Collective Behavior and Individual Rules

This April the theme for "Math Awareness Month" is complexity. It is a huge theme, comprising a wide range of diverse topics, and not all of it directly connected to mathematics. But one of its most interesting components is the emergence of highly structured behavior of large assemblies governed by relatively simple rules of local interaction, and this shows great potential as a point for mathematical attack. Early studies involved only small groups of animals, but as computers advanced rapidly in power these were followed by striking if largely speculative computer simulations. However, in recent years technology, for example increasing sophistication of digital photography, has brought about a resurgence of work on real populations. Among the most impressive studies are those of bird flocks—pigeons, starlings, and ducks—but there have also been others concerned with schools of fish and large groups of insects such as locusts.

The images on the cover were supplied by Ryan Lukeman (now on the faculty of St. Francis Xavier University in Nova Scotia), and are from his Ph.D. thesis at the University of British Columbia. Lukeman took thousands of photographs of surf scoters moving around in salt water near docks in downtown Vancouver. The background shows a segment of his photographs. The inset gives some idea of how the photographs were processed. Lukeman tells us,

> The figures illustrate four steps in processing our photographs. (1) The top figure is a typical image from a series of hundreds of images taken at 3 fps to capture the motion of surf scoters. (2) I then pass the image as a three-layered matrix into Matlab, and use image processing to extract individual birds from the image. Image processing involves several different kinds of analysis, followed by manual work to ensure that every individual has been marked. (3) The center of mass of each duck is computed, then overlaid on the original image for visual checking. (4) By linking positions in successive frames using particle tracking software, trajectories are constructed for each individual in the frame. I also correct the image distortion using simple trigonometry. The corrected positions and velocities are plotted at one instant in time in the last frame.
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> Generally, useful data on collective animal motion is very difficult to obtain in the field. Computer simulations have been valuable, but the real challenge is to determine which of these models are biologically relevant. We want to infer from field observations the real mechanisms governing collective motion.
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> Computers have advanced far enough over the past thirty years to produce stunning simulations, but many of these have been based on imagined rather than biologically verified schemes. Recent advances in digital imaging have now made it possible to obtain high-quality dynamic data on large groups. These data are crucial to construct biologically realistic models, as opposed to strictly hypothetical ones.
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> The work we did in obtaining trajectories of hundreds of ducks within flocks allowed us to come to concrete conclusions about the nature of their interactions. We found that repulsion forces were an order of magnitude larger than attraction and alignment forces, and that there was also a strong interaction with neighbors directly in front.

More information on the project can be found in the article by Lukeman, Li, and Edelstein-Keshet in the July 13, 2010, issue of the *Proceedings of the National Academy of Science*.

Here is a brief list of suggestions of other things to look at:

Boids. One of the most famous computer simulations was Craig Reynolds' boids. The webpage `http://www.red3d.com/cwr/boids/` is comprehensive.

Starlings. The images on the Web are fascinating. The main page for the 2007 STARFLAG meeting is a good place to start: `http://angel.elte.hu/starling/meeting.html`.

Much of the best work is by Andrea Cavagna and his group. Here are two popular articles about it: `http://lansingwbu.blogspot.com/2010/01/how-do-thousands-of-starlings-flock.html`. `http://www.americanscientist.org/issues/num2/2011/1/flights-of-fancy/1`.

Pigeons. You can get a good idea of work on pigeons from: `http://www.wired.com/wiredscience/2010/04/pigeon-flock-pecking-order/`.

Fish. Some of the most beautiful images on the Web: `http://pinewooddesign.co.uk/2008/06/25/thousands-of-golden-rays-glide-silently-through-the-ocean-photos/`.

Insects. There are two interesting groups at Princeton: `http://icouzin.princeton.edu/`; `http://paw.princeton.edu/issues/2010/05/12/pages/9635/index.xml`.

Other biological collectives. Even bacteria do it. `http://tglab.princeton.edu/`.

—Bill Casselman
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