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of the American Mathematical Society

June/July 2011 **Volume 58, Number 6**

About the Cover: Forty-six years of topology from the differentiable viewpoint *(see page 819)*

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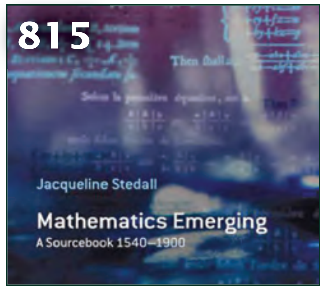
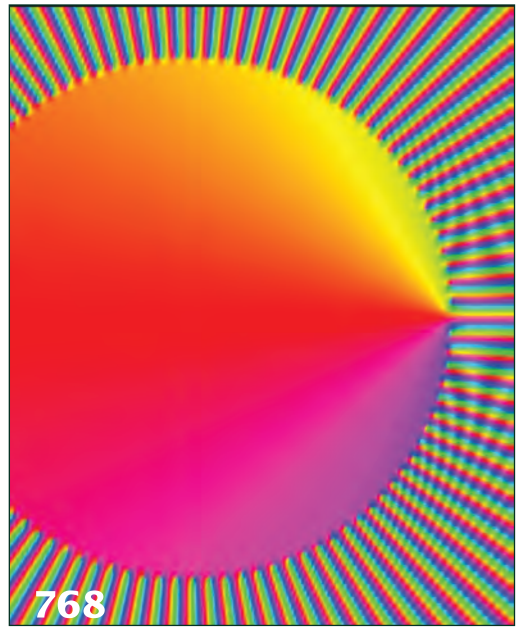
June/July 2011

Communications

Commentary





Features

The June/July issue features three quite distinct aspects of modern mathematics. John Milnor (recent Abel laureate and a father of differential topology) recounts the modern history of the subject of differential topology. Elias Wegert and Gunter Semmler present some new graphical representations of complex functions. And Joseph Grcar recounts the complex (and little-known) history of the technique of Gaussian elimination. Among the Communications articles is a remembrance of the remarkable mathematician Johannes Duistermaat.

—Steven G. Krantz, Editor

Notices
of the American Mathematical Society

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Departments

From the AMS Secretary

I thank Randi D. Ruden for her splendid editorial work, and for helping to assemble this issue. She is essential to everything that I do.

—Steven G. Krantz, Editor

Math Sciences 2025

The first decade of the current century witnessed a rapid increase in the pace of mathematical discovery. The proof of the Poincaré Conjecture, the development of the Schramm-Loewner process describing conformally invariant scaling limits, and the Green-Tao proof of arbitrarily long arithmetic progressions in the primes are vivid examples that quickly spring to mind. Simultaneously there has been a significant shortening of the time scale over which new discoveries have impact on science, technology, and society. For example, a 2006 *BusinessWeek* cover story describes how algorithms derived from recent research in combinatorics and number theory are driving electronic commerce, Internet search, and cybersecurity. A particularly dramatic illustration is the speed with which the theorems of Donoho, Candès, and Tao on compressed sensing enabled development of new imaging equipment that significantly outperforms existing standards. Just three years after publication of their ideas in 2004, compressed sensing was named by *Technology Review* as one of the top ten emerging technologies of 2007. We are truly in a golden age for the mathematical sciences, and the prospects for the future are very exciting. (This paragraph is based on an excellent summary of the decade that was provided in 2008 by Peter March and colleagues to the Board on Mathematical Sciences and Their Applications (BMSA) of the National Academies.)

It is healthy for disciplines to step back occasionally and evaluate how well they're operating and where they're headed. Such an evaluation—sponsored by the National Science Foundation's Division on Mathematical Sciences (DMS) and carried out by a BMSA committee—is currently under way. This study is called "The Mathematical Sciences in 2025" to suggest that we are looking over the horizon to infer what our discipline and community might look like in 2025 and what we need to attend to now to prepare.

Previous such studies include the landmark 1984 "David Report", which documented serious federal underfunding of the mathematical sciences, and the 1998 "Odom Report", which again highlighted the great potential contributions of the mathematical sciences and the funding shortfalls that were limiting our ability to seize those opportunities.

But those reports did more than focus on federal funding. At their heart, both gave the discipline a chance to examine itself and reflect on its progress and health. They sparked healthy discussions within the community. And, perhaps most importantly, they provided evidence that federal funding agencies could use in arguing for greater support of the mathematical sciences. Such evidence is valuable in Washington. It demonstrates that the field is thoughtful about its future, is a responsible steward of federal funds, and has clear ideas about its future growth.

When Tony Chan was appointed as NSF Assistant Director of Mathematics and Physical Sciences in 2006, he observed that the other fields in his portfolio carried out such self-evaluations with regularity and that they were powerful tools in arguing for resources within the NSF and for building support within Congress. He and Peter March, then DMS director, approached the BMSA to commission a study. BMSA members developed careful plans and a funding proposal, and the study committee held its first meeting in September 2010.

DMS and BMSA agreed that, for maximum credibility, the study should be chaired by a nonmathematician, following the model of the David and Odom reports. As longtime president of Caltech, Tom Everhart is familiar with the mathematical sciences and their roles within science, engineering, and education, but he is not perceived as an advocate for one field over another. The rest of the committee was assembled to span core and applied mathematics and statistics, along with fields with which the mathematical sciences interface: e.g., financial engineering, computer simulation, machine learning, theoretical physics, and theoretical computer science. The result is a strong committee that can see both the inner workings of our community and also the setting in which it has influence.

The official charge for this study reads as follows:

The study will produce a forward-looking assessment of the current state of the mathematical sciences and of emerging trends that will affect the discipline and its stakeholders as they look ahead to the quarter-century mark. Specifically, the study will assess:

- *The vitality of research in the mathematical sciences, looking at such aspects as the unity and coherence of research, significance of recent developments, rate of progress at the frontiers, and emerging trends.*
- *The impact of research and training in the mathematical sciences on science and engineering, on industry and technology, on innovation and economic competitiveness, on national security, and other areas of national interest.*
- *The study will make recommendations to [DMS] on how to adjust its portfolio of activities to improve the vitality and impact of the discipline.*

We intend this study to develop a strategic view that is useful to the NSF and other federal agencies; to chairs, deans, and academic administrators; to the mathematics and statistics communities; to the science and engineering community more broadly; and to the leadership of business, industry, government laboratories, and federal mission agencies. If you have insights you'd like to pass on to the committee, please visit `http://www.nas.edu/mathsci2025`, where you can upload a pdf or Word document or directly enter shorter input. The last date for submissions is August 31, 2011. This site also contains more detail about the study committee and its meetings to date.

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Letters to the Editor

Quality versus Quantity

I read with much interest Reinhard Siegmund-Schultze's book *Mathematicians Fleeing from Nazi Germany* [reviewed in the November 2010 *Notices*]. It contains many valuable sources, but it is more a list of events, persons, and documents than a historical text. A historian must not limit himself to compiling a list of events, but he must insert them into a broader context in which they acquire their real meaning. The task of a historian is to give significance to situations, casting light on the relations among them. Obviously, this structure cannot be objective like a mathematical definition; it is, on the contrary, highly qualitative. But that is not a problem. Poincaré talked about "topology as a purely qualitative science of space". A relation of "homotopy", for example, is strongly qualitative; so is the definition of "being unidimensional" in terms of the Möbius ribbon. Thus a historian must not fear that the structure in which he inserts the events has a fuzzy border. This is a philosophical problem, but from a pragmatic point of view, a good structure works for all practical purposes. The enumerations of facts and sources is only a preliminary stage of making history. I would prefer fewer names, but a more articulated and deeper review of the dramatic events during the Nazi age.

—*Davide Bondoni*
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(Received March 26, 2011)

Enriques's Quest

The February 2011 *Notices* contains two fascinating articles about the Fundamental Theorem of Irregular Algebraic Surfaces and, in particular, a key ingredient in its proof, the theorem of Completeness of the Characteristic System. The first article, written by Babbitt and Goodstein, elaborates on the personalities of the three geometers most involved: Enriques, Castelnuovo, and Severi. The second, written by Mumford, shows that, after all, Enriques's final attempt in 1936 to prove Completeness is essentially correct. Both articles are to be commended for their interest and scholarship.

On the other hand, the first article makes this statement on p. 245: "When he [Grothendieck] visited Harvard, Mumford pointed out to him that, as an immediate corollary, this [construction of the Picard Scheme] gave the long-sought algebro-geometric proof of a conjecture [the Completeness Theorem] he had never heard of!"

That statement may seem odd, as Grothendieck himself gave a different idea in his Bourbaki exposé on the Hilbert scheme. In Remarque 5.5, Grothendieck speaks explicitly about the characteristic homomorphism of classical algebraic geometry. In the next subsection, he notes that the Completeness Theorem follows from his theory of the Picard scheme, not immediately, but when combined with Cartier's Reducedness Theorem for complex group schemes.

Exposé 221 is dated May 1961, just before Grothendieck went to Harvard, and Bourbaki's exposés are written up for distribution on their given date. Furthermore, Grothendieck was always very careful about giving credit. In Exposé 221, he does not mention Mumford's name, but he does in connection with other work in his exposés 232 and 236 on the Picard scheme and in his commentaries.

In an email to me, Mumford wrote: "This is most interesting…I suppose what happened may be that AG wasn't aware of how central the completeness theorem was to Italian geometry and how much controversy it had created. You must be right that he did know that his (and Cartier's—I had forgotten Cartier was involved too) results implied the completeness theorem. But my guess is that Zariski and I might have told him how big a deal this was historically speaking, how it had been used from 1905 on to tie together all the main surface invariants."

Curiously, neither article cites Mumford's book, *Lectures on Curves on an Algebraic Surface*, where he explains Grothendieck's construction and Cartier's theorem slowly and carefully from scratch. Further, Mumford introduces certain Bockstein operators in order to extend the theory to positive characteristic.

—*Steven L. Kleiman*
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(Received February 21, 2011)

Response to Kleiman

David Mumford's seminal work, *Lectures on Curves on an Algebraic Surface*, served as an essential roadmap for our paper on Enriques and his lifelong pursuit of a satisfactory geometric proof of the Completeness Theorem. We regret the omission of his book in our paper but wish to acknowledge that it was the combination of Enriques's 1945 letter to Beniamino Segre and Mumford's discussion of the key shortcoming of the Italians' attempted proofs in the Introduction of his book that led to the Enriques's Quest project in the first place.

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(Received April 10, 2011)

Cooperation in Mathematics Education

I read Professor Wu's article "The Mis-Education of Mathematics Teachers" in the March 2011 *Notices of the AMS* with interest. I agree with the message, beginning with the two conditions for the body of knowledge teachers need. I would like to provide some additional thoughts from the perspective of a mathematics educator who has worked successfully with mathematicians for twenty-five years while in departments of mathematics and for twelve years while in colleges of education following five years of teaching high school mathematics.

I agree that the input of mathematicians is a necessary condition for developing courses for teachers, but offer that

1. mathematicians should work cooperatively with mathematics educators, not in isolation or at odds with mathematics educators;
2. such efforts by mathematicians should be viewed as significant contributions in evaluating tenure and promotions in their departments;
3. previous recommendations addressing teacher preparation and noteworthy curriculum materials developed by mathematicians—and how they did or did not bring about desirable changes—should be considered; and,
4. just as mathematics educators who are leading the process need to be well prepared in mathematics, mathematicians should have extended experience observing and interacting in a wide range of schools, especially those where the challenges are greatest.

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(Received March 23, 2011)

Hurricanes as Weatherquakes and their Response to Climate Change

In a recent article, M. E. Walter introduces the interesting concept of "weatherquake", in analogy with earthquakes [1]. Tornados and hurricanes are mentioned as examples of such meteorological events. Then Walter enunciates the *weatherquake hypothesis*, summarized by the statement that there is no preferred scale for the size of weatherquakes, which implies that their size has to be power-law distributed. For simplicity, size is labeled as intensity, although I find it more clear to talk about energy (nevertheless, the distinction is not relevant for the purposes here). So the probability density of the size z verifies $N_z(z) = \tau\, z^{-(1+\tau)}$ for $1 < z < \infty$ (in arbitrary units). Defining a magnitude x as $x = \alpha \ln z$, it turns out that x is exponentially distributed, $N_x(x) = \beta\, e^{-\beta x}$, with $\beta = \tau/\alpha$.

In Ref. [2] we have found that the weatherquake hypothesis is in some sense verified by tropical cyclones (from which hurricanes constitute a subset), that is, there is a range $1 < z < b < \infty$ for which the power law holds, although a better description is given for the whole range by $N_z(z) = A\, z^{-(1+\tau)}\, e^{-z/w}$ [3]. The extra exponential term arises as a boundary effect and does not reflect an intrinsic characteristic of tropical-cyclone evolution, in contrast to the power-law exponent.

Combining the weatherquake hypothesis with *Gallet's conjecture*, which associates an increase in the proportion of extreme weather events with atmospheric warming, Walter proposes as plausible that the characteristic constant β in $N_x(x)$, and therefore the power-law exponent $1+\tau$, decreases in a warmer environment. What we have found for tropical cyclones is that τ (and then β) is nearly independent of sea surface temperature (SST), but the characteristic energy cutoff w increases with SST, both in the North Atlantic and in the Northeastern Pacific ocean basins. Our result is in agreement on the one hand with Gallet's conjecture but also with common knowledge in thermodynamics of phase transitions, where critical exponents such as τ are extremely robust under perturbations.

Another example of weatherquake arises in rainfall, for which similar features hold [4].

Let me thank information flow from my campus neighbor P. Puig.

[1] M. E. Walter, Earthquakes and weatherquakes: Mathematics and climate change, *Notices of the American Mathematical Society*, **57**:1278–1284, 2010.
[2] A. Corral, A. Ossó, and J. E. Llebot, Scaling of tropical-cyclone dissipation, *Nature Phys.*, **6**:693–696, 2010.
[3] A. Corral, Variability of North Atlantic hurricanes: Seasonal versus individual-event features, preprint, 2011.
[4] O. Peters, A. Deluca, A. Corral, J. D. Neelin, and C. E. Holloway, Universality of rain event size distributions, *J. Stat. Mech.*, P11030, 2010.

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Submitting Letters to the Editor

The *Notices* invites readers to submit letters and opinion pieces on topics related to mathematics. Electronic submissions are preferred (`notices-letters@ams.org`); see the masthead for postal mail addresses. Opinion pieces are usually one printed page in length (about 800 words). Letters are normally less than one page long, and shorter letters are preferred.

Phase Plots of Complex Functions: A Journey in Illustration

Elias Wegert and Gunter Semmler

This work was inspired by the article "Möbius transformations revealed" by Douglas Arnold and Jonathan Rogness [3]. There the authors write:
"*Among the most insightful tools that mathematics has developed is the representation of a function of a real variable by its graph. … The situation is quite different for a function of a complex variable. The graph is then a surface in four-dimensional space, and not so easily drawn. Many texts in complex analysis are without a single depiction of a function. Nor is it unusual for average students to complete a course in the subject with little idea of what even simple functions, say trigonometric functions, 'look like'.*"

In the printed literature there are a few laudable exceptions to this rule, such as the prize-winning *Visual Complex Analysis* by Tristan Needham [28], Steven Krantz's textbook [21] with a chapter on computer packages for studying complex variables, and the MAPLE-based (German) introduction to complex function theory [15] by Wilhelm Forst and Dieter Hoffmann.

But looking behind the curtain, one encounters a different situation that is evolving very quickly. Some of us have developed our own techniques for visualizing complex functions in teaching and research, and one can find many beautiful illustrations of complex functions on the Internet.

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Both authors were supported by the Deutsche Forschungsgemeinschaft with grant We1704/8-2.

Figure 1. The phase plot of an analytic function in the unit disk.

This article is devoted to "phase plots" or "phase portraits", a special tool for visualizing and exploring analytic functions. Figure 1 shows such a fingerprint of a function in the complex unit disk. The explanation of this illustration is deferred to a later section, where it is investigated in detail.

Phase plots have been invented independently by a number of people, and it is impossible to give credit to someone for being the first. Originally, they were mainly used in teaching as simple and effective methods for visualizing complex functions. Over the years, and in particular during the process of writing and rewriting this manuscript, the topic developed its own dynamic, and gradually these

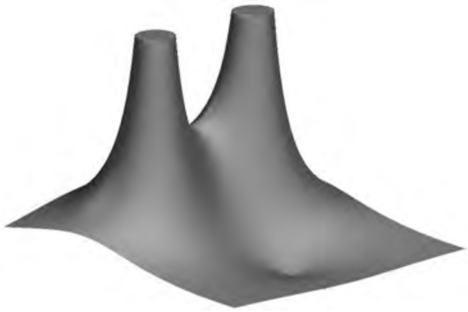

Figure 2. The analytic landscape of $f(z) = (z-1)/(z^2+z+1)$.

innocent illustrations transmuted to sharp tools for dissecting complex functions.

So the main purpose of this article is not only to present nice pictures that allow one to recognize complex functions by their individual faces, but also to develop the mathematical background and demonstrate the utility and creative uses of phase plots. That they sometimes also facilitate a new view on known results and may open up new perspectives is illustrated by a universality property of the Riemann zeta function that, in the setting of phase plots, can be explained to (almost) anyone.

The final section is somewhat special. It resulted from a self-experiment carried out to demonstrate that phase plots are sources of inspiration that can help to establish new results. The main finding is that any meromorphic function is associated with a *dynamical system* that generates a *phase flow* on its domain and converts the phase plot into a *phase diagram*. These diagrams will be useful tools for exploring complex functions, especially for those who prefer thinking geometrically.

Visualization of Functions

The graph of a function $f : D \subset \mathbb{C} \to \mathbb{C}$ lives in four real dimensions, and since our imagination is trained in three-dimensional space, most of us have difficulties in "seeing" such an object.[1]

Some books on complex function theory have nice illustrations showing the *analytic landscape* of a function, which is the graph of its modulus (see Figure 2). The concept was not introduced by Johann Jensen in 1912, as is sometimes claimed, but probably earlier by Edmond Maillet [27] in 1903 (see also Otto Reimerdes's paper [32] of 1911).

Differential geometric properties of analytic landscapes have been studied in quite a number of early papers (see Ernst Ullrich [37] and the references therein). Jensen [20] and others also considered the graph of $|f|^2$, which is a smooth surface. The second edition of "Jahnke-Emde" [19] made analytic landscapes popular in applied mathematics.

Figure 3. The color circle and the color-coded phase of points close to the origin.

Analytic landscapes involve only one part of the function f, its *modulus* $|f|$; the *argument* $\arg f$ is lost. In the era of black-and-white illustrations our predecessors sometimes compensated for this shortcoming by complementing the analytic landscape with lines of constant argument. Today we can achieve this much better using colors. Since coloring is an essential ingredient of phase plots, we consider it in some detail.

Recall that the argument $\arg z$ of a complex number z is unique up to an additive multiple of 2π. In order to make the argument well defined its values are often restricted to the interval $(-\pi, \pi]$, or, even worse, to $[0, 2\pi)$. This ambiguity disappears if we replace $\arg z$ with the *phase* $z/|z|$ of z. Though one usually does not distinguish between the notions of "argument" and "phase", it is essential here to keep these concepts separate.

The phase lives on the complex unit circle $\mathbb{T}$, and points on a circle can naturally be encoded by *colors*. We thus let color serve as the lacking fourth dimension when representing graphs of complex-valued functions (see Figure 3). The *colored analytic landscape* is the graph of $|f|$ colored according to the phase of f. Since the modulus of analytic functions typically varies over a wide range, one does better to use a logarithmic scaling of the vertical axis. This representation is also more natural since $\log |f|$ and $\arg f$ are conjugate harmonic functions (see Figure 4). Colored analytic landscapes came to life with easy access to computer graphics and by now quite a number of people have developed software for their visualization. Andrew Bennett [7] has an easy-to-use Java implementation, and an executable Windows program can be downloaded from Donald Marshall's website [26]. We further refer to Chapter 12 of Steven Krantz's book [21], as well as to the websites run by Hans Lundmark [25] and Tristan Needham [29]. Very beautiful pictures of (uncolored) analytic landscapes can be found on the "The Wolfram Special Function Site" [43].[2]

[1] *One exception is Thomas Banchoff, who visualized four-dimensional graphs of complex functions in* [6].

[2] *As of March 2011 Wolfram's tool visualizes the modulus and the principal value of the argument, but not phase.*

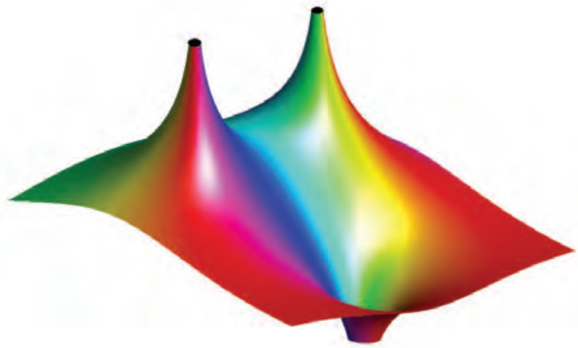

Figure 4. The colored logarithmic analytic landscape of $f(z) = (z-1)/(z^2+z+1)$.

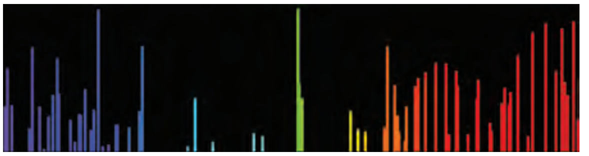

Figure 5. A typical spectrum of polar light.

Though color printing is still expensive, colored analytic landscapes also appear in the printed literature (see, for example, the outstanding mathematics textbook [1] by Arens et al. for engineering students).

With colored analytic landscapes the problem of visualizing complex functions could be considered solved. However, there is yet another approach that is not only simpler but even more general.

Instead of drawing a graph, one can depict a function directly on its domain by color-coding its values, thus converting it to an *image*. Such *color graphs* of functions f live in the product of the domain of f with a color space.

Coloring techniques for visualizing functions have been familiar for many decades, for example in depicting altitudes on maps, but mostly they represent *real-valued* functions using a one-dimensional color scheme. It is reported that two-dimensional color schemes for visualizing complex valued functions have been in use for more than twenty years by now (Larry Crone [9], Hans Lundmark [25]), but they became popular only with Frank Farris's review [13] of Needham's book and its complement [14]. Farris also coined the name "domain coloring".

Domain coloring is a natural and universal substitute for the graph of a function. Moreover, it easily extends to functions on Riemann surfaces or on surfaces embedded in $\mathbb{R}^3$ (see Konstantin Poelke and Konrad Polthier [31], for instance).

It is worth mentioning that we human beings are somewhat limited with respect to the available color spaces. Since our visual system has three different color receptors, we can only recognize colors from a three-dimensional space. Mathematicians of the species *Gonodactylus oerstedii*[3] could use domain coloring techniques to even visualize functions with values in a twelve-dimensional space (Welsch and Liebmann [41], p. 268; for details see Cronin and King [10]).

[3] *The Mantis shrimp (Gonodactylus oerstedii) has twelve separate visual pigments.*

Indeed, many people are not aware that natural colors in fact provide us with an *infinite dimensional space*—at least theoretically. In reality "color" always needs a carrier. "Colored light" is an electromagnetic wave that is a mixture of monochromatic components with different wavelengths and intensities. A simple prismatic piece of glass reveals how light is composed from its *spectral components*. Readers interested in further information are recommended to visit the fascinating Internet site of Dieter Zawischa [44]. The wavelengths of visible light fill an interval between 375 nm and 750 nm approximately, and hence color spectra form an infinite-dimensional space (see Figure 5).

How many color dimensions are distinguishable in reality depends on the resolution of the measuring device. A simple model of the human eye, which can be traced back to Thomas Young in 1802, assumes that our color recognition is based on three types of receptors that are sensitive to red, green, and blue, respectively.

Since, according to this assumption, our visual color space has dimension three, different spectra of light induce the same visual impression. Interestingly, a mathematical theory of this effect was developed as early as in 1853 by Hermann Grassmann, the ingenious author of the "Ausdehnungslehre", who found three fundamental laws of this so-called *metamerism* [16] (see Welsch and Liebmann [41]).

Bearing in mind that the world of real colors is infinite dimensional, it becomes obvious that its compression to at most three dimensions cannot lead to completely satisfying results, which explains the variety of color schemes in use for different purposes. The most popular color systems in our computer-dominated world are the RGB, CMYK, and HSV schemes.

In contrast to domain colorings that color-code the complete values $f(z)$ by a two-dimensional color scheme, *phase plots* display only $f(z)/|f(z)|$ and thus require just a *one-dimensional* color space with a *circular topology*. As will be shown in the next section, they nevertheless contain almost all relevant information about the depicted analytic or meromorphic function.

In Figure 6 the Riemann sphere $\widehat{\mathbb{C}}$ (with the point at infinity on top) is colored using two typical schemes for phase plots (left) and domain

Figure 6. Color schemes for phase plots and domain coloring on the Riemann sphere.

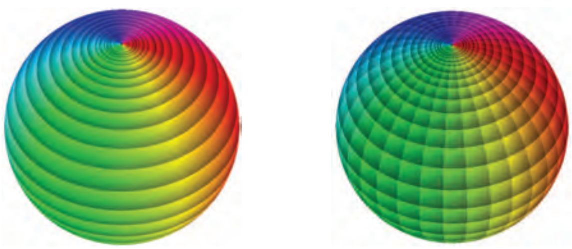

Figure 7. Two color schemes involving sawtooth functions of gray.

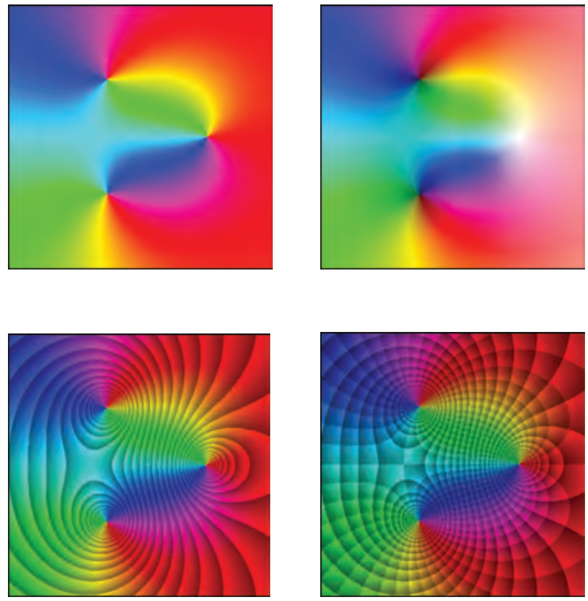

Figure 8. Four representations of the function $f(z) = (z-1)/(z^2+z+1)$.

coloring (right), respectively. Somewhat surprisingly, the number of people using phase plots seems to be quite small. The website of François Labelle [24] has a nice gallery of nontrivial pictures,[4] including Euler's gamma and Riemann's zeta functions.

Since the phase of a function occupies only one dimension of the color space, there is plenty of room for depicting additional information. It is recommended to encode this information by a gray scale, since color (hue) and brightness are *visually orthogonal*. Figure 7 shows two such color schemes on the Riemann w-sphere. The left scheme is a combination of phase plots and standard domain coloring. Here the brightness b does not depend monotonously on $\log|w|$ but is a sawtooth function thereof, like, for example, $b(w) = \log|w| - \lfloor \log|w| \rfloor$. This coloring works equally well, no matter in which range the values of the function are located.

In the right scheme the brightness is the product of two sawtooth functions depending on $\log|w|$ and $w/|w|$, respectively. The discontinuities of this shading generate a logarithmically scaled polar grid. *Pulling back* the coloring from the w-sphere to the z-domain of f by the mapping $w = f(z)$ resembles a *conformal grid mapping*, another well-known technique for depicting complex functions (see Douglas Arnold [2]). Note that pulling back a grid instead of pushing it forward avoids multiple coverings. Of course all coloring schemes can also be applied to functions on Riemann surfaces.

[4] *Labelle justifies the sole use of phase by reasons of clarity and aesthetics.*

For comparison, Figure 8 shows the four representations of $f(z) := (z-1)/(z^2+z+1)$ in the square $|\mathrm{Re}\, z| \le 2$, $|\mathrm{Im}\, z| \le 2$ corresponding to the color schemes of Figure 6 and Figure 7, respectively. Though these pictures (in particular the upper two) look quite similar, which makes it simple to use them in parallel, the philosophy and the mathematics behind them is quite different. I shall comment on this issue in the final section.

The Phase Plot

The phase of a complex function $f : D \to \hat{\mathbb{C}}$ is defined on $D_0 := \{z \in D : f(z) \in \mathbb{C}^\times\}$, where $\mathbb{C}^\times$ denotes the complex plane punctured at the origin. Nevertheless we shall speak of phase plots $P : D \to \mathbb{T}, z \mapsto f(z)/|f(z)|$ on D, considering those points where the phase is undefined as singularities. Recall that $\mathbb{T}$ stands for the (colored) unit circle.

