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of the American Mathematical Society

May 2011

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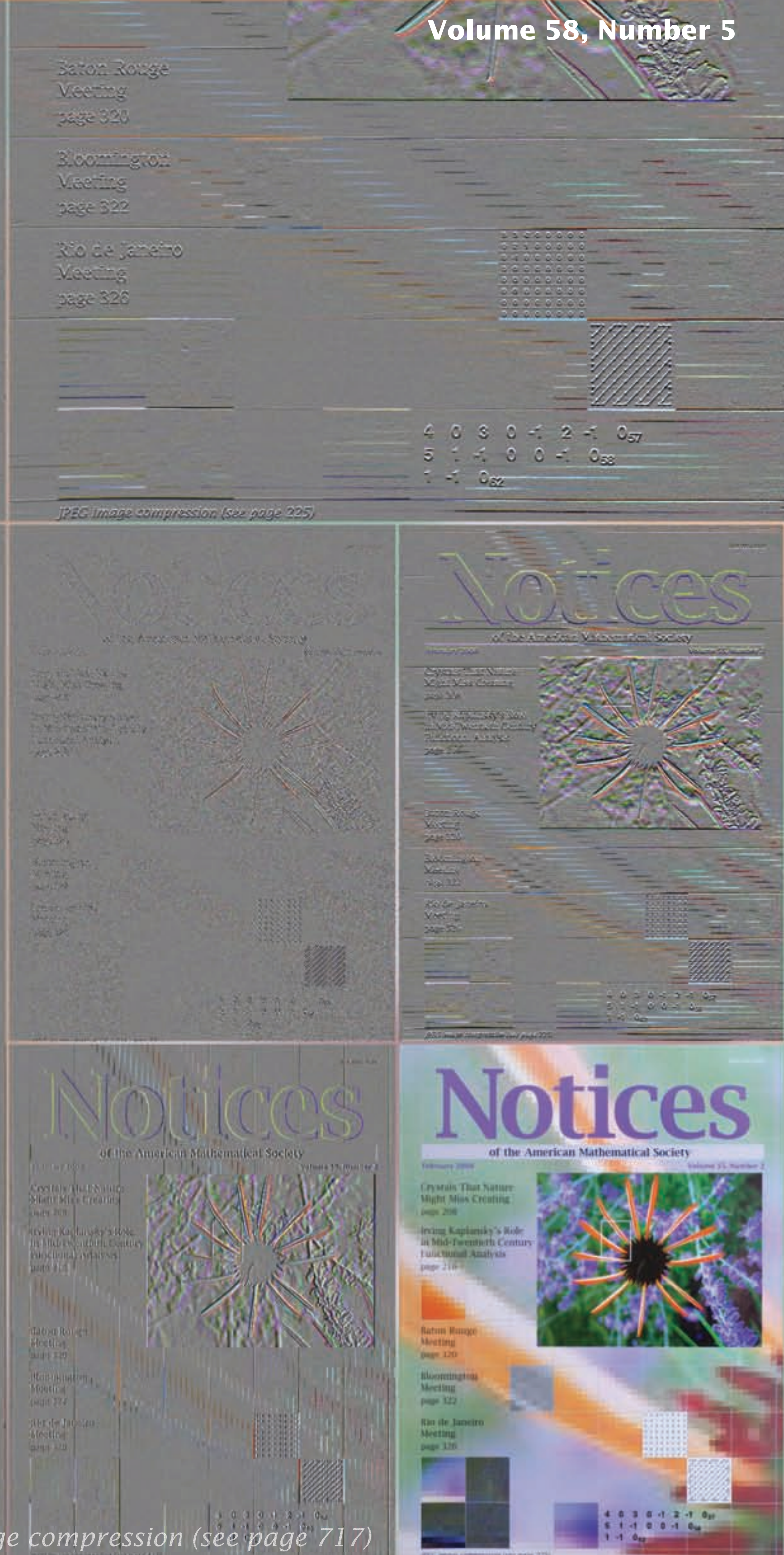
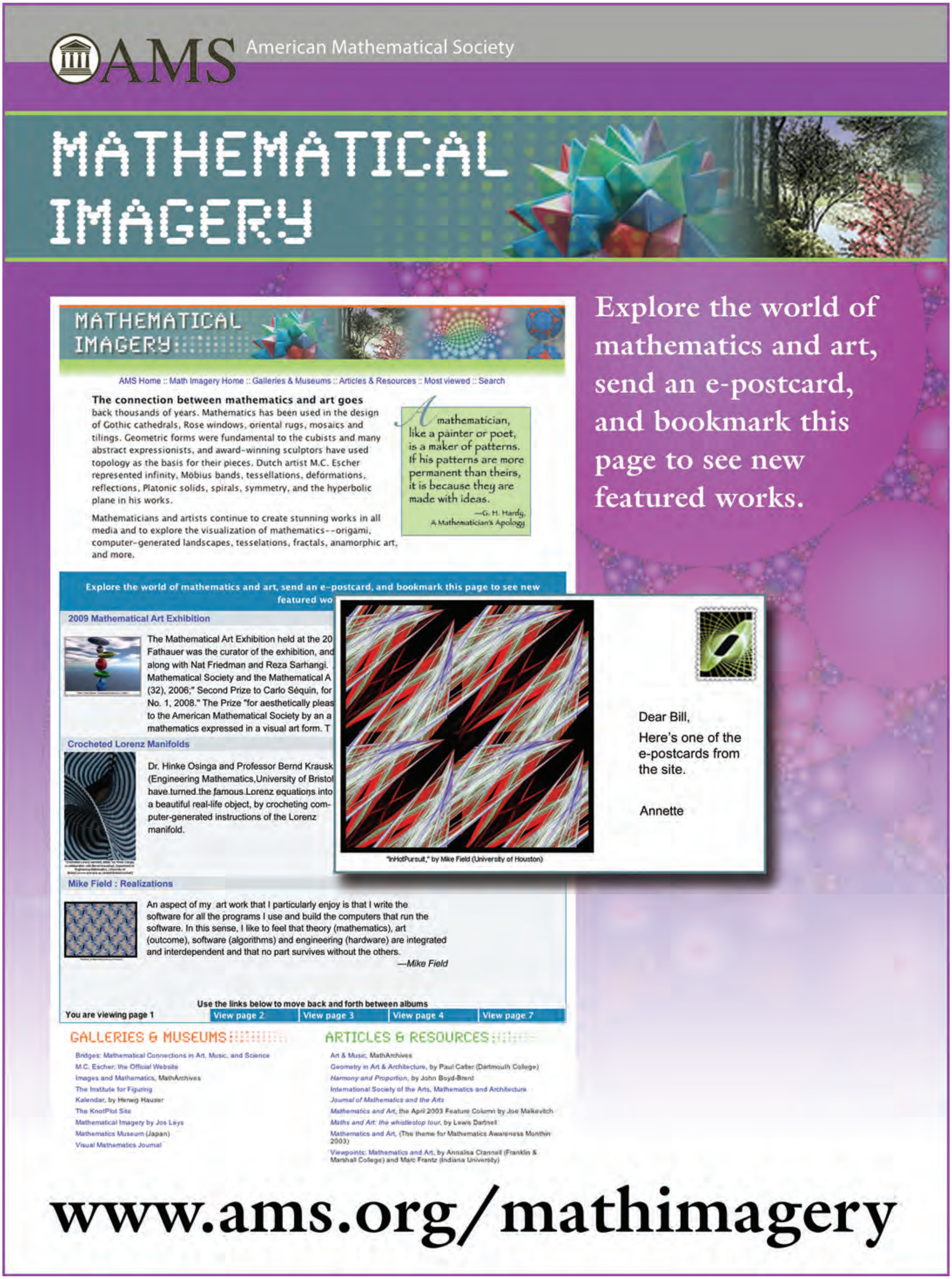


About the Cover: Wavelets in image compression (see page 717)



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The connection between mathematics and art goes back thousands of years. Mathematics has been used in the design of Gothic cathedrals, Rose windows, oriental rugs, mosaics and tilings. Geometric forms were fundamental to the cubists and many abstract expressionists, and award-winning sculptors have used topology as the basis for their pieces. Dutch artist M.C. Escher represented infinity, Möbius bands, tessellations, deformations, reflections, Platonic solids, spirals, symmetry, and the hyperbolic plane in his works.
Mathematicians and artists continue to create stunning works in all media and to explore the visualization of mathematics--origami, computer-generated landscapes, tesselations, fractals, anamorphic art, and more.
A mathematician, like a painter or poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with ideas.
—G. H. Hardy, A Mathematician's Apology
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2009 Mathematical Art Exhibition
The Mathematical Art Exhibition held at the 20
Fathauer was the curator of the exhibition, and
along with Nat Friedman and Reza Sarhangi.
Mathematical Society and the Mathematical A
(32), 2006;" Second Prize to Carlo Séquin, for
No. 1, 2008." The Prize "for aesthetically pleas
to the American Mathematical Society by an a
mathematics expressed in a visual art form. T
Crocheted Lorenz Manifolds
Dr. Hinke Osinga and Professor Bernd Krausk
(Engineering Mathematics,University of Bristol
have turned the famous Lorenz equations into
a beautiful real-life object, by crocheting computer-generated instructions of the Lorenz manifold.
"InHotPursuit," by Mike Field (University of Houston)
Dear Bill,
Here's one of the e-postcards from the site.
Annette
Mike Field : Realizations
An aspect of my art work that I particularly enjoy is that I write the software for all the programs I use and build the computers that run the software. In this sense, I like to feel that theory (mathematics), art (outcome), software (algorithms) and engineering (hardware) are integrated and interdependent and that no part survives without the others.
—Mike Field
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Mathematics and Art, (The theme for Mathematics Awareness Month 2003)
Viewpoints: Mathematics and Art, by Annalisa Crannell (Franklin & Marshall College) and Marc Frantz (Indiana University)
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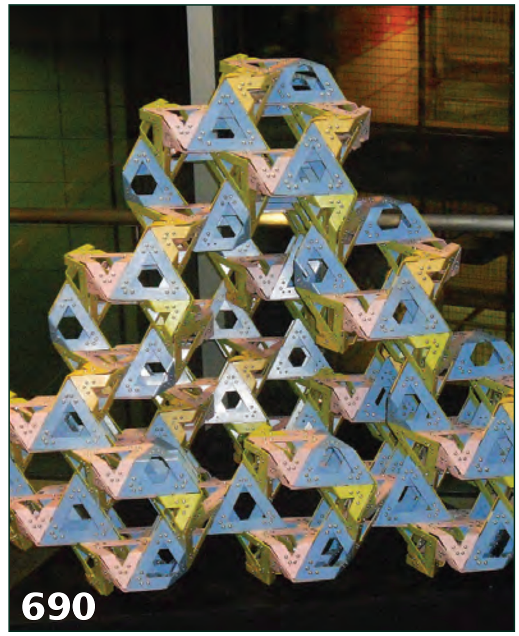
Notices
of the American Mathematical Society

May 2011

Communications

Commentary



Features

The May issue showcases diverse features of the mathematical life. John Ewing, former executive director of the AMS and now president of Math for America, tells us of the dangers of value-added modeling. Michael Saclolo writes of a mathematical analysis of the poetry of medieval troubadours. Catherine Bénéteau and Patrick Van Fleet explain how wavelets can be used to augment undergraduate mathematics education. Finally, we share memories of noted mathematician Leon Ehrenpreis.

—Steven G. Krantz, Editor

Notices of the American Mathematical Society

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Departments

From the AMS Secretary

I thank Randi D. Ruden for her splendid editorial work, and for helping to assemble this issue. She is essential to everything that I do.

—Steven G. Krantz
Editor

Letters to the Editor

New Bias Discovered: *Proc. AMS* Favors Commutative Rings to Astronomy

I was amazed by the article of Joseph F. Grcar, "Topical bias in generalist mathematics journals", *Notices* **57** (11).

A bias "which seems not to be discovered previously" consists in the following: of two papers reviewed in the *Zentralblatt* or in *Math Reviews*, a paper in pure mathematics is more likely to be published in *Proceedings of the AMS* than a paper in optics, statistics, astronomy, or geophysics.

In my opinion, this observation is trivial: just ask any mathematician or astronomer which of the two papers is more likely to appear in *Proc AMS*. And s/he will give you the right answer without any statistical analysis.

The author does not take into account an evident thing: *Zentralblatt* and *Math Reviews* try to cover not only mathematics but also other areas where mathematics is applied. By the way, the original name of *Zentralblatt* was *Zentralblatt für Mathematik [und ihre Grenzgebiete]* which clearly shows the intention to cover "boundary areas". So the set of the papers from which the author takes his statistics is initially biased: it contains many papers whose primary subject is not mathematics.

A person who wrote a paper in astronomy with some mathematical content has a choice: to submit it to an astronomy journal or to a mathematical journal. An author of a paper in pure mathematics does not have such choice. That's why mathematical journals seem biased towards "pure mathematics".

—*Alex Eremenko*
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(Received January 29, 2011)

AMS Rescues Commutative Algebra—Is Geometry Next?

My article is meant to confront preconceptions with data. I thank Alex Eremenko for the opportunity to explore an example.

He suggests *Proceedings of the AMS (PAMS)* favors commutative algebra to subjects including statistics because papers of the subjects may appear in journals of other fields, thus reducing their presence in *PAMS*. If this were correct, then the same bias for commutative algebra would be seen at other generalist journals, such as *Comptes Rendus Mathématique* (*C. R. Math*). However, *C. R. Math* publishes commutative algebra and statistics in proportions 22 : 262, which is roughly the reciprocal of 267 : 40 at *PAMS*. (As in my article, papers have primary or secondary subjects in classes 13 and 62 for 2000–09.) Thus Eremenko's interpretation is inconsistent with the data. The limitations of his suggestion are demonstrated further if he were to try explaining why *PAMS* favors commutative algebra to geometry.

The importance of topical bias is not the cause but the impact. Someone wrote to me that it is "discriminatory to many mathematicians" if tenure committees prefer publications in journals that have topical biases. Another person wrote that *PAMS* favors the abstract to the concrete, but "a less monolithic mathematical community is in all of our interests".

—*Joe Grcar*
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P.S. There are two minor inaccuracies in Eremenko's letter. My article says biases have not been "discussed" (rather than "discovered"), and my article does state the full title of *Zentralblatt*.

(Received February 3, 2011)

Honoring Professor Chern: A Bridge Builder between Nations

While there are many issues of interest and concern to both China and the United States, there is one reason for both countries to celebrate. 2011 is the one hundredth anniversary of the birth in China of the great Chinese-American mathematician, the late Shiing-Shen Chern. Although Chern was born and passed away in China, the bulk of his illustrious professional career was spent in the United States, where he was a professor of mathematics, first at the University of Chicago and then at the University of California at Berkeley.

Throughout his life Chern did everything that he could to bring people together. One hopes that both the U.S. and Chinese governments will see fit to jointly honor this wonderful mathematician and teacher. The American Mathematical Society should encourage both governments to jointly honor the memory of Shiin-Shen Chern who was, after all, a vice president of the AMS.

Personally, I would like to see two new institutes for mathematics—one in the United States and one in China—emphasizing mathematics and mathematically related fields and dedicated to providing research opportunities for both young mathematicians and for teachers of school mathematics. Such institutes could serve as fitting permanent memorials to a person whose like we are not likely to see again.

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(Received January 31, 2011)

The Law of the Constant Ratio

The use of publication and citation databases, namely Web of Science and Scopus, by funding agencies and governmental institutions around the world for evaluation of the scientific performance of countries, universities, research institutions, and individuals, appears as a global trend and a global issue. Since this trend appears to be unavoidable, it is important to have better measures for comparing scientific performance in different fields of science.

A suitable tool for better qualitative comparison of scientific performance in different fields of science is the law of the constant ratios (`http://dx.doi.org/10.1007/s11192-005-0240-0`) (Podlubny, I., *Scientometrics* **64**(1):2005, 95–99), which says that the ratio of the total numbers of citations in any two broad fields of science remains close to constant. One citation in mathematics corresponds to five citations in engineering and technology, to eight citations in biology, to nine citations in earth and space sciences, to thirteen citations in social and behavioral sciences, to fifteen citations in chemistry, to nineteen citations in physics, and to seventy-eight citations both in clinical medicine and in biomedical research (these nine broad fields are taken from the U.S. National Science Foundation classification). This approach has already been successfully used for compiling the first multidisciplinary list of highly cited researchers (`http://dx.doi.org/10.3152/147154406781775887`) (Podlubny, I., and Kassayova, K., *Research Evaluation* **15**(3):2006, 154–162).

Based on the same source of data (`http://www.nsf.gov/statistics/seind04/`), we extend the law of the constant ratio to the number of articles, too. Namely, one article in mathematics corresponds to three articles in earth and space science; to four articles in engineering and technology, in biology, and in chemistry; to five articles in physics; to nine articles in social and behavioral sciences; to ten articles in biomedical research; and to eighteen articles in clinical medicine.

This means that quantitative comparison of researchers working in different fields of science can be done by normalizing their corresponding citation and paper counts with respect to mathematics, and not by using just total counts.

The law of the constant ratio for citation counts would also allow creation of multidisciplinary rankings based on the h-index using normalized citation counts.

This is impossible when absolute citation counts are used, since nonnormalized citation counts naturally result in significant differences in h-indices between the fields (`http://dx.doi.org/10.1038/448737a`) (Ball, Ph., *Nature* **448**:2007, 737).

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(Received January 22, 2011)

A Retired Gentleman

After reading the very fine memorial for Professor Helson in *Notices* **58**(2), I would like to add a personal recollection.

In 1999 a dapper little gentleman appeared at the office. I was serving again as chairman at Saint Mary's College in Moraga, California, a tiny school about twenty minutes from Berkeley.

He was applying for the position we had posted for a part-time instructor in math. He claimed he was retired from Cal and had seen our opening. Given the disjunction of our scale from that at Cal, we may conclude immediately that money was not the consideration. He was then hired to fill in temporarily.

He came into our department and was comfortable at once. He appeared at our events and many times gave the impression that he was a junior member at the bottom of the ladder—invoking seniority or experience was not his way. He simply liked being a teacher.

As he taught several courses, it became apparent to me that this extraordinary fellow had a massive background, a wide acquaintanceship, and a remarkable charm. I soon put him in charge of our senior seminar. This course is meant to help the majors prepare a research paper and then to present it to the entire department orally. Given his experience with research and his devotion to clarity in presentation of results, he took over the seminar with great success, and with great benefit to our seniors.

It was our privilege here in Moraga to enjoy Henry Helson for a couple of years.

—*Brother L. Raphael, FSC*
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(Received February 10, 2011)

A Disagreement about Hilbert

The review of the book *Logicomix* in the December 2010 issue of *Notices* includes (page 1429) a comment on Hilbert and his son corresponding to a statement in the book (page 327): "When the boy was diagnosed with schizophrenia, at age fifteen, his father sent him off to an asylum, where he spent the rest of his life. Hilbert never visited his son."

This account disagrees with Constance Reid's book, *Hilbert*, which asserts that he entered the insane asylum at about age twenty-one (pp. 138–139), was released a few years later (p. 151), and occasionally lived at home afterwards (pp. 151, 210, 215).

—*George Glauberman*
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(Received February 11, 2011)

Discrete Wavelet Transformations and Undergraduate Education

Catherine Bénéteau and Patrick J. Van Fleet

Wavelet theory was an immensely popular research area in the 1990s that synthesized ideas from mathematics, physics, electrical engineering, and computer science. In mathematics, the subject attracted researchers from areas such as real and harmonic analysis, statistics, and approximation theory, among others. Applications of wavelets abound today—perhaps the most significant contributions of wavelets can be found in signal processing and digital image compression. As the basic tenets of wavelet theory were established, they became part of graduate school courses and programs, but it is only in the last ten years that we have seen wavelets and their applications being introduced into the undergraduate curriculum.

The introduction of the topic to undergraduates is quite timely—most of the foundational questions posed by wavelet researchers have been answered, and several current applications of wavelets are firmly entrenched in areas of image processing. Several authors [7, 2, 8, 14] have written books in which they present the basic results of wavelet theory in a manner that is accessible to undergraduates. Others have authored books [15, 13, 1, 9] that include applications as part of their presentation.

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This material is based upon work supported by the National Science Foundation under grant DUE-0717662.

In much the same way that wavelet theory is the confluence of several mathematical disciplines, we have discovered that *the discrete wavelet transformation* is an ideal topic for a modern undergraduate class—the derivation of the discrete wavelet transformation draws largely from calculus and linear algebra, provides a natural conduit to Fourier series and discrete convolution, and allows near-immediate access to current applications. Students learn about signal denoising, edge detection in digital images, and image compression, and use computer software in both the derivation and implementation of the discrete wavelet transformation. In the process of investigating applications, students learn *how* the application often drives the development of mathematical tools. Finally, the design of more advanced *wavelet filters* allows the students to gain experience "working in the transform domain" and provides motivation for several important concepts from an undergraduate real analysis class.

In what follows, we outline the basic development of discrete wavelet transformations and discuss their connection with Fourier series, convolution, and filtering. In the process, we illustrate the application of discrete wavelet transformations to digital images. We next construct one pair of filters that are used by the JPEG2000 image compression standard. We conclude the paper by illustrating how wavelet filters are implemented by the JPEG2000 standard.

Let us begin by discussing the simplest discrete wavelet transformation.

The Discrete Haar Wavelet Transformation

The discrete Haar wavelet transformation is the most elementary of all discrete wavelet transforms and serves as an excellent pedagogical tool for the development of more sophisticated discrete wavelet transformations and their application to digital signals or images.

We motivate this transformation as follows: Suppose we wish to transmit a list (vector) of N numbers, N even, to a friend via the Internet. To reduce transfer time, we decide to send only a length $N/2$ approximation of the data. One way to form the approximation is to send pairwise averages of the numbers. For example, the vector $[6, 12, 15, 15, 14, 12, 120, 116]^T$ can be approximated by the vector $[9, 15, 13, 118]^T$. Of course, it is impossible to determine the original N numbers from this approximation, but if, in addition, we transmit the $N/2$ averaged *directed distances*, then our friend could completely recover the original data. For our example, the averaged directed distances are $[3, 0, -1, -2]^T$.

We define the one–dimensional *discrete Haar wavelet transformation* as the linear transformation

$$\mathbf{x} \to \left[\begin{array}{c}\mathbf{a}\\ \hline \mathbf{d}\end{array}\right],$$

where $\mathbf{x} \in \mathbf{R}^N$, N even, and the elements of $\mathbf{a}, \mathbf{d} \in \mathbf{R}^{N/2}$ are

$$a_k = \frac{x_{2k-1} + x_{2k}}{2},$$
$$d_k = \frac{-x_{2k-1} + x_{2k}}{2},$$

where $k = 1, \ldots, N/2$. Given these averages and differences, we can recover the original data.

In matrix form, the transformation can be expressed as

$$\tilde{W}_N = \left[\begin{array}{ccccccc} \frac{1}{2} & \frac{1}{2} & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & & 0 & 0 \\ & & & & \ddots & & \\ 0 & 0 & 0 & 0 & \cdots & \frac{1}{2} & \frac{1}{2} \\ \hline -\frac{1}{2} & \frac{1}{2} & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & \frac{1}{2} & & 0 & 0 \\ & & & & \ddots & & \\ 0 & 0 & 0 & 0 & \cdots & -\frac{1}{2} & \frac{1}{2} \end{array}\right]$$

$$(1) \qquad = \left[\begin{array}{c}\tilde{H}_{N/2}\\ \hline \tilde{G}_{N/2}\end{array}\right]$$

with

$$(2) \qquad \tilde{W}_N^{-1} = 2\tilde{W}_N^T = 2\left[\begin{array}{c|c}\tilde{H}_{N/2}^T & \tilde{G}_{N/2}^T\end{array}\right].$$

What is the advantage of transforming $\mathbf{x}$ into $\mathbf{a}$ and $\mathbf{d}$? The vector $\mathbf{a}$ gives us an approximation of the original vector $\mathbf{x}$, while $\mathbf{d}$ allows us to identify large (or small) differences between the pairwise averages and the original data. Identifying large differences might be of interest if we were attempting to detect edges in a digital image, for example. On the other hand, we might be inclined to convert differences that are small in absolute value to zero—this action, assuming the original data were largely homogeneous, has the effect of producing a large number of zeros in the *quantized* differences vector $\tilde{\mathbf{d}}$. In this case, the modified transform is more amenable to data coders in image compression applications. Of course, we have no hope of recovering the original vector $\mathbf{x}$ from the modified transform, but depending on the application, the sacrifice of loss of resolution for storage size might be desirable.

The Two-Dimensional Discrete Haar Wavelet Transformation

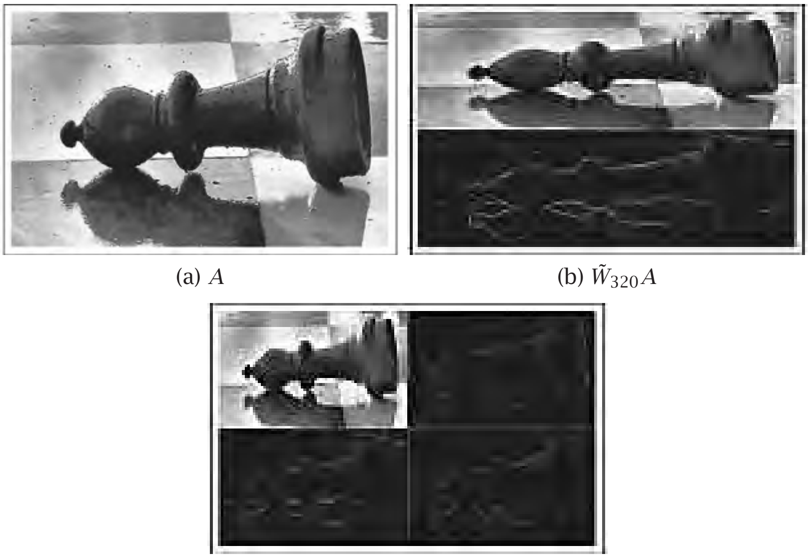
If we view a grayscale digital image as an $M \times N$ matrix A whose entries are integers between 0 and 255 inclusive, where 0 represents black and 255 is white, and integers in between indicate varying degrees of intensities of gray, then application of the one-dimensional Haar wavelet transform $\tilde{W}_M$ acts on the *columns* of A. Each column is transformed to a column whose top half gives an approximation, or *blur*, of the original column and whose bottom half holds the differences or *details* in the data.

We can process the rows of A as well by multiplying $\tilde{W}_M A$ by $\tilde{W}_N^T$. We define the *two-dimensional discrete Haar wavelet transformation* of the matrix A to be $\tilde{W}_M A \tilde{W}_N^T$. Of course, both M and N must be even integers. An image (stored in A), $\tilde{W}_M A$, and $\tilde{W}_M A \tilde{W}_N^T$ are plotted in Figure 1.

We can better understand the image displayed in Figure 1(c) if we express the two–dimensional transformation in block form. Indeed, using (1) and (2), we see that

$$\begin{aligned} \tilde{W}_M A \tilde{W}_N^T &= \left[\begin{array}{c}\tilde{H}_{M/2}\\ \hline \tilde{G}_{M/2}\end{array}\right] A \left[\begin{array}{c|c}\tilde{H}_{N/2}^T & \tilde{G}_{N/2}^T\end{array}\right] \\ &= \left[\begin{array}{c|c}\tilde{H}_{M/2} A \tilde{H}_{N/2}^T & \tilde{H}_{M/2} A \tilde{G}_{N/2}^T \\ \hline \tilde{G}_{M/2} A \tilde{H}_{N/2}^T & \tilde{G}_{M/2} A \tilde{G}_{N/2}^T\end{array}\right] \\ &= \left[\begin{array}{c|c}\mathcal{B} & \mathcal{V} \\ \hline \mathcal{H} & \mathcal{D}\end{array}\right]. \end{aligned}$$

It is a straightforward computation to see that if we partition A into 2×2 blocks $A_{j,k}$, $1 \le j \le M/2$, $1 \le k \le N/2$, the (j, k) elements of $\mathcal{B}$, $\mathcal{V}$, $\mathcal{H}$, $\mathcal{D}$ are



(a) A

(b) $\tilde{W}_{320}A$

(c) $\tilde{W}_{320}A\tilde{W}_{512}^T$

Figure 1. A 320 x 512 digital grayscale image, the product $\tilde{W}_M A$, and the two-dimensional discrete Haar wavelet transformation $\tilde{W}_M A \tilde{W}_N^T$.

constructed using the four values in $A_{j,k}$. Indeed, if

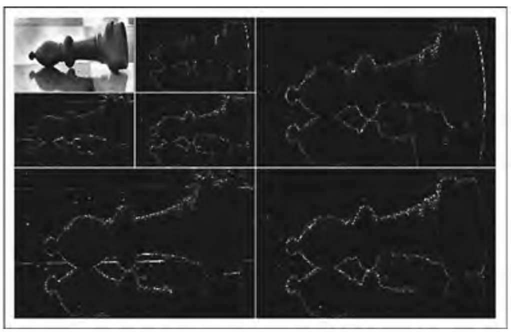
$$A_{j,k} = \begin{bmatrix} a & b \\ c & d \end{bmatrix},$$

then each (j,k) element of $\mathcal{B}$, $\mathcal{V}$, $\mathcal{H}$, $\mathcal{D}$ is

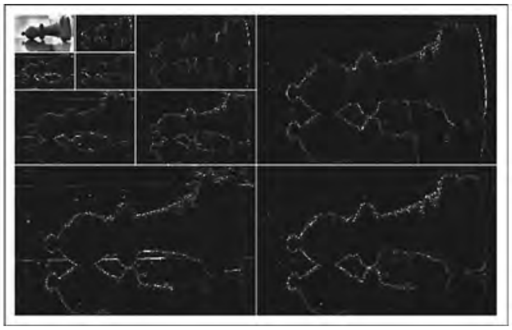
$$\begin{array}{cc} \dfrac{a+b+c+d}{4} & \dfrac{(a+c)-(b+d)}{4} \\ \dfrac{(a+b)-(c+d)}{4} & \dfrac{(a+d)-(b+c)}{4}, \end{array}$$

respectively. In this way we see that the elements of $\mathcal{B}$ are simply the averages of the elements in the blocks $A_{j,k}$, and the elements in $\mathcal{V}$, $\mathcal{H}$, and $\mathcal{D}$ are averaged differences in the vertical, horizontal, and diagonal directions, respectively.

If M and N are divisible by 2^p, then we can *iterate* the transformation a total of p times. Figure 1(c) shows one iteration of the discrete Haar wavelet transformation. We can perform a second iteration by applying the two-dimensional discrete Haar wavelet transformation to $\mathcal{B}$. Figure 2 shows two and three iterations of the discrete Haar wavelet transformation applied to A. In applications such as image compression, the iterated transformation produces more details portions, and in the case in which the original data are largely homogeneous, then we can expect a large number of near-zero values in the details portions of the iterated transform. These small values can be quantized to zero, and the data coder should be able to compress the modified transform in a highly efficient manner.



(a) Two iterations



(b) Three iterations

Figure 2. Multiple iterations of the discrete Haar wavelet transformation applied to the matrix A from Figure 1(a).

Alternatively, we can use the averages portion $\mathcal{B}$ in applications as well. Suppose we stored a four-iteration version of an image on a web server. Then

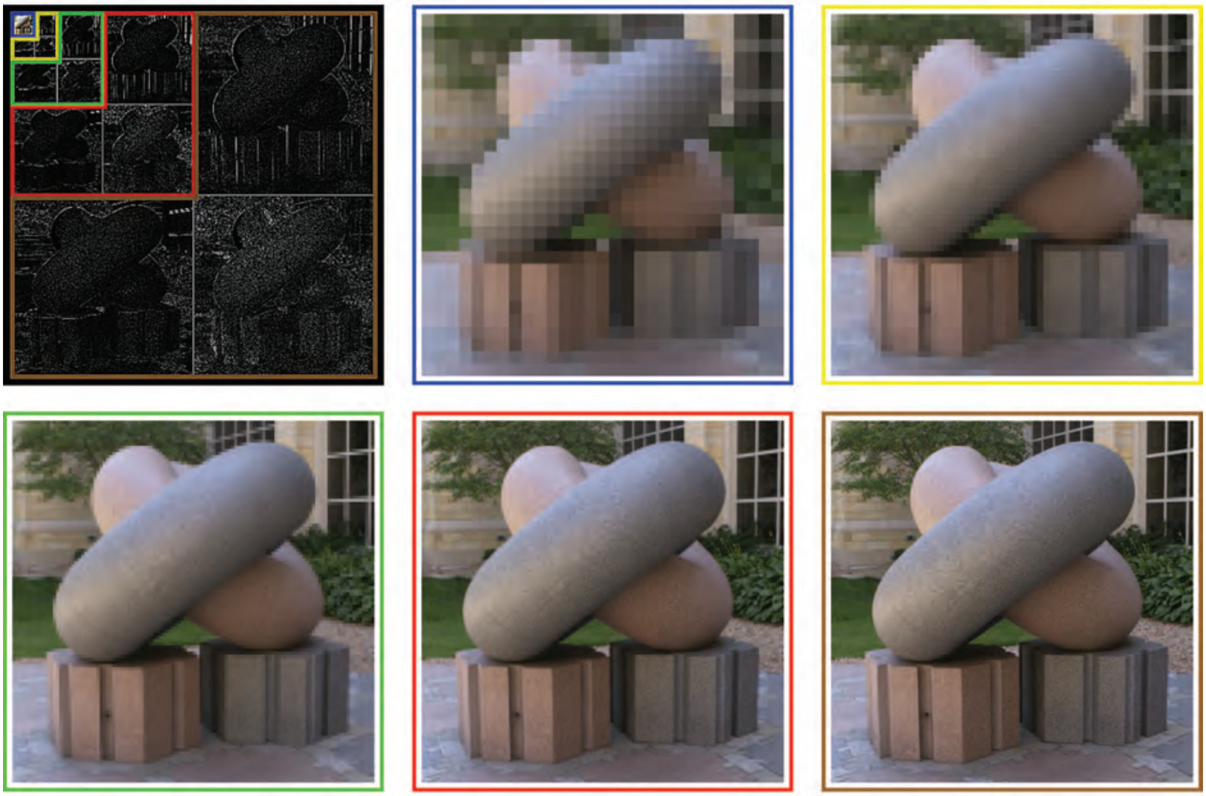


Figure 3. The wavelet transform of a digital image of Helaman Ferguson's *Four Canoes* is shown at the top left. The portion framed in blue is enlarged and transmitted. Next, the portion of the transform outlined in yellow is transmitted and, with $\mathcal{B}_4$, inverted and enlarged to create the image at top right. This process is repeated for the green, red, and brown portions of the transform to progressively create a sequence of images whose resolution increases by a factor of two at each step. The original image, outlined in brown, is at bottom right.

$\mathcal{B}_4$, the fourth iteration averages portion of the transform, could serve as a thumbnail image for the original. If a user requests the original image, we could first transmit $\mathcal{B}_4$ and then progressively send detail portions at each iterative level. The recipient can apply the inverse transform as detail portions are received to sequentially produce a higher-resolution version of the original image. Figure 3 illustrates the process.

Equation (2) tells us that the discrete Haar wavelet transformation is almost orthogonal. Indeed, if we define $W_N = \sqrt{2}\tilde{W}_N$, then $W_N^T = W_N^{-1}$, and our transformation is orthogonal. The inverse of the transformation now has a simple form, which allows us to easily reconstruct the signal from its transformed version.

Relation to Discrete Convolution and Filters

We have motivated the construction of W_N by insisting that W_N transform the original data $\mathbf{x}$ into an averages portion $\mathbf{a}$ and details portion $\mathbf{d}$. We could just as well develop W_N via discrete convolution.

Recall that if $\mathbf{h}$ and $\mathbf{x}$ are bi-infinite vectors, then the convolution of $\mathbf{h}$ with $\mathbf{x}$ is the bi-infinite vector $\mathbf{y}$, denoted by $\mathbf{y} = \mathbf{h} * \mathbf{x}$, where the nth component, $n \in \mathbb{Z}$, of the vector is

$$y_n = \sum_{k\in\mathbb{Z}} h_k x_{n-k}.$$

If we assume that $x_n = 0$ for $n < 1$ and $n > N$, where N is an even integer, then we can form $W_N\mathbf{x}$ by

- Computing $\mathbf{u} = \mathbf{h} * \mathbf{x}$, where the only nonzero values of $\mathbf{h}$ are $h_0 = h_1 = \sqrt{2}/2$.
- Computing $\mathbf{v} = \mathbf{g} * \mathbf{x}$, where the only nonzero values of $\mathbf{g}$ are $g_0 = \sqrt{2}/2$ and $g_1 = -\sqrt{2}/2$.
- *Downsampling* $\mathbf{u}$ and $\mathbf{v}$ by removing the odd components of each vector.
- Truncating the downsampled vectors appropriately to obtain $\mathbf{a}$ and $\mathbf{d}$.

The vectors $\mathbf{h}$ and $\mathbf{g}$ are often called *filters*. In fact, $\mathbf{h}$ and $\mathbf{g}$ are special kinds of filters. Let $\mathbf{x}$ be a vector whose elements $x_k = 1$, $k \in \mathbb{Z}$, and suppose

$\mathbf{y}$ is a vector whose elements are $y_k = (-1)^k$, $k \in \mathbb{Z}$. Then $\mathbf{h} * \mathbf{x} = \sqrt{2}\mathbf{x}$ and $\mathbf{h} * \mathbf{y}$ is the bi-infinite zero vector. Thus $\mathbf{h}$ is an example of a *lowpass filter*—it passes low-frequency signals through largely unchanged but attenuates the amplitudes of high-frequency data. In a similar manner we see that $\mathbf{g} * \mathbf{x}$ is the bi-infinite zero vector and $\mathbf{g} * \mathbf{y} = \sqrt{2}\mathbf{y}$. Here $\mathbf{g}$ is an example of a *highpass filter*—it allows high-frequency signals to pass largely unchanged but attenuates the amplitudes of low-frequency data. Thus in filtering terms, the discrete Haar wavelet transform is constructed by applying a lowpass filter and a highpass filter to the input data, downsampling both results, and then appropriately truncating the downsampled vectors. We will learn that all discrete wavelet transforms are built from a lowpass (*scaling*) filter $\mathbf{h}$ and a highpass (*wavelet*) filter. Fourier series are valuable tools for constructing these filters.

Fourier Series and Discrete Wavelet Transformations

A course on discrete wavelet transformations is an excellent venue for the introduction of Fourier series into the undergraduate curriculum. There are two main uses of Fourier series in such a course. We can determine whether a finite-length filter $\mathbf{f} = [f_0, \ldots, f_L]^T$ is lowpass or highpass by plotting the modulus of the Fourier series

$$F(\omega) = \sum_{k=0}^{L} f_k e^{ik\omega},$$

and, as we will see later, we can characterize a (bi)orthogonal discrete wavelet transformation by investigating an identity related to the Fourier series of the scaling filter(s).

For the discrete Haar wavelet transformation, the finite-length filters

$$\mathbf{h} = [h_0, h_1]^T = \left[\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right]^T$$

$$\mathbf{g} = [g_0, g_1]^T = \left[\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right]^T$$

give rise to the Fourier series

$$\begin{aligned} H(\omega) &= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}e^{i\omega} \\ &= \sqrt{2}e^{i\omega/2}\cos\left(\frac{\omega}{2}\right) \end{aligned} \tag{3}$$

$$\begin{aligned} G(\omega) &= \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}e^{i\omega} \\ &= -\sqrt{2}ie^{i\omega/2}\sin\left(\frac{\omega}{2}\right) \end{aligned} \tag{4}$$

for $\omega \in \mathbb{R}$. To determine the nature of the filter, we plot $|H(\omega)|$ and $|G(\omega)|$. Since $|H(\omega)|$ and $|G(\omega)|$ are both 2π-periodic, even functions, we plot them on the interval $[0, \pi]$. The functions are displayed in Figure 4.

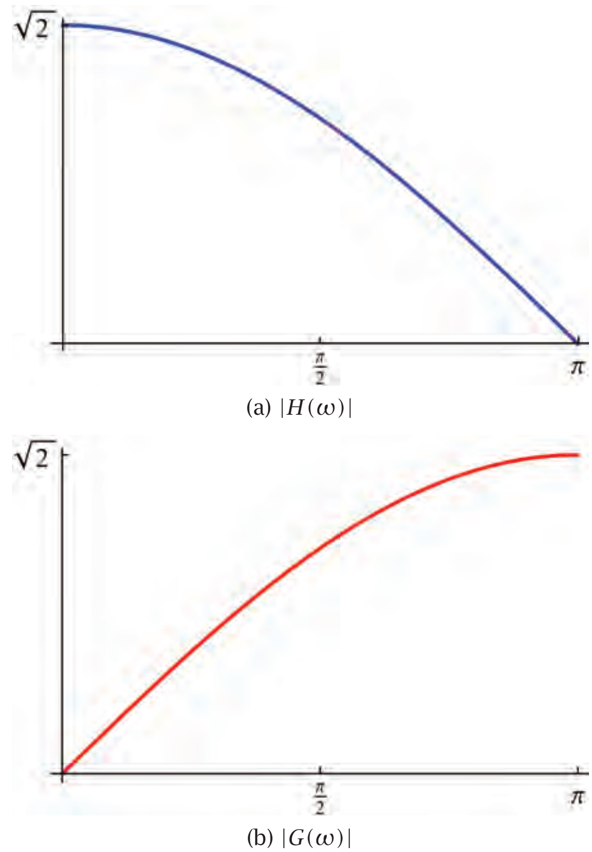


Figure 4. The functions $|H(\omega)|$ and $|G(\omega)|$.

At the lowest frequency, $\omega = 0$, $|H(0)| = \sqrt{2}$. At the highest frequency, $\omega = \pi$, $|H(\pi)| = 0$. In the frequency domain, these two conditions indicate that $\mathbf{h}$ is a lowpass filter. In a similar manner, $|G(0)| = 0$ and $|G(\pi)| = \sqrt{2}$ are conditions that imply that $\mathbf{g}$ is a highpass filter.

For $\omega \in \mathbb{R}$, we use (3) to write

$$|H(\omega)|^2 + |H(\omega+\pi)|^2 = 2\cos^2\left(\frac{\omega}{2}\right) + 2\cos^2\left(\frac{\omega+\pi}{2}\right) = 2. \tag{5}$$

In a similar manner, we use (3) and (4) to obtain

$$|G(\omega)|^2 + |G(\omega+\pi)|^2 = 2 \tag{6}$$

and

$$H(\omega)\overline{G(\omega)} + H(\omega+\pi)\overline{G(\omega+\pi)} = 0. \tag{7}$$

In the next section, we will see that the first step in a general strategy for finding longer, more sophisticated filters $\mathbf{h}$ and $\mathbf{g}$ is to find $\mathbf{h}$ so that (5) is satisfied and then make a judicious choice of $\mathbf{g}$ so that (6) and (7) hold as well. The additional lowpass conditions $|H(0)| = \sqrt{2}$ and $|H(\pi)| = 0$ with our choice of $\mathbf{g}$ ensure that the highpass conditions $|G(0)| = 0$, $|G(\pi)| = \sqrt{2}$ hold, and in this way, we see that all lowpass, highpass, and matrix orthogonality conditions placed on filters

$\mathbf{h}$ and $\mathbf{g}$ can be completely characterized using Fourier series.

Daubechies Orthogonal Scaling Filters

Suppose we apply the one-dimensional discrete Haar wavelet transformation to the vector $\mathbf{v} = [0, 0, 100, 100]^T$. We obtain the transformed vector $\sqrt{2}\,[0, 100 |\, 0, 0]^T$. The differences portion of the transformed vector consists only of zeros, missing the large "edge" between 0 and 100 in $\mathbf{v}$. The problem is that the Haar scaling/wavelet filter pair is too short. Let's consider longer filter pairs. These filters and the corresponding wavelets were discovered by Ingrid Daubechies (see, e. g., [4]).

We wish to construct an orthogonal matrix W_N that uses longer scaling and wavelet filters. It turns out that we need to consider even-length filters, so let's start with a scaling filter $\mathbf{h} = [h_0, h_1, h_2, h_3]^T$ and wavelet filter $\mathbf{g} = [g_0, g_1, g_2, g_3]^T$. We insist that

$$|H(0)| = |h_0 + h_1 + h_2 + h_3| = \sqrt{2} \tag{8}$$

$$H(\pi) = h_0 - h_1 + h_2 - h_3 = 0 \tag{9}$$

and

$$G(0) = g_0 + g_1 + g_2 + g_3 = 0 \tag{10}$$

$$|G(\pi)| = |g_0 - g_1 + g_2 - h_3| = \sqrt{2} \tag{11}$$

so that the filters $\mathbf{h}$, $\mathbf{g}$ are lowpass and highpass, respectively.

We also wish to retain the structure of the transformation by suitably truncating two downsampled, convolution products. In this case, the longer lengths of our filters force us to "wrap" rows at the bottom of each block $H_{N/2}$, $G_{N/2}$ of the matrix W_N. For example, in the case in which $N = 8$, W_N takes the form

$$W_8 = \left[\begin{array}{cccccccc} h_3 & h_2 & h_1 & h_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_3 & h_2 & h_1 & h_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h_3 & h_2 & h_1 & h_0 \\ h_1 & h_0 & 0 & 0 & 0 & 0 & h_3 & h_2 \\ \hline g_3 & g_2 & g_1 & g_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & g_3 & g_2 & g_1 & g_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & g_3 & g_2 & g_1 & g_0 \\ g_1 & g_0 & 0 & 0 & 0 & 0 & g_3 & g_2 \end{array}\right]. \tag{12}$$

The wrapping rows cause problems in applications since it inherently assumes that data are periodic, but on the other hand, it simplifies our final requirement that W_N be an orthogonal matrix. In this case, we must have

$$h_0^2 + h_1^2 + h_2^2 + h_3^2 = 1 \tag{13}$$

$$h_0h_2 + h_1h_3 = 0. \tag{14}$$

It turns out that if real numbers $h_0, \ldots, h_3$ satisfy (9), (13), and (14), then (8) holds. We will choose the positive square root so that

$$h_0 + h_1 + h_2 + h_3 = \sqrt{2}. \tag{15}$$

We now set $\mathbf{g} = [h_3, -h_2, h_1, -h_0]^T$. It is straightforward to show that if (9), (13), (14), and (15) hold, then the highpass conditions (10), (11) are satisfied and

$$\begin{aligned} g_0^2 + g_1^2 + g_2^2 + g_3^2 &= 1 \\ g_0g_2 + g_1g_3 &= 0. \end{aligned}$$

Moreover, we can show that

$$\begin{aligned} h_0g_0 + h_1g_1 + h_2g_2 + h_3g_3 &= 0 \\ h_1g_3 + h_0g_2 &= 0 \\ h_3g_1 + h_2g_0 &= 0 \end{aligned}$$

so that W_N is an orthogonal matrix. However, there are infinitely many solutions to the system

$$\boxed{\begin{aligned} h_0^2 + h_1^2 + h_2^2 + h_3^2 &= 1 \\ h_0h_2 + h_1h_3 &= 0 \\ h_0 + h_1 + h_2 + h_3 &= \sqrt{2} \\ h_0 - h_1 + h_2 - h_3 &= 0 \end{aligned}}\,. \tag{16}$$

Here is where the Fourier series approach really makes an impact. Daubechies chose to impose the additional condition $H'(\pi) = 0$. This condition has the effect of "flattening" the graph of the modulus $|H(\omega)|$ even more at $\omega = \pi$; thus intuitively the filter $\mathbf{h}$ does an even better job of attenuating the amplitudes of the high-frequency portions of the input data. If we expand the left-hand side of $H'(\pi) = 0$, we obtain the following linear equation

$$h_1 - 2h_2 + 3h_3 = 0, \tag{17}$$

and we add (17) to our system (16).

It turns out that there are two real solutions to the system, one of which is a reflection of the other. The solution can be solved by hand (see, e. g., [12]) to obtain the *Daubechies four-term scaling filter*:

$$h_0 = \frac{1+\sqrt{3}}{4\sqrt{2}}, \quad h_1 = \frac{3+\sqrt{3}}{4\sqrt{2}}$$
$$h_2 = \frac{3-\sqrt{3}}{4\sqrt{2}}, \quad h_3 = \frac{1-\sqrt{3}}{4\sqrt{2}}.$$

The modulus of $H(\omega)$ is plotted in Figure 5. Compare this graph with that of Figure 4.

In order to construct longer even-length filters $\mathbf{h} = [h_0, \ldots, h_L]^T$, L odd, we must write down general orthogonality conditions and lowpass conditions. We can ensure the orthogonality of W_N using the following theorem.

