Using Mathematics to Improve Fluid Intelligence

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In the past several decades the mathematical community has witnessed a fervent debate about the relationship between the development of basic mathematical skills and higher-order thinking [9]. This debate addresses both the learning processes and facilitating the retention of what is learned.

One important educational tool in promoting the learning process is testing. My experience suggests that, to be most effective, testing should be both cumulative and time restricted. By cumulative, I mean individual tests, including quizzes, that include items from material covered earlier in the term. When students realize that testing will be cumulative, there is strong motivation to understand, practice, and review all the material taught from the beginning of the course. As opposed to tests on chapters or modules, cumulative tests center on the essence of education, recognizing that the integration of knowledge is the very heart of learning. By the very hierarchical nature of mathematics, understanding of new material depends upon what has been learned before, and so the learning of the new topic becomes intimately tied to the knowledge of the previous topics. The effectiveness of cumulative vs. narrowly focused testing is supported by studies we have conducted in mathematics (using control groups) and by research of other educators [1, 5, 7]. Related work by other respected scholars on test-enhanced learning has appeared in [4, 6] as well.

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Time-restricted tests provide additional benefits to the educational process. Time limits build students' concentration skills; the student must fully focus attention on the task. This requirement addresses the contemporary habit of living with constant disruptions, which is reflected in students' thought processes that meander, lacking the ability to focus. Frequent, time-restricted testing in mathematics trains students to fully concentrate on a task, targeting the development of their ability to learn and retain knowledge. Recent collaborative research that I have conducted has shown that using frequent and time-restricted tests in mathematics improves students' outcomes not only in mathematics, but also in "unrelated" subjects such as reading comprehension [7, 8]. One of my colleagues has personally reported similar results in the social sciences. These examples seem very likely attributable to students' improved concentration skills.

Beyond this, we have found that timed tests, much more than paper and pencil routines, enhance students' ability to do mental mathematics by training them to instantly build images of multi-step problems in their minds and solve them rapidly. For example, to solve the equation $x/2 - 7 = -3$, they create an image of the problem, perform the additive and multiplicative properties mentally and arrive at $x = 8$ as the solution. In trigonometry, to simplify the identity $1 + \tan^2 x$, they can rapidly create an image $1 + \tan^2 x$ as $1 + \sin^2 x/\cos^2 x$, equal to $\cos^2 x/\cos^2 x + \sin^2 x/\cos^2 x$ which results in $\cos^2 x + \sin^2 x/\cos^2 x$ leading to $1/\cos^2 x$ which equals $\sec^2 x$.

In calculus also, to calculate the derivative $(e^{\sin 2x})'$, they form a mental image of the chain rule on composition of functions, and form the product of $e^{\sin 2x} \cdot (\sin^{2x})'$, which is $e^{\sin 2x} \cdot (\cos 2x) \cdot 2$. This is what mathematicians do when they confront problems of such characteristics. They do problems in their minds. I have seen very good students begin to reach this point.

Finally, timed tests build students' automaticity of basic skills in mathematics. This may sound like nothing more than inducing stress during tests, but many cognitive scientists have determined that stress within the context of a learning experience induces focused attention and improves memory of relevant information [3]. This enables the mind to perform at that level of conceptual thought devoted to higher-order thinking and problem-solving activities. The comfort that one experiences by achieving fluency in basic skills far transcends any initial stress as such fluency removes educational barriers which exist on the way to performing at higher domains of thought.

Recently there has been exciting seminal research suggesting that fluid intelligence can be improved with training on working memory [2]. Fluid intelligence is considered to be one of the most important factors in learning and comprises those sets of abilities associated with abstract reasoning and higher-order thinking. Intelligence has always been thought to have a strong hereditary component, immutable even with training, but the new research shows that training in continuous performance tasks (dual *n*-back tasks) stimulates brain activity leading to improved results as reflected through intelligence tests. Psychologists have experimented with dual *n*-back tasks to provide simultaneous auditory and visual stimuli on subjects in time-restricted intervals. Such tasks rely heavily on attentional control, which is required in the performance of demanding working-memory tasks. The new research on fluid intelligence is also important in that it shows that the training effect occurs across all ability levels, i.e., people with low IQ, as well as those at the higher end of the spectrum. The results of the new research are important contributions in learning sciences, as they show that cognitive training improves fluid intelligence. Beyond that, these findings also have important implications in the mathematical sciences.

In mathematics we have a natural paradigm for training the brain to deal in a focused manner with demanding tasks. This is what I refer to as concentration, automaticity, and mental mathematics. Our earlier research suggests that work in mathematics using frequent, cumulative, and time-restricted testing can improve the working memory. Training students to perform multi-level problems mentally in timed intervals has a close resemblance to dual *n*-back tasks, in the related working-memory research in psychology: both demand full concentration, speed, and accuracy in the processing of stimuli. If this correlation is valid, and training in working memory can correlate with gains in fluid intelligence, then disciplined training in mathematics utilizing cumulative, time-restricted testing can improve fluid intelligence and students' ability to reason and solve problems in any field and in all disciplines.

While more research is required—including the relative value of this protocol at various levels of mathematical study—the possible implications for mathematics education are dramatic. As we continue to explore this premise, one cannot help but reflect once again upon Plato's penetrating insight on the richness and value of training in mathematics.

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