To begin with we remark that meromorphic functions are characterized almost uniquely by their phase plot.

Theorem 1. *If two nonzero meromorphic functions f and g on a connected domain D have the same phase, then f is a positive scalar multiple of g.*

Proof. Removing from D all zeros and poles of f and g, we get a connected domain D_0. Since, by assumption, $f(z)/|f(z)| = g(z)/|g(z)|$ for all $z \in D_0$, the function f/g is holomorphic and real-valued in D_0, and so it must be a (positive) constant.

It is obvious that the result extends to the case in which the phases of f and g coincide merely on an open subset of D.

In order to check whether two functions f and g with the same phase are equal, it suffices to compare their values at a single point that is neither a

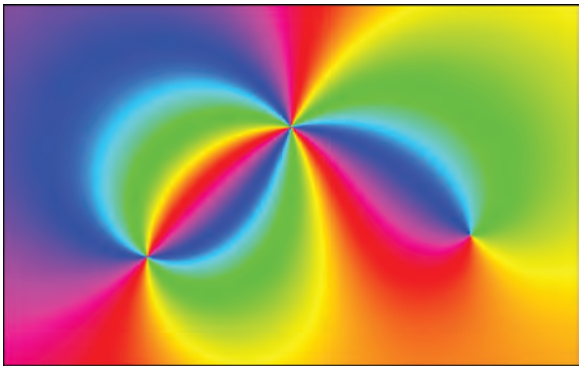

Figure 9. A function with a simple zero, a double zero, and a triple pole.

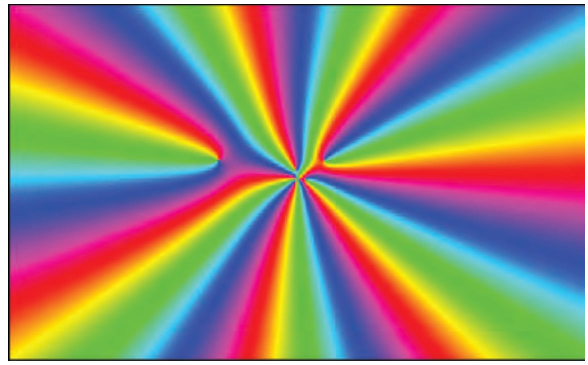

Figure 10. This function has no poles. How many zeros are in the displayed rectangle?

zero nor a pole. For purists there is also an intrinsic test that works with phases alone: Assume that the nonconstant meromorphic functions f and g have the same phase plot. Then it follows from the open mapping principle that $f \neq g$ if and only if the phase plots of $f + c$ and $g + c$ are different for one, and then for all, complex constants $c \neq 0$.

Zeros and Poles

Since the phases of zero and infinity are undefined, zeros and poles of a function are singularities of its phase plot. What does the plot look like in a neighborhood of such points?

If a meromorphic function f has a zero of order n at z_0 it can be represented as

$$f(z) = (z - z_0)^n g(z),$$

where g is meromorphic and $g(z_0) \in \mathbb{C}^\times$. It follows that the phase plot of f close to z_0 resembles the phase plot of z^n at 0, rotated by the angle $-(1/n)\arg g(z_0)$. The same reasoning, with a negative integer n, applies to poles.

Note in Figure 9 that the colors are arranged in opposite orders for zeros and poles. It is now clear that the phase plot not only shows the location of zeros and poles but also reveals their *multiplicity*.

A useful tool for locating zeros is the *argument principle*. In order to formulate it in the context of phase plots we translate the definition of winding number into the language of colors: Let $\gamma : \mathbb{T} \to D_0$ be a closed oriented path in the domain D_0 of a phase plot $P : D_0 \to \mathbb{T}$. Then the usual winding number of the mapping $P \circ \gamma : \mathbb{T} \to \mathbb{T}$ is called the *chromatic number* of γ with respect to the phase plot P and is denoted by $\operatorname{chrom}_P \gamma$ or simply by $\operatorname{chrom} \gamma$.

Less formally, the chromatic number counts how many times the color of the point $\gamma(t)$ moves around the complete color circle when $\gamma(t)$ traverses γ once in the positive direction.

Now the argument principle can be rephrased as follows: *Let D be a Jordan domain with positively oriented boundary ∂D and assume that f is meromorphic in a neighborhood of D. If f has n zeros and p poles in D (counted with multiplicity), and none of them lies on ∂D, then*

$$n - p = \operatorname{chrom} \partial D.$$

Looking at Figure 10 in search of zeros immediately brings forth new questions, for example: Where do the isochromatic lines end up? Can they connect two zeros? If so, do these lines have a special meaning? What about "basins of attraction"? Is there always a natural (cyclic) ordering of zeros? What can be said about the global structure of phase plots? We shall return to these issues later.

The Logarithmic Derivative

Along the *isochromatic lines* of a phase plot the argument of f is constant. The Cauchy-Riemann equations for a continuous branch of the logarithm $\log f = \log |f| + \mathrm{i} \arg f$ imply that these lines are orthogonal to the level lines of $|f|$, i.e., the isochromatic lines are parallel to the gradient of $|f|$. According to the chosen color scheme, we have red on the right and green on the left when walking on a yellow line in the ascending direction.

To go a little beyond this qualitative result, we denote by s the unit vector parallel to the gradient of $|f|$ and set $n := \mathrm{i}s$. With $\varphi := \arg f$ and $\psi := \log |f|$ the Cauchy-Riemann equations for $\log f$ imply that the directional derivatives of φ and ψ satisfy

$$\partial_s \psi = \partial_n \varphi > 0, \quad \partial_n \psi = -\partial_s \varphi = 0,$$

at all points z of the phase plot where $f(z) \neq 0$ and $f'(z) \neq 0$. Since the absolute value of $\partial_n \varphi$ measures the density of the isochromatic lines, we can visually estimate the growth of $\log |f|$ along these lines from their density. Because the phase plot delivers no information on the absolute value, this does not say much about the growth of $|f|$. But taking into account the second Cauchy-Riemann equation and

$$|(\log f)'|^2 = (\partial_n \varphi)^2 + (\partial_s \varphi)^2,$$

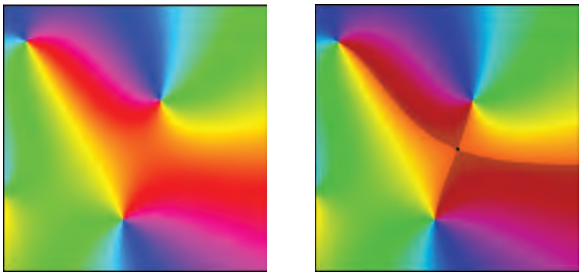

Figure 11. Zeros of f' are color saddles.

Figure 12. The analytic landscape of $f(z) = e^{1/z}$.

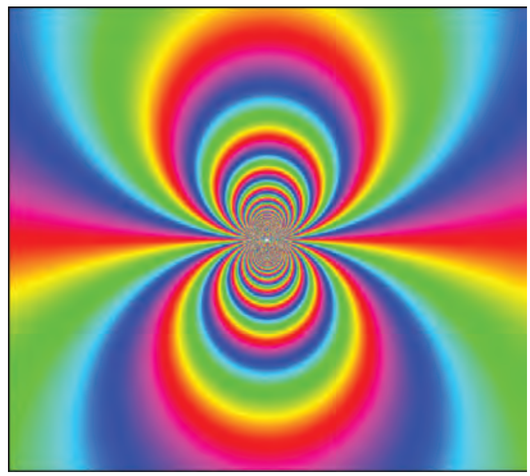

Figure 13. A phase plot depicting the essential singularity of $f(z) = e^{1/z}$.

we obtain the correct interpretation of the density $\partial_n \varphi$: it is the modulus of the *logarithmic derivative*,

$$\partial_n \varphi = |f'/f|. \tag{1}$$

So, finally, we need not worry about branches of the logarithm. It is worth mentioning that $\partial_n \varphi(z)$ behaves asymptotically like $k/|z - z_0|$ if z approaches a zero or pole of order k at z_0.

But this is not yet the end of the story. What about zeros of f'? Equation (1) indicates that something should be visible in the phase plot. Indeed, points z_0 where $f'(z_0) = 0$ and $f(z_0) \neq 0$ are "*color saddles*", i.e., intersections of isochromatic lines.

If f' has a zero of order k at z_0, then $z \mapsto f(z) - f(z_0)$ has a zero of order $k + 1$ at z_0. Consequently, f can be represented as

$$f(z) = f(z_0) + (z - z_0)^{k+1} g(z),$$

where $g(z_0) \neq 0$. It follows that $f(z)$ travels $k + 1$ times around $f(z_0)$ when z moves once around z_0 along a small circle. In conjunction with $f(z_0) \neq 0$ this can be used to show that there are exactly $2k + 2$ isochromatic lines emanating from z_0 where the phase of f is equal to the phase of $f(z_0)$. Alternatively, one can also think of $k + 1$ smooth isochromatic lines intersecting each other at z_0. Color saddles appear as diffuse spots such as at the center of the left picture in Figure 11. To locate them precisely it is helpful to modify the color scheme so that it gets a jump at some point t of the unit circle. If $t := f(z_0)/|f(z_0)|$ is chosen, then the phase plot shows a sharp saddle at the zero z_0 of f' as in the right picture.

Essential Singularities

Have you ever seen an essential singularity? Figure 12 is the picture that usually illustrates this situation.

Despite the massive tower, this is not very impressive, and with regard to the Casorati-Weierstrass theorem or the Great Picard theorem one would expect something much wilder. Why does the analytic landscape not reflect this behavior? For the example the answer is easy: the function has a tame modulus, every contour line is a single circle through the origin. Now look at the phase plot in Figure 13. But must there not be a symmetry between modulus and phase? In fact not. There is such a symmetry of modulus and *argument* (for nonvanishing functions), but phase plots depict the phase and not the argument—and this makes a difference.

So much for the example, but what about the general case? Perhaps there are also functions that conceal their essential singularities in the phase plot?

In order to show that this cannot happen, we assume that $f : D\{z_0\} \to \mathbb{C}$ is analytic and has an essential singularity at z_0.

By the Great Picard theorem, there exists a color $c \in \mathbb{T}$ such that any punctured neighborhood U of z_0 contains infinitely many points $z_k \in U$ with $f(z_k) = c$. Moreover, the set of zeros of f' in D is at most countable, and hence we can choose c such that no saddle point $z \in U$ with $f'(z) = 0$ and $f(z) \neq 0$ lies on an isochromatic line with color c. As was shown in the preceding section, the modulus of f is strictly monotone along such lines, so that two distinct points z_k cannot lie on the same isochromatic line.

Consequently any neighborhood of an essential singularity contains a countable set of pairwise disjoint isochromatic lines with color c. Combining

Figure 14. Phase plot of $f(z) = e^z$.

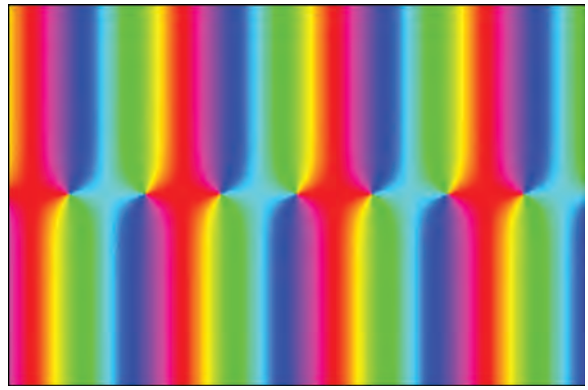

Figure 15. Phase plot of $f(z) = \sin z$.

this observation with the characterization of phase plots near poles and removable singularities, we obtain the following result.

Theorem 2. *An isolated singularity z_0 of an analytic function f is an essential singularity if and only if any neighborhood of z_0 intersects infinitely many isochromatic lines of the phase plot with one and the same color.*

Note that a related result does not hold for the argument, since then, in general, the values of $\arg f(z_k)$ are different. For example, any two isochromatic lines of the function $f(z) = \exp(1/z)$ have a different argument.

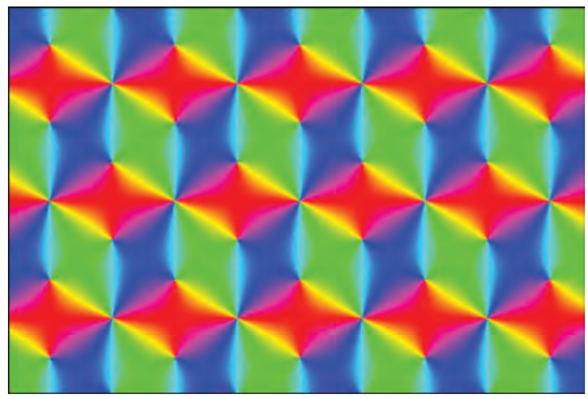

Figure 16. Phase plot of a Weierstrass $\wp$-function.

Periodic Functions

Obviously, the phase of a periodic function is periodic, but what about the converse?

Though there are only *two* classes (simply and doubly periodic) of nonconstant periodic meromorphic functions on $\mathbb{C}$, we can observe *three* different types of periodic phase plots.

"Striped" phase plots such as Figure 14 always depict exponential functions $f(z) = e^{az+b}$ with $a \neq 0$. Functions with simply p-periodic phase (see Figure 15, for example) need not be periodic, but have the more general form $e^{\alpha z/p}\, g(z)$ with $\alpha \in \mathbb{R}$ and a p-periodic function g.

The first result basically follows from the fact that the function $\arg f$ is harmonic and has parallel straight contour lines, which implies that $\arg f(x + iy) = \alpha x + \beta y + \gamma$. Since $\log |f|$ is conjugate harmonic to $\arg f$, it necessarily has the form $\log |f(x + iy)| = -\alpha y + \beta x + \delta$.

If the phase of f is p-periodic, then we have

$$h(z) := \frac{f(z+p)}{f(z)} = \frac{|f(z+p)|}{|f(z)|} \in \mathbb{R}_+,$$

and since h is meromorphic on $\mathbb{C}$, it must be a positive constant e^{α}. Now it follows easily that $g(z) := f(z) \cdot e^{-\alpha z/p}$ is periodic with period p.

Finally, if the phase plot of f is doubly periodic (see Figure 16), i.e., $f/|f|$ has periods p_1 and p_2 with $p_1/p_2 \notin \mathbb{R}$, then it does not necessarily follow that f is an elliptic function. A counterexample is $f(z) := e^{\alpha z} g(z)$, where $g \neq$ const has periods p_1 and p_2, and $\alpha \in \mathbb{C}\setminus\{0\}$ is a solution of the linear system

$$\operatorname{Im}(\alpha p_j) = 2k_j\pi, \quad j = 1, 2,$$

with $k_1, k_2 \in \mathbb{Z}$. It may be an interesting problem to prove that all functions with doubly periodic phase plots are indeed of this form.

Partial Sums of Power Series

Figure 17 shows a strange image that, in similar form, occurred in a numerical experiment as the phase plot of a Taylor polynomial. Since it looks so special, one could attribute it to a programming error. A moment's thought reveals what is going on here. This example demonstrates again that looking at phase plots can immediately provoke new questions.

Indeed the figure illustrates a general result (see Titchmarsh [36], Section 7.8) that was proven by Robert Jentzsch in 1914:

If a power series $a_0 + a_1 z + a_2 z^2 + \cdots$ has a positive finite convergence radius R, then the zeros of its partial sums cluster at every point z with $|z| = R$.

The reader interested in the life and personality of Robert Jentzsch is referred to the recent

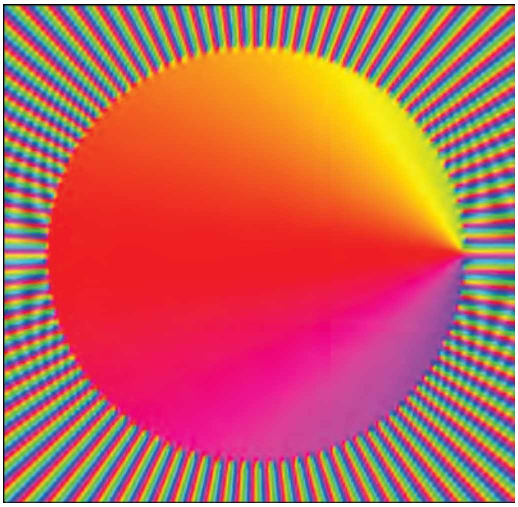

Figure 17. A Taylor polynomial of $f(z) = 1/(1-z)$.

paper [11] by Peter Duren, Anne-Katrin Herbig, and Dmitry Khavinson.

Boundary Value Problems

Experimenting with phase plots raises a number of new questions. One such problem is to find a criterion for deciding which color images are *analytic phase plots*, i.e., phase plots of analytic functions.

Since phase plots are painted with the restricted palette of *saturated* colors from the color circle, Leonardo's Mona Lisa will certainly never appear. But for analytic phase plots there are much stronger restrictions: By the uniqueness theorem for harmonic functions an arbitrarily small open piece determines the plot entirely.

So let us pose the question a little differently: What are appropriate data that can be prescribed to construct an analytic phase plot, say, in a Jordan domain D? Can we start, for instance, with given colors on the boundary ∂D? If so, can the boundary colors be prescribed arbitrarily, or are they subject to constraints?

In order to state these questions more precisely we introduce the concept of a *colored set* K_C, which is a subset K of the complex plane together with a mapping $C : K \to \mathbb{T}$. Any such mapping is referred to as a *coloring* of K.

For simplicity we consider here only the following setting of boundary value problems for phase plots with continuous colorings:

Let D be a Jordan domain and let B be a continuous coloring of its boundary ∂D. Find all continuous colorings C of $\overline{D}$ such that the restriction of C to ∂D coincides with B and the restriction of C to D is the phase plot of an analytic function f in D.

If such a coloring C exists, we say that the coloring B *admits a continuous analytic extension to* $\overline{D}$.

The restriction to continuous colorings automatically excludes zeros of f in D. It does, however, *not* imply that f must extend continuously onto $\overline{D}$—and in fact it is essential not to require the continuity of f on $\overline{D}$ in order to get a nice result.

Theorem 3. *Let D be a Jordan domain with a continuous coloring B of its boundary ∂D. Then B admits a continuous analytic extension to $\overline{D}$ if and only if the chromatic number of B is zero. If such an extension exists, then it is unique.*

Proof. If $C : \overline{D} \to \mathbb{T}$ is a continuous coloring, then a simple homotopy argument (contract ∂D inside D to a point) shows that the chromatic number of its restriction to ∂D must vanish.

Conversely, any continuous coloring B of ∂D with chromatic number zero can be represented as $B = e^{i\varphi}$ with a continuous function $\varphi : \partial D \to \mathbb{R}$. This function admits a unique continuous harmonic extension Φ to $\overline{D}$. If Ψ denotes a harmonic conjugate of Φ, then $f = e^{i\Phi - \Psi}$ is analytic in D. Its phase $C := e^{i\Phi}$ is continuous on $\overline{D}$ and coincides with B on ∂D. Recall that f need not be continuous on $\overline{D}$.

Theorem 3 *parameterizes* analytic phase plots that extend continuously on $\overline{D}$ by their boundary colorings. This result can be generalized to phase plots that are continuous on $\overline{D}$ with the exception of finitely many singularities of zero or pole type in D. Admitting now boundary colorings B with arbitrary chromatic number we get the following result:

For any finite collection of given zeros with orders $n_1, \ldots, n_j$ and poles of orders $p_1, \ldots, p_k$ the boundary value problem for meromorphic phase plots with prescribed singularities has a (unique) solution if and only if the boundary coloring B satisfies

$$\operatorname{chrom} B = n_1 + \cdots + n_j - p_1 - \cdots - p_k.$$

The Riemann Zeta Function

After these preparations we are ready to pay a visit to "zeta", the mother of all analytic functions. Figure 18 is a phase plot in the square $-40 \le \operatorname{Re} z \le 10$, $-2 \le \operatorname{Im} z \le 48$. We see the pole at $z = 1$, the *trivial zeros* at the points $-2, -4, -6, \ldots$, and several zeros on the *critical line* $\operatorname{Re} z = 1/2$. Also we observe that the isochromatic lines are quite regularly distributed in the left half plane.

Saying that zeta is the mother of all functions alludes to its *universality*. Our starting point is the following strong version of Voronin's universality theorem due to Bagchi [4] (see also Karatsuba and Voronin [22], Steuding [35]):

Let D be a Jordan domain such that $\overline{D}$ is contained in the strip

$$R := \{z \in \mathbb{C} : 1/2 < \operatorname{Re} z < 1\},$$

and let f be any function that is analytic in D, continuous on $\overline{D}$, and has no zeros in $\overline{D}$. Then f can be

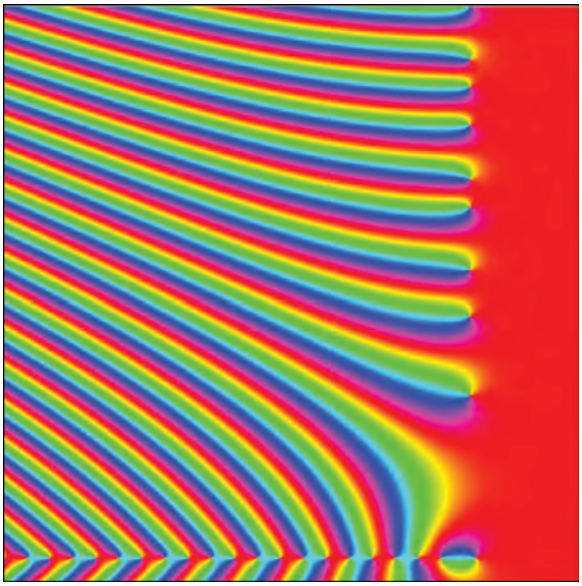

Figure 18. The Riemann zeta function.

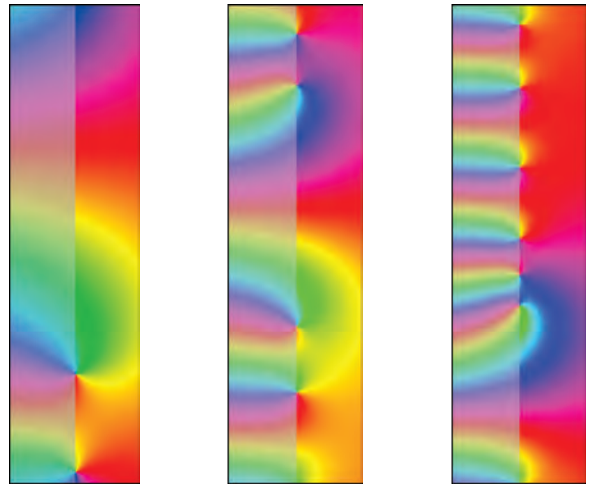

Figure 20. The Riemann zeta function at $\operatorname{Im} z = 171$, 8230 and 121415.

uniformly approximated on $\overline{D}$ by vertical shifts of zeta, $\zeta_t(z) := \zeta(z + \mathrm{i}t)$ with $t \in \mathbb{R}$.

Recall that a continuously colored Jordan curve J_C is a continuous mapping $C : J \to \mathbb{T}$ from a simple closed curve J into the color circle $\mathbb{T}$. A *string* S is an equivalence class of all such colored curves with respect to rigid motions of the plane. Like colored Jordan curves, strings fall into different classes according to their *chromatic number*. Figure 19 depicts a representative of a string with chromatic number one.

We say that a string S *lives in a domain* D if it can be represented by a colored Jordan curve J_C with $J \subset D$. A string can *hide itself* in a phase plot $P : D \to \mathbb{T}$ if, for every $\varepsilon > 0$, it has a representative J_C such that $J \subset D$ and

$$\max_{z \in J} |C(z) - P(z)| < \varepsilon.$$

In less technical terms, a string can hide itself if it can move to a place where it is invisible since it blends in almost perfectly with the background.

In conjunction with Theorem 3, the following universality result for the phase plot of the Riemann zeta function can easily be derived from Voronin's theorem.

Figure 19. A representative of a string.

Theorem 4. *Let S be a string which lives in the strip R. Then S can hide itself in the phase plot of the Riemann zeta function on R if it has chromatic number zero.*

In view of the extreme richness of Jordan curves and colorings, this result is a real miracle. Figure 20 shows phase plots of zeta in the critical strip. The regions with saturated colors belong to R. The rightmost figure depicts the domain considered on p. 342 of Conrey's paper [8].

What about the converse of Theorem 4? If there exist strings with *nonzero chromatic number* that can hide themselves in the strip R, their potential hiding places must be Jordan curves with nonvanishing chromatic number in the phase plot. By the argument principle, this would imply that zeta has zeros in R. If we assume this, for a moment, then such strings indeed exist: they are perfectly hidden and wind themselves once around such a zero. So the converse of Theorem 4 holds if and only if R contains no zeros of zeta, which is known to be equivalent to the *Riemann hypothesis* (see Conrey [8], Edwards [12]).

Phase Flow and Diagrams

Mathematical creativity is based on the interplay of problem posing and problem solving, and it is our belief that the former is even more important than the latter: often the key to solving a problem lies in asking the right questions.

Illustrations have a high density of information and stimulate imagination. Looking at pictures helps in getting an intuitive understanding of mathematical objects and finding interesting questions, which then can be investigated using rigorous mathematical techniques.

This section intends to demonstrate how phase plots can produce novel ideas. The material presented here is the protocol of a self-experiment that has been carried out by the first author in order to check the creative potential of phase plots.

Let us start by looking at Figure 1 again. It depicts the phase of a *finite Blaschke product*, which

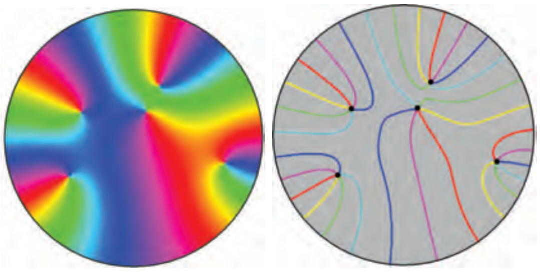

Figure 21. The phase plot of a Blaschke product with five zeros.

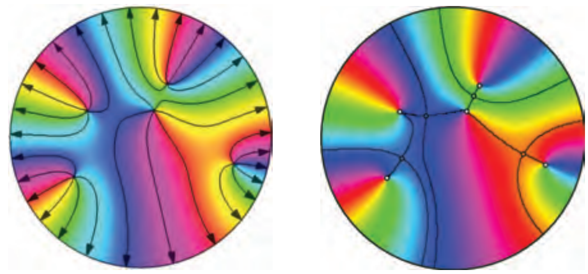

Figure 22. Phase transport to the boundary, zeros of f and f' with invariant manifolds.

is a function of the form

$$f(z) = c \prod_{k=1}^{n} \frac{z - z_k}{1 - \overline{z_k} z}, \quad z \in \mathbb{D},$$

with $|z_k| < 1$ and $|c| = 1$. Blaschke products are fundamental building blocks of analytic functions in the unit disc and have the property $|f(z)| = 1$ for all $z \in \mathbb{T}$. The function shown in Figure 1 has eighty-one zeros z_k in the unit disk.

Looking at Figure 1 for a while leaves the impression of a cyclic ordering of the zeros. Let us test this with another example having only five zeros (Figure 21, left). The picture seems to confirm the expectation: if we focus attention to the yellow color, any of these lines connects a zero with a certain point on the boundary, thus inducing a cyclic ordering.

However, looking only at one specific color is misleading. Choosing another one, for instance blue, can result in a different ordering. So what is going on here? More precisely: What is the *global structure* of the phase plot of a Blaschke product? This could be a good question.

An appropriate mathematical framework in which to develop this idea is the theory of *dynamical systems*. We here sketch only the basic facts; for details see [39].

With any meromorphic function f in a domain D we associate the dynamical system

$$\dot{z} = g(z) := \frac{f(z)\,\overline{f'(z)}}{|f(z)|^2 + |f'(z)|^2}. \qquad (2)$$

The function g on the right-hand side of (2) extends from D_0 to a smooth function on D. This system induces a *flow* Φ on D, which we designate as the *phase flow* of f.

The fixed points of (2) are the zeros of f (repelling), the poles of f (attracting), and the zeros of f' which are not zeros of f (saddles). The remaining orbits are the components of the isochromatic lines of the phase plot of f when the fixed points are removed. Thus the orbits of the phase flow endow the phase plot with an additional structure and convert it into a *phase diagram* (for details see [39]).

Intuitively, the phase flow Φ transports a colored substance ("phase") from the zeros to the poles and to the boundary of the domain (see Figure 22). The left part of Figure 23 illustrates how "phase" of measure 2π emerging from the zero in the highlighted domain is transported along the orbits of Φ until it is finally deposited along (parts of) the boundary.

In the general case, where $f : D \to \widehat{\mathbb{C}}$ is meromorphic on a domain D and $G \subset D$ is a Jordan domain with boundary J in D_0, the phase transport from any zero (pole) of f in G generates a (signed) *measure* on J. The result is a *quantitative version of the argument principle* that tells us in which way the phase of the zeros (poles) is distributed along J (see Figure 23, right).

