2011 Eisenbud Prize

HERBERT SPOHN received the 2011 AMS Leonard Eisenbud Prize for Mathematics and Physics at the 117th Annual Meeting of the AMS in New Orleans in January 2011.

Citation

The Eisenbud Prize for 2011 is awarded to Herbert Spohn for his group of works on stochastic growth processes:

1. "Exact height distribution for the KPZ equation with narrow wedge initial condition" (with T. Sasamoto, *Nuclear Phys. B* **834** (2010), no. 3, 523–542).

2. "One-dimensional Kardar-Parisi-Zhang equation: An exact solution and its universality" (with T. Sasamoto, *Phys. Rev. Lett.* **104**, 230–602 (2010)).

3. "Scaling limit for the space-time covariance of the stationary totally asymmetric simple exclusion process" (with P. L. Ferrari, *Comm. Math. Phys.* **265** (2006), no. 1, 1–44).

We also cite, outside the six-year window:

4. "Scale invariance of the PNG droplet and the Airy process" (with M. Prähofer, *J. Statist. Phys.* **108** (2002), no. 5–6, 1071–1106).

Stochastically growing interfaces is a subject of intense study in both probability theory and nonequilibrium statistical physics. Three of the most widely studied models that describe the height of a growing interface, $h = h(x,t)$, are (1) the asymmetric simple exclusion process (ASEP), an interacting particle system introduced by F. Spitzer some forty years ago; (2) the polynuclear growth model (PNG); and (3) the KPZ equation, a formal nonlinear stochastic PDE for the height function h.[1] The main question is to describe the asymptotic properties of the height function h.

In early work with M. Prähofer on the PNG model, Spohn identified the underlying process that is expected to describe the fluctuations of h for a large class of growth models; this process they called the *Airy process.* In work with P. Ferrari, Spohn established that in the scaling limit the PNG model and a special case of ASEP have the same scaling function for their covariance. This established what physicists call KPZ universality for these two discrete models. In recent work with T. Sasamoto, Spohn both gave meaning to solutions of the KPZ equation and found the one-point distribution of the height function. As strange as it sounds, Sasamoto and Spohn showed that the KPZ equation lies in the KPZ universality class! We remark that this last work was also done independently by G. Amir, I. Corwin, and J. Quastel. We note that many of the predictions of Spohn and others are now being experimentally tested (and verified) in the laboratory.[2]

Herbert Spohn

There are many questions left to answer regarding the Airy process and its universality, but what is clear is that the work of Herbert Spohn has opened up many new lines of research in mathematics and physics.

Biographical Sketch

Herbert Spohn is professor of mathematical physics at Zentrum Mathematik of the Technische Universität München (TUM). Spohn was born in 1946 at Tübingen, Germany, grandson of the mathematician Konrad Knopp. He earned his Vordiplom in physics at the Technische Universität Stuttgart in 1969 and his Diplom and Ph.D. in physics at the Ludwig-Maximilians-Universität (LMU) München. After postdoc years at Yeshiva, Princeton, Rutgers, and Leuven and an extended stay at the IHES, Paris, in 1982 he joined the statistical physics group at LMU, headed by Herbert Wagner, as associate

[1] *The KPZ equation was introduced in the mid-1980s by M. Kardar, G. Parisi, and Y.-C. Zhang.*

[2] *K. Takeuchi and M. Sano, "Universal fluctuations of growing interfaces: Evidence in turbulent liquid crystals" (*Phys. Rev. Lett. **104**, *230–601 (2010)).*

professor of solid state physics. In 1998 he became professor of applied probability in conjunction with statistical physics at TUM.

Spohn received the 1993 Max-Planck Research Award, jointly with Joel Lebowitz, and the 2011 Dannie Heineman Prize for Mathematical Physics. He headed the International Association of Mathematical Physics.

Response

I am deeply honored to receive the 2011 Leonard Eisenbud Prize for Mathematics and Physics. I view this as an appreciation of the work of so many physicists and mathematicians on exact solutions for growth models in the KPZ universality class.

On the macroscopic scale, matter is commonly organized in equilibrium phases, which in themselves are spatially homogeneous. Distinct phases are separated by a layer typically a few atomic spacings wide. Just think of a water droplet in contact with its vapor. Such interfaces are of basic scientific and technological interest. In particular one wants to know how they change in the course of time. The corresponding evolution equations have been studied widely in mathematics. To mention only one prominent example of interface motion: for a droplet immersed in three-space, the local interface velocity is taken to be proportional to the local mean curvature, hence the dynamics is motion by mean curvature. The droplet shrinks, but it also may pinch off into two or several pieces.

On the mesoscopic scale, still large compared to atomic spacings, one observes fluctuations on top of the large-scale motion. The PDE for the motion turns into a stochastic PDE of some sort. From a statistical physics perspective the interest shifts to shape fluctuations, with the hope to discover universal (i.e., to a large extent model independent) statistical laws. Roughly speaking, this is the main focus of our research with one twist. We consider an interface which borders a stable against an unstable phase. This imbalance drives the interface motion. In a widely cited paper of 1986, Kardar, Parisi, and Zhang proposed a stochastic PDE to model this particular interface evolution.

In 1999 Baik, Deift, and Johansson studied the length of the longest increasing subsequence of a random permutation with the startling result that the fluctuations are governed by the Tracy-Widom distribution, discovered before as the fluctuations of the largest eigenvalue of a GUE random matrix in the limit of large N. Shortly later, Johansson noted a related mathematical structure for the single-step growth model with wedge initial data (alias totally asymmetric simple exclusion process with step initial conditions). We realized that random permutations are isomorphic to a driven interface model, the polynuclear growth model, and, based on the work of Baik and Rains, we proved that, while the scaling exponent is always 1/3, the precise statistical fluctuations depend on initial conditions. This feature was not at all anticipated but is beautifully confirmed by the recent experiments of Takeuchi and Sano. The novel advance concerns the KPZ equation in one dimension. Arguably, we obtained for the first time an exact time-dependent solution of a nonlinear stochastic PDE. In the long time limit one finds the same probability distributions as for the stochastic lattice growth models, supporting universality.

The analysis of one-dimensional stochastic growth models in the KPZ class borders at domains which before were considered to be fairly distinct: random matrix theory, Dyson's Brownian motion, statistical mechanics of line ensembles, directed polymers in a random medium, combinatorics of tilings, representation theory and Schur functions, integrable models and Bethe ansatz, interacting stochastic particle systems. Such crossroads have often been the source of further advances.

I would like to use the occasion to thank Joachim Krug, Michael Prähofer, Patrik Ferrari, and Tomohiro Sasamoto. Their deep insights and unfailing encouragement were instrumental for achieving our results.

About the Prize

The Eisenbud Prize was established in 2006 in memory of the mathematical physicist Leonard Eisenbud (1913–2004) by his son and daughter-in-law, David and Monika Eisenbud. Leonard Eisenbud, who was a student of Eugene Wigner, was particularly known for the book *Nuclear Structure* (1958), which he coauthored with Wigner. A friend of Paul Erdős, he once threatened to write a dictionary of "English to Erdős and Erdős to English". He was one of the founders of the Physics Department at the State University of New York, Stony Brook, where he taught from 1957 until his retirement in 1983. His son David was president of the AMS during 2003–2004. The Eisenbud Prize for Mathematics and Physics honors a work or group of works that brings the two fields closer together. Thus, for example, the prize might be given for a contribution to mathematics inspired by modern developments in physics or for the development of a physical theory exploiting modern mathematics in a novel way. The US$5,000 prize will be awarded every three years for a work published in the preceding six years.

The Eisenbud Prize is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2011 prize, the members of the selection committee were Barry Simon, Yakov G. Sinai, and Craig A. Tracy.

Previous recipients of the Eisenbud Prize are Hirosi Ooguri, Andrew Strominger, and Cumrun Vafa (2008).