedge product. On seven occasions, du Sautoy writes "Lie group" when he means "finite group of Lie type". Three of the nineteen digits of $|Co_1|$ on page 317 are incorrect.

In the chapter on the Alhambra, du Sautoy expresses annoyance at his guide's claim that the Arabs invented zero (page 71). He asserts that "the Indians discovered zero. The Arabs were just good messengers bringing the idea from the East to the West." Perhaps this statement refers to the Hindu-Arabic zero in use today, but, if one goes back further in time, a case can be made that zero originated in the Middle East. A tablet that was unearthed at Kish, about eighty kilometers southeast of present-day Baghdad, indicates that the Babylonians used zero as a placeholder in their system of numeration around 700 BCE. Indeed, Hans Freudenthal advanced the theory that the Babylonian zero migrated to India. Bartel van der Waerden deemed this proposal "quite possible", but it remains a conjecture.

On page 240, du Sautoy declares that Burnside's $p^a q^b$ theorem "proved to be just what was needed to identify the simple groups with a small number of symmetries". The example he gives, the list of noncyclic simple groups of orders up to 200, is an unsatisfactory illustration of his assertion, given that Hölder determined the same list in 1892, twelve years before Burnside proved the $p^a q^b$ theorem. In his next paragraph, du Sautoy asserts that Burnside used the $p^a q^b$ theorem in [3] to determine all simple groups up to order 1092. As remarked earlier, Burnside did not pursue this fruitless path in his second edition; he provided only a reference to this determination, which he published in 1895, nine years before the $p^a q^b$ theorem. Special cases of the theorem, such as $p^a q$ with $p < q$, were known in 1895, and Burnside used them alongside his interesting observation that, Z_2 excepted, no simple group of even order n can exist unless $12 \mid n$, $16 \mid n$, or $56 \mid n$.

We are informed on page 305 that Graham Higman (no relation to Donald Higman) and McKay were in Oxford when they constructed J_3. The next page reminds us of "Graham Higman and John McKay in Oxford". British place names are recorded at every opportunity, with enough detail to separate Oxford from neighboring Chilton, but, with few exceptions, American locations are omitted. As a result, du Sautoy's readers will gain no impression of the disproportionate amount of group theory that has been done at Cal Tech, Rutgers, and the Universities of Chicago, Illinois, and Michigan. By contrast, Cambridge University merits three lines in the index.

Du Sautoy's reportage is spotty in places. Michael O'Nan, Richard Lyons, the sporadic groups they discovered (ON and Ly), and the constructions of these groups by Sims are not mentioned. The construction of the Baby Monster is not ascribed to Jeffrey Leon and Sims. The contribution of David Wales to the construction of the sporadic group Ru is acknowledged, but, for the 604 800 element group J_2, the third and final addition to Dickson's list, only Wales's coauthor, Marshall Hall Jr., is credited. In this context du Sautoy alludes to a "rather tense stand-off that got the whole group theory community talking." Should a group be named after its predictor, or its constructor? "Hall would get rather upset if the group he'd constructed was called simply the second Janko group." There may be some legitimacy to the controversy, but du Sautoy chose an unsuitable example. As Griess has pointed out [8, p. 237], Hall discovered J_2 independently, so, in this case, joint credit is due regardless of the larger controversy. (Chapter 17 of [6] supplemented by [7, p. 110] is a good place to sort out matters of divination and fabrication.)

On pages 310 and 315, du Sautoy spins an entertaining tale of how McKay alerted Conway to the possibility of a simple group associated with the Leech lattice. It was the summer of 1966, and both mathematicians were in Moscow for the International Congress of Mathematicians. Conway, who was manning the *pirozhki* stand McKay approached, handed over a roll, and McKay handed back the Leech. This is almost too good a story to scrutinize, but the problem is that du Sautoy's unsourced version does not agree with the firsthand accounts that McKay and John Leech gave in telephone interviews a quarter century closer to the event [14, pp. 118–119]. Ultimately, it is of no consequence whether the exchange took place in Moscow in 1966 or, as McKay and Leech both say, in Cambridge in 1967. This discrepancy, however, does illustrate the confusion that can arise from the absence of documentation.

On page 330, du Sautoy states that, "By the mid 1970s, a total of 25 different sporadic groups had been discovered or conjectured to exist.... The feeling was that 25 might be the limit of what was possible." Is this portrayal of a consensus accurate? Referring to the late 1970s, by which time Janko had predicted a twenty-sixth sporadic group, du Sautoy remarks, "It seemed that only two of the 26 [sporadic groups] were still unclaimed [i.e., not yet constructed]" (page 335). Was it really so apparent then that there would be no additional sporadic groups to construct? According to Griess [9],

> By the late 1970s the classification program had been making a lot of progress, and there were increasing expectations of eventual closure. However, there was not a firm conjecture that the list of simple groups known or suspected to exist at that time must be the complete list. There seemed to be no certainty that the number of sporadic groups must be a *particular number* (such as 26).

> During the early 1970s there was a gap of about two and a half years in which no new sporadic group was discovered (between the discoveries of the Lyons group and the Rudvalis group). Within that time period, no one felt confidence about predicting numbers of sporadic groups to come. The fourth Janko group was conjectured to exist in 1975. During the remainder of the 1970s it was too soon after Janko's latest discovery to feel strongly that 26 must be the right number.

Du Sautoy reports that Daniel Gorenstein declared CFSG to be "all over" in February 1981, when, *to quote du Sautoy*, "a paper...proved that there couldn't be two different groups that looked like the Monster." This assertion may refer to a footnote in [7, p. 1]. It is true that the uniqueness theorem for M was an essential ingredient of CFSG—in 1900, Ida Schottenfels proved that A_8 and $PSL(3,4)$ are nonisomorphic simple groups of order 20160 (and Dickson proved that there are infinitely many orders to which there correspond pairs of nonisomorphic simple groups). However, du Sautoy's use of the term *paper* is misleading. Gorenstein, at the end of his footnote, added the cautionary remark that the work to which he referred was a manuscript in preparation. When a paper was finally published in 1985, it contained an outline of a procedure for determining uniqueness but no claim that the method had actually been implemented. Griess, Ulrich Meierfrankenfeld, and Yoav Segev established uniqueness of the Monster in 1989 [13, p. 341]. (Additionally, this result finally yielded the irreducible character of degree 196883 whose assumed existence had been the basis of the calculation of the character table of M a decade earlier.)

In his concluding pages, du Sautoy's tone becomes noticeably downcast. Although the aftermath of CFSG was a period of "intense activity" [13, pp. 345–347], du Sautoy portrays the opposite when he asserts, "A sense of anticlimax descended on group theory." With pronouncements such as, "The mathematicians who truly understood all the intricacies of [CFSG] were getting old," "Very special techniques could die out with the passing of this generation of practitioners," and "Few young and aspiring mathematicians were attracted to the field," du Sautoy is neither accurate nor attentive to the lessons that can be gleaned from the histories of CFSG and moonshine. After all, the passing of Hermite, Weierstrass, Dedekind, and Klein did not bequeath a dire future for the modular function. A more fitting outlook is that of Solomon [13, §10], who concludes, "We await the visionaries of new generations...who will shed unexpected new light on this ever-fascinating subject." Visionaries and, perhaps, moonbeams.

Based on subject matter and level, Mark Ronan's *Symmetry and the Monster: One of the Greatest Quests of Mathematics* [11] may be regarded as a competitor of du Sautoy's book. It was released slightly earlier and reviewed (favorably) by Griess in the *Notices* [8]. As the titles suggest, *Journey* and *Quest* treat similar mathematical topics. Sometimes the resemblance is uncanny. Walter Feit's description of his wardrobe, "I now possess five new pairs of trousers, two new jackets plus new shoes," written shortly after he arrived in New York, is perfectly unremarkable, but both authors saw fit to quote it. To Ronan, a simple group is a "symmetry atom". To du Sautoy, it is a "symmetry building block". Ronan nicknames Jacques Tits "The Man from Uccle", a pun on the 1960s television series and a reference to the Belgian town where Tits was born. Also playing on words, du Sautoy labels Griess "the mathematical Doctor Frankenstein". Whereas Ronan is a disciplined chronicler who likes to share interesting tales, du Sautoy is a raconteur who aims to educate his readers by means of stories that unfold vividly.

For anyone who is interested in the CFSG program, both books are recommended as informative supplements to articles such as [1], [2], [4], [12], and [13]. If I were forced to choose only one of these books, and if CFSG were the only criterion, then I would give the nod to Ronan's more focused approach. But du Sautoy's work is also up to the task, and his broad sweep and imaginative writing will have great appeal. He has gathered a wide range of loosely related topics and has very cleverly assembled them into a coherent, absorbing narrative. Readers of the *Notices* will find *Symmetry: A Journey into the Patterns of Nature* enjoyable and worthwhile.

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