The question about the structure of phase plots of Blaschke products can now be rephrased in the setting of dynamical systems: What are the *basins of attraction* of the zeros of f with respect to the (reversed) phase flow?

The key to solving this problem is given by the *invariant manifolds* of the saddle points, i.e., the points $a_j \in \mathbb{D}$, where $f'(a_j) = 0$ and $f(a_j) \neq 0$. Removing all *unstable manifolds* of the points a_j from $\mathbb{D}$ results in an open set B, which is the union of connected components B_j. Any component B_j contains exactly one zero b_j of f, where multiple zeros are counted only once.

The intersection of every set $\overline{B_j}$ with $\mathbb{T}$ is not empty and consists of a finite number of arcs A_{ji}. The complete set of these arcs covers the unit circle, and two arcs are either disjoint or their intersection is a singleton. These *separating points* are the endpoints of unstable manifolds that originate from saddle points. For later use we renumber the arcs A_{ji} as $A_1, A_2, \ldots, A_s$ in a counterclockwise direction.

It is obvious that the number s of separating points cannot be less than the number of distinct zeros of f. In order to get an upper bound of s, f has m distinct zeros with multiplicities $\beta_1, \ldots, \beta_m$ and k saddle points where f' has zeros of multiplicities $\alpha_1, \ldots, \alpha_k$, respectively. Then we have

$$\alpha_1 + \cdots + \alpha_k = n - 1, \quad \beta_1 + \cdots + \beta_m = n.$$

The first equation follows from the well-known fact that the derivative of a Blaschke product of order

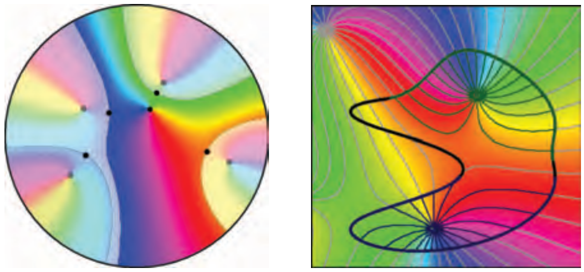

Figure 23. The phase flow and the argument principle.

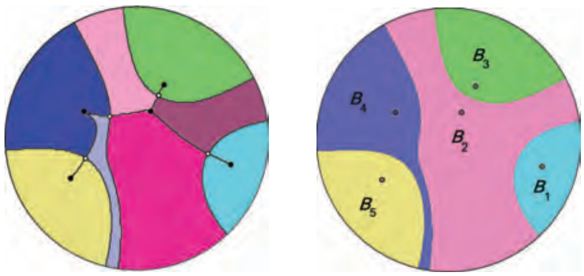

Figure 24. Invariant manifolds of the saddles and basins of attraction of the zeros.

n has exactly $n-1$ zeros in $\mathbb{D}$, this time counting multiplicity. From any saddle point a_j exactly α_j+1 rays emerge that belong to the unstable manifold of a_j. Since any separating point must be the endpoint of one such line, the total number s of separating points cannot be greater than $k+\alpha_1+\cdots+\alpha_k=k+n-1$. Thus we finally get

$$m \le s \le n+k-1.$$

Examples show that both estimates are sharp.

It turns out ([34]) that the global topological structure of the phase plot is completely characterized by the sequence S of integers, which associates with any of the arcs $A_1,\ldots,A_s$ (in consecutive order) the number of the corresponding zero. This sequence depends on the specific numbering of the zeros and the arcs, but an appropriate normalization makes it unique. For example, the Blaschke product depicted in Figure 24 is represented by the sequence $S=(1,2,3,2,4,5,4,2)$.

Let us now return to Figure 1 again. Picturing once more that "phase" is a substance emerging from sources at the zeros that can exit the domain only at its boundary, is it not then quite natural that phase plots of Blaschke products *must* look like they do?

And if you are asking yourself what "natural" means, then this is already another question.

Concluding Remarks

Phase plots result from splitting the information about the function f into two parts (phase and modulus), and one may ask why we do not separate f into its real and imaginary parts. One reason is that often zeros are of special interest; their presence can easily be detected and characterized using the phase, but there is no way to find these from the real or imaginary part alone.

And what is the advantage of using $f/|f|$ instead of $\log|f|$? Of course, zeros and poles can be seen in the analytic landscape, but they are much better represented in the phase plot. In fact there is a subtle asymmetry between modulus and argument (respectively, phase). For example, Theorem 3 has no counterpart for the modulus of a function.[5]

Since phase plots and standard domain coloring produce similar pictures, it is worth mentioning that they are based on different concepts and have a distinct mathematical background.

Recall that standard domain coloring methods use the complete values of an analytic function, whereas phase plots depict only its phase. Taking into account that phase can be considered as a periodization of the argument, which is (locally) a harmonic function, reveals the philosophy behind phase plots: Analytic functions are considered as harmonic functions, endowed with a set of singularities having a special structure. Algebraically, phase plots forget about the linear structure of analytic functions, while their multiplicative structure is preserved.

This approach has at least two advantages. The first one is almost trivial: phase has a small range, the unit circle, which allows visualizing all functions with one and the same color scheme. Moreover, a one-dimensional color space admits a better resolution of singularities. Mathematically more important is the existence of a simple parameterization of *analytic* and *meromorphic* phase plots by their boundary values and their singularities (Theorem 3). There is no such result for domain colorings of analytic functions.

The following potential fields of applications demonstrate that phase plots may be a useful tool for anyone working with complex-valued functions.

1. A trivial but useful application is *visual inspection* of functions. If, for example, it is not known which branch of a function is used in a certain software, a glance at the phase plot may help. In particular, if several functions are composed, software implementations with different branch cuts can lead to completely different results. You may try this with the MATHEMATICA functions `Log(Gamma)` and `LogGamma`. Another useful exercise in teaching is to compare the phase plots of $\exp(\log z)$ and $\log(\exp z)$.

2. A promising field of application is visual analysis and synthesis of *transfer functions* in *systems*

[5] *There is such a result for* outer functions*, but it is impossible to see if a function is outer using only the boundary values of its modulus.*

theory and *filter design*. Since here the modulus (gain) is often more important than phase, it is recommended to use the left color scheme of Figure 7.

3. Further potential applications lie in the area of *Laplace* and complex *Fourier transforms*, in particular to the method of steepest descent (or stationary phase).

4. Phase plots also allow one to guess the *asymptotic behavior* of functions (compare, for example, the phase plots of $\exp z$ and $\sin z$) and to find *functional relations*. A truly challenging task is to rediscover the functional equation of the Riemann zeta function from phase plots of ζ and Γ (see [40], pp. 44–46).

5. Complex *dynamical systems*, in the sense of iterated functions, have been investigated by Felix Huang [18] and Martin Pergler [30] using domain coloring methods. The problem of scaling the modulus disappears when using phase plots; see the pictures of François Labelle [24] and Donald Marshall [26].

6. Geir Arne Hjelle [17] developed an interactive Java applet that visualizes interpolation problems for Blaschke products using phase portraits.

7. The utility of phase plots is not restricted to analytic functions. Figure 25 visualizes the function

$$h(z) := \mathrm{Im}\,(\mathrm{e}^{-\frac{\mathrm{i}\pi}{4}}\, z^n) + \mathrm{i}\,\mathrm{Im}\,(\mathrm{e}^{\frac{\mathrm{i}\pi}{4}}\,(z-1)^n),$$

with $n = 4$. This is Wilmshurst's example [42] of a harmonic polynomial of degree n having the maximal possible number of n^2 zeros. For background information we recommend the paper on gravitational lenses by Dmitry Khavinson and Genevra Neumann [23]. To understand the construction of the depicted function it is important to keep track of the zeros of its real and imaginary parts. In Figure 25 these (straight) lines are visualized using a modified color scheme that has jumps at the points $1, \mathrm{i}, -1$ and $-\mathrm{i}$ on the unit circle.

Besides these and other concrete applications one important feature of phase plots is their potential to bring up interesting questions and produce novel ideas. If you would like to try out phase plots on your own problems, you may start with the following self-explaining MATLAB code:[6]

```
xmin=-0.5; xmax=0.5; ymin=-0.5; ymax=0.5;
xres = 400; yres = 400;
x = linspace(xmin,xmax,xres);
y = linspace(ymin,ymax,yres);
[x,y] = meshgrid(x,y); z = x+i*y;
f = exp(1./z);
p = surf(real(z),imag(z),0*f,angle(-f));
set(p,'EdgeColor','none');
caxis([-pi,pi]), colormap hsv(600)
view(0,90), axis equal, axis off
```

[6] *This is a contribution to Nick Trefethen's project of communicating ideas by exchanging ten-line computer codes.*

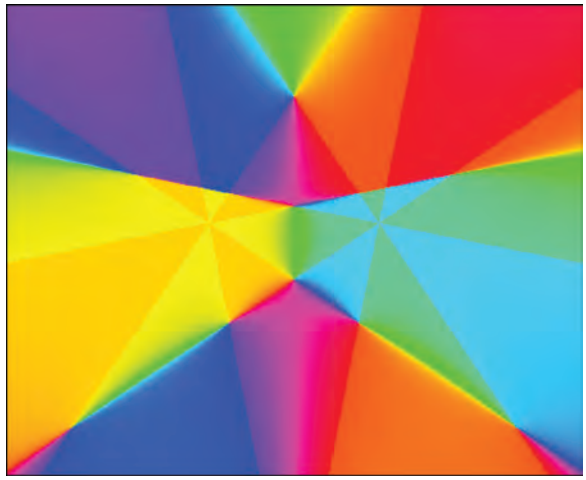

Figure 25. A modified phase plot of Wilmshurst's example for $n = 4$.

Though the phase of a function is at least of the same importance as its modulus, it has not yet been studied to the same extent as the latter. It is our conviction that *phase plots are problem factories*, which have the potential to change this situation. The forthcoming book [37] aims to give a comprehensive introduction to complex functions using phase portraits. The reactions to a mathematical calendar [33] featuring phase portraits of analytic functions have shown that the beauty of these images may also stimulate the interest of a more general public.

Technical Remark

All images in this article were created using MATHEMATICA and MATLAB.

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Mathematicians of Gaussian Elimination

Joseph F. Grcar

Gaussian elimination is universally known as "the" method for solving simultaneous linear equations. As Leonhard Euler remarked, it is the most natural way of proceeding ("der natürlichste Weg" [Euler, 1771, part 2, sec. 1, chap. 4, art. 45]). Because Gaussian elimination solves linear problems directly, it is an important technique in computational science and engineering, through which it makes continuing, albeit indirect, contributions to advancing knowledge and to human welfare. What is *natural* depends on the context, so the algorithm has changed many times with the problems to be solved and with computing technology.

Gaussian elimination illustrates a phenomenon not often explained in histories of mathematics. Mathematicians are usually portrayed as "discoverers", as though progress is a curtain that gradually rises to reveal a static edifice which has been there all along awaiting discovery. Rather, Gaussian elimination is living mathematics. It has mutated successfully for the last two hundred years to meet changing social needs.

Many people have contributed to Gaussian elimination, including Carl Friedrich Gauss. His method for calculating a special case was adopted by professional hand computers in the nineteenth century. Confusion about the history eventually made Gauss not only the namesake but also the originator of the subject. We may write *Gaussian elimination* to honor him without intending an attribution.

This article summarizes the evolution of Gaussian elimination through the middle of the twentieth century [Grcar, 2011a,b]. The sole development in ancient times was in China. An independent origin in modern Europe has had three phases. First came the schoolbook lesson, beginning with Isaac Newton. Next were methods for professional hand computers, which began with Gauss, who apparently was inspired by work of Joseph-Louis Lagrange. Last was the interpretation in matrix algebra by several authors, including John von Neumann. There may be no other subject that has been molded by so many renowned mathematicians.

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Bildarchiv Preussischer Kulturbresitz/Art Resource/NY. Photo by Olaf M. Tebmer.

Figure 1. It began in cuneiform tablets like VAT 8389. Vorderasiatisches Museum in Berlin, 12.1 by 7.9 cm.

Ancient Mathematics

Problems that can be interpreted as simultaneous linear equations are present, but not prevalent, in the earliest written mathematics. About a thousand cuneiform tablets have been published that record mathematics taught in the Tigris and Euphrates valley of Iran and Iraq in 2000 BC [Robson, 2008, table B.22]. Most tablets contain tables of various kinds, but some tablets are didactic (Figure 1). The first problem on VAT 8389 asks for the areas of two fields whose total area is 1800 sar, when the rent for one field is 2 silà of grain per 3 sar, the rent for the other is 1 silà per 2 sar, and the total rent on the first exceeds the other by 500 silà. If you do not remember these numbers, then you are not alone. The author of the tablet frequently reminds readers to "keep it in your head" in the literal translation by Høyrup [2002, pp. 77–82].[1] For perspective, when problems on these tablets can be written as simultaneous equations, usually one equation is nonlinear [Bashmakova and Smirnova, 2000, p. 3], which suggests that linear systems were not a large part of the Babylonian curriculum.

Simultaneous linear equations are prominent in just one ancient source. The *Jiuzhang Suanshu*, or *Nine Chapters of the Mathematical Art*, is a collection of problems that were composed in China over 2000 years ago [Shen et al., 1999] (Figure 2). The first of eighteen similar problems in chapter 8 asks for the grain yielded by sheaves of rice stalks from three grades of rice paddies. The combined yield is 39 dou of grain for 3 sheaves from top-grade paddies, 2 from medium-grade, and 1 from low-grade; and similarly for two more collections of sheaves. Mathematicians in ancient China made elaborate calculations using counting rods to represent numbers, which they placed inside squares arranged in a rectangle called a counting table (an ancient spreadsheet essentially). The right column in the following rectangle represents the first collection of paddies in the problem.

1	2	3	top
2	3	2	medium
3	1	1	low
26	34	39	yield

The solution was by Gaussian elimination. The right column was paired with each of the other columns to remove their top numbers, etc. The arithmetic retained integers by multiplying each column in a pair by the number atop the other and then subtracting right from left.

		3
4	5	2
8	1	1
39	24	39

Hart [2011, pp. 70–81] explains the entire calculation.

The *Nine Chapters* and similar, apparently derivative, work in Asia in later centuries [Hart, 2011; Libbrecht, 1973; Martzloff, 1997] are the only treatments of general linear problems until early modern Europe. The main surviving work of Greek and Roman algebra is the *Arithmetica* problem book by Diophantus, which is believed to be from the third century. Problem 19 in chapter (or book) 1 is to find four numbers such that the sum of any three exceeds the other by a given amount [Heath, 1910, p. 136]. Diophantus reduced the repetitive conditions to one with a new unknown, such as the sum of all the numbers, from which the other unknowns can be found. The earliest surviving work of ancient Hindu mathematics is the *Āryabhaṭīya* of Āryabhaṭa from the end of the fifth century. His linear problems are reminiscent of Diophantus but more general, being for any quantity of unknowns. Problem 29 in chapter 2 is to find several numbers given their total less each number [Plofker, 2009, p. 134]. The immediate sources for European algebra were Arabic texts. Al-Khwarizmi of Baghdad and his successors could solve the equivalent of quadratic equations, but they do not appear to have considered simultaneous linear equations beyond the special cases of Diophantus. For example, Rāshid [1984, p. 39] cites a system of linear equations that Woepcke [1853, p. 94, prob. 20] traces to a problem in the *Arithmetica*.

Figure 2. A depiction of Liu Hui, who said the *Nine Chapters* were already old when he wrote explanations of them in the third century.

Schoolbook Elimination

Diophantus and Āryabhaṭa solved what can be interpreted as simultaneous linear equations without the generality of the *Nine Chapters*. An obvious prerequisite is technology able to express the problems of interest. The *Nine Chapters* had counting tables, and Europe had symbolic algebra. Possessing the expressive capability does not mean it is used immediately. Some time was needed to develop the concept of equations [Heeffer, 2011], and even then, of 107 algebras printed between 1550 and 1660 in the late Renaissance, only four books had *simultaneous* linear equations [Kloyda, 1938].

The earliest example found by Kloyda was from Jacques Peletier du Mans [1554]. He solved a problem of Girolamo Cardano to find the money held by three men when each man's amount plus a fraction of the others' is given. Peletier first took the approach of Cardano. This solution was by verbal reasoning in which the symbolic algebra is a

[1] *Exercise: Solve the problem in your head by any means and then consult Høyrup [2002, pp. 81–82] for the scribe's method. Discuss.*

convenient shorthand. The discourse has variables for just two amounts, and it represents the third by the given formula of the other two. Peletier [p. 111] then re-solved the problem almost as we do, starting from three variables and equations and using just symbolic manipulation (restated here with modern symbols):

$$\begin{aligned} 2R + A + B &= 64 \\ R + 3A + B &= 84 \\ R + A + 4B &= 124 \\ 2R + 4A + 5B &= 208 \\ 3A + 4B &= 144 \\ 3R + 4A + 2B &= 148 \\ 3R + 2A + 5B &= 188 \\ 6R + 6A + 7B &= 336 \\ 6R + 6A + 24B &= 744 \\ 17B &= 408 \end{aligned}$$

Peletier's overlong calculation suggests that removing unknowns systematically was a further advance. That step was soon made by Jean Borrel, who wrote in Latin as Johannes Buteo [1560, p. 190]. Borrel and the *Nine Chapters* both used the same double-multiply elimination process (restated with modern symbols):

$$\begin{aligned} 3A + B + C &= 42 \\ A + 4B + C &= 32 \\ A + B + 5C &= 40 \\ 11B + 2C &= 54 \\ 2B + 14C &= 78 \\ 150C &= 750 \end{aligned}$$

Lecturing on the algebra in Renaissance texts became the job of Isaac Newton (Figure 3) upon his promotion to the Lucasian professorship. In 1669–1670 Newton wrote a note saying that he intended to close a gap in the algebra textbooks: "This bee omitted by all that have writ introductions to this Art, yet I judge it very propper & necessary to make an introduction compleate" [Whiteside, 1968–1982, v. II, p. 400, n. 62]. Newton's contribution lay unnoticed for many years until his notes were published in Latin in 1707 and then in English in 1720. Newton stated the recursive strategy for solving simultaneous equations whereby one equation is used to remove a variable from the others.

> And you are to know, that by each Æquation one unknown Quantity may be taken away, and consequently, when there are as many Æquations and unknown Quantities, all at length may be reduc'd into one, in which there shall be only one Quantity unknown.
> — Newton [1720, pp. 60–61]

Newton meant to solve any simultaneous algebraic equations. He included rules to remove one variable from two equations which need not be linear: substitution (solve an equation for a variable and place the formula in the other) and equality-of-values (solve in both and set the formulas equal).

Figure 3. Isaac Newton (1643–1727) closed a gap in algebra textbooks. 1689 portrait by Godfrey Kneller.

While Newton's notes awaited publication, Michel Rolle [1690, pp. 42ff.] also explained how to solve simultaneous, specifically linear, equations. He arranged the work in two columns with strict patterns of substitutions. We may speculate that Rolle's emphasis survived in the "method of substitution" and that his "columne du retour" is remembered as "backward" substitution. Nevertheless, Newton influenced later authors more strongly than Rolle.

In the eighteenth century many textbooks appeared "all more closely resembling the algebra of Newton than those of earlier writers" [Macomber, 1923, p. 132]. Newton's direct influence is marked by his choice of words. He wrote "extermino" in his Latin notes [Whiteside, 1968–1982, v. II, p. 401, no. 63] that became "exterminate" in the English edition and the derivative texts. A prominent example is the algebra of Thomas Simpson [1755, pp. 63ff.]. He augmented Newton's lessons for "the Extermination of unknown quantities" with the rule of addition and/or subtraction (linear combination of equations).

Among many similar algebras, Sylvestre Lacroix (Figure 4) made an important contribution to the nomenclature. His polished textbooks presented the best material in a consistent style [Domingues, 2008], which included a piquant name for each concept. Accordingly, Lacroix [1804, p. 114] wrote, "This operation, by which one of the unknowns is removed, is called *elimination*" (Cette opération, par laquelle on chasse une des inconnues, se nomme *élimination*). The first algebra printed in the United States was a translation by John Farrar of Harvard College [Lacroix, 1818]. As derivative texts were written, "this is called elimination" became a fixture of American algebras.

Gaussian elimination for the purpose of schoolbooks was thus complete by the turn of the nineteenth century. It was truly *schoolbook* elimination,


J. G. Reinis, The Portrait Medallions of David D'Angers.

Figure 4. Sylvestre Lacroix (1765–1843) called it *élimination*. Image of an 1841 bas-relief by David d'Angers.

Figure 5. Carl Friedrich Gauss (1777–1855) devised the first professional method, replacing "common elimination". Lithograph by Siegfried Detlev Bendixen for the 1828 frontispiece of *Astronomische Nachrichten*.

because it had been developed to provide readily undertaken exercises in symbolic algebra.

Professional Elimination

A societal need to solve simultaneous linear equations finally arose when Adrien-Marie Legendre [1805] and Gauss [1809] (Figure 5) invented what Legendre named the method of least squares ("méthode des moindres quarrés," modern "carrés"). It was a method to draw statistical inferences for the unknowns in overdetermined, simultaneous linear equations by minimizing the sum of the squares of the discrepancies. Gauss became a celebrity by using unstated methods to calculate the orbit of the "lost" dwarf planet Ceres. Then Legendre succinctly stated what is now called the general linear model, which led to an unfortunate priority dispute once Gauss disclosed his calculations [Stigler, 1986]. The approximate solutions in the least-squares sense were obtained from Legendre's "equations of the minimum" or Gauss's "normal equations", which could be solved, they respectively said, by "ordinary methods" or "common elimination", meaning schoolbook elimination.[2] Such was the importance of the least-squares method that soon Gaussian elimination would evolve with the technology of professional hand computation.

Gauss himself was an incorrigible computer, who estimated that his prodigious calculations involved a million numbers [Dunnington, 2004, p. 138]. His least-squares publications mostly dealt with statistical justifications, but in one passage he described his own calculations. He reformulated the problem to reexpress the sum of squares in the canonical form of Lagrange [1759]. Gauss [1810, p. 22] wrote the overdetermined equations as

$$\begin{array}{l} n + a\,p + b\,q + c\,r + \ldots = w \\ n' + a'\,p + b'\,q + c'\,r + \ldots = w' \\ n'' + a''p + b''q + c''r + \ldots = w'' \\ \qquad\qquad \text{etc.,} \end{array}$$

where $p, q, r, \ldots$ are the unknown parameters of the linear model, while $n, a, b, c, \ldots$ are numbers that differ with each observed instance of the model (as indicated by the growing quantities of primes). The $w, w', w'', \ldots$ are the discrepancies whose sum of squares, Ω, is to be minimized. Gauss chose an unnamed notation,

$$[xy] = xy + x'y' + x''y'' + \ldots,$$

which he used to represent the numeric coefficients in the normal equations, equivalently, in the quadratic form Ω. Gauss then extended his bracket notation to

$$[xy, 1] = [xy] - \frac{[ax][ay]}{[aa]},$$

$$[xy, 2] = [xy, 1] - \frac{[bx, 1][by, 1]}{[bb, 1]},$$

and so on, in terms of which he constructed linear combinations of successively fewer parameters:

$$\begin{array}{llll} A = [an] & + [aa]p & + [ab]q & + [ac]r + \ldots \\ B = [bn, 1] & & + [bb, 1]q & + [bc, 1]r + \ldots \\ C = [cn, 2] & & & + [cc, 2]r + \ldots \\ & \text{etc.} & & \end{array}$$

[2] *The quoted phrases are: "d'équations du minimum" and "méthodes ordinares" [Legendre, 1805, p. 73], "eliminationem vulgarem" [Gauss, 1809, p. 214], and "Normalgleichungen" [Gauss, 1822, p. 84]. Following Gauss, the name normal equations is still given to the first-order differential conditions for the minimum.*

In modern notation of numerical analysis rather than statistics, for a matrix A of full column rank and suitably sized column vectors b and x, the least-squares problem is $\min_x \|b - Ax\|_2$*, and the normal equations are* $A^t Ax = A^t b$*, to which elimination was applied.*

These formulas complete the squares of successive parameters in the quadratic form, leaving

$$\Omega = \frac{A^2}{[aa]} + \frac{B^2}{[bb,1]} + \frac{C^2}{[cc,2]} + \cdots + [nn,\mu],$$

where μ is the quantity of variables. Thus $A = 0$, $B = 0$, $C = 0$, … can be solved in reverse order to obtain values for $p, q, r, \ldots$, also in reverse order, at which Ω attains its minimum, $[nn,\mu]$.

Solving least-squares problems by Gauss's method required the numbers $[xy,k]$ for increasing indices k. Gauss halved the work of schoolbook elimination, because he needed x, y only in alphabetic order. More importantly, his notation discarded the equations of symbolic algebra, thereby freeing computers to organize the work more efficiently. When Gauss calculated, he simply wrote down lists of numbers using his bracket notation to identify the values [Gauss, 1810, p. 25].

The subsequent evolution of Gaussian elimination illustrates the meta-disciplinary nature of mathematics [Grcar, 2011c]. People such as Gauss with significant knowledge of fields besides mathematics were responsible for these developments. The advances do not superficially resemble either schoolbook elimination or what is taught at universities today, but they are no less important, because through them social needs were met.

Professional elimination began to develop due to the economic and military value of cartography. Gauss took government funds to map the German principality where he lived. Maps were drawn from networks of triangles using angles that were measured by surveyors. The angles had to be adjusted to make the triangles consistent. To that end, Gauss [1826] devised a method to find least-squares solutions of *under*determined equations by a quadratic-form calculation similar to the overdetermined case.[3] Once Friedrich Bessel [1838] popularized Gauss's approach, cartographic bureaus adopted the bracket notation. Gauss's calculations became part of the mathematics curriculum for geodesists.

> By whatever method of elimination is performed, we shall necessarily arrive at the same final values …; but when the number of equations is considerable, the method of substitution, with Gauss's convenient notation, is universally followed.
>
> — Chauvenet [1868, p. 514] (Figure 6)

The first innovations after Gauss came from Myrick Doolittle [1881] (Figure 7). He was a computer at the United States Coast Survey who could solve forty-one normal equations in a week. This feat was astounding, because in principle it requires about $23000 \approx n^3/3$ arithmetic operations for $n = 41$. Doolittle dispensed with Gauss's brackets and instead identified the numbers by their positions in tables. He replaced the divisions in the bracket formulas by reciprocal multiplications. Doolittle's tables colocated values to help him use the product tables of August Leopold Crelle [1864]. An unrecognized but prescient feature of Doolittle's work combined graphs with algebra to reduce computational complexity. He used the cartographic triangulation to suggest an ordering of the overdetermined equations (equivalently, an arrangement of coefficients in the normal equations) to preserve zeroes in the work. The zeroes obviated many of the 23000 operations.

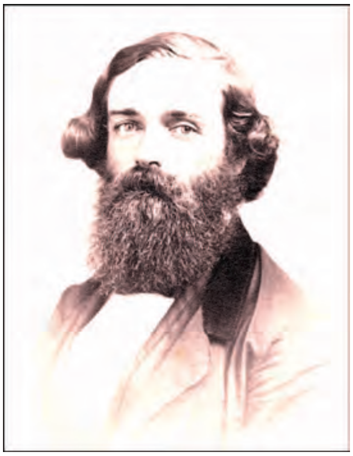
Figure 6. William Chauvenet (1820–1870) and others taught Gauss's computing methods. Portrait circa 1839 by Eunice Makepeace Towle.

The next innovations were made by the French military geodesist André-Louis Cholesky [Benoit, 1924] (Figure 8). He, too, addressed the normal equations of the underdetermined, angle-adjustment problem. Although Cholesky is remembered for the square roots in his formulas,[4] his innovation was to order the arithmetic steps to exploit a feature of multiplying calculators. The machines were mass-produced starting in 1890 [Apokin, 2001], and Cholesky personally used a Dactyle (Figure 9). A side effect of long multiplication is that the machines could internally accumulate sums of products.[5] Calculations thus arranged were quicker to perform because fewer

[3] *For A of full row rank, these problems were* $\min_{Ax=b} \|x\|_2$*, where x are the angle adjustments and $Ax = b$ are the consistency conditions. The solution was given by correlate equations $x = A^t u$, where $AA^t u = b$ are again normal equations to be solved.*

[4] *Cholesky originated a construction that is now interpreted as decomposing a symmetric and positive definite matrix into a product LL^t, where L is a lower triangular matrix and ^{t} is transposition.*

[5] *The sums of products became known as dot-, inner-, or scalar-products of vectors after the adoption of matrix notation. Calculations employing sums were described as "abbreviated" or "compact".*



Courtesy of Antiochiana, Antioch College.

Figure 7. Myrick Hascall Doolittle (1830–1911), circa 1862.

© École Polytechnique.