Theorem 1. *Suppose* $A(\omega) = \sum_{k\in\mathbb{Z}} a_k e^{ik\omega}$ *and* $B(\omega) = \sum_{k\in\mathbb{Z}} b_k e^{ik\omega}$ *are Fourier series. Then*

$$|A(\omega)|^2 + |A(\omega+\pi)|^2 = 2 \tag{18}$$

if and only if

$$\sum_{k\in\mathbb{Z}} a_k a_{k-2n} = \delta_{0,n} \tag{19}$$

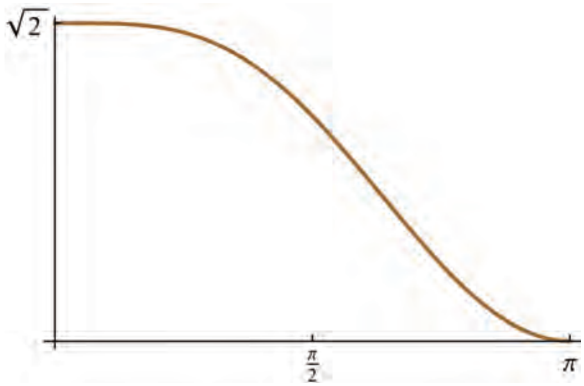


Figure 5. $|H(\omega)|$ for the length four Daubechies scaling filter.

for $n \in \mathbb{Z}$, and

$$A(\omega)\overline{B(\omega)} + A(\omega+\pi)\overline{B(\omega+\pi)} = 0 \tag{20}$$

if and only if

$$\sum_{k\in\mathbb{Z}} a_k b_{k-2n} = 0 \tag{21}$$

for all $n \in \mathbb{Z}$.

The proof of this theorem is straightforward. Note that equations (19) and (21) guarantee orthogonality of the matrix W_N.

Suppose $H(\omega)$ satisfies (18). If we take

$$G(\omega) = -e^{iL\omega}\overline{H(\omega+\pi)}, \tag{22}$$

then $G(\omega)$ satisfies (18), and $H(\omega)$, $G(\omega)$ satisfy (20). It is an easy exercise to show that the Fourier coefficients of $G(\omega)$ are $g_k = (-1)^k h_{1-k}$, $k \in \mathbb{Z}$.

The condition $H'(\pi) = 0$ leads naturally to the following generalization: If we want to produce longer scaling filters $\mathbf{h}$, each time we increase the filter length by two, we require an additional derivative condition at π. For example, the Daubechies scaling filter of length six is, in part, constructed by requiring the conditions that $H'(\pi) = 0$ and $H''(\pi) = 0$. Each time an additional derivative condition is imposed at π, the modulus of the corresponding Fourier series becomes flatter at the highest frequency $\omega = \pi$. So the general derivative conditions are

$$H^{(m)}(\pi) = 0, \quad m = 0, \ldots, \frac{L-1}{2}. \tag{23}$$

We also add the lowpass condition

$$H(0) = \sum_{k=0}^{L} h_k = \sqrt{2} \tag{24}$$

so that our general system is given by (19), (23), and (24). Daubechies proved that there are $M = 2^{\lfloor \frac{L+2}{4} \rfloor}$ real-valued scaling filters $\mathbf{h} = [h_0, h_1, \ldots, h_L]^T$, L is odd, that satisfy this system. Moreover, if we construct $G(\omega)$ using (22), then $G(0) = 0$ and $|G(\pi)| = \sqrt{2}$, as desired.

While this derivation of the system is easy to understand, solving it numerically for large filter lengths can be difficult. It should be noted that Daubechies's classical derivation of these filters is centered around the construction of a trigonometric polynomial that leads to the solution of (18). Particular roots of this polynomial are then used to find a Fourier series $H(\omega)$ that solves (23). This process (see [5] or [9]) works well for filters of long length, but the mathematics required to understand it may be better suited for advanced undergraduate or beginning graduate students.

Orthogonal Transforms in Applications

Now that we have the means for constructing orthogonal filters $\mathbf{h}$, $\mathbf{g}$ of any odd length, it is natural to consider the advantages and disadvantages of their implementation in applications.

Let's first consider filter length. Certainly, the time to compute $W_N\mathbf{v}$ is inversely proportional to the length of $\mathbf{h}$. On the other hand, it can be shown [4] that the derivative conditions (23) lead to filters $\mathbf{g}$ that annihilate (modulo wrapping rows) data obtained by uniformly sampling a polynomial of degree $(L-1)/2$.

In image compression we typically transform the image, then quantize the transform, and finally encode the result. It can be shown that if the transform is orthogonal, then the error incurred when quantizing the transform coefficients is exactly the same as the error between the original image and the compressed image.

There are disadvantages to using orthogonal transformations. In lossless image compression, the quantization step is omitted. In this case, we need to use a transformation that maps integers to integers. It is possible to modify orthogonal transforms (see [3]) and produce integer-to-integer mappings, but it is easier to modify the biorthogonal transformations described in the next section.

Unfortunately, the wrapping rows that appear in the W_N constructed from Daubechies scaling and wavelet filters pose a problem in many applications. In (12), there is one wrapping row in each block of the transform. Figure 6 illustrates the problem when the discrete Daubechies wavelet transformation is applied to $\mathbf{v} \in \mathbb{R}^{16}$, where $v_k = k$, $k = 1, \ldots, 16$. The wrapping rows cause the first two elements to be combined with the last two elements of $\mathbf{v}$ to form the last weighted average and last weighted difference. If the data are periodic, this makes perfect sense, but in many signal/image processing applications, the data are not periodic. The wrapping row problem, caused by truncating downsampled convolution products, is typical in applied mathematics.

Biorthogonal Scaling Filters

Fortunately, there is a way to deal with the wrapping row problem, and that is to develop filters that are *symmetric*.

An odd-length filter $\mathbf{h}$ is called symmetric if $h_k = h_{-k}$, while an even-length filter is called symmetric if $h_k = h_{1-k}$. Daubechies [5] proved that the only symmetric, finite length, orthogonal filter is the Haar filter, and we have already discussed some of its limitations. How can we produce symmetric filters while preserving some of the desirable properties of orthogonal filters? These desirable properties are finite length (computational speed), orthogonality (ease of inverse), and the ability to produce a good approximation of the original data with the scaling filter. Since the inverse need only be computed once, it was Daubechies's idea to relinquish orthogonality and to construct instead a discrete *biorthogonal* wavelet transformation.

The idea is to construct *two* sets of filters instead of one. In other words, we construct two wavelet transform matrices $\tilde{W}_N$ and W_N so that $\tilde{W}_N^{-1} = W_N^T$. In block form, we have

$$\begin{aligned}\tilde{W}_N W_N^T &= \left[\begin{array}{c}\tilde{H}_{N/2}\\ \hline \tilde{G}_{N/2}\end{array}\right]\cdot\left[\begin{array}{c|c}H_{N/2}^T & G_{N/2}^T\end{array}\right]\\ &= \left[\begin{array}{c|c}\tilde{H}_{N/2}H_{N/2}^T & \tilde{H}_{N/2}G_{N/2}^T\\ \hline \tilde{G}_{N/2}H_{N/2}^T & \tilde{G}_{N/2}G_{N/2}^T\end{array}\right]\\ &= \left[\begin{array}{c|c}I_{N/2} & 0_{N/2}\\ \hline 0_{N/2} & I_{N/2}\end{array}\right].\end{aligned}$$

Let $\tilde{\mathbf{h}}$ and $\mathbf{h}$ denote two filters and suppose $H(\omega)$, $\tilde{H}(\omega)$ are the associated Fourier series. We require both filters to be lowpass so we have $\tilde{H}(0) = H(0) = \sqrt{2}$ and $\tilde{H}(\pi) = H(\pi) = 0$. Instead of orthogonality, we insist that the filters satisfy the general biorthogonal condition

$$\tilde{H}(\omega)\overline{H(\omega)} + \tilde{H}(\omega+\pi)\overline{H(\omega+\pi)} = 2. \tag{25}$$

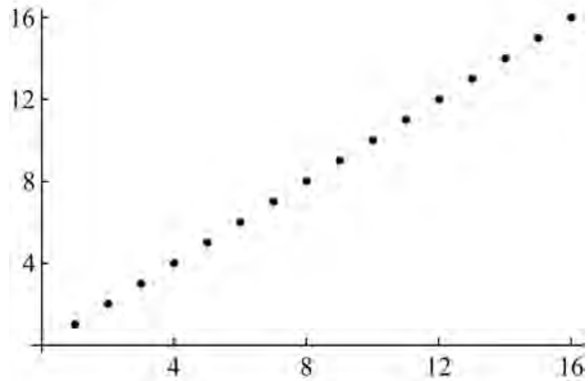
Although the filters are no longer orthogonal, they are close to orthogonal, and a proof very similar to that of Theorem 1 shows that (25) holds if and only if

$$\sum_{k\in\mathbb{Z}} \tilde{h}_k h_{k-2n} = \delta_{0,n}, \tag{26}$$

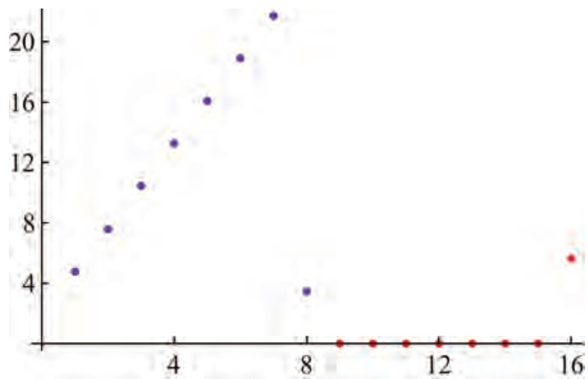
where $n \in \mathbb{Z}$. Equation (26) ensures us, *if* we wrap rows, that $\tilde{H}_{N/2}H_{N/2}^T = I_{N/2}$. If we take

$$\tilde{G}(\omega) = -e^{i\omega}\overline{H(\omega+\pi)} \tag{27}$$

$$G(\omega) = -e^{i\omega}\overline{\tilde{H}(\omega+\pi)}, \tag{28}$$



(a) $\mathbf{v}$



(b) $W_{16}\mathbf{v}$

Figure 6. The discrete wavelet transform, constructed using the Daubechies length four scaling filter, applied to v.

then we can easily verify that $\tilde{G}(0) = G(0) = 0$, $\tilde{G}(\pi) = G(\pi) = \sqrt{2}$, and

$$\tilde{G}(\omega)\overline{G(\omega)} + \tilde{G}(\omega+\pi)\overline{G(\omega+\pi)} = 2 \tag{29}$$

$$\tilde{H}(\omega)\overline{G(\omega)} + \tilde{H}(\omega+\pi)\overline{G(\omega+\pi)} = 0 \tag{30}$$

$$\tilde{G}(\omega)\overline{H(\omega)} + \tilde{G}(\omega+\pi)\overline{H(\omega+\pi)} = 0. \tag{31}$$

Equation (29) implies that $\tilde{G}_{N/2}G_{N/2}^T = I_{N/2}$, and (30), (31), in conjunction with (20) from Theorem 1, establishes $\tilde{H}_{N/2}G_{N/2}^T = \tilde{G}_{N/2}H_{N/2}^T = 0_{N/2}$. We can also expand the Fourier series in (27) and (28) to obtain the wavelet filter coefficients

$$\tilde{g}_k = (-1)^k h_{1-k} \quad \text{and} \quad g_k = (-1)^k \tilde{h}_{1-k}, \tag{32}$$

where $1-k$ ranges over the nonzero indices of $\tilde{\mathbf{h}}$, $\mathbf{h}$. Thus, if we solve (25), we can find all the filters we need to build $\tilde{W}_N$ and W_N. Of course (25) represents a quadratic system in terms of the scaling filter coefficients, and such systems are difficult to solve. Daubechies's solution to this problem was to simply pick the scaling filter $\tilde{\mathbf{h}}$ and then solve the *linear system* (25) for the scaling filter $\mathbf{h}$. Moreover, since we have given up orthogonality in this new construction, we can insist that both scaling filters be symmetric. Let's look at an example.

Example 1. Suppose we want $\tilde{\mathbf{h}}$ to be a symmetric, length three, lowpass filter $\tilde{\mathbf{h}} = [\tilde{h}_{-1}, \tilde{h}_0, \tilde{h}_1]^T = [\tilde{h}_1, \tilde{h}_0, \tilde{h}_1]^T$. We seek two numbers $\tilde{h}_0$, $\tilde{h}_1$ so that $\tilde{H}(0) = \tilde{h}_1 + \tilde{h}_0 + \tilde{h}_1 = \sqrt{2}$ and $\tilde{H}(\pi) = -\tilde{h}_1 + \tilde{h}_0 - \tilde{h}_1 = 0$. The filter[1] $\tilde{\mathbf{h}}$ we seek is

$$[\tilde{h}_{-1}, \tilde{h}_0, \tilde{h}_1]^T = \frac{\sqrt{2}}{4}[1, 2, 1]^T.$$

To find a symmetric filter $\mathbf{h}$, we must satisfy the biorthogonality condition (25), together with the lowpass constraints $H(0) = \sqrt{2}$ and $H(\pi) = 0$. It turns out that $\mathbf{h}$ must have an odd length of $5, 9, 13, \ldots$. We choose symmetric $\mathbf{h} = [h_2, h_1, h_0, h_1, h_2]^T$ and use the lowpass condition $H(\pi) = 0$ in conjunction with (26) to obtain the system

$$\begin{aligned} h_0 - 2h_1 + 2h_2 &= 0 \\ h_0 + \ \ h_1 \qquad &= \sqrt{2} \\ h_1 + 2h_2 &= 0. \end{aligned}$$

Solving this system gives $h_2 = -\frac{\sqrt{2}}{8}$, $h_1 = \frac{\sqrt{2}}{4}$, and $h_0 = \frac{3\sqrt{2}}{4}$. We can next use (32) to find the coefficients of the wavelet filters $\mathbf{g}$ and $\tilde{\mathbf{g}}$. We have:

$\tilde{\mathbf{g}}$	$\mathbf{g}$
$\tilde{g}_{-1} = \frac{\sqrt{2}}{8}$	
$\tilde{g}_0 = \frac{\sqrt{2}}{4}$	$g_0 = \frac{\sqrt{2}}{4}$
$\tilde{g}_1 = -\frac{3\sqrt{2}}{4}$	$g_1 = -\frac{\sqrt{2}}{2}$
$\tilde{g}_2 = \frac{\sqrt{2}}{4}$	$g_2 = \frac{\sqrt{2}}{4}$
$\tilde{g}_3 = \frac{\sqrt{2}}{8}$	

Note that there is some symmetry in the highpass filters: $\tilde{g}_k = \tilde{g}_{2-k}$ and $g_k = g_{2-k}$. Here are examples of the biorthogonal transformation matrices with $N = 8$. We form the matrices by placing the 0th element in the upper left corner of each block with positive indexed elements placed consecutively to the right and negative indexed elements placed consecutively to the left with wrapping if necessary.

$$\tilde{W}_8 = \left[\begin{array}{cccccccc}
\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} \\
0 & \frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{4} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{4} \\
\hline
\frac{\sqrt{2}}{4} & -\frac{3\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{8} & 0 & 0 & 0 & \frac{\sqrt{2}}{8} \\
0 & \frac{\sqrt{2}}{8} & \frac{\sqrt{2}}{4} & -\frac{3\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{8} & 0 & 0 \\
0 & 0 & 0 & \frac{\sqrt{2}}{8} & \frac{\sqrt{2}}{4} & -\frac{3\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{8} \\
\frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{8} & 0 & 0 & 0 & \frac{\sqrt{2}}{8} & \frac{\sqrt{2}}{4} & -\frac{3\sqrt{2}}{4}
\end{array}\right]$$

[1] *Do you see how Pascal's triangle can be used to generate symmetric, lowpass filters?*

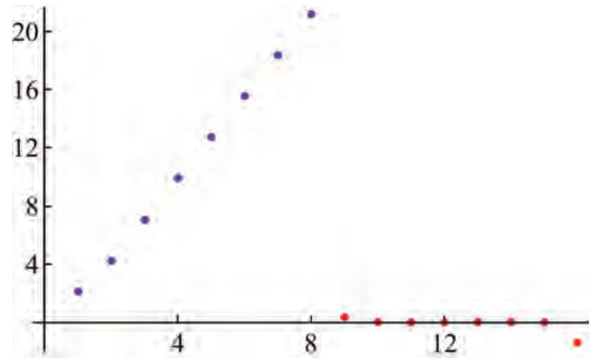


Figure 7. The modified biorthogonal wavelet transformation applied to the vector v from Figure 6(a).

and

$$W_8 = \left[\begin{array}{cccccccc}
\frac{3\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{8} & 0 & 0 & 0 & -\frac{\sqrt{2}}{8} & \frac{\sqrt{2}}{4} \\
-\frac{\sqrt{2}}{8} & \frac{\sqrt{2}}{4} & \frac{3\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{8} & 0 & 0 & 0 \\
0 & 0 & -\frac{\sqrt{2}}{8} & \frac{\sqrt{2}}{4} & \frac{3\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{8} & 0 \\
-\frac{\sqrt{2}}{8} & 0 & 0 & 0 & -\frac{\sqrt{2}}{8} & \frac{\sqrt{2}}{4} & \frac{3\sqrt{2}}{4} & \frac{\sqrt{2}}{4} \\
\hline
\frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{4} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{4} & 0 \\
\frac{\sqrt{2}}{4} & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{2}
\end{array}\right]$$

It is straightforward to verify that $\tilde{W}_8^{-1} = W_8^T$. Note that $\tilde{W}_8$ and W_8 still have wrapping rows, but we can exploit the symmetry of the scaling filters to construct a transformation that diminishes the adverse effects of the wrapping rows. The idea is quite simple, and we illustrate it now with an example.

Example 2. Consider again the vector $\mathbf{v} \in \mathbb{R}^{16}$ where $v_k = k$. We form the vector $\mathbf{v}_p \in \mathbb{R}^{30}$ where

$$\mathbf{v}_p = [v_1, \ldots, v_{16} | v_{15}, v_{14}, \ldots, v_2]^T.$$

We next compute $\left[\begin{array}{c} \mathbf{a}_p \\ \hline \mathbf{d}_p \end{array}\right] = \tilde{W}_{30}\mathbf{v}_p$. The periodic nature of $\mathbf{v}_p$ means that the weighted averages and differences formed by the wrapping rows use consecutive elements from $\mathbf{v}$. For example, the first element of $\mathbf{a}_p$ is $\frac{\sqrt{2}}{4}v_2 + \frac{\sqrt{2}}{2}v_1 + \frac{\sqrt{2}}{4}v_2$ instead of the first element of $\tilde{W}_{16}$, which is $\frac{\sqrt{2}}{4}v_{16} + \frac{\sqrt{2}}{2}v_1 + \frac{\sqrt{2}}{4}v_2$. The elements of the transform that do not involve wrapping rows are the same as elements in $\tilde{W}_{16}\mathbf{v}$. We keep the first eight elements of $\mathbf{a}_p$ and $\mathbf{d}_p$ and concatenate them to form our transform. The result is plotted in Figure 7. Compare this result to that plotted in Figure 6(b).

We can certainly construct biorthogonal filter pairs of different lengths—we can either select $\tilde{\mathbf{h}}$, then set up and solve a linear system for $\mathbf{h}$, or we can use the closed formulas for $\tilde{H}(\omega)$ and $H(\omega)$

obtained by Daubechies in [6]. The biorthogonal filter pair from the above example is particularly interesting since it plays a significant role in the JPEG2000 lossless image compression standard. Next we present one more modification of the transformation from Example 2 to understand how the discrete biorthogonal wavelet transformation is used in the JPEG2000 image compression standard.

Discrete Wavelet Transformations and JPEG2000

In 1992 JPEG (Joint Photographic Image Experts Group) became an international standard for compressing digital images. In the late 1990s work began to improve the JPEG compression standard, and a new algorithm was developed. This algorithm uses the biorthogonal wavelet transform instead of the discrete cosine transform. Because of patent issues, JPEG2000 is not yet supported by popular web browsers, but it is likely that once the patent issues are resolved, JPEG2000 will replace JPEG as the most popular standard for image compression.

The basic JPEG2000 algorithm works as follows. Let us assume that we have a grayscale image A. We first subtract 128 from each entry of A. We apply as many iterations of the modified biorthogonal wavelet transformation (see Example 2) as we wish (and as possible, depending on the dimensions of the image). The filter pair used in the discrete biorthogonal wavelet transform depends on whether we choose to perform *lossy* or *lossless* compression. In the case of the former, the transformed image is quantized with many elements either reduced in magnitude or converted to zero. Of course, once we quantize the transformed image, we have no hope of recovering the original image, but this loss of resolution is not as important as the coder's ability to compress the quantized transform. In lossless compression, no quantization is performed. In both cases, the next step is to code the data. JPEG2000 uses an arithmetic coding algorithm known as Embedded Block Coding with Optimized Truncation (EBCOT) to actually encode the (quantized) transformed data. This last step is the actual compression step. After transmission of the data, the original image can be reconstructed by decoding, then applying the inverse transformation as many times as iterated. With lossless compression, we can recover the image exactly.

One of the many advantages of JPEG2000 over JPEG is its ability to allow the user to perform the lossless compression. The biorthogonal filter pair for this task is a modified version of the $\tilde{\mathbf{h}}$ and $\mathbf{h}$ from Example 1. The filter $\mathbf{h}$ is multiplied by $\sqrt{2}/2$ and $\tilde{\mathbf{h}}$ is multiplied by $\sqrt{2}$. Moreover, the filters are "reversed"—$\mathbf{h}$ is used to build $\tilde{W}_N$. For example, the modified transform for vectors of length 8 is:

$$\tilde{W}_8 = \left[\begin{array}{cccccccc} \frac{3}{4} & \frac{1}{4} & -\frac{1}{8} & 0 & 0 & 0 & -\frac{1}{8} & \frac{1}{4} \\ -\frac{1}{8} & \frac{1}{4} & \frac{3}{4} & \frac{1}{4} & -\frac{1}{8} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{8} & \frac{1}{4} & \frac{3}{4} & \frac{1}{4} & -\frac{1}{8} & 0 \\ -\frac{1}{8} & 0 & 0 & 0 & -\frac{1}{8} & \frac{1}{4} & \frac{3}{4} & \frac{1}{4} \\ \hline -\frac{1}{2} & 1 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 1 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{2} & 1 & -\frac{1}{2} & 0 \\ -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 1 \end{array}\right].$$

We use the ideas of Example 2 to further modify the transformation so that it better handles output produced by wrapping rows. This modification is equivalent to computing $\tilde{W}_8^p$ to $\mathbf{v}$ where

$$\tilde{W}_8^p = \left[\begin{array}{cccccccc} \frac{3}{4} & \frac{1}{2} & -\frac{1}{4} & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{8} & \frac{1}{4} & \frac{3}{4} & \frac{1}{4} & -\frac{1}{8} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{8} & \frac{1}{4} & \frac{3}{4} & \frac{1}{4} & -\frac{1}{8} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{8} & \frac{1}{4} & \frac{5}{8} & \frac{1}{4} \\ \hline -\frac{1}{2} & 1 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 1 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{2} & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{array}\right]. \tag{33}$$

This modified transform is still invertible, and it now does a good job of handling edge effects. However, the coding step of the JPEG2000 algorithm works best if the transformation maps integers to integers. How can we modify (33) for general size N so that it maps integers to integers? We could multiply the transformed data by 8, but that increases the range of the output, which is not desirable for coders. The algorithm used by JPEG2000 for lossless compression is called *lifting* and is due to Sweldens [11].

The Lifting Scheme

The idea behind lifting is quite simple. For a nice tutorial, see [10], and more details are provided in the books [12, 8].

To understand how lifting works, we apply (33) to $\mathbf{v} \in \mathbb{Z}^8$. The resulting product is $\tilde{W}_8^p\mathbf{v} =$

$$\left[\frac{\mathbf{a}}{\mathbf{d}}\right] = \left[\begin{array}{c} -\frac{1}{8}v_3+\frac{1}{4}v_2+\frac{3}{4}v_1+\frac{1}{4}v_2-\frac{1}{8}v_3 \\ -\frac{1}{8}v_1+\frac{1}{4}v_2+\frac{3}{4}v_3+\frac{1}{4}v_4-\frac{1}{8}v_5 \\ -\frac{1}{8}v_3+\frac{1}{4}v_4+\frac{3}{4}v_5+\frac{1}{4}v_6-\frac{1}{8}v_7 \\ -\frac{1}{8}v_5+\frac{1}{4}v_6+\frac{3}{4}v_7+\frac{1}{4}v_8-\frac{1}{8}v_7 \\ \hline -\frac{1}{2}v_1+v_2-\frac{1}{2}v_3 \\ -\frac{1}{2}v_3+v_4-\frac{1}{2}v_5 \\ -\frac{1}{2}v_5+v_6-\frac{1}{2}v_7 \\ -\frac{1}{2}v_7+v_8-\frac{1}{2}v_7 \end{array}\right], \tag{34}$$

where we have rewritten a_1, a_4, and d_4 so that they display the same symmetry as the other elements in the product. Note that the center value of the elements of $\mathbf{a}/(\mathbf{d})$ are built from the odd (even) elements of $\mathbf{v}$. If we form "even" and "odd" vectors

$$\mathbf{e} = [e_1, e_2, e_3, e_4]^T = [v_2, v_4, v_6, v_8]^T$$
$$\mathbf{o} = [o_1, o_2, o_3, o_4]^T = [v_1, v_3, v_5, v_7]^T,$$

then

$$d_k = e_k - \frac{1}{2}(o_{k+1} - o_k), \tag{35}$$

where $k = 1, 2, 3, 4$ and $o_5 := o_4$. The elements of the top half of the transformation can then be *lifted* from $\mathbf{d}$:

$$a_k = o_k + \frac{1}{4}(d_k + d_{k-1}), \tag{36}$$

where $k = 1, 2, 3, 4$ and $d_0 := d_1$.

This method is computationally more efficient than a fast matrix multiplication. Moreover, we can easily modify this transformation so that it maps integers to integers. Instead of computing the elements of $\mathbf{d}$ using (35), we compute $\mathbf{d}^*$, whose elements are

$$d_k^* = e_k - \left\lfloor \frac{1}{2}(o_k + o_{k+1}) \right\rfloor \tag{37}$$

and then compute $\mathbf{a}^*$, whose elements are

$$a_k^* = o_k + \left\lfloor \frac{1}{4}(d_k^* + d_{k-1}^*) + \frac{1}{2} \right\rfloor. \tag{38}$$

Adding $1/2$ to the term above eliminates bias. Certainly, the elements of $\mathbf{a}^*$ and $\mathbf{d}^*$ are integer-valued and only slightly different from those given by (35), (36), respectively. Moreover, the process is completely invertible. Given $\mathbf{d}^*$ and $\mathbf{a}^*$, we can certainly recover the values o_k using (38), and then given $\mathbf{o}$ and $\mathbf{d}^*$, we can use (37) to recover $\mathbf{e}$.

Concluding Remarks

In this article, we have provided a very brief introduction to discrete wavelet transformations and have shown how the topic effectively lends itself to an intriguing undergraduate applied mathematics course.

By introducing students to the discrete Haar wavelet transformation, we are able to not only construct a matrix representation of a tool used in several applications but also provide students with a concrete example of how convolution and Fourier series can lead to a more general mathematical model for the construction of more sophisticated scaling and wavelet filters.

The Daubechies filters are better suited for applications, but still suffer from the effects of truncating infinite convolution products. This problem leads to the development of biorthogonal scaling filters. Still more modification is needed to handle boundary effects, and students learn that these modifications are not only tractable but also lead to the lifting scheme. This computational method is not only faster than sparse matrix multiplication but can also be modified to produce the seemingly impossible invertible transformation that involves rounding! This final modification is an important tool used by the JPEG2000 image compression standard to perform lossless compression.

We have not discussed in this paper the classical approach to wavelet theory—the mathematics there is quite elegant (the books [2, 14, 9] are aimed at undergraduates). We believe, however, that an introduction to the discrete wavelet transformation, with immediate immersion in applications, is an effective way to introduce students to applications that are ubiquitous in today's digital age, to hone problem-solving skills, to promote and improve scientific computing, and to motivate concepts in upper-level undergraduate courses such as real and complex analysis.

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Mathematical Intimidation: Driven by the Data

John Ewing

Mathematicians occasionally worry about the misuse of their subject. G. H. Hardy famously wrote about mathematics used for war in his autobiography, *A Mathematician's Apology* (and solidified his reputation as a foe of applied mathematics in doing so). More recently, groups of mathematicians tried to organize a boycott of the Star Wars project on the grounds that it was an abuse of mathematics. And even more recently some fretted about the role of mathematics in the financial meltdown.

But the most common misuse of mathematics is simpler, more pervasive, and (alas) more insidious: mathematics employed as a rhetorical weapon—an intellectual credential to convince the public that an idea or a process is "objective" and hence better than other competing ideas or processes. This is mathematical intimidation. It is especially persuasive because so many people are awed by mathematics and yet do not understand it—a dangerous combination.

The latest instance of the phenomenon is valued-added modeling (VAM), used to interpret test data. Value-added modeling pops up everywhere today, from newspapers to television to political campaigns. VAM is heavily promoted with unbridled and uncritical enthusiasm by the press, by politicians, and even by (some) educational experts, and it is touted as the modern, "scientific" way to measure educational success in everything from charter schools to individual teachers.

Yet most of those promoting value-added modeling are ill-equipped to judge either its effectiveness or its limitations. Some of those who are equipped make extravagant claims without much detail, reassuring us that someone has checked into our concerns and we shouldn't worry. Value-added modeling is promoted because it has the right pedigree—because it is based on "sophisticated mathematics". As a consequence, mathematics that ought to be used to illuminate ends up being used to intimidate. When that happens, mathematicians have a responsibility to speak out.

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Background

Value-added models are all about tests—standardized tests that have become ubiquitous in K–12 education in the past few decades. These tests have been around for many years, but their scale, scope, and potential utility have changed dramatically. Fifty years ago, at a few key points in their education, schoolchildren would bring home a piece of paper that showed academic achievement, usually with a percentile score showing where they landed among a large group. Parents could take pride in their child's progress (or fret over its lack); teachers could sort students into those who excelled and those who needed remediation; students could make plans for higher education.

Today, tests have more consequences. "No Child Left Behind" mandated that tests in reading and mathematics be administered in grades 3–8. Often more tests are given in high school, including high-stakes tests for graduation. With all that accumulating data, it was inevitable that people would want to use tests to evaluate everything educational—not merely teachers, schools, and entire states but also new curricula, teacher training programs, or teacher selection criteria. Are the new standards better than the old? Are experienced teachers better than novice? Do teachers need to know the content they teach? Using data from tests to answer such questions is part of the current "student achievement" ethos—the belief that the goal of education is to produce high test scores. But it is also part of a broader trend in modern society to place a higher value on numerical (objective) measurements than verbal (subjective) evidence. But using tests to evaluate teachers, schools, or programs has many problems. (For a readable and comprehensive account, see [Koretz 2008].) Here are four of the most important problems, taken from a much longer list.

1. **Influences.** Test scores are affected by many factors, including the incoming levels of achievement, the influence of previous teachers, the attitudes of peers, and parental support. One cannot immediately separate the influence of a particular teacher or program among all those variables.
2. **Polls.** Like polls, tests are only samples. They cover only a small selection of material from a larger domain. A student's score is meant to represent how much has been learned on all material, but tests (like polls) can be misleading.
3. **Intangibles.** Tests (especially multiple-choice tests) measure the learning of facts and procedures rather than the many other goals of teaching. Attitude, engagement, and the ability to learn further on one's own are difficult to measure with tests. In some cases, these "intangible" goals may be more important than those measured by tests. (The father of modern standardized testing, E. F. Lindquist, wrote eloquently about this [Lindquist 1951]; a synopsis of his comments can be found in [Koretz 2008, 37].)
4. **Inflation.** Test scores can be increased without increasing student learning. This assertion has been convincingly demonstrated, but it is widely ignored by many in the education establishment [Koretz 2008, chap. 10]. In fact, the assertion should not be surprising. Every teacher knows that providing strategies for test-taking can improve student performance and that narrowing the curriculum to conform precisely to the test ("teaching to the test") can have an even greater effect. The evidence shows that these effects can be substantial: One can dramatically increase test scores while at the same time actually *decreasing* student learning. "Test scores" are not the same as "student achievement".

This last problem plays a larger role as the stakes increase. This is often referred to as Campbell's Law: "The more any quantitative social indicator is used for social decision-making, the more subject it will be to corruption pressures and the more apt it will be to distort and corrupt the social processes it is intended to measure" [Campbell 1976]. In its simplest form, this can mean that high-stakes tests are likely to induce some people (students, teachers, or administrators) to cheat…and they do [Gabriel 2010]. But the more common consequence of Campbell's Law is a distortion of the education experience, ignoring things that are not tested (for example, student engagement and attitude) and concentrating on precisely those things that are.

Value-Added Models

In the past two decades, a group of statisticians has focused on addressing the first of these four problems. This was natural. Mathematicians routinely create models for complicated systems that are similar to a large collection of students and teachers with many factors affecting individual outcomes over time.

Here's a typical, although simplified, example, called the "split-plot design". You want to test fertilizer on a number of different varieties of some crop. You have many plots, each divided into subplots. After assigning particular varieties to each subplot and randomly assigning levels of fertilizer to each whole plot, you can then sit back and watch how the plants grow as you apply the fertilizer. The task is to determine the effect of the fertilizer on growth, distinguishing it from the effects from the different varieties. Statisticians have developed standard mathematical tools (mixed models) to do this.

Does this situation sound familiar? Varieties, plots, fertilizer…students, classrooms, teachers? Dozens of similar situations arise in many areas, from agriculture to MRI analysis, always with the same basic ingredients—a mixture of fixed and random effects—and it is therefore not surprising that statisticians suggested using mixed models to analyze test data and determine "teacher effects".

This is often explained to the public by analogy. One cannot accurately measure the quality of a teacher merely by looking at the scores on a single test at the end of a school year. If one teacher starts with all poorly prepared students, while another starts with all excellent, we would be misled by scores from a single test given to each class. To account for such differences, we might use two tests, comparing scores from the end of one year to the next. The focus is on how much the scores increase rather than the scores themselves. That's the basic idea behind "value-added".

But value-added models (VAMs) are much more than merely comparing successive test scores. Given many scores (say, grades 3–8) for many students with many teachers at many schools, one creates a mixed model for this complicated situation. The model is supposed to take into account all the factors that might influence test results—past history of the student, socioeconomic status, and so forth. The aim is to predict, based on all these past factors, the growth in test scores for students taught by a particular teacher. The *actual* change represents this more sophisticated "value-added"—good when it's larger than expected; bad when it's smaller.

The best-known VAM, devised by William Sanders, is a mixed model (actually, several models), which is based on Henderson's mixed-model equations, although mixed models originate much earlier [Sanders 1997]. One calculates (a huge computational effort!) the best linear unbiased predictors for the effects of teachers on scores. The precise details are unimportant here, but the process is similar to all mathematical modeling, with underlying assumptions and a number of choices in the model's construction.

History

When value-added models were first conceived, even their most ardent supporters cautioned about their use [Sanders 1995, abstract]. They were a new tool that allowed us to make sense of mountains of data, using mathematics in the same way it was used to understand the growth of crops or the effects of a drug. But that tool was based on a statistical model, and inferences about individual teachers might not be valid, either because of faulty assumptions or because of normal (and expected) variation.

Such cautions were qualified, however, and one can see the roots of the modern embrace of VAMs in two juxtaposed quotes from William Sanders, the father of the value-added movement, which appeared in an article in *Teacher Magazine* in the year 2000. The article's author reiterates the familiar cautions about VAMs, yet in the next paragraph seems to forget them:

> Sanders has always said that scores for individual teachers should not be released publicly. "That would be totally inappropriate," he says. "This is about trying to improve our schools, not embarrassing teachers. If their scores were made available, it would create chaos because most parents would be trying to get their kids into the same classroom."
>
> Still, Sanders says, it's critical that ineffective teachers be identified. "The evidence is overwhelming," he says, "that if any child catches two very weak teachers in a row, unless there is a major intervention, that kid never recovers from it. And that's something that as a society we can't ignore" [Hill 2000].

Over the past decade, such cautions about VAM slowly evaporated, especially in the popular press. A 2004 article in *The School Administrator* complains that there have not been ways to evaluate teachers in the past but excitedly touts value-added as a solution:

> Fortunately, significant help is available in the form of a relatively new tool known as value-added assessment. Because value-added isolates the impact of instruction on student learning, it provides detailed information at the classroom level. Its rich diagnostic data can be used to improve teaching and student learning. It can be the basis for a needed improvement in the calculation of adequate yearly progress. In time, once teachers and administrators grow comfortable with its fairness, value-added also may serve as the foundation for an accountability system at the level of individual educators [Hershberg 2004, 1].

And newspapers such as *The Los Angeles Times* get their hands on seven years of test scores for students in the L.A. schools and then publish a series of exposés about teachers, based on a value-added analysis of test data, which was performed under contract [Felch 2010]. The article explains its methodology:

> The *Times* used a statistical approach known as value-added analysis, which rates teachers based on their students' progress on standardized tests from year to year. Each student's performance is compared with his or her own in past years, which largely controls for outside influences often blamed for academic failure: poverty, prior learning and other factors.
>
> Though controversial among teachers and others, the method has been increasingly embraced by education leaders and policymakers across the country, including the Obama administration.

It goes on to draw many conclusions, including:

> Many of the factors commonly assumed to be important to teachers' effectiveness were not. Although teachers are paid more for experience, education and training, none of this had much bearing on whether they improved their students' performance.

The writer adds the now-common dismissal of any concerns:

> No one suggests using value-added analysis as the sole measure of a teacher. Many experts recommend that it count for half or less of a teacher's overall evaluation.

> Nevertheless, value-added analysis offers the closest thing available to an objective assessment of teachers. And it might help in resolving the greater mystery of what makes for effective teaching, and whether such skills can be taught.

The article goes on to do exactly what it says "no one suggests"—it measures teachers solely on the basis of their value-added scores.

What Might Be Wrong with VAM?

As the popular press promoted value-added models with ever-increasing zeal, there was a parallel, much less visible scholarly conversation about the limitations of value-added models. In 2003 a book with the title *Evaluating Value-Added Models for Teacher Accountability* laid out some of the problems and concluded:

> The research base is currently insufficient to support the use of VAM for high-stakes decisions. We have identified numerous possible sources of error in teacher effects and any attempt to use VAM estimates for high-stakes decisions must be informed by an understanding of these potential errors [McCaffrey 2003, xx].

In the next few years, a number of scholarly papers and reports raising concerns were published, including papers with such titles as "The Promise and Peril of Using Valued-Added Modeling to Measure Teacher Effectiveness" [RAND, 2004], "Re-Examining the Role of Teacher Quality in the Educational Production Function" [Koedel 2007], and "Methodological Concerns about the Education Value-Added Assessment System" [Amrein-Beardsley 2008].

What were the concerns in these papers? Here is a sample that hints at the complexity of issues.

> • In the real world of schools, data is frequently missing or corrupt. What if students are missing past test data? What if past data was recorded incorrectly (not rare in schools)? What if students transferred into the school from outside the system?
> • The modern classroom is more variable than people imagine. What if students are team-taught? How do you apportion credit or blame among various teachers? Do teachers in one class (say mathematics) affect the learning in another (say science)?
> • Every mathematical model in sociology has to make rules, and they sometimes seem arbitrary. For example, what if students move into a class during the year? (Rule: Include them if they are in class for 150 or more days.) What if we only have a couple years of test data, or possibly more than five years? (Rule: The range three to five years is fixed for all models.) What's the rationale for these kinds of rules?
> • Class sizes differ in modern schools, and the nature of the model means there will be more variability for small classes. (Think of a class of one student.) Adjusting for this will necessarily drive teacher effects for small classes toward the mean. How does one adjust sensibly?
> • While the basic idea underlying value-added models is the same, there are in fact many models. Do different models applied to the same data sets produce the same results? Are value-added models "robust"?
> • Since models are applied to longitudinal data sequentially, it is essential to ask whether the results are consistent year to year. Are the computed teacher effects comparable over successive years for individual teachers? Are value-added models "consistent"?

These last two points were raised in a research paper [Lockwood 2007] and a recent policy brief from the Economic Policy Institute, "Problems with the Use of Student Test Scores to Evaluate Teachers", which summarizes many of the open questions about VAM.

> For a variety of reasons, analyses of VAM results have led researchers to doubt whether the methodology can accurately identify more and less effective teachers. VAM estimates have proven to be unstable across statistical models, years, and classes that teachers teach. One study found that across five large urban districts, among teachers who were ranked in the top 20% of effectiveness in the first year, fewer than a third were in that top group the next year, and another third moved all the way down to the bottom 40%. Another found that teachers' effectiveness ratings in one year could only predict from 4% to 16% of the variation in such ratings in the following year. Thus, a teacher who appears to be very ineffective in one year might have a dramatically different result the following year. The same dramatic fluctuations were found for teachers ranked at the bottom in the first year of analysis. This runs counter to most people's notions that the true quality of a teacher is

> likely to change very little over time and raises questions about whether what is measured is largely a "teacher effect" or the effect of a wide variety of other factors [Baker 2010, 1].

In addition to checking robustness and stability of a mathematical model, one needs to check validity. Are those teachers identified as superior (or inferior) by value-added models actually superior (or inferior)? This is perhaps the shakiest part of VAM. There has been surprisingly little effort to compare valued-added rankings to other measures of teacher quality, and to the extent that informal comparisons are made (as in the *LA Times* article), they sometimes don't agree with common sense.

None of this means that value-added models are worthless—they are not. But like all mathematical models, they need to be used with care and a full understanding of their limitations.

How Is VAM Used?

Many studies by reputable scholarly groups call for caution in using VAMs for high-stakes decisions about teachers.

> **A RAND research report:** The estimates from VAM modeling of achievement will often be too imprecise to support some of the desired inferences [McCaffrey 2004, 96].

> **A policy paper from the Educational Testing Service's Policy Information Center:** VAM results should not serve as the sole or principal basis for making consequential decisions about teachers. There are many pitfalls to making causal attributions of teacher effectiveness on the basis of the kinds of data available from typical school districts. We still lack sufficient understanding of how seriously the different technical problems threaten the validity of such interpretations [Braun 2005, 17].

> **A report from a workshop of the National Academy of Education:** Value-added methods involve complex statistical models applied to test data of varying quality. Accordingly, there are many technical challenges to ascertaining the degree to which the output of these models provides the desired estimates [Braun 2010].

And yet here is the *LA Times*, publishing value-added scores for individual teachers by name and bragging that even teachers who were considered first-rate turn out to be "at the bottom". In an episode reminiscent of the Cultural Revolution, the *LA Times* reporters confront a teacher who "was surprised and disappointed by her [value-added] results, adding that her students did well on periodic assessments and that parents seemed well-satisfied" [Felch 2010]. The teacher is made to think about why she did poorly and eventually, with the reporter's help, she understands that she fails to challenge her students sufficiently. In spite of parents describing her as "amazing" and the principal calling her one of the "most effective" teachers in the school, she will have to change. She recants: "If my student test scores show I'm an ineffective teacher, I'd like to know what contributes to it. What do I need to do to bring my average up?"

Making policy decisions on the basis of value-added models has the potential to do even more harm than browbeating teachers. If we decide whether alternative certification is better than regular certification, whether nationally board certified teachers are better than randomly selected ones, whether small schools are better than large, or whether a new curriculum is better than an old by using a flawed measure of success, we almost surely will end up making bad decisions that affect education for decades to come.

This is insidious because, while people debate the use of value-added scores to judge teachers, almost no one questions the use of test scores and value-added models to judge policy. Even people who point out the limitations of VAM appear to be willing to use "student achievement" in the form of value-added scores to make such judgments. People recognize that tests are an imperfect measure of educational success, but when sophisticated mathematics is applied, they believe the imperfections go away by some mathematical magic. But this is not magic. What really happens is that the mathematics is used to disguise the problems and intimidate people into ignoring them—a modern, mathematical version of the Emperor's New Clothes.

What Should Mathematicians Do?

The concerns raised about value-added models ought to give everyone pause, and ordinarily they would lead to a thoughtful conversation about the *proper* use of VAM. Unfortunately, VAM proponents and politicians have framed the discussion as a battle between teacher unions and the public. Shouldn't teachers be accountable? Shouldn't we rid ourselves of those who are incompetent? Shouldn't we put our students first and stop worrying about teacher sensibilities? And most importantly, shouldn't we be driven by the data?

This line of reasoning is illustrated by a recent fatuous report from the Brookings Institute, "Evaluating Teachers: The Important Role of Value-Added" [Glazerman 2010], which dismisses the many cautions found in all the papers mentioned above, not by refuting them but by asserting their unimportance. The authors of the Brookings paper agree that value-added scores of teachers are unstable (that is, not highly correlated year to year) but go on to assert:

> The use of imprecise measures to make high-stakes decisions that place societal or institutional interests above those of individuals is widespread and accepted in fields outside of teaching [Glazerman 2010, 7].

To illustrate this point, they use examples such as the correlation of SAT scores with college success or the year-by-year correlation of leaders in real estate sales. They conclude that "a performance measure needs to be good, not perfect". (And as usual, on page 11 they caution not to use value-added measures alone when making decisions, while on page 9 they advocate doing precisely that.)

Why must we use value-added even with its imperfections? Aside from making the unsupported claim (in the very last sentence) that "it predicts more about what students will learn…than any other source of information", the only apparent reason for its superiority is that value-added is based on data. Here is mathematical intimidation in its purest form—in this case, in the hands of economists, sociologists, and education policy experts.

Of course we should hold teachers accountable, but this does not mean we have to pretend that mathematical models can do something they cannot. Of course we should rid our schools of incompetent teachers, but value-added models are an exceedingly blunt tool for this purpose. In any case, we ought to expect more from our teachers than what value-added attempts to measure.

A number of people and organizations are seeking better ways to evaluate teacher performance in new ways that focus on measuring much more than test scores. (See, for example, the Measures of Effective Teaching project run by the Gates Foundation.) Shouldn't we try to measure long-term student achievement, not merely short-term gains? Shouldn't we focus on how well students are prepared to learn in the future, not merely what they learned in the past year? Shouldn't we try to distinguish teachers who inspire their students, not merely the ones who are competent? When we accept value-added as an "imperfect" substitute for all these things because it is conveniently at hand, we are not raising our expectations of teachers, we are lowering them.

And if we drive away the best teachers by using a flawed process, are we really putting our students first?

Whether naïfs or experts, mathematicians need to confront people who misuse their subject to intimidate others into accepting conclusions simply because they are based on some mathematics. Unlike many policy makers, mathematicians are not bamboozled by the theory behind VAM, and they need to speak out forcefully. Mathematical models have limitations. They do not by themselves convey authority for their conclusions. They are tools, not magic. And using the mathematics to intimidate—to preempt debate about the goals of education and measures of success—is harmful not only to education but to mathematics itself.

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Remembering Leon Ehrenpreis (1930–2010)

Daniele C. Struppa

Leon Ehrenpreis: A Note on His Mathematical Work

Leon Ehrenpreis passed away on August 16, 2010, at the age of eighty, after a life enriched by his passions for mathematics, music, running, and the scriptures. I first met Leon in 1978, when I was a doctoral student under Carlos Berenstein (himself a student of Leon's), at a time when Carlos had invited him to lecture at the complex analysis seminar at the University of Maryland.

I had just begun to attempt my first (of many) readings of his work on the fundamental principle [11] and I was in awe of his presence. And yet, I discovered a gentle human being, with a passion for mathematics and a genuine desire to help the newcomer (me, in that case) to understand its mysteries.

What follows in this section is a quick overview of Ehrenpreis's work, mostly biased by my own interest and work on the fundamental principle. It is of course impossible to pay full tribute to the complexity of Ehrenpreis's work in such a short note. The subsequent sections are short remembrances, both mathematical and personal, of Leon's life, from some of his friends and collaborators.

Leon Ehrenpreis's mathematical career started with his 1953 dissertation [4], which he wrote while doing his doctoral work at Columbia University under the guidance of Chevalley. The dissertation, whose main results later appeared in [8], already shows some of Ehrenpreis's most characteristic

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Photograph by François Rouvière.

Leon explaining his AU-spaces at the 2008 conference held in Stockholm in honor of Jan Boman's seventy-fifth birthday.

skill in his willingness to tackle bold generalizations, as he extends distribution theory to the case in which Euclidean spaces are replaced by locally compact Hausdorff spaces denumerable at infinity and the derivations $\partial/\partial x_i$ are replaced by a countable family of local, closed operators.

He then plunged immediately into the topics that would become the main drivers for his fundamental principle. In an impressive series of papers from 1954 to 1960, Ehrenpreis addressed the *problem of division* in a variety of contexts. Specifically, he investigated in [5] the issue of surjectivity of

Photo by Fabrizio Colombo.

Leon with his student Carlos Berenstein, Carlos's student Daniele Struppa, and Daniele's student Irene Sabadini at the 2004 conference held in Fairfax in honor of Carlos's sixtieth birthday.

partial differential operators with constant coefficients in the space $\mathcal{D}'_F$ of distributions of finite order and in the space $\mathcal{E}'$ of infinitely differentiable functions. These were the heroic years of the application of the theory of distributions to the study of the solvability of linear constant coefficients partial differential equations, and Ehrenpreis's results joined those of Hörmander [23] and Malgrange [27] to help develop the modern form of this theory. In [6] Ehrenpreis tackled the case of some special convolution equations and extended the results of L. Schwartz on exponential solutions to such equations [30], an analysis that he continued later on in [7], where he considered a larger class of spaces.

All of this culminated with his striking announcement [9] in 1960 of what became known as the fundamental principle of Ehrenpreis-Palamodov (this result was in fact also independently and concurrently discovered by Palamodov; see [28] and the Russian references therein) and whose complete proof appeared in his most important monograph [11].

The fundamental principle can be seen as a far-reaching generalization of the well-known representation theorem for solutions of ordinary, constant coefficients linear differential equations, which we teach in any introductory class on differential equations.

In order to state it (at least in a particular case), let us consider the space $\mathcal{E}$ of infinitely differentiable functions on $\mathbb{R}^n$, and let $P_1(D), \dots, P_r(D)$ be r differential operators with constant coefficients, whose symbols are the polynomials $P_1, \cdots, P_n$. Now let $\mathcal{E}^{\vec{P}}$ denote the space of infinitely differentiable functions that are solutions of the system $P_1(D)f = \cdots = P_r(D)f = 0$. The fundamental principle roughly states that it is possible to obtain a topological isomorphism between the space $\mathcal{E}^{\vec{P}}$ and the dual of a space of holomorphic functions satisfying suitable growth conditions on the union of certain varieties V_k, where $\{V_k\}$ is a finite sequence of algebraic subvarieties of $V = \{z\colon P_1(z) = \cdots = P_r(z) = 0\}$. The proof of the theorem is quite complicated, and even decades after Ehrenpreis's original announcement, it is still the subject of analysis and reinterpretations (see, for example, the chapters dedicated to the fundamental principle in [2] and in the third edition of Hörmander's monograph on several complex variables [25]). The crux of the proof, however, is the understanding of the nature of the growth conditions that must be satisfied by holomorphic functions on the variety V for the duality to take place. Such growth conditions must be imposed not just on the functions, but also on suitable derivatives that somehow incorporate the multiplicities related to the polynomials P_j.

Maybe the most important consequence of this result is the representation theorem, which states that every function f in $\mathcal{E}^{\vec{P}}$ can be represented as

$$f(x) = \sum_{k=1}^{t} \int_{V_k} Q_k(x) \exp(iz, x)\, d\mu_k(z),$$

where the Q_ks are polynomials and μ_ks are bounded measures supported by V_k.

It is important to point out that the fundamental principle actually extends to rectangular systems of differential equations and that it holds for a very large class of spaces (what Ehrenpreis called *analytically uniform spaces*), a category that includes Schwartz's distributions, Beurling spaces, holomorphic functions, etc. (when the objects in these spaces are generalized functions, the representation theorem stated above has to be suitably interpreted, of course).

One of the common themes in most of Ehrenpreis's work in those years was a sort of philosophical principle that held that one could extend most structural results that hold for holomorphic functions to spaces of solutions of systems of linear constant coefficient differential equations satisfying suitable conditions.

Maybe the most striking such result was his beautiful and elegant proof of the classical Hartogs' theorem on the removability of compact singularities for holomorphic functions. By essentially using the arguments he had developed in the course of his study of division problems, Ehrenpreis demonstrated [10] that Hartogs' phenomenon is not a peculiarity of holomorphic functions in several variables but in fact holds for more general spaces of functions that are solutions of systems of differential equations (or even convolution equations), satisfying some specific algebraic properties. The paper is beautiful and elegant and shows Ehrenpreis's talent for simplicity and anticipates the kind of algebraic treatment of systems of differential equations that became so important in years to come.

The interest of Ehrenpreis in removability of singularity phenomena is also apparent in another beautiful series of papers ([12], [13], [14], [15]) dealing with elegant and surprising variations on the edge-of-the-wedge theorem.

I am obviously unable to touch upon all the important work of Ehrenpreis, but it would be impossible not to mention his interest in the Radon transform, which absorbed much of his mathematical efforts in the last ten or so years. His work on this topic culminated in his second monograph [16], in which Ehrenpreis introduces a generalized definition of the Radon transform in terms of very general geometric objects defined on a manifold (what he calls spreads). Just like his previous monograph [11], this more recent work brings a new and invigorating perspective to his subject matter. And as in the case of [11], this work will provide mathematicians with ideas and challenges that should keep them busy for a long time to come. As Ehrenpreis's former student Carlos Berenstein states in his review [1] of [16] "[this is] a book that is worth studying, although mining may be a more appropriate word, as the reader may find the clues to the keys he's searching for to open up subjects that are seemingly unrelated to this book. Thus, one finds at the end that the title is justified."

Hershel Farkas

A Remembrance

Leon Ehrenpreis was one of the leading mathematical figures of the past century. In many areas of mathematics, specifically differential equations, Fourier analysis, number theory, and geometric analysis, his name is a household word. He was a person who could discuss mathematical problems from a very wide variety of subfields and directly and indirectly had a great influence and impact on mathematicians and mathematics. He had the ability of getting to the core of a problem and usually the ability to give suggestions for possible solutions; sometimes he left these for others, worked jointly with other people toward the solution, or just solved the problem himself. Ehrenpreis's diversity extended way beyond mathematics. He was a pianist, a marathon runner, a Talmudic scholar, and above all a fine and gentle soul. My personal association with Leon Ehrenpreis goes back over fifty years. We began as teacher-student when I was still doing my undergraduate degree. I heard about a famous, brilliant young mathematician who was going to give a course entitled *Mathematics and the Talmud* and I decided to register for it. After the semester I asked him whether he would be willing to write a letter supporting my application to graduate school, and he told me that I would be better off asking a person more familiar with my mathematical abilities. After that I did not have much personal contact with Leon until after I finished my graduate work and our paths crossed again. I was working on problems related to compact Riemann surfaces, and we would meet now and then. Leon was also very much interested in this subject (Ehrenpreis conjecture, Schottky problem) and always expressed an interest in what I was doing. We even wrote two papers together ([20], [17]), but this is not the measure of the influence he had on me. In fact, my book with Irwin Kra, *Theta Constants, Riemann Surfaces and the Modular Group*, [21] grew out of a conversation with Leon. A visit to New York would never be complete without meeting Leon. In fact I arrived in New York on August 15 planning to meet with Leon in the middle of the week to discuss my current work, but he passed away on Monday, August 16. Mathematics has lost one of its finest practitioners. He will be sorely missed by the mathematical community as both a scholar and a gentleman.

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Takahiro Kawai

The Fundamental Principle, Hamburger's Theorem and Date Line

When I was a graduate student (late sixties), the "fundamental principle of Ehrenpreis" [9] was really a guiding principle for many young analysts, including me. It aimed at, and succeeded in, analyzing general systems of linear differential equations with constant coefficients, in the days when a single equation or a determined system was the main target of specialists in differential equations, and the cohomological machinery was not in the toolboxes of most of them. Thus the publication of his book [11] had been yearned for; actually I asked a bookseller to get it by airmail as soon as it appeared—it was an exceptional case as one U.S. dollar equaled 360 yen in 1970. Unfortunately I could not indulge in reading [11] as I had hoped; in 1970 and 1971 I was totally absorbed in the collaboration with Sato and Kashiwara to complete [29], which was a successor to [9] in the sense that it presented the microlocal structure theorem

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Photograph by Yael Ehrenpreis Meyer.

Leon with Jan Boman at the Stockholm conference.

for a general system of linear differential equations, not necessarily with constant coefficients, at a generic point of its characteristic variety. Still, I have continued to enjoy browsing through [11] occasionally; the book is full of the ideas and dreams of Leon Ehrenpreis. When I browse in the pages of [11], I feel as if I were chatting with Leon. The paper [26] is an example of outcomes of such chatting.

When I first met Leon in 1973 in Kyoto, I got the impression that he was a kind man of sincerity. The impression has continued until now. In ending this memory I note two incidents that endorse the impression. When I explained the prototype of $\mathbb{R}$-holonomic complexes to Leon in 1980, he immediately noticed its relevance to Hamburger's theorem and he kindly expounded the theorem and related works of Hecke and Weil. I cannot forget the warm atmosphere full of intellectual curiosity, which led to our paper [19]. Another incident I note is that I once happened to notice that he had not taken anything for two days and that the reason was that he was dubious about the date of the fast day in Kyoto due to the effect of the International Date Line.

Thus I have learned much from Leon both in mathematics and in daily life. Many, many thanks, Leon!

Peter Kuchment

Leon Ehrenpreis—A Friend and Colleague.[1]

It is hard for me to believe that one now has to use the past tense when talking about Leon Ehrenpreis. The thought of him being gone has not yet sunk in.

I have been familiar with some of Leon's outstanding results (e.g., existence of fundamental solutions) for a very long time, at least since the early 1970s. His work on what he called the fundamental principle, as well as related papers and books by B. Malgrange and V. Palamodov, played a crucial role in my work with L. Zelenko in the late 1970s on periodic PDEs. We thus had to learn Leon's techniques in detail. It was not an easy task, in part since Leon always preferred unusual terms and notations, but once one got through these minor hurdles, one was always rewarded by an abundance of wonderful ideas.

I met Leon for the first time in 1989 at the integral geometry and tomography conference in Arcata, California. It was my very first trip outside the former USSR, and it felt like being in a dream. First of all, the constructive proof that the world outside the iron curtain truly did exist was a revelation! Another shock during my first visit and my emigration soon afterward was that names like Leon Ehrenpreis, Peter Lax, Louis Nirenberg, and other luminaries, which obviously existed only on book covers, or at least referred to semi-gods somewhere well above this Earth, corresponded to mere mortals, who not only existed in the flesh but were not that old at all and still very active. Meeting Leon in Arcata was my first experience of this kind. (I also met Sigurdur Helgason there, but his existence had been proven to me earlier indirectly, when I communicated with him while translating his book *Groups and Geometric Analysis* [22] into Russian.) Another happy discovery was that, unknown to me at that time, Leon had worked on integral geometry, and I had just started working in computed tomography, in which integral geometry is one of the main mathematical tools. This is how our collaboration started.

Just like everyone else, I loved Leon from the first encounter. His unfailing cheerful disposition and his abundant eagerness to discuss any kind of mathematics at any time made every occasion we met feel like a holiday. And we met quite a few times over the past twenty years, Leon visiting me in Kansas and Texas on a regular basis, seeing each other at conferences, and communicating by phone and email. I have also had the privilege of writing joint papers together with him and contributing (jointly with Todd Quinto) a chapter to his book [16] on the Radon transform.

Everyone who knew Leon Ehrenpreis realized that his brain was engaged all the time in some variant of intellectual activity—be it mathematics, or the Talmud, or music. It is well known that he was an avid runner and had run the New York marathon every year since its inception in 1970 till 2007. He also liked to run during his visits, so when he visited me in Wichita, Kansas, I would sometimes pick a room for him in a hotel seven

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[1] *I am grateful to P. Deift, M. Mogilevsky, and D. C. Struppa for useful comments and corrections.*

miles away from the campus, with a sufficiently attractive route to run between the two. So, after his lecture, or just a working day, he would give me his things to take back to the hotel, while he would run. Every time I would meet him after the run, he would have some new ideas (and he had so many great ideas!) about the problem we were working on at the time. Once, when he came back and I was waiting for him in the hotel's lobby, the receptionist at the front desk asked him: "Did you really run all the way from the campus?" Leon's reply was: "What else could I do? He refused to give me a ride"—and he pointed at me. I think I lost all the receptionist's respect at that time. This was not a one-time event; Leon always liked to crack or to hear a good joke. He was smiling most of the time that I saw him. It was a joy to discuss with him not only mathematics, but also religion, music, or anything else. What made this even more enjoyable was that in my experience he never imposed his opinions, beliefs, or personal problems (and he unfortunately had quite a few) on others. It was relaxing to talk to him. He must have been a wonderful rebbe. Over the years, he became our dear family friend.

Leon always liked a good story and had some of his own to tell. My favorite, which I tell to students often, was about him teaching a calculus class many years ago. As any good teacher would do, he tried to lead his students, whenever possible, to the discovery of new things. So, he once said: "Let us think, how could we try to define the slope of a curve?" "What is there to think about?" was the reply from one smart student, "it says on page 52 of our textbook that this is the derivative." "Well," replied Leon, "I haven't read till page 52 yet." The result was that the class complained to the administration that they were given an unqualified teacher. So much for inspiring teaching; it can backfire!

I miss Leon, a great mathematician and human being, and hope that he dwells happily in the house of the Lord forever.

Photograph by Yael Ehrenpreis Meyer.

Leon at the Stockholm conference in front of the Royal Palace.

Eric Todd Quinto

A Remembrance

Before I met Leon Ehrenpreis I was a little apprehensive. He had theorems named after him, and I knew he was a brilliant mathematician. His work on existence of solutions to PDEs helped revolutionize the field, and I wondered what its creator would be like. Leon immediately put me at ease with his warm enthusiasm and love for mathematics. When we talked about mathematics he described the big picture, and he illustrated his points with examples that brought the ideas down to earth. Leon's enthusiasm was contagious, and it got even skeptical listeners involved and intrigued. I especially appreciated Leon's respect and concern for the person he was talking with; he wanted the listener to be as engaged and interested as he was. His daughter Yael told me that he was exactly the same way when he helped her with math homework.

Leon's mathematics reflected his emphasis on unifying principles. In Leon's book [16] *The Universality of the Radon Transform*, he developed several overarching ideas and used them to understand properties of the transforms, such as range theorems and inversion methods. One such overarching idea he defined is the nonparametric Radon transform using spreads (a collection of foliations of Euclidean space, the leaves of which depend on a *spread* parameter). For the hyperplane transform, the spread is defined by a specific hyperplane L_0 through the origin (or element of the Grassmannian $G(n, n-1)$, and the spread variable is a point s in $L_0^\perp$: the spread is defined for each L_0 by $s \mapsto s + L_0$. After ten pages of the book, he is considering other manifolds and spread variables, and he is relating the geometric construction to differential equations and groups. The book draws connections between several fields, including complex variables, PDE, harmonic analysis, number theory, and distribution theory—all of which have benefited from his contributions over the years. I remember one talk he gave at Harvard while he was writing the book. He pointed to about three hundred pages of notes on the lectern and told us with a broad smile and hearty laugh that he was going to talk about all that. Worried that we would be there for a while, we laughed a little nervously. However, if I am remembering right, he covered a lot of material

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but ended more or less on time. Peter Kuchment and I wrote an appendix to the book, and this, too, was a pleasure.

Leon was a rabbi, and some of our most lively conversations occurred when I asked him simple questions about Jewish law. Leon would launch into an energetic description of what the rabbis thought about a point and how they resolved seeming inconsistencies. He always made sure the conversation was a dialogue and that I was engaged. He cared deeply about Jewish law, and he wanted to share it with me.

I will miss Leon's theorems and mathematical taste, as well as our conversations. His ideas are important, but so was his way of communicating those points and sharing his enjoyment of them. I will miss Leon's enthusiasm for math and life, his smile, and his *menschlichkeit*.

Shlomo Sternberg

A Personal Recollection

I will leave it to others to describe the extremely distinguished mathematical career of Leon Ehrenpreis. In fact, Leon, Victor Guillemin, and I wrote one joint paper together [18] of which I am still proud, but this reflects only a very tiny fraction of his great mathematical achievements. I will concentrate on the personal.

I first met Leon Ehrenpreis in the early 1950s at Johns Hopkins University, where I was a student and he an instructor. We became close personal friends, and then, when I married Aviva, he became a close friend of ours, a friendship that lasted from the time that we were teenagers until his death. Thinking back through the years, I can't recall a single time, no matter how trying the circumstances may have been, whether casual or serious, that his voice, his eyes, his whole demeanor conveyed less than deep warmth, profound generosity, an optimism, a hopefulness that was pure Leon.

When we were young, "pure Leon" might include a dash of madcap charm, a directness, a boyish whimsy, a ruefulness, that belied his distinguished mathematical achievements. His style was not professorial. He was not into style or image—then or ever. Leon retained and presented an honesty, a disarming forthrightness, a genuineness, a profound generosity and sheer vitality that he carried with him all of his life. He was certainly extremely generous to us at crucial junctures in our lives.

And it was, and is, this vitality that perhaps for us best describes Leon. The years passed; life transpired with its joys and sorrows. For Leon and his family, the sorrows were of such immensity that would otherwise crush anyone. But Leon bore his with unimaginable courage and responsibility. Courage that, we dare say, none of us could have possibly comprehended, let alone mustered. But despite it all, and no matter what transpired, Leon retained every bit of the vitality of our earlier years. His mathematical work continued. His, along with Ahava's, loving care and unstinting dedication to his family continued. His kindness and loyalty to us, his friends, continued. It was who he was.

To think now that this vitality is gone from us is very hard to accept. We'll always cherish our recollections of him—even as an attempt, however illusive, to conjure up a whiff of his vitality. Leon was one of a kind. We know he is irreplaceable. And we miss him.

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Alan Taylor

Remembrances of Leon Ehrenpreis

I first met Leon in the summer of 1966 at an AMS summer conference, "Entire Functions and Related Parts of Analysis", in La Jolla, California. I had just completed my thesis, which was about problems in spaces of entire functions that fit his theory of *analytically uniform spaces*, and I had studied some of his important papers on division problems and their applications to constant coefficient partial differential operators. So it was very exciting to meet and talk with him. It was also where I had my first encounter with his kindness, as he suggested that I apply to spend a postdoctoral year at the Courant Institute, where he was a professor. It was during that year, 1968-1969, that I really got to spend a lot of time talking with him and met his student at that time, Carlos Berenstein. Thinking back on those times is bittersweet; it is sad now because of Leon's death and doubly so because Carlos is too ill to share memories of that special year. But it also reminds me of this most interesting and fun year of my professional life. Carlos was finishing his doctoral thesis at Courant and helping Leon with the final editing of his book on Fourier analysis [11]. Leon had just moved to Yeshiva University in uptown New York City, where he was giving a course on the book. Both Carlos and his wife and I and my family lived in New York University-owned housing in Greenwich Village, so each Thursday Carlos and I would take the A train uptown to spend the day with Leon, attending his class and talking about mathematics.

I really saw Leon's style of doing mathematics in that class. He was always interested in the

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Leon discussing mathematics with Dennis Sullivan and John Morgan at Columbia University, 2006.

fundamental reasons that theorems were true and in illustrative examples but less interested in the details. It seemed to me that he could look at almost any problem in analysis from the point of view of Fourier analysis. Indeed, his book on Fourier analysis, in addition to presenting the proof of his most important contribution, the fundamental principle, contains chapters on general boundary value problems, lacunary series, and quasianalytic functions in which this general point of view is explored. This style of inquiry is present in much of his work, for example, that on the "Edge of the Wedge theorem" and its extension to some systems of constant coefficient partial differential equations. Leon was doing mathematics 100% of the time I spent around him and I think it was true always, especially when riding the train and in his jogging. After that year, I saw Leon about every year or two, mostly when he would visit our department in Ann Arbor. I was lucky that his wife, Ahava (nee) Sperka, was from a family that lived in a Detroit suburb, so that family visits were always a good occasion for Leon to visit Ann Arbor and our department.

I want to pass along two things I learned about Leon over the years that I think are worth remembering. First, I learned from my advisor, Lee Rubel, that Leon came from a remarkable undergraduate class at City College of New York, a fact that Harold Shapiro also mentions in his article [31] remembering Allen Shields, my colleague for many years at Michigan. In that class, a year or two around the year 1949, there was a remarkable group of young mathematicians who interacted with one another. The group included Leon Ehrenpreis, Donald Newman, Lee Rubel, Jacob Schwartz, Allen Shields, Harold S. Shapiro, Leo Flatto, Martin Davis, and Robert Aumann. What an outstanding group of mathematicians! Also in the group was David Finkelstein (physics). Several times I heard stories from Rubel and Shields about the stimulating atmosphere and competition between these very talented youths.

A second vivid memory I have is of a conversation with Leon sometime in the 1970s, in large part because it was so influential on my own work. Throughout the 1970s, I often talked with Leon about the problem of determining which constant coefficient partial differential operators had a continuous linear right inverse on the space of infinitely differentiable functions. At some point I asked him how, being a student of Claude Chevalley, he had come to work on problems that led to the fundamental principle. He told me that Chevalley had suggested he write to Laurent Schwartz asking for thesis-problem suggestions. Schwartz had written back with a list of several questions about partial differential operators along with what he knew about them at the time. These included the fundamental questions, answered by Leon, Malgrange, and Hörmander in the 1950s, that now underlie the modern theory of linear constant coefficient partial differential operators. He also told me that the list from Schwartz included the right inverse question that we had been discussing, as well as the question of surjectivity of such operators on the space of real analytic functions. (The surjectivity on real analytic functions was solved in the negative by De Giorgi and Cattabriga [3] with a characterization of the surjective operators being given by Hörmander in 1973.) The right inverse question was solved by Meise, Vogt, and myself around 1988, and questions that arose from this work have occupied me for the last twenty years. So it is indeed the case that Leon Ehrenpreis had a profound influence on my entire professional life.

Leon was a kind man, always interested in talking about mathematics. I never saw him without a smile on his face, and I never heard an unkind word from him. He was generous with his mathematical suggestions and a great friend. I cannot say that I was personally close to him in that I knew much about his private life, but I still remember many little things. Like the fact that he always drank tea when we worked, that he wanted to have enough children to form a baseball team, and that he always called me "Alan", my boyhood name. I will greatly miss him.

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How a Medieval Troubadour Became a Mathematical Figure

Michael P. Saclolo

Lyric poetry of the Middle Ages may seem far removed from subgroups of the symmetric group or primitive roots of finite fields. However, one piece of medieval poetry has led to work in these mathematical disciplines, namely a sestina written in the Romance language of Old Occitan by a troubadour named Arnaut Daniel:

Lo ferm voler q'el cor m'intra
no'm pot ies becs escoissendre ni ongla
de lausengier, qui pert per mal dir s'arma
e car non l'aus batr'ab ram ni ab verga
si vals a frau lai o non aurai
oncle
jauzirai joi, en vergier o dinz cambra

Qan mi soven de la cambra
on a mon dan sai que nuills hom non intra
anz me son tuich plus que fraire ni oncle
non ai membre no'm fremisca, neis
l'ongla
aissi cum fai l'enfas denant la verga
tal paor ai no'l sia trop de l'arma

Del core li fos non de l'arma
e cossentis m'a celat deniz sa cambra
que plus mi nafra'l cor que colps de verga
car lo sieus sers lai on ill es non intra
totz temps serai ab lieis cum carns et ongla
e non creirai chastic d'amic ni d'oncle

Anc la seror de mon oncle
non amei plus ni tant per aqest'arma
c'aitant vezis cum es lo detz de l'ongla
s'a liei plagues volgr'esser de sa cambra
de mi pot far l'amors q'inz el cor m'intra
mieills a son vol c'om fortz de frevol verga

Pois flori la seca verga
Ni d'en Adam mogron nebot ni oncle
tant fin'amors cum cella q'el cor m'intra
non cuig fos anc en cors ni eis en arma
on q'ill estei fors on plaz' o dins
cambra
mos cors no' is part de lieis tant cum ten l'ongla

The firm desire that enters
my heart no beak can tear out, no nail
of the slanderer, who speaks his dirt and loses his soul.
And since I dare not beat him with branch or rod,
then in some secret place, at least, where I'll have no
uncle
I'll have my joy of joy, in a garden or a chamber.

When I am reminded of the chamber
where I know, and this hurts me, no man enters--
no, they're all more on guard than brother or uncle--
there's no part of my body that does not tremble, even
my nail,
as the child shakes before the rod,
I am that afraid I won't be hers enough, with all my soul.

Let me be hers with my body, not my soul,
let her hide me in her chamber,
for it wounds my heart more than blows from a rod
that where she dwells her servant never enters;
I will always be as close to her as flesh and nail,
and never believe the reproaches of brother and uncle.

Not even the sister of my uncle
did I love more, or as much, by my soul,
for as familiar as finger with nail
I would, if it pleased her, be with her chamber.
It can do more as it wills with me, this love that enters
my heart, than a strong man with a tender rod.

Since the flower was brought forth on the dry rod,
and from En Adam descended nephews and uncles,
a love so pure as that which enters
my heart never dwelt in body, nor yet in soul.
Wherever she stands, outside in the town or inside her
chamber,
my heart is not further away than the length of a nail.

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C'aissi s'enpren e s'enongla
mos cors e lei cum l'escorss'en la verga
q'ill m'es de joi tors e palaitz e cambra
e non am tant fraire paren ni oncle
q'en paradis n'aura doble joi m'arma
si ja nuills hom per ben amar lai intra

Arnautz tramet sa chanson d'ongl'e d'oncle
a grat de lieis que de sa verg'a l'arma
son Desirat cuit pretz en cambra intra

For my heart takes root in her and grips with its nail,
hold on like bark on the rod,
to me she is joy's tower and palace chamber,
and I do not love brother as much, or father, or uncle;
and there'll be double joy in Paradise for my soul,
if a man is blessed for loving well there and enters.

Arnaut sends his song of the nail and the uncle,
to please her who rules his soul with her rod,
to his Desired, whose glory in every chamber enters.

Note the rearrangement of the final word from each line of the stanzas to the next. The English translation by Frederick Goldin ([6]) to the right of the original illustrates the same phenomenon.

Arnaut's sestina is an example of courtly love poetry. The last three-line stanza unequivocally declares the poet's intention to please his noble lady. What interests us here, however, is how the six six-line stanzas, or sextets, use the same six ending words in a permutation given by the cycle (124536). The arrangement of the ending words in each sextet corresponds to a power of this cycle, and so if we were to compose a seventh sextet, we would arrive at one with the original arrangement of the six ending words. Poets and, lately, mathematicians have been fascinated by this poetic form, and the desire to generalize it has led to the investigation of the mathematical structures involved.

Arnaut Daniel and the Troubadours

I begin at the source, the poet. Information about Arnaut's life comes primarily from his *Vida*. The *Vidas*, very short biographical sketches of poet-troubadours in surviving manuscripts of their works (see, for example, [2]), were written primarily to promote interest in the troubadours, and therefore the information contained therein may not be entirely factual. What is commonly accepted, however, is that Arnaut lived in the latter part of the twelfth century. The *Vida* tells that he hailed from a castle named Ribérac in the bishopric of Périgueux in the Dordogne region of France. He was of noble birth and presumably had early contact with the nobles he would later entertain as a troubadour. The *Vida* mentions Arnaut as courting a married noble lady from Gascony, although it quickly points out that this forbidden love was believed to have remained unattained. For an extended account of Arnaut's life, refer to the introduction to the book edited by James Wilhelm [21].

As a troubadour, Arnaut was part of a group of musical poets, who entertained courtly audiences of southern France. Their language of Old Occitan counts modern Provençal as a descendant. Catalan, the language of Catalunya in northern Spain and in the Pyrenees region of France, is also closely related. The troubadours' northern French-speaking counterparts are called the *trouvères*. These northern poets were influenced by the troubadours and adopted the latter group's style and themes.

The troubadours and the trouvères celebrated *courtly love*. In this highly ritualistic literary tradition, full of rules and codes, the object of often forbidden desire is an idealized lady of the Court. The poetry composed and performed by the troubadours expresses her unattainability.

Arnaut himself was revered as a poet. One of his earliest and most prominent admirers was Dante Alighieri, the Italian poet and celebrated author of the *Divine Comedy*, who lived about a century after Arnaut. Dante mentions Arnaut several times in *De vulgari eloquentia*. He even depicts Arnaut as doing penance in *Purgatorio* in the *Divine Comedy*, imploring Dante in Occitan not to forget him.

Today, our most direct link to courtly love is its long and rich literary tradition. It is an understatement to say that the poetry of the troubadours and the trouvères influenced many poetic forms we enjoy today. Wilhelm discusses sources and influences in the introduction to [21].

The Sestina

The poetic form presented earlier is called the sestina, and Arnaut is credited with its invention ([4]). Both Dante and Petrarch composed in the form. Rudyard Kipling, Ezra Pound, Algernon Charles Swinburne, and Sir Philip Sidney are among poets who composed sestinas in English. Sidney composed an example of a double sestina.

As its base, the sestina has the structure of six sextets followed by a tercet called a *tornada* or an *envoi*; the former is an Occitan term meaning "turned" or "twisted", while the latter is French and means "send-off". The *envoi* traditionally incorporates all six ending words of the sextets. As previously mentioned, the ending words of each line of a sextet are rearranged in the subsequent sextet according to the cycle (124536). Of course, any permutation of order six can be used to compose a poem that shows a different rearrangement of the six ending words, while producing the same number of stanzas and lines. But the particular permutation used by Arnaut inspired poets and mathematicians alike. Poets sought either to compose in the same style or to develop variations. Mathematicians, instead, posed the question: What permutations have the "same" properties as the one used in the sestina? This is the story we are telling here.

Generalizing the Sestina: the Quenine

Interest in the sestina and the structure presented by the permutation began among literary figures in the twentieth century among literary figures. According to A. Tavera ([17]), it was Eugène Vinaver, Tavera's mentor and professor of French at the University of Manchester, who made him aware of the spiral shape of the permutation. To see this, number the six ending words of the sextet, top to bottom, 1 to 6. Now draw an inward "spiral" starting from the integer 6 heading towards 1 then 5, 2, 4, and finally 3 as in Figure 1. The resulting order of the six integers, following the trajectory of the spiral, is precisely the order of the six ending words in the subsequent strophe.

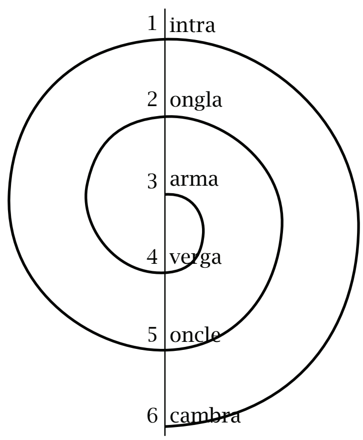

Figure 1. Spiral representation of a sestina. The six ending words of the strophes of Arnaut's sestina are listed in its original order.

The mathematical question arising from the particular permutation and its spiral representation was taken up by the poet Raymond Queneau and his colleague, the poet, writer, and mathematician Jacques Roubaud. Queneau was a member of the French mathematical society, the Société Mathématique de France. Moreover, he and Roubaud were central figures in the development of OULIPO ("OUvroir de LIttérature POtentielle"), a literary movement and association that Queneau and François Le Lionnais founded in 1960. Composed of mostly European members, the goal of OULIPO is to explore and compose what the group calls "potential" literature by using constraints drawn from mathematics and other technical disciplines. An example is George Pérec's novel, *La Disparition* (translated into English as *A Void*), a lipogrammatic novel, redacted without the letter *e* [9]. Queneau himself played with combinatorial possibilities in his collection of sonnets *Cent Mille Milliard de Poèmes* (*One Hundred Thousand Billion Poems*) [10]. For more information on the work of OULIPO, see, for example, the text translated and edited by Warren Motte [7].

Queneau noticed that the spiral representation can be extended to permutations of a set of any size n. Then he asked which positive integers n give an order n "spiral" permutation. He determined [11], apparently by rote calculation, a list of thirty-one positive integers less than 100 that have this property. Queneau called the generalized sestina for such n the *n-ine*. Thus the sestina is the same as a 6-ine, and any one-line poem is degenerately a 1-ine. The smallest nonadmissible number is $n = 4$, for the associated permutation that results, (124), fixes the 3. Later the n-ine became the *quenine*, coined by Roubaud in honor of Queneau. The French word for the sestina is *sextine*. By analogy one might call the generalization *n-ina* or *quenina* in English, but as yet there is no established tradition of this convention either in mathematical writing or poetry written in English.

The Search for Quenines

In this section I shall highlight the various developments to find positive integers n that give a generalized sestina, culminating with the definitive classification. First I shall set up some basic notations and definitions.

For x a positive integer, σ_n is defined by:

$$\sigma_n(x) = \begin{cases} x/2 & \text{if } x \text{ is even,} \\ n - (x-1)/2 & \text{if } x \text{ is odd.} \end{cases} \tag{1}$$

The following terminology related to $\sigma_n(x)$ was formalized by Monique Bringer in [2]. The function $\sigma_n(x)$ is called a **spiral** permutation. Integers n such that $\sigma_n(x)$ is of order n are called **admissible**. Roubaud also referred to such n as **Queneau numbers**. A spiral permutation with admissible n is also a **quenine** or **n-ine**. In honor of Queneau and Daniel, Bringer called the cyclic subgroup of the symmetric group generated by $\sigma_n(x)$ the **Queneau-Daniel** group.

Thus the search for quenines became the search for admissible n. Moreover, it was the study of the inverse permutation to σ_n that eventually produced the desired results. Let δ_n be the inverse permutation to σ_n defined by:

$$\delta_n(x) = \begin{cases} 2x & \text{if } 2x \le n, \\ 2n + 1 - 2x & \text{otherwise.} \end{cases} \tag{2}$$

As σ_n is of order n if and only if δ_n is also of order n, finding admissible n by way of δ_n gives the admissible n for σ_n.

Queneau

Queneau apparently did not realize this connection, at least not in published works (see [11], [12], [13]). He recognized early on that not every n is admissible, citing some basic counterexamples. In the case of $n = 4$, 7, or 10, for instance, σ_n fixes at least one number. For $n = 8$, σ_8 is of order 4. As for some initial characterization of nonadmissible n, he mentioned the cases in which $n = 2xy + x + y$ for x, y integers greater than or equal to 1 or if $n = 2^k$ for $k > 1$ as examples of such n, since the resulting permutation subgroups are of order less than n. (Consider the orbit of 2 under δ_n for example.) Alas, these conditions do not exhaust the cases in which n is not admissible.

After Queneau

Roubaud and his student Bringer continued the search for admissible n. In [14] Roubaud was interested in the various permutations employed by the troubadours. In considering the sestina, he attempted to connect the particular permutation used by Daniel to the assonant pairing of ending words in the stanzas, reiterating Queneau's problem of generalizing the sestina. But it was Bringer (in [2]) who took on Queneau's problem directly and was among the first to characterize such admissible and nonadmissible integers n. Bringer found that a necessary condition for n to be admissible is for $2n + 1$ to be prime. She also found that if 2 is a primitive root of the finite field $\mathbb{Z}/(2n+1)\mathbb{Z}$ (i.e., where $2n+1$ is prime), then n is admissible. (Note: While the mathematics that follows in the rest of the article is generally accessible, those who would like to read background and reference materials may look at [5] about finite fields and at classics such as [6], for a short treatment on symmetric groups, and [4], for a treatment of primitive roots.) The following is a summary of Bringer's results, listing conditions for both admissible and nonadmissible n.

Theorem 1 (Bringer)**.** (1) *If n is admissible, then $2n + 1$ is prime.*
(2) *If n and $2n + 1$ are prime, then n is admissible.*
(3) *If $n = 2p$ where p and $4p + 1 = 2n + 1$ are prime, then n is admissible.*
(4) *$n = 2^k - 1$ is not admissible.*
(5) *$n = 4k$ is not admissible.*

To show that $2n + 1$ is prime for n to be admissible, suppose the contrary. Then there exists a positive integer $q > 1$ that divides $2n+1$. Hence, if m is in the orbit of q under δ_n, then $m \equiv \pm 2^k q \pmod{2n + 1}$. This implies that q divides m since it divides both $2n + 1$ and $\pm 2^k q$. Therefore the orbit of q only contains divisors of q. But, then, $2, \dots, q - 1$ do not divide q and thus cannot be in the orbit of q, a contradiction. Hence $2n + 1$ must be prime.

Bringer also points out that the necessary condition that $2n + 1$ is *prime* is equivalent to Queneau's prior remark that n is not admissible when $n = 2xy + x + y$, where x and y are integers greater than or equal to 1. For if $2n + 1$ were prime, suppose that $n = 2xy + x + y$ have noninteger solutions different from $x = -1$ and $y = -(n + 1)$ (which are always solutions), then $2n + 1 = 4xy + 2x + 2y + 1 = (2x + 1)(2y + 1)$. $2x + 1$ and $2y + 1$ are not equal to 1 (unless $n = 1$) and divide $2n + 1$, which is not possible. Now if $n = 2xy + x + y$ does not have integer solutions, suppose that $2n + 1$ is not prime. Then $2n + 1 = pq$ for p and $q \neq 1$ and both odd, i.e., $p = 2p' + 1$ and $q = 2q' + 1$. But it follows then that $n = 2p'q' + p' + q'$, which is false, and so $2n + 1$ must be prime for n to be admissible. Bringer also remarks that $2n + 1$ being prime is not sufficient, citing a counterexample, $n = 20$.

Recall that, earlier, Queneau made the claim that if $n = 2^k$, $k \neq 1$, then n is not admissible. This particular situation falls under the last item above of Bringer's list of results, but in [2] she includes a separate proof of this case. To see this, note that $\sigma_n(2) = 1$, $\sigma_n^2(2) = n = 2^k$. It follows that $\sigma_n^{k+1}(2) = 2$. The permutation cycle generated by 2 is of order $k + 1$, which is strictly less than 2^k if $k \neq 1$.

In 1993 Roubaud summarized what was known until that time about the search for quenines or admissible n. In [15] he even proposes extensions to the quenine. He also attempts to complete Bringer's result, stating a seemingly complete characterization of admissible n. He claims that a necessary and sufficient condition for n to be admissible is for 2 to be of order n or $2n$ in the multiplicative group of integers modulo $2n + 1$. Jean-Guillaume Dumas, in [3], points out that this is not true when n is even, citing the counterexample $n = 8$. Also 2 is of order 8 (mod 17). But, as we have seen, the "octine" is not possible as $n = 8$ gives a spiral permutation of order 4.

From the Middle Ages to the Twenty-First Century

Roubaud was almost correct, and Dumas provided the solution. In [3] Dumas gives a complete characterization of admissible numbers, and therefore quenines. He even adds to the list started by Queneau, naming all quenines under one thousand. (For example, the highest admissible n under 1000 is $n = 998$. Thus one can compose a poem with 998 strophe-ending words permuting "spirally" for 998 stanzas.) Extending Bringer's work, he investigates the properties of the finite field $\mathbb{F}_p$, for such n where $p = 2n + 1$ is prime. His characterization of Queneau numbers n is the following:

Theorem 2 (Dumas)**.** *Let n be an integer such that $p = 2n + 1$ is prime. Then n is admissible if and only if:*

(1) *either 2 is of order $2n$ (i.e., 2 is a primitive root) of $\mathbb{F}_p$*
(2) *or n is odd and 2 is of order n in $\mathbb{Z}_p$.*

Recall that the first characterization above of admissible n was previously elucidated by Bringer. The following, however, is the entire proof by Dumas.

Proof. Beginning with the necessary condition, consider an admissible n. As the order of the 2 in $\mathbb{F}_p$ divides $2n$, the number of invertible elements of $\mathbb{F}_p$, then 2 can only be of order $2n$, n, or strictly less than n. Now suppose 2 is of order $j < n$. Then $\delta_n^j(2) \equiv \pm 2^j 2 \equiv \pm 2 \pmod{2n+1}$. So there are two cases to consider. First, if $\delta_n^j(2) = 2$, then the orbit of 2 only contains $j < n$ elements, which cannot be true since it is assumed that n is admissible. Thus, in the other case $\delta_n^j(2) = -2$. But by definition $1 \leq \delta_n^j(x)$, so $1 \leq 2n + 1 - 2 \leq 2$, or $n \leq 1$. But 2 is of order 2 (mod $2(1) + 1 = 3$). Therefore, if n is admissible, then 2 is of order n or $2n$ (mod $2n + 1$). But it remains to exclude the instance in which $n = 2k$ is even and 2 is of order n (mod $2n + 1$). But in this case $2^k \equiv -1 \pmod{2n+1}$. From here it follows that $\delta_n^k(2) = \pm 2^k 2 \equiv \pm 2 \pmod{2n+1}$. As $k = \frac{n}{2} < n$, then as before neither situation can exist if n is admissible.

Next, we prove the sufficient condition. Let us call ω the cardinality of the smallest orbit of the elements of the set $N = \{1, 2, 3, \ldots, n\}$ by δ_n. Now suppose that $u \in N$ is of order ω. Now there exists k such that $\delta_n^\omega(u) \equiv u \equiv (-1)^k 2^\omega u \pmod{2n+1}$. And, since u is invertible, $(-1)^k 2^\omega \equiv 1 \pmod{2n+1}$. This means that $2^\omega \equiv \pm 1 \pmod{2n+1}$. If $2^\omega \equiv 1$, then ω is greater than the order of 2. Therefore $\omega \geq n$ and so $\omega = n$ for the permutation is at most of order n. In the case that $2^\omega \equiv -1$, then 2 is of order $2n$ or an odd n. If 2 is of order $2n$, then $2^n \equiv -1 \pmod{2n+1}$. So, $2^{n+\omega} = 1$, and therefore $\omega = n$. If the order of 2 is n with n odd, then $(2^\omega)^2 = (-1)^2 = 1$, which means that n divides 2ω. As n is odd, Gauss's lemma implies that n divides ω. So once more, $\omega = n$. □

Another characterization of the quenine provided by Dumas is a corollary to the theorem above. According to Dumas it restates a conjecture by Joerg Arndt.

Corollary 3. *Let $\mathbb{F}_p$ be as above. Then n is admissible if and only if:*

(1) *either 2 is of order $2n$ in $\mathbb{F}_p$, and $n \equiv 1$ or $2 \pmod 4$,*
(2) *or 2 is of order n in $\mathbb{F}_p$, and $n \equiv 3 \pmod 4$.*

The characterizations of admissible n obviously provide a test of whether a quenine of a certain order n exists. At this point there is no way of *generating* such n.

Extensions and Connections

There are some very natural ways to extend mathematically the results inspired by the quenine. One generalization to δ_n, suggested by Roubaud, is to replace multiplication by 2 by any k. Call this permutation $\delta_{n,k}(x)$, the *k-quenines*. For example, the 3-*quenine* has formula:

$$\text{(3)} \qquad \delta_{n,3}(x) = \begin{cases} 3x & \text{if } 3x \leq n, \\ 2n + 1 - 3x & \text{if } n < 3x \leq 2n, \\ 3x - (2n+1) & \text{otherwise.} \end{cases}$$

In this case, when $n = 8$, the permutation $\delta_{8,3}$ is of order 8. Roubaud and Pérec developed an extension to σ_n, inspired by the absence of a quenine of order 10 ([15]). The permutation, π_n, is called the **pérecquine** after Pérec and is defined by the formula:

$$\text{(4)} \qquad \pi_n(x) = \begin{cases} 2x & \text{if } 2x \leq n, \\ 2x - (n+1) & \text{otherwise.} \end{cases}$$

As he did with the quenine, Dumas formalizes these extensions in [3] and provides analogous spiral-like representations for the permutations involved. With the pérecquine he also characterizes all admissible n, finding that π_n is of order n if and only if 2 is of order n (mod $n + 1$).

Finally, Dumas extends Theorem 2 to all admissible n to $\delta_{n,k}$ and all primitive roots.

Theorem 4 (Dumas)**.** *Let n be an integer such that $p = 2n + 1$ is prime, and let $k \leq n$. Let $\mathbb{F}_p$ be the finite field containing $p = 2n+1$ elements; then $\delta_{n,k}$ is of order n if and only if:*

(1) *Either k is of order $2n$ (i.e., k is a primitive root) in $\mathbb{F}_p$, or*
(2) *n is odd and k is of order n in $\mathbb{F}_p$.*

Once more, $\delta_{n,k}$ is the inverse permutation to $\sigma_{n,k}$. The proof is similar to that of Theorem 2 except for the condition $k \leq n$.

Corollary 5. *Let n be a positive integer such that $2n+1$ is prime. Then there exists k, $1 \leq k \leq n$, such that $\delta_{k,n}$ is of order n.*

A Connection to Artin's Conjecture on Primitive Roots

Artin's conjecture states that a positive integer k that is not a perfect square is a primitive root of p, for infinitely many primes p. Moreover, the set of prime numbers p such that k is a primitive root has a positive asymptotic density in the set of primes. For example, when $k = 2$ (and, in general, if $k = ab^2$ and a is not congruent to 1 (mod 4)), then the density is Artin's constant

$$C := \prod_p \left(1 - \frac{1}{p(p-1)}\right) \approx 0.373955, \tag{5}$$

where the product is taken over all primes including 2. The conjecture then implies that there are infinitely many admissible or Queneau numbers n, where $p = 2n + 1$. Indeed, the sequence of such n corresponding to that of primes p for which 2 is a primitive root (sequence A001122 of N. J. A. Sloane of the On-Line Encyclopedia of Integer Sequences) appears to be a (proper) subset of the set of all Queneau numbers. There is a brief treatment of Artin's conjecture in the preface to his collected works [1], while the expository articles by Murty [8] and Stevenhagen [16] trace related history and summarize some developments in proving the conjecture.

Conclusion

The mathematical ideas inspired by Arnaut's sestina may not contribute anything to one's understanding of courtly love, to the culture of the troubadours, or even to the meaning of Arnaut's poem itself. However, to those who understand the mathematics involved, these results inform the reading of the poetry by providing another dimension to the appreciation of its composition. Whether or not Arnaut was aware of the spiral structure of the permutation employed in the sestina, this structure provided the natural link and spark to mathematical activity. Perhaps we shall never know what inspired Daniel to choose $\sigma_{n,6}$ to compose his sestina. His choice, however, inspired modern mathematicians to create and contribute to their own discipline. Now it is the poets' turn.

Acknowledgments

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WHAT IS...

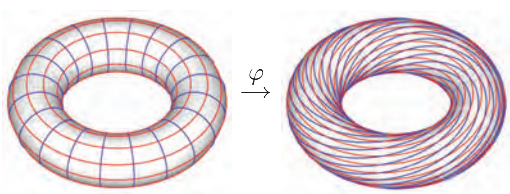
an Infra-nilmanifold Endomorphism?

Karel Dekimpe

Ever since the term "infra-nilmanifold endomorphism" arose in the 1960s, there has been confusion about its exact meaning, and different authors have used the term to refer to different concepts. Partly because of this confusion, two major results in dynamical systems, one on Anosov diffeomorphisms (1974) and one on expanding maps (1981), turn out to be incorrect.

With this historical background as motivation, let me introduce the reader gradually to the world of infra-nilmanifold endomorphisms, starting with one of the best known examples, Arnold's cat map φ on the torus $\mathbb{T}^2 = \mathbb{R}^2/\mathbb{Z}^2$. This map is given by

$$\varphi : \mathbb{T}^2 \to \mathbb{T}^2 : (x, y) + \mathbb{Z}^2 \mapsto (2x + y, x + y) + \mathbb{Z}^2.$$



In this picture one sees that φ stretches any small rectangle on the torus in one direction and shrinks it in the other. In dynamical systems one refers to such a map as an Anosov diffeomorphism. This diffeomorphism exhibits uniformly hyperbolic behavior: Around any point there are some directions in which the map is expanding and complementary directions in which it is contracting ([1] contains further details).

It is an open problem to determine the class of manifolds admitting an Anosov diffeomorphism, and it is also quite hard to construct examples of these mappings. The most obvious method to obtain new examples is to generalize Arnold's cat map to higher dimensions. The cat map lifts to a linear map l_φ of $\mathbb{R}^2$ that is given by the matrix $A = \left[\begin{smallmatrix} 2 & 1 \\ 1 & 1 \end{smallmatrix}\right]$, and φ being Anosov is equivalent to A having no eigenvalues of modulus 1. More generally, if $A \in \mathrm{GL}(n, \mathbb{Z})$, then the linear map $l_\varphi : \mathbb{R}^n \to \mathbb{R}^n$ determined by A induces a diffeomorphism $\varphi : \mathbb{T}^n \to \mathbb{T}^n : \mathbf{r} + \mathbb{Z}^n \mapsto l_\varphi(\mathbf{r}) + \mathbb{Z}^n$ on the n-torus. Such maps, called *toral automorphisms*, are prototype examples of infra-nilmanifold automorphisms, which we will describe below. When A has no eigenvalue of modulus 1, A is said to be hyperbolic, and the corresponding toral automorphism, which is called a hyperbolic toral automorphism, is an Anosov diffeomorphism. It is easy to construct a hyperbolic matrix $A \in \mathrm{GL}(n, \mathbb{Z})$, for any $n \geq 2$; hence any torus $\mathbb{T}^n$ of dimension $n \geq 2$ admits an Anosov diffeomorphism. Moreover, by a result of J. Franks, it is known that any Anosov diffeomorphism ψ on a torus $\mathbb{T}^n$ is topologically conjugate to such a hyperbolic toral automorphism. This means that there exists a hyperbolic toral automorphism φ of $\mathbb{T}^n$ and a homeomorphism h of $\mathbb{T}^n$ such that $\psi = h \circ \varphi \circ h^{-1}$.

The linear map l_φ can be seen as an automorphism of the abelian Lie group $\mathbb{R}^n$, leaving $\mathbb{Z}^n$ invariant. Now, $\mathbb{Z}^n$ is a discrete and cocompact (meaning that the quotient $\mathbb{R}^n/\mathbb{Z}^n$ is compact) subgroup of $\mathbb{R}^n$. More generally, let G be any Lie group, and assume that $l_\varphi \in \mathrm{Aut}(G)$ is an automorphism of G, such that there exists a discrete and cocompact subgroup Γ of G, with $l_\varphi(\Gamma) = \Gamma$. Then the space of right cosets $\Gamma\backslash G$ is a closed manifold, and l_φ induces a diffeomorphism $\varphi : \Gamma\backslash G \to \Gamma\backslash G : \Gamma g \mapsto \Gamma l_\varphi(g)$.

If we want this diffeomorphism to be Anosov, l_φ must be hyperbolic. It is known that this can happen only when G is nilpotent. So we restrict ourselves to that case, where the resulting manifold $\Gamma\backslash G$ is said to be *a nilmanifold*. Such a diffeomorphism φ, induced by an automorphism l_φ, is called a *nilmanifold automorphism* and is said to be a hyperbolic nilmanifold automorphism, when l_φ is hyperbolic. Just as in the case of tori, any hyperbolic infra-nilmanifold automorphism is Anosov. Again there is more: by

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a result of A. Manning, we know that any Anosov diffeomorphism on a given nilmanifold $\Gamma\backslash G$ is topologically conjugate to a hyperbolic nilmanifold automorphism on that nilmanifold.

At this point, one might think that the algebraic approach of finding Anosov diffeomorphisms ends with the class of nilmanifolds. However, there are manifolds that are not nilmanifolds but are finitely covered by a nilmanifold and that also admit Anosov diffeomorphisms obtained in an algebraic way. These manifolds are called *infra-nilmanifolds*. To define such a manifold, one constructs the affine group $\mathrm{Aff}(G)$ of a nilpotent Lie group G as being the semidirect product $G \rtimes \mathrm{Aut}(G)$. An element of $\mathrm{Aff}(G)$ is of the form (g, α), with $g \in G$ its "translational part" and $\alpha \in \mathrm{Aut}(G)$ its "linear" part. The element (g, α) acts on $x \in G$ by $(g, \alpha) \cdot x = g\alpha(x)$. Note that, for $G = \mathbb{R}^n$, we recover the usual affine group $\mathrm{Aff}(\mathbb{R}^n)$.

Let Γ be a torsion-free subgroup of $\mathrm{Aff}(G)$ such that $\Lambda = \Gamma \cap G$ is a discrete and cocompact subgroup of G and the index $[\Gamma : \Lambda]$ is finite. The orbit spaces $\Gamma\backslash G$ obtained in this way are the infra-nilmanifolds, and each of them is finitely covered by a nilmanifold $\Lambda\backslash G$. The most famous example of an infra-nilmanifold is, without any doubt, the Klein bottle which has $\mathbb{T}^2$ as a double cover. To construct the Klein bottle, one needs the group $\Lambda = \mathbb{Z}^2$ of translations, which together with a glide reflection, generates the needed group Γ.

Now assume that, for a given infra-nilmanifold $\Gamma\backslash G$, there exists a $l_\varphi \in \mathrm{Aut}(G)$, with $l_\varphi \Gamma l_\varphi^{-1} = \Gamma$ (where the conjugation is computed inside the affine group $\mathrm{Aff}(G)$); then again l_φ induces a diffeomorphism $\varphi : \Gamma\backslash G \to \Gamma\backslash G$, now called an *infra-nilmanifold automorphism*. Note that, for $\Lambda \subseteq G$, one can see that $l_\varphi \Lambda l_\varphi^{-1} = l_\varphi(\Lambda)$, showing that this new situation is a generalization of the previous one. When l_φ is hyperbolic, the corresponding infra-nilmanifold automorphism is Anosov. For a long time it was claimed that any Anosov diffeomorphism on an infra-nilmanifold is topologically conjugate to such a hyperbolic infra-nilmanifold automorphism. (And even more, it is conjectured that only infra-nilmanifolds admit an Anosov diffeomorphism.) However, as pointed out in [2], there do exist Anosov diffeomorphisms on certain infra-nilmanifolds that are not of this type. The idea is to look for an element $\delta = (g, l_\varphi) \in \mathrm{Aff}(G)$, with $g \neq 1$ such that $\delta\Gamma\delta^{-1} = \Gamma$. This δ also induces a diffeomorphism $\psi : \Gamma\backslash G \to \Gamma\backslash G : \ \Gamma x \mapsto \Gamma g l_\varphi(x)$, which is Anosov when l_φ is hyperbolic. Let us call ψ a (hyperbolic) *affine automorphism* of the infra-nilmanifold. Some authors, including me, have used the term infra-nilmanifold automorphism to refer to this more general type of diffeomorphisms. It is an open question whether all Anosov diffeomorphisms on an infra-nilmanifold can be obtained in this way.

Up to now, we have discussed only diffeomorphisms. However, if $\Gamma\backslash G$ is an infra-nilmanifold and $l_\varphi \in \mathrm{Aut}(G)$ is such that $l_\varphi \Gamma l_\varphi^{-1} \subseteq \Gamma$, then l_φ still induces a map φ on $\Gamma\backslash G$. This map will no longer be a diffeomorphism, but a self-covering. So such a map φ is a self-map of the infra-nilmanifold $\Gamma\backslash G$ that lifts to a Lie group automorphism of G, and, moreover, any self-map of $\Gamma\backslash G$ that lifts to an automorphism of G is of this form. M. W. Hirsch was the first person to use the term *infra-nilmanifold endomorphism* for these maps. However, when using the term infra-nilmanifold endomorphism, most researchers refer to a paper of J. Franks ([3]). It is important to realize that, although Franks himself attributes the terminology of an infra-nilmanifold endomorphism to Hirsch, the definition used by J. Franks is not consistent with the one used by Hirsch, and, in fact, Franks's definition is even mathematically incorrect, causing several researchers to misinterpret this notion. However, it is obvious from the rest of Franks's paper that he also intended to use the term infra-nilmanifold endomorphism for a map φ lifting to (and hence induced by) an automorphism as above.

These more general infra-nilmanifold endomorphisms are also important in dynamical systems: e.g., when l_φ has only eigenvalues of modulus > 1, φ is an expanding map. As a corollary to his famous paper on groups of polynomial growth (1981), M. Gromov proved that any expanding map is topologically conjugate to an infra-nilmanifold endomorphism. Unfortunately, this corollary depended not only on Gromov's own main result but also on false results by other researchers, and recently a counterexample to this statement was constructed ([2]). However, again we can consider the slightly more general class of maps, obtained by taking $\delta \in \mathrm{Aff}(G)$, with $\delta\Gamma\delta \subseteq \Gamma$ and constructing the map ψ as above. We say that ψ is an *affine endomorphism* of the infra-nilmanifold. It turns out that any expanding map is topologically conjugate to such an expanding affine endomorphism.

We have seen that the class of infra-nilmanifold endomorphisms provides interesting examples of maps, but one should really study the more general class of affine endomorphisms of an infra-nilmanifold. More evidence for this claim follows from the fact that any self-map of an infra-nilmanifold is homotopic to a map induced by an affine map δ of the corresponding Lie group. So perhaps after more than forty years of lack of clarity about the notion of infra-nilmanifold endomorphisms, it is time to consider instead what we have referred to here as affine endomorphisms.

Further Reading

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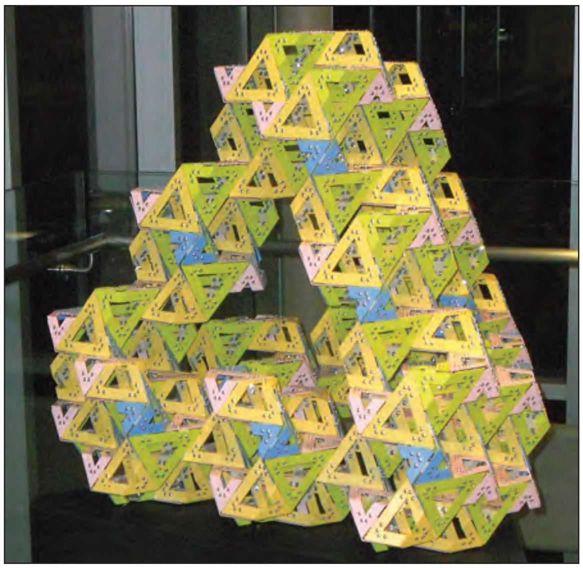
The Story of Gyrangle

Reza Sarhangi



Photograph courtesy of George Hart.

Figure 1. Gyrangle.

The first USA Science and Engineering Festival was a call by President Obama to invite the nation to the National Mall in Washington, D.C. for a celebration of science and engineering achievements and their contributions to our society. For two beautiful sunny days, Saturday and Sunday, October 23–24, 2010, thousands of people attended the festival at the National Mall and enjoyed talking to scientists and engineers to learn about their new inventions and discoveries.

People of all ages played with scientific toys and machines, and participated in interesting and exciting activities presented in hundreds of booths located all over the Mall.

Mathematicians appeared in this national celebration by presenting mathematical ideas and activities at several booths, including ones for the AMS and the Mathematical Association of America (MAA). The award-winning mathematics writer Ivars Peterson, director of publications and communications at the MAA, was among the hosts in the MAA booth. One gift from the MAA booth was a folded one-sheet booklet, "A Field Guide to Math on the National Mall", highlighting mathematical aspects of some interesting sites in the Washington, D.C., area.

The sculptor, mathematician, and computer scientist George Hart led a public sculpture barn-raising of his latest work, *Gyrangle* (Figure 1), at the AMS booth. The 38-inch-high sculpture consists of hundreds of laser-cut steel units bolted together in a novel way. To be exact, there are 490 flat or folded hollow equilateral triangles in four

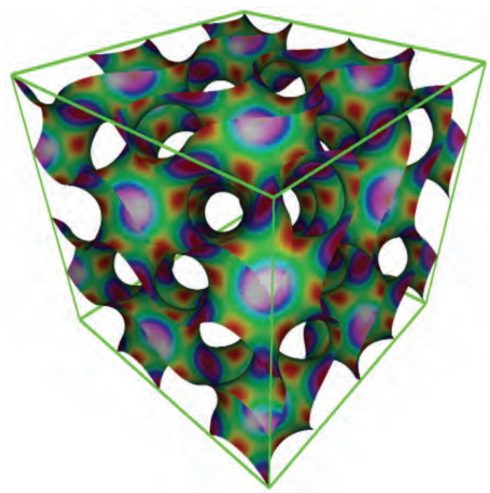


Figure 2. Gyroid surface.

colors. It illustrates a discrete version of the gyroid surface, made entirely from equilateral triangles.

The gyroid is a smooth, infinite, triply periodic, minimal surface discovered by Alan Schoen in 1970. Channels run through it in many directions and connect at an angle to other channels. The direction of connection travels in a spiral along each tunnel, giving rise to the name "gyroid". Figure 2, downloaded from Wikipedia, illustrates a cubical portion of the infinite gyroid surface.

The gyroid is a smooth surface, but Hart discovered a way to triangulate it entirely with equilateral triangles. Both the faces and vertices in the construction exhibit a property known as uniformity.

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Photographs courtesy of Reza Sarhangi.

Figure 3 (left) George Hart, (right) Annette Emerson and Mike Breen, AMS public awareness officers.

Although the gyroid itself contains no straight lines, Hart's triangular discretization contains infinite straight lines embedded in various directions. It divides all space into two congruent but mirror-image volumes. Another interesting property is that the faces do not meet edge-to-edge, but instead each triangle shares six half-edges with six neighboring triangles. The construction is previously unpublished, so the sculpture serves as the first presentation of this discovery. As it illustrates a gyroid made of triangles, Hart coined the term *Gyrangle* for it.

The full infinite surface would be visually repetitive (and infinitely large) so is not itself suitable for a sculpture. Instead, Hart chose a tetrahedral portion of space as the outer form of the sculpture and inhabited it with the Gyrangle surface as a kind of three-dimensional texture. To seal off the edges he introduced a folded triangle unit that closes off the boundary. For visual and mathematical interest, the 100 pounds of laser-cut steel triangles are painted in four colors according to the directions of their normal vectors. It is one of a series of many sculptures Hart has created for assembly at "sculpture barn-raising" events. He prepares the components and invites the public to participate in the construction.

During this barn-raising at the Mall, Hart helped hundreds of children and adults screw pieces together. At the same time, he continuously answered questions about the mathematics, computer science algorithms, and fabrication aspects of the design. The AMS representatives at the booth were Annette Emerson and Mike Breen (see the photo at right in Figure 3). They patiently helped children successfully fasten screws and connect pieces together, while also answering their very interesting but occasionally difficult questions, such as "Why are we making this thing?" and "What is it used for?" The public often sees mathematics only as something useful, but here the goal was to show another aspect of mathematics: mathematics is beautiful!

Mathematical sculpture is a rich area of visual art that is approached in various ways. Some sculptors have no formal background in mathematics, yet their artworks present patterns or structures that make sense to mathematicians. On the other hand, there are mathematicians who use techniques from computer science or the arts to visualize abstract mathematical forms. This gives the public an opportunity to "see" mathematical properties that were previously accessible only to specialized mathematicians. In either approach, sculpture may present harmony and balance in a new way and bring an appreciation of mathematics to the public, in a way similar to how we enjoy music and feel its importance in our lives.

The AMS financially supported the construction of this unique piece of mathematical art and donated it to the Department of Mathematics at Towson University, Towson, Maryland. The department is known for educational activities concerning mathematical connections to the arts and culture. The president of the Bridges Organization, the author, is a faculty member of this department. The Bridges Organization is a nonprofit organization, which runs the international Bridges conferences on mathematical connections in art, music, and science (`http://www.BridgesMathArt.org`); see "Bridges Pécs 2010", by Paul Gailiunas, *Notices*, February 2011, pages 289–290. These annual conferences, held in various locations around the world, attract faculty members from mathematics and science, as well as from the arts, communications, and music. Hart is on the board of directors of the Bridges Organization.

Hart's sculptures are recognized around the world for their mathematical depth and creative use of materials. He is a pioneer in using computer technology and solid free-form fabrication to create sculptures. Examples of his artwork can be seen at major universities, such as the Massachusetts

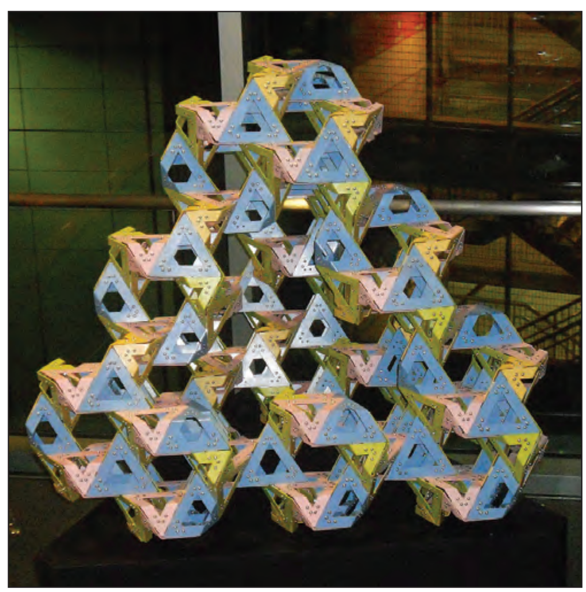
Photographs courtesy of Reza Sarhangi.

Figure 4. Finishing assembly of Gyrangle at Towson University.

Institute of Technology, the University of California at Berkeley, and Princeton University. After having served as a faculty member at Columbia University and Stony Brook University, Hart is now chief of content at the Museum of Mathematics, which will open in New York City in 2012.

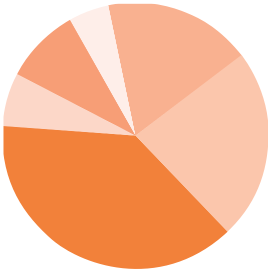
After two days of hard work to assemble Gyrangle at the Mall, only two-thirds of the construction was completed. The partly finished sculpture was transported to Towson University, where faculty and students helped to complete it (Figure 4). The sculpture graces the glass lobby of the Department of Mathematics (Figure 5).

Thank you, George Hart and the American Mathematical Society, for your great gift to Towson University!



Photograph courtesy of George Hart.

Figure 5. A view of Gyrangle showing some of its hexagonal tunnels.

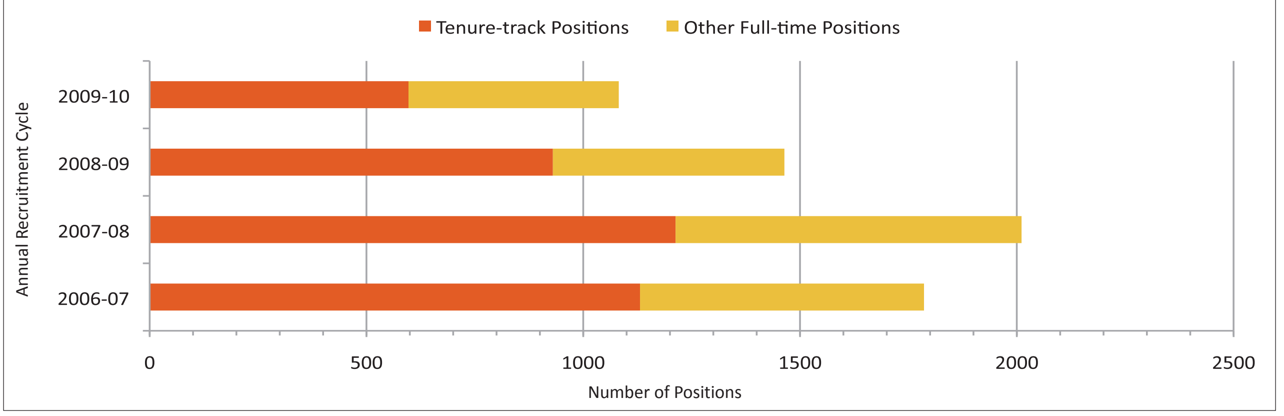


Report on 2009-2010 Academic Recruitment and Hiring

James W. Maxwell and Colleen Rose

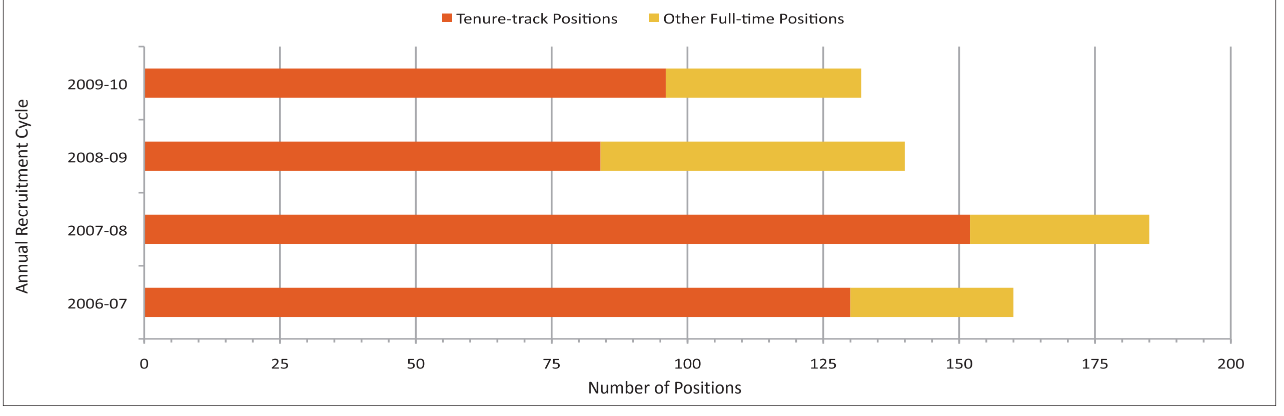
The recent declines in full-time positions under recruitment in mathematics departments continued during the 2009-2010 academic recruitment cycle. The total number of positions under recruitment by all mathematics departments combined was 1,081. This number is down 26% from the 2008-2009 total and down 46% from the 2007-2008 total. (Note: Throughout this report, the term tenure-track encompasses positions that come with tenure as well as those which provide the option of earning tenure at some point after appointment.)

Figure A.1: Positions Under Recruitment in All Mathematics Departments Combined



The doctoral statistics and biostatistics departments, Group IV, also saw a decline in the level of recruitment, though not as precipitous a decline as for the mathematics departments. Group IV consists of 93 departments and so constitutes a much smaller employment pool than the approximately 1,400 mathematics departments. In 2009 there were approximately 1,800 full-time faculty in Group IV compared to approximately 22,400 full-time faculty in all mathematics departments combined.

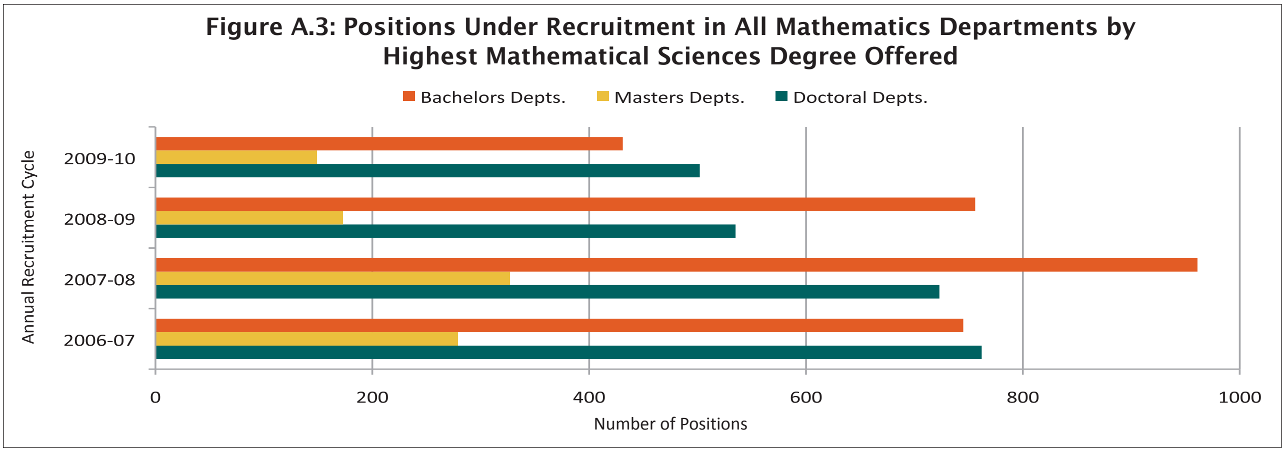
Figure A.2: Positions Under Recruitment in Doctoral Statistics and Biostatistics Departments Combined

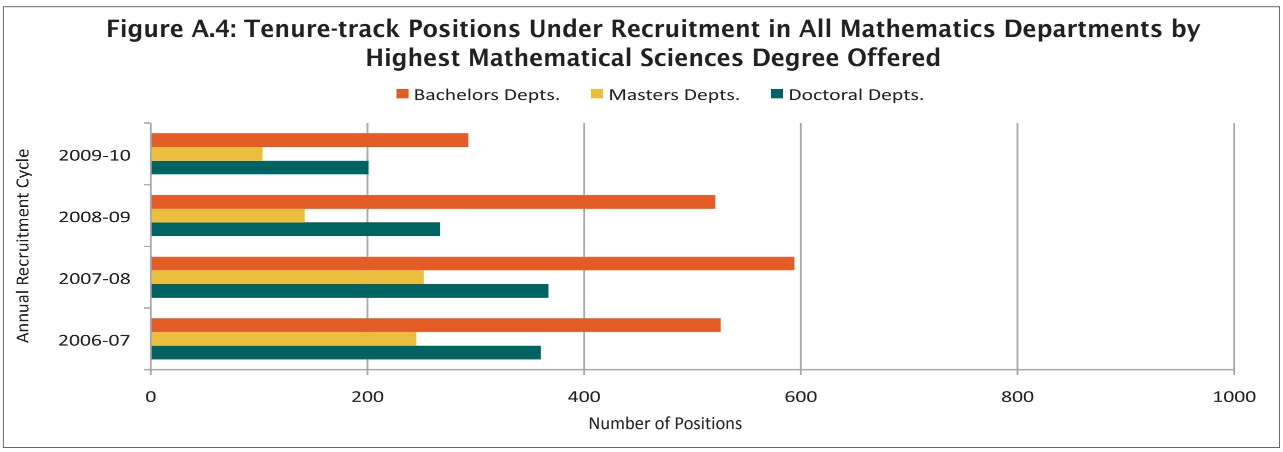


James W. Maxwell is AMS associate executive director for special projects. Colleen A. Rose is AMS survey analyst.

Positions Under Recruitment

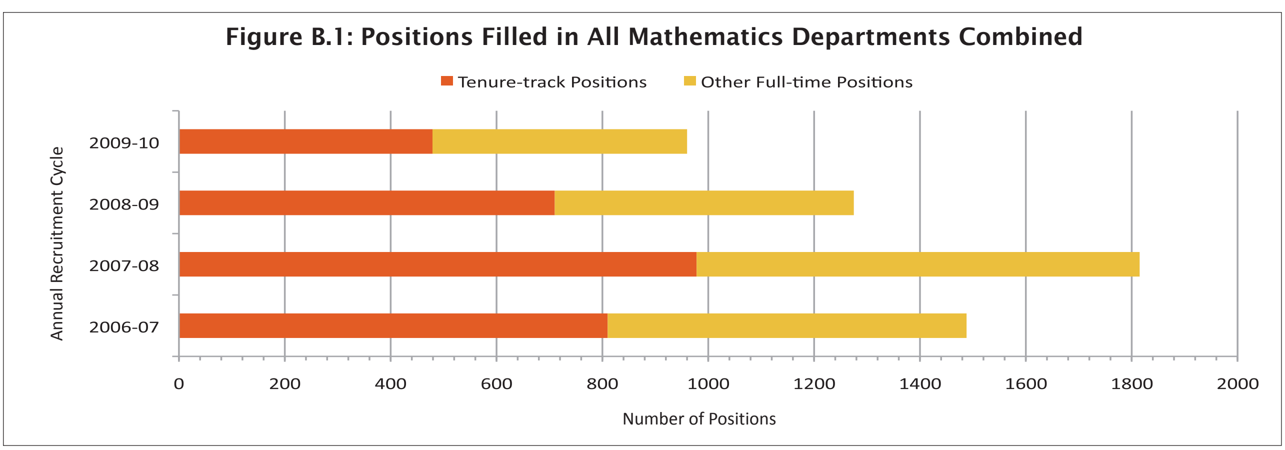
The percentage decline in positions under recruitment varied significantly among the various reporting groups within the combined mathematics departments. There was a 6% decline for the doctoral mathematics departments combined, a 14% decline for masters mathematics departments (Group M) and a 43% decline for bachelors mathematics departments (Group B). Similarly, the declines in tenure-track positions under recruitment for these same department groupings were 25%, 27%, and 44%, respectively.





Positions Filled

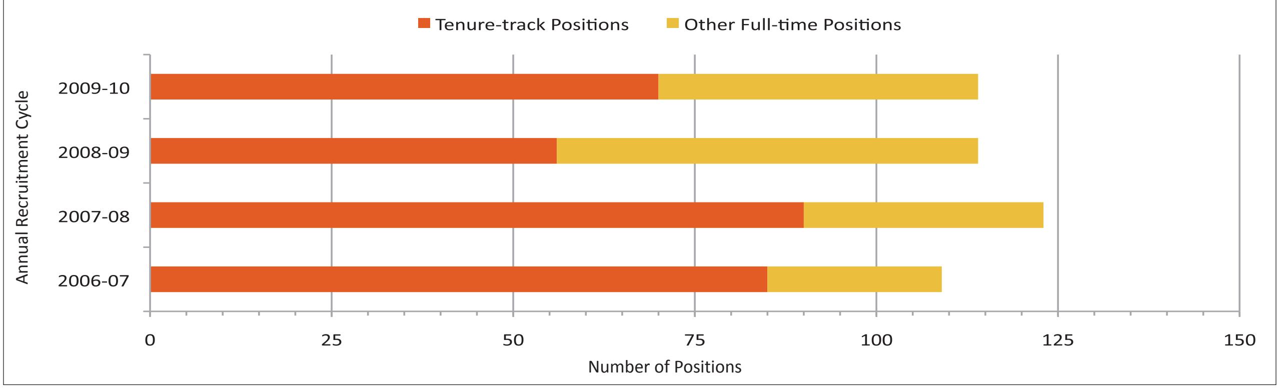
A total of 960 positions were filled during the 2009-2010 academic cycle for employment beginning in fall 2010 by all mathematics departments combined. This total is down 25% from the 2008-2009 total and down 47% from the 2007-2008 total.



Positions Filled

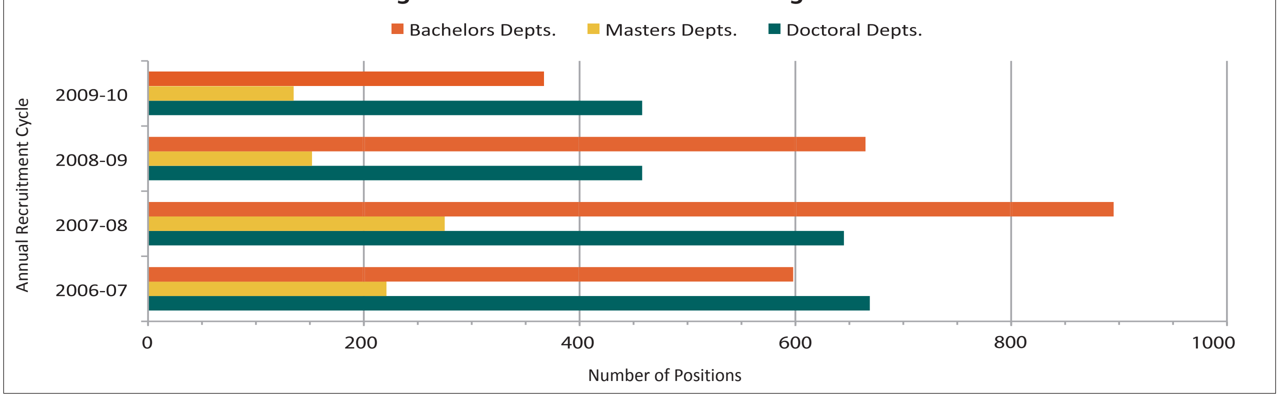
The situtation for doctoral statistics departments and biostatistics departments combined was much different, as demonstrated by the accompanying figure.

Figure B.2: Positions Filled in Doctoral Statistics & Biostatistics Departments Combined



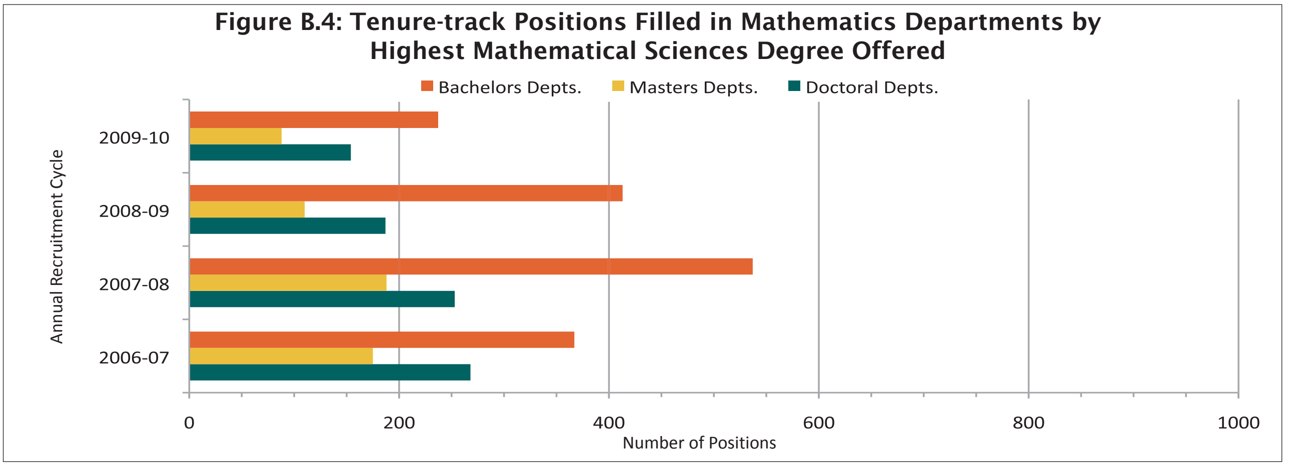
The decline in positions filled for fall 2010 also varied widely among the various reporting groups within all mathematics departments. For the doctoral mathematics departments combined, the number of positions filled was 458, unchanged from the fall 2009 count but down 29% from the fall 2008 count. For Group M the count was 135, down 11% from fall 2009 and down 51% from the fall 2008 count. For Group B the count was 367, down 45% from fall 2009 and down 59% from fall 2008.

Figure B.3: Positions Filled in Mathematics Departments by Highest Mathematical Sciences Degree Offered



The total tenure-track positions filled during the 2009-2010 academic cycle by all mathematics departments combined was 479, down 33% from the 2008-2009 total of 710. This total is down 51% from the 2007-2008 figure of 978, the highest total over the past ten years.

Positions Filled



This survey is part of the Annual Survey of the Mathematical Sciences. All 2010 numbers are estimates based on 721 usable responses from the 1,487 departments that make up the total population of mathematical sciences departments eligible to receive the survey. Since the survey was conducted online, it required that we have a valid email address in our records. This excluded 165 departments from participation in the survey.

The interested reader may view additional details on the results of this survey and prior year trends by visiting the AMS website at www.ams.org/annual-survey. Survey results for the doctoral departments in statistics and biostatistics are available there.

Other Information

Survey Response Rates

Faculty Recruitment & Hiring Response Rates

Department*	Number	Percent
Group I (Public)	19 of 25	76
Group I (Private)	19 of 23	83
Group II	46 of 56	82
Group III	60 of 80	75
Group IV (Statistics)	36 of 57	63
Group IV (Biostatistics)	13 of 35	38
Group Va	12 of 18	67
Group M	105 of 180	58
Group B	411 of 1014	41

* Departments and programs that do not formally "house" faculty and their salaries are excluded from this survey.

Other Sources of Data

Visit the AMS website at www.ams.org/annual-survey/other-sources for a listing of additional sources of data on the Mathematical Sciences.

Group Descriptions

Group I is composed of 48 departments with scores in the 3.00–5.00 range. Group I Public and Group I Private are Group I departments at public institutions and private institutions, respectively.

Group II is composed of 56 departments with scores in the 2.00–2.99 range.

Group III contains the remaining U.S. departments reporting a doctoral program, including a number of departments not included in the 1995 ranking of program faculty.

Group IV contains U.S. departments (or programs) of statistics, biostatistics, and biometrics reporting a doctoral program.

Group V contains U.S. departments (or programs) in applied mathematics/applied science, operations research, and management science that report a doctoral program.

Group Va is applied mathematics/applied science; Group Vb, which was no longer surveyed as of 1998–99, was operations research and management science.

Group M contains U.S. departments granting a master's degree as the highest graduate degree.

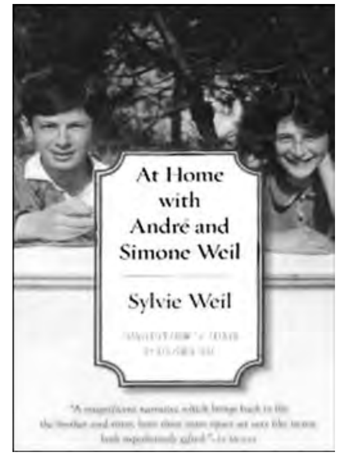
Group B contains U.S. departments granting a baccalaureate degree only.

Listings of the actual departments which compose these groups are available on the AMS website at www.ams.org/annual-survey/groups_des.

Book Review

At Home with André and Simone Weil

Reviewed by Michèle Audin



At Home with André and Simone Weil
Sylvie Weil
(English translation of Chez les Weils*, translated by Benjamin Ivry)*
Northwestern University Press, October 2010
US$24.95, 192 pages
ISBN-13: 978-0810127043

This book is about the family of the mathematician André Weil (1906–1998), one of the most influential number theorists in the twentieth century. The family consisted of a grandfather and a grandmother (Bernard and Selma Weil), a father (André Weil) and a mother (Eveline Weil), a stepson (Alain de Possel), and two daughters (the book's author Sylvie and her sister Nicolette). And yes, there was an aunt, too, Simone Weil (1909–1943), the philosopher. The book tells Sylvie's story and that of her family—of how it was to be André Weil's daughter and Simone Weil's niece, and of how Sylvie made her way, in life and in this family.

André Weil's Daughter…

Mathematicians will have no difficulty understanding that it was not very easy to be André Weil's daughter: Having a father who was a brilliant and famous mathematician, as well as the main character in the Bourbaki adventure, complicated Sylvie's efforts to find her own way in life. In addition, he had a circuitous career. The daughter of a French Jewish refugee, Sylvie Weil was born in the United States in 1942, left for Brazil in 1945, then left Brazil for Chicago in 1947. She was raised by her parents, of course, but also by her grandparents, in particular by her two grandmothers. While her father was a professor in America, Sylvie studied in Paris, which meant, for instance, that at age eleven she took, alone, the "direct" Air France flight Chicago-Montréal-Gander-Shannon-Paris.

…and Simone Weil's Niece

And there was an aunt, too, a deceased one: Sylvie was the baby daughter who arrived just a few months before her aunt, the philosopher Simone Weil, who died at age thirty-four. This was another heavy inheritance, inside the Weil family and outside it, as the baby daughter became a little girl, then a young woman, looking very much like her aunt—an aunt, who, moreover, was considered a saint. The little girl was surely a comfort to her mourning grandparents, but this heritage must have been quite a burden for her. A professor of French literature in several American universities and a writer,[1] Sylvie obviously found her own way. But she had to learn about and to understand the aunt she so closely resembled, through the sometimes difficult relationships in the family.

Visiting André Weil in Jail in 1940

Because he had decided not to serve as a soldier during World War II, André Weil was jailed from February to May 1940 (this is also the reason that he could not get a position in France after the war and why the life of the Weil family was geographically so complicated). In his jail cell, he worked quite a lot, and he was supported by his

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[1] *Author of* A New York il n'y a pas de tremblements de terre, Le jardin de Dima, Les vendanges de Rachi, *among others, and two novels for young adults translated in the U.S.,* My Guardian Angel *and* Elvina's Mirror.

Bourbaki friends and by his family, who would visit him, bring him books, and so on.[2]

Of course, Sylvie, who would be born in 1942, was not present then. But, from a simple sheet of paper on which André's mother, father, sister, and wife had each written a sentence for him, Sylvie is able to reconstitute a portrait of this group visiting him in his jail cell. She is also able to evoke the different personalities of these four people. This is a beautiful example of the subtle way she conveys delicate things in the book.

Riemann's Zeta Function

As this is a review for the *Notices*, let me make it unbalanced and concentrate more on the André side than on the Simone one. Readers should, of course, not expect to learn any mathematics here. However, there is a formula in the book,

$$1+\frac{1}{2^s}+\frac{1}{3^s}+\cdots+\frac{1}{n^s}+\cdots,$$

the definition of the Riemann zeta function, which André Weil wrote on a piece of paper for his daughter when she was learning how to add fractions. This is the only formula in the book, but it signifies his constant interest in Dirichlet series and the Riemann hypothesis. This is another hint about how the book proceeds, by light brush strokes.

The Sugar Bowl

One of the themes of the book is the failure of Simone and André Weil to adapt to ordinary life and the real world. Many touching details in the book illustrate this failure. For example, the knowledge of where the sugar bowl was kept in André and Eveline Weil's house was incompatible with the higher subjects occupying André's brain. Reading the book, we understand that this was not just absentmindedness, as one might think, but something more profound.

Walks

But André Weil was a father, too, and Sylvie recalls quite a few stories about him and their father-daughter relationship. They used to have long walks together, in the Luxembourg Gardens in Paris, in São Paulo, in Chicago, or in Princeton, the little girl with her tall father. And there is also the story of the (adult) daughter accompanying her old father to Japan in 1994, where André Weil would be awarded the Kyoto Prize (together with Akira Kurosawa).

I will not comment on the English translation, by Benjamin Ivry, but let me mention the design of the cover, in which, unfortunately, the siblings André and Simone are pulled apart by the title of the book.

By now it should be clear that this book is not a biography, neither of André Weil nor of Simone Weil. But it does allow us to enter into the Weil home and to have a tactful glance at a few colorful Jewish ancestors, at Selma and Bernard, at the privacy of Simone, André, and Eveline—and at Sylvie.

I hope the readers of the *Notices* will enjoy, as much as I did, reading this both reserved and intimate account of how it was to be at home with the Weil family.

References

[1] A. Weil, *Œuvres scientifiques, Volume I*, Springer, 1979.

[2] ———, *The Apprenticeship of a Mathematician*, Birkhäuser, Basel, 1992, translated from the French by Jennifer Gage.

[2] *For André Weil's 1940 story, see* [2]*; for the mathematics he made in prison, see the enlightening comments in his complete works* [1].

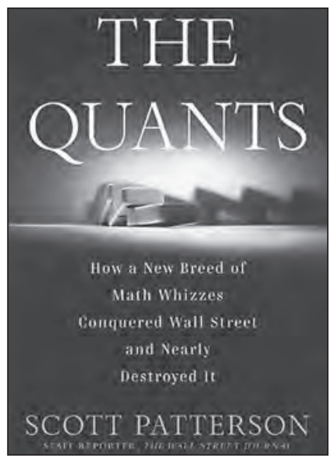
The Value of Nothing: A Review of *The Quants*

Reviewed by David Steinsaltz

The Quants
Scott Patterson
Crown Business, 2010
US$27.00, 352 pages
ISBN-13: 978-0307453372

Scott Patterson, former financial journalist for *The Wall Street Journal*, has written a book-length love letter to quantitative finance and its practitioners. To judge by some comments posted on amazon.com and elsewhere, though—"naïve", "mathematically illiterate", "sensationalism", "bumbling idiot"—the love is not requited. Of course, the book is not really about the sort of people who write comments on the websites of online retailers. The "quants" of Patterson's title are a handful of capitalist potentates, supremely successful and influential practitioners of mathematically inspired finance. Putting to one side a certain oversensitivity to criticism and the unquestionably sensationalist subtitle—"How a New Breed of Math Whizzes Conquered Wall Street and Nearly Destroyed It"—many who identify with the job title "quant" are very far removed from the world of Patterson's conquering heroes and from Patterson's enthusiasm for them. While the book makes little pretence of reflecting their careers or their experience, it probably offends by implying—with little evidence—that the managers and the menials share a unified mathematical culture and mindset.

But the managers, and their real and perceived relationship to mathematics, do make an important story. Economic historians teach us that one indispensable ingredient in a financial crisis is an excuse for ignoring the lessons of the past, for overriding the traditional safeguards, for believing that "this time is different" [15]. The most recent round of excuses was provided, if not directly by mathematicians, then under the banner of mathematics, and the crisis that ensued was of terrifying proportions. For this reason alone *The Quants* would deserve the attention of the mathematical community. Few readers will be bored, and most will learn some things that are worth knowing about the world and about the place of mathematics in the world.



This book joins a long list of recent popular or semipopular titles on quantitative finance and its practitioners. It is not the best in all respects, certainly not as a technical primer. Its approach can seem infuriatingly nonanalytical and apolitical, even willfully obtuse at times. But it is intelligent and serious, by and large, and its relentless focus on the look and feel of the rarefied quant world, although a limited perspective, is a valuable one, and one that requires the skills of a talented journalist, which Patterson obviously is.

The quants, as Patterson describes them, "couldn't care less about a company's 'fundamentals', amorphous qualities such as the morale of its employees or the cut of its chief executive's jib. That was for the dinosaurs of Wall Street [...] who focused on factors such as what a company actually made and whether it made it well. Quants were agnostic on such matters, devoting themselves instead to predicting whether a company's stock would move up or down based on a dizzying array

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of numeric variables." It's an old story, actually. A similar conflict embroiled the earliest attempts, three centuries ago, to expand the nascent probability theory beyond its disreputable origins in games of chance. Historian Lorraine Daston writes, "The mathematicians created a new approach to the subject that challenged the previous *practice* of risk, legal and otherwise. […] It was as if the jurists and the commercial class they wrote for lived in a world of fine-grained detail where regularities were partial at best […] The mathematicians, in contrast, apparently lived in a world strictly governed by invariable laws that could be expressed as the function of a small number of variables" [4, Chapter 3.1].

In that twilight struggle, B. Gnedenko has argued [11], the probabilists were routed. From the early gambling studies there followed a profusion of "papers devoted to applications in various branches of the natural sciences and public life. Many of these had so little validity that they were considered 'mathematically scandalous affairs'. Disenchantment followed and among Western European mathematicians probability theory began to be thought of as some kind of mathematical entertainment hardly deserving serious attention."

Probability's association with gambling endangers more than just respectability. Human beings have natural intuitions about risk that are systematically violated by cards, dice, and roulette wheels. As evolutionary psychologists have remarked, "If humans had evolved in casinos where their winnings translated into reproductive success, selection probably would have eliminated the gambler's fallacy" [13]. In the real world, probability theory is a specialized adjunct to more natural human intuitions, not a substitute for them.

In Patterson's account, modern quantitative finance originated with Ed Thorp, a mathematician who applied the Kelly criterion to blackjack in his 1962 book *Beat the Dealer* before turning the same principles to finance in *Beat the Market* (1967). Patterson makes clear (as do other sources) that Thorp himself has always been the farthest thing from a gambler by temperament, but some of his intellectual heirs revel in high-stakes poker parties and junkets to Las Vegas casinos. Describing the credit derivatives group at Deutsche Bank around 2000, Patterson writes "In their downtime, Weinstein's traders would randomly bet on just about anything in sight: a hundred on the flip of a coin, whether it would rain in the next hour, whether the Dow would close up or down." The financial markets are "the world's biggest casino". While "investors" put up the money, the quants "place bets": Bets on trade patterns, bets on currency exchange rates, bets on company growth and defaults, and bets on the bets that other traders would make.

There is nothing new about the accusation that financial transactions involving risk—insurance, stock purchases, futures contracts—differ only in name from gambling. Indeed (see [4, Chapter 3.2]) life insurance in the sixteenth and seventeenth centuries was often a short-term bet on the life of some famous person. Usually the accusation is lobbed from the left, to be dismissed by the financiers as propaganda, ignorant of the vital work performed by capital markets. It is a surprise to see how many of today's leading financiers—not all of them mathematical types—have come to embody this accusation. "Every day they went head-to-head on Wall Street, facing off in a computerized game of high-stakes poker in financial markets around the globe, measuring one another's wins and losses from afar, but here [in their quant poker games] was a chance to measure their mettle face-to-face." It is not the least of the paradoxes that Patterson's protagonists eagerly seek risk in gambling, while their core mathematical models presume that investors pay a premium to dispose of risk. The contradiction does not seem to register on Patterson, who throughout seems entranced by these sharp operators using financial markets as a gambling den.

After probability and compound interest, the key financial principle underlying the quant models is arbitrage, the financial perpetual motion machine generated by price discrepancies. In principle, if you find the same asset being sold at different prices in different markets—Patterson uses the example of gold trading for $1,000 in New York and $1,050 in London—you can buy in one market and sell in the other to generate riskless profit. Back in the day, you would need to float your gold bricks over the Atlantic, but today the only limit on your profits would be the amount of money you could borrow and the amount of gold you could buy before the New York market raises the price. *Statistical arbitrage* expands the possibilities, by allowing for randomness. It attempts to extract profits from discrepancies in the future expectations of combinations of assets. Patterson is at his best when describing these strategies, both the mechanisms and the psychology that gives birth to them. These depend, in different ways, on inefficiencies in pricing mechanisms, creating short-term disequilibria that can generate profits as they reset.

Unless they don't. The book is punctuated by crises, large and small, in which the expectation fails. A casino owner needs only to pump enough money through the system and let the law of large numbers take care of the rest. In the financial markets all the "bets" are correlated, in hard-to-estimate ways, and the probabilities are only vaguely defined, estimated by analogy with the past. Not to mention that the quant strategies themselves alter the patterns of the markets. Pumping large quantities of borrowed money through these strategies can lead to a meltdown.

Arbitrage is a bit like dumpster diving. Lars Eighner formulated [5] three rules for safely consuming discarded comestibles. The third rule: answer the question "Why was this discarded?" The principle of "efficient markets" says that market prices already incorporate all publicly available information, so there should not be any opportunities for statistical arbitrage. This principle need not be true—indeed, there are good reasons, well discussed in this book, to believe it is not—but mathematical market models generally depend on it. Thus the computations of quantitative finance are largely based on the principle that these computations are a waste of time. Patterson repeatedly circles back to this paradox, which clearly troubles many of the quants. How can they consistently beat the market average? Some explanations on offer:

- **They're smarter than other people.** This seems to be, unsurprisingly, their favored explanation. Traders who translate new information into prices can profit, a process that Patterson compares to throwing meat into a pool full of piranhas. The meat (new information) disappears quickly, but the piranhas (the traders) do get fed. Now, Patterson describes the quants —with some notable exceptions—as unconcerned with anything so coarse as commerce, but their pattern-seeking is another way to integrate information, including past information and unrecognized persistent biases. This could earn them a steady profit.
- **Weaker regulatory constraints.** If you find a pristine can of soup in the dumpster on its sell-by date, you know why the supermarket couldn't use it; unconstrained by rigid health directives, an ordinary person could be fairly confident that the contents are worth having. Retail banks, pension funds, municipal governments, and many other institutional investors are tightly regulated, forced to make decisions based on crude categories. Laws constrain the sorts of risks they are allowed to take with the funds entrusted to them. By loaning the capital to hedge funds or investment banks the regulations vanish, a benefit sometimes called *regulatory arbitrage*. Considerable theoretical ingenuity has gone into producing financial instruments, such as the now infamous auction-rate securities that duplicate traditional banking functions within an unregulated securities framework, sometimes called "shadow banking".
- **Special privileges.** A significant portion of quant profits come from their access to information and trades closed to ordinary investors. They have top-of-the-line computer systems for bringing in market information, processing it, and executing trades instantaneously. They work for major international investment banks, or they win preferred-customer treatment from the banks. They may also profit from inside knowledge of how their own vastly leveraged trades are moving the markets. Many of the most lucrative trades are simply closed to outsiders. Michael Lewis's *The Big Short* [10] narrates the years-long struggle of investors, who predicted the collapse of mortgage-backed securities (MBS), to make the necessary contacts to sell the MBS short. The big quant shops would have made the short sale in microseconds.
- **They act crazy.** Patterson's quants cultivate an aura of strangeness. Some gamble manically. Many defy Wall Street dress conventions. They organize company paintball tournaments. One rages at bad news and destroys computer monitors. One makes a show of busking in a Wall Street subway station. They rave about the Truth of "Alpha".[1]

  Patterson describes all this with amusement, as some blend of residual math-nerd culture and plutocrat eccentricity. He never considers its strategic value. If Trader A makes a large sale of stock Z, he signals that he doesn't think Z is worth holding at the current price, causing the price to drop. George Akerlof analyzed this problem in his famous study of the used car market and came to the conclusion that this information imbalance—A knows why he sold, but B doesn't—depresses prices and can lead to a market-destroying downward spiral. If A appears crazy, or at least inscrutable, his trades will have less influence on the market. Eccentricity functions like the poker player's dark glasses.
- **They're not.** If I were to sell budget earthquake insurance in California, I could make a fortune—until the earthquake. Economist Joseph Stiglitz

[1] α *is just the intercept term in a linear regression of an individual asset price against an overall market index, but the quants, or Patterson, or both, seem to confuse it with the cabalistic aleph.*

> writes: "In today's dynamic world, this market discipline broke down. The financial wizards invented highly risky products that gave about normal returns for a while—with the downside not apparent for years. Thousands of money managers boasted that they could 'beat the market', and there was a ready population of shortsighted investors who believed them" [17].
>
> As long as high profits over a few years are enough to be labeled a success, there is an incentive to define the earthquakes out of the model. Until the earthquake strikes he's a genius, and after the quake he still gets to keep his genius bonus. At the height of the last crisis one of Patterson's heroes, Citadel Investment chief Ken Griffin, estimated that his company had a 55 percent chance of surviving [12]. How high do the annual returns need to be, to be worth risking everything on a coin flip?

This is not, primarily, a book about mathematical models. It includes some nice profiles of mathematicians and some slightly dodgy accounts of random walks and Lévy processes. For all that it has to say about mathematical thinking or the application of mathematical models, though, it could be *Harry Potter and the Volatility Smile*, with BlackBerries instead of wands. It's all gesture and evocation, as in: "The quants pulled out their calculators, cracked open their calculus books, and came up with solutions." Indeed, nearly every figure of any significance in the book is referred to as a "wizard" or a "whiz". There is "quant alchemy" and the "dark art of securitization".

What we get instead is a psychohistory of unresolved adolescent conflict, shading into a pervasive sexual menace. "The money was huge, the women were beautiful, and everyone was brilliant and inside the secret. […] At Deutsche Bank, risk wasn't [expurgated] *managed*. Risk was [expurgated]-slapped, risk was tamed and told what to do." The quants identify the dinosaur traders of Wall Street with all the privileged bullies who humiliated them on the playground and mocked their mathematical interests in high school. They are burning for revenge, and Patterson channels their fantasies into his narration: "A friend sent Muller congratulatory flowers for his new job. The bouquet was delivered to his desk on the trading floor. It was raw meat to the grizzled traders around him: *Look at the California quant boy and his pretty flowers.*"[2] In the 1980s we read, "quants were seen as second-class citizens at most trading firms, computer nerds who didn't have the balls to take the kinds of risks that yielded the real money." Thus begins one of the more fascinating vignettes of the book, told from the perspective of one Aaron Brown, who was "sick of seeing the same rich kids he'd suckered at Harvard lord it over the quants in trading-floor games such as Liar's Poker." Brown killed the game off by spreading a winning strategy among his fellow quants. "No longer would they stand at the end of the line and be victimized by the [BSDs]."[3]

In such an environment women can only be peripheral objects: wives and girlfriends distracting the quants with the blandishments of forty-bedroom home and hearth; or a secretary, whose firing gives a clue to the mental state of her (male) boss. The one female quant in the book, Kim Elsesser—also, though herself quite senior, the book's token representative of nonkingpin quant-dom—disappears after a few amusing anecdotes "to study gender issues in the workplace at UCLA". It hardly surprised me after this to read of the recent lawsuit charging persistent gender discrimination at Goldman Sachs [3] or that the number of women working in U.S. finance has been dropping steadily [18].

The tone shifts in the aftermath of the crash. Skeptical academics take the stage (along with the irrepressible and omnipresent Nassim Taleb). Paul Wilmott, who wrote back in 2000 about the dangerous misuse of mathematics in finance [20], produced with Emanuel Derman a "modelers' Hippocratic Oath": five vows of humility, concluding with "I understand that my work may have enormous effects on society and the economy, many of them beyond my comprehension." Estimable sentiments, but absent a suitable educational program, what is it worth to swear an oath to "understand" something? It seems about as effective as combating the modelers' anglophone parochialism by requiring the vow: "I understand Chinese."

Has the corner of mathematics called "mathematical finance" actually influenced financial practice? Haug and Taleb [8] have debunked the mystique of the Black-Scholes-Merton formula, arguing (persuasively, if somewhat self-contradictorily) that the formula has never been applied in the real world, that equivalent calculations have been known and applied since the dawn of time, and that it is fundamentally misleading. Li's copula formula (which Patterson describes as a phenomenon without really explaining) has been blamed for the collateralized debt obligation (CDO) fiasco [19], but it is so banal in itself that it seems more like a decoration than a real impetus to the CDO market.

---

[2] *In this case, though, it is possible to compare Patterson's dialogue directly to his source. There is evidence of considerable embellishment.*

[3] [*BSD*] *is here a crass expression for the bankers' anatomical endowment that seems to be a term of art in the finance world.*

It takes more than Hardyan "uselessness" [7] to buy a mathematician a clear conscience these days, though. University mathematicians have served the finance industry with a steady stream of students trained to look past the reality of the world to an abstract realm of interchangeable entities and to accept models axiomatically, without skepticism. At least as important, we have participated in what G. Bowker [2] has termed "legitimacy exchange" (though with a view to the huge blow suffered by the reputations of both groups, perhaps it might be called a "credibility default swap"). The bank's team of MIT Ph.D.s is a token of seriousness, like the Picasso in the lobby, the marble columns, and the expensive wristwatch. In return for lending the reputation of their subject, academic mathematicians are compensated by sharing the financiers' reputation as important people doing important work, and the enviable status as a conduit to high-paying careers, whether or not they also pocketed consulting fees.

Faced with dwindling public esteem, defenders of quantitative finance have sought to mortgage their intellectual stature for a legitimacy loan from the engineering profession. Patterson picks up on this, writing "few—aside from zealots such as Taleb—were calling for them to be cast out of Wall Street. That would be tantamount to banishing civil engineers from the bridge-making profession after a bridge collapse." (It is not clear exactly who Patterson is quoting here, but he or his source is channeling the very similar comments published in 2008 by mathematician S. Shreve [16].) M. Hellwig, in the course of an otherwise careful analysis of sources of the crisis [9], averred that "One might as well blame the architect of the World Trade Center for not having taken the risk into account that kerosene-filled airplanes might be flown into the building." As it happens, the lead structural engineer for the World Trade Center told reporters in 1993 [14] that the towers *had* been designed to withstand a jet collision. The collapse has since been extensively studied as an engineering failure, but thousands survived because of the safety margin that was built into the building. Cranking the 9/11 metaphor up another notch, David Hand has compared bank executives to people who break into a 747 cockpit and crash the plane [6].

Mathematicians are not engineers, and not only because one can hardly imagine top civil engineering academics reacting with nonchalance if their best graduates were not building bridges but finding bridges prone to collapse so they can cash in buying insurance on them. Or boasting of how their elite training and inherent brilliance enabled them to hoodwink the inspectors into approving shoddy plans. The engineering analogy was eloquently demolished by Nicolas Bouleau, financial mathematician and professor at the École Nationale des Ponts et Chausées. "Engineering culture and financial culture" is a chapter in his book [1]. (An earlier version of this chapter was published in the late 1990s.) The traditions and professional ethic of engineering "charges them with examining painstakingly what will happen in case of an accident, a fracture or conflagration, and to effect a repair." Engineers, he wrote, plan for the inevitability of failure and for its consequences.

Financial engineers don't install emergency exits, and on the evidence of this book they cannot imagine the need. Is this ostentatious math-nerd naïveté genuine or merely a camouflage? Surely, one thinks, as the disasters and near-disasters pile up, as the difference between brilliance and bankruptcy is the whim of a single deep-pocketed investor, or a flight to liquidity, or the faith of creditors, or a government decision to bail out a counterparty or temporarily ban short-selling, surely they cannot still believe that there is an ineffable truth to the markets, to be captured in probabilistic models and calculated to the fifth decimal place. It is disappointing that Patterson never poses the question of premeditation. The people who end up with the billions are portrayed as innocent bystanders, just little boys on the seashore collecting pretty pebbles. We are even invited to pity their stressful days and sleepless nights during the crash: "A quant nightmare. Markets were at the mercy of unruly forces such as panicked investors and government regulators."

The mighty quant barons of this story are not, it must be emphasized, the entire world of quantitative finance, and one longs for a latter-day quant Max Weber to map the lines of power and the percolation of ideology through the institutions. Until he or she arrives, Patterson has produced at least one plausible journalistic portrait of the past few decades of quantitative finance, one that is at least consistent with what we see in other recent books. It is a picture from outside the mathematical community, and it shows us how, whatever we may believe personally, the successes of financial mathematics will be largely privatized, while the failures will be hung around all of our necks. "Quant alchemy" indeed. By the end, I couldn't help thinking of H. G. Wells's famous takedown of Winston Churchill after the First World War: "He believes quite naïvely that he belongs to a peculiarly gifted and privileged class of beings to whom the lives and affairs of common men are given over, the raw material of brilliant careers." The quant aristocrats have had their Gallipoli. How will they adapt? How will the mathematical community respond? After the last quant has done his turn on the economic stage, it now seems hard to imagine that anyone will want to erect a statue to his memory.

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Revitalizing the 1960 Mathematics Major

Alan Tucker

I entered college around 1960 in a golden era for the mathematics major. At that time, 5 percent of freshmen expressed an interest in becoming math majors (although only half as many earned math degrees). Math grads were in demand for new careers in aerospace and other technological industries, along with traditional careers in insurance and teaching. Many of my math major friends planned to go to medical or law school, as well as graduate school in mathematics or other quantitative disciplines. For many smart freshmen with unsure career plans, mathematics was a default major. This piece sketches my reading of why the math major changed, along with an argument for why it needs to be revitalized.

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Most United States universities in the 1950s had many tenured non-Ph.D. faculty in mathematics, and many offered a dated mathematics curriculum, e.g., a typical junior math course was tensor calculus. A dramatic increase in the number of math Ph.D.s was needed both to upgrade the quality of faculty and to match the rapid growth of college enrollments. The mathematics major curriculum required a massive revision to reverse the high failure rate in first-year graduate courses in Lebesgue integration and such. The proposed solution appeared in *Pregraduate Preparation for Research Mathematicians* (1963), the first report of the MAA's new Committee on the Undergraduate

Program in Mathematics (CUPM). The widely adopted 1965 CUPM report, *A General Curriculum for Mathematics in Colleges*, presented a gentler version of the 1963 recommendations with the goal that mathematics majors at all institutions should have some preparation for graduate study in mathematics (it was assumed that one-third of math majors would earn an M.S. in the mathematical sciences).

From 1960 to 1965, the number of math bachelor's degrees grew from 11,000 to 20,000, and math Ph.D.s from 300 to 700, according to AMS and CBMS survey data. The 1968 NRC COSRIMS report predicted 60,000 B.S.s and 2,200 Ph.D.s in math by 1975. The bachelors' number actually decreased, dropping to 12,000 by 1980 and staying around there to this day. Ph.D. numbers did grow but more slowly. Many math majors struggled with the more abstract curriculum, and employers complained that math graduates often seemed ill prepared to work on industry's problems. Further, new programs in computer science lured away many potential math majors. Although math majors were now better prepared for success in graduate school, the catholic 1960 image of the math major had faded. From 1975 on, only 1 percent of college freshmen expressed an interest in a math major.

Now let me turn to the situation from the faculty viewpoint, based on conversations with my father A. W. Tucker, who was chairman of the Princeton Mathematics Department around 1960. His view of academic life was that he was paid to teach but given the free time to do research, which he loved. That was a time when universities had modest graduate programs and the focus was on undergraduates; federal support of research was just starting. All Princeton faculty, including department chairs and even the dean of the college, taught two courses per semester. One semester each year my father was the lead professor in the big calculus course that most freshmen took. When I asked him years later when I was a department chair why he assumed this extra duty, he looked at me like I was crazy and said, "The most important thing the Princeton Mathematics Department did was to teach freshman calculus, and so as department chairman it was obvious that I should lead that effort." As chair, he felt it was also his job to get to know most of the math majors personally. [For more vignettes about those days, see my personal website: `www.ams.sunysb.edu/~tucker`.]

My father believed that one person could be blamed (or credited) with reorienting university faculty toward graduate students and research—Clark Kerr. In seeking to recruit star faculty from the Ivy League schools to Berkeley, Kerr hit upon the winning strategy of offering reduced teaching loads and little undergraduate teaching. I remember seeing one after another of our faculty neighbors get lured away, while my father and the rest of the Princeton administration watched helplessly as they stuck to their two-courses-per-semester, undergraduate-focused tradition. Other public universities followed the Berkeley example, and finally the Ivies gave in. Soon all faculty at universities wanted to be treated like stars with reduced teaching loads. A huge increase in precalculus and other low-level mathematics enrollments at public universities, along with the pressure to get research grants, has further diminished the attention given to math majors over the past forty years.

I believe that research mathematics departments will have trouble flourishing, especially at public universities, without flourishing math majors. This was a key message in the 1999 AMS *Towards Excellence* report, which cited examples of several universities with flourishing math majors. As one model, in the programs cited at UCLA and Chicago, research faculty collaborate with nonresearch faculty or mathematically trained advisors to give math majors personal attention.

Mathematics departments cannot compete with the natural sciences for resources in research dollars. Because regular faculty do limited teaching in calculus-and-below courses, which represent over 80 percent of math enrollments, mathematics departments cannot count on the huge enrollments in these courses to justify their size. Indeed, administrators are increasingly using four-course-per-semester lecturers, adjuncts, and technology to provide cheaper instruction than graduate students and instructors—and they often do a better job; in the process, math teaching assistantships and faculty lines are eliminated.

Without meaning to diminish the importance of research mathematics, I believe that a broad-based, thriving major in a core discipline such as mathematics is valued and respected by administrators as an important university service to society and its economic well-being. The value of studying mathematics is perhaps more in its mental training than its content. The wide-ranging accomplishments of math majors speak for themselves: from economist John Maynard Keynes to biologist Eric Lander, who led the Human Genome Initiative, to Jim Simons, professor turned hedge fund guru turned philanthropist, and even to basketball star Michael Jordan.

Alan Tucker has taught for forty years at SUNY-Stony Brook, where 6 percent of bachelor's graduates are in mathematics, pure and applied.

JPBM Communications Award

The 2011 Communications Award of the Joint Policy Board for Mathematics (JPBM) was presented at the Joint Mathematics Meetings in New Orleans, Louisiana, in January 2011.

The JPBM Communications Award is presented annually to reward and encourage journalists and other communicators who, on a sustained basis, bring mathematical ideas and information to nonmathematical audiences. JPBM represents the American Mathematical Society, the American Statistical Association, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics. The award carries a cash prize of US$1,000.

Cheryl Heuton and Nicolas Falacci

Previous recipients of the JPBM Communications Award are: James Gleick (1988), Hugh Whitemore (1990), Ivars Peterson (1991), Joel Schneider (1993), Martin Gardner (1994), Gina Kolata (1996), Philip J. Davis (1997), Constance Reid (1998), Ian Stewart (1999), John Lynch and Simon Singh (special award, 1999), Sylvia Nasar (2000), Keith J. Devlin (2001), Claire and Helaman Ferguson (2002), Robert Osserman (2003), Barry Cipra (2005), Roger Penrose (2006), Steven H. Strogatz (2007), Carl Bialik (2008), George Csicsery (2009), and Marcus du Sautoy (2010).

The 2011 JPBM Communications Award was presented to Nicolas Falacci and Cheryl Heuton. The text that follows presents the selection committee's citation, a brief biographical sketch, and the recipients' responses on receiving the award.

Citation

The 2011 JPBM Communications Award is awarded to Nicolas Falacci and Cheryl Heuton for their positive portrayal of the power and fun of mathematics through their hit TV series, *Numb3rs*.

Nicolas Falacci and Cheryl Heuton created the extraordinary TV series *Numb3rs*, featuring an FBI agent and his brother, a mathematical genius. Through its six-season run on CBS, the series featured the use of mathematical thinking and modeling to solve crimes. *Numb3rs* provided the general public with a glimpse of the mathematical world, its depth and its power, in a way that connected with a broad spectrum of viewers. With creativity and cleverness, their work, which includes over one hundred episodes, made its fans aware of the ubiquity of mathematics in their daily lives.

[Falacci and Heuton have been recognized by the National Science Board with its Public Service Award, and they are the recipients of the Carl Sagan Public Understanding of Science Award.]

Biographical Sketch

Nicolas Falacci was born 1959 in Hyannis, Massachusetts. He attended the undergraduate film program at New York University's Tisch School of the Arts and received his B.F.A. in 1981. He sold his first feature-length screenplay in 1989 to Columbia Pictures and producer Joel Silver. He continued writing film projects for various studios and producers, mostly in the science fiction genre. That same year, while pursuing his favorite pastime of rock climbing in the Los Angeles area, he met Cheryl Heuton. Within a couple of years, the two of them moved to New York City, married, and began writing together.

Cheryl Heuton was born 1957 in Whittier, California. She grew up in the north San Diego area and attended the University of California, San Diego. She worked as a reporter for local weekly newspapers, then went on to become an editorial writer for the *Los Angeles Herald-Examiner* and later the *Long Beach Press Telegram*. She was nominated for a Pulitzer Prize for her series of articles about the mentally ill homeless.

As a writing team, Cheryl and Nick sold their first feature script to Warner Brothers, then went on to write film projects for New Line, MGM, Imagine, Sony, and HBO.

In 2003 they pitched CBS Television an idea for a television series centered around a mathematician. Production on *Numb3rs* began in 2004, and the show debuted on CBS in January 2005. A ratings success, *Numb3rs* was renewed for a total of six seasons. During those six years, Cheryl and Nick worked on the show as executive producers. Each season they wrote and supervised numerous episodes.

In early 2010 Nick directed the 119th and final episode of *Numb3rs*. The show continues to be broadcast in syndication in the United States and in numerous foreign countries, including the United Kingdom, Germany, Sweden, Australia, Japan, and Brazil.

Response from Nick Falacci and Cheryl Heuton

While we pursued a career in film and television writing, we both have a lifelong passion and interest in science. I, specifically, arrived at NYU intent on achieving a double major in film and…physics. Once I was informed of the required workload, especially the number of math classes I would have to take, I abandoned my scientific aspirations on the spot and focused my energy on filmmaking.

Cheryl and I discovered our shared love of science on our first date, when we realized we were both tremendous fans of James Burke's "The Day the Universe Changed". Though we never discussed a specific intention to write about scientists, we found ourselves naturally inclined to create characters with backgrounds in engineering, math, and science. One of our feature scripts was based on the true story of the Glomar Explorer, an amazing engineering feat by the Navy to salvage a Russian submarine three miles beneath the surface of the ocean. We developed a network television series about the extraordinary crash and accident analysts at the National Transportation Safety Board.

It was probably only a matter of time before Cheryl and I would be drawn to the world of mathematics and mathematicians. Both long-time skeptics, we were fascinated by the rigorous rational thinking of mathematicians. We were continually and delightedly surprised by the seemingly endless capacity of mathematics to help mankind understand the nature of the world and fuel the development of technology. With the help of the writing of various authors like John Allen Paulos, we discovered the unique way that mathematicians view the world. The more we explored and researched the topic, the more we were convinced that television audiences would find mathematicians as fascinating as we did.

Noting the popularity of crime dramas, specifically the ones based on forensic sciences, we felt that this type of storytelling could provide the opportunity to contrast and collide the thinking that goes on within a criminal investigation by police detectives with the extreme deductive reasoning of a mathematician. Our research led us to the real-life collision of math and police work: This happened by way of Kim Rossmo, a Canadian mathematician, homicide detective, and, more importantly, one of the pioneers of geographic profiling.

The notion of a mathematician solving major crime investigations was a reality. We had a strong suspicion that a lot of other people would be as fascinated by this unexpected, yet exciting confluence of disciplines as we were.

We are extremely honored to have been selected to receive the JPBM Communications Award. Neither of us, obviously, are mathematicians, and neither of us pursued our careers with any plan to popularize mathematics on network television. So much of what brought *Numb3rs* to fruition was, as mathematicians or cosmologists might say, a happy coincidence.

By creating *Numb3rs*, we have experienced two extremely rewarding accomplishments: the excitement of creating a successful television drama and the profound satisfaction of introducing an audience of ten to twelve million viewers each week to the elegance and power of mathematics and its direct impact on our daily lives.

We wish to acknowledge our utmost gratitude and appreciation for the people at CBS who believed in the show from the very beginning; the other *Numb3rs* writers who took on the daunting task of incorporating mathematics into a crime procedural drama week in, week out; our entire production staff, who embraced the notion and premise of the show; our enthusiastic researchers and our extraordinarily talented consultants who helped us navigate the world of mathematics; and, of course, Caltech, for its vigorous and wholehearted support of the show and for making us welcome on their campus.

MAA Prizes Presented in New Orleans

At the Joint Mathematics Meetings in New Orleans, Louisiana, in January 2010, the Mathematical Association of America (MAA) presented several prizes.

Gung and Hu Award for Distinguished Service

The Yueh-Gin Gung and Dr. Charles Y. Hu Award for Distinguished Service to Mathematics is the most prestigious award made by the MAA. It honors distinguished contributions to mathematics and mathematical education, in one particular aspect or many, whether in a short period or over a career.

Joseph Gallian is Distinguished Professor of Teaching and Professor of Mathematics at the University of Minnesota, Duluth. His service to mathematics takes form in three major activities: his work with Research Experiences for Undergraduates (REUs), his work with Project NExT, and his service to professional organizations and the mathematical community at large. He was one of the early proponents of undergraduates conducting mathematical research, and his REU at Duluth, which began in 1977, is widely regarded as the premier REU. The quality of the work at his REU is evidenced by the 150 papers by participants that grew out of their REU work and that have appeared in the *Journal of Algebra, Journal of Combinatorial Theory, Discrete Mathematics*, and others. In 2002 he was recognized by the Council on Undergraduate Research (CUR) with a Fellow Award given to members who have demonstrated sustained excellence in research with undergraduates.

Gallian has been involved with Project NExT since 1994, and became its codirector in 1998, assuming primary responsibility for many parts of the program and participating in developing the workshop program. He has coordinated Mathematics Awareness Month twice and served on more than forty national committees. He was a member of the CUR for eleven years, serving as chair of the mathematics and computer science division for part of that time. He has served as associate editor of *Mathematics Magazine* and *American Mathematical Monthly* and has directed or codirected five conferences. He has been a referee for forty journals.

Gallian has received teaching awards from the University of Minnesota Duluth, the Carnegie Foundation for the Advancement of Teaching, and the Mathematical Association of America (Haimo Award). He has received the MAA Trevor Evans and Carl B. Allendoerfer Awards and has been an MAA Polya Lecturer. He has served as vice president and president of MAA. He has also published more than one hundred articles in mathematical journals and other publications.

Haimo Awards for Teaching

The Deborah and Franklin Tepper Haimo Awards for Distinguished College or University Teaching were established in 1991. These awards honor college or university teachers who have been widely recognized as extraordinarily successful and whose teaching effectiveness has been shown to have had influence beyond their own institutions.

The 2011 Haimo Awards were presented to Erica Flapan, Karen Rhea, and Zvezdelina Stankova.

Erica Flapan has been teaching at the collegiate level for more than twenty-five years. She is currently Lingurn H. Burkhead Professor of Mathematics at Pomona College. Her outstanding teaching was recognized early; she received awards before earning her Ph.D., as a postdoctoral fellow, and as the recipient of the 2005 Irving Foundation Distinguished Faculty Fellowship for mentoring students of color at Pomona and the 2010 Southern California–Nevada Section Award for Distinguished College or University Teaching. She is best known for her dynamic, energetic classes that foster the participation of all students in the room. She was co-principal investigator of a grant to bridge mathematics and chemistry at Pomona, which helped to fund the creation of an Advanced Problem Solving course designed to give aspiring chemistry students needed mathematical strengthening. Her work in connecting mathematics and chemistry is further illustrated by her widely acclaimed book *When Topology Meets Chemistry: A Topological Look at Molecular Chirality* (Cambridge University Press and the Mathematical Association of America, 2000). Flapan teaches in several summer programs aimed at broadening interest in advanced mathematics among high school and undergraduate students and encouraging them to pursue graduate degrees. Her colleagues and students say that she serves both as a formal and informal advisor to many students; she is known as a tireless advocate and strong voice in support of diversity. Flapan received her B.A. from Hamilton College in 1977 and her Ph.D. from the University of Wisconsin at Madison in 1983. Her research interests are in low-dimensional topology and its applications to chemistry and molecular biology. She has recently coauthored a book (with James Pommersheim and Tim Marks) titled *Number Theory: A Lively Introduction with Proofs, Applications, and Stories* (John Wiley & Sons, 2010).

Karen Rhea has been a faculty member in colleges and universities for about thirty years, the last decade at the University of Michigan. In 1998, while a professor at the University of Southern Mississippi, she was awarded the MAA Louisiana-Mississippi Section's Award for Distinguished College or University Teaching of Mathematics. She is director of the introductory program at Michigan, which serves about 4,500 students annually in precalculus and the first year of calculus and which is widely viewed as one of the most successful programs of its scope in the country. At the beginning of each academic year she runs an intense training week for all new instructors in the introductory program. She has received praise for helping instructors find their own voices, and she continues to work with them after they leave the program. Many of the instructors she works with go on to become faculty elsewhere, broadening her influence on mathematics instruction. Her students say that her enthusiasm and eagerness to teach make class interesting and that she presents the material as clearly as possible. She has contributed to the general development of the calculus curriculum and to the discourse on how to teach calculus effectively through her work with the Harvard Calculus Consortium. Not only has her work made significant contributions to the content of the introductory courses at Michigan but it also has contributed to the development of a calculus curriculum that aims to get students actively involved in their own learning throughout the country. Rhea has given numerous talks and workshops, has served on the MAA Committee on Professional Development, and was recently appointed to the Committee on the Teaching of Undergraduate Mathematics.

Zvezdelina Stankova's goals in teaching are to develop students' ability to do independent thinking, no matter what the level of the student, from the middle and high school students in the Math Circles she works with through the senior undergraduate mathematics majors of Mills College and the University of California, Berkeley. As a full-time faculty member at Mills College, Stankova has also taught one course per year at the University of California, Berkeley, for eleven years, and, in 1998, she founded the Berkeley Math Circle, a weekly program for fifty Bay Area middle and high school students. She has been the Berkeley Math Circle's director and a frequent lecturer since the beginning. In addition, she has been directly involved in the creation of Math Circles in seven more cities in the United States and Canada and has contributed to the creation of Circles in twelve other cities. She was coeditor of the book *A Decade of the Berkeley Math Circle* (American Mathematical Society, 2008). With Paul Zeitz and Hugo Rossi, she cofounded the Bay Area Mathematical Olympiad, an annual competition among 250 students from forty-five schools in the Bay Area. Several of these students have gone on to be members of the U.S.A. Mathematical Olympiad team. Stankova has been actively involved in the U.S. participation in the International Mathematical Olympiad, including as an instructor in the training camps of the USAMO. Her students at every level are enthusiastic about her teaching and mentoring, indicating that her classes are challenging and fun. They rave about her teaching ability, her enthusiasm for mathematics, and her capacity to dramatically change their attitudes toward mathematics and their perceptions of their own mathematical abilities. Stankova was drawn to mathematics through her Math Circle in Bulgaria, and she consequently earned silver medals at the International Mathematical Olympiads. She completed a B.A./M.A. degree at Bryn Mawr in 1992 and received her Ph.D. in algebraic geometry from Harvard University in 1998.

Chauvenet Prize

The Chauvenet Prize recognizes a member of the MAA who has written an outstanding expository article. First awarded in 1925, the prize is named for William Chauvenet, who was a professor of mathematics at the United States Naval Academy.

Bjorn Poonen of the Massachusetts Institute of Technology received the 2011 Chauvenet Prize for his article "Undecidability in number theory", *Notices of the American Mathematical Society* **55** (2008), no. 3, 344–350. Hilbert's tenth problem (in his famous list of twenty-three problems posed in 1900) asks if there is an algorithm to decide whether, for a given polynomial $p(x_1,\ldots, x_n)$ in n variables with integer coefficients, there exist integers $a_1,\ldots, a_n$ such that $p(a_1,\ldots, a_n) = 0$. Although a proof that no such algorithm exists was found in 1970, "Hilbert's tenth", with its many ramifications and generalizations, continues to stimulate mathematicians today. For example, it is known that there is no algorithm to decide the existence of integer solutions to integer-coefficient polynomial equations in eleven variables, but what about just two variables? We don't know yet. Poonen's masterful exposition strikes a perfect balance between technicality (for the experts) and accessibility (for the rest of us). His story involves Turing machines, quantum computers, Diophantine sets, undecidability, prime-producing polynomials, and the Riemann hypothesis. It is a treat to find such a diversity of ideas wrapped up neatly into a single fascinating package. Poonen received his A.B. in mathematics and physics from Harvard University in 1989 and his Ph.D. in mathematics from the University of California, Berkeley, in 1994.

He taught at Berkeley until 2008, then moved to MIT. His research focuses on number theory and algebraic geometry, and he has also worked in combinatorics, probability, and computer science. He is the founding managing editor of *Algebra and Number Theory.*

Euler Book Prize

The Euler Book Prize is given to the author(s) of an outstanding book about mathematics. Mathematical monographs at the undergraduate level, histories, biographies, works of mathematical fiction, and anthologies are among those types of books eligible for the prize. The prize was given for the first time in 2007, the 300th anniversary of the birth of Leonhard Euler.

TIMOTHY GOWERS has been awarded the 2011 Euler Book Prize for his editorial work on *The Princeton Companion to Mathematics* (Princeton University Press, 2008). The book is a vast compendium of mathematical information in the form of essays by experts on a wide variety of fields, in most cases bringing the reader up to date on developments in recent decades in a way that nonexperts can understand and appreciate. The Committee understood that the book is the work of many, but singled out Gowers because of his extraordinary achievement in putting this whole volume together (over 1,000 pages of text) and also for writing a beautiful 76-page introduction, as well as 68 of the 288 individual entries. The organization is thematic, with sections on the origins of modern mathematics, mathematical concepts, the various branches of the subject, the big problems, biographical essays, and a section on the influence of mathematics on other fields. The book has something for everyone who has any interest in mathematics. Many sections can be read with great benefit and considerable pleasure by mathematical amateurs and students. Overwhelmed as we are in the twenty-first century by the enormous size of mathematics, the professional mathematician can benefit from finding out what colleagues are doing in branches of mathematics that did not exist when many of us were in school. Anyone who wonders about "mirror symmetry", "quantum groups", "vertex operator algebras", "automorphic forms", or "Ricci flow", topics referred to in the current mathematical literature or even in the newspapers, can find help here. Gowers's early research was in the geometry of Banach spaces. He solved several old problems in the area, some posed by Banach himself. More recently, he has worked in additive combinatorics: his new proof of Szemerédi's theorem has been particularly influential. In 1996 he was awarded a European Mathematical Society prize, and he received a Fields Medal in 1998.

David P. Robbins Prize

This prize was established in memory of David P. Robbins by members of his family. Robbins, who died in 2003, received his Ph.D. in 1970 from MIT. He was a long-time member of the Institute for Defense Analysis Center for Communication Research and a prolific mathematician whose work (much of it classified) was in discrete mathematics. The prize is for a paper with the following characteristics: it shall report on novel research in algebra, combinatorics, or discrete mathematics and shall have a significant experimental component, and it shall be on a topic which is broadly accessible and shall provide a simple statement of the problem and clear exposition of the work. This prize is awarded every three years.

The 2011 David P. Robbins Prize was awarded to MIKE PATERSON, Warwick University; YUVAL PERES, Microsoft Research, University of Washington, University of California, Berkeley; MIKKEL THORUP, AT&T Labs; PETER WINKLER, Dartmouth College; and URI ZWICK, Tel Aviv University, for their innovative work on two papers: "Overhang", *American Mathematical Monthly* **116**, January 2009, and "Maximum Overhang", *American Mathematical Monthly* **116**, December 2009. The two papers together solve, to within a constant factor, the classic problem of stacking blocks on a table to achieve the maximum possible overhang, i.e., reaching out the furthest horizontal distance from the edge of the table. The January paper was written by Paterson and Zwick, and the December paper was written by all five authors. The January paper proves the surprising result that n blocks can be (cunningly) stacked using suitable counterbalancing to achieve an overhang proportional to $n(1/3)$. (Many people have assumed that the overhang of about log n, given by the standard calculus exercise, is optimal.) The December paper gave a complementary argument, showing that an overhang proportional to $n(1/3)$ is, in fact, the largest possible for any balanced stack. The papers describe an impressive result in discrete mathematics; the problem is easily understood and the arguments, despite their depth, are easily accessible to any motivated undergraduate.

Certificates of Meritorious Service

Each year the MAA presents Certificates of Meritorious Service for service at the national level or for service to a section of the MAA. Those honored in 2011 are: JOSEPH GALLIAN (University of Minnesota, Duluth), North Central Section; JOHN HAGOOD (Northern Arizona University), Southwest Section; ALLEN HIBBARD (Central College, Pella, Iowa), Iowa Section; JOSEPH MALKEVITCH (York College, CUNY), Metro New York Section; JENNY MCNULTY (University of Montana), Pacific Northwest Section; and GERALD PORTER (University of Pennsylvania), Eastern Pennsylvania and Delaware Section.

AWM Awards Given in New Orleans

The Association for Women in Mathematics (AWM) presented three awards at the Joint Mathematics Meetings in New Orleans, Louisiana, in January 2011.

Shafer Prize

The Alice T. Schafer Prize for Excellence in Mathematics by an Undergraduate Woman was established in 1990. The prize is named in honor of Alice T. Schafer, one of the founders of AWM and one of its past presidents. Schafer passed away in September of 2009.

The 2011 Schafer Prize was awarded to Sherry Gong of Harvard University. A senior, her performance in her classes has been outstanding. She began with Harvard's famous problem-solving class, in which she achieved a score above 100, and since her sophomore year she has taken numerous graduate mathematics courses, earning A's in all of them. Her recommenders were universally amazed by her ability to master sophisticated mathematics rapidly, whether in a class or independently mastering background for a research project. Gong has been involved in four different research projects and is the author or coauthor of three papers. She spent summer 2008 at the Duluth REU researching cyclotomic polynomials; her paper was published in the *Journal of Number Theory*. In 2009 she worked with a group at MIT that did research on computing the dimension of the space of characters of the Lie algebra of Hamiltonian vector fields on a symplectic vector space; their work will be published shortly. She and an economist have published a paper in *Integers* on congruence conditions characterizing primes. Most recently she did research on periodic cyclic cohomology of group algebras of torsion free groups at Vanderbilt. As a high school student, Gong medaled repeatedly in the International Mathematical Olympiad, winning a gold medal in 2007. After entering college, she returned to the Mathematical Olympiad Summer Program as a grader and also served as a grader for the Mathematical Olympiad of Central America and the Caribbean. In 2010 Gong served as one of the coaches for the USA team for the Girls' Mathematical Olympiad in China. Five of the eight girls on the team won gold medals, and the head coach describes Gong as "a young lady with a great heart, thoughtful and gentle", who pushed the students with "acute mathematical insights and inspiring personality". Her mentors describe a remarkable young mathematician, exceptionally talented and original, with one commenting she is already "comparable to some of the best mathematical minds I know".

Louise Hay Award

Established in 1991, the Louise Hay Award for Contributions to Mathematics Education recognizes outstanding achievements in any area of mathematics education. Louise Hay was widely recognized for her contributions to mathematical logic and her devotion to students.

The 2011 award was presented to Patricia Campbell of the University of Maryland, College Park, in recognition of her leadership and contributions in research, teaching, and service to mathematics education. Throughout her career, she has engaged and challenged her students, university colleagues, professional colleagues, school administrators, and classroom teachers to advance the teaching of mathematics. As a leader in the field of mathematics education, Campbell is esteemed especially for her contributions to the teaching and learning of mathematics in urban settings and for working in schools that serve predominately minority populations from low-income backgrounds. She has worked in schools to improve student learning for two decades. From 1989–1997, she led Project IMPACT, a professional development effort that demonstrated the feasibility of school-wide mathematics reform, supplementing summer professional development with in-school mathematics specialists in order to increase achievement in schools with predominately minority populations. She has collaborated in developing the MARS Project (Mathematics: Application and Reasoning Skills), which has addressed a complex set of problems in Baltimore public schools, targeting poor student achievement through system-wide teacher development in mathematics. With Campbell as the principal investigator, the MARS Project was awarded a five-million-dollar grant through the NSF Local Systemic Initiative program.

Through her current research, Campbell continues to pursue her efforts to ensure quality education for all children. As part of the research component of the Mid-Atlantic Center for Mathematics Teaching and Learning, she leads a research project that is poised to assess the impact of Grade 4–8 teachers' knowledge of mathematics and mathematics pedagogy on student achievement. Her current work in the area of mathematics leadership at the elementary level builds on her prior efforts and her evaluation of the work and role of elementary mathematics specialists and will contribute significantly to the research in this area. She has been a member of the board of directors of the National Council of Teachers of Mathematics and has served many roles within the organization. As a teacher, mentor, and colleague, she has gained the appreciation of her students and colleagues for her commitment, skill, and energy for the cause of mathematics education and for challenging them to think more deeply about the tough issues they must confront.

M. Gweneth Humphreys Award for Mentorship of Undergraduate Women in Mathematics

This award is named for M. Gweneth Humphreys (1911–2006). Humphreys graduated with honors in mathematics from the University of British Columbia in 1932, earning the prestigious Governor General's Gold Medal at graduation. After receiving her master's degree from Smith College in 1933, Humphreys earned her Ph.D. at age 23 from the University of Chicago in 1935. She taught mathematics to women for her entire career. This award, funded by contributions from her former students and colleagues at Randolph-Macon Woman's College, recognizes her commitment to and her profound influence on undergraduate students of mathematics.

The inaugural award was presented to Rhonda Hughes of Bryn Mawr College in recognition of her outstanding mentoring of undergraduate women in mathematics.

Hughes is a dedicated and motivating teacher at all undergraduate levels, from basic calculus to advanced PDEs. She ably identifies research topics that match the students' interests and abilities. Especially noteworthy is her encouragement of students whose potential had previously gone unnoticed, even by the students themselves. The results bear witness to the strength of Hughes's belief in her students, to the force of her personality, and to the contagious quality of her enthusiasm for mathematics. Particularly stunning are the accounts of students who began college convinced they were "bad at mathematics", but who were charmed by Hughes into taking calculus with her. They take more mathematics, still not convinced of their own abilities but warming to the subject. The students end up majoring in mathematics, doing a research project, and proceeding to graduate programs and careers in mathematics. The overall numbers are striking, and the mathematics program at Bryn Mawr has flourished in the time Hughes has been there, with large increases in majors.

Hughes has brought her knowledge of how to encourage young women in mathematics to the national level. She has served AWM in many ways, including as president in 1987–88. She has also served often as organizer, panelist, and speaker in activities aimed at increasing the participation of women and minorities in mathematics. After having identified the challenges that some young women face, especially minority women, Hughes began programs to help negotiate crucial transitions, the first one being from undergraduate course work to the math major. In 1998 she and Sylvia Bozeman of Spelman College created the ongoing EDGE program, which addresses the transition from college to graduate school with marked success.

2011 Mathematics Programs That Make a Difference

Each year, the AMS Committee on the Profession (CoProf) selects outstanding programs to be designated as *Mathematics Programs That Make a Difference*. For 2011 the programs honored are the Center for Women in Mathematics at Smith College and the Department of Mathematics at North Carolina State University.

CoProf created the Mathematics Programs That Make a Difference designation in 2005 as a way to bring recognition to outstanding programs that successfully address the issue of underrepresented groups in mathematics. Each year CoProf has identified two exemplary programs that:

1. aim to bring more individuals from underrepresented minority backgrounds into some portion of the pipeline beginning at the undergraduate level and leading to an advanced degree in mathematics or retain them in the pipeline;
2. have achieved documentable success in doing so; and
3. are replicable models.

Previously designated Mathematics Programs That Make a Difference are: the graduate program at the University of Iowa and the Summer Institute in Mathematics for Undergraduates/Research Experience for Undergraduates at Universidad de Puerto Rico, Humacao (2006); Enhancing Diversity in Graduate Education (EDGE) and the Mathematical Theoretical Biology Institute (2007); the Mathematics Summer Program in Research and Learning (Math SPIRAL) at the University of Maryland and the Summer Undergraduate Mathematical Science Research Institute at Miami University (Ohio) (2008); the Department of Statistics at North Carolina State University and the Department of Mathematics at the University of Mississippi (2009); the Department of Computational and Applied Mathematics at Rice University and the Summer Program in Quantitative Sciences, Harvard School of Public Health (2010).

The selection committee for the 2011 Mathematics Programs That Make a Difference consisted of: Matthias Heinkenschloss, Bryna Kra, Susan Loepp (chair), Rick Miranda, Richard Tapia, and Eric Tchetgen Tchetgen.

Below are CoProf's citations, followed by brief descriptions of the programs prepared by *Notices* staff.

Center for Women in Mathematics, Smith College

Citation

Be it resolved that the American Mathematical Society and its Committee on the Profession recognize the Center for Women in Mathematics and the Center's Postbaccalaureate Program, Smith College, for its significant efforts to encourage women to continue in the study of mathematics.

The program's supportive and encouraging environment has resulted in remarkable success. Talented undergraduate women, who have discovered their passion for mathematics late in their undergraduate careers or after they have obtained a degree in a field other than mathematics, are recruited for the program. The program's participants take courses, attend weekly colloquia, and participate in research projects, resulting in a valuable and effective community experience. Graduates of the program are accepted for and succeed in mathematics graduate programs at an astonishingly high rate. The program has made a remarkable contribution to the national effort to produce more women Ph.D.s in the mathematical sciences.

The AMS commends the members of the Department of Mathematics at Smith College for their high level of commitment and their successful efforts to improve the diversity of the profession of mathematics in the United States.

Program Description

After earning a bachelor's degree in theater, with a minor in mathematics, at Smith College in 2005, Samantha Oestreicher spent two years working in the costume shop in a college theater department and freelancing in costuming and backstage work in theaters in New England. Fast-forward to the fall of 2010, and she is starting her third year of graduate school in mathematics at the University of Minnesota. Her performance is outstanding: she passed her preliminary examinations early, participates actively in a research seminar, and is an exemplary teaching assistant. She held an internship at Los Alamos National Laboratory and now has a research assistantship on a grant from the National Science Foundation (NSF).

That Oestreicher made her way back to mathematics and then excelled so quickly is an indication of her talent and motivation. Helping that talent to flower and that motivation to strengthen was the postbaccalaureate program at the Center for Women in Mathematics at Smith College. Founded in 2007 with funding from the NSF, the center serves women who discover their desire to do mathematics only after having exited the traditional educational track and who need to build up or refresh their backgrounds to be able to enter graduate school. "If it wasn't for Smith's program and the NSF funding, then I would never have made it as far as I have," Oestreicher said. "Smith's program gave me a totally fresh start and the opportunity to do math for a living."

The center's postbaccalaureate program, the first in mathematics in the United States, attracts students from all over the country. In this one-year program, students take upper-level courses in the Smith mathematics department and also have the opportunity to take courses at Amherst, Hampshire, and Mount Holyoke colleges and the University of Massachusetts at Amherst. In addition, the center designed two special courses for the students. The first, Dialogues in Mathematics, introduces the students to the culture and profession of mathematics through various means: meetings with colloquium speakers to talk about mathematics and mathematical careers, preparation for the GRE examination, discussions of strategies for graduate school, and examination of issues pertinent to the mathematical community. In the other course, the Advanced Mathematics Seminar, the students work in small groups with faculty, generally on a research topic. The topics have included graph theory, combinatorics, and computational geometry, as well as biological and medical applications of mathematics. The center's NSF grant provides tuition and a living stipend for some of the postbaccalaureate students who are U.S. citizens or permanent residents.

Among the center's other activities is a Junior Program for undergraduate women from outside Smith College who want to spend one or both semesters of their junior year in a mathematically intense environment. The NSF grant provides financial aid to participants who are U.S. citizens or permanent residents. In addition, the center hosts an annual conference called Women in Mathematics in New England (WIMIN). In 2009 the conference was one of the Regional Undergraduate Mathematics conferences sponsored by the Mathematical Association of America and attracted 105 participants and twenty-four student talks. Most of the center's postbaccalaureate students attend the Joint Mathematics Meetings, the Joint Statistical Meeting, and/or the Hudson River Undergraduate Mathematics Conference.

The center is well integrated into the Smith College mathematics department and capitalizes on some of the department's assets, such as the Math Forum, a welcoming and comfortable space where students and teachers gather for conversation, study, and relaxation, as well as tea and cookies. At the same time, the center has had a positive, revitalizing influence on the department. By bringing together a group of active, serious, and motivated women, the center inspires Smith mathematics majors and increases the level of activity in the department. The center has fostered a vertically integrated community that is developing into a focal point for women mathematicians in New England. It is also having a national impact, as other departments around the country establish programs emulating those at the Smith center.

Data about the center's students demonstrate its success. By the fall of 2010, twenty-four women had completed the postbaccalaureate program. Of those, seventeen entered graduate school in the mathematical sciences immediately after completing the program, and five will have entered after brief deferrals; almost all of these students went to Group I or II departments. Only one student has dropped out of the graduate program. Ten new students joined the program in the fall of 2010.

The September 2007 issue of the *Notices* carried an Opinion column by the directors of the then newly created Center for Women in Mathematics. The final paragraph of their piece captures well the philosophy behind the center: "Yes, we marvel at the prodigies whose unwavering interest and aptitude in mathematics are evident from the start. But we must also open our minds to the vast numbers of students—women and minorities (and others from all walks of life)—who can be mathematicians but may not make that choice until late in their college careers or after."

Program directors: Ruth Haas and Jim Henle

Website: `http://www.math.smith.edu/center`

Department of Mathematics, North Carolina State University

Citation

Be it resolved that the American Mathematical Society and its Committee on the Profession recognize the Department of Mathematics, North Carolina State University, for its significant efforts to encourage students from underrepresented groups to continue in the study of mathematics.

The department's exceptional commitment to its students includes early research opportunities, financial support provided by grant funding, and extraordinary faculty mentoring. In addition, the department offers impressive undergraduate programs and postdoctoral opportunities. Since 1999 the department has produced almost 13 percent of all African-American Ph.D.s nationally. The department at North Carolina State University has made, and continues to make, a remarkable contribution to the national effort to produce more minority Ph.D.s in the mathematical sciences.

The AMS commends the members of the Department of Mathematics at North Carolina State University for their high level of commitment and their successful efforts to improve the diversity of the profession of mathematics in the United States.

Program Description

In how many different ways can a mathematics department succeed? Count the successes of the Mathematics Department at North Carolina State University, and you will have a good approximation. The recipient of last year's AMS Award for an Exemplary Program or Achievement in a Mathematics Department, NC State has excelled in just about everything mathematics departments do. This time, the department was chosen for the Programs That Make a Difference distinction for one particular achievement: its extraordinary record in serving students who have traditionally been underrepresented in mathematics, especially African-Americans.

The numbers speak for themselves. According to the Annual Survey of mathematics departments that the AMS and collaborating organizations prepare each year, Group I and II mathematics departments produced seventy-three African-American Ph.D.s during the academic years 1999–2000 to 2008–2009 (this counts U.S. citizens only). Nine of these seventy-three, or 12.3 percent, graduated from the NC State mathematics department. And NC State is not an enormous producer of mathematics Ph.D.s overall; in that period, it accounted for just 2 percent of the total number of Ph.D.s produced by Group I and II departments. NC State's impressive record has continued, with a total of four additional African-Americans receiving their Ph.D.s in the department in the 2009–2010 and 2010–2011 academic years. When it comes to female African-American Ph.D.s in mathematics, NC State might very well be the top producer in the nation: In the same nine-year time period as above, twenty-six mathematics doctorates were awarded to female African-Americans by Group I and II departments, and six, or nearly a quarter, earned their degrees at NC State.

What accounts for this achievement? One factor is the department's commitment, starting more than a decade ago, to recruiting domestic students into its graduate program. Today 96 percent of its graduate students are U.S. citizens or permanent residents (and 49 percent are women). One of the strategies the department uses is an annual recruiting weekend, in which students are invited to the campus to learn about the graduate program. Through its long-standing contacts with historically black colleges and universities, the department identifies strong African-American applicants, encourages them to apply, and invites them to the recruiting weekend.

Another factor is the substantial support the department offers to students. An S-STEM grant from the National Science Foundation provides financial support for all of the department's African-American students during the first two years in graduate school. For students whose backgrounds need additional shoring up, extra mentoring is available. In addition, the department has a Research Experience for Early Graduate Students (REG) program, which provides stipends for first- and second-year graduate students to work with faculty on summer research projects. Unlike other graduate students, S-STEM students are eligible for two summers of REG participation.

A third factor in the department's success lies in the broad and flexible nature of its graduate program. Students have a wide variety of research areas to choose from, and many opportunities are available to work on interdisciplinary problems with researchers in other university departments, in industry, and in government laboratories. The mathematics department wanted the structure of its qualifying examinations to reflect the increasingly interdisciplinary nature of mathematical research, so it created a set of fifteen examinations, and each student must complete three of them.

The final, and perhaps most important, factor is the faculty's commitment to students. The department's success with students from underrepresented groups is not due to outsized efforts by a small number of faculty members. Indeed, the thirteen African-Americans who received their Ph.D.s in the department during the academic years 1999–2000 to 2010–2011 were spread among twelve different advisors.

The department's commitment to underrepresented students does not stop with the graduate program. The department stands out nationally as a predominantly white institution in which a large number of African-Americans earn bachelor's degrees in mathematics. Alongside its Research Experiences for Undergraduates program, the department has started REU+, which is geared toward students from historically black colleges and universities. At the postdoctoral level, the department has spearheaded a new national program called the Alliance for Building Faculty Diversity in the Mathematical Sciences, which offers postdoctoral fellowships to new Ph.D.s from traditionally underrepresented groups. Fellows will typically spend two years in one of the alliance institutions and one year in an NSF-funded mathematics institute.

After NC State was chosen for the AMS Exemplary Program Award, department head Loek Helminck was interviewed for an article in the *Notices*. Asked how the department managed to achieve so much, he replied: "You have to create a culture in which people believe this is the right direction for the department so that there is broad faculty participation. You start with a few people and you show that a program is successful, then one by one you have faculty start to participate…They can totally change their minds."

Department head: Loek Helminck
Website: `http://www.math.ncsu.edu`

2011 Award for an Exemplary Program or Achievement in a Mathematics Department

The Award for an Exemplary Program or Achievement in a Mathematics Department was established by the AMS Council in 2004 and was given for the first time in 2006. The purpose is to recognize a department that has distinguished itself by undertaking an unusual or particularly effective program of value to the mathematics community, internally or in relation to the rest of society. Departments of mathematical sciences in North America that offer at least a bachelor's degree in mathematical sciences are eligible. Through the generous support of an anonymous donor, the award carries a cash prize of US$5,000.

The award is presented by the AMS Council acting on the recommendation of a selection committee. For the 2011 award, the members of the selection committee were: Carlos Castillo-Chavez, Amy Cohen (chair), William Jacob, and Philip Kutzko.

The previous recipients of the award are Harvey Mudd College (2006), the University of California, Los Angeles (2007), the University of Iowa (2008), the University of Nebraska, Lincoln (2009), and North Carolina State University (2010).

The recipient of the 2011 Award for an Exemplary Program or Achievement in a Mathematics Department is the Math Center at the University of Arizona. What follows is the selection committee's citation.

Citation

The American Mathematical Society is pleased to recognize the undergraduate Math Center at the University of Arizona with the 2011 Award for an Exemplary Program or Achievement by a Mathematics Department. The Math Center is a national leader in the effort to recruit, mentor, and graduate undergraduate math majors, especially those students from backgrounds traditionally underrepresented in mathematics. Since 2004 the Math Center has

- increased the total number of majors by 69 percent, from 281 to 474,
- increased the total number of female majors by 72 percent, from 102 to 175,
- increased the total number of majors from underrepresented minority groups by 125 percent, from 48 to 108, with the result that about 25 percent of math majors at the University of Arizona come from these groups.

This impressive achievement comes about not as a result of any substantial change in the structure of the math major but rather by the tireless recruitment activities of the Center, the range of activities and opportunities provided to UA math majors, and the collegial and welcoming environment created by the Center. Recruiting is done on a personal basis. There is a yearly high school calculus class visitation project in which a team consisting of a faculty member, a graduate student, and an undergraduate student visits AP calculus classes at local high schools. There is an MAA-[Mathematical Association of America] funded five-day calculus workshop for incoming students enrolled in calculus. And, perhaps most remarkably, there is the personal effort of Professor William Vélez, Associate Head for Undergraduate Affairs for the UA Math Department, who has, for the last fifteen years, conducted a program of making twenty-minute individual appointments with every minority student enrolled in calculus. Once a student becomes a math major, the Center offers a variety of programs, ranging from a substantial undergraduate research program to a highly popular math club to encourage and support the student's progress toward success. Finally, a concerted effort is made to ensure that the student is informed of the full range of opportunities available in academe, government, and industry to someone with computational skills. Indeed, since 2007 the program has broadened its scope to target students in other fields who might be interested in a double major, thus increasing the fraction of double majors from 30 percent to 40 percent over the last three years. The numbers show that this program is succeeding: over the last two years, about a quarter of UA math majors have gone on to graduate programs, and almost 20 percent have gone into middle and high school teaching. As further evidence of this success, the May 2010 outstanding undergraduate researcher and the December 2010 outstanding graduating senior in the College of Science were both math majors.

The small numbers of women and minorities at all levels in our profession, together with an increasing alienation of all U.S. students from the mathematical sciences, is well documented and is cause for concern both on moral and on practical grounds. In that context, the Math Center at the University of Arizona stands out both as an example of what can be done and as a model of what must be done, the very definition of an exemplary program.

About the Cover

Wavelets in image compression

This month's cover was inspired by the article "Discrete wavelet transformations and undergraduate education", by Catherine Bénéteau and Patrick Van Fleet. The February 2008 issue of the *Notices* contained an article by David Austin on the JPEG format, including at the end a few remarks about the newer JPEG2000 specification, which is based on wavelets. This month's cover illustrates what the first step in a simple wavelet compression would be like, when applied to Austin's cover for the 2008 issue. The effect is much like one of the figures in the article by Bénéteau and Van Fleet.

In making the cover, we considered several sophisticated options, but in the end chose to apply Haar wavelets for a simple visual effect. Wavelets are complicated, and it would have been almost impossible to make a single cover image that showed clearly something more subtle. There is no natural way to interpret differences of colors graphically. Magnitudes in the difference images have been shifted and magnified in order to make the process more visible, and this has introduced some apparently random noise.

Although in the end we didn't use anything fancy, we found the book *Ripples in Mathematics*, listed in the Bénéteau and Van Fleet article, as well as the article "Factoring wavelet transforms into lifting steps" by Ingrid Daubechies and Wim Sweldens (*Journal of Fourier Analysis and Applications*, 1998), to be valuable guides to the lifting algorithm.

—Bill Casselman
Graphics Editor
(notices-covers@ams.org)

Arizona's Math Center Wins AMS Award

Allyn Jackson

Just take one more math course: That's what William Velez tells as many students as he can. He doesn't say this only to those who might major in mathematics. And he doesn't say it only to students he meets in person. Velez, a professor of mathematics and director of the University of Arizona's Math Center, sends out email messages to, for example, all students who got an A or B in precalculus in the previous semester, maybe 150 students in all. He congratulates them on their success, explains why mathematics is useful in many professions, and repeats the mantra: Just take one more math course. He doesn't get many immediate responses. "But four months later," he says, "I'll get a message from one of the students saying, 'I enjoyed that math course, can I come and talk to you?'"

This high-tech yet personalized recruitment of students has become the hallmark of the Math Center. As the focal point for the undergraduate mathematics major at Arizona, the center has grown organically as an expression of how the mathematics department sees its mission within the university. When the center gets students to take more mathematics, it can count on excellent instruction delivered by a faculty committed to undergraduate education. The Math Center's impact is reflected in an arresting statistic: the number of mathematics majors at Arizona has more than doubled since 2004. For its dedication to students and its outstanding success in increasing the number of mathematics majors at the University of Arizona, the Math Center was awarded the 2011 AMS Award for an Exemplary Program or Achievement in a Mathematics Department.

Tables, Chairs, and a Killer Database

> We usually gather in the Math Center to do homework in groups. My musician girlfriend came there one time and was shocked to find a group of real analysis students (myself included) yelling, screaming, and throwing things about a calculus problem... I told her this happened all the time.
>
> *—Jonathan Cain, mathematics major, class of 2011*

In terms of space and budget, the Math Center is nothing glorious. It's a big room on the second floor of the mathematics building, outfitted with blackboards, several tables, and plenty of chairs. There is a small office for the coordinator of the center, Laurie Varecka, a former graduate student in the department who has taught beginning courses there and used to coordinate the teaching assistant program. Asked about how the center is funded, department head William McCallum says with a chuckle, "It doesn't really have any funding." The center has no outside grants; it is supported entirely by the mathematics department budget, mainly through dedicated staff and faculty time. It has a small pot of money for things like pizza for students after talks sponsored by MathCats, the club for undergraduate mathematics majors.

The Math Center was started around twenty years ago by an adjunct faculty member, Robin Steinberg (now at Pima Community College), who was given the task of looking after the department's majors. A few years later, McCallum became director of the center. "I had been making noises in the department about how we needed to do something much more systematic and organized for the math majors, that we should really create a home for them," McCallum said. "And I was rewarded by being given the job of doing that!" He began to develop the center as a place where mathematics majors could drop in to ask questions and get advice. The idea was to supplement the guidance of faculty advisors—who see advisees perhaps once a semester—by providing contact on a day-to-day basis and help with things like navigating the university bureaucracy. The center became a forum where students could gather and talk to each other or to professors who passed through. It built a small library of mathematics books for students to peruse, and it began holding events, such as celebrations of Mathematics Awareness Month.

In August 2003 Velez was appointed Associate Head for Undergraduate Programs and thereby

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director of the Math Center. He brought with him his long experience and successful track record in recruiting and mentoring mathematics students, particularly those from minority groups that are underrepresented in the field. The center directorship gave him a platform from which to greatly expand his recruitment work—and it gave him the time to do it: With release time from the department, he teaches only one course per year. Around the same time that he started as director, the department had decided to create a database to get a better picture of the population of math majors—the number of minority students, the number of women students, the number of freshmen, how many and which courses they took, what grades they received, and so forth. The Math Center built this database, which has become central to Velez's recruitment efforts. Much of the information was available within the mathematics department, but at some point the Math Center was also able to tap into an extensive data system run centrally by the university. "We can mine the university data to an extent unthinkable before," Velez said.

As an example of what he is able to do with the database, he described a project he carried out in December 2010. He went through the database to identify students who had gotten As or Bs in second-semester calculus. He then made a list of which ones had not subsequently enrolled in a math course. He sent out email messages to each of those students, encouraging them to continue taking math and suggesting what courses they might take next. Projects such as this—zeroing in on a particular set of students and crafting an email suited to their situation—are of course very time-consuming. Velez is setting them up to run automatically in the future.

One much larger such mailing has been automated and has run for the past few years. For this mailing, the Math Center obtains the names and email addresses of all students who have been accepted to the University of Arizona, about 4,000 or 5,000 students. In March each year it sends messages to these individuals suggesting that they study mathematics and explaining the value of the mathematics major. The messages include a link to the "Math Moments" posters on the AMS website. In addition, for the past couple of years, Velez has added a "hook" to the message: He offers to send a document called "Resources for Calculus Students" that he wrote a few years back. Originally intended for minority students, the document is now given to all calculus students at Arizona. This hook does not elicit a great number of replies, but enough students have responded that Velez is convinced his message is being read.

A few statistics bear out the success of the Math Center's approach. In 2004 the department had 281 mathematics majors—already a respectable number that points to the emphasis the department puts on undergraduate education. By early 2011 the number of majors had more than doubled, to just over 600. The number of female majors also doubled, from about 100 to about 200. But perhaps most impressive of all is the increase in the number of majors from underrepresented minority groups: In 2004 there were 48 such majors, and by early 2011 the number had risen to 125, an astounding 160 percent increase. Today about 20 percent of math majors at the University of Arizona come from these groups. The department also has about 600 math minors. These results came about not through changes in the major program but rather through the Math Center's systematic recruitment, which sends the message that mathematics welcomes many kinds of people with diverse interests and goals.

Photo courtesy of the Dept. of Mathematics, University of Arizona.

Associate professor and faculty advisor Ted Laetsch conversing with a group of undergraduate math majors.

Inviting Students into Mathematics

> The Math Center provides support to all undergraduate math majors at the UA. From the moment I set foot on campus during orientation to my junior year in college, I have always felt the presence of the mathematics community on campus.
>
> *—Lynette Guzman, mathematics major, class of 2012*

Born in Tucson to parents who were natives of Mexico, Velez received his degrees at the University of Arizona. When he joined the Arizona mathematics department as a faculty member in 1977, he was the only Chicano—and he is still the only one (though there are now two Hispanic faculty members). In the late 1980s he started to notice the small number of mathematics majors coming from minority groups underrepresented in mathematics—there was perhaps one such student graduating every other year. And this was in Arizona, a state with a large population of Chicanos, Hispanics, and Native Americans. Velez began

Photo courtesy of the Dept. of Mathematics, University of Arizona.

Jaron Gubernick, president of the undergraduate mathematics club MathCats, juggling at an event sponsored by the Math Center.

directly contacting students from these groups and encouraging them to take mathematics. For the last fifteen years he has made twenty-minute individual appointments with every minority student enrolled in calculus.

Velez takes the issue of underrepresented minorities very personally. "I am part of the Chicano community here," he says. "I interact with this community a lot. It's impressive, the cleverness of these people to survive. If we could tap that cleverness and entice children to come into mathematics, we would see wonderful mathematicians." At the same time, this community is beleaguered by social problems such as drugs, violence, and poor schools. To mathematicians who might be moved to help such students, these problems can seem like insurmountable obstacles. But students who make it to the university have passed the first hurdle, Velez says. What they need is explicit encouragement—an "invitation", as he puts it—to come into mathematics and to learn that it is a fulfilling and valuable area to study. He took on the directorship of the Math Center as a way to institutionalize his concern for minority students. "When I retire, I don't want concern for minority students to retire with me," he says.

In fact, Velez's brand of "aggressive advising" works for all students, not just those from underrepresented minorities, as is evident from the 50 percent increase in the total number of math majors one year after he became director of the Math Center. He believes mathematics faculty need to be much more proactive in conveying to students the excitement of mathematics and the career opportunities the major opens up. "How do students know there is a math major if we don't tell them?" he asks. In addition, he notes, the prevailing attitude in many departments is that the mathematics major has one goal only: to prepare students for graduate school in mathematics. Velez thinks that that attitude must change. "Mathematics departments don't understand how mathematics has invaded all of the sciences," he says. "This opens opportunities for math majors to go to graduate school in other areas…. Also, mathematicians don't understand what students can do with a math major when they enter the workforce. This prevents mathematicians from being more aggressive in recruiting and advising students.''

An active member and former president of SACNAS (Society for the Advancement of Chicanos and Native Americans in Science), Velez has lectured widely on undergraduate education, mentoring, and increasing diversity in mathematics and science. He has also written many articles on these subjects, including several for the *Notices* (the most recent was "Not business as usual'', an Opinion column in the May 2003 issue). He has received many honors for his work mentoring students, including the President's Award for Excellence in Science, Mathematics and Engineering Mentoring in 1997.

A Committed Department

> The Math Center builds our community and makes people come together…. As students, we look past the competition we have with each other and realize that we are all there for the same reason and that we can realize our goals together.
>
> — *Jonathan Cain, mathematics major, class of 2011*

Last fall Math Center coordinator Laurie Varecka was making a course substitution for a math major who was to graduate at the end of the year, and she decided to check his completed degree requirements. She noticed that he was missing one credit in one area. "That would have been enough to prevent him from graduating," she says. She quickly got in touch with him. As it turned out, he was doing his independent study with a departmental faculty advisor, who was very willing to be flexible and increase the number of hours for the course to help the student graduate on time. "That's the kind of thing that makes me happy, when we are able to catch things like that and help students," she says. Such mindfulness shows students the Math Center is there for them and will go the extra mile to help them succeed. "A lot of the work [of the Math Center] is just keeping track of the students," McCallum says. "Laurie sends out emails with events they might be interested in. When the times of the year arrive when students are supposed to register for classes, she makes sure that they meet their advisors. A lot of it is just paying attention and making them feel that someone is interested in their welfare."

This attention starts early, with an orientation session for freshman and transfer math majors, held the Saturday before classes begin. At the session, students learn about the math major and the options within it and also find out about resources available through the Math Center. Over lunch the students can have informal conversations with McCallum, Varecka, Velez, and the other faculty members who attend. As students progress through the major, they benefit from a department in which attention to undergraduate education is the norm. They also have many opportunities to work with department faculty on "research experience" projects, for which research assistantship support is available. In fact, Velez says that there are more faculty willing to do research with undergraduates than there are undergraduates wishing to do research. Many math majors participate in research experience programs in other disciplines, such as biology, chemistry, and geosciences, where their mathematical skills are very useful. In addition, students can count on help from the Math Center in finding summer internships and other opportunities to gain professional experience.

How does the department inspire in its faculty such devotion to students? As department head, McCallum has been asked this question, particularly during site visits for the department's two VIGRE grants from the National Science Foundation. "I don't think anybody in the department thinks that anything special is going on to make that happen," he said. "It's just part of the culture." When he visits other mathematics departments, he realizes that often there is much more distance between faculty and students and much less attention paid to undergraduates. "And I think, gee, how did that happen?" he says. The Arizona department had an exceptional dedication to undergraduates when McCallum first joined the faculty in 1987. "A culture like that, once established, is self-maintaining because it attracts faculty who like that culture," he remarks. "It's not that every single faculty member in the department is totally devoted to undergraduates, and that is not necessary to have the right culture. You just need enough."

Nevertheless, there are some specific programs in the department that nurture that culture. One is a program of "teaching postdocs"—postdoctoral positions for people who have Ph.D.s in mathematics but who are primarily interested in teaching careers. "They contribute tremendously to the life of the department, and specifically to the educational environment for the undergraduates," McCallum says. Another program, called Undergraduate Teaching Assistantships (UTA), gives math majors who are interested in teaching careers opportunities to work with faculty on instructional and curricular issues and perhaps even to teach a course. "It's sort of the analogue for teaching of a research experience," McCallum explains.

The UTA program dovetails nicely with the department's long tradition of devoting thought to the undergraduate curriculum and course development. Some faculty members—including McCallum, Velez, and Deborah Hughes Hallett—have made significant contributions to undergraduate education at the national level. McCallum was on the AMS Committee on Education from 2002 through 2008 and served as chair for five years; Hughes Hallett also served on that committee from 2008 through 2010.

The department also has a strong commitment to improving precollege mathematics education. This shows in small ways, such as visits by department faculty to high school classrooms in Tucson, and in much larger and more formal structures, such as the Institute for Mathematics and Education. Funded primarily by the university, with additional support from the NSF and private foundations, the institute was started by McCallum in 2006; the current director is faculty member Joceline Lega. It sponsors a variety of local, national, and international projects designed to promote collaborations that aim to improve mathematics education, mainly at the precollege level. The department's Center for Recruitment and Retention of Mathematics Teachers recruits undergraduates into the education option of the mathematics major. It also supports beginning teachers by pairing them with mentors who are senior teachers and by sending undergraduates out into the schools to work as tutors.

The educational activity in the department, paired with its strong research profile, make for a unique atmosphere that is especially inviting to individuals interested in both teaching and research. "There is no division really," McCallum says. "Some people are focusing their energies more on their research, obviously, and some people are focusing their energies more on educational activities. But you can move back and forth between those two, and you don't get labeled particularly that you are one or the other. That appeals to me personally, because that is what I've done in my career."

In the middle of all of this activity stands the Math Center, the linchpin of a vibrant and successful department. Much of the center's success is due to Velez's zeal for recruiting; Varecka's thorough and thoughtful approach and the structure of the center are also key. But a large part of its success is the climate in the department. Velez notes that the Exemplary Program Award was given to the Math Center, not to him personally. "It is the resources the department has invested in the major program that has allowed me to do the things I have done," he says. "My job is to bring the students in the door. If I didn't have the support of gifted teachers on the faculty, we could not keep these students as majors. I am part of an unusual department."

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ε Raoul A. De Villiers
ε*Paul F. J. Dhooghe
ε Beverly E. J. Diamond
Meighan Irene Dillon
ε Gerald P. Dinneen
ε*Dragomir Ž. Djoković
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ε*Boro Doering
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ε James S. Dukelow Jr.
ε Steve N. Dulaney
ε Stephen C. Dumars
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* Kanat Durgun
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ε*Allan L. Edmonds
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Maurice C. Figueres
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Chihchy Fwu
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ε Fernando Galaz-Fontes
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ε D. Ghisa
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Ross B. Gingrich
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ε*Helmut Groemer
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ε Robert Andrew Grossman
ε Edward A. Grove
Gerd Grubb
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ε*Andras Hajnal
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ε*Alfred W. Hales
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Margo Halsted
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ε Heiko Harborth
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ε Andrew William Harrell
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ε Troy L. Hicks
Takeyuki Hida
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ε*Hugh M. Hilden
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ε*Nancy Hingston
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ε*Friedrich E. P. Hirzebruch
ε*Peter David Hislop
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ε*Jonathan P. E. Hodgson
ε*Helmut H. W. Hofer
ε*Michael E. Hoffman
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ε Charles S. Holmes
ε*Philip John Holmes
ε*Roger H. Homer
ε Jennifer L. Hopkins
ε Jim E. Hoste
ε Fredric T. Howard
ε Everett W. Howe
ε*J. S. Hsia
ε*Pao-sheng Hsu
Robert Hubata
ε Archibald Perrin Hudgins
ε*Denise Huet
ε*George A. Hufford
ε*Anne Hughes
ε*Mark E. Huibregtse
Gustavus E. Huige
ε*Birge K. Huisgen-Zimmermann
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ε*Tom Ilmanen
ε Ettore Ferrari Infante
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* Ron Irving
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Mattias Jonsson
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ε Vladislav Kargin
ε*Johan Karlsson
ε Guido Karrer
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ε Shunji Kawamoto
ε Kiran S. Kedlaya
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ε*Edward L. Keller
ε*Wayne G. Kellner
ε*John B. Kelly
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ε Efim Khalimsky
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Andrei Khruzin
ε Michael K. H. Kiessling
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Benjamin G. Klein
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ε Stanislav V. Klimenko
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ε K. R. K. Knorr
ε Hai-Ping Ko
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ε*Sundaresan Kondagunta
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ε*Jurg Kramer
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ε*Gary R. Krumpholz
ε*Wei-Eihn Kuan
ε Sharon Kunoff
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ε*Robert P. Kurshan
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ε Marwan Lahoud
ε Kee Y. Lam
ε*John Patrick Lambert
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ε*Peter S. Landweber
ε Leo J. Lange
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ε Ke-Seung Lee
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ε Liangpan Li
Yanyan Li
* Jaung Liang
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ε*Peter A. Linnell
ε*Sally Irene Lipsey
ε William G. Lister
ε Andrew C-F Liu
ε*Ming Chit Liu
ε*Tsai-Sheng Liu
* George W. Lofquist
ε*Charles J. Lombardo
* John M. Long
ε*William C. Lordan
* Michael P. Loss
ε László Lovász
ε*Sylvia Chin-Pi Lu
ε Tsu-Ming Lu
ε*Jonathan D. Lubin
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ε Sorin Lugojan
ε*Norman Y. Luther
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ε Michael C. Mackey
ε Manohar L. Madan
ε James Joseph Madden
ε*Adolf G. Mader
* Mehran Mahdavi
ε*Joseph Malkevitch
ε Salvador Malo
ε Michael Maltenfort
* Alfred P. Maneki
ε John L. Manferdelli
ε*Jason Manning
ε Pauline Mann-Nachbar
ε*Margaret O. Marchand
ε*Stefano Marchiafava
ε*Murray Angus Marshall
ε Nathaniel F. G. Martin
ε Thomas Harry Martin
ε*Samuel Masih
ε Attila Máté
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ε John Patterson Mayberry
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* Robert M. McConnel
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ε*Anders Melin
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ε Nadine L. Menninga
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ε*Richard A. Moore
ε W. Keith Moore
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ε Kanji Namba
ε Takao Namiki
ε Isaac Namioka
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ε Manmath Nayak
ε Jan V. Neerven
ε*Joseph Neggers
ε Philip C. Nelson
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ε*Umberto Neri
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James A. Nickel
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Anand Pillay
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David Quesada
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ε*Louis B. Rall
Dinakar Ramakrishnan
ε*Melapalayam S. Ramanujan
ε*George N. Raney
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ε*Salvatore Rao
ε*Louise Arakelian Raphael
ε Douglas C. Ravenel
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Charles W. Rezk
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Horst P. Richter
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ε Ronald Edgar Rietz
ε Alcibiades Rigas
ε Robert D. Rigdon
* Jose Rio
ε Thomas W. Rishel
ε Joel W. Robbin
ε Alain M. Robert
ε*Anne Drinkwater Roberts
ε Joel L. Roberts
ε*Joseph B. Roberts
ε Lois J. Roberts
ε Paul C. Roberts
Margaret M. Robinson
ε Tom Roby
ε Pedro Martins Rodrigues
John Roe
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ε Dale P. O. Rolfsen
ε*Guillermo Romero Melendez
ε Nicholas J. Rose
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ε*Jonathan M. Rosenberg
ε Roger D. Rosenkrantz
ε Kenneth A. Ross
* Adrian S. Roth
ε Mitchell J. Rothstein
James Rovnyak
ε*Virginia G. Rovnyak
William W. Rowell
ε*James Samuel Royer
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ε*Henry Sharp Jr.
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ε Michael Sherbon
ε Amin Shokrollahi
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Martha J. Siegel
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ε Joseph H. Silverman
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ε*Patrick J. Sime
* Yakov G. Sinai
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ε*Man-Keung Siu
ε*Walter S. Sizer
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ε Michael Slattery
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ε Boris Solomyak
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ε Sergio Spagnolo
ε Ralf J. Spatzier
ε Birgit Speh
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ε Paul H. Stanford
ε*Lee James Stanley
ε Christopher W. Stark
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ε David R. Steinsaltz
Ellen M. Stenson
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ε John Colin Stillwell
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ε H. A. Stone
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ε*Kelly John Suman
ε Patrick Suppes
ε*Andrew V. Sutherland
ε*William J. Sweeney
ε*Roman Sznajder
ε Kazuaki Taira
ε*Lajos F. Takács
ε Yoshinori Takei
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ε Daniel Joseph Tancredi
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ε*James J. Tattersall
ε*S. James Taylor
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Chuu-Lian Terng
ε*Paul M. Terwilliger
ε John Alexander Thacker
ε Edward Linus Thome
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ε*Andre Toom
Craig A. Tracy
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ε*Selden Y. Trimble V
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ε Tamotsu Tsuchikura
ε*Kazô Tsuji
ε*Joann Stephanie Turisco
Helene R. Tyler
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* Yasushi Unai Unai
ε Harald Upmeier
ε*John A. W. Upton
ε Wilfredo O. Urbina
ε*Johannes A. Van Casteren
ε Jan Harm Van der Walt
ε*H. N. Van Eck
ε Aernout C. D. Van Enter
ε*A. H. Van Tuyl
ε Alphonse Thomas Vasquez
ε Juan L. Vazquez
ε James Vickers
ε*Jose L. Viviente
ε*Michael Voichick
Paul S. Voigt
ε*Paul A. Vojta
Emil J. Volcheck
ε Hans W. Volkmer
Lisa Wade
ε Daniel F. Waggoner
ε William M. Wagner
* Jonathan M. Wahl
ε Masato Wakayama
ε*Nolan R. Wallach
Lawrence J. Wallen
ε*John Thomas Walsh
ε James Robert Ward Jr.
* Seth L. Warner
ε*Bette L. Warren
ε Lawrence C. Washington
ε*Arthur G. Wasserman
Robert H. Wasserman
ε*Michiaki Watanabe
ε Shôji Watanabe
* William C. Waterhouse
ε*David S. Watkins
ε*Mark E. Watkins
ε*David L. Webb
ε Elias Wegert
ε*Hans F. Weinberger
ε*Joel L. Weiner
ε*Michael I. Weinstein
ε David M. Wells
ε Greg Wene
ε John C. Wenger
ε*Henry C. Wente
ε John Wermer
ε Elisabeth M. Werner
ε*John E. Wetzel
ε*Brian Cabell White
ε*Charles M. White
ε Tad P. White
ε Alan Dean Wiederhold
ε*Roger A. & Sylvia Margaret Wiegand
ε*Susan Gayle Williams
Charles K. Williamson
ε*John R. Willis
* Robert Lee Wilson
ε Samuel Ronald Windsor
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ε Nick John Wood
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ε*Hiroyoshi Yamaki
ε Deane Yang
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ε Mitsuru Yasuhara
ε*Suresh Yegnashankaran
ε*J. Michael Yohe
ε*Donald F. Young
ε Gabjin Yun
ε*Charles T. Zahn
ε*V. E. Zakharov
ε Jean-Claude Zambrini
ε*François Zara
ε Jose Zero
ε*David E. Zitarelli
ε Paul Zorn
ε Steven M. Zucker
ε*Paul F. Zweifel
ε Paul J. Zwier
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Mathematics People

Butcher Receives Inaugural Jones Medal

John Butcher of the University of Auckland, New Zealand, has received the inaugural Jones Medal for lifetime achievement in the mathematical sciences. The prize citation reads in part: "John Butcher is without doubt one of the leading world experts on numerical methods for the solution of ordinary differential equations (ODEs), and the world expert on Runge-Kutta methods. … The entire subject today is organized around the concept of B-series, named after John, the expansion of the integrator as a power series in the time step, with coefficients that are polynomial in the Runge-Kutta coefficients and in the vector field appearing in the ODE and its derivatives: these appear in certain combinations known as elementary differentials of the vector field which are represented as rooted trees."

The Jones Medal is named for Vaughan Jones and was established by the Royal Society of New Zealand. It will be awarded biennially for lifetime achievement in pure or applied mathematics or statistics by a person with substantial connections to New Zealand.

—From a Royal Society of New Zealand announcement

Polterovich Awarded Coxeter-James Prize

Iosif Polterovich of the University of Montreal has been awarded the 2011 Coxeter-James Prize of the Canadian Mathematical Society (CMS). He was recognized for his work in spectral geometry, as well as his contributions to isospectral domains with mixed boundary conditions, the asymptotics of eigenvalues of the Laplacian, and isoperimetric inequalities for eigenvalues.

The prize was named after geometer Donald Coxeter and Ralph Duncan James, both former presidents of the CMS. It recognizes young mathematicians who have made outstanding contributions to mathematical research.

—From a CMS announcement

Scholze Awarded 2011 Clay Research Fellowship

Peter Scholze of the University of Bonn has been awarded a five-year Clay Research Fellowship by the Clay Mathematics Institute (CMI). He received his M.Sc. at the University of Bonn in 2010 under the supervision of Michael Rapoport and is currently working on his Ph.D. thesis at Bonn. His major interest is arithmetic geometry, and he is working on the bad reduction of Shimura varieties and the Langlands program.

Clay Research Fellows are appointed for terms ranging from two to five years; graduating doctoral students and mathematicians within three years of receiving the doctoral degree are eligible for the fellowships. The primary selection criteria for the fellowship are the exceptional quality of the candidate's research and the candidate's promise to become a mathematical leader.

—From a CMI announcement

Mazzucato Awarded Michler Prize

Anna Mazzucato of Pennsylvania State University has been awarded the 2011 Ruth I. Michler Memorial Prize by the Association of Women in Mathematics (AWM). She was honored for her "wide range of mathematical talents". Her research involves the analysis of partial differential equations, particularly those arising from continuum mechanics of deformable solids and incompressible fluids, and associated inverse problems. In 1994 she earned her Laurea (B.S./M.S.) in mathematical physics at Universitá degli Studi di Milano. She received her Ph.D. in mathematics at the University of North Carolina, Chapel Hill, in 2000, where she studied the Navier-Stokes and other nonlinear evolution equations under the direction of Michael Taylor.

The Michler Prize grants a midcareer woman in academia a residential fellowship in the Cornell University mathematics department without teaching obligations.

—From an AWM announcement

Spohn Awarded Heineman Prize

Herbert Spohn of the Technical University of Munich has been awarded the 2011 Dannie Heineman Prize for Mathematical Physics. He was honored for his "seminal contributions to nonequilibrium statistical mechanics as exemplified by his exact solutions of growth models and stationary states of open systems. Combining mathematical rigor with physical insight, his work elucidates the transition from microscopic to macroscopic behavior."

The prize carries a cash award of US$10,000 and is presented in recognition of outstanding publications in the field of mathematical physics. The prize was established in 1959 by the Heineman Foundation for Research, Educational, Charitable, and Scientific Purposes, Inc., and is administered jointly by the American Institute of Physics (AIP) and the American Physical Society (APS). The prize is presented annually.

—From an APS announcement

National Academy of Engineering Elections

The National Academy of Engineering (NAE) has elected sixty-eight new members and nine foreign associates. Among them are four whose work involves the mathematical sciences. They are: William J. Cook, Chandler Family Chair Professor in Industrial and Systems Engineering, Georgia Institute of Technology, Atlanta, "for theoretical and computational contributions to discrete optimization"; Daphne Koller, professor of computer science, Stanford University, "for contributions to representation, inference, and learning in probabilistic models with applications to robotics, vision, and biology"; Terrence J. Sejnowski, Francis Crick Professor and director of the Computational Neurobiology Laboratory, Salk Institute for Biological Studies, La Jolla, California, "for contributions to artificial and real neural network algorithms and applying signal processing models to neuroscience"; and Mihalis Yannakakis, Percy K. and Vida L. W. Hudson Professor of Computer Science, Columbia University, "for contributions to algorithms and computational complexity."

—From an NAE announcement

Mathematics Opportunities

NSF Postdoctoral Research Fellowships

The National Science Foundation (NSF) awards Mathematical Sciences Postdoctoral Research Fellowships (MSPRF) for appropriate research in areas of the mathematical sciences, including applications to other disciplines. Awardees are permitted to choose research environments that will have maximal impact on their future scientific development. Awards are made in the form of either Research Fellowships or Research Instructorships. The Research Fellowship option provides full-time support for any eighteen academic-year months in a three-year period, in intervals not shorter than three consecutive months. The Research Instructorship option provides a combination of full-time and half-time support over a period of three academic years, usually one academic year full time and two academic years half time. Under both options, the award includes six summer months; however, no more than two summer months of support may be received in any calendar year. Under both options, the stipend support for twenty-four months (eighteen academic-year months plus six summer months) will be provided within a forty-eight-month period.

The deadline for proposals is **October 19, 2011**. See `http://www.nsf.gov/pubs/2008/nsf08582/nsf08582.htm`.

—From an NSF announcement

International Mathematics Competition for University Students

The Eighteenth International Mathematics Competition (IMC) for University Students will be held July 28 through August 3, 2011, at American University in Blagoevgrad, Bulgaria. Participating universities are invited to send several students and one teacher; individual students are welcome. Students completing their first, second, third, or fourth years of university education are eligible. The competition will consist of two sessions of five hours each. Problems will come from the fields of algebra, analysis (real and complex), geometry, and combinatorics. The working language will be English. See the website `http://www.imc-math.org.uk/` or contact John Jayne, University College London, Gower Street, London WC1E 6BT, United Kingdom; telephone: +44-20-7679-7322; email: `j.jayne@ucl.ac.uk`.

—John Jayne
University College London

Inside the AMS

Epsilon Awards for 2010

The AMS Epsilon Fund for Young Scholars was established in 1999 to provide financial assistance to summer programs in the United States and Canada for mathematically talented high school students. These programs have provided mathematically talented youngsters with their first serious mathematical experiences. The name for the fund was chosen in remembrance of the late Paul Erdős, who was fond of calling children "epsilons".

The AMS has chosen ten summer mathematics programs to receive Epsilon grants for activities in the summer of 2011. The grants will support program expenses and student scholarships and, in some cases, scholarships only. The programs were chosen on the basis of mathematical excellence and enthusiasm. Award amounts were governed by the varying financial needs of each program and totaled US$100,000.

The 2011 grants are awarded to: All Girls/All Math, University of Nebraska, Lincoln; Canada/USA Mathcamp, Reed College, Portland, Oregon; Lamar Achievement in Mathematics Program (LAMP), Lamar University, Beaumont, Texas; MathPath, Colorado College, Colorado Springs; PROMYS, Boston University; PROTaSM (Puerto Rico Opportunities for Talented Students in Mathematics), University of Puerto Rico, Mayagüez Campus; Research Science Institute, Massachusetts Institute of Technology; Ross Mathematics Program, The Ohio State University; Texas State Honors Summer Math Camp, Texas State University, San Marcos; Young Scholars Program, University of Chicago.

The grants for summer 2011 are paid for by the AMS Epsilon Fund for Young Scholars. The AMS Epsilon Fund for Young Scholars has been funded by contributions of AMS members and friends; the goal of the endowment is to provide at least US$100,000 in support each summer. For further information about the Epsilon Fund for Young Scholars, visit the website `http://www.ams.org/giving-to-ams/` or contact `development@ams.org`. Information about how to apply for Epsilon grants is available at `http://www.ams.org/employment/epsilon.html`. A fairly comprehensive listing of summer programs for mathematically talented high school students (including those with and without Epsilon grants) is available at `http://www.ams.org/employment/mathcamps.html`.

—AMS Development Office

AMS Holds Workshop for Department Chairs

The AMS held its annual workshop for department chairs prior to the Joint Mathematics Meetings in New Orleans, Louisiana, in January 2011. This one-day session is designed in a workshop format to stimulate discussion and allow the sharing of ideas and experiences among attending department chairs, which allows attendees to address departmental challenges from new perspectives. The 2011 workshop was led by Timothy Hodges, University of Cincinnati; John Meakin, University of Nebraska-Lincoln; Helen Roberts, Montclair State University; and Stephen Robinson, Wake Forest University.

Workshop sessions have included a range of issues facing departments, including planning and budgeting, personnel management, assessment, outreach, faculty development, communications, and departmental leadership. This year's workshop focused on leading a department in challenging times, assessment, new approaches to nonmajor mathematics classes, and faculty evaluation as an opportunity.

The 2011 workshop was the largest ever for the AMS, with sixty department chairs and leaders from across the country attending.

—Anita Benjamin
AMS Washington Office

From the AMS Public Awareness Office

Adriana Salerno

Ph.D. + epsilon

The AMS has launched a blog, Ph.D. + epsilon, written by Adriana Salerno, assistant professor at Bates College. As an early-career mathematician, she will blog about her experiences and challenges. The AMS invites other early-career mathematicians to follow the blog and share comments at `http://www.ams.org/blog/phdplus/`.

American Association for the Advancement of Science Annual Meeting

The AMS Public Awareness Office ran the Who Wants to Be a Mathematician game during Family Science Days and hosted an exhibit at the 2011 meeting of the American Association for the Advancement of Science in Washington, D.C., February 17–21. See descriptions of some of the events related to mathematics that took place at the meeting at `http://www.ams.org/meetings/aaas2011`.

2011 Mathematical Art Exhibition album

The fifty-eight works in the 2011 Mathematical Art Exhibition, held at the Joint Mathematics Meetings, are in an album on Mathematical Imagery. The works include digital prints, origami, crochet, jewelry, and sculpture; the images may be sent as e-postcards. See `http://www.ams.org/mathimagery`.

Deaths of AMS Members

Hugh N. Albright, professor, LaSalle University, died on February 24, 2011. Born on February 27, 1928, he was a member of the Society for 40 years.

Winifred A. Asprey of Poughkeepsie, New York, died on October 19, 2007. Born on April 8, 1917, she was a member of the Society for 61 years.

Thierry E. Aubin, of Paris, France, died on March 21, 2009. Born on May 6, 1942, he was a member of the Society for 29 years.

John E. Brothers, professor, Indiana University, died on November 15, 2010. Born on July 6, 1937, he was a member of the Society for 47 years.

Elizabeth H. Cuthill, of Solomons, Maryland, died on January 11, 2011. Born on October 16, 1923, she was a member of the Society for 62 years.

David F. Dawson, of Denton, Texas, died on February 7, 2011. Born on September 16, 1926, he was a member of the Society for 59 years.

Denis A. Higgs, of Toronto, Canada, died on February 25, 2011. Born on May 6, 1932, he was a member of the Society for 41 years.

Greg Hjorth, professor, University of Melbourne, died on January 13, 2011. Born on June 14, 1963, he was a member of the Society for 18 years.

Jaroslav Jezek, professor, Charles University, died on February 13, 2011. Born on March 5, 1945, he was a member of the Society for 33 years.

George E. Lang, professor, Fairfield University, died on May 9, 2007. Born on June 29, 1942, he was a member of the Society for 38 years.

Gerard McDonald, professor, Loyola University of Chicago, died on September 18, 2010. Born on November 27, 1946, he was a member of the Society for 31 years.

Ernst Snapper, professor, Dartmouth College, died on February 5, 2011. Born on December 2, 1913, he was a member of the Society for 69 years.

Alfred J. van der Poorten, of Australia, died on October 9, 2010. Born on May 16, 1942, he was a member of the Society for 41 years.

Thomas Williams, of Mt. Joy, Pennsylvania, died on September 30, 2010. Born on October 24, 1937, he was a member of the Society for 12 years.

Francis U. Williamson, of France, died on January 8, 2011. Born on June 20, 1938, he was a member of the Society for 27 years.

A. Wayne Wymore, of Tucson, Arizona, died on February 24, 2011. Born on February 1, 1927, he was a member of the Society for 58 years.

Correction

Reference [17] in the January 2011 *Notices* article "Real face of János Bolyai" omitted the name of one of the authors of the cited article. The reference should have read: E. Kiss and J. Sandor, On a congruence by János Bolyai connected by pseudoprimes, *Mathematica Pannonia*, **15**(2) (2004), 283–288.

—*Notices* staff

Reference and Book List

*The **Reference** section of the* Notices *is intended to provide the reader with frequently sought information in an easily accessible manner. New information is printed as it becomes available and is referenced after the first printing. As soon as information is updated or otherwise changed, it will be noted in this section.*

Contacting the *Notices*

The preferred method for contacting the *Notices* is electronic mail. The editor is the person to whom to send articles and letters for consideration. Articles include feature articles, memorial articles, communications, opinion pieces, and book reviews. The editor is also the person to whom to send news of unusual interest about other people's mathematics research.

The managing editor is the person to whom to send items for "Mathematics People", "Mathematics Opportunities", "For Your Information", "Reference and Book List", and "Mathematics Calendar". Requests for permissions, as well as all other inquiries, go to the managing editor.

The electronic-mail addresses are notices@math.wustl.edu in the case of the editor and notices@ams.org in the case of the managing editor. The fax numbers are 314-935-6839 for the editor and 401-331-3842 for the managing editor. Postal addresses may be found in the masthead.

Upcoming Deadlines

April 30, 2011: Nominations for AWM Gweneth Humphreys Award. See www.awm-math.org, telephone 703-934-0163, or email awm@awm-math.org.

May 1, 2011: Applications for May review for National Academies Research Associateship Programs. See the National Academies website at http://sites.nationalacademies.org/PGA/RAP/PGA_050491 or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email rap@nas.edu.

May 1, 2011: Applications for National Academies Christine Mirzayan Graduate Fellowship Program for fall 2011. See http://sites.nationalacademies.org/PGA/policyfellows/index.htm.

May 1, 2011: Applications for AWM Travel Grants. See http://www.awm-math.org/travelgrants.html#standard.

May 15–June 15, 2011: Submission period for DMS Workforce Program in the Mathematical Sciences. See http://www.nsf.gov/funding/pgm_summ.jsp?pims_id=503233.

August 1, 2011: Applications for August review for National Academies Research Associateship Programs. See the National Academies website at http://sites.nationalacademies.org/PGA/RAP/PGA_050491 or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email rap@nas.edu.

October 1, 2011: Applications for AWM Travel Grants. See http://www.awm-math.org/travelgrants.html#standard.

October 1, 2011: Nominations for the 2012 Emanuel and Carol Parzen Prize. Contact Thomas Wehrly, Department of Statistics, 3143 TAMU, Texas A&M University, College Station, Texas 77843-3143.

October 19, 2011: Proposals for NSF Postdoctoral Research Fellowships. See "Mathematics Opportunities" in this issue.

November 1, 2011: Applications for November review for National Academies Research Associateship Programs. See the National Academies website at http://sites.nationalacademies.org/PGA/RAP/PGA_050491 or contact Research

Where to Find It

A brief index to information that appears in this and previous issues of the *Notices*.

Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email rap@nas.edu.

December 21, 2011: Nominations for the Schauder Medal. Contact Lech Gorniewicz, manager of Schauder Center, tmna@mat.uni.torun.pl.

Book List

The Book List highlights books that have mathematical themes and are aimed at a broad audience potentially including mathematicians, students, and the general public. When a book has been reviewed in the Notices, *a reference is given to the review. Generally the list will contain only books published within the last two years, though exceptions may be made in cases where current events (e.g., the death of a prominent mathematician, coverage of a certain piece of mathematics in the news) warrant drawing readers' attention to older books. Suggestions for books to include on the list may be sent to* notices-booklist@ams.org.

*Added to "Book List" since the list's last appearance.

Apocalypse When?: Calculating How Long the Human Race Will Survive, by Willard Wells. Springer Praxis, June 2009. ISBN-13: 978-03870-983-64.

**At Home with Andre and Simone Weil,* by Sylvie Weil. (Translation of *Chez les Weils*, translated by Benjamin Ivry.) Northwestern University Press, October 2010. ISBN 978-08101-270-43. (Reviewed in this issue.)

The Best Writing on Mathematics: 2010, edited by Mircea Pitici. Princeton University Press, December 2010. ISBN-13: 978-06911-484-10.

The Black Swan: The Impact of the Highly Improbable, by Nassim Nicholas Taleb. Random House Trade Paperbacks, second edition, May 2010. ISBN-13: 978-08129-738-15. (First edition reviewed March 2011.)

Bright Boys: The Making of Information Technology, by Tom Green. A K Peters, April 2010. ISBN-13: 978-1-56881-476-6.

**The Calculus Diaries: How Math Can Help You Lose Weight, Win in Vegas, and Survive a Zombie Apocalypse,* by Jennifer Ouellette. Penguin, reprint edition, August 2010. ISBN 978-01431-173-77.

The Calculus of Friendship: What a Teacher and Student Learned about Life While Corresponding about Math, by Steven Strogatz. Princeton University Press, August 2009. ISBN-13: 978-0691-13493-2. (Reviewed June/July 2010.)

The Calculus of Selfishness, by Karl Sigmund. Princeton University Press, January 2010. ISBN-13: 978-06911-427-53.

The Clockwork Universe: Isaac Newton, the Royal Society, and the Birth of the Modern World, by Edward Dolnick. Harper, February 2011. ISBN-13: 978-00617-195-16. (Reviewed April 2011.)

Complexity: A Guided Tour, by Melanie Mitchell. Oxford University Press, April 2009. ISBN-13: 978-01951-244-15. (Reviewed April 2011.)

The Cult of Statistical Significance: How the Standard Error Costs Us Jobs, Justice, and Lives, by Stephen T. Ziliak and Deirdre N. McCloskey, University of Michigan Press, February 2008. ISBN-13: 978-04720-500-79. (Reviewed October 2010.)

Duel at Dawn: Heroes, Martyrs, and the Rise of Modern Mathematics, by Amir Alexander. Harvard University Press, April 2010. ISBN-13: 978-06740-466-10. (Reviewed November 2010.)

Euler's Gem: The Polyhedron Formula and the Birth of Topology, by David S. Richeson. Princeton University Press, September 2008. ISBN-13: 978-06911-267-77. (Reviewed December 2010.)

The Grand Design, by Stephen Hawking and Leonard Mlodinow. Bantam, September 2010. ISBN-13: 978-05538-053-76.

Here's Looking at Euclid: A Surprising Excursion through the Astonishing World of Math, by Alex Bellos. Free Press, June 2010. ISBN-13: 978-14165-882-52.

Hidden Harmonies (The Lives and Times of the Pythagorean Theorem), by Robert and Ellen Kaplan. Bloomsbury Press, January 2011. ISBN-13:978-15969-152-20.

**Hot X: Algebra Exposed*, by Danica McKellar. Hudson Street Press, August 2010. ISBN 978-15946-307-05.

How to Read Historical Mathematics, by Benjamin Wardhaugh. Princeton University Press, March 2010. ISBN-13: 978-06911-401-48.

Isaac Newton on Mathematical Certainty and Method, by Niccolò Guicciardini. MIT Press, October 2009. ISBN-13: 978-02620-131-78.

**Le Operazioni del Calcolo Logico,* by Ernst Schröder. Original German version of *Operationskreis des Logikkalkuls* and Italian translation with commentary and annotations by Davide Bondoni. LED Online, 2010. ISBN 978-88-7916-474-0.

Logicomix: An Epic Search for Truth, by Apostolos Doxiadis and Christos Papadimitriou. Bloomsbury USA, September 2009. ISBN-13: 978-15969-145-20. (Reviewed December 2010.)

Loving + Hating Mathematics: Challenging the Myths of Mathematical Life, by Reuben Hersh and Vera John-Steiner. Princeton University Press, January 2011. ISBN-13: 978-06911-424-70.

The Math Book: From Pythagoras to the 57th Dimension, 250 Milestones in the History of Mathematics, by Clifford A. Pickover. Sterling, September 2009. ISBN-13: 978-14027-579-69.

A Mathematician's Lament: How School Cheats Us Out of Our Most Fascinating and Imaginative Art Form, by Paul Lockhart. Bellevue Literary Press, April 2009. ISBN-13:978-1-934137-17-8.

Mathematicians: An Outer View of the Inner World, by Mariana Cook. Princeton University Press, June 2009. ISBN-13: 978-06911-3951-7. (Reviewed August 2010.)

Mathematicians Fleeing from Nazi Germany: Individual Fates and Global Impact, by Reinhard Siegmund-Schultze. Princeton University Press, July 2009. ISBN-13: 978-06911-4041-4. (Reviewed November 2010.)

A Motif of Mathematics: History and Application of the Mediant and the Farey Sequence, by Scott B. Guthery. Docent Press, September 2010. ISBN-13:978-4538-105-76.

Mrs. Perkins's Electric Quilt: And Other Intriguing Stories of Mathematical Physics, Paul J. Nahin, Princeton University Press, August 2009. ISBN-13: 978-06911-354-03.

Naming Infinity: A True Story of Religious Mysticism and Mathematical Creativity, by Loren Graham and Jean-Michel Kantor. Belknap Press of Harvard University Press, March 2009. ISBN-13: 978-06740-329-34.

Nonsense on Stilts: How to Tell Science from Bunk, by Massimo Pigliucci. University of Chicago Press, May

2010. ISBN-13: 978-02266-678-67. (Reviewed April 2011.)

**Number Freak: From 1 to 200—The Hidden Language of Numbers Revealed,* by Derrick Niederman. Perigee Trade, August 2009. ISBN-10: 03995-345-98.

**Numbers: A Very Short Introduction*, by Peter M. Higgins. Oxford University Press, February 2011. ISBN 978-0-19-958405-5.

Numbers Rule: The Vexing Mathematics of Democracy, from Plato to the Present, by George G. Szpiro. Princeton University Press, April 2010. ISBN-13: 978-06911-399-44. (Reviewed January 2011.)

The Numerati, by Stephen Baker. Houghton Mifflin, August 2008. ISBN-13: 978-06187-846-08. (Reviewed October 2009.)

Our Days Are Numbered: How Mathematics Orders Our Lives, by Jason Brown. Emblem Editions, April 2010. ISBN-13: 978-07710-169-74.

Perfect Rigor: A Genius and the Mathematical Breakthrough of the Century, by Masha Gessen. Houghton Mifflin Harcourt, November 2009. ISBN-13: 978-01510-140-64. (Reviewed January 2011.)

Pioneering Women in American Mathematics: The Pre-1940 Ph.D.'s, by Judy Green and Jeanne LaDuke. AMS, December 2008. ISBN-13: 978-08218-4376-5.

Plato's Ghost: The Modernist Transformation of Mathematics, by Jeremy Gray. Princeton University Press, September 2008. ISBN-13: 978-06911-361-03. (Reviewed February 2010.)

Probabilities: The Little Numbers That Rule Our Lives, by Peter Olofsson. Wiley, March 2010. ISBN-13: 978-04706-244-56.

Problem-Solving and Selected Topics in Number Theory in the Spirit of the Mathematical Olympiads, by Michael Th. Rassias. Springer, 2011. ISBN-13: 978-1-4419-0494-2.

**Proofiness: The Dark Arts of Mathematical Deception*, by Charles Seife. Viking, September 2010. ISBN 978-06700-221-68.

Proofs from THE BOOK, by Martin Aigner and Günter Ziegler. Expanded fourth edition, Springer, October 2009. ISBN-13: 978-3-642-00855-9.

**The Quants: How a New Breed of Math Whizzes Conquered Wall Street and Nearly Destroyed It*, by Scott Patterson. Crown Business, January 2011. ISBN 978-03074-533-89. (Reviewed in this issue.)

Recountings: Conversations with MIT Mathematicians, edited by Joel Segel. A K Peters, January 2009. ISBN-13: 978-15688-144-90.

Roger Boscovich, by Radoslav Dimitric (Serbian). Helios Publishing Company, September 2006. ISBN-13: 978-09788-256-21.

The Shape of Inner Space: String Theory and the Geometry of the Universe's Hidden Dimensions, by Shing-Tung Yau (with Steve Nadis). Basic Books, September 2010. ISBN-13: 978-04650-202-32. (Reviewed February 2011.)

The Solitude of Prime Numbers, by Paolo Giordano. Pamela Dorman Books, March 2010. ISBN-13: 978-06700-214-82. (Reviewed September 2010.)

Solving Mathematical Problems: A Personal Perspective, by Terence Tao. Oxford University Press, September 2006. ISBN-13: 978-0199-20560-8. (Reviewed February 2010.)

The Strangest Man, by Graham Farmelo. Basic Books, August 2009. ISBN-13: 978-04650-182-77.

Street-Fighting Mathematics: The Art of Educated Guessing and Opportunistic Problem Solving, by Sanjoy Mahajan. MIT Press, March 2010. ISBN-13: 978-0262-51429-3.

Survival Guide for Outsiders: How to Protect Yourself from Politicians, Experts, and Other Insiders, by Sherman Stein. BookSurge Publishing, February 2010. ISBN-13: 978-14392-532-74.

Symmetry: A Journey into the Patterns of Nature, by Marcus du Sautoy. Harper, March 2008. ISBN: 978-00607-8940-4. (Reviewed February 2011.)

Symmetry in Chaos: A Search for Pattern in Mathematics, Art, and Nature, by Michael Field and Martin Golubitsky. Society for Industrial and Applied Mathematics, second revised edition, May 2009. ISBN-13: 978-08987-167-26.

Teaching Statistics Using Baseball, by James Albert. Mathematical Association of America, July 2003. ISBN-13: 978-08838-572-74. (Reviewed April 2010.)

What's Luck Got to Do with It? The History, Mathematics and Psychology of the Gambler's Illusion, by Joseph Mazur. Princeton University Press, July 2010. ISBN: 978-069-113890-9.

Officers of the Society 2010 and 2011 Updates

Except for the members at large of the Council, the month and year of the first term and the end of the present term are given. For members at large of the Council, the last year of the present term is listed.

Council

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Eric M. Friedlander 2/11–1/13

President Elect
Eric M. Friedlander 2/10–1/11

Immediate Past President
George E. Andrews 2/11–1/12

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Associate Treasurer
John M. Franks 2/11–1/12
Linda Keen 2/09–1/11

Members at Large
All terms are for three years and expire on January 31 following the year given.

2010
Rebecca F. Goldin
Bryna Kra
Irena Peeva
Joseph H. Silverman
Sarah J. Witherspoon

2011
Aaron Bertram
William A. Massey
Panagiotis E. Souganidis
Michelle L. Wachs
David Wright

2012
Alejandro Adem
Richard Hain
Jennifer Schultens
Janet Talvacchia
Christophe Thiele

2013
Matthew Ando
Estelle Basor
Patricia Hersh
Tara S. Holm
T. Christine Stevens

Members of Executive Committee

Members of the Council, as provided for in Article 7, Section 4 (last sentence), of the Bylaws of the Society.

Ruth M. Charney 2/07–1/11
Ralph L. Cohen 2/11–1/15
Craig L. Huneke 2/08–1/12
Bryna Kra 2/10–1/14
Joseph H. Silverman 2/09–1/13

Publications Committees

***Bulletin* Editorial Committee**
Susan J. Friedlander 7/05–1/12

***Colloquium* Editorial Committee**
Paul J. Sally Jr. 2/05–1/12

***Journal of the AMS* Editorial Committee**
Karl Rubin 8/09–1/14

***Mathematical Reviews* Editorial Committee**
Ronald M. Solomon 2/10–1/13

***Mathematical Surveys* and *Monographs* Editorial Committee**
Ralph L. Cohen 2/09–1/13

***Mathematics of Computation* Editorial Committee**
Chi-Wang Shu 2/02–1/12

***Proceedings* Editorial Committee**
Ken Ono 2/10–1/14

***Transactions* and *Memoirs* Editorial Committee**
Robert Guralnick 2/05–1/13

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Mark Green 2/10–1/15
Jane M. Hawkins (*ex officio*) 2/11–1/13
William H. Jaco 2/11–1/16
Linda Keen 2/99–1/11
Ronald J. Stern 2/09–1/14
Karen Vogtmann 2/08–1/13
Carol S. Wood 2/02–1/12

Call for NOMINATIONS

The selection committees for these prizes request nominations for consideration for the 2012 awards, which will be presented at the Joint Mathematics Meetings in Boston, MA, in January 2012. Information about past recepients of these prizes may be found in the November 2009 issue of the *Notices*, pp. 1326–1345 and at http://www.ams.org/prizes-awards.

ALBERT LEON WIHTEMAN MEMORIAL PRIZE

The Albert Leon Whiteman Memorial Prize now is awarded every third year, for notable exposition on the history of mathematics. The ideas expressed and understandings embodied in that exposition should reflect exceptional mathematical scholarship.

DISTINGUISHED PUBLIC SERVICE AWARD

This award was established by the AMS Council in response to a recommendation from its Committee on Science Policy. The award is presented to a research mathematician who made a distinguished contribution to the profession in the preceding five years. The US$4,000 prize is awarded every two years.

Nominations should be submitted to the Secretary, Robert J. Daverman, American Mathematical Society, 238 Ayres Hall, Department of Mathematics, University of Tennessee, Knoxville TN 37996-1320. Include a short description of the work that is the basis of the nomination, with complete bibliographic citations when appropriate. A brief curriculum vita of the nominee also should be included. The nominations will be forwarded by the Secretary to the appropriate prize selection committee, which will make final decisions on the awarding of these prizes.

Deadline for nominations is June 30, 2011.

AMS
American Mathematical Society
www.ams.org

Call for Nominations

2012

Frank and Brennie Morgan AMS-MAA-SIAM Prize

for Outstanding Research in Mathematics by an Undergraduate Student

The prize is awarded each year to an undergraduate student (or students having submitted joint work) for outstanding research in mathematics. Any student who is an undergraduate in a college or university in the United States or its possessions, or Canada or Mexico, is eligible to be considered for this prize.

The prize recipient's research need not be confined to a single paper; it may be contained in several papers. However, the paper (or papers) to be considered for the prize must be submitted while the student is an undergraduate; they cannot be submitted after the student's graduation. The research paper (or papers) may be submitted for consideration by the student or a nominator. All submissions for the prize must include at least one letter of support from a person, usually a faculty member, familiar with the student's research. Publication of research is not required.

The recipients of the prize are to be selected by a standing joint committee of the AMS, MAA, and SIAM. The decisions of this committee are final. The 2012 prize will be awarded for papers submitted for consideration no later than June 20, 2011, by (or on behalf of) students who were undergraduates in December 2010.

Questions may be directed to:

Barbara T. Faires
Secretary
Mathematical Association of America
Westminster College
New Wilmington, PA 16172

Telephone: 724-946-6268
Fax: 724-946-6857
Email: faires@westminster.edu

Nominations and submissions should be sent to:

Morgan Prize Committee
c/o Robert J. Daverman, Secretary
American Mathematical Society
238 Ayres Hall
Department of Mathematics
University of Tennessee
Knoxville, TN 37996-1320

THE MATHEMATICAL ASSOCIATION OF AMERICA

siam

Mathematics Calendar

Please submit conference information for the Mathematics Calendar through the Mathematics Calendar submission form at http://www.ams.org/cgi-bin/mathcal-submit.pl.
The most comprehensive and up-to-date Mathematics Calendar information is available on the AMS website at http://www.ams.org/mathcal/.

May 2011

* 2–6 **Advanced Course on Krein-de Branges Spaces of Entire Functions and Old and New Spectral Problems**, Centre de Recerca Matemàtica, Bellaterra, Barcelona, Spain.
Scientific committee: Hakan Hedenmalm, Nikolai Makarov, Joaquim Ortega-Cerdà, Mikhail Sodin.
Information: http://www.crm.cat/acdebrange.

* 7–8 **Southeast Geometry Conference**, University of South Carolina, Columbia, South Carolina.
Description: The 21st annual Southeast Geometry Conference will feature 50-minute talks on topics including geometric analysis and extremization problems. Young researchers and grad students are especially encouraged to participate in the conference, which will include a poster session. If you plan to attend the conference, please let the organizers know for planning purposes. Some NSF support will be available for graduate students, postdocs, and junior researchers.
Local Organizers: Ralph Howard (USC), Thomas Ivey (College of Charleston). Confirmed Speakers: Jacob Bernstein (Stanford), Jason Cantarella (UGA), John Pardon (Princeton), Ray Treinen (Kansas State).
Information: http://www.math.sc.edu/~howard/SEGC2011/.

* 9–13 **DIMACS/CCICADA Workshop on Risk-Averse Algorithmic Decision Making**, DIMACS Center, CoRE Building, Rutgers University, Piscataway, New Jersey.
Short Description: Properly assessing riskiness of available options is central to decision making under uncertainty. Measures of risk are often important in the context of solving dynamic optimization problems. This workshop will concentrate on recent work developing alternative measures and their properties, especially in the context of dynamic optimization. Since a risk measure has to be efficiently computed to be useful, there is a need to develop efficient algorithms that address dynamic risk measures in both the objective function and the constraints. This calls for new methods of ADT. It also calls for new methods for modeling constraints in a dynamic setting in a practically useful manner and for ways to understand the interplay among risk measures, problem formulation, and computational tractability. There is a further need to develop optimization techniques and algorithms for computing risk measures, and to understand the limitations that existing optimization algorithms impose on the viability of a risk measure when efficiency in computing is important. Consequently, the WS will also focus on practically useful modeling techniques and efficient algorithmic approaches for incorporating risk measures in the constraints and/or objective function.
Organizers: Melike Baykal-Gursoy, Rutgers University; gursoy@rci.rutgers.edu; David Brown, Duke University, dbbrown@duke.edu; Aleksandar Pekec, Duke University, pekec@duke.edu; Andrzej Ruszczynski, Rutgers University, rusz@business.rutgers.edu; Dharmashankar Subramanian, IBM Watson Labs, dharmash@us.ibm.com.
Sponsors: DIMACS Special Focus on Algorithmic Decision Theory and The Homeland Security Center for Command, Control, and Interoperability Center for Advanced Data Analysis (CCICADA). Presented under the auspices of the Special Focus on Algorithmic Foundations of the Internet and the Center for Dynamic Data Analysis (DyDAn).
Local Arrangements: Workshop Coordinator, DIMACS Center, workshop@dimacs.rutgers.edu, 732-445-5928.
Deadlines: Pre-registration deadline: May 2, 2011.
Information: http://dimacs.rutgers.edu/Workshops/RiskMeasures/.

This section contains announcements of meetings and conferences of interest to some segment of the mathematical public, including ad hoc, local, or regional meetings, and meetings and symposia devoted to specialized topics, as well as announcements of regularly scheduled meetings of national or international mathematical organizations. A complete list of meetings of the Society can be found on the last page of each issue.
An announcement will be published in the *Notices* if it contains a call for papers and specifies the place, date, subject (when applicable), and the speakers; a second announcement will be published only if there are changes or necessary additional information. Once an announcement has appeared, the event will be briefly noted in every third issue until it has been held and a reference will be given in parentheses to the month, year, and page of the issue in which the complete information appeared. Asterisks (*) mark those announcements containing new or revised information.
In general, announcements of meetings and conferences carry only the date, title of meeting, place of meeting, names of speakers (or sometimes a general statement on the program), deadlines for abstracts or contributed papers, and source of further information. If there is any application deadline with respect to participation in the meeting, this fact should be noted. All communications on meetings and conferences in the mathematical sciences should be sent to the Editor of the *Notices* in care of the American Mathematical Society in Providence or electronically to notices@ams.org or mathcal@ams.org.
In order to allow participants to arrange their travel plans, organizers of meetings are urged to submit information for these listings early enough to allow them to appear in more than one issue of the *Notices* prior to the meeting in question. To achieve this, listings should be received in Providence **eight months** prior to the scheduled date of the meeting.
The complete listing of the Mathematics Calendar will be published only in the September issue of the *Notices*. The March, June/July, and December issues will include, along with new announcements, references to any previously announced meetings and conferences occurring within the twelve-month period following the month of those issues. New information about meetings and conferences that will occur later than the twelve-month period will be announced once in full and will not be repeated until the date of the conference or meeting falls within the twelve-month period.
The Mathematics Calendar, as well as Meetings and Conferences of the AMS, is now available electronically through the AMS website on the World Wide Web. To access the AMS website, use the URL: http://www.ams.org/.

* 10–14 **Riemannian Geometry and Applications "RIGA" 2011**, University of Bucharest, Faculty of Mathematics and Computer Science, Bucharest, Romania.

Description: This is the third Conference Riemannian Geometry and Applications "RIGA", which is devoted to geometry of Riemannian and pseudo-Riemannian manifolds, submanifold theory, structures on manifolds, complex geometry and contact geometry, Finsler, Lagrange and Hamilton geometries, applications to other fields.

Organizing committee: Ion Mihai (University of Bucharest), Adela Mihai (University of Bucharest), Radu Miron (Romanian Academy), Valentin Vlad (Romanian Academy). Invited speakers: Alfonso Carriazo (University of Seville), Bang-Yen Chen (Michigan State University), Franki Dillen (Katholieke Universiteit Leuven), Leopold Verstraelen (Katholieke Universiteit Leuven).

Information: `http://fmi.unibuc.ro/ro; fmi.unibuc.ro/ro/manifestari`.

* 16–18 **Fifth International Conference on Recent Advances in Applied Dynamical Systems**, Shanghai Normal University, Shanghai, People's Republic of China.

Description: This is part of a conference series co-organized by Zhejiang Normal University, Shanghai Normal University and Guangzhou University, chaired by Professor Jibin Li (ZJNU), Maoan Han (SHNU) and Jianshe Yu (GZU), and the previous conferences have been held in Jinhua (2005, 2007, 2010), Guangzhou (2009).

Scientific Committee Co-Chairs: Shui-Nee Chow (Georgia Institute of Technology, USA), Peter Bates (Michigan State University, USA).

Organizing Committee: Honorary Chair: Benyu Guo (Shanghai Normal University, China), Chair: Maoan Han (Shanghai Normal University, China), Co-Chairs: Jibin Li (Zhejiang Normal University and KUST, China) Jianshe Yu (Guangzhou University, China).

Information: `http://jxshix.people.wm.edu/2011shanghai/`.

* 16–20 **Ramification in Algebra and Geometry at Emory**, Emory University, Atlanta, Georgia.

Description: This conference is primarily focused on the interaction between algebra and geometry within the following thematic areas: The arithmetic of linear algebraic groups and homogeneous spaces, cohomological invariants, essential dimension, division algebras and the Brauer group, Chow groups and motivic decomposition, generic Galois extensions, and Brauer-Manin obstructions.

Information: `http://www.mathcs.emory.edu/RAGE/`.

* 18–19 **Workshop on "Entanglement and linking"**, Centro di Ricerca Matematica "Ennio De Giorgi", Collegio Puteano, Piazza dei Cavalieri 3, 56126, Pisa, Italy.

Description: This workshop will be mainly devoted to differential geometric, topological and hydrodynamical aspects of ordinary and higher-order linking, braiding and their asymptotic versions (helicity and its higher order analogues); integral formulae for higher order linking numbers and the quest for generalized "helicity bounds energy" results and generalizations of Whitehead products, link-homotopy invariants of links and the homotopy invariants of associated maps to configuration space, with applications to magnetohydrodynamics of plasmas, vortices, and relaxations of braided magnetic fields.

Invited Speakers: P. Akhmetiev, Izmiran, Russia; M. Berger, Exeter Univ., United Kingdom; G. Hornig, Dundee Univ., United Kingdom; C.-C. Hsieh, Academia Sinica, TPE; L. Kauffman, Univ. Illinois, Chicago, USA; R. Komendarczyk, Tulane Univ., USA; M. Lefranc, Lille Univ., France; J. Parsley, Wake Forest Univ., USA; R. Ricca, Univ. Milano-Bicocca, Italy.

Short Talks: A. Benvegnù, Univ. Verona, Italy; F. Maggioni, Univ. Bergamo, Italy; C. Shonkwiler, Haverford College, USA.

* 23–27 **7th conference on Stochastic Analysis, Random Fields and Applications**, Centro Stefano Franscini (Monte Verita), Ascona, Switzerland.

Description: May 23–24 will be devoted to stochastic methods in financial models, May 23–27 to stochastic analysis, random fields and other applications than finance.

Information: `http://mathaa.epfl.ch/prob/ascona/2011/index.html`.

* 26–29 **Waterloo Workshop in Computer Algebra, WWCA 2011 Celebrating Herbert S. Wilf's 80th birthday, W80**, Wilfrid Laurier University, Waterloo, Ontario, Canada.

Description: The event features 27 Invited Speakers.

Information: `http://www.cargo.wlu.ca/W80`.

June 2011

* 6–8 **Perspectives in Mathematics and Life Sciences**, Fundación Euroárabe (C/San Jerónimo 27), Granada, Spain.

Coordinator: Juan Soler.

Organizing Committee: Juan J. Nieto, Juan Calvo, Óscar Sánchez, José L. López, Tomás Alarcón.

Scientific Committee: Ángel Calsina, Carmen Cadarso, Manuel Doblaré, Miguel A. Herrero, Ángel Sánchez, Juan Soler.

Speakers: Bascompte, Jordi; Bellomo, Nicola; Beresticky, Henri; Berlyand, Leonid; Carleton, Alan; Caselles, Vicent; Cuesta, José; Deco, Gustavo; Degond, Pierre; Frangi, Alejandro; García Ojalvo, Jordi; Grenier, Emmanuel; Jabin, Pierre-Emmanuel; Meinhardt, Hans; Mimura, Masayasu; Perthame, Benoit; Ruíz i Altaba, Ariel.

Information: `http://www.ugr.es/~kinetic/biomat`.

* 8–9 **Workshop on "Numerical knots: models & simulations"**, Centro di Ricerca Matematica "Ennio De Giorgi", Collegio Puteano, Piazza dei Cavalieri 3, 56126 Pisa, Italy.

Description: In this workshop we will focus on modeling open and closed chains in a variety of biological and physical contexts, on the analysis of the topological, geometric, and other properties of the modeled chains. Examples include lattice models, wormlike chains, ideal knots, shape and scale of knotting, localization, constrained environments, and knot energies with applications to DNA, proteins, viral capsids and polymer melts.

Plenary Speakers: Jason Cantarella, UG, Athens, USA; Cristian Micheletti, SISSA, Trieste, Italy; Enzo Orlandini, INFN, IT; Eric Rawdon, STU, St. Paul, USA; Rob Scharein, SFSU, San Francisco, USA; Doros Theodorou, NTUA, Athens, Greece; Mariel Vazquez (to be confirmed), SFSU, San Francisco, USA; Peter Virnau, UM, Mainz, Germany.

Short Talks: Eleni Panagioutou, NTUA, Athens, Greece; Luca Tubiana, SISSA, Trieste, Italy; Christos Tzoumanekas (to be confirmed), NTUA, Athens, Greece; Guillaume Witz (to be confirmed), EPFL, Lausanne, Zwitzerland.

Thematic Workshop Conveners: Ken Millett, Eric Rawdon, Rob Scharein (to be confirmed).

Contact: email: `millett@math.ucsb.edu`. (This event is part of the intensive research period "Knots Applications").

Information: `http://www.crm.sns.it/event/206/`.

* 13–15 **The Mathematics of Porous Media**, Radisson Blu Resort, Split, Croatia.

Description: The aim of this International Conference is to celebrate the 60th birthday of Professor C. J. (Hans) van Duijn. With his work, started in the 70s, Hans has essential and pioneering contributions to the Mathematics of Porous Media. He is still an active researcher in this field, in spite of taking the lead of the Eindhoven University of Technology in the past seven years. The programme will consist of keynote lectures, two lectures dealing with aspects of the scientific work of C. J. van Duijn, shorter lectures (by invitation) and poster presentations. The number of participants is limited to 35. At the moment no parallel sections are planned. We address the invitation to participate in this conference to the friends and colleagues of of Hans van Duijn, but further to all younger researchers and Ph.D. students who work in this exiting field or want to learn more about it.

Information: http://www.am.uni-erlangen.de/modeling-simulation-optimization/mpm2011.html.

* 13–16 **Poisson Geometry and Applications**, Figueira da Foz, Portugal.
Description: The aim of the conference is to bring together researchers working in Poisson geometry and related topics (like integrable systems, geometric mechanics and quantization).
Invited speakers: J. Cariñena (Zaragoza), M. Crainic (Utrecht), I. Cruz (Oporto), D. Iglesias-Ponte (La Laguna), Y. Kosmann-Schwarzbach (Paris), C. Laurent-Gengoux (Coimbra), R. Loja Fernandes (Lisbon), C. M. Marle (Paris), J. C. Marrero (La Laguna), E. Martinez (Zaragoza), F. Petalidou (Thessaloniki), T. Ratiu (Lausanne), V. Rubtsov (Angers), F. Schätz (Lisbon), M. Stiénon (Penn-State Univ.), P. Xu (Penn-State Univ.), M. Zambon (Madrid).
Registration: To register please follow the instructions in the conference website: http://www.mat.uc.pt/~poissonga/ (click on the link "On-line registration" in the left side menu). Please send your questions or comments to: poissonga@mat.uc.pt.
Information: http://www.mat.uc.pt/~poissonga/.

* 14–23 **Advanced Course on Dynamical Systems**, Centre de Recerca Matemàtica, Bellaterra, Barcelona, Spain.
Scientific committee: George Elliott, Andrew S. Toms.
Information: http://www.crm.cat/acdynamical.

* 15–16 **Workshop on "DNA knots"**, Centro di Ricerca Matematica "Ennio De Giorgi", Collegio Puteano, Piazza dei Cavalieri 3, 56126 Pisa, Italy.
Description: Viruses pack DNA very tightly in capsids; upon release and circularization in the globular phase, the DNA knots produced can inform about the packing geometry. DNA is also packed very tightly in eukaryotic chromosomes, and some DNA knotting and linking may be present in the chromosome organization.This workshop will have talks discussing various aspects of DNA knotting and the relationship of knotting to biological understanding.
Plenary Speakers: Javier Arsuaga, SFSU, San Francisco, USA; Dorothy Buck, Imperial College London, UK; Isabel Darcy, Univ. Iowa, USA; Cristian Micheletti, SISSA, Trieste, Italy; Andrzej Stasiak, Univ. Lausanne, Switzerland; De Witt Sumners, Florida State Univ., USA; Mariel Vazquez (to be confirmed), SFSU, San Francisco, USA.
Short Talks: Marco Baiesi (to be confirmed), Univ. Padova, Italy; Mauro Mauricio, Imperial College London, United Kingdom; Angelo Rosa (to be confirmed), IIT, Genova, Italy; Luca Tubiana, SISSA, Trieste, Italy; Karin Valencia, Imperial College London, United Kingdom.
Thematic Workshop Conveners: D. Buck, C. Micheletti, D. Sumners (to be confirmed).
Contact: email: sumners@math.fsu.edu (This event is part of the intensive research period "Knots and Applications").
Information: http://www.crm.sns.it/event/207/.

* 15–18 **International Conference on Mathematical Methods and Models in Biosciences (BIOMATH 2011)**, Bulgarian Academy of Sciences, Sofia, Bulgaria.
Description: The conference is devoted to recent research in biosciences based on applications of mathematics as well as mathematics applied to or motivated by such applications. It is a multidisciplinary meeting forum for researchers who apply mathematical tools and methods of analysis to the study of phenomena in the broad fields of biology, ecology, medicine, biotechnology and bioengineering. Advanced mathematical tools relevant to deal with life science problems are also welcome. Contributed talks in any of these fields are invited for presentation at the conference.
Abstract deadline: April 30, 2011.
Information: http://www.biomath-bg.eu.

* 20–23 **2011 Workshop on Mixed Integer Programming (MIP2011)**, University of Waterloo, Waterloo, Ontario, Canada.
Description: The Mixed Integer Programming (MIP) workshop series brings together the integer optimization research community in an annual meeting, to discuss very recent developments in the field. It consists of a single track of invited talks, which are scheduled to leave ample time for discussion and interaction among the participants. To encourage participation of students and junior researchers, no registration fee is charged, and financial support is available. The workshop features a poster session.
Deadline: For poster abstract submissions and applications for travel support is March 15, 2011.
Information: http://www.math.uwaterloo.ca/~mip2011.

* 20–24 **2011 Casablanca International Workshop on Mathematical Biology**, University Hassan II, Casablanca, Morocco.
Description: This workshop intends to bring together expert researchers from around the world to exchange ideas and share their research results about all aspects of Mathematical Biology in general and the use of optimal control theory in biology and medicine in particular. This meeting also intends to promote and explore new collaborations among the international scientific community and in particular, researchers in Africa and Morocco. The workshop will include nine plenary talks by leading researchers in mathematical biology, parallel sessions of invited and contributed talks, a graduate students and postdoctoral fellows poster session and an expert panel to discuss current problems and future directions in mathematical modeling of biological processes.
Information: http://sites.google.com/a/asu.edu/cicwmb/.

* 21–24 **Summer School Geodesic Flow, Negative Curvature and Isomorphism Conjectures**, Georg-August Universität, Göttingen, Germany.
Description: School (for graduate students) on geodesic flow in negative curvature (broadly defined) and its applications. Second part of the school on the Farrell-Jones isomorphism conjecture (one of the applications) and more general aspects of geometric group theory.
Speakers: Patrick Eberlein (University of North Carolina), Wolfgang Lück (Bonn), Romain Tessera (Lyon).
Information: http://www.uni-math.gwdg.de/geoflow.

* 26–July 1 **II Jaen Conference on Approximation Theory, Computer Aided Geometric Design, Numerical Methods and Applications**, Ubeda, Jaen, Spain.
Description: The objective of this conference is to provide a useful and nice forum for researchers in the subjects to meet and discuss. In this sense, the conference programs have been designed to keep joined the group during five days with a program full of scientific and social activities. The Conference will take place in Úbeda, a World Heritage Site.
Invited speakers: Allan Pinkus, Technion, Israel; Francisco Marcellán, Universidad Carlos III de Madrid, Spain; Hrushikesh N. Mhaskar, California State University, USA; Igor Shevchuk, University of Kyiv, Ukraine; Kirill A. Kopotun, University of Manitoba, Canada; Ronald DeVore, Texas A&M University, USA; Tom Lyche, University of Oslo, Norway; Zeev Ditzian, University of Alberta, Canada. We invite you to contribute a talk or a poster. We provide the possibility of organizing minisymposia on subjects of current interest. The conference will be dedicated to Prof. Dany Leviatan on the occasion of his retirement.
Information: http://www.ujaen.es/revista/jja/jca/.

* 27–July 1 **3rd Workshop on Fluids and PDE**, University of Campinas, Campinas, SP Brazil.
Description: This is the third edition in a series of workshops aimed at fostering collaboration among the growing community of researchers working in Brazil on mathematical fluid mechanics, in particular on the incompressible Navier-Stokes and Euler equations, as well as stimulating further collaboration with the international community. This edition of the workshop will feature a series of tutorial lectures (the first day) aimed at graduate students and young researchers, followed by specialized seminars by leading experts in the field. There is space for contributed short talks as well as poster

presentations. Register at the website; to submit an oral communication or a poster and to apply for funding the registration must be received no later than May 27th, 2011.

Information: http://www.ime.unicamp.br/~fluid/WFEDP3.

July 2011

* 4–8 **2011 Taiwan International Conference on Geometry: Special Lagrangians and Related Topics**, Department of Mathematics, National Taiwan University, Taipei, Taiwan.

Description: This conference will start a series of bi-yearly international conferences on differential geometry in Taiwan. An important area in geometry will be specified as the main theme each time. Our purpose is to create a discussion and interaction platform in the chosen area, and at the same time to foster future cooperations and introduce new people into the field. Additional short courses may also be arranged around the same time as the conference.

Topic: Special Lagrangians and Related Topics will include special Lagrangians, Lagrangian mean curvature flow, J-holomorphic curve techniques for Lagrangians, and the calibrated geometries in general. The tentative topic for 2013 is "Geometry and General Relativity."

Information: http://www.tims.ntu.edu.tw/workshop/Default/index.php?WID=111.

* 6–8 **DIMACS Workshop on Competitive Algorithms for Packet Scheduling, Buffering and Routing in the Internet**, DIMACS Center, CoRE Building, Rutgers University Piscataway, New Jersey.

Description: The Internet employs the store-and-forward packet switching paradigm, where packets are forwarded in a hop-by-hop manner toward their destinations, and are stored at each intermediate router until they can be sent to the next intermediate router, chosen according to the routing policy. The main advantage of the packet switching paradigm is the ability to share a common infrastructure for multiple connections using statistical multiplexing, which allows one to create networks at much lower cost with greater throughput, flexibility, and robustness. In this workshop we will consider optimization problems under various parameters that arise in the design and management of packet networks, and the algorithms to solve them together with their competitive analysis. We are interested in algorithms that perform the various tasks needed in a packet network, such as routing, buffering or scheduling. We are interested both in work that focuses on a single component of the network, such as a single buffer or a single switch (router), and in work that considers the network as a whole, such as work on routing or the interaction of the routing and scheduling decisions. Finally, we are interested both in work that uses "classical" competitive analysis, and in work that suggests methods to render the worst case approach of competitive analysis less worst-case oriented.

Organizers: Alex Kesselman, Google; Alex Lopez-Ortiz, University of Waterloo, alopez-ou at waterloo.ca; Yishay Mansour, Tel Aviv University; Adi Rosen, CNRS, Paris.

Deadlines: Pre-registration deadline: June 29, 2011.

Information: http://dimacs.rutgers.edu/Workshops/Packet/.

* 9–10 **Directions in Matrix Theory 2011**, Department of Mathematics, University of Coimbra, Portugal.

Description: This workshop aims to be a forum of discussion of frontline areas of Matrix Theory and its Applications. The workshop will bring together renowned researchers of diverse fields fostering the exchange of experiences and insights from different perspectives.

Information: http://www.mat.uc.pt/~cmf/Directions2011/DMT2011.htm.

* 11–15 **Workshop in Mathematical Physics**, Agia Napa, Cyprus.

Description: Part of Sigma Phi International Conference in Statistical Physics.

Topics: Classical and quantum integrable systems, Symmetries of Differential equations, Variational methods, Evolution equations, Geometric methods in Classical Mechanics, Poisson and Symplectic Geometry.

Information: http://www.sigmaphi2011.org/events-detail.aspx?IDEvento=35.

* 12–13 **Workshop on "Topological dynamics in physics and biology"**, Centro di Ricerca Matematica "Ennio De Giorgi", Collegio Puteano, Piazza dei Cavalieri 3, 56126 Pisa, Italy.

Description: With this workshop we shall investigate the role of topological properties of knot theory in some physical and biological systems, by exploring how topological invariants influence and constrain the dynamics and the energy minimization of physical systems, from optics to polymer physics, from magnetic and nematic systems to soap films.

Invited Speakers: Dorothy Buck, Imperial College, UK; Tetsuo Deguchi, Ochanumizu Univ., Japan; Mark Dennis, Bristol Univ., UK; Giovanni Dietler, EPFL, Switzerland; Ray Goldstein (to be confirmed), Cambridge Univ., UK; Cristian Micheletti, SISSA, Trieste, Italy; Michael Monastyrsky, ITEP, Federal Republic of Russia; Enzo Orlandini, INFN, Padova, Italy; Adriana Pesci (to be confirmed), Cambridge Univ., UK; Andrzej Stasiak, Univ. Lausanne, Switzerland; De Witt Sumners, Florida State Univ., USA.

Contact: email: renzo.ricca@unimib.it. (This event is part of the intensive research period "Knots and Applications").

Information: http://www.crm.sns.it/event/221/.

* 18–22 **Infinity Conference**, Centre de Recerca Matemàtica, Bellaterra, Barcelona, Spain.

Scientific committee: Sy Friedman, John Baldwin, Maria Luisa Bonet, Juan Carlos Martínez, Michael Rathjen.

Information: http://www.crm.cat/cinfinity.

* 24–29 **ICDEA2011: 17th International Conference on Difference Equations and Applications**, Université du Québec à Trois-Rivières, Québec, Canada.

Description: The conference is organized by the Department of Mathematics and Computer Science and the Department of Physics of the Université du Québec à Trois-Rivières (UQTR), under the auspices of the International Society of Difference Equations (ISDE). The purpose of the conference is to bring together experts and students in the theory and applications of difference equations and discrete dynamical systems.

Plenary speakers: Will be experts chosen from the many areas of difference equations as well as experts in discrete dynamical systems and their interplay with nonlinear science. Contributed talks in any area of difference equations are welcome.

Scientific Committee: George Sell (chair), Saber Elaydi, Jean Mawhin, Linda Allen, and Adel Antippa.

Local organizing committee: Adel Antippa (chair), Louis Marchildon (chair), Ismaïl Biskri (chair), Boucif Amar Bensaber, Pierre Bénard, Jean-Sébastien Boisvert, Jérémie Gobeil, Alain Goupil, Jérémie Mathieu, Mhamed Mesfioui, Sébastien Tremblay.

Information: For more information please contact Dr. Adel Antippa (email: afa@adelantippa.com) or visit http://www.icdea2011.org.

* 24–30 **Dynamical models in life sciences**, University of Evora, Evora, Portugal.

Description: The aim of this summer school is to bring together specialists and students in mathematics, biology, physics and engineering with a common goal: the better understanding of the mathematics behind life sciences and the better understanding of life sciences using a deeper knowledge of mathematics. This will be a joint event of the Portuguese Centro Internacional de Matem · tica, the European Society for Mathematical and Theoretical Biology and the European Mathematical Society in the historical city of Evora, Portugal, providing the perfect environment for the interaction

between senior scientists in the field and students. It will consist of 7 mini-courses aimed to Ph.D. students and junior post-docs, in mathematics, biology and related areas.
Information: http://c3.glocos.org/ssmtb/.

* 25–29 **NSF-CBMS Regional Research Conference, Mathematical Epidemiology and its Applications**, East Tennessee State University, Johnson City, Tennessee.
Description: Carlos Castillo-Chavez and Fred Brauer will give ten keynote lectures, and there will be breakout sessions and formation of working groups. A poster session will also be held for participants to display their work. Full support will be offered to between 35 and 40 participants. See webpage for more details.
Information: http://www.etsu.edu/cas/math/cbms.aspx.

August 2011

* 18–20 **The First Strathmore University Mathematics Conference. "Strengthening Mathematics Research, Applications and Education in Africa"**, Strathmore University, Nairobi, Kenya.
Description: The Centre of Advanced Research in Mathematical Sciences (CARMS) of Strathmore University will hold its "First International Conference in Mathematical Sciences".
Theme: "Strengthening Mathematics Research, Applications and Education in Africa". The first morning of the conference will also serve as a launch for CARMS. The theme of the launch is "The Role of Mathematics and Statistics in Vision 2030, Kenya". The conference will feature plenary sessions, invited speakers, poster presentations, and be preceded by workshops. The five proposed workshops cover a wide area of interests. The main objective of this conference is to provide a forum for researchers, educators, students and industries interested in mathematical sciences. This process will involve local, regional and international mathematicians.
Information: http://www.strathmore.edu/maths/.

* 31–September 3 **XX International Fall Workshop on Geometry and Physics**, Instituto de Ciencias Matemáticas, ICMAT (CSIC-UAM-UC3M-UCM), Madrid, Spain.
Description: The Fall Workshops on Geometry and Physics have been held yearly since 1992, and bring together Spanish and Portuguese geometers and physicists, along with an ever increasing number of participants from outside the Iberian peninsula. The meetings aim to provide a forum for the exchange of ideas between researchers of different fields in differential geometry, applied mathematics and physics, and always include a substantial number of enthusiastic young researchers amongst the participants.
Program: Includes: Mini courses: Alberto Ibort (Univ. Carlos III), Franco Magri (Univ. Milano Bicocca).
Invited speakers: Mauro Carfora (Univ. Pavia), Piotr T. Chrusciel (Univ. Vienna), Marisa Fernández (Univ. PaÌs Vasco), Gary Gibbons (Univ. Cambridge), Janusz Grabowski (Polish Academy of Sciences), Renate Loll (Utrecht Univ.), Patrizia Vitale (Naples Univ.).
Registration/Deadline: The on-line registration will be opened on March 1, 2011.
Information: http://www.icmat.es/congresos/xxifwgp/.

September 2011

* 5–9 **AGMP&MP2 Summer School: Mathematical Topics in Quantum Mechanics and Quantum Information**, Tjärnö, Sweden.
Description: The aim of the Summer School is to introduce Ph.D. students and junior researchers of mathematical and physical specialities to mathematical methods of quantum mechanics and quantum information.
Information: http://www.agmp.eu/3q11/.

* 5–10 **Toric Topology and Automorphic Functions**, Pacific National University, Khabarovsk, Russia.
Program Committee: Victor Buchstaber (Steklov Mathematical Institute, Moscow, chairman); Mikhail Guzev (Institute of Applied Mathematics, Vladivostok, vice-chairman); Frederic Cohen (Rochester University, USA); Mikiya Masuda (Osaka City University, Japan); Taras Panov (Moscow State University); Iskander Taimanov (Sobolev Institute of Mathematics); Alexander Podgaev (Pacific National University).
Important dates: Deadline for registration: April 30, 2011. Deadline for submission of abstracts: May 30, 2011. Recommended arrival day: September 4, 2011. September 5–10, 2011: Conference.
Information: See http://iam.khv.ru/ttaf-2011/the1st_announcement.htm.

* 11–15 **Israeli-Polish Mathematical Meeting**, University of Łódz´, Poland.
Description: Israeli-Polish Mathematical Meeting is a joint initiative of the Polish Mathematical Society (PTM) and Israel Mathematical Union (IMU). The main purpose of the conference is to stimulate the exchange of new ideas in all aspects of mathematics. Mathematicians from other countries are also cordially invited to participate. The meeting program will cover many topics pertaining to mathematical research conducted in Israel and Poland. The meeting will consist of plenary lectures, thematic sections and other informal scientific discussions and social activities as well. All the talks will be in English. The meeting will be hosted by the University of Łódz´, Faculty of Mathematics and Computer Science, Łódz´, Poland. The conference will start in the morning of Monday, September 12, 2011 and end in the afternoon of Thursday, September 15, 2011.
Information: http://imuptm.math.uni.lodz.pl.

* 12–16 **Satellite Meeting Unravelling and Controlling Discrete Dynamical Systems**, Vienna, Austria.
Description: The Satellite Meeting Unravelling and Controlling Discrete Dynamical Systems is organized during the 8th European Conference on Complex Systems (ECCS).
Goals: Are fourfold: foster the interest in discrete dynamical systems; communicate the most recent advances on the means to quantitatively grasp and control discrete dynamical systems; perpetuate the dynamical systems point of view on the intriguing dynamics of cellular automata; gain insight into the dynamics of models based upon discrete modelling paradigms.
Deadline: For abstract submission: May 10, 2011.
Information: http://www.disdysys.ugent.be/.

October 2011

* 7–9 **ICMA2011: The Third Conference on Mathematical Modeling and Analysis of Populations in Biological Systems**, Trinity University, San Antonio, Texas.
Description: This conference will build upon the success of the previous two conferences that were held on the campuses of the University of Arizona in 2007 and University of Alabama Huntsville in 2009. Furthermore, the proceedings of ICMA I were published in two issues of the *Journal of Biological Dynamics* and one issue in the *Journal of Difference equations*. The proceedings of ICMA II will be published in two issues of the *Journal of Biological Dynamics*. The main focus of this conference will be on ecology and evolution. In addition, there will be a focus on ecological and evolutionary problems in related fields, such as environmental science, epidemiology, etc. We will invite researchers working on model derivation and analysis, data testing, and model simulations in these fields.
Scientific Committee: Chris Cosner, Jim Cushing (chair), Maia Martcheva, Gail Wolkowicz, and Sebastian Schrieber.
Organizing Committee: Saber Elaydi (chair), Jim Cushing, Jia Li, David Ribble, Peter Olofsson, and Cabral Balreira.
Information: For more information contact Dr. Saber Elaydi, Conference Director (email: selaydi@trinity.edu) and visit http://web.trinity.edu/x8339.xml.

* 10–14 **AIM Workshop: Weighted singular integral operators and non-homogenous harmonic analysis**, American Institute of Mathematics, Palo Alto, California.
Description: This workshop, sponsored by AIM and the NSF, will focus on recent developments on weighted inequalities for singular integral operators and the connection with questions in geometric measure theory and PDEs.
Information: `http://aimath.org/ARCC/workshops/singularintops.html`.

* 24–26 **Algebra Geometry Mathematical Physics**, University of Haute Alsace, Mulhouse, France.
Main topics: Include, but are not limited to: Deformation theory and quantization, Hom-algebras and n-ary algebraic structures, Hopf algebra, quantum algebra, Integrable systems, Jet theory and Weil bundles, Lie theory and applications, Noncommutative and nonassociative algebra, Representation theory, Number theoretical methods in string theory, Quantum geometry, Spectral and computational methods for Maxwell equations, Ternary algebras and applications.
Information: `http://www.agmp.eu/mul11/`.

* 24–28 **AIM Workshop: The Kardar-Parisi-Zhang equation and universality class**, American Institute of Mathematics, Palo Alto, California.
Description: This workshop, sponsored by AIM and the NSF, will be devoted to the study of the the Kardar-Parisi-Zhang equation and universality class.
Information: `http://aimath.org/ARCC/workshops/kpzequation.html`.

* 31–November 4 **AIM Workshop: Geometry of large networks**, American Institute of Mathematics, Palo Alto, California.
Description: This workshop, sponsored by AIM and the NSF, is devoted to geometric models of large networks. It intends to bring together mathematicians, computer scientists, and engineers.
Information: `http://aimath.org/ARCC/workshops/largenetworks.html`.

November 2011

* 7–11 **AIM Workshop: The Klein project**, American Institute of Mathematics, Palo Alto, California.
Description: This workshop, sponsored by AIM and the NSF, will be devoted to the development of the Klein Project, specifically to the preparation of "vignettes" that characterise important ideas within the field of the mathematical sciences using specific examples.
Information: `http://aimath.org/ARCC/workshops/kleinproject.html`.

* 17–18 **Jornadas de Criptografía (Spanish Cryptography Days)**, Murcia, Spain.
Description: As part of the activities of the Centenary of the Real Sociedad Española de Matemáticas the Algebra Group of Murcia organize the Jornadas de Criptografía SCD2011 with the aim of bringing together researchers in cryptography, including mathematicians and computer scientists, as well as industrial and government organizations interested in new developments and applications of cryptography.
Information: `http://www.um.es/docencia/jsimon/CongresoCripto/CryptographyMeeting.html`.

December 2011

* 5–9 **AIM Workshop: Stability, hyperbolicity, and zero localization of functions**, American Institute of Mathematics, Palo Alto, California;
Description: This workshop, sponsored by AIM and the NSF, will be devoted to the emerging theory of stability and hyperbolicity of functions, in particular the cases of polynomials and analytic functions of one or several variables.
Information: `http://aimath.org/ARCC/workshops/hyperbolicpoly.html`.

January 2012

* 4–7 **Joint Mathematics Meetings**, Boston, Massachusetts.
Information: `http://www.ams.org/meetings/national/jmm/2138_intro.html`.

* 23–27 **AIM Workshop: Set theory and C^*-algebras**, American Institute of Mathematics, Palo Alto, California.
Description: This workshop, sponsored by AIM and the NSF, will be devoted to applications of set theory to C^*-algebras.
Information: `http://aimath.org/ARCC/workshops/settheorycstar.html`.

March 2012

* 3–4 **AMS Western Section Meeting**, University of Hawaii, Honolulu, Hawaii.
Information: `http://www.ams.org/meetings/sectional/sectional.html`.

* 10–11 **AMS Southeastern Section Meeting**, University of South Florida, Tampa, Florida.
Information: `http://www.ams.org/meetings/sectional/sectional.html`.

* 12–16 **AIM Workshop: Classifying fusion categories**, American Institute of Mathematics, Palo Alto, California.
Description: This workshop, sponsored by AIM and the NSF, will be devoted to the classification problem for fusion categories, including those with additional structure, e.g., ribbon and modular fusion categories. More specifically, we will focus on the development and application of both theoretical and computational techniques for classifying fusion categories that are "small" in various senses.
Information: `http://aimath.org/ARCC/workshops/fusioncat.html`.

* 17–18 **AMS Eastern Section Meeting**, George Washington University, Washington, District of Columbia.
Information: `http://www.ams.org/meetings/sectional/sectional.html`.

* 30–April 1 **AMS Central Section Meeting**, University of Kansas, Lawrence, Kansas.
Information: `http://www.ams.org/meetings/sectional/sectional.html`.

The following new announcements will not be repeated until the criteria in the next to the last paragraph at the bottom of the first page of this section are met.

May 2012

* 20–2 **7th European Conference on Elliptic and Parabolic Problems**, Gaeta, Italy.
Description: Besides Elliptic and Parabolic issues, the topics of the conference include Geometry, Free Boundary Problems, Fluid Mechanics, Evolution Problems in general, Calculus of Variations, Homogenization, Control, Modeling and Numerical Analysis. In addition to the plenary talks parallel sessions and minisymposia will be organized.
Information: `http://www.math.uzh.ch/gaeta2012`.

Classified Advertisements

Positions available, items for sale, services available, and more

ALABAMA

SHELTON STATE COMMUNITY COLLEGE
Department of Mathematics
Instructor, Mathematics

Shelton State Community College is accepting applications for: Instructor, Mathematics. For Information, visit `http://www.sheltonstate.edu`, or call the Office of Human Resources at 205-391-2272 EOE

TEXAS

TEXAS A&M UNIVERSITY
Senior Faculty Position in Applied and Computational Mathematics

As part of Texas A&M University's recognition of the increasing importance of the modeling and computational sciences, the Department of Mathematics (`http://www.math.tamu.edu`) is recruiting for a senior faculty position in applied and computational mathematics. This position is one of three new senior lines dedicated to computational science that were created as part of an initiative led by the Institute for Applied Mathematics and Computational Science (`http://iamcs.tamu.edu`). Considerable startup funding is available.

Computational science has become inherently multidisciplinary. As a result, successful candidates for this position should be able to demonstrate a strong record of research accomplishments and leadership both within the mathematics discipline and in multidisciplinary initiatives. Documentation of such success should include a record of publication in both mathematics and a multidisciplinary application area and examples of collaboration and program building. Special emphasis will be placed on applied analysis and scientific computation. Areas of particular interest are multiscale modeling and simulations as well as uncertainty analysis.

Additional information can be obtained by contacting the search committee chair, Dr. Jay R. Walton (`jwalton@math.tamu.edu`).

Individuals wishing to be considered for this position should send a copy of their CV and a letter of interest to:

Dr. Jay R. Walton, Chair
IUMRI Mathematics Search Committee
Department of Mathematics
3368 TAMU
Texas A&M University
College Station, Texas 77843-3368

Electronic submissions will also be accepted and should be sent to: `jwalton@math.tamu.edu`, with IUMRI Mathematics Position in the Subject Line. Additional information and letters of reference will be solicited after a preliminary review. Review of the applicant pool will begin April 1, 2011. Start dates are flexible, and the position will remain open until filled.

Texas A&M University is an Equal Opportunity Employer and has a policy of being responsive to the needs of dual-career couples.

000031

CHILE

PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE
Departamento de Matemticas

The Department of Mathematics invites applications for three tenure-track positions at the assistant professor level beginning either March or August 2012. Applicants should have a Ph.D. in mathematics, proven research potential either in pure or applied mathematics, and a strong commitment to teaching and research. The regular teaching load for assistant professors consists of three one-semester courses per year, reduced to two during the first two years. The annual salary will be US$45,000 (calculated at the current exchange rate of 500 chilean pesos per dollar).

Please send a letter indicating your main research interests, potential collaborators in our department (`http://www.mat.puc.cl`), detailed curriculum vitae, and three letters of recommendation to:

Director Departamento de Matemticas,
Pontificia Universidad Catolica de Chile,
Av. Vicua Mackenna 4860 Santiago, Chile;
fax: (56-2) 552-5916;
email: `mmusso@mat.puc.cl`

For full consideration, complete application materials must arrive by June 30, 2011.

000027

Suggested uses for classified advertising are positions available, books or lecture notes for sale, books being sought, exchange or rental of houses, and typing services.

The 2011 rate is $3.25 per word. No discounts for multiple ads or the same ad in consecutive issues. For an additional $10 charge, announcements can be placed anonymously. Correspondence will be forwarded.

Advertisements in the "Positions Available" classified section will be set with a minimum one-line headline, consisting of the institution name above body copy, unless additional headline copy is specified by the advertiser. Headlines will be centered in boldface at no extra charge. Ads will appear in the language in which they are submitted.

There are no member discounts for classified ads. Dictation over the telephone will not be accepted for classified ads.

Upcoming deadlines for classified advertising are as follows: June/July 2011 issue–April 28, 2011; August 2011 issue–May 27, 2010; September 2011 issue–June 28, 2011; October 2011 issue–July 28, 2011; November 2011 issue–August 30, 2011; December issue–September 28, 2011.

U.S. laws prohibit discrimination in employment on the basis of color, age, sex, race, religion, or national origin. "Positions Available" advertisements from institutions outside the U.S. cannot be published unless they are accompanied by a statement that the institution does not discriminate on these grounds whether or not it is subject to U.S. laws. Details and specific wording may be found on page 667 (vol. 56).

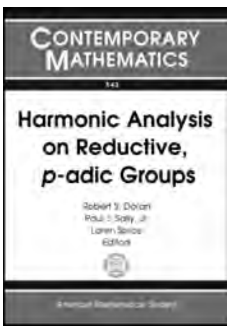
Situations wanted advertisements from involuntarily unemployed mathematicians are accepted under certain conditions for free publication. Call toll-free 800-321-4AMS (321-4267) in the U.S. and Canada or 401-455-4084 worldwide for further information.

Submission: Promotions Department, AMS, P.O. Box 6248, Providence, Rhode Island 02940; or via fax: 401-331-3842; or send email to `classads@ams.org`. AMS location for express delivery packages is 201 Charles Street, Providence, Rhode Island 20904. Advertisers will be billed upon publication.

New Publications Offered by the AMS

To subscribe to email notification of new AMS publications, please go to http://www.ams.org/bookstore-email.

Algebra and Algebraic Geometry



Harmonic Analysis on Reductive, p-adic Groups

Robert S. Doran, *Texas Christian University, Ft. Worth, TX*, **Paul J. Sally, Jr.**, *University of Chicago, IL*, and **Loren Spice**, *Texas Christian University, Ft. Worth, TX*, Editors

This volume contains the proceedings of the AMS Special Session on Harmonic Analysis and Representations of Reductive, p-adic Groups, which was held on January 16, 2010, in San Francisco, California.

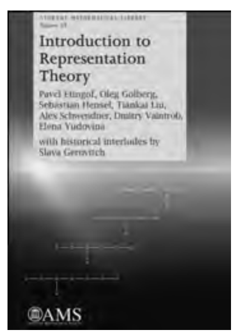
One of the original guiding philosophies of harmonic analysis on p-adic groups was Harish-Chandra's Lefschetz principle, which suggested a strong analogy with real groups. From this beginning, the subject has developed a surprising variety of tools and applications. To mention just a few, Moy-Prasad's development of Bruhat-Tits theory relates analysis to group actions on locally finite polysimplicial complexes; the Aubert-Baum-Plymen conjecture relates the local Langlands conjecture to the Baum-Connes conjecture via a geometric description of the Bernstein spectrum; the p-adic analogues of classical symmetric spaces play an essential role in classifying representations; and character sheaves, originally developed by Lusztig in the context of finite groups of Lie type, also have connections to characters of p-adic groups.

The papers in this volume present both expository and research articles on these and related topics, presenting a broad picture of the current state of the art in p-adic harmonic analysis. The concepts are liberally illustrated with examples, usually appropriate for an upper-level graduate student in representation theory or number theory. The concrete case of the two-by-two special linear group is a constant touchstone.

Contents: P. N. Achar and **C. L. R. Cunningham**, Toward a Mackey formula for compact restriction of character sheaves; **J. D. Adler**, **S. DeBacker**, **P. J. Sally, Jr.**, and **L. Spice**, Supercuspidal characters of SL_2 over a p-adic field; **A.-M. Aubert**, **P. Baum**, and **R. Plymen**, Geometric structure in the representation theory of reductive p-adic groups II; **B. Casselman**, The construction of Hecke algebras associated to a Coxeter group; **J. Hakim** and **J. M. Lansky**, Distinguished supercuspidal representations of SL_2; **J.-L. Kim** and **J.-K. Yu**, Twisted Levi sequences and explicit types on Sp_4; **F. Murnaghan**, Regularity and distinction of supercuspidal representations; **M. Nevins**, Patterns in branching rules for irreducible representations of $SL_2(k)$, for k a p-adic field; **R. Portilla**, Parametrizing nilpotent orbits in p-adic symmetric spaces; **S. Spallone**, An integration formula of Shahidi; **M. H. Weissman**, Managing metaplectiphobia: Covering p-adic groups.

Contemporary Mathematics, Volume 543

June 2011, 277 pages, Softcover, ISBN: 978-0-8218-4985-9, LC 2011000976, 2010 *Mathematics Subject Classification:* 22E50, 11F70, 22E35, 20G25, 20C33, 20G40, 20G05, **AMS members US$79.20**, List US$99, Order code CONM/543



Introduction to Representation Theory

Pavel Etingof, *Massachusetts Institute of Technology, Cambridge, MA*, **Oleg Golberg**, **Sebastian Hensel**, *Universität Bonn, Germany*, **Tiankai Liu**, *Massachusetts Institute of Technology, Cambridge, MA*, **Alex Schwendner**, *Two Sigma Investments, New York, NY*, **Dmitry Vaintrob**, *Harvard University, Cambridge, MA*, and **Elena Yudovina**, *University of Cambridge, United Kingdom*; with historical interludes by **Slava Gerovitch**, Massachusetts Institute of Technology, Cambridge, MA

Very roughly speaking, representation theory studies symmetry in linear spaces. It is a beautiful mathematical subject which has many applications, ranging from number theory and combinatorics to geometry, probability theory, quantum mechanics, and quantum field theory.

The goal of this book is to give a "holistic" introduction to representation theory, presenting it as a unified subject which studies representations of associative algebras and treating the representation theories of groups, Lie algebras, and quivers as special cases. Using this approach, the book covers a number of standard topics in the representation theories of these structures. Theoretical material in the book is supplemented by many problems and exercises which touch upon a lot of additional topics; the more difficult exercises are provided with hints.

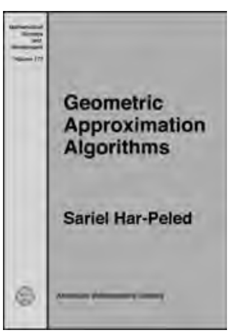
The book is designed as a textbook for advanced undergraduate and beginning graduate students. It should be accessible to students with a strong background in linear algebra and a basic knowledge of abstract algebra.

Contents: Introduction; Basic notions of representation theory; General results of representation theory; Representations of finite groups: Basic results; Representations of finite groups: Further results; Quiver representations; Introduction to categories; Homological algebra; Structure of finite dimensional algebras; References for historical interludes; Bibliography; Index.

Student Mathematical Library, Volume 59

August 2011, approximately 232 pages, Softcover, ISBN: 978-0-8218-5351-1, 2010 *Mathematics Subject Classification:* 16Gxx, 20Gxx, **AMS members US$33.60**, List US$42, Order code STML/59

Applications



Geometric Approximation Algorithms

Sariel Har-Peled, *University of Illinois at Urbana-Champaign, IL*

Exact algorithms for dealing with geometric objects are complicated, hard to implement in practice, and slow. Over the last 20 years a theory of geometric approximation algorithms has emerged. These algorithms tend to be simple, fast, and more robust than their exact counterparts.

This book is the first to cover geometric approximation algorithms in detail. In addition, more traditional computational geometry techniques that are widely used in developing such algorithms, like sampling, linear programming, etc., are also surveyed. Other topics covered include approximate nearest-neighbor search, shape approximation, coresets, dimension reduction, and embeddings. The topics covered are relatively independent and are supplemented by exercises. Close to 200 color figures are included in the text to illustrate proofs and ideas.

Contents: The power of grids—closest pair and smallest enclosing disk; Quadtrees—hierarchical grids; Well-separated pair decomposition; Clustering—definitions and basic algorithms; On complexity, sampling, and ε-nets and ε-samples; Approximation via reweighting; Yet even more on sampling; Sampling and the moments technique; Depth estimation via sampling; Approximating the depth via sampling and emptiness; Random partition via shifting; Good triangulations and meshing; Approximating the Euclidean traveling salesman problem (TSP); Approximating the Euclidean TSP using bridges; Linear programming in low dimensions; Polyhedrons, polytopes, and linear programming; Approximate nearest neighbor search in low dimension; Approximate nearest neighbor via point-location; The Johnson-Lindenstrauss lemma; Approximate nearest neighbor (ANN) search in high dimensions; Approximating a convex body by an ellipsoid; Approximating the minimum volume bounding box of a point set; Coresets; Approximation using shell sets; Duality; Finite metric spaces and partitions; Some probability and tail inequalities; Miscellaneous prerequisite; Bibliography; Index.

Mathematical Surveys and Monographs, Volume 173

July 2011, approximately 358 pages, Hardcover, ISBN: 978-0-8218-4911-8, LC 2011002940, 2010 *Mathematics Subject Classification:* 68U05, 68W25; 68P05, 52Cxx, **AMS members US$78.40**, List US$98, Order code SURV/173

Geometry and Topology

Grassmannians, Moduli Spaces and Vector Bundles

David A. Ellwood, *Clay Mathematics Institute, Cambridge, MA*, and **Emma Previato**, *Boston University, MA*, Editors

This collection of cutting-edge articles on vector bundles and related topics originated from a CMI workshop, held in October 2006, that brought together a community indebted to the pioneering work of P. E. Newstead, visiting the United States for the first time since the 1960s. Moduli spaces of vector bundles were then in their infancy, but are now, as demonstrated by this volume, a powerful tool in symplectic geometry, number theory, mathematical physics, and algebraic geometry. In fact, the impetus for this volume was to offer a sample of the vital convergence of techniques and fundamental progress, taking place in moduli spaces at the outset of the twenty-first century.

This volume contains contributions by J. E. Andersen and N. L. Gammelgaard (Hitchin's projectively flat connection and Toeplitz operators), M. Aprodu and G. Farkas (moduli spaces), D. Arcara and A. Bertram (stability in higher dimension), L. Jeffrey (intersection cohomology), J. Kamnitzer (Langlands program), M. Lieblich (arithmetic aspects), P. E. Newstead (coherent systems), G. Pareschi and M. Popa (linear series on Abelian varieties), and M. Teixidor i Bigas (bundles over reducible curves).

These articles do require a working knowledge of algebraic geometry, symplectic geometry and functional analysis, but should appeal to practitioners in a diversity of fields. No specialization should be necessary to appreciate the contributions, or possibly to be stimulated to work in the various directions opened by these path-blazing ideas; to mention a few, the Langlands program, stability criteria for vector bundles over surfaces and threefolds, linear series over abelian varieties and Brauer groups in relation to arithmetic properties of moduli spaces.

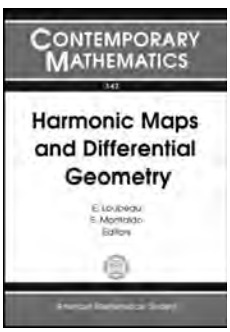
This item will also be of interest to those working in algebra and algebraic geometry.

Titles in this series are co-published with the Clay Mathematics Institute (Cambridge, MA).

Contents: **J. E. Andersen** and **N. L. Gammelgaard**, Hitchin's projectively flat connection, Toeplitz operators and the asymptotic expasnion of TQFT curve operators; **M. Aprodu** and **G. Farkas**, Koszul cohomology and applications to moduli; **D. Arcara** and **A. Bertram**, Reider's theorem and Thaddeus pairs revisited; **L. Jeffrey**, Intersection pairings in singular moduli spaces of bundles; **J. Kamnitzer**, The Beilinson-Drinfeld Grassmannian and symplectic knot homology; **M. Lieblich**, Arithmetic aspects of moduli spaces of sheaves on curves; **P. E. Newstead**, Existence of α-stable coherent systems on algebraic curves; **G. Pareschi** and **M. Popa**, Regularity on abelian varieties III: Relationship with generic vanishing and applications; **M. Teixidor i Bigas**, Vector bundles on reducible curves and applications.

Clay Mathematics Proceedings, Volume 14

June 2011, 180 pages, Softcover, ISBN: 978-0-8218-5205-7, 2010 *Mathematics Subject Classification:* 13D03, 14D20, 14F22, 14H60, 14K10, 22E57, 47B35, 53B10, 53D30, 57R56, **AMS members US$44**, List US$55, Order code CMIP/14



Harmonic Maps and Differential Geometry

E. Loubeau, *Université de Bretagne, Brest, France*, and **S. Montaldo**, *University of Cagliari, Italy*, Editors

This volume contains the proceedings of a conference held in Cagliari, Italy, from September 7–10, 2009, to celebrate John C. Wood's 60th birthday.

These papers reflect the many facets of the theory of harmonic maps and its links and connections with other topics in Differential and Riemannian Geometry. Two long reports, one on constant mean curvature surfaces by F. Pedit and the other on the construction of harmonic maps by J. C. Wood, open the proceedings. These are followed by a mix of surveys on Professor Wood's area of expertise: Lagrangian surfaces, biharmonic maps, locally conformally Kähler manifolds and the DDVV conjecture, as well as several research papers on harmonic maps. Other research papers in the volume are devoted to Willmore surfaces, Goldstein-Pedrich flows, contact pairs, prescribed Ricci curvature, conformal fibrations, the Fadeev-Hopf model, the compact support principle and the curvature of surfaces.

This item will also be of interest to those working in analysis.

Contents: **J. C. Wood**, Thirty-nine years of harmonic maps; **A. Gerding**, **F. Pedit**, and **N. Schmitt**, Constant mean curvature surfaces: An integrable systems perspective; **J. C. Wood**, Explicit constructions of harmonic maps; **P. Baird** and **M. Wehbe**, Discrete harmonic map heat flow on a finite graph; **G. Bande** and **D. Kotschick**, Contact pairs and locally conformally symplectic structures; **E. Musso**, Congruence curves of the Goldstein-Petrich flows; **H. Ma** and **Y. Ohnita**, Differential geometry of Lagrangian submanifolds and Hamiltonian variational problems; **L. Ornea** and **M. Verbitsky**, A report on locally conformally Kähler manifolds; **M. Rigoli**, **M. Salvatori**, and **M. Vignati**, k-Hessian differential inequalities and the compact support principle; **H. Urakawa**, The geometry of biharmonic maps; **G. Calvaruso**, Constructing metrics with prescribed geometry; **J. Bolton** and **L. Fernández**, On the regularity of the space of harmonic 2-spheres in the 4-sphere; **S. Heller**, Conformal fibrations of $\mathbb{S}^3$ by circles; **K. Leschke**, Harmonic map methods for Willmore surfaces; **F. Mercuri**, **S. Montaldo**, and **I. I. Onnis**, Some remarks on invariant surfaces and their extrinsic curvature; **P. Baird**, **E. Loubeau**, and **C. Oniciuc**, Harmonic and biharmonic maps from surfaces; **J. Jost** and **F. M. Şimşir**, Non-divergence harmonic maps; **R. Slobodeanu**, A note on higher-charge configurations for the Faddeev-Hopf model; **J. Q. Ge** and **Z. Z. Tang**, A survey on the DDVV conjecture; **G. Bande** and **A. Hadjar**, On the characteristic foliations of metric contact pairs; **C.-L. Bejan**, A note on η-Einstein manifolds; **M. I. Munteanu** and **A. I. Nistor**, Minimal and flat surfaces in $\mathbb{H}^2 \times \mathbb{R}$ with canonical coordinates; **R. C. Voicu**, Ricci curvature properties and stability on 3-dimensional Kenmotsu manifolds; **S. Gudmundsson** and **M. Svensson**, On the existence of harmonic morphisms from three-dimensional Lie groups.

Contemporary Mathematics, Volume 542

May 2011, approximately 281 pages, Softcover, ISBN: 978-0-8218-4987-3, LC 2011000414, 2010 *Mathematics Subject Classification:* 53-06, 53-XX, 58Exx; 15A45, 35-XX, 49S05, 57R17, 58D10, 81T20, **AMS members US$79.20**, List US$99, Order code CONM/542

New AMS-Distributed Publications

Algebra and Algebraic Geometry

An Introduction to Groups and Lattices

Finite Groups and Positive Definite Rational Lattices

Robert L. Griess, Jr., *University of Michigan, Ann Arbor, MI*

Rational lattices occur throughout mathematics, as in quadratic forms, sphere packing, Lie theory, and integral representations of finite groups. Studies of high-dimensional lattices typically involve number theory, linear algebra, codes, combinatorics, and groups. This book presents a basic introduction to rational lattices and finite groups and to the deep relationship between these two theories.

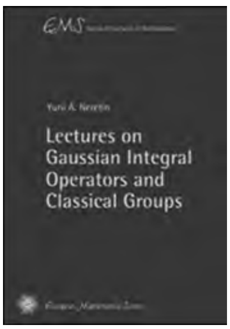
Robert L. Griess, Jr. is a professor of mathematics at the University of Michigan. His various honors include a Guggenheim Fellowship, an invited lecture at the International Congress of Mathematicians, membership in the American Academy of Arts and Sciences, and the 2010 AMS Leroy P. Steele Prize for his seminal construction of the Monster group.

A publication of International Press. Distributed worldwide by the American Mathematical Society.

Contents: Introduction; Bilinear forms, quadratic forms and their isometry groups; General results on finite groups and invariant lattices; Root lattices of types A, D, E; Hermite and Minkowski functions; Constructions of lattices by use of codes; Group theory and representations; Overview of the Barnes-Wall lattices; Construction and properties of the Barnes-Wall lattices; Even unimodular lattices in small dimensions; Pieces of eight; References; Index; Appendix A: The finite simple groups.

International Press

February 2011, 251 pages, Softcover, ISBN: 978-1-57146-206-0, 2010 *Mathematics Subject Classification:* 11H56, 20C10, 20C34, 20D08, **AMS members US$44**, List US$55, Order code INPR/95



Lectures on Gaussian Integral Operators and Classical Groups

Yurii A. Neretin, *University of Vienna, Austria*

This book is an elementary self-contained introduction to some constructions of representation theory and related topics of differential geometry and analysis.

Topics covered include the theory of various Fourier-like integral operators such as Segal–Bargmann transforms, Gaussian integral operators in L^2 and in the Fock space, integral operators with theta-kernels, the geometry of real and p-adic classical groups and symmetric spaces.

The heart of the book is the Weil representation of the symplectic group (real and complex realizations, relations with theta-functions and modular forms, p-adic and adelic constructions) and representations in Hilbert spaces of holomorphic functions of several complex variables.

This book is addressed to graduate students and researchers in representation theory, differential geometry, and operator theory. Prerequisites are standard university courses in linear algebra, functional analysis, and complex analysis.

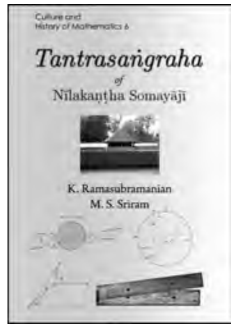
A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.

Contents: Gaussian integral operators; Pseudo-Euclidean geometry and groups $\mathrm{U}(p,q)$; Linear symplectic geometry; The Segal–Bargmann transform; Gaussian operators in Fock spaces; Gaussian operators. Details; Hilbert spaces of holomorphic functions in matrix balls; The Cartier model; Gaussian operators over finite fields; Classical p-adic groups. Introduction; Weil representation over a p-adic field; Addendum; Bibliography; Notation; Index.

EMS Series of Lectures in Mathematics, Volume 13

February 2011, 571 pages, Softcover, ISBN: 978-3-03719-080-7, 2010 *Mathematics Subject Classification:* 22-02, 22E46, 53C35, 22E50, 47G10, 32M15, 51N30, 14K25, 47B50, **AMS members US$62.40**, List US$78, Order code EMSSERLEC/13

General Interest



Tantrasaṅgraha of Nīlakantha Somayājī

K. Ramasubramanian, *ITT Bombay, Powai, India*, and **M. S. Sriram**, *University of Madras, Chennai, India*

It is truly remarkable that Nīlakantha and Copernicus, contemporaries living 4000 miles apart, should have independently made profound revisions to the classical epicyclic models of the planetary system at the same time. Here, for the first time, Westerners can read Nīlakantha's great work, the Tantrasaṅgraha*, and learn the mathematical principles of Indian astronomy in their most developed form. This work has a major place in the canon of the history of science, enlarging our worldwide view of the landmark human discoveries.*

– **David Mumford**

Tantrasaṅgraha, composed in 1500 CE by the renowned Kerala astronomer Nīlakantha Somayājī, ranks along with *Āryabhatīya* of Āryabhata and *Siddhāntaśiromani* of Bhāskarācārya as a seminal work that significantly influenced further work on astronomy in India. This text introduces a major revision of the traditional planetary models, leading to a unified theory of planetary latitudes and a better formulation of the equation of center for the interior planets (Mercury and Venus) than was previously available.

Several important innovations in mathematical technique are also included in *Tantrasaṅgraha*, especially related to the computation of accurate sine tables, the use of series for evaluating the sine and cosine functions, and a systematic treatment of the problems related to the diurnal motion of the celestial objects. The spherical trigonometry relations presented in the text—applied to a variety of problems such as the computation of eclipses and elevation of Moon's cusps—are also exact.

This edition of *Tantrasaṅgraha* will appeal to historians of astronomy as well as those who are keen to know about the actual computational procedures employed in Indian astronomy. All 432 verses have been translated into English and supplemented with detailed explanations through the use of mathematical equations, tables, and figures.

A publication of Hindustan Book Agency. Distributed on an exclusive basis by the AMS in North America. Online bookstore rights worldwide.

Contents: Mean longitudes of planets; True longitudes of planets; Gnomonic shadow; Lunar eclipse; Solar eclipse; Vyatīpāta; Reduction to observation; Elevation of lunar horns; *Appendices:* Representation of numbers; Spherical trigonometry; Coordinate systems; Solution of the ten problems: A couple of examples from *Yuktibhāṣā*; Derivation of the maximum declination of the Moon; The traditional Indian planetary model and its revision by Nīlakantha; Glossary; Bibliography; Index; Index of half-verses.

Hindustan Book Agency

February 2011, 642 pages, Hardcover, ISBN: 978-93-80250-09-0, 2010 *Mathematics Subject Classification:* 01A32, 85-03, **AMS members US$65.60**, List US$82, Order code HIN/48

Meetings & Conferences of the AMS

IMPORTANT INFORMATION REGARDING MEETINGS PROGRAMS: AMS Sectional Meeting programs do not appear in the print version of the *Notices*. However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on the AMS website. See `http://www.ams.org/meetings/`. Final programs for Sectional Meetings will be archived on the AMS website accessible from the stated URL and in an electronic issue of the *Notices* as noted below for each meeting.

Las Vegas, Nevada

University of Nevada

April 30 – May 1, 2011

Saturday – Sunday

Meeting #1071

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: February 2011
Program first available on AMS website: March 17, 2011
Program issue of electronic *Notices*: April 2011
Issue of *Abstracts*: Volume 32, Issue 3

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Elizabeth Allman, University of Alaska, *Evolutionary trees and phylogenetics: An algebraic perspective*.

Danny Calegari, California Institute of Technology, *Stable commutator length in free groups*.

Hector Ceniceros, University of California Santa Barbara, *Immersed boundaries in complex fluids*.

Tai-Ping Liu, Stanford University, *Hilbert's sixth problem*.

Special Sessions

Advances in Modeling, Numerical Analysis and Computations of Fluid Flow Problems, **Monika Neda**, University of Nevada, Las Vegas.

Arithmetic Dynamics, **Arthur Baragar**, University of Nevada, Las Vegas, and **Patrick Ingram**, University of Waterloo.

Computational Algebra, Groups, and Applications, **Benjamin Fine**, Fairfield University, **Gerhard Rosenberger**, University of Hamburg, Germany, and **Delaram Kahrobaei**, City University of New York.

Computational and Mathematical Finance, **Hongtao Yang**, University of Nevada, Las Vegas.

Discrete Dynamical Systems in Graph Theory, Combinatorics, and Geometry, **Eunjeong Yi** and **Cong X. Kang**, Texas A&M University at Galveston.

Extremal Combinatorics, **Jozsef Balogh**, University of California San Diego, and **Ryan Martin**, Iowa State University.

Flow-Structure Interaction, **Paul Atzberger**, University of California Santa Barbara, and **Hector D. Ceniceros**, University of California Santa Barbara.

Geometric Group Theory and Dynamics, **Matthew Day**, **Danny Calegari**, and **Joel Louwsma**, California Institute of Technology, and **Andy Putman**, Rice University.

Geometric PDEs, **Matthew Gursky**, Notre Dame University, and **Emmanuel Hebey**, Université de Cergy-Pontoise.

Knots, Surfaces and 3-manifolds, **Stanislav Jabuka**, **Swatee Naik**, and **Chris Herald**, University of Nevada, Reno.

Lie Algebras, Algebraic Transformation Groups and Representation Theory, **Andrew Douglas** and **Bart Van Steirteghem**, City University of New York.

Multilevel Mesh Adaptation and Beyond: Computational Methods for Solving Complex Systems, **Pengtao Sun**, University of Nevada, Las Vegas, and **Long Chen**, University of California Irvine.

Nonlinear PDEs and Variational Methods, **David Costa** and **Hossein Tehrani**, University of Nevada, Las Vegas, and **Zhi-Qiang Wang**, Utah State University.

Partial Differential Equations Modeling Fluids, **Quansen Jiu**, Capital Normal University, Beijing, China, and **Jiahong Wu**, Oklahoma State University.

Recent Advances in Finite Element Methods, **Jichun Li**, University of Nevada, Las Vegas.

Recent Developments in Stochastic Partial Differential Equations, **Igor Cialenco**, Illinois Institute of Technology, and **Nathan Glatt-Holtz**, Indiana University, Bloomington.

Set Theory, **Douglas Burke** and **Derrick DuBose**, University of Nevada, Las Vegas.

Topics in Modern Complex Analysis, **Zair Ibragimov**, California State University, Fullerton, **Zafar Ibragimov**, Urgench State University, and **Hrant Hakobyan**, Kansas State University.

Ithaca, New York

Cornell University

September 10–11, 2011

Saturday – Sunday

Meeting #1072

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: June 2011
Program first available on AMS website: July 28, 2011
Program issue of electronic *Notices*: September 2011
Issue of *Abstracts*: Volume 32, Issue 4

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: May 24, 2011
For abstracts: June 29, 2011

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Mladen Bestvina, University of Utah, *Title to be announced.*

Nigel Higson, Pennsylvania State University, *Title to be announced.*

Gang Tian, Princeton University, *Title to be announced.*

Katrin Wehrheim, Massachusetts Institute of Technology, *Title to be announced.*

Special Sessions

Analysis, Probability, and Mathematical Physics on Fractals (Code: SS 10A), **Luke Rogers**, University of Connecticut, **Robert Strichartz**, Cornell University, and **Alexander Teplyaev**, University of Connecticut.

Difference Equations and Applications (Code: SS 1A), **Michael Radin**, Rochester Institute of Technology.

Gauge Theory and Low-dimensional Topology (Code: SS 7A), **Weimin Chen**, University of Massachusetts-Amherst, and **Daniel Ruberman**, Brandeis University.

Geometric Aspects of Analysis and Measure Theory (Code: SS 9A), **Leonid Kovalev** and **Jani Onninen**, Syracuse University, and **Raanan Schul**, State University of New York at Stony Brook.

Geometric and Algebraic Topology (Code: SS 15A), **Boris Goldfarb** and **Marco Varisco**, University at Albany, SUNY.

Geometric Structures on Manifolds with Special Holonomy, and Applications in Physics (Code: SS 16A), **Tamar Friedmann**, University of Rochester, **Colleen Robles**, Texas A&M University, and **Sema Salur**, University of Rochester.

Geometry of Arithmetic Groups (Code: SS 11A), **Mladen Bestvina**, University of Utah, and **Ken Brown**, **Martin Kassabov**, and **Tim Riley**, Cornell University.

Kac-Moody Lie Algebras, Vertex Algebras, and Related Topics (Code: SS 14A), **Alex Feingold**, Binghamton University, and **Antun Milas**, State University of New York at Albany.

Mathematical Aspects of Cryptography and Cyber Security (Code: SS 8A), **Benjamin Fine**, Fairfield University, **Delaram Kahrobaei**, City University of New York, and **Gerhard Rosenberger**, Passau University and Hamburg University, Germany.

Multivariable Operator Theory (Code: SS 13A), **Ronald G. Douglas**, Texas A&M University, and **Rongwei Yang**, State University of New York at Albany.

Parabolic Evolution Equations of Geometric Type (Code: SS 4A), **Xiaodong Cao**, Cornell University, and **Bennett Chow**, University of California San Diego.

Partial Differential Equations of Mixed Elliptic-Hyperbolic Type and Applications (Code: SS 3A), **Marcus Khuri**, Stony Brook University, and **Dehua Wang**, University of Pittsburgh.

Representations of Local and Global Groups (Code: SS 12A), **Mahdi Asgari**, Oklahoma State University, and **Birgit Speh**, Cornell University.

Set Theory (Code: SS 2A), **Paul Larson**, Miami University, Ohio, **Justin Moore**, Cornell University, and **Ernest Schimmerling**, Carnegie Mellon University.

Species and Hopf Algebraic Combinatorics (Code: SS 6A), **Marcelo Aguiar**, Texas A&M University, and **Samuel Hsiao**, Bard College.

Symplectic Geometry and Topology (Code: SS 5A), **Tara Holm**, Cornell University, and **Katrin Wehrheim**, Massachusetts Institute of Technology.

Winston-Salem, North Carolina

Wake Forest University

September 24–25, 2011

Saturday – Sunday

Meeting #1073

Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: June 2011
Program first available on AMS website: August 11, 2011
Program issue of electronic *Notices*: September 2011
Issue of *Abstracts*: Volume 32, Issue 4

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: June 7, 2011
For abstracts: August 2, 2011

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Benjamin B. Brubaker, Massachusetts Institute of Technology, *Square ice, symmetric functions, and their connections to automorphic forms*.

Shelly Harvey, Rice University, *Title to be announced*.

Allen Knutson, Cornell University, *Title to be announced*.

Seth M. Sullivant, North Carolina State University, *Title to be announced*.

Special Sessions

Algebraic and Geometric Aspects of Matroids (Code: SS 1A), **Hoda Bidkhori**, **Alex Fink**, and **Seth Sullivant**, North Carolina State University.

Applications of Difference and Differential Equations to Biology (Code: SS 2A), **Anna Mummert**, Marshall University, and **Richard C. Schugart**, Western Kentucky University.

Combinatorial Algebraic Geometry (Code: SS 6A), **W. Frank Moore**, Wake Forest University and Cornell University, and **Allen Knutson**, Cornell University.

Extremal Combinatorics (Code: SS 7A), **Tao Jiang**, Miami University, and **Linyuan Lu**, University of South Carolina.

Geometric Knot Theory and its Applications (Code: SS 12A), **Yuanan Diao**, University of North Carolina at Charlotte, **Jason Parsley**, Wake Forest University, and **Eric Rawdon**, University of St. Thomas.

Low-Dimensional Topology and Geometry (Code: SS 13A), **Shelly Harvey**, Rice University, and **John Etnyre**, Georgia Institute of Technology.

Modular Forms, Elliptic Curves, and Related Topics (Code: SS 11A), **Matthew Boylan**, University of South Carolina, and **Jeremy Rouse**, Wake Forest University.

New Developments in Graph Theory (Code: SS 10A), **Joshua Cooper** and **Kevin Milans**, University of South Carolina, and **Carlos Nicolas** and **Clifford Smyth**, University of North Carolina at Greensboro.

Noncommutative Algebra (Code: SS 5A), **Ellen E. Kirkman** and **James J. Kuzmanovich**, Wake Forest University.

Nonlinear Boundary Value Problems (Code: SS 9A), **Maya Chhetri**, University of North Carolina at Greensboro, and **Stephen B. Robinson**, Wake Forest University.

Nonlinear Dispersive Equations (Code: SS 4A), **Sarah Raynor**, Wake Forest University, **Jeremy Marzuola**, University of North Carolina at Chapel Hill, and **Gideon Simpson**, University of Toronto.

Recent Advances in Infectious Disease Modeling (Code: SS 8A), **Fred Chen** and **Miaohua Jiang**, Wake Forest University.

Symmetric Functions, Symmetric Group Characters, and Their Generalizations (Code: SS 3A), **Sarah Mason**, Wake Forest University, **Aaron Lauve**, Loyola University-Chicago, and **Ed Allen**, Wake Forest University.

Lincoln, Nebraska

University of Nebraska-Lincoln

October 14–16, 2011

Friday – Sunday

Meeting #1074

Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: August 2011
Program first available on AMS website: September 1, 2011
Program issue of electronic *Notices*: October 2011
Issue of *Abstracts*: Volume 32, Issue 4

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: June 28, 2011
For abstracts: August 23, 2011

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Lewis Bowen, Texas A&M University, *Title to be announced*.

Emmanuel Candes, Stanford University, *Title to be announced* (Erdős Memorial Lecture).

Alina Cojocaru, University of Illinois at Chicago, *Title to be announced*.

Michael Zieve, University of Michigan, *Title to be announced*.

Special Sessions

Algebraic Geometry and Graded Commutative Algebra (Code: SS 8A), **Susan Cooper** and **Brian Harbourne**, University of Nebraska-Lincoln.

Algorithmic and Geometric Properties of Groups and Semigroups (Code: SS 10A), **Susan Hermiller** and **John Meakin**, University of Nebraska-Lincoln.

Association Schemes and Related Topics (Code: SS 1A), **Sung Y. Song**, Iowa State University, and **Paul Terwilliger**, University of Wisconsin, Madison.

Asymptotic Behavior and Regularity for Nonlinear Evolution Equations (Code: SS 4A), **Petronela Radu** and **Lorena Bociu**, University of Nebraska-Lincoln.

Coding Theory (Code: SS 7A), **Christine Kelley** and **Judy Walker**, University of Nebraska-Lincoln.

Computational and Applied Mathematics (Code: SS 13A), **Ludwig Kohaupt**, Beuth University of Technology Berlin, Germany, and **Yan Wu**, Georgia Southern University.

Continuous and Numerical Analysis in the Control of PDE's (Code: SS 9A), **George Avalos**, **Mohammad Rammaha**, and **Daniel Toundykov**, University of Nebraska-Lincoln.

Dynamic Systems on Time Scales with Applications (Code: SS 3A), **Lynn Erbe** and **Allan Peterson**, University of Nebraska-Lincoln.

Extremal and Probabilistic Combinatorics (Code: SS 5A), **Stephen Hartke** and **Jamie Radcliffe**, University of Nebraska-Lincoln.

Invariants in Knot Theory and Low-dimensional Topology (Code: SS 14A), **Mark Brittenham**, University of Nebraska-Lincoln, and **Robert Todd**, University of Nebraska at Omaha.

Local Commutative Algebra (Code: SS 11A), **H Ananthnarayan**, University of Nebraska-Lincoln, **Inês B. Henriques**, University of California Riverside, and **Hamid Rahmati**, Syracuse University.

Matrices and Graphs (Code: SS 12A), **In-Jae Kim**, Minnesota State University, **Adam Berliner**, St. Olaf College, **Leslie Hogben**, Iowa State University, and **Bryan Shader**, University of Wyoming.

Quantum Groups and Representation Theory (Code: SS 2A), **Jonathan Kujawa**, University of Oklahoma, and **Natasha Rozhkovskaya**, Kansas State University.

Recent Progress in Operator Algebras (Code: SS 6A), **Allan P. Donsig** and **David R. Pitts**, University of Nebraska-Lincoln.

Salt Lake City, Utah

University of Utah

October 22–23, 2011

Saturday – Sunday

Meeting #1075

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: August 2011
Program first available on AMS website: September 8, 2011
Program issue of electronic *Notices*: October 2011
Issue of *Abstracts*: Volume 32, Issue 4

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: July 5, 2011
For abstracts: August 30, 2011

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Graeme Milton, University of Utah, *Title to be announced.*

Lei Ni, University of California San Diego, *Title to be announced.*

Igor Pak, University of California Los Angeles, *Title to be announced.*

Monica Visan, University of California Los Angeles, *Title to be announced.*

Special Sessions

Celestial and Geometric Mechanics (Code: SS 5A), **Lennard Bakker** and **Tiancheng Ouyang**, Brigham Young University.

Commutative Algebra (Code: SS 3A), **Chin-Yi Jean Chan**, Central Michigan University, and **Lance E. Miller** and **Amurag K. Singh**, University of Utah.

Electromagnetic Wave Propagation in Complex and Random Environments (Code: SS 4A), **David Dobson**, University of Utah, and **Peijun Li**, Purdue University.

Geometric Evolution Equations and Related Topics (Code: SS 2A), **Andrejs Treibergs**, University of Utah, Salt Lake City, **Lei Ni**, University of California San Diego, and **Brett Kotschwar**, Arizona State University.

Geometric, Combinatorial, and Computational Group Theory (Code: SS 1A), **Eric Freden**, Southern Utah University, and **Eric Swenson**, Brigham Young University.

Port Elizabeth, Republic of South Africa

Nelson Mandela Metropolitan University

November 29 – December 3, 2011

Tuesday – Saturday

Meeting #1076

First Joint International Meeting between the AMS and the South African Mathematical Society.
Associate secretary: Matthew Miller
Announcement issue of *Notices*: June 2011

Program first available on AMS website: Not applicable
Program issue of electronic *Notices*: Not applicable
Issue of *Abstracts*: Not applicable

Deadlines

For organizers: See the information at `http://www.nmmu.ac.za/sams-ams2011/ssprop.htm`
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/internmtgs.html`.

Invited Addresses

Mark J. Ablowitz, University of Colorado, *Title to be announced*.

James Raftery, University of Kwazulu Natal, *Title to be announced*.

Daya Reddy, University of Cape Town, *Title to be announced*.

Peter Sarnak, Princeton University, *Title to be announced*.

Robin Thomas, Georgia Institute of Technology, *Title to be announced*.

Amanda Weltman, University of Cape Town, *Title to be announced*.

Boston, Massachusetts

John B. Hynes Veterans Memorial Convention Center, Boston Marriott Hotel, and Boston Sheraton Hotel

January 4–7, 2012

Wednesday – Saturday

Meeting #1077

Joint Mathematics Meetings, including the 118th Annual Meeting of the AMS, 95th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: October 2011
Program first available on AMS website: November 1, 2011
Program issue of electronic *Notices*: January 2012
Issue of *Abstracts*: Volume 33, Issue 1

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: September 22, 2011

Honolulu, Hawaii

University of Hawaii

March 3–4, 2012

Saturday – Sunday

Meeting #1078

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: March 2012
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: August 3, 2011
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Tampa, Florida

University of South Florida

March 10–11, 2012

Saturday – Sunday

Meeting #1079

Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: March 2012
Issue of *Abstracts*: To be announced

Deadlines

For organizers: August 10, 2011
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Special Sessions

Discrete Models in Molecular Biology (Code: SS 1A), **Alessandra Carbone**, Université Pierre et Marie Curie and Laboratory of Microorganisms Genomics, **Natasha Jonoska**, University of South Florida, and **Reidun Twarock**, University of York.

Washington, District of Columbia

George Washington University

March 17–18, 2012

Saturday – Sunday

Meeting #1080

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: March 2012
Issue of *Abstracts*: To be announced

Deadlines

For organizers: August 17, 2011
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Lawrence, Kansas

University of Kansas

March 30 – April 1, 2012

Friday – Sunday

Meeting #1081

Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: August 31, 2011
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Special Sessions

Combinatorial Commutative Algebra (Code: SS 1A), **Christopher Francisco** and **Jeffrey Mermin**, Oklahoma State University, and **Jay Schweig**, University of Kansas.

Rochester, New York

Rochester Institute of Technology

September 22–23, 2012

Saturday – Sunday

Meeting #1082

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: February 22, 2012
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

New Orleans, Louisiana

Tulane University

October 13–14, 2012

Saturday – Sunday

Meeting #1083

Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: October 2012
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 13, 2012
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Akron, Ohio

University of Akron

October 20–21, 2012

Saturday – Sunday

Meeting #1084

Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 22, 2012
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Tucson, Arizona

University of Arizona, Tucson

October 27–28, 2012

Saturday – Sunday

Meeting #1085

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 27, 2012
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Ken Ono, Emory University, *Title to be announced* (Erdős Memorial Lecture).

San Diego, California

San Diego Convention Center and San Diego Marriott Hotel and Marina

January 9–12, 2013

Wednesday – Saturday
Joint Mathematics Meetings, including the 119th Annual Meeting of the AMS, 96th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: October 2012
Program first available on AMS website: November 1, 2012
Program issue of electronic *Notices*: January 2012
Issue of *Abstracts*: Volume 34, Issue 1

Deadlines

For organizers: April 1, 2012
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Chestnut Hill, Massachusetts

Boston College

April 6–7, 2013

Saturday – Sunday
Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 6, 2012
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Ames, Iowa

Iowa State University

April 27–28, 2013

Saturday – Sunday
Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: April 2013
Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 27, 2012
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Special Sessions

Operator Algebras and Topological Dynamics (Code: SS 1A), **Christopher Francisco** and **Jeffrey Mermin**, Oklahoma State University, and **Jay Schweig**, University of Kansas.

Alba Iulia, Romania

June 27–30, 2013

Thursday – Sunday

First Joint International Meeting of the AMS and the Romanian Mathematical Society, in partnership with the "Simion Stoilow" Institute of Mathematics of the Romanian Academy.

Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: Not applicable
Program issue of electronic *Notices*: Not applicable
Issue of *Abstracts*: Not applicable

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Riverside, California

University of California Riverside

November 2–3, 2013

Saturday – Sunday

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 2, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Baltimore, Maryland

Baltimore Convention Center, Baltimore Hilton, and Marriott Inner Harbor

January 15–18, 2014

Wednesday – Saturday

Joint Mathematics Meetings, including the 120th Annual Meeting of the AMS, 97th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Matthew Miller
Announcement issue of *Notices*: October 2013
Program first available on AMS website: November 1, 2013
Program issue of electronic *Notices*: January 2013
Issue of *Abstracts*: Volume 35, Issue 1

Deadlines

For organizers: April 1, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Tel Aviv, Israel

June 16–19, 2014

Monday – Thursday

Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

San Antonio, Texas

Henry B. Gonzalez Convention Center and Grand Hyatt San Antonio

January 10–13, 2015

Saturday – Tuesday

Joint Mathematics Meetings, including the 121st Annual Meeting of the AMS, 98th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: October 2014
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2015
Issue of *Abstracts*: Volume 36, Issue 1

Deadlines

For organizers: April 1, 2014
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Seattle, Washington

Washington State Convention & Trade Center and the Sheraton Seattle Hotel

January 6–9, 2016

Wednesday – Saturday

Joint Mathematics Meetings, including the 122nd Annual Meeting of the AMS, 99th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: October 2015
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2016
Issue of *Abstracts*: Volume 37, Issue 1

Deadlines

For organizers: April 1, 2015
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Atlanta, Georgia

Hyatt Regency Atlanta and Marriott Atlanta Marquis

January 4–7, 2017

Wednesday – Saturday

Joint Mathematics Meetings, including the 123rd Annual Meeting of the AMS, 100th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Georgia Benkart
Announcement issue of *Notices*: October 2016
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2017
Issue of *Abstracts*: Volume 38, Issue 1

Deadlines

For organizers: April 1, 2016
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

San Diego, California

San Diego Convention Center

January 10–13, 2018

Wednesday – Saturday

Joint Mathematics Meetings, including the 124th Annual Meeting of the AMS, 101st Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Matthew Miller
Announcement issue of *Notices*: October 2017
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 1, 2017
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Meetings and Conferences of the AMS

Associate Secretaries of the AMS

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The Meetings and Conferences section of the *Notices* gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited in the table of contents on this page for more detailed information on each event. Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. **Information in this issue may be dated. Up-to-date meeting and conference information can be found at** `www.ams.org/meetings/`**.**

Meetings:

Important Information Regarding AMS Meetings

Potential organizers, speakers, and hosts should refer to page 100 in the January 2011 issue of the *Notices* for general information regarding participation in AMS meetings and conferences.

Abstracts

Speakers should submit abstracts on the easy-to-use interactive Web form. No knowledge of LaTeX is necessary to submit an electronic form, although those who use LaTeX may submit abstracts with such coding, and all math displays and similarily coded material (such as accent marks in text) must be typeset in LaTeX. Visit `http://www.ams.org/cgi-bin/abstracts/abstract.pl`. Questions about abstracts may be sent to `abs-info@ams.org`. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

Conferences: (see `http://www.ams.org/meetings/` for the most up-to-date information on these conferences.)

June 12–July 2, 2011: Mathematics Research Research Communities, Snowbird, Utah. (Please see `http://www.ams.org/amsmtgs/mrc.html` for more information.)

July 4–7, 2011: von Neumann Symposium on Multimodel and Multialgorithm Coupling for Multiscale Problems, Snowbird, Utah. (Please see `http://www.ams.org/meetings/amsconf/symposia/symposia-2011` for more information.)

July 24–29, 2011: Conference on Applied Mathematics, Modeling, and Computational Science, Waterloo, Canada (held in cooperation with the AMS).

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