Figure 8. André-Louis Cholesky (1875–1918), circa 1895.

intermediate results had to be recorded on paper. Cholesky's tables for n equations held only $\mathcal{O}(n^2)$ numbers compared to Doolittle's $\mathcal{O}(n^3)$.

Statistical analysis in the form of regression analysis became the primary use for least-squares problems after the First World War. Doolittle's method was so prominent that it successfully made the transition to statistics. For example, United States government employees insisted on using Doolittle's method when they moved from the Coast Survey to the Department of Agriculture [Tolley and Ezekiel, 1927]. The department used least-squares methods for regression analyses in econometrics [Fox, 1989]. Cholesky's method was superior, but it received little notice [Brezinski, 2006] because it appeared in geodesy after statistics had begun to develop its own methods. The "square root method" did become widely used [Laderman, 1948] once it was reinvented by the American statistician Paul Dwyer [1945].

Photo by Freddy Haeghens, f.haeghens@skynet.be.

Figure 9. Dactyle calculator based on the pinwheel design of the Russian-Swedish entrepreneur Willgodt Odhner. Dactyle resold machines made by Brunsviga in Braunschweig and from 1905 by Chateau Bros. in Foncine-le-Haut.

The methods of Gauss, Doolittle, and Cholesky sufficed only for normal equations. In modern terms, the matrix of coefficients must be symmetric and positive definite. The need to routinely solve more kinds of simultaneous linear equations gradually developed in engineering. Unaware of the earlier work by Cholesky, a mathematician at the Massachusetts Institute of Technology, Prescott Crout [1941] (Figure 10), reorganized schoolbook Gaussian elimination to accumulate sums of products. Crout's method was publicized by a leading manufacturer of calculating machines (Figure 11).

Matrix Interpretation

The milieu of using symbolic algebra to modify Gaussian elimination ended with the adoption of matrix algebra. Several authors had developed matrices in the second half of the nineteenth century [Hawkins, 1975, 1977a,b, 2008]. Although matrices were not needed to compute by hand, the new representational technology showed that all the proliferating elimination algorithms were trivially related through matrix decompositions. Eventually matrices would help organize calculations for the purpose of programming electronic computers.

This development leads from the astronomical observatory of the Jagiellonian University to the numerical analysis textbooks that are presently in your campus bookstore. Manual computing motivated astronomer Tadeusz Banachiewicz [1938a,b] (Figure 12) to independently invent matrices in the form called Cracovians. They have a column-by-column product, which is the natural way to calculate with columns of figures by hand.

> It must, however, be conceded that in practice it is easier to multiply column by column than to multiply row by column …. It may, in fact, be said that the computations are made by Cracovians and the theory by matrices.
>
> — Jensen [1944]

Courtesy of MIT Museum.

Figure 10. Prescott Durand Crout (1907–1984), circa 1936.

Courtesy of Adam Strzałkowski.

Figure 12. Tadeusz Banachiewicz (1882–1954) used Cracovian algebra to describe computing methods. Photo from 1946.

Courtesy of John Wolff, www.vicnet.net.au/~wolff.

Figure 11. Crout's method was performed with machines such as this Marchant calculator, model 10ACT. It was manufactured in Oakland, California, in the 1930s and 1940s.

Banachiewicz advocated using Cracovians to represent calculations as early as 1933. The idea was realized in Arthur Cayley's matrix algebra by two people. Henry Jensen [1944] (Figure 13) of the Danish Geodætisk Institut used pictograms, □ = ◺◹, to emphasize that three algorithms for solving normal equations amounted to expressing a square matrix as a triangular product: the "Gaussian algorithm" (the calculation with Gauss's brackets), the Cracovian method, and Cholesky's method. A noteworthy aspect of Jensen's presentation was suggested by Frazer et al. [1938]: to represent arithmetic operations through multiplication by "elementary matrices". In the same year Paul Dwyer [1944] (Figure 14) of the University of Michigan showed that Doolittle's method was an "efficient way of building up" some "so-called triangular" matrices. He found no similar interpretation except in the work of Banachiewicz. The coincident papers of Jensen and Dwyer are the earliest to depict Gaussian elimination in roughly the modern form, that is, in terms of Cayleyan matrices.

A deep use for the matrix interpretation came from John von Neumann (Figure 15) and his collaborator Herman Goldstine. They and others were in the process of building the first programmable, electronic computers. Concerns over the efficacy of the machines motivated von Neumann and Goldstine to study Gaussian elimination. The initial part of their analysis introduced the matrix decomposition.

> We may therefore interpret the elimination method as … the combination of two tricks: First, it decomposes A into a product of two [triangular] matrices … [and second] it forms their inverses by a simple, explicit, inductive process.
>
> — von Neumann and Goldstine [1947]

Von Neumann and Goldstine used matrix algebra to establish bounds on the rounding errors of what they anticipated would be the mechanized algorithm once computers became available. When the matrix is symmetric and positive definite, their bound remains the best that has been achieved. Although Gaussian elimination is observed to be accurate, a comparable error bound has yet to be established in the general case.[6]

The next step to the campus bookstore was aided by *Mathematical Reviews*. John Todd (Figure 16) found Jensen's paper through *MR* and

[6] *No exercise: The end result of much study is, do not try to prove what von Neumann did not.*

Figure 13. Henry Jensen (1915–1974) was inspired by Banachiewicz to use Cayleyan algebra.

Figure 14. Paul Sumner Dwyer (1901–1982) independently used matrix algebra. Photo circa 1960s.

Figure 15. John von Neumann (1903–1957) saw "the combination of two tricks". Photo from March 1947.

lectured on it at King's College London [Taussky and Todd, 2006]. An auditor communicated the matrix formalism to staff at the National Physical Laboratory. Among them was Alan Turing (Figure 17), who evidently learned of the matrix interpretation both from Jensen through Todd and also during a visit to von Neumann and Goldstine, whom Turing [1948] cited. He described Gaussian elimination in the manner of von Neumann and Goldstine by treating the general case of schoolbook elimination and in the manner of Jensen with elementary matrices. Turing wrote with a brevity of expression that made ideas clear without overworking them.

The invention of electronic computers created a discipline that was at first populated by those who made scientific calculations [Traub, 1972; Wilkinson, 1970]. Among them, George Forsythe (Figure 18) was a visionary mathematician who is reputed to have named "computer science" [Knuth, 1972]. Gauss's involvement lent credence to the subject matter of the new discipline. The terminology that geodesists had used to describe the calculations of Gauss suggested an origin for what then was named simply "elimination". In an address to the American Mathematical Society, Forsythe [1953] misattributed "high school" elimination to Gauss and appears to have been the first to call it "Gaussian elimination" [Grcar, 2011a, tab. 1]. The name was widely used within a decade.

The university mathematics curriculum adopted matrix descriptions more slowly. Linear algebra itself was not commonly taught until the 1960s. When Fox [1964] and Forsythe and Moler [1967] wrote influential numerical analysis textbooks that featured the matrix interpretation, then they reprised Turing's presentation.

Coda

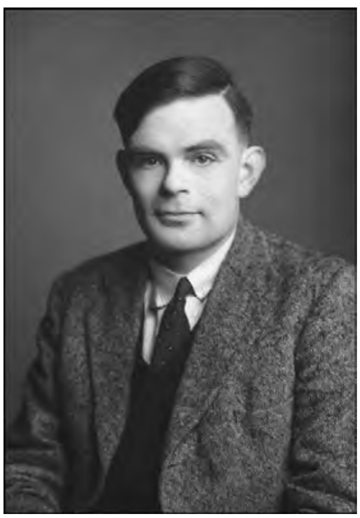
An *algorithm* is a series of steps for solving a mathematical problem. The matrix interpretation of Gaussian elimination seldom becomes an algorithm in a straightforward way, because the speed of computing depends on whether the calculation is well adapted to the problem and the computer. Just as Gauss developed the first professional method for least-squares calculations and then Doolittle developed a method for use with multiplication tables, other methods were developed more recently to solve the equations of finite-element analysis [Irons, 1970] with parallel computers [Duff and Reid, 1983]. While Cholesky and Crout emphasized sums of products for calculating machines, the arithmetic steps can be

Courtesy of the Archives, California Institute of Technology.

Figure 16. John Todd (1911–2007) lectured on Jensen's work in 1946.

Josè Mercado/Stanford News Service.

Figure 18. George Forsythe (1917–1972) called it "Gaussian elimination".



Courtesy of the Royal Society Archives, © Godfrey Argent Studio.

Figure 17. Alan Turing (1912–1954) described it in the way universities now teach it. Photograph circa 1951.

reordered automatically to suit different computer architectures [Whaley and Dongarra, 1998]. More radical transformations are possible that reduce the work to solve n equations below $\mathcal{O}(n^3)$ arithmetic operations [Strassen, 1969; Cohn and Umans, 2003; Demmel et al., 2007]. Perhaps the only certainty about future algorithms is their name. Rather than being a Platonic archetype, Gaussian elimination is an evolving technique.

Acknowledgments

I am grateful to the editor and the referees for advice that improved this article. It is a pleasure to acknowledge the scholarship of Kloyda [1938], Macomber [1923], and Whiteside [1968–1982] (Figures 19, 20, 21), which made it possible to identify Newton's contribution.

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Courtesy of Sisters of Saint Francis, Rochester, Minnesota.

Figure 19. Sister Mary Thomas à Kempis Kloyda (1896–1977) discovered the few examples in Renaissance texts. She was a pioneering woman of American mathematics [Green and LaDuke, 2009]. Her dissertation is supported by an unrivaled collection of primary sources that is all the more impressive for being assembled before the age of electronic information.

Courtesy of the Shasta County Historical Society.

Figure 20. Gertrude Louise Macomber [1923] observed that Newton's lesson for solving simultaneous equations was "the earliest appearance of this method on record". She anticipated the importance of historical antecedents to American textbooks and bested more prominent later academicians who neglected to examine the primary sources. Shasta County High School yearbook, Redding, California, 1920.

Courtesy of Dr. Piers Bursill-Hall, Cambridge University.

Figure 21. Derek Thomas Whiteside (1932–2008) [Guicciardini, 2008] remembered what Newton wrote in a monumental collection of his papers.

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Remembering Johannes J. Duistermaat (1942–2010)

Victor Guillemin, Álvaro Pelayo, San Vũ Ngọc, and Alan Weinstein, Coordinating Editors

We are honored to pay tribute to Johannes (Hans) J. Duistermaat (December 20, 1942–March 19, 2010), a world leading figure in geometric analysis and one of the foremost Dutch mathematicians of the twentieth century, by presenting a collection of contributions by some of Hans's colleagues, collaborators, and students. Duistermaat's first striking contribution was his article "Fourier integral operators II" with Hörmander (published in *Acta Mathematica*), a work that he did after his doctoral dissertation. Several influential results in analysis and geometry have the name Duistermaat attached to them, for instance, the Duistermaat-Guillemin trace formula (1975), Duistermaat's global action-angle theorem (1980), the Duistermaat-Heckman theorem (1982) and the Duistermaat-Grunbaum bispectral theorem (1986). Duistermaat's papers offer an unusual display of originality and technical mastery.

Hans Duistermaat passed away on March 19, 2010, in The Netherlands. Hans had been affiliated with the Mathematics Institute at Utrecht University as Professor of Pure and Applied Mathematics since 1974. He officially retired in 2007 but continued to be active in research until his unexpected death. During the last five years of his career, he held a KNAW Professorship from the Royal Academy of Sciences of the Netherlands, which freed him from teaching and administrative duties. He was also "Ridder in de Orde van de Nederlandse Leeuw" (Knight of the Order of the Dutch Lion) and the recipient of numerous other awards. A conference in honor of the sixty-fifth birthday of Hans Duistermaat took place in Utrecht in 2007, and the proceedings are to be published by Birkhäuser, Boston. The Dutch Research School in Mathematics (WONDER) has decided to establish the Hans Duistermaat Chair for Visiting Professors. With the passing of Hans Duistermaat, mathematics has lost an exceptional and original figure. Hans had a powerful, sharp personality, and he loved a good argument, but, at the same time, he was unusually kind and generous toward others.

Photograph by J. W. H. Kolk.

Hans Duistermaat (December 20, 1942–March 19, 2010) at a gathering following J. A. C. Kolk's thesis defense in 1977.

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Hans Duistermaat considered himself an analyst, but his influence on mathematics extends well beyond the field of analysis. He liked geometry very much because he liked to understand the intrinsic nature of the problems that he tackled. The mathematics of Hans is a rare combination of elegance and practical relevance. He did not want to be a theoretician and loved to revisit the most classical examples and demonstrate their universal qualities. And when a theoretical problem appeared before him, amply motivated, his capacity for abstraction was extraordinary.

Hans was genuinely modest, both at a human and at a professional level. He did not care about honors, but when he received them he was at first surprised and then gratified. He refused to have a conference celebrating his sixtieth birthday, but at the insistence of the mathematical community he agreed to have a sixty-fifth birthday celebration provided that the conference lectures consisted of purely scientific communications.

His modesty was matched only by his scientific honesty. Hans thought that most of the so-called new ideas could be traced back to the works of other famous figures in mathematics. His passion for history, evident to all those who knew him, drove him back to the original works of Lie, Cartan, Poincaré, and even Huygens and Newton. Consequently, and contrary to current trends, he published sparingly. Richard Cushman, one of his colleagues in Utrecht, confessed to us that Hans had accumulated a massive amount of unpublished notes that could provide the material for dozens of articles. Fortunately, Hans's taste for history did not take over his enthusiasm for advancing mathematics. He was known to be a strong supporter of young people and always encouraged them to push the boundaries of knowledge without fear.

Next, we give a brief glimpse of (some of the) mathematics that Hans Duistermaat encountered in his distinguished trajectory as a mathematician. He was awarded a doctorate in mathematics at the University of Utrecht in 1968. Although H. Freudenthal is listed as his advisor, Duistermaat's doctoral thesis was directed by G. K. Braun, who died one year prior to the thesis defense. Following his thesis, he had the great insight of studying the work of Hörmander regarding a technique to study linear partial differential equations: Fourier integral operators.

After reading certain parts of Hörmander's work, Hans decided to take a postdoctoral position at Lund where Hörmander was teaching, so he could better understand Hörmander's work. For a year, Hörmander lectured about his new theory to a small group of mathematicians, including a young and energetic Duistermaat. Duistermaat's extensive knowledge of Lie's work allowed him to contribute in an essential way to the global theory of Fourier integral operators. It was undoubtedly his first stroke of brilliance: when he returned to the Netherlands in 1970, Hans dedicated all his energy to exploring and understanding the applications of the theory of Fourier integral operators, and after several months of work he started mailing manuscripts with the results of his investigations to Hörmander. After many months of unanswered letters, Hans received a letter from Hörmander saying: "now I understand what is going on—manuscript will follow."

Hans eventually received a manuscript from Hörmander, and with his usual modesty he said that he "recognized" his contributions here and there in the text. Duistermaat had revealed himself to the mathematical community as a pioneer in the field of microlocal analysis. This work [10], published by *Acta Mathematica* in 1972, has become an essential reference in the field. He also wrote about this topic in his notes from Nijmegen and the Courant Institute [4].

Hans witnessed, for the first time during his stay in Lund, a difference in the views about Maslov's theory between Hörmander and Leray, who belonged to different schools of thought. The latter understood the meaning of the WKB ansatz in terms of Lagrangian distributions, but his intuitive approach to this theory, motivated by concrete problems in physics (e.g., the spectral theory of the helium atom), was, for mathematicians at least, difficult to follow. On the other hand, Hörmander, in the Bourbaki vein, wanted a clean and mathematically precise theory that would make no reference to physics. In France, Leray was very interested in Maslov's approach.

Hans stayed open to both views; even better, his phenomenal quickness allowed him to benefit from, and make contributions to, both approaches. Only two years after his seminal article with Hörmander, "Fourier integral operators II", he published in *Communications in Pure and Applied Mathematics* another article on oscillatory integrals [2] that has become a standard reference in the field, in which Maslov's formulation (including the small parameter $\hbar$) is mathematically justified. Moreover, Hans became quite interested in the Maslov index that appears in stationary phase. He was the first to establish a link between the index defined by Hörmander and the Morse index of the variational problem.

Meanwhile, Hans Duistermaat and Victor Guillemin had started collaborating on a paper [7] regarding the link between the spectrum of positive elliptic operators and periodic bicharacteristics, using Fourier integral operators, a paper that was published in *Inventiones Mathematicae* in 1975.

Hans made important contributions concerning harmonic analysis on Lie groups and locally symmetric spaces. It is impossible not to mention


Photograph by Á. Pelayo.

Duistermaat playing chess against himself. Picture taken in the living room of the Duistermaats' residence in the Netherlands in 2006.

his work [9] with Gert Heckman, "On the variation in the cohomology of the symplectic form of the reduced phase space", another classic also published in *Inventiones Mathematicae* in 1982. This work had a tremendous influence on the development of a number of topics in symplectic geometry and analysis. Atiyah stated that this article was one of his main motivations for his work with R. Bott on equivariant cohomology [1].

Formula of Duistermaat-Heckman

$$\int_M e^{J_X} e^{\sigma} = \sum_j \int_{N_j} \frac{e^{i_j{}^* J_X}\, e^{i_j{}^*\sigma}}{\det \frac{LX+\Omega}{2\pi i}}.$$

Formula of Duistermaat-Heckman

The Duistermaat-Heckman formula is one of the most famous results in the history of symplectic geometry. Some years after he and Heckman proved this theorem, Hans, in collaboration with Guillemin, Meinrenken, and Wu, looked at the issue of commutation of quantization and symplectic reduction in [8], an important paper in the subject. The beautiful book *The Heat Kernel Lefschetz Fixed Point Formula for the Spin-c Dirac Operator* [5], published in 1996, consists of a collection of notes on Dirac operators, the index theorem (proved by the heat equation), and equivariant cohomology, which he had written for himself to understand the important developments that were taking place at the time. Hans wrote these notes with the goal of being able to work effectively on the subject himself rather than with the goal of publishing them, but many people encouraged him. Had it not been for his colleagues' enthusiasm, it is very likely that these notes would never have been made publicly available.

In the most recent years, Hans Duistermaat worked mainly on three projects. The first project, with Á. Pelayo, concerned the structure theory of symplectic manifolds equipped with symplectic torus actions and its relations to Kodaira's work on complex analytic surfaces. This project gave rise to four papers, starting with a paper in which a classification of torus actions with coisotropic orbits is given [13]. The second project concerned QRT (Quispel, Roberts, and Thompson) maps and elliptic surfaces. Hans devoted an enormous amount of energy to this project, which resulted in a six-hundred-page book [6] (his work on this topic was prompted by an interaction with J. M. Tuwankotta); the book presents a self-contained and complete treatment of the subject, analyzing QRT maps using Kodaira's theory of elliptic surfaces. His final project, with N. Joshi, had the goal of gaining a full understanding of Painlevé differential equation $d^2 y/dx^2 = 6y^2 + x$.

By the time of Duistermaat's passing, he and Joshi had strengthened considerably the previous asymptotic results about the Painlevé equation (which is the first in Painlevé's classification of those algebraic second-order ordinary differential equations such that every isolated singularity of moderate growth of every solution is a pole) and corrected older, incomplete results. The paper "Okamoto's space for the first Painlevé equation in Boutroux coordinates", with N. Joshi, available in the *Mathematics arXiv* since October 27, 2010, regards this project.

These brief highlights of selected results due to Hans Duistermaat should not hide the fact that Hans made other remarkable contributions on which we have not elaborated. He always had deep motivations for working on every topic he studied. Many of his motivations came from classical mechanics, a topic which he was passionate about. He had a predilection for Hamiltonian systems. His 1980 article on the globalization of action-angle coordinates [3] is still regarded as the best reference on the subject and prompted much subsequent research. His works on symplectic reduction are rooted in his very detailed study of the bifurcations of Hamiltonian periodic orbits, and his works on elliptic curves are motivated by the theory of completely integrable Hamiltonian systems and Lagrangian fibrations and their applications.

Hans had a rare combination of talents: he was often motivated by "applied" problems and loved examples, but at the same time he was a technical and theoretical master. As his contributions show, he was able to continuously use his knowledge of classical examples to make long-lasting theoretical breakthroughs. We should also not forget his pedagogical contributions, for example, in his beautiful volumes on real analysis, in collaboration with his colleague J. A. C. Kolk [12]. His course on Fourier integral operators [4] quickly became a standard reference in the subject, and his book on "Spin-c" is a terrific treatment for those who want to enter the subject. Finally, the book on Lie groups [11], again in collaboration with Kolk, was very quickly recognized by the community as a clear and important reference.

Duistermaat's body of work has been and continues to be an influential driving force in geometric analysis. If you have not done so yet, we encourage you to explore one of Duistermaat's books or papers; you will not regret it. We hope that these pages provide a glimpse of Duistermaat's profound influence on mathematics. The contributions by Atiyah, Cushman, Heckman, van den Ban and Kolk, Nirenberg, Sjamaar, and Sjöstrand that follow provide further testimony of Hans Duistermaat's influence on mathematics.

P.S. An article in memory of Hans Duistermaat, written by Erik van den Ban and Johan Kolk, has also appeared in *Nieuw Archief voor Wiskunde* and is available at:

`http://www.math.leidenuniv.nl/~naw/serie5/dee1011/dec2010/nawdec235.pdf`

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The remainder of this text consists of contributions by a few of Hans's students and close colleagues, discussing aspects of both the mathematician and the person.

Michael Atiyah

My main memory of Hans is of a colleague with a dry sense of humor and a passionate attachment to mathematics. His enduring love was analysis, centering around PDE and Lie groups. He was a scholar who knew and appreciated the detailed work of our predecessors in past centuries, but he was also modern in outlook, using microlocal analysis and pushing forward the frontiers. He is perhaps most famous for a small spin-off, the Duistermaat-Heckman theorem, giving conditions under which the stationary-phase approximation is exact.

The Duistermaat-Heckman theorem asserts that the stationary-phase approximation is exact for a Hamiltonian arising from the action of a circle on a compact symplectic manifold. The result is striking and the proof is simple and elegant. It stimulated Raoul Bott and me to examine it carefully and to show how it could be interpreted in terms of equivariant cohomology and fixed point formulae. It also connects up with work of Witten on Hodge-Morse theory. This leads on formally to a number of interesting cases in infinite dimensions that occur in quantum physics. Altogether it was very influential.

Very many years ago, when we were both young and hearty, we spent a week together in the Canadian Rockies, after a conference in Vancouver. For someone from the Netherlands where height above (or below) sea level is measured in single digits, the Rockies, where height is measured in thousands, were an irresistible and inevitable attraction. We were not professional mountaineers with ice-axe and rope but enthusiastic hill-walkers with boots and energy. The scenery was fabulous, with the beautifully named Emerald Lake imprinted on my memory. Hans was the perfect companion, slowing me down with his measured words which conveyed his deep appreciation of the landscape. I still visualise him in that role.

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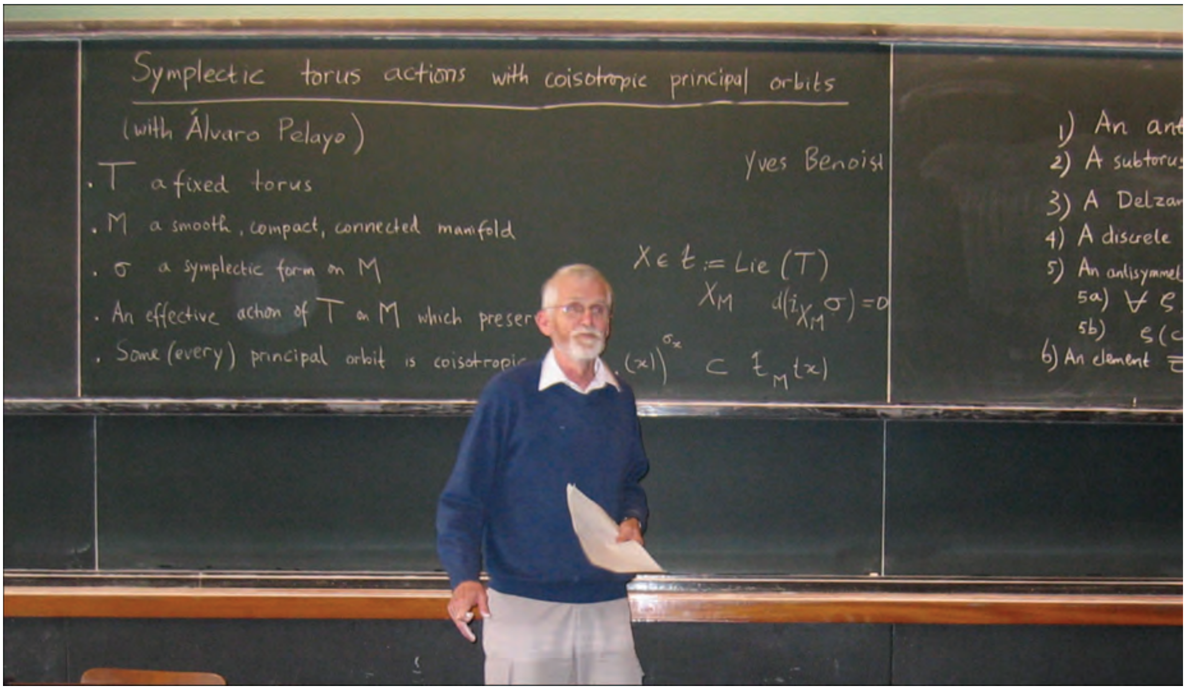

Photograph by Michael Pignon.

Duistermaat lecturing on symplectic differential geometry at the Colin de Verdière Fest in 2006.

Richard Cushman

Hans was a mathematical polymath. His scientific papers deal with many areas of geometric analysis. I will focus on one of his contributions to the geometric study of Hamiltonian systems, namely, monodromy.

To describe what monodromy is we look at a completely integrable, two-degree-of-freedom Hamiltonian system on four-dimensional Euclidean space. In other words, we assume that there is a function, called an integral, which is constant on the motions of the original Hamiltonian system in addition to the Hamiltonian. The integral map of such a system is given by assigning to each point in Euclidean 4-space the value of the Hamiltonian and the extra integral. We suppose that this integral map is proper and the preimage of each point is connected. Then the theorem of action-angle coordinates states that the preimage of a suitably small open 2-disk in the set of regular values of the integral map is symplectically diffeomorphic to a product of a 2-torus and 2-disk. In [1] Hans showed that this local theorem need not hold globally on the set of regular values. In particular, if we have a smooth closed curve in the set of regular values of the integral map, which is diffeomorphic to a circle that is not contractible to a point, then its preimage under the integral map is the total space of a 2-torus bundle over a circle. In general this bundle is twisted, that is, it need not be diffeomorphic to a product of a circle and a 2-torus. A 2-torus bundle over a circle may be constructed from a product bundle over a closed interval with a typical fiber a 2-torus, which is Euclidean 2-space modulo the lattice of points with integer coordinates, by identifying each of its two end 2-tori by an integer 2×2 matrix with determinant 1.

When Hans was starting to write [1], he asked me to find an example of a Hamiltonian system with monodromy. The next day I told him that the spherical pendulum has monodromy and gave him a proof. When writing up the paper Hans found a much simpler geometric argument. In the early days, showing that a particular integrable system had monodromy was not easy. In [2] Hans did this for the normal form of the Hamiltonian Hopf bifurcation.

Eight years after the discovery of monodromy in classical mechanics came its discovery in quantum mechanics [3]. In particular, Hans and I showed that the quantum analogue of monodromy appears in the joint spectrum of the energy and angular momentum operators of the quantized spherical pendulum as the failure of the local lattice structure of this joint spectrum to be a global lattice. This discovery has now been recognized as fundamental by chemists who study the spectra

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of molecules and has led to a very active area of scientific research.

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Gert Heckman

In my recent contribution to a memoriam for Hans Duistermaat in the *Notices of the (Dutch) Royal Mathematical Society*, I described the important role that Hans played in the early days of my career. The interested reader can also find this on my website (http://www.math.ru.nl/~heckman/). Here I would like to talk about our contact in the last year of his life.

It started with a master class I have been teaching over the past few years for high school students in their final grade. In collaboration with Maris van Haandel, a high school teacher in Wageningen (with a Ph.D. in functional analysis), we taught, over the course of six Wednesday afternoon sessions, the basics of calculus, finishing with a discussion of the proofs of the three Kepler laws of planetary motion. It was a wonderful experience to work with these talented high school students.

In the process of going over the various proofs of the ellipse law in the literature, Maris and I found yet another proof that we liked best for use in our class. When I mentioned this proof to Hans he reacted positively, but, when I told him that the original proof by Newton was incomplete (as I had read somewhere in the literature), Hans was critical and suggested that I read Newton's original text. This was a great advice, since we were to discover—after some translation into modern language—how beautiful and complete the original proof of Newton really was. We explained this in an article in the *Mathematical Intelligencer*.

In September 2009 Maris and I organized a training weekend for high school teachers on the possibilities of teaching the Kepler laws in high school. For the Friday evening we invited Hans to give an informal lecture on Poincaré and his influence on classical mechanics, and he happily accepted. However, a few weeks before the training weekend, I got an email from Hans telling me that

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Photograph by Michael Pignon.

(Left to right) Yves Colin de Verdière, Hans Duistermaat, and Victor Guillemin. Picture taken at the Colin de Verdière Fest in Grenoble in 2006.

there was a problem. That same Friday evening there was a ping-pong competition, and his club simply could not do without him. We arranged the following deal: Hans would write up his lecture and explain some of the details to me, then I would give his lecture.

When I visited Hans in Utrecht to discuss his lecture it occurred to me that he had grown older. He mentioned very briefly that he had not been too well of late: that spring his spleen had been removed, but things were now under control. Hans gave me a wonderful two hours of private tutoring on Poincaré's work on celestial mechanics: the restricted three-body problem, the Poincaré slice, the return map, the twist map, the stable and unstable manifold and their intersection, leading to the homoclinic disorder. Looking back at his notes, it struck me how his interest in Poincaré is related to his early (and in my opinion best) work with Victor Guillemin on spectral asymptotics of positive elliptic operators.

On top of this Hans gave me yet another hour of his time, answering and commenting on some of my questions on Lagrange points and related spectral asymptotics. It was a truly delightful morning with Hans at his best. The next and last time I spoke with Hans was in December 2009. With enthusiasm he told about his ping-pong evening, how his club had won and had been promoted to a higher division. I did not know what to say when Erik van den Ban called me up that Friday in March and told me that Hans had passed away early that morning. All I can do now is to repeat what I wrote in my previous eulogy, namely that I am very lucky that Hans shared his godsent talent with me so generously.

Erik van den Ban and Johan Kolk

On March 19, 2010, mathematics lost one of its leading geometric analysts, Johannes Jisse Duistermaat. At age sixty-seven he passed away, after a short illness following a renewed bout of lymphoma the doctors thought they had controlled. "Hans", as Duistermaat was universally known among friends and colleagues, was not only a brilliant research mathematician and an inspiring teacher, but also an accomplished chess player and very fond of several physical sports.

Hans dropped the subject of thermodynamics because the thesis had led to dissent between mathematicians and physicists at Utrecht University. Nevertheless, this topic exerted a decisive influence on his further development: in its study, Hans had encountered contact transformations. These he studied thoroughly by reading S. Lie, who had initiated their theory. In 1969–70 he spent one year in Lund, where L. Hörmander was developing the theory of Fourier integral operators; this class of operators contains partial differential operators as well as classical integral operators as special cases. Hans's knowledge of the work of Lie turned out to be an important factor in the formulation of this theory. His mathematical reputation was then firmly established by a long joint article with Hörmander concerning applications of the theory to linear partial differential equations. In 1972 Duistermaat was appointed full professor at the Catholic University of Nijmegen, and in 1974 at Utrecht University, as the successor to Freudenthal.

It is characteristic of Hans's work, that on the basis of a complete clarification of the underlying geometry, deep and powerful results are obtained in the area of geometric analysis: in his case, in ordinary or partial differential equations, discrete integrable systems, classical mechanics, analysis on Lie groups, and symplectic differential geometry. Furthermore, after a period of intense concentration on a particular topic, he would move to a different area of mathematics, thereby bringing acquired insights often to new fruition. Nevertheless, on closer inspection, most of these transitions are not as abrupt as they look at first sight.

For example, Hans's interest in semisimple Lie groups was stimulated by efforts to improve, in the more specific setting of such groups, error estimates for the asymptotics of the eigenvalues of elliptic partial differential operators, obtained in previous joint work with V. W. Guillemin. In turn, some results derived along the way acted as a catalyst for the Duistermaat-Heckman formula in symplectic differential geometry. The latter field forms the structural basis for large parts of classical mechanics.

In the later part of his life, Hans had an intense interest in application of mathematics elsewhere in society. For instance, he worked with geophysicists on the inversion of seismic data and on modeling the polarity reversals of the Earth's magnetic field; with mathematical economists on barrier functions and on options; and with biomedical technologists on computer vision.

In 2004 Hans was honored with a special professorship at Utrecht University endowed by the Royal Netherlands Academy of Arts and Sciences. This position allowed him to focus exclusively on research. He frequently described his work at that time as "an obsession". It concerned QRT mappings; these arise in mathematical physics, and Hans studied them with methods from dynamical systems and algebraic geometry. His final work was concerned with Painlevé functions.

While Hans clearly exerted a substantial influence on mathematics through his own research, often with co-authors, and through that of his twenty-three Ph.D. students, the eleven books that he was involved with cover a wide spectrum of mathematical exposition, both in topic and level of sophistication. But in this case, again, there is a common characteristic: every result, however commonplace it might be, had to be fully understood and explained in its proper context. In addition to this, when writing, he insisted that the original works of the masters be studied. He often expressed his admiration for the depth of their treatment, but he could also be quite upset about incomplete proofs that had survived decennia of careless inspection.

On many occasions Hans worked together with colleagues from the United States. Even though several U.S. departments offered him positions and he enjoyed coming to the United States for sabbatical periods, he preferred Utrecht as his permanent base. There he had easy access to mathematicians with a wide range of expertise, while he could still enjoy his favorite physical activities, such as biking and skating.

Hans took teaching seriously and considered it an indispensable counterbalance for his tremendous concentration when involved in research. As a lecturer, he was quite aware that not every student was as gifted as he. Despite the fact that he could ignore all restrictions of time and demanded

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Johan Kolk is professor of mathematics at Utrecht University. His email address is `j.a.c.kol@uu.nl`.

serious work from the students, he was very popular among them. On several occasions he gave nonscheduled courses on their request.

In mathematics, Hans's life was a search for exhaustive solutions to important problems. This quest he pursued with impressive single-mindedness, persistence, power, and success. In our minds he remains very vivid, one of the most striking among the mathematicians we have met. We, as his students, co-authors, and colleagues at Utrecht University, deeply mourn his loss; yet we can take comfort in memories of many years of true and inspiring friendship.

Louis Nirenberg

I first met Hans Duistermaat when he visited Courant Institute in New York as a postdoc around the year 1971. It was September, and New York was hot and muggy, like a steam bath. Almost the first thing Hans said was: "It smells like Jakarta"—he was raised there.

Our families became good friends immediately. He was full of warmth and good humor and radiated intelligence. It soon became clear that he was very talented. Lars Hörmander was visiting about the same time and they wrote their famous paper, "Fourier integral operators II". Hans's research then spread to a wide variety of subjects—in which he made fundamental discoveries.

Over the years we would meet occasionally at conferences. Once, my wife and I had rented an apartment in London, and Hans, who was visiting Oxford, or perhaps Cambridge, came to stay with us for a weekend. At one point I quoted a line from a London guidebook: "It is inadvisable to leave London without having had tea at the Ritz." We decided to do so, but I pointed out that he would have to wear a suit. He promptly went out and bought one.

It was always an enormous pleasure to be together with him, always full of fun. All his friends miss him as well as his mathematics.

Reyer Sjamaar

I got to know Hans Duistermaat as an undergraduate when I took his freshman analysis course at Utrecht. I quickly became hooked and realized that I wanted one day to become his graduate student. This came to pass and I finished in 1990 my thesis work on a combination of two of Hans's favorite topics, Lie groups and symplectic

Photograph by Marius Crainic.

Duistermaat and Alan Weinstein. Picture taken at a thesis defense in Utrecht, 2009.

geometry. He was an older friend as much as an advisor, and I greatly enjoyed my ride with him. Although I was born too late to join him in the kite fighting he engaged in during his boyhood in Indonesia, we did go speedskating together for a few winters at the local rink. Portions of the Utrecht analysis sequence are now available in the form of Duistermaat and Kolk's beautiful textbook *Multidimensional Real Analysis*.

A good way to honor Duistermaat's memory is to display one of his mathematical gems, which the reader can find in Chapter 1 of his *Lie Groups*, another book he cowrote with Joop Kolk. This work is much closer to Lie's original point of view pertaining to differential equations than modern treatments such as Bourbaki, which are more algebraic in spirit. Nevertheless the book is notable for several innovations, particularly its proof of Lie's third fundamental theorem in global form, which in outline runs as follows.

Given a finite-dimensional real Lie algebra $\mathfrak{g}$ with a norm, the space $P(\mathfrak{g})$ of continuous paths $[0,1] \to \mathfrak{g}$ equipped with the supremum norm is a Banach space. For each γ in $P(\mathfrak{g})$ let A_γ be the continuously differentiable path of linear endomorphisms of $\mathfrak{g}$ determined by the linear initial-value problem

$$A'_\gamma(t) = \mathrm{ad}(\gamma(t)) \circ A_\gamma(t), \qquad A_\gamma(0) = \mathrm{id}_{\mathfrak{g}}.$$

Duistermaat and Kolk prove that the multiplication law

$$(\gamma \cdot \delta)(t) = \gamma(t) + A_\gamma(t)\delta(t)$$

turns $P(\mathfrak{g})$ into a Banach Lie group. Next they introduce a subset $P(\mathfrak{g})_0$, which consists of all paths γ that can be connected to the constant path 0 by a family of paths γ_s which is continuously differentiable with respect to s and has the property that

$$\int_0^1 A_{\gamma_s}(t)^{-1} \frac{\partial \gamma_s}{\partial s}(t)\, dt = 0$$

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for $0 \leq s \leq 1$. The subset $P(\mathfrak{g})_0$ is a closed connected normal Banach Lie subgroup of $P(\mathfrak{g})$ of finite codimension, and the quotient $P(\mathfrak{g})/P(\mathfrak{g})_0$ is a simply connected Lie group with Lie algebra $\mathfrak{g}$!

For the details I refer you to the book; also be sure to read the historical and bibliographical notes at the end of the chapter. One of the many virtues of this proof is that it is manifestly functorial: a Lie algebra homomorphism $\mathfrak{g} \to \mathfrak{h}$ induces a continuous linear map on the path spaces $P(\mathfrak{g}) \to P(\mathfrak{h})$, which is a homomorphism of Banach Lie groups and maps the subgroup $P(\mathfrak{g})_0$ to $P(\mathfrak{h})_0$, and therefore descends to a Lie group homomorphism. A few years after publication, the Duistermaat-Kolk proof became at Alan Weinstein's suggestion a central feature of Marius Crainic and Rui Loja Fernandes's resolution of two long-standing problems in differential geometry; see "Integrability of Lie brackets", *Ann. of Math.* (2) **157** (2003), no. 2, 575–620, and "Integrability of Poisson brackets", *J. Differential Geom.* **66** (2004), no. 1, 71–137. My Cornell colleague Leonard Gross has a paper in preparation that adapts the Duistermaat-Kolk argument to certain infinite-dimensional situations.

Johannes Sjöstrand

Hans spent an academic year in Lund, maybe the one of 1969–70. This was a very active time in the development of (what would somewhat later be called) microlocal analysis, and Hans had precious knowledge in symplectic geometry. I especially remember his short lecture notes on the subject, containing precisely what was needed for a Ph.D. student in one of his first years. Hans's curiosity and his interest in original historical mathematical texts were a great stimulation.

It was a great pleasure to collaborate with him on a subject close to my thesis. Here, his original suggestion was to introduce and study microlocally defined projections onto the kernel and the cokernel of pseudodifferential operators, rather than adding auxiliary operators as in my thesis. This collaboration was not very central to Hans's work and he soon changed his interests, but for me it remained useful through the years.

Later I had the pleasure of meeting Hans in Paris and at other places, though less often during the last decades. All this time he has remained a very important person for me, partly because of his great mathematical results and principally because of his very strong and incisive personality that one cannot forget.

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For more on Hans's scientific work, one may consult *Geometric Aspects of Analysis and Mechanics: A Conference in Honor of the 65th birthday of Hans Duistermaat*, to be published by Birkhäuser, Boston. WONDER (The Dutch Research School in Mathematics) has decided to establish the Hans Duistermaat Chair for Visiting Professors.

Ph.D. Students of Johannes J. Duistermaat

J. A. C. Kolk, 1977
G. J. Heckman, 1980
E. P. van den Ban, 1982
S. J. van Strien, 1982
H. E. Nusse, 1983
J. C. van der Meer, 1985
M. Poel, 1986
P. J. Braam (University of Oxford), 1987
P. H. M. van Mouche, 1988
R. Sjamaar, 1990
H. van der Ven, 1993
J. B. Kalkman, 1993
J. Hermans, 1995
O. Berndt, 1996
E. A. Cator, 1997
M. V. Ruzhansky, 1998
C. C. Stolk, 2000
B. W. Rink, 2003
A. M. M. Manders, 2003
H. Lokvenec-Guleska, 2004
T. Gantumur, 2006
P. T. Eendebak, 2007
A. Q. Vélez, 2009

Differential Topology Forty-six Years Later

John Milnor

In the 1965 Hedrick Lectures,[1] I described the state of differential topology, a field that was then young but growing very rapidly. During the intervening years, many problems in differential and geometric topology that had seemed totally impossible have been solved, often using drastically new tools. The following is a brief survey, describing some of the highlights of these many developments.

Major Developments

The first big breakthrough, by Kirby and Siebenmann [1969, 1969a, 1977], was an obstruction theory for the problem of triangulating a given topological manifold as a PL (= piecewise-linear) manifold. (This was a sharpening of earlier work by Casson and Sullivan and by Lashof and Rothenberg. See [Ranicki, 1996].) If B_{Top} and B_{PL} are the stable classifying spaces (as described in the lectures), they showed that the relative homotopy group $\pi_j(B_{\text{Top}}, B_{\text{PL}})$ is cyclic of order two for $j = 4$, and zero otherwise. Given an n-dimensional topological manifold M^n, it follows that there is an obstruction $\mathbf{o} \in H^4(M^n; \mathbb{Z}/2)$ to triangulating M^n as a PL-manifold. In dimensions $n \geq 5$ this is the only obstruction. Given such a triangulation, there is a similar obstruction in $H^3(M^n; \mathbb{Z}/2)$ to its uniqueness up to a PL-isomorphism that is topologically isotopic to the identity. In particular, they proved the following.

Theorem 1. *If a topological manifold M^n without boundary satisfies*

$$H^3(M^n; \mathbb{Z}/2) = H^4(M^n; \mathbb{Z}/2) = 0 \quad \textit{with} \quad n \geq 5\,,$$

then it possesses a PL-manifold structure that is unique up to PL-isomorphism.

(For manifolds with boundary one needs $n > 5$.) The corresponding theorem for all manifolds of dimension $n \leq 3$ had been proved much earlier by Moise [1952]. However, we will see that the corresponding statement in dimension 4 is false.

An analogous obstruction theory for the problem of passing from a PL-structure to a smooth structure had previously been introduced by Munkres [1960, 1964a, 1964b] and Hirsch [1963]. (See also [Hirsch-Mazur, 1974].) Furthermore, Cerf had filled in a crucial step, proving by a difficult geometric argument that the space of orientation-preserving diffeomorphisms of the three-sphere is connected. (See the Cartan Seminar Lectures of 1962/63, as well as [Cerf, 1968].) Combined with other known results, this led to the following.

Theorem 2. *Every PL-manifold of dimension $n \leq 7$ possesses a compatible differentiable structure, and this structure is unique up to diffeomorphism whenever $n < 7$.*

For more information see **Further Details** at the end of this article.

The next big breakthrough was the classification of simply connected closed topological 4-manifolds by Freedman [1982]. He proved, using wildly nondifferentiable methods, that such a manifold is uniquely determined by

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[1] *These lectures have recently been digitized by MSRI and should be available soon. Thanks to Dusa McDuff for unearthing the original tapes.* (*With regard to Wilder's introduction, compare* [Milnor, 1999].)

(1) the isomorphism class of the symmetric bilinear form $H^2 \otimes H^2 \to H^4 \cong \mathbb{Z}$, where $H^k = H^k(M^4; \mathbb{Z})$, together with

(2) the Kirby-Siebenmann invariant $\mathbf{o} \in H^4(M^4; \mathbb{Z}/2) \cong \mathbb{Z}/2$.

These can be prescribed arbitrarily, except for two restrictions: The bilinear form must have determinant ± 1; and in the "even case" where $x \cup x \equiv 0 \pmod{2H^4}$ for every $x \in H^2$, the Kirby-Siebenmann class must be congruent to $(1/8)$th of the signature. As an example, the Poincaré hypothesis for 4-dimensional topological manifolds is an immediate consequence. For if M^4 is a homotopy sphere, then both H^2 and the obstruction class must be zero.

One year later, Donaldson [1983] used gauge theoretic methods to show that many of these topological manifolds cannot possess any smooth structure (and hence by Theorem 2 cannot be triangulated as PL-manifolds). More explicitly, if M^4 is smooth and simply connected with positive definite bilinear form, he showed that this form must be diagonalizable; hence M^4 must be homeomorphic to a connected sum of copies of the complex projective plane. There are many positive definite bilinear forms with determinant 1, and with signature divisible by 16 in the even case, which are not diagonalizable. (See, for example, [Milnor-Husemoller, 1973].) Each of these corresponds to a topological manifold M^4 with no smooth structure but such that $M^4 \times \mathbb{R}$ does have a smooth structure that is unique up to diffeomorphism.

The combination of Freedman's topological results and Donaldson's analytic results quickly led to rather amazing consequences. For example, it followed that there are uncountably many nonisomorphic smooth or PL structures on $\mathbb{R}^4$. (Compare [Gompf, 1993].) All other dimensions are better behaved: For $n > 4$, Stallings [1962] showed that the topological space $\mathbb{R}^n$ has a unique PL-structure up to PL-isomorphism. Using the Moise result for $n < 4$ together with the Munkres-Hirsch-Mazur obstruction theory, it follows that the differentiable structure of $\mathbb{R}^n$ is unique up to diffeomorphism for all $n \neq 4$.

A satisfactory theory of three-dimensional manifolds took longer. The first milestone was the geometrization conjecture by Thurston [1982, 1986], which set the goal for what a theory of three-manifolds should look like. This conjecture was finally verified by Perelman [2002, 2003a, 2003b], using a difficult argument based on the "Ricci flow" partial differential equation. (Compare the expositions of Morgan-Tian [2007] and Kleiner-Lott [2008].) The three-dimensional Poincaré hypothesis followed as a special case.

The Poincaré Hypothesis: Three Versions

First consider the purely topological version.

Theorem 3. *The topological Poincaré hypothesis is true in all dimensions.*

That is, every closed topological manifold with the homotopy type of an n-sphere is actually homeomorphic to the n-sphere. For $n > 4$ this was proved by Newman [1966] and by Connell [1967], both making use of the "engulfing method" of Stallings [1960]. For $n = 4$ it is of course due to Freedman. For $n = 3$ it is due to Perelman, using Moise [1952] to pass from the topological to the PL case, and then using the Munkres-Hirsch-Mazur obstruction theory to pass from PL to smooth. □

Theorem 4. *The piecewise-linear Poincaré hypothesis is true for n-dimensional manifolds except possibly when n = 4.*

That is, any closed PL manifold of dimension $n \neq 4$ with the homotopy type of an n-sphere is PL-homeomorphic to the n-sphere. For $n > 4$ this was proved by Smale [1962]; while for $n = 3$ it follows from Perelman's work, together with the Munkres-Hirsch-Mazur obstruction theory. □

The differentiable Poincaré hypothesis is more complicated, being true in some dimensions and false in others, while remaining totally mysterious in dimension 4. We can formulate the question more precisely by noting that the set of all oriented diffeomorphism classes of closed smooth homotopy n-spheres (= topological n-spheres) forms a commutative monoid S_n under the connected sum operation. In fact this monoid is actually a finite abelian group except possibly when $n = 4$. Much of the following outline is based on [Kervaire-Milnor, 1963], which showed in principle how to compute these groups[2] in terms of the stable homotopy groups of spheres for $n > 4$. Unfortunately, many proofs were put off to part 2 of this paper, which was never completed. However, the missing arguments have been supplied elsewhere; see especially [Levine, 1985].

Using Perelman's result for $n = 3$, the group S_n can be described as follows for small n (Table 1). (Here, for example, $2 \cdot 8$ stands for the group $\mathbb{Z}/2 \oplus \mathbb{Z}/8$, and 1 stands for the trivial group.)

Thus the differentiable Poincaré hypothesis is true in dimensions 1, 2, 3, 5, 6, and 12, but unknown in dimension 4. I had conjectured that it would be false in all higher dimensions. However, Mahowald has pointed out that there is at least

[2] *The Kervaire-Milnor paper worked rather with the group Θ_n of homotopy spheres up to h-cobordism. This makes a difference only for $n = 4$, since it is known, using the h-cobordism theorem of Smale [1962], that $S_n \xrightarrow{\cong} \Theta_n$ for $n \neq 4$. However, the difference is important in the four-dimensional case, since Θ_4 is trivial, while the structure of S_4 is the great unsolved problem.*

Table 1

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
S_n	1	1	1	**?**	1	1	28	2	$2\cdot2\cdot2$	6	992	1	3	2	$2\cdot8128$	2	$2\cdot8$	$2\cdot8$

one more exceptional case: *The group S_{61} is also trivial.* (Compare **Further Details** below.)

Problem. Is the group S_n nontrivial for all $n > 6$, $n \neq 12,\ 61$?

(Any precise computation for large n is impossible at the present time, since the stable homotopy groups of spheres have been computed completely only up to dimension 64. However, it seems possible that enough is known to decide this question one way or another.)

Denote the stable homotopy groups of spheres by

$$\Pi_n = \pi_{n+q}(\mathbb{S}^q) \qquad \text{for} \quad q > n+1\,,$$

and let $J_n \subset \Pi_n$ be the image of the stable Whitehead homomorphism $J : \pi_n(\mathbf{SO}) \to \Pi_n$. (See [Whitehead, 1942].) This subgroup J_n is cyclic of order[3]

$$|J_n| \ = \begin{cases} \text{denominator}\left(\frac{B_k}{4k}\right) & \text{for } n = 4k-1\,, \\ 2 & \text{for } n \equiv 0,\,1 \pmod 8\,,\text{and} \\ 1 & \text{for } n \equiv 2,\,4,\,5,\,6 \pmod 8\,, \end{cases}$$

where the B_k are Bernoulli numbers, for example,

$$B_1 = \frac{1}{6},\ B_2 = \frac{1}{30},\ B_3 = \frac{1}{42},$$
$$B_4 = \frac{1}{30},\ B_5 = \frac{5}{66},\ B_6 = \frac{691}{2730}\,,$$

and where the fraction $\frac{B_k}{4k}$ must be reduced to lowest terms. (Compare [Milnor-Stasheff, 1974, Appendix B].)

According to Pontrjagin and Thom, the stable n-stem Π_n can also be described as the group of all framed cobordism classes of framed manifolds. (Here one considers manifolds smoothly embedded in a high-dimensional Euclidean space, and a *framing* means a choice of trivialization for the normal bundle.) Every homotopy sphere is stably parallelizable, and hence possesses such a framing. If we change the framing, then the corresponding class in Π_n will be changed by an element of the subgroup J_n. Thus there is an exact sequence

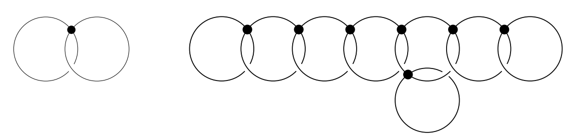
$$(1) \qquad 0 \to S_n^{\mathbf{bp}} \to S_n \to \Pi_n/J_n\,,$$

where $S_n^{\mathbf{bp}} \subset S_n$ stands for the subgroup represented by homotopy spheres that bound parallelizable manifolds. This subgroup is the part of S_n that is best understood. It can be partially described as follows.

Theorem 5. *For $n \neq 4$ the group $S_n^{\mathbf{bp}}$ is finite cyclic with an explicitly known generator. In fact this group is:*

- *trivial when n is even,*
- *either trivial or cyclic of order two when $n = 4k-3$, and*
- *cyclic of order* $2^{2k-2}(2^{2k-1}-1)$ numerator $\left(\frac{4B_k}{k}\right)$ *when $n = 4k-1 > 3$.*

(This last number depends on the computation of $|J_{4k-1}|$ as described above.) In the odd cases, setting $n = 2q-1$, an explicit generator for the $S_{2q-1}^{\mathbf{bp}}$ can be constructed using one basic building block, namely the tangent disk-bundle of the q-sphere, together with one of the following two diagrams.



Here each circle represents one of our $2q$-dimensional building blocks, which is a $2q$-dimensional parallelizable manifold with boundary, and each dot represents a plumbing construction in which two of these manifolds are pasted across each other so that their central q-spheres intersect transversally with intersection number $+1$. The result will be a smooth parallelizable manifold with corners. After smoothing these corners we obtain a smooth manifold X^{2q} with smooth boundary.

For q odd, use the left diagram, and for q even use the right diagram. In either case, if $q \neq 2$, the resulting smooth boundary ∂X^{2q} will be a homotopy sphere representing the required generator of $S_{2q-1}^{\mathbf{bp}}$. (The case $q = 2$ is exceptional since ∂X^4 has only the homology of the 3-sphere. In all other cases where $S_{2q-1}^{\mathbf{bp}}$ is trivial, the boundary will be diffeomorphic to the standard $(2q-1)$-sphere.)

The exact sequence (1) can be complemented by the following information.

Theorem 6. *For $n \not\equiv 2 \pmod 4$, every element of Π_n can be represented by a topological sphere.*

[3] *This computation of $|J_{4k-1}|$ is a special case of the* ***Adams conjecture*** [Adams, 1963, 1965]. *The proof was completed by Mahowald [1970], and the full Adams conjecture was proved by Quillen [1971], Sullivan [1974], and by Becker-Gottlieb [1975]. Adams also showed that J_n is always a direct summand of Π_n.*

Hence the exact sequence (1) *takes the more precise form*

(2) $$0 \to S_n^{\mathbf{bp}} \to S_n \to \Pi_n/J_n \to 0\,.$$

However, for $n = 4k-2$*, it rather extends to an exact sequence*

(3) $$0 = S_{4k-2}^{\mathbf{bp}} \to S_{4k-2} \to \Pi_{4k-2}/J_{4k-2} \xrightarrow{\Phi_k} \mathbb{Z}/2 \to S_{4k-3}^{\mathbf{bp}} \to 0\,.$$

Brumfiel [1968, 1969, 1970] sharpened this result by showing that the exact sequence (2) is split exact, except possibly in the case where n has the form 2^k-3. (In fact it could fail to split only in the cases $n = 2^k - 3 \geq 125$. See the discussion below.)

The **Kervaire homomorphism** Φ_k in (3) was introduced in [Kervaire, 1960]. (The image $\Phi_k(\theta) \in \mathbb{Z}/2$ is called the ***Kervaire invariant*** of the homotopy class θ.) Thus there are two possibilities:

- If $\Phi_k = 0$, then $S_{4k-3}^{\mathbf{bp}} \cong \mathbb{Z}/2$, generated by the manifold ∂X^{4k-2} described above, and every element of Π_{4k-2} can be represented by a homotopy sphere.
- If $\Phi_k \neq 0$, then $S_{4k-3}^{\mathbf{bp}} = 0$. This means that the boundary of X_{4k-2} is diffeomorphic to the standard $\mathbb{S}^{4k-3}$. We can glue a $4k-2$ ball onto this boundary to obtain a framed $(4k-2)$-manifold that is not framed cobordant to any homotopy sphere. In this case the kernel of Φ_k forms a subgroup of index two in Π_{4k-2}/J_{4k-2} consisting of those framed cobordism classes that can be represented by homotopy spheres.

The question as to just when $\Phi_k = 0$ was the last major unsolved problem in understanding the group of homotopy spheres. It has recently been solved in all but one case by Hill, Hopkins, and Ravenel:

Theorem 7. *The Kervaire homomorphism* Φ_k *is nonzero for* $k = 1, 2, 4, 8, 16$*, and possibly for* $k = 32$*, but is zero in all other cases.*

In fact Browder [1969] showed that Φ_k can be nonzero only if n is a power of two, and Barratt, Jones, and Mahowald [1984] completed the verification that Φ_k is indeed nonzero for $k = 1, 2, 4, 8, 16$. Finally, Hill, Hopkins, and Ravenel [2010] have shown that $\Phi_k = 0$ whenever $k > 32$. (Their basic tool is a carefully constructed generalized cohomology theory of period 256.)

Thus only the case $k = 32$, with $4k-2 = 126$, remains unsettled. In particular, for $n \neq 4, 125, 126$, if the order $|\Pi_n|$ is known, then we can compute the number $|S_n|$ of exotic n-spheres precisely. In fact, if we exclude 4, 126, and numbers of the form $2^k - 3 \geq 125$, then the group S_n can be described completely whenever the structure of Π_n is known.[4]

Further Details

Here is a brief outline of the current knowledge of Π_n. Since the direct summand J_n is known precisely, we need only look at the quotient Π_n/J_n. The most difficult part is the 2-primary component, which has been computed by Kochman [1990], with corrections by Kochman and Mahowald [1995], in all cases with $n \leq 64$. The 3 and 5 primary components have been computed in a much larger range by Ravenel [1986]. The primary components for $p \geq 7$ are trivial for $n < 82$. (In fact, for any p, the p-primary component of Π_n/J_n is trivial whenever $n < 2p(p-1)-2$ and is cyclic of order p when $p = 2p(p-1)-2$.)

Thus the stable stem Π_n is precisely known for $n \leq 64$, and hence the group S_n is precisely known for $n \leq 64$, $n \neq 4$. In Table 2 the notation b_k stands for the order of the subgroup $S_{4k-1}^{\mathbf{bp}} \subset S_{4k-1}$, whereas a notation such as $2^3 \cdot 4$ stands for the direct sum of three copies of $\mathbb{Z}/2$ and one copy of $\mathbb{Z}/4$. The trivial group is indicated by a heavy dot. All entries corresponding to the subgroups $S_n^{\mathbf{bp}}$ have been underlined. (Note that $S_{4k-3}^{\mathbf{bp}}$ is cyclic of order two, indicated by a $\underline{2}$ in the 1 or 5 column, except in the cases $k = 1, 2, 4, 8, 16$.) Within this range, the group S_n is trivial only in the cases

$$n = 1, 2, 3, 5, 6, 12, 61 \qquad (\text{and possibly } 4).$$

The corresponding values of b_k are not difficult to compute but grow very rapidly. See Table 3 (with approximate values for $k > 5$). Note that those b_k for which k has many divisors tend to be somewhat larger.

In conclusion, here is an argument that was postponed above.

Outline Proof of Theorem 2. It is not difficult to check that the group $\pi_0(\mathrm{Diff}^+(\mathbb{S}^n))$ consisting of all smooth isotopy classes of orientation preserving diffeomorphisms of the unit n-sphere is abelian. Define Γ_n to be the quotient of $\pi_0(\mathrm{Diff}^+(\mathbb{S}^{n-1}))$ by the subgroup consisting of those isotopy classes that extend over the closed unit n-disk. There is a natural embedding $\Gamma_n \subset S_n$ that sends each $(f) \in \Gamma_n$ to the "twisted n-sphere" obtained by gluing the boundaries of two n-disks together under f. It followed from [Smale, 1962] that $\Gamma_n = S_n$ for $n \geq 5$, and from [Smale, 1959] that $\Gamma_3 = 0$. Since it is easy to check that $\Gamma_1 = 0$ and $\Gamma_2 = 0$, we have

$$\Gamma_n = S_n \qquad \text{for every} \qquad n \neq 4\,.$$

[4]*One also needs the fact that the kernel of* Φ_k *is always a direct summand of* Π_{4k-2} (*at least for* $4k-2 \neq 126$). *Mahowald tells me that this is true in dimension* 62*, and the remaining four cases are straightforward.*

Table 2

n	0	1	2	3	4	5	6	7
0 +	–	•	•	•	?	•	•	$\underline{b}_2$
8 +	2	$\underline{2} \cdot 2^2$	6	$\underline{b}_3$	•	3	2	$\underline{b}_4 \cdot 2$
16 +	2	$\underline{2} \cdot 2^3$	$2 \cdot 8$	$\underline{b}_5 \cdot 2$	24	$\underline{2} \cdot 2^2$	2^2	$\underline{b}_6 \cdot 2 \cdot 24$
24 +	2	$\underline{2} \cdot 2$	$2 \cdot 6$	$\underline{b}_7$	2	3	3	$\underline{b}_8 \cdot 2^2$
32 +	2^3	$\underline{2} \cdot 2^4$	$2^3 \cdot 4$	$\underline{b}_9 \cdot 2^2$	6	$\underline{2} \cdot 2 \cdot 6$	$2 \cdot 60$	$\underline{b}_{10} \cdot 2^4 \cdot 6$
40 +	$2^4 \cdot 12$	$\underline{2} \cdot 2^4$	$2^2 \cdot 24$	$\underline{b}_{11}$	8	$\underline{2} \cdot 2^3 \cdot 720$	$2^3 \cdot 6$	$\underline{b}_{12} \cdot 2^3 \cdot 12$
48 +	$2^3 \cdot 4$	$\underline{2} \cdot 6$	$2^2 \cdot 6$	$\underline{b}_{13} \cdot 2^2 \cdot 4$	$2^2 \cdot 6$	$\underline{2} \cdot 2^4$	$2 \cdot 4$	$\underline{b}_{14} \cdot 3$
56 +	2	$\underline{2} \cdot 2^3$	2^2	$\underline{b}_{15} \cdot 2^2$	4	•	$2 \cdot 12$	$\underline{b}_{16} \cdot 2^3$

Table 3

k	2	3	4	5	6	7	8	9
b_k	28	992	8128	261632	1.45×10^9	6.71×10^7	1.94×10^{12}	7.54×10^{14}

k	10	11	12	13	14	15	16
b_k	2.4×10^{16}	3.4×10^{17}	8.3×10^{21}	7.4×10^{20}	3.1×10^{25}	$5.\times10^{29}$	1.8×10^{31}

On the other hand, Cerf proved[5] that $\pi_0(\mathrm{Diff}^+(\mathbb{S}^3)) = 0$ and hence that $\Gamma_4 = 0$ (although S_4 is completely unknown). Using results about S_n as described above, it follows that $\Gamma_n = 0$ for $n < 7$, and that Γ_n is finite abelian for all n.

The Munkres-Hirsch-Mazur obstructions to the existence of a smooth structure on a given PL-manifold M^n lie in the groups $H^k(M^n;\Gamma_{k-1})$, whereas obstructions to its uniqueness lie in $H^k(M^n;\Gamma_k)$. (Unlike most of the constructions discussed above, this works even in dimension 4.) Evidently Theorem 2 follows. □

For further historical discussion see Milnor [1999, 2007, 2009].

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I Want to Be a Mathematician: A Conversation with Paul Halmos

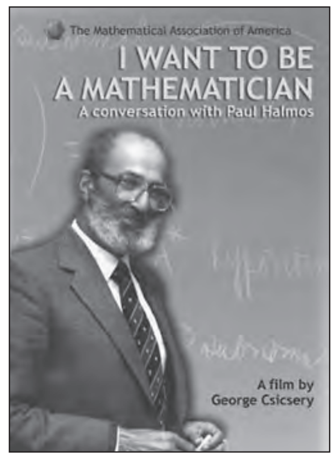
Reviewed by John Ewing

I Want to Be a Mathematician: A Conversation with Paul Halmos
A film by George Csicsery
Mathematical Association of America, 2009
DVD: US$39.95 List; $29.95 Member of MAA
ISBN-978-0-88385-909-4

> I spent most of a lifetime trying to be a mathematician—and what did I learn? What does it take to be one? I think I know the answer: you have to be born right, you must continually strive to become perfect, you must love mathematics more than anything else, you must work at it hard and without stop, and you must never give up.
>
> —*Paul Halmos,* I Want to Be a Mathematician: An Automathography in Three Parts, *Mathematical Association of America, Washington, 1988, p. 400*

What does it mean to be a mathematician? That's not a fashionable question these days. Career counselors caution young people about identifying themselves too closely with their careers. Sociologists confidently predict that most people will have many careers in the future and quote some statistic to prove it's already under way (an average of at least eight careers—one wonders how "career" is defined and what sort of precision is contained in the phrase "an average of at least").

Paul Halmos came into mathematics in an earlier age, when immersing oneself in a career was still fashionable. He lived life as a mathematician as fully as anyone might live such a life. That doesn't mean he spent every moment doing mathematics; he didn't. But almost every part of his life was influenced by being a mathematician—



social, professional, family, friends, and travel.

In a bygone age, this was not unusual. Some decades ago, the annual meeting of the American Mathematical Society was held in the period between Christmas and New Year's. Stories tell of mathematicians who packed up their families and drove halfway across the country to attend that meeting as a perfectly ordinary way to celebrate the holidays. Decades later, summer meetings of the AMS/MAA (and then only the MAA) were events to which families tagged along to join up with other mathematical families for a summer vacation…at a mathematics meeting! Social life in mathematics departments revolved around colloquia and seminar dinners, as well as parties. When job candidates came for interviews, the whole department was part of the evaluation process, including (often) at meals and social gatherings. Daily mathematical lunches were frequent, daily teas with faculty and students were expected, your colleagues often were among your closest friends—not your only friends, to be sure, but among them.

This may sound dull to a modern ear. Your job is not supposed to dominate your life, and you are told to keep some distance between your job and the other parts of your life. But, for Paul Halmos, "mathematician" wasn't a job; it was a profession—a lifestyle that dominated all parts of life. He not only led the life of a mathematician, he taught others to lead it as well.

I don't mean he gave advice to mathematicians, although he certainly did that. Paul Halmos wrote about writing, talking, publishing, traveling,

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teaching, editing, and many other activities mathematicians engage in. Every talk should contain a proof. Teaching should be about asking questions, not giving answers. The hardest part about being an advisor is to get a student started—to suggest a problem for the student to work on. Mathematics is not a deductive science—that's a cliché. Letters of recommendation should be governed by two rules: always tell the truth, but sometimes not the whole truth; and no matter how much good you want to say, be sure to say something bad. Elections at departments tend to produce second-rate leadership. Surveys are hard to write, but good expositions—the low road—are harder still. A good book review is a chatty expository essay on a currently interesting subject. This is a small sample of Halmos's advice that, even when it wasn't adopted, prompted mathematicians to think about their craft.

But Paul Halmos gave a different kind of advice by the life he led. He organized mathematical lunches nearly every day, banging on doors with his famous cane to gather a group of mathematicians. Lunch conversation ranged from mathematical gossip (who would win the Fields Medal) to tricky problems (what can the value of pi—that is, the ratio of circumference to diameter—be for various convex sets). He came to tea most days and made conversation easily with everyone—new faculty and old, graduate students and visitors, mathematicians and physicists, who were all part of the mix. He attended nearly every colloquium, whether related to his field or not. He faithfully went to regional weekend workshops. He and his wife, Ginger, attended almost every party for colloquium speakers and job candidates; they held many social events themselves, often mixing mathematicians, historians, philosophers, and musicians together. Mathematics and mathematicians were ever present in his life.

The DVD under review provides a glimpse of the man as he chats about his life and offers snippets of advice. It contains a 45-minute interview by Peter Renz, along with some additional conversations with mathematicians who knew Halmos. A large segment of the interview focuses on his views about teaching and his enthusiasm for the Moore method, which may be slightly misleading. Halmos's enthusiasm was genuine, but he also was a superb lecturer, and the Moore method never defined him as a teacher. Much of the conversation points to the advice he wrote about in other places, and the DVD contains scanned images of seven related papers by Halmos—well worth reading. (Alas, some of this material is not carefully scanned, which is a bit jarring when reading the words of someone who strove to be perfect.) The DVD also includes a scanned image of a large excerpt from his book *I Want to Be a Mathematician: An Automathography*, after which this present work was named.

The last few pages of his book speak eloquently both about his own life and about the life of a mathematician.

> All these prescriptions and descriptions about how to be a mathematician arose, inevitably, from my own attempts to become one. Nobody can tell you what mathematicians should do, and I am not completely sure I know what in fact they do—all I can really say is what I did.…I taught, I wrote, and I talked mathematics for fifty years, and I am glad I did. I wanted to be a mathematician. I still do.
>
> *—Halmos,* I Want to Be a Mathematician*, pp. 401–402*

And he was.

WHAT IS...

a Resplendent Structure?

Roman Kossak

Resplendent structures were introduced by Jon Barwise and John Schlipf in [1]. The reader may want to consult a dictionary for the meaning of the word "resplendent", but checking a thesaurus is even more informative. The long list of synonyms starts with "splendid, brilliant, dazzling, ..." and ends with "majestic". The last section of [1] is devoted to historical remarks, but the authors do not mention why this particular name was chosen. Before we can explain what makes resplendent structures resplendent, we need a short introduction to model theory.

Graphs, groups, fields, Lie algebras, and many other mathematical structures consist of a set (the universe of the structure) with a set of functions and relations on it. Such objects are called *first-order structures*. In the discussion below we will just call them structures. Since every function can be interpreted as a relation by considering its graph instead, to simplify the discussion we will consider only relational structures, i.e., structures with no functions. In general there are no restrictions on the number of relations on a structure, but to avoid some technicalities, in this article we will assume that structures have only finitely many relations.

The real number field $(\mathbb{R}, +, \times)$ consists of a universe $\mathbb{R}$ and two ternary relations $+$ and $\times$. A *reduct* of a structure is obtained by forgetting some of its relations. For example, the group $(\mathbb{R}, +)$ is a reduct of $(\mathbb{R}, +, \times)$. An *expansion* is obtained by adding relations. For example, $(\mathbb{R}, +, \times, <)$ and $(\mathbb{R}, +, \times, \exp)$ are both expansions of $(\mathbb{R}, +, \times)$. There is an important difference between these two expansions. The relation $x < y$ is defined in $(\mathbb{R}, +, \times)$ by the formula $(x \neq y) \wedge \exists z(y = x+z\times z)$; hence any property of the real numbers expressed in terms of $+$, $\times$, and $<$ can be expressed in terms of $+$ and $\times$ only. The status of exponentiation is different. It follows from the Tarski-Seidenberg elimination of quantifiers for real closed fields that the relation $y = \exp(x)$ cannot be defined in terms of $+$ and $\times$, so the latter expansion is an essential one. Here by definability we mean definability in first-order logic. Each structure determines its language, which consists of names of all of its relations and all properly formed formulas that can be written using those names, variables, parentheses, the equality symbol =, Boolean connectives, and quantifiers. Such formulas are called *first order*. If $\mathcal{M}$ is a structure with a universe M, then an n-ary relation $R \subseteq M^n$ is *definable* if there is a formula $\varphi(\bar{x}, \bar{y})$ of the language of $\mathcal{M}$ and some m-tuple $\bar{b}$ from M such that R is the set of those n-tuples $\bar{a}$ for which $\varphi(\bar{a}, \bar{b})$ is true in $\mathcal{M}$.

A *theory* is a set of sentences in a given first-order language. If T is a theory and all sentences of T are true in a structure $\mathcal{M}$, then we say that $\mathcal{M}$ is a *model* of T.

Suppose that $\mathcal{M}$ and $\mathcal{N}$ are structures such that the universe of $\mathcal{M}$ is a subset of the universe of $\mathcal{N}$ and the relations of $\mathcal{M}$ are restrictions to the universe of $\mathcal{M}$ of the relations of $\mathcal{N}$. Then we say that $\mathcal{N}$ is an *elementary extension* of $\mathcal{M}$ if for each first-order sentence $\varphi(\bar{a})$ involving a finite string of parameters $\bar{a}$ from $\mathcal{M}$, $\varphi(\bar{a})$ is true in $\mathcal{M}$ if and only if it is true in $\mathcal{N}$. If $\mathcal{N}$ is an elementary extension of $\mathcal{M}$, then we also say that $\mathcal{M}$ is an *elementary substructure of* $\mathcal{N}$. One can show that $(\mathbb{R}, <)$ is an elementary extension of $(\mathbb{Q}, <)$, but $(\mathbb{R}, \times)$ is not an elementary extension of $(\mathbb{Q}, \times)$,

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since, for example, the sentence $\exists x\ [x^2 = 2]$ is true in the former structure but not in the latter.

Let $\mathcal{M}$ be a structure, and let R be a relation symbol not in the language of $\mathcal{M}$. Let $\varphi(R)$ be a statement in the language of $\mathcal{M}$ with this new symbol. Is there a relation R_M on the universe of $\mathcal{M}$ such that $\varphi(R)$ is true in the expanded structure $(\mathcal{M}, R_M)$ when R is interpreted as R_M? For example, if $(G, +)$ is a group, is there a ternary relation $\times_G$ on G such that $(G, +, \times_G)$ is a field? In other words, if $\varphi(\times)$ is the conjunction of the field axioms in the language with $+$ and $\times$ as ternary relations, is there an expansion of $(G, +)$ satisfying $\varphi(\times)$? There may be some obvious obstacles. Suppose $(G, +)$ is a six-element group. There are no six-element fields, so G cannot be a universe of a field. The fact that a universe of a structure has exactly six elements can be expressed by a first-order sentence ψ. The sentence ψ is true in $(G, +)$, and it is *inconsistent* with $\varphi(\times)$ in the sense that there is no structure in which both $\varphi(\times)$ and ψ would be true. For an infinite example, let us consider $(\mathbb{Z}, +)$. The sentence $[1 + 1 = 0 \vee \exists x(x + x = 1)]$ is true in every field but is false in $(\mathbb{Z}, +)$; hence no expansion of $(\mathbb{Z}, +)$ is a field. But what happens if there are no such inconsistencies?

Let $\bar{a}$ be a tuple of elements from the universe of a structure $\mathcal{M}$. By $\mathrm{Th}(\mathcal{M}, \bar{a})$ we denote the set of all sentences $\varphi(\bar{a})$ in the language of $\mathcal{M}$ that are true in $\mathcal{M}$. Suppose that $\varphi(R, \bar{a})$ is a sentence in the language of $\mathcal{M}$ with new relation symbol R and with parameters $\bar{a}$ such that the set of sentences $\mathrm{Th}(\mathcal{M}, \bar{a}) \cup \{\varphi(R, \bar{a})\}$ is consistent, meaning that there is a structure in which all those sentences are true. Then—and this is an exercise in model theory—$\mathcal{M}$ has an elementary extension $\mathcal{N}$ such that for some relation R_N on the universe of $\mathcal{N}$ the sentence $\varphi(R, \bar{a})$ is true in the expanded structure $(\mathcal{N}, R_N)$. In other words, a relation satisfying a consistent first-order property $\varphi(R, \bar{a})$ always exists on the universe of some elementary extension of $\mathcal{M}$. We are interested in the situation in which a relation like that can be already found on $\mathcal{M}$. This leads to the following definition.

Let $\mathcal{M}$ be a first-order structure. We say that $\mathcal{M}$ is *resplendent* if for any first-order sentence $\varphi(R, \bar{a})$ with a new relation symbol and a tuple $\bar{a}$ of parameters from $\mathcal{M}$ such that the set of sentences $\mathrm{Th}(\mathcal{M}, \bar{a}) \cup \{\varphi(R, \bar{a})\}$ is consistent, $\varphi(R, \bar{a})$ is true in $(\mathcal{M}, R_M)$ for some relation R_M on the universe of $\mathcal{M}$. In other words, if there is a relation satisfying some first-order requirement in an elementary extension of $\mathcal{M}$, then there is also such a relation on $\mathcal{M}$.

Technically, any finite structure is resplendent. This is an interesting exercise in model theory but is otherwise an uninteresting fact. More interesting examples are not hard to find, but one has to use some model theory. The crucial fact is that every structure has a resplendent elementary extension of the same cardinality. The result itself is not difficult to prove, but it has many important consequences. Let us see how it implies that the structure $(\mathbb{Q}, <)$ is resplendent. Let $(D, <)$ be a resplendent countable elementary extension of $(\mathbb{Q}, <)$. By elementarity of the extension, $(D, <)$ is a dense linear ordering without endpoints. Since, up to isomorphism, there is only one countable dense linear ordering without endpoints, $(D, <) \cong (\mathbb{Q}, <)$. Hence $(\mathbb{Q}, <)$ is resplendent. This argument works in a more general setting. Let κ be a cardinal number. A theory T in a first-order language is *κ-categorical* if, up to isomorphism, there is exactly one model of T of cardinality κ. Arguing as above, one can show that if T is κ-categorical and $\mathcal{M}$ is a model of T of cardinality κ, then $\mathcal{M}$ is resplendent. It follows that, among others, the countable random graph, any uncountable algebraically closed field, and any infinite-dimensional vector space over a finite field are all resplendent.

For an example of a structure that is not resplendent let us consider the ring $(\mathbb{Z}, +, \times)$ and the sentence $\varphi(I)$ in the language of rings with additional unary relation symbol I:

$$\exists x\, I(x) \wedge \exists x \neg I(x)$$
$$\wedge \forall x, y[x + 1 = y \rightarrow (I(x) \leftrightarrow I(y))].$$

The sentence is saying that I is a proper nonempty subset of the universe that is closed under successors and predecessors. Clearly, there are no such subsets of $\mathbb{Z}$. If $(K, +, \times)$ is a proper elementary extension, then $\mathbb{Z}$ is a proper convex subset of K; hence $\varphi(I)$ is true in $(K, +, \times, \mathbb{Z})$. It follows that $(\mathbb{Z}, +, \times)$ is not resplendent. A similar argument shows that the field $(\mathbb{R}, +, \times)$ is not resplendent (but recall that the field of complex numbers is).

So how resplendent are resplendent structures? Each infinite resplendent structure has nontrivial automorphisms, and each countable resplendent structure has continuum many. Moreover, each infinite resplendent structure is isomorphic to a proper elementary substructure of itself. There is no room here for a full discussion, but let us just make one comment about proofs of such results. In the whole spectrum of models of a given theory T, resplendent models typically form a smaller class, often with a well-defined complete set of isomorphism invariants. Often, to show that a resplendent model $\mathcal{M}$ has some property P, one constructs a resplendent elementary extension $\mathcal{N}$ of $\mathcal{M}$ that has property P and has the same isomorphism invariants as $\mathcal{M}$. It follows that $\mathcal{N}$ is isomorphic to $\mathcal{M}$; hence $\mathcal{M}$ has property P.

Countable resplendent structures are even more than resplendent; they are *chronically resplendent*. The definition is the same as that of resplendency, except that we also require that the expansion $(\mathcal{M}, R_M)$ is also resplendent. This feature is particularly useful in applications.

Let $\mathcal{M}$ be a countable structure and suppose that A is a relation on the universe of $\mathcal{M}$ such that the expanded structure $(\mathcal{M}, A)$ is resplendent and A is not definable in $\mathcal{M}$. Then A has continuum many automorphic images in $\mathcal{M}$. Here is a natural example. Let $(K, +_K, \times_K)$ be a proper countable elementary extension of $(\mathbb{Z}, +, \times)$. It can be shown that for all such extensions $(K, +_K)$ is resplendent. We know that $\times$ (as a relation on $\mathbb{Z}$) is not definable in $(\mathbb{Z}, +)$.[1] Since the extension is elementary, $\times_K$ is not definable from $+_K$ in K. We conclude that $\times_K$ has continuum many automorphic images in $(K, +_K)$; hence there are continuum many different multiplications $\circ$ on K such that $(K, +_K, \circ)$ has all the first-order properties of $(\mathbb{Z}, +, \times)$.

Resplendent models have many applications in model theory, but here is an application of a more algebraic nature. Let $\mathcal{R} = (R, +, \times)$ be an ordered field. A set $I \subseteq R$ is an *integer part* of $\mathcal{R}$ if I is a discretely ordered subring such that 1 is the least positive element and for each $x \in R$ there is some $i \in I$ such that $i \leq x < i + 1$. By a result of Mourgues and Ressayre [3], every real closed field has an integer part. Of course, the real field $(\mathbb{R}, +, \times)$ has the unique integer part $(\mathbb{Z}, +, \times)$, but other fields can have many. By the already-mentioned Tarski-Seidenberg elimination of quantifiers, every definable subset of a real closed field is a finite union of intervals. Hence, no integer part of a real closed field can be first-order definable. It follows that if $\mathcal{R}$ is a resplendent countable real closed field, then $\mathcal{R}$ has continuum many integer parts. In a recent paper [2], D'Aquino, Knight, and Starchenko proved that a countable real closed field has an integer part that is a nonstandard model of Peano Axioms (PA) if and only if the field is resplendent. One direction is straightforward once we note that in the definition of resplendency the first-order property $\varphi(R, \bar{a})$ can be replaced by any effectively presented (computable) list of such properties (still with a fixed tuple of parameters $\bar{a}$). An example would be a list of sentences $\Phi(I) = \{\varphi_n(I) : n < \omega\}$ expressing that I is an integer part that satisfies the Peano Axioms (which can be effectively listed). Clearly, all sentences in $\Phi(I)$ are true in the structure $(\mathbb{R}, +, \times, \mathbb{Z})$. The axioms of real closed fields are complete, which implies that for each real closed field $\mathcal{R}$, $\mathrm{Th}(\mathcal{R}) = \mathrm{Th}(\mathbb{R}, +, \times)$. It follows that Φ is consistent with $\mathrm{Th}(\mathcal{R})$; hence every resplendent real closed field has an expansion satisfying $\Phi(I)$, and, by the results mentioned before, any countable resplendent real closed field has continuum many such expansions. It is interesting that a similar argument shows that every resplendent real closed field has integer parts satisfying countably many different theories (all contradicting the Peano Axioms). This follows from a theorem of Shepherdson [4] according to which any discretely ordered ring satisfying the axioms of Peano Arithmetic restricted to formulas without quantifiers is an integer part of a real closed field.

[1] *It follows, for example, from the fact that $(\mathbb{Z}, +)$ is a decidable structure and $(\mathbb{Z}, +, \times)$ is not.*

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Mathematics Emerging: A Sourcebook 1540–1900

Reviewed by David Pengelley

Mathematics Emerging: A Sourcebook 1540–1900
Jacqueline Stedall
Oxford University Press, 2008
Hardcover, 680 pp., US$100.00
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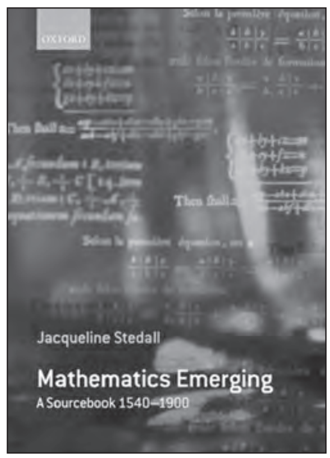
What is a mathematics sourcebook, and why should we welcome a new one? A sourcebook collects together excerpts from primary historical sources, each book with a particular emphasis, point of view, and intended audience. Over the past several decades a number of sourcebooks have appeared, and while some are specific to a particular subject area, a goodly number, like the one under review, are quite general [13]. Many also include commentary on the sources, providing context and direction.

Why should primary sources be important to today's mathematician or student of mathematics? And does Jacqueline Stedall's new sourcebook offer something especially different from the others?

Earlier mathematics is often as valid and valuable as the new, while perspective, conceptualization, motivation, notation, and ways of validating may have been dramatically different. Primary sources provide our main window into how mathematics was practiced in the past and often display key insights leading to important developments. In fact, today's top mathematicians are often quite familiar with the primary sources of their field, obtaining insight and inspiration from these sources



for their research. And primary sources seem to be chock full of phenomena that raise fascinating questions, leading us to why primary sources are also good for students.

Learning mathematics from primary sources offers students the benefits that the humanities have always possessed from this focus. Studying primary sources can foster motivation, broaden perspective, reveal context, draw attention to subtleties, hone deductive skills and conversion between verbal descriptions and modern mathematical formulations, provide excitement, bring students closer to the practice of research, show the genesis and progression of ideas, and display the human face of mathematics. Perhaps most importantly, the challenges of comprehending a primary source raise students from the role of spectator to practitioner. And students gain a more profound comprehension, because the initial simplicity of a theory, along with the questions that arise naturally from studying primary sources and that engender cognitive dissonance requiring conceptual understanding to resolve, can be a tremendous help in learning.

For these reasons, among others, increasingly many instructors are having students engage primary sources directly [5, 6, 7, 11, 12]. And, fortunately, a community of dedicated scholars and teachers has been working worldwide for

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many years to make primary sources accessible and usable [7, 11], so that now for school and undergraduate teaching levels, a meaningful fraction of the most useful sources has been and is being translated and collected in sourcebooks and in other organized ways [13]. These range from compiled raw resources for individual instructor selection all the way to coherent sequences of primary sources with commentary and exercises designed for teaching entire courses [9, 10]. Materials have also been organized into modular student project formats suitable for insertion into existing courses to supplement or replace parts or all of a textbook [1, 2, 3].

How does *Mathematics Emerging: A Sourcebook 1540–1900* enhance what we already have from other sourcebooks? The author, Jacqueline Stedall, explains that the book emerged from a course she designed with Raymond Flood, Peter Neumann, and Robin Wilson "to provide [Oxford University] mathematics undergraduates with some historical background to the material that is now taught universally to students in their final years at school and the first years at college or university: the core subjects of calculus, analysis, and abstract algebra, along with others such as mechanics, probability, and number theory. All of these evolved into their present form in a relatively limited area of western Europe from the mid-sixteenth century onward, and it is there that we find the major writings that relate in a recognizable way to contemporary mathematics. Hence the relatively narrow focus of this book."

Indeed, this aim, utilizing sources encompassing a decent breadth of typical undergraduate mathematics at both lower and upper levels, distinguishes this book clearly from all other sourcebooks. Probably three quarters of the sources featured are from the eighteenth and nineteenth centuries, whereas other sourcebooks tend more to emphasize mathematics from earlier periods, largely for audiences at school or beginning university level. And even from the centuries of possible overlap, the author has chosen many fresh materials to emphasize, often very nice lesser-known alternatives to now-standard sources in other books, so all told there is relatively little duplication of sources. I will analyze the detailed coverage below.

The author is quite clear that this sourcebook is also distinguished from others by the philosophy that "almost every source is given in its original form, not just in the language in which it was first written, but as far as practicable in the layout and typeface in which it was read by contemporaries." This welcome departure from previous sourcebooks is made not just for strong aesthetic reasons nor merely to entice readers to try their own hand at reading the originals, but reflects also the author's stated aim of encouraging readers to interpret historical materials directly for themselves, including emphasizing what one learns from the notation and layout of an original. Supplementing this admirable exhortation, English translations are nonetheless provided alongside all excerpts in other languages. The appearance of almost every excerpt as an image from an original publication, accompanied by a modern typeset English translation, indeed creates a consistently thrilling historical and emotional experience for the reader and particularly commends this sourcebook.

Stedall acknowledges the "relatively brief" nature of her commentary around each source extract and also any feeling on the reader's part that a source should have been included but wasn't, by encouraging the reader to "become not just a reader but a historian," seeking out other sources or opinions for study and comparative judgments. With this I heartily agree in principle, but I will discuss some places where I feel either that some commentary is misleading or confusing or that entire subjects have been given much too short a shrift for comfort. Nonetheless, the overall scope of the book's coverage is prodigiously valuable at over 600 pages of mostly primary source material. And the presource commentaries are, although sometimes brief, generally informative and helpful; the reader feels the confidence of having a guide.

As to Stedall's detailed choices of actual excerpt passages from within her chosen sources, I have two thoughts, related to her introductory remarks. First, she comments that some may object that "brief extracts" give only a partial picture, but she believes that "for beginners a complete picture would be overwhelming." Actually, I do not on balance find her extracts too brief at all for her aims; in fact, often they are refreshingly generous. However, I do not agree that beginners cannot benefit from a complete in-depth picture of a topic from primary sources. That is exactly how I teach with them myself, often having students learn a prescribed subject (not an overview of all of modern mathematics) in great depth from extensive study of primary texts [9, 10, 11]. In fact, I would like to see all our specific subject courses built around in-depth study of primary sources [12].

Second, Stedall comments that her extracts will "find mathematicians groping in the dark, experimenting with new ideas, making hypotheses and guesses, proving correct theorems wrongly, and even on occasion proving incorrect theorems wrongly too" and that the excerpts display the "process of discovery". I think her extracts succeed admirably in achieving this worthy aim, even including some sources of criticism and controversy. But the choice of passages with this aim sometimes makes me uncomfortable that the balance thereby shifts a bit more away from concrete important results and proofs than I would personally prefer; I sometimes feel that, while the chosen excerpt introduced new thinking, new concepts, and new notation, I and my students got no new mathematical results out of it. And I find the excerpts vary from appetizers to the impenetrable, although this may have been intentional to display full variety. Perhaps in the end my discomfort reflects nothing more than the richly different ways in which primary sources are valuable: I tend to teach with them to see how concrete results were first discovered, formulated, and proven, and I incline to provide more commentary, while Stedall's focus in the course underlying this book is to provide students with historical background for a great breadth of mathematics already studied in other courses and to encourage them to go further on their own via other materials to fill out the picture for themselves.

The eighteen chapters of Stedall's book arrange her source excerpts roughly chronologically, but beyond the first two, each chapter focuses on a sequence of sources from just one particular topic in a particular time period. This method of organization works well. I will address her chosen materials largely by topic.

• Stedall wisely sets her stage with the actors arithmetic, geometry, numbers, and algebra in a *Beginnings* chapter of a few sources from prior to her formal starting date of 1540. These sources span Babylonian times through Sacrobosco's thirteenth-century *Algorismus*, giving a very brief overview of the mathematical "common knowledge" of western Europe prior to the beginning of her story in the Renaissance.

• The second chapter, "Fresh Ideas", continues with European sources in four arenas bridging the sixteenth and seventeenth centuries: calculation, notation, analytic geometry, and indivisibles. Improvements in calculation are highlighted by reading Stevin on decimal fractions and Napier on logarithms. Unfortunately, the commentary introducing Stevin leaves the inexpert reader with the impression that Stevin was the first to invent decimal fractions around 1585. While I understand the author's reasons for focusing on European sources in this book, it is then particularly incumbent at least to mention earlier developments in other places: in this case, even outside China, decimal fractions had an extensive well-known history in the Arabic world [4, Ch. 2.3], [8, pp. 226–268, 278, 345–346], beginning with the text of al-Uqlīdīsī in the year 952 [14]. The translation of Stevin is the only one in the book in which I am a bit disappointed, seeming both stilted and a little inaccurate.

For Napier's logarithms, a particularly challenging text given how different his point of view was from today's, the commentary is hampered rather than aided by the attached diagram, whose notation seems quite disconnected from the commentary and source.

But the rest of the chapter is excellent, with commentary providing in-depth discussion and connections to larger context for sources highlighting improvements in notation by Harriot and Descartes; the invention of analytic geometry by Viète, Fermat, and Descartes; and the theory of indivisibles by Cavalieri and Wallis; along with a critique by Hobbes. One of the strengths of the book shows itself already here in the stimulating back and forth of controversy, with Hobbes's critique sounding much like Bishop George Berkeley's later attack on the calculus of Leibniz and Newton.

The succeeding sixteen topic-oriented chapters seem to me somewhat unbalanced, since fully ten of them fall within the breadth of what we today call analysis. Indeed, I will argue that some other areas of mathematics have been unfairly shortchanged, but it is also undeniably the case that the seventeenth through nineteenth centuries were largely dominated by the development of analysis, and the profusion of sources in this book allows us to revel in all its glorious twists and turns.

• *Analysis*: The scope of over seventy-five sources in these ten chapters is enormous, ranging from Fermat and Descartes finding tangents to curves in the 1630s to Lebesgue on integration more than two and a half centuries later. And along the way, these analysis sources, many of which are not the best-known ones, follow various branches and influences. For instance, Newton's *Principia* has a chapter to itself, showing the strong interplay between geometry, physics, and limiting processes, and provoking issues we are not used to today. And there are entire sequences of sources focusing on each of power series, the function concept, foundations of calculus, applications involving differential equations, limits and continuity, derivatives and integrals, complex analysis, and convergence and completeness, offering multiple perspectives on each. I cannot adequately emphasize the exuberance one feels being inside the grand developments of analysis through these admirably chosen source excerpts with very good introductory commentary (although I sometimes wished for more to help me through certain difficult parts), enhanced by the give and take of controversy included in the sources and by a focus on struggles, not just results, leaving the reader to make judgments. All told, these sources provide a wealth of understanding of much of the development of analysis, and students who have already studied some modern analysis will benefit enormously from them.

• *Algebra*: This is the second most extensively treated topic in the book, featured in three chapters. The first follows solving polynomial equations via sources from Cardano to Lagrange, while the second shows the nineteenth-century emergence of abstract algebra from Cauchy and Galois through Cayley to Kummer and Dedekind on ideals. Even the latter two sources will be somewhat accessible and enlightening to students who have already studied some modern algebra. The third algebra chapter is a tour of three centuries and eleven sources, today united under the umbrella "linear algebra", from determinants through eigenvalues and matrices to vector spaces. It is fascinating to see how these themes proceeded independently in the problems of each era, in contrast with our more unified understanding today. I only feel a little sorry for poor Arthur Cayley. He was disparaged in an earlier chapter for his work on groups having "never [come] anywhere near the depth or sophistication of Cauchy or Galois." Now he is belittled, unfairly in my view, by the book's statement that in his development of an algebraic theory of matrices, e.g., the Cayley-Hamilton theorem, he "succeeded in performing a few low level calculations, but failed to see the larger picture, or work with anything like the sophistication of his continental contemporaries."

• *Probability*: This chapter is a refreshing addition in a sourcebook. It does not just provide the early, discrete developments of Pascal and Bernoulli, but quickly moves to sources on more advanced analytical topics like DeMoivre on the normal distribution and confidence intervals, and Bayes's theorem. We end the chapter reading a sophisticated 1812 application by Laplace in which he analyzes differences in the ratios of boy to girl baptisms in London and Paris and speculates that his results suggest that French farmers choose preferentially to abandon their female infants to adoption. It is a fascinating source connecting advanced mathematics to social issues. This chapter would greatly enhance an advanced probability course.

• *Foundations*: Here we have sources by Cantor and Dedekind on the real and natural numbers and their cardinalities, and by Hilbert on the axiomatization of Euclidean geometry. But in this chapter one feels acutely the lack of one of the missing topics in the book, since non-Euclidean geometries are mentioned only once in the entire book, in passing here. One realizes that the book contains nothing on projective, algebraic, or hyperbolic geometries; only in this final chapter would the reader even learn that non-Euclidean geometries exist. And while mentioning geometry, a real disappointment is that its modern child topology is nowhere in the book, not for lack of wonderful sources from Leibniz's or Euler's time forward. Finally, a chapter titled "Foundations" also makes one realize that mathematical logic is not to be found, despite its underpinnings arising strongly from the nineteenth century.

• *Number theory*: This is the one remaining topic represented with a chapter and promised in the introduction. But it is far and away the shortest chapter of all and ends around 1650 with Fermat. The author claims that "most of modern number theory is beyond the scope of this book," but I feel this is no more true of number theory than of the analysis and algebra to which thirteen chapters of sources were devoted. There is a plethora of wonderful number theory sources by Euler, Lagrange, Legendre, Gauss, Riemann, etc., which are actually more accessible than many of the challenging sources in the book and which could introduce the reader to the distribution of primes via the quadratic reciprocity law, prime number theorem, and zeta function, or to Fermat's last theorem [9, 10]. I think an opportunity has been missed here, with number theory being shortchanged as something of an afterthought despite being explicitly claimed as a subject of focus for the book.

In addition to sources and commentary, the book has another nice feature, a list of relevant mathematicians, their institutional affiliations and connections to other people, to help tie the big picture together, as well as a list of historical institutions and journals, a useful bibliography, a list of a few modern digital archives, and an index. But there are some things that could have benefited from more attention to detail in final editing. For instance, it is amusing to learn that Lagrange replaced himself at the Berlin Academy and a little disappointing that the likes of Legendre, Lobachevsky, and Roberval are not in the list of people. And it is frustrating in the text to have people sometimes introduced only by surname, as if we should already know them well. This can even lead to confusion; for instance, unless we look him up, we might wonder the first few times we read about Mercator whether he is the same Mercator who developed the map projection (he isn't). And there are a very few errors, e.g., Euler's *Institutiones Calculi Differentialis* is not volume II of his *Introductio in Analysin Infinitorum*. The *Introductio* contained no calculus and is a spectacular triumph for developing the properties, expansions, and identities of the important functions of analysis using only infinite algebra.

Finally, there is one disappointment in the otherwise beautiful production of this sourcebook, namely, the very poor visual quality of a number of the photographically reproduced sources. I know firsthand that there are real challenges associated with reproductions, but I simply do not think that so much faintness and distortion from bindings was necessary if the authors had devoted a bit more effort to finding good-quality original publications.

Despite these few weaknesses, the overall quality and magnitude of the endeavor, along with the distinctive provision of original source images, are outstanding, so *Mathematics Emerging* is a highly valuable and uniquely positioned new contribution, providing something extensive for upper-level undergraduates for the first time. I am extremely impressed by its coverage of analysis, for which it provides voluminous enrichment for instructors and students at many levels, and it accomplishes in large measure the same for students of modern algebra and perhaps also probability. The areas where it offers relatively little or nothing at all and could have offered so much more by choosing a better balance against analysis are number theory, modern geometries, topology, and logic.

Every scholarly library should own *Mathematics Emerging*. What about its usefulness for teachers or directly for students in the classroom? The Oxford course on which it was based is highly unusual, as are the courses I teach basing student learning of content directly on primary sources. And the sources chosen here were not selected to provide a primary learning path through the mathematics, but rather as a supplement to other study. If you engage undergraduates, the book by itself will greatly enhance your teaching, as it has for me, and you will be changed slowly by it as you read different parts at different times. Personally I would not hesitate to use *Mathematics Emerging* as the principal student text for an upper undergraduate history of mathematics course, supplemented by other sources in certain areas. And I will very confidently and happily assign targeted excerpts or chapters in many other regular courses in the curriculum. It all comes down to whether we will know and teach mathematics as a living, breathing subject with a bountiful and very human life history. This book is a huge enabling step.

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About the Cover

Forty-six years of topology from the differentiable viewpoint

This month's cover was stimulated by John Milnor's article in this issue. It is a photograph of the sketch by him that appeared on the cover of his classic text, *Topology from the Differentiable Viewpoint,** published in 1965 by the University Press of Virginia.

Milnor tells us, "The original sketch of the grouchy vector field picture may exist in some box, but the probability of finding it is near zero. The motivation for the picture is described in §6 (and the end of §5) of the book. I was just sketching a vector field in the disk, pointing out around the boundary, with one saddle point and one index +2 point. It came out looking decidedly cross."

We thank Milnor and the University Press of Virginia for permission to reproduce the cover image.

—Bill Casselman
Graphics editor
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DOCEAMUS

doceamus . . . let us teach

The Common Core State Standards—Education Reform and Us

Solomon Garfunkel

We now live in a country with common core state standards (CCSS) in K–12 mathematics. While technically these standards were created by the National Governors Association and the Council of Chief State Officers and funded by private foundations, in reality they are a major part of the Obama administration's education policy. I don't mean to say that they were written by people in the administration—only that the idea of common standards is an article of faith of the U.S. Department of Education. And of course one can see the logic. With *No Child Left Behind* we had, for all practical purposes, fifty sets of standards and fifty sets of high-stakes tests. Students were being anointed as mathematically proficient in one state, when they couldn't score in the twenty-fifth percentile in another, or get a passing grade on more generic assessments such as the National Assessment of Educational Progress (NAEP).

As I write this piece forty-four states have agreed to abide by the CCSS, and the betting line is forty-five. In effect we now have a set of national standards. In addition, two major state assessment consortia have been funded by the Department of Education. They are charged with preparing a bank of tests to be ready by 2014–2015; so in effect we will have national tests as well. In other words, we are within five years of standards, tests, and curricula all marching in the same direction.

What does all of this mean for the mathematics research community? Why should the community care? For one thing, today's K–12 students will be the college and university students of tomorrow. We care about their preparation and their motivation. For another thing, these students are the next generation of our citizens. We care about their ability to run the world (and pay for our Social Security). And we care about the future of mathematics. We want a mathematically literate student body that becomes a mathematically literate citizenry. We want more and better math majors and more and better majors in related scientific and engineering disciplines. Increased mathematical literacy will make the profession better and the world a better place. So it's important and it demands attention.

While it would be easy to find fault with the standards as written, one must recognize the difficult nature of the writing task. The standards authors had to appeal to many different constituencies in as many states as possible. And, given the push from the Department of Education, they were given much too short a timetable. But that is now quite clearly beside the point. We will be living

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with this version of the standards for at least the next ten years, and it will determine what and how K–12 mathematics is taught and tested.

Of course, you should read the standards for yourselves and draw your own conclusions. Here's my take. The content standards themselves are in part a list of mathematical topics that will seem quite familiar. In fact, they feel rather old-fashioned, more or less your grandfather's standards. At this writing there are no exemplary tasks contained in the document, which would have been of great help in making the meaning of the statements clearer to teachers. The high school standards in particular are written in a language many teachers simply don't speak.

But again, that is beside the point. These are our standards now. And there is some good news here. The Standards for Mathematical Practice are well formed and important. They require that students

- make sense of problems and persevere in solving them.
- reason abstractly and quantitatively.
- construct viable arguments and critique the reasoning of others.
- model with mathematics.
- use appropriate tools strategically.
- attend to precision.
- look for and make use of structure.
- look for and express regularity in repeated reasoning.

Moreover, states signing on to the CCSS are permitted to add up to 15% of their own standards. This flexibility is an opportunity for us to ensure that the mathematics taught K–12 contains the breadth of ideas and applications that demonstrate the importance and excitement of our subject. By getting involved at the school, district, and state level we can provide the examples so desperately missing—examples that challenge students to reason, connect, and communicate their understanding.

A conference was recently held in Washington, DC, to discuss the implications of living in this new standards-based world, especially for textbook and course development. Several important recommendations have come from a report of that conference. The two most important ones (to my mind) are the recommendations that the standards remain a living document and that the mathematical practices, as opposed to a laundry list of mathematical topics, remain prominent.

But the real question for the mathematics community is what is our role in all this; what do these standards mean to us? For some time, the standards, the tests, and the curricula designed to prepare students for those tests will define K–12 mathematics. We need to be diligent—careful that our students' vision of mathematics does not become too narrow, too focused on algebraic symbol manipulation. This is where the practices must come in. Mathematics needs to make sense; mathematical modeling needs to be introduced early and often; we must make room for more recent mathematical ideas—graph theory, game theory, operations research, et al. As a nation we must not sacrifice our creativity for a higher ranking on skills-based tests.

Although college placement exams are heavy on checking that students can recall their high school algorithms, college faculty often decry their students' lack of content knowledge. We must demand more of students and of the education system we are revamping. During the last standards debate of the late 1980s and early 1990s, the research community was mostly silent and unaware. The voices that were heard were often strident, because the only people who seemed to care, cared too much. We can no longer afford mathematics education turning into a political campaign—big on slogans and all about how the message is framed.

It is worth noting that previous reform movements have come from inside. The mathematics and the mathematics education community decided on the need for change and the need for new standards. The funding for the curriculum development, staff development, and implementation work that followed came primarily from the National Science Foundation (NSF). Not this time. This time the impetus for change has come from outside—from the federal and state governments. And the funding for the follow-up work will not come primarily from NSF, but from the U.S. Department of Education.

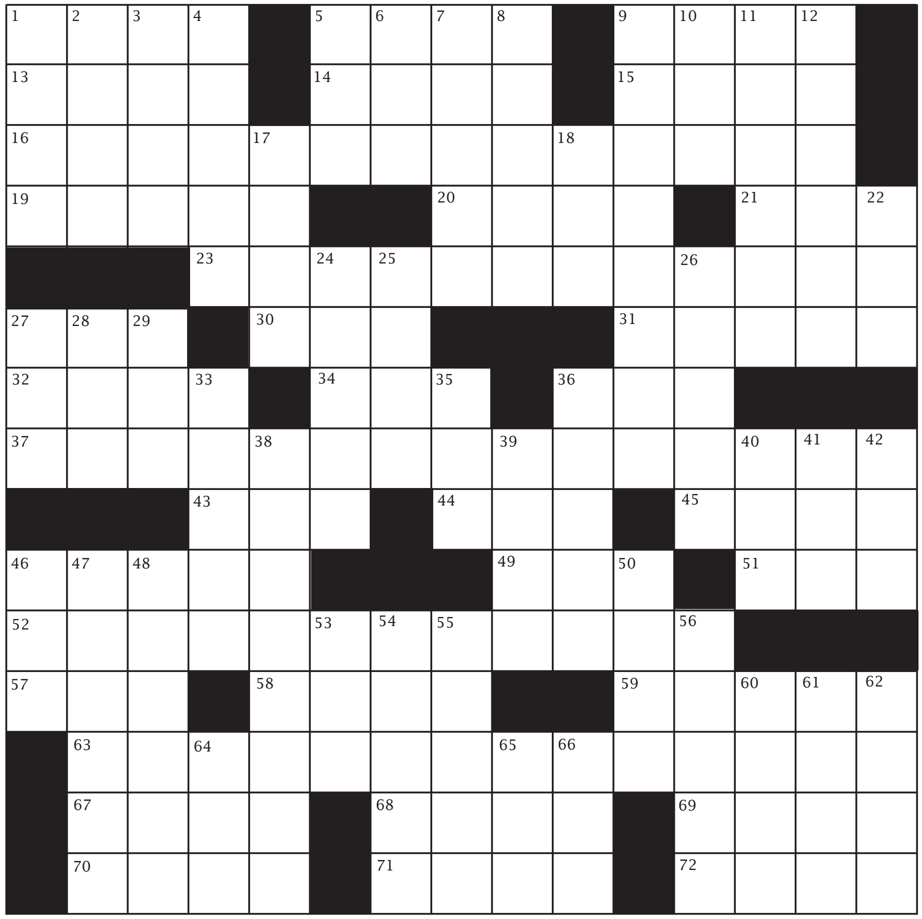
The point of this piece is to simply say, "be aware of what's going on in mathematics education on the larger stage." We are entering a new era of reform. If it is to be a successful one, then we all need to take part. If we don't, we risk having a small minority make decisions that will affect us all for years and years to come. We can't afford it.

CROSSWORD

inspiring dreams...

by Ned White

Answers: Page 869



ACROSS
1 Mennen lotion
5 Electric rockers?
9 Poet banished by Augustus
13 Cross in Cordoba
14 "How _______ you?!"
15 ABBA's "Pretty Ballerina"
16 After dreaming of a secret cell of militant extremists, Ramanujan was inspired to work on _______________.
17 With 60-Across, words spoken by 47-Across on June 23, 1993, that resulted in sustained applause
19 Lucy's landlady, on '50s TV
20 It's normally played with 80 balls
21 Ω, sometimes
23 After dreaming of his advances being rebuffed by a buxom baroness, Euler was inspired to explore ___________ to illustrate syllogistic reasoning.
27 T, with a key
30 Medium's claim
31 Thatcher's supply
32 Salon, e.g.
34 Goth foe
36 Recipe amt.
37 After dreaming of having lucid conversations with bundles of wheat, Jean-Pierre Serre was inspired to pursue the cohomology of ________________.
43 Org. once chaired by actress Jane Alexander
44 Plum or peach
45 "Holy cow!"
46 Whistling _______
49 Foundation supporter
51 Campaign supporter, briefly
52 After dreaming of an endless Federer-Nadal match, Georg Cantor was inspired to seek a precise definition of _________________
57 Mai-______
58 Shortly
59 Hoarse
63 After dreaming of being at the Super Bowl, Archimedes was inspired to invent the _______________ pulley.
67 Superlative suffix
68 When bats awake
69 It holds the line
70 General ______ chicken
71 Storage place
72 Same on the Seine

DOWN
1 Adolescent outburst
2 It's raised on a neck
3 Backside
4 Tlaloc worshiper
5 Put on
6 Saturn, but not Uranus
7 Daffy Duck, for one
8 Gave up
9 Going as planned
10 Through
11 Moonstruck
12 Like some hopes
17 Role for Reese in "Legally Blonde"
18 Like Sibelius' Symphony No. 7
22 Stack for eds.
24 Actor Milo of "The Verdict"
25 Like cotton candy
26 Make good on
27 Mo. for awarding of Nobel Prizes
28 Catullus' "Odi et _____"
29 "Gotcha!"
33 Some bottled spirits
35 Greatest possible
36 When repeated, words of solace
38 Stages, as a historic event
39 Heros
40 Type of treatment
41 UV index watcher
42 "One _____." (shortly)
46 27-Across counterpart
47 Shortly
48 TV series that produced 117 episodes in Vancouver, B.C., after "The"
50 1977 product launch with a pivoting head
53 Press coverage, slangily
54 Odious types
55 The blahs
56 Orléans oath opener
60 Surfboard fin
61 Answer to a judge
62 Cheerleaders' bit
64 Bear, in Baja
65 Internet access inits.
66 Ring finale, in brief

The State of High School Textbooks

Guershon Harel and W. Stephen Wilson

Do textbooks matter? According to Education Market Research [1], in 2001–2002, K–12 school districts spent more than $4 billion on textbooks, and in mathematics an estimated $1.95 billion was spent nationwide in 2008 on mathematics instructional materials—a robust 25.8 percent increase from 2005. Tyson-Bernstein & Woodward [3] found that, while textbooks are a dominant part of teaching and learning in all subjects, in mathematics the reliance on textbooks is even greater. Textbooks are the primary source for planning daily mathematics instruction by teachers, according to Weiss et al. [4].

Given the high expenditure on mathematics textbooks and the central role they play in mathematics instruction, evaluating their quality and effectiveness would seem necessary. Recently, we had the opportunity to review the high school textbook series from four different publishers [2], [5]. We found the experience an enlightening but depressing one. We found an interesting combination of mathematical errors, obliviousness to basic foundational work, and a deliberate avoidance of symbolic manipulation in algebra and of a coherent postulate-based approach to geometry.

Our examination focused on two topics in algebra, forms of linear functions and equations and forms of quadratic functions and equations, and one topic in geometry, parallel lines and the triangle sum theorem. These topics were chosen because they are viewed as central to the high school curriculum. The examination was to ensure that they are coherently developed, completely covered, and mathematically correct, and provide solid foundation for further study in mathematics.[1]

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Linear Functions

The algebraic concepts and skills associated with linear functions are crucial for the rest of the study of algebra and beyond. Appropriate definitions and justifications for concepts such as slope provide the basis for understanding linear functions and equations. These issues were carefully examined, as were the presence of all forms of linear functions and equations, how these are connected to each other, and the opportunities given to apply them to solve problems.

Understanding linear functions is fundamental to a good Algebra 1 course. The connection between the graph of a linear function and the algebraic version is important. We were disappointed. No program produced the basics here. Slope, although defined, is never shown to be well defined. It is never shown that the graph of an algebraic linear function really is a line in the coordinate plane, and it is never shown that a line in the coordinate plane really is the graph of an algebraic linear function. The worst aspect of this was that it seemed the textbook authors were unaware that something was missing. There are no

[1] *We are, of course, not the first to notice mathematical problems with high school texts. For example, in the National Mathematics Advisory Panel final report in 2008,* Foundations for Success, *in Appendix B of the Conceptual Knowledge and Skills Task Group Report, Chapter 3, pages 63–65, there is a very nice summary of problems in high school algebra texts.*

comments such as "it can be shown." They don't always even state these things clearly. A typical sequence is:

"A function whose graph forms a straight line is called a linear function."

"A function is linear if it is described by a linear equation. A linear equation is any equation that can be written in the standard form shown below:

$$Ax + By = C$$

where A, B, and C are real numbers and A and B are not both 0."

Ignoring the little detail that if $B = 0$ we don't have a function, there is the rather large leap of faith: straight line graphs come from linear equations. There is no attempt to show this. For that matter, there seems to be no awareness that it should be shown. The worst part is that this is as good as it gets, because they actually tell you that functions that give lines come from linear equations.

From another book that is less sure of itself, we have: "A **linear equation** is an equation that forms a line when it is graphed. Linear equations are *often* written in the form $Ax + By = C$." (Our emphasis.) Again, no attempt to show that the equation gives rise to a line or to explain what equations give lines "not so often". But, it gets worse when they try to be more specific:

"The **standard form of a linear equation** is $Ax+By = C$, where $A \geq 0$, A and B are not both zero, and A, B, and C are integers with greatest common factor of 1."

Gone are real numbers. However, just when you think things can't get worse, open up another book and, after using a calculator to recursively plot points, you get:

"The points you plotted in the example showed a **linear relationship** between floor numbers and their heights."

This is it for the definition of linear relationship for this program.

For our final example, we have "**linear functions**—those with straight-line graphs, data patterns showing a constant rate of change in the dependent variable, and rules like $y = a + bx$." They explain why they use $y = a + bx$ instead of $y = mx + b$: "Statisticians prefer the general form $y = a + bx$." Mathematicians use $y = mx + b$. This is made explicit in the text and that is the explanation for the choice.

At best the relationship between a linear equation and the graph of a linear function as a line is assumed. At worst, it is murky. A real mathematical connection is never made, though.

Slope

In general, the slope of a line in the coordinate plane is defined as the change in y divided by the change in x, or the rise divided by the run. The problem, of course, is that the definition only involves two points on the line, and you really want to get the same answer (slope) if you use two different points. This is shown to be the case using similar triangles. However, texts never seem to see the need to show that slope is well defined, so we never see a proof. Obviously this must get pointed out to some authors, but they don't always know what to make of it. So an alternative definition of slope is that it is the rise divided by the run for "any two points on the line". They didn't quite get the message, though. Now you have to show that this is the same number for all pairs of points, just like before.

Linear equations and functions can be written in many forms, for example $Ax+By = C$, $y = mx+b$, $y - y_0 = m(x - x_0)$, and $y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x - x_1)$.

The letter "m" in $y = mx + b$ is assumed to be the slope, but this connection is rarely made mathematically.

Not all texts introduce all of these forms, and even when they do, they don't always do the general algebraic version. So a text might show how to find the equation for a line with slope 3 that goes through the point (2,1) but won't show how to find the equation for a line with slope "m" that goes through the point (x_0, y_0) or show how to rewrite $y - y_0 = m(x - x_0)$ or $Ax + By = C$, $B \neq 0$, in the form $y = mx + b$.

There seems to be a general tendency to avoid algebraic manipulation if at all possible. The closest any book comes to proving that slope is well defined is "You've probably noticed by now that the rate of change of a linear function is constant."

Quadratic Functions

The ability to put quadratic functions in vertex form allows access to symmetry and finding the maximum or the minimum of the function. This opens up a new world of problems that can be solved, namely max/min problems. The approach to max/min problems is examined for both the basic algebra and the conceptual development, which includes a coherent definition of a quadratic function and how the line of symmetry is explained and justified.

Core to Algebra 2 is a complete understanding of quadratic functions. Problems similar to those found for linear functions also occur here. The texts tend to love the symmetry of quadratic functions, and we'll now take a look at how they deal with it.

Our first book actually shows the symmetry for $y = x^2$. It then immediately states that "This shows that parabolas are symmetric curves. The **axis of symmetry** is the line through the vertex of a parabola that divides the parabola into two congruent halves." Of course this makes you want to back up a few pages and find out what a parabola is. We are told that the graph of $y = x^2$ is a curve called a parabola.

Next they show how to do vertical and horizontal translations of the function $y = x^2$ to get other quadratic functions. They could show that these new functions $y = (x - h)^2 + k$ are also symmetric, since they are just translations, but this is not mentioned. Next, functions are multiplied by a constant to reflect, stretch, and/or compress them. We now have the vertex form of a quadratic function: $y = a(x - h)^2 + k$, which this book uses for its definition of a quadratic. It is now described as a parabola, or, more accurately, the fact that the graph is a parabola is taken for granted in the following way that also gives us the **vertex** that we also wanted to look up: "If a parabola opens upward, it has a lowest point. If a parabola opens downward, it has a highest point. This lowest or highest point is the **vertex of a parabola**." Up to this point, we didn't even know these were parabolas. Neither symmetry nor the name was carried along through the transformations.

This particular text is as good as it gets. Next it mentions the **standard form** of a quadratic function as $f(x) = ax^2 + bx + c$ and then expands the vertex form and solves for b and c in terms of h and k, as well as h in terms of b and c. Consequently, they have computed the x-coordinate for the line of symmetry as $\frac{-b}{2a}$, assuming there was a line of symmetry. A good, knowledgeable teacher might help make sense of all of this.

Our second example takes for the definition of a **quadratic function**:

$$f(x) = ax^2 + bx + c, \text{ where } a \neq 0.$$

And we are told "The graph of a quadratic function is called a **parabola**." We are then shown a graph of one (1) example of a quadratic, $f(x) = 3x^2 - 12x + 6$, and the book concludes: "The **axis of symmetry** is a line through the graph of a parabola that divides the graph into two congruent halves." And we are given the equation for the axis of symmetry: $x = \frac{-b}{2a}$. There is no computation or justification either for the existence of the axis of symmetry or the value of x that gives it. It does have the advantage that it is clean, quick, and simple.

For our next example we again start with $y = x^2$ and look at vertical and horizontal translations. We are told that the shape of this graph is a **parabola** and that parabolas always have a **line of symmetry**. The proof that $y = x^2$ is symmetric is just $(-x)^2 = x^2$, but this is not done. Reflections and stretches (as transformations) are done in general, but not specifically for quadratic functions. A hundred and fifty pages later **quadratic functions** are defined as second-degree polynomial functions, and it is asserted that all quadratic functions come from transformations of $y = x^2$. In fact, they have shown no such thing. They have shown that quadratics in the vertex form come from such transformations, but they have not shown that all second-degree polynomials can be written in vertex form. In addition, they now call all quadratic functions parabolas. Symmetry is not mentioned as these transformations occur. Symmetry could be carried along, but they do not bother.

In our final example, "quadratic function" is used for several pages without definition. Eventually the general form is coughed up. We are told that the graphs of all quadratics are called parabolas. It is left to an exercise to show that $y = ax^2$ is symmetric, and this is generalized to $y = ax^2 + c$. The next step is harder. The authors just assume symmetry for $y = ax^2 + bx$. This makes it easy to compute the line of symmetry and to get the line of symmetry for the general form from that.

It isn't hard to show that $ax^2 + bx$ has a symmetric graph, but it isn't done. Is it skipped intentionally, or do the authors just not realize what they are doing?

The second author's early 1960s (small-town Kansas) Algebra 2 textbook shows that $x = \frac{-b}{2a}$ must give a maximum or a minimum for $y = ax^2 + bx + c$ by plugging in both $x = \frac{-b}{2a}$ and $x = \frac{-b}{2a} \pm k$ and showing that the difference between the two y values is ak^2. This also shows symmetry. None of the textbooks we looked at would dare venture into this level of algebraic manipulation.

To solve max/min problems for quadratic functions it is essential to be able to move back and forth between the standard form, $y = ax^2 + bx + c$, and the vertex form, $y = a(x - h)^2 + k$. Once this is done, symmetry is quite easy to see, but only if it is pointed out. It is not pointed out. As with linear functions, this is much more likely to be done with numbers, if at all, rather than for the general algebraic version.

Triangle Sum Theorem

The development and application of the triangle sum theorem (that the sum of the angles of a triangle is 180 degrees) was examined. Despite the simplicity of both the theorem and the proof, they depend on a great many preliminary results, postulates, and definitions. It is an excellent way to see if the material is structured properly. For example, the theorem depends on a solid understanding of parallel lines, the lines that cross them, and the angles associated with them all. The examination focused on the coherence and logical progression of the material leading up to the theorem.

Euclidean geometry is perhaps the only place in high school mathematics where a (relatively) complete and rigorous mathematical structure can be taught. However, deductive geometry can be treated in numerous ways and in different levels of rigor. Our examination was based on the view that an adequate level of rigor is necessary and possible in high school. Deciding what constitutes an "adequate level of rigor" is crucial. In dealing with this question, we used Euclid's *Elements* as a framework. In a program consistent with this framework, subtle concepts and axioms, such as those related to "betweenness" and "separation" are dealt with intuitively, but the progression from definitions and axioms to theorems and from one theorem to the next is coherent and logical and exhibits a clear mathematical structure.

Two of the geometry books we reviewed do not have a clear logical structure for the material taught, and there is no clear development of a demarcation line between empirical reasoning and deductive reasoning. In one of these books, all the material is presented through problems. There is nothing wrong with this approach, except that all the problems seem to be of equal "status". However, although some problems are essential to the development of a geometric structure, others are not. For a teacher to discern the essential mathematical progression, he or she must identify all the critical problems—many of which appear in the homework sections—and know in advance what the intended structure is. Missing one or two of these problems would result in an incomplete or deficient structure.

In this book, important theorems in geometry are not justified. Moreover, with the way the material is sequenced, some of these theorems cannot be justified. Specifically, the construction of perpendicular lines requires congruence of triangles, which appears in earlier courses but mostly in empirical, not deductive, forms. Congruence appears later, after parallel lines, and is based on similarity, which, in turn, is based on parallel lines. Thus the construction that is fundamentally needed for parallel lines can only be justified by results that are based on parallel lines! In other words, the argument is circular.

Also, due to the program's choice of starting with parallel lines rather than congruence, there is loss of an opportunity to convey a critical mathematical lesson about the role of postulates in the development of mathematical structures—that a whole constellation of theorems can be proved without the use of the parallel postulates. This lesson—a landmark in the historical development of mathematics—should be within the grasp of high school students.

In another book the approach amounts to empirical observations of geometric facts; it has little or nothing to do with deductive geometry. There is definitely a need for intuitive treatment of geometry in any textbook, especially one intended for high school students. But the experiential geometry presented in the first nearly 800 pages of the book is not utilized to develop geometry as a deductive system. Most, if not all, assertions appear in the form of conjectures, and most of the conjectures are not proved. It is difficult, if not impossible, to systematically differentiate which of the conjectures are postulates and which are theorems. It is difficult to learn from this text what a mathematical definition is or to distinguish between a necessary condition and a sufficient condition. Students are also expected to discover definitions given pictures as hints.

The development that leads up to the proof of the triangle sum theorem in the other two books does not include circular reasoning. However, there is repeated misuse of the concept of postulate, and some important theorems are stated without proof. In addition, this development is interrupted by two sections on analytic geometry, with theorems that are either incorrectly labeled as postulates or appear without proof. In the process of developing a deductive structure for synthetic geometry, the text introduced a "foreign object", analytic geometry, which does not belong to the development of this structure.

Conclusion

The texts discussed are not unusual. Middle and elementary school textbooks are no better, perhaps even worse. How can mathematics be viewed as logical when foundational work is missing, and, worse, it is not even pointed out as something that could or should be done?

The purpose is to inform the college mathematics teaching community about the sorry state of high school textbooks. There is much to complain about in college textbooks as well, but at least they are usually written by mathematicians with some sense of mathematical integrity.

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What Is New in LaTeX? IV. WYSIWYG LaTeX

G. Grätzer

Leslie Lamport Is Alive and Well and Lives on the West Coast

But were he not, he would turn in his grave, because, as he wrote in [5]:

"The function of typographic design is to help the reader understand the author's ideas. For a document to be easy to read, its visual structure must reflect its logical structure. Quotations and computer programs, being logically distinct structural elements, should be distinguished visually from one another…. Since LaTeX can't understand your prose, you must explicitly indicate the logical structure by typing special commands…. As you are writing your document, you should be concerned with its logical structure, not its visual appearance."

So LaTeX and WYSIWYG are the polar opposites. How can I write about WYSIWYG LaTeX?

WYSIWYG vs. LaTeX

When you work with a word processor, you see your document on the computer display as it will look printed with its various fonts, font sizes, font shapes and weights, and so on. WYSIWYG: What You See Is What You Get. The following example, typed in Word (in Times font), has bold and italics:

> It is **not** unusual for a continuous function to exhibit *pathological* properties.

This is how it looks on the monitor; this is how it looks when printed.

To get the *same text* (in CM font) in LaTeX, type:

```
It is \textbf{not} unusual for a
continuous function to exhibit
\textit{pathological} properties.
```

`\textbf` and `\textit` are LaTeX *markup commands*. In the document you type (called the *source file*), all characters appear in the same font. To indicate how you want the typeset text to be shaped, add markup commands.

In order to typeset math, you need *math markup commands*. As a simple example, take the formula $\frac{a^2+b}{c}$. To mark up this formula in LaTeX, type

```
$\frac{a^{2}+b}{c}$
```

LaTeX Front Ends

LaTeX is really superior in typesetting high-quality mathematics; to help produce the necessary source file, many LaTeX front ends have been developed. They show you a table of Greek letters, you click on ε, and the code for it, `\varepsilon`, is written in the source file. Symbol tables and command completion ease the burden of remembering the hundreds of LaTeX commands. Today most mathematicians work in front ends, for instance, *WinEdt* for Windows and *TeXShop* for the Mac.

Already in the 1980s there was *Leo* (from ABK Software in Boulder, CO); it was capable of presenting `\alpha` in the source code as α on the monitor. It used the DOS operating system; even today, there are Windows users who still use *Leo* in a DOS compatibility box in Windows XP.

In 1992 MacKichan Software, Inc., announced *Scientific Word*, the first visual front end for LaTeX; see my review [1], written in 1993. *Scientific Word* is not WYSIWYG but WYSIWYM: What You See Is What You Mean. Sections, theorems, symbols are properly visually represented, but this representation is not identical to the LaTeX output. I hope to review

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Scientific Word again when Version 6 is unveiled later this year.

LyX, an Open Source WYSIWYM Front End

At the beginning there was *Lyrics* (using `.lyx` documents), released in mid-1995 by Matthias Ettrich, soon renamed *LyX*. Version 1.0 was released in 1999. A major update, Version 2.0, has just been released. The fifteen-year development witnessed huge changes.

Getting Started with LyX

Google "LyX"; the LyX home page is the top hit. Go to the home page, click on download, and follow the on-screen instructions for Windows, Mac, or Linux; it is easy. Then start LyX, choose Help, and select Tutorial. The LyX team put together a short and instructive tutorial; you learn the basic features by fixing up—*lyxifying*—a sample file `example_raw.lyx`.

There is only one problem: `example_raw.lyx` is very difficult to find. (Google `example_raw.lyx`, and you see how many people have had the same problem.) On a Windows computer, try searching for it and get no hit. Find it in
`Program Files\LyX2.0\Resources\examples`
On the Mac, Control Click on the LyX application, and choose
`Show Package Contents`
Open the `Contents` folder, then the `Resources` folder, finally the `examples` folder, and you find the file.

Work through the tutorial and then graduate to the third item on the Help menu: the *LyX User's Guide*. Many open source projects find excellent programmers. Few find talented people who can provide good documentation. LyX is one of them. This guide is well written and clear.

The guide is continued in the fourth item on the Help menu, *Additional LyX Features*, in the fifth item on the Help menu, *Embedded Objects* (the title is not very user friendly, but the content is down to earth, explaining figures, tables, etc.), and in the sixth item, *Math*, math is explained; especially useful is Chapter 18, describing the use of the AMS multiline formulas. There are nine more manuals listed in Help.

A Taste of LyX

In a short article such as this, I can give but the briefest introduction to a complex application such as LyX. But I can give you some examples of how it works. So open LyX, and in the `File` menu choose `New` for a new document. In the `View` menu, choose `Toolbars` and select `View/Update`, `Extra`, `Math`, and `Math Panels`. You then have two toolbars at the top of the window, two math toolbars at the bottom. Hovering over the icons of the four toolbars, little floating text boxes appear explaining what they do. For good measure, under `Toolbars`, choose `Other Toolbars` and select `Greek`.

Since you want to do math, you will want to use the AMS document class `amsart`. In the `Document` menu, choose `Settings...`. A new window comes up. Here the `Document class` is a drop-down menu; choose `article (AMS)`. Close the window. Save the document.

Type the title:
`A taste of LyX`
as you would type it in any word processor. With the cursor in the line, go to the upper left of the window, where you find the "environment drop-down menu"; click on it, and select `Title`. See how `A taste of LyX` is now displayed as appropriate for the title of an AMS article.

In the next line, type `by Very Beginner`, choose the `Author` environment, and watch the transformation.

Now type
`My first sentence.`
To emphasize "first": select it, click the toggle emphasis icon (with the exclamation mark), and you see *first*. To make "sentence" bold, select it, and use the usual keyboard command for bold.

Now start doing math. Type:
`The Greek letter delta:`
There are three ways to get $delta$:

(1) Click the `Insert math mode` icon (it looks like this, $\frac{a+b}{c}$; it is in the first toolbar on the top, sixth icon from the right). In the article, you get a little blue box surrounded by four red corners, indicating math mode. Now type `\delta ` (with a space following) and the character δ appears surrounded by the red corners of the math mode.
(2) Click the `Greek` icon (it looks like this: α; it is in the second toolbar on the bottom, ninth icon from the left) and a table pops up with all the Greek letters. Click on δ in the table, and in the document you get δ surrounded by the red corners of the math mode.
(3) In the Greek toolbar, click on δ.

Now type
`A fraction with an exponent:`
to learn how to get the formula $\frac{a^2+b}{c}$:

(1) Click on the icon, `Insert standard fraction` (it looks like this: $\frac{a}{b}$) and you get in math mode two blue boxes, one above and one below a horizontal line.
(2) In the box above, you want $a^2 + b$; click on the `Superscript` icon and type `a` and `2` in the appropriate boxes, escape the superscript mode by pressing the space key, and type `+ b`.
(3) Type `c` in the box below the line. By magic, $\frac{a^2+b}{c}$ appears in the document.

All this is so visual and intuitive that it becomes second nature in a very short time. Alternatively, in math mode, you can type the LaTeX code:
`\frac` space `a^2` space `+ b` down-arrow `c` space space space
to get the fraction.

But the proof of the pudding is in multiline math environments; let me illustrate this with an align environment:

$$x^2 + 3 = y,$$
$$u = \frac{a}{b}.$$

To place this in your LyX document:

(1) In the `Insert` menu, choose `Math`, and then `AMS align Environment`.
(2) You get two boxes, the two columns to be aligned. In the first, place $x^2 + 3$ as before, in the second, `=y,`.
(3) Press Command-Return to start the next line; you get two boxes again—type `u` in the first and $= \frac{a}{b}$. in the second. You are done.

If You Done It, It Ain't Bragging

Baseball great "Dizzy" Dean would agree: LyX is not bragging about WYSIWYM, it does indeed deliver. Look at this very recent real-life example, the LyX version of Theorem 1 of [4]:

Theorem 1. *In a semimodular lattice, let*

$$C\colon 0 = c_0 \prec c_1 \prec \cdots \prec c_n = 1,$$
$$D\colon 0 = d_0 \prec d_1 \prec \cdots \prec d_n = 1.$$

Then there is a permutation π of the set $\{1, \dots, n\}$ such that $[c_{i-1}, c_i]$ is projective to $[d_{\pi(i)-1}, d_{\pi(i)}]$, for all i.

Compare it with the PDF of the typeset TeX file:

Theorem 1. *In a semimodular lattice, let*

$$C\colon 0 = c_0 \prec c_1 \prec \cdots \prec c_n = 1,$$
$$D\colon 0 = d_0 \prec d_1 \prec \cdots \prec d_n = 1.$$

Then there is a permutation π of the set $\{1, \dots, n\}$ such that $[c_{i-1}, c_i]$ is projective to $[d_{\pi(i)-1}, d_{\pi(i)}]$, for all i.

The LyX file informs you that this is Theorem 1 and properly displays all the math.

And So Much More…

Go to the LyX site for a listing of the main features and, in the news section, the thirty main new features of Version 2.0. Just to whet your appetite, this is what Version 1.6 already offers: full support for math, including AMS math, table editor, references, index and bibliography (including BibTeX), spellchecking, thesaurus, and revision tracking. It provides support for writing documents in many languages (Unicode), including languages such as Arabic, Chinese, Hebrew, and Japanese. It supports version control, XeTeX and LuaTeX (see [2]).

Version 2.0 adds advanced search, diff (compare documents), reference styles, multiple indices, better outliner, enhanced table features, colors, search for DVI/PDF files, experimental XHTML+MathML export, and so many others.

Where We Agree and Where We Part Ways

I agree with the LyX community—those who write the compelling code, those who write the excellent documentation, those who use it every day in their work—that LyX is a first-rate product. Whether it is for you, only you can decide, but you owe it to yourself to give LyX a try.

I view LyX as a LaTeX front end (although it is clearly more, it is also a front end for DocBook and—soon—for XHTML+MathML), not as the LyX folks promote it: "a document processor that encourages an approach to writing based on the structure of your documents and not simply their appearance." I would encourage all LyX users (especially those who use mathematical formulas) to be thoroughly familiar with LaTeX, to use LyX to produce the document, but to export it to LaTeX for the final touch up. I realize that for many documents (for instance, exams) fine tuning is not necessary.

As examples of fine tuning, let me mention two types of errors in the final submission of manuscripts that the authors can very easily correct—but they are hard to do for the editors: lines that are too long and bad line breaks. An author can eliminate a too-long line by simply rephrasing the sentence or by adding an optional hyphen. A bad line break can also be easily corrected with a nonbreakable space. These are awkward to do in LyX—it would be very easy for the designers of LyX to add a few small features that would facilitate such corrections.

Acknowledgement

Special thanks to Uwe Stöhr, who helped me get started, and to Russ Woodroofe—he generously gave his time to save some of mine. I received corrections and additions to the article by Richard G. Heck Jr. and Jean-Marc Lasgouttes. Finally, Barbara Beeton helped as always.

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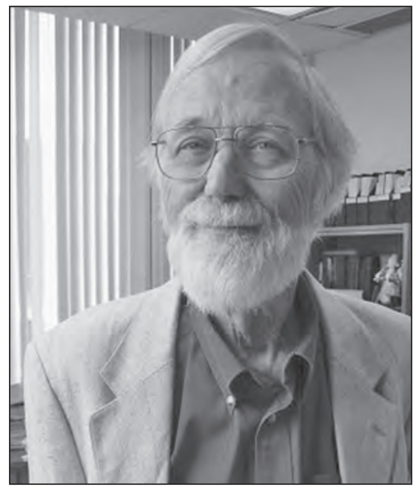
Milnor Receives 2011 Abel Prize

The Norwegian Academy of Science and Letters has awarded the Abel Prize for 2011 to John Willard Milnor, Stony Brook University, "for pioneering discoveries in topology, geometry and algebra." The Abel Prize recognizes contributions of extraordinary depth and influence to the mathematical sciences and has been awarded annually since 2003. It carries a cash award of 6,000,000 Norwegian kroner (approximately US$1 million). John Milnor received the Abel Prize from His Majesty King Harald at an award ceremony in Oslo, Norway, on May 24, 2011.

Biographical Sketch

John Milnor was born in Orange, New Jersey, in 1931. He spent his undergraduate and graduate student years at Princeton, receiving his Ph.D. in 1954. He has served on the faculties of Princeton University and the Institute for Advanced Study, as well as at the University of California Los Angeles and the Massachusetts Institute of Technology. He joined Stony Brook University and became its first director of the Institute for Mathematical Sciences in 1989; he is now codirector of the institute. His research interests have included such subjects as game theory, differential geometry, algebraic topology, differential topology, quadratic forms, and algebraic *K*-theory. For the past twenty-five years his main focus has been on dynamical systems, particularly low-dimensional holomorphic dynamical systems. Milnor has received many awards and honors. He received the Fields Medal in 1962 for his work in differential topology. This year he was awarded the 2011 Leroy P. Steele Prize for Lifetime Achievement by the AMS. Milnor had previously won two other Steele Prizes: for Mathematical Exposition (2004) and for Seminal Contribution to Research (1982). In 1989 he received the Wolf Prize in Mathematics. Milnor was elected as a member of the National Academy of Sciences in 1963 and was awarded the National Medal of Science in 1967. Since 1994 he has been a foreign member of the Russian Academy of Sciences, and in 2004 he became a member of the European Academy of Sciences, Arts and Letters.



John Milnor

Citation

All of Milnor's works display marks of great research: profound insights, vivid imagination, elements of surprise, and supreme beauty. Milnor's discovery of exotic smooth spheres in seven dimensions was completely unexpected. It signaled the arrival of differential topology and an explosion of work by a generation of brilliant mathematicians; this explosion has lasted for decades and changed the landscape of mathematics. With Michel Kervaire, Milnor went on to give a complete inventory of all the distinct differentiable structures on spheres of all dimensions; in particular they showed that the 7-dimensional sphere carries exactly twenty-eight distinct differentiable structures. They were among the first to identify the special nature of 4-dimensional manifolds, foreshadowing fundamental developments in topology. Milnor's disproof of the long-standing *Hauptvermutung* overturned expectations about combinatorial topology dating back to Poincaré. Milnor also discovered homeomorphic smooth manifolds with nonisomorphic tangent bundles, for which he developed the theory of microbundles. In 3-manifold theory, he proved an elegant unique factorization theorem. Outside topology, Milnor made significant contributions to differential geometry, algebra, and dynamical systems. In each area Milnor touched upon, his insights and approaches have had a profound impact on subsequent developments. His monograph on isolated hypersurface singularities is considered the single most influential work in singularity theory; it gave us the Milnor number and the Milnor fibration. Topologists started to actively use Hopf algebras and coalgebras after the definitive work by Milnor and J. C. Moore. Milnor himself came up with new insights into the structure of the Steenrod algebra (of cohomology operations) using the theory of Hopf algebras. In algebraic *K*-theory, Milnor introduced the degree 2 functor; his celebrated conjecture about the functor—eventually proved by Voevodsky—spurred new directions in the study of motives in algebraic geometry. Milnor's introduction of the growth invariant of a group linked combinatorial group theory to geometry, prefiguring Gromov's theory of hyperbolic groups. More recently, John Milnor turned his attention to dynamical systems in low dimensions. With Thurston, he pioneered "kneading theory" for interval maps, laying down the combinatorial foundations of interval dynamics, creating a focus of intense research for three decades. The Milnor-Thurston conjecture on entropy monotonicity prompted efforts to fully understand dynamics in the real quadratic family, bridging real and complex dynamics in a deep way and triggering exciting advances. Milnor is a wonderfully gifted expositor of sophisticated mathematics. He has often tackled difficult, cutting-edge subjects for which no account in book form existed. Adding novel insights, he produced a stream of timely yet lasting works of masterly lucidity. Like an inspired musical composer who is also a charismatic performer, John Milnor is both a discoverer and an expositor.

About the Prize

The Niels Henrik Abel Memorial Fund was established in 2002 to award the Abel Prize for outstanding scientific work in the field of mathematics. The prize is awarded by the Norwegian Academy of Science and Letters, and the choice of Abel Laureate is based on the recommendation of the Abel Committee, which consists of five internationally recognized mathematicians.

Previous recipients of the Abel Prize are: Jean-Pierre Serre (2003), Michael Atiyah and I. M. Singer (2004), Peter Lax (2005), Lennart Carleson (2006), S. R. S. Varadhan (2007), John G. Thompson and Jacques Tits (2008), Mikhail L. Gromov (2009), and John Tate (2010).

—From announcements of the Norwegian Academy of Science and Letters

Valiant Receives 2010 Turing Award

The Association for Computing Machinery (ACM) has announced that LESLIE G. VALIANT of Harvard University is the recipient of the 2010 A. M. Turing Award "for transformative contributions to the theory of computation, including the theory of probably approximately correct (PAC) learning, the complexity of enumeration and of algebraic computation, and the theory of parallel and distributed computing." The award carries a cash prize of US$250,000 and will be presented to Valiant at the annual ACM Awards Banquet on June 4, 2011, in San Jose, California.

Citation

Over the past thirty years, Leslie Valiant has made fundamental contributions to many aspects of theoretical computer science. His work has opened new frontiers, introduced ingenious new concepts, and presented results of great originality, depth, and beauty. Time and again, Valiant's work has literally defined or transformed the computer science research landscape.

Valiant's greatest single contribution may be his 1984 paper "A Theory of the Learnable", which laid the foundations of computational learning theory. He introduced a general framework as well as concrete computational models for studying the learning process, including the famous "probably approximately correct" (PAC) model of machine learning. This has developed into a vibrant research area and has had enormous influence on machine learning, artificial intelligence, and many areas of computing practice, such as natural language processing, handwriting recognition, and computer vision.

Valiant has made many seminal contributions to computational complexity. He introduced the notion of complexity of enumeration, in terms of the complexity class #P. The most surprising consequence of this study was that natural enumeration problems can be intractable even when the corresponding decision problem is tractable. Another fundamental contribution to computational complexity was Valiant's theory of algebraic computation, in which he established a framework for understanding which algebraic formulas can be evaluated efficiently. In analogy with the Boolean complexity classes P and NP, his theory characterizes the difficulty of computing fundamental functions in linear algebra, namely the determinant and permanent. Together with his work on #P, this set the stage for some of the most exciting subsequent developments in computational complexity, such as the development of interactive proofs for problems beyond NP.

Leslie G. Valiant

A third broad area in which Valiant has made important contributions is the theory of parallel and distributed computing. His design of randomized routing strategies laid the groundwork for a rich body of research that exposed how randomization can be used to offset congestion effects in communication networks. He proposed the bulk synchronous model of parallel computation. He also posed a number of influential challenges leading to the construction of parallel algorithms for seemingly inherently sequential problems. Finally, the superconcentrators constructed by Valiant in the context of computational complexity established the fundamental role of expander graphs in computation.

Biographical Sketch

Leslie Valiant received his Ph.D. in computer science from the University of Warwick in 1974.

He taught at Carnegie Mellon University, Leeds University, and the University of Edinburgh and is currently the T. Jefferson Coolidge Professor of Computer Science and Applied Mathematics at Harvard University's School of Engineering and Applied Sciences (SEAS). He is the recipient of the 1986 Nevanlinna Prize from the International Mathematical Union, the 1997 Knuth Prize from the ACM Special Interest Group on Algorithms and Computation Theory (SIGACT) and the IEEE Technical Committee on the Mathematical Foundations of Computing, and the 2008 European Association for Theoretical Computer Science (EATCS) Award. He is a Fellow of the Royal Society (London), a Fellow of the American Association for Artificial Intelligence, and a member of the National Academy of Sciences of the USA. His research interests include complexity theory, computational learning, and parallel computation, as well as computational neuroscience, evolution, and artificial intelligence.

About the Award

The A. M. Turing Award was named for Alan M. Turing, the British mathematician who articulated the mathematical foundation and limits of computing and who was a key contributor to the Allied cryptanalysis of the German Enigma cipher during World War II. Since its inception in 1966 the Turing Award has honored the computer scientists and engineers who created the systems and underlying theoretical foundations that have propelled the information technology industry.

The award is given to an individual selected for contributions of a technical nature made to the computing community. The contributions should be of lasting and major technical importance to the computer field. Financial support for the Turing Award is provided by the Intel Corporation and Google Inc.

—ACM announcement

Mathematics People

2011–2012 AMS Centennial Fellowship Awarded

The AMS has awarded its Centennial Fellowship for 2011–2012 to Andrew S. Toms of Purdue University. The fellowship carries a stipend of US$79,000, an expense allowance of US$7,900, and a complimentary Society membership for one year.

Andrew Toms was born in Montreal in 1975. He received his Ph.D. from the University of Toronto in 2002. After holding faculty positions at the University of New Brunswick and York University, he was appointed associate professor in the Department of Mathematics at Purdue University in 2010. Recently he was the recipient of two Canadian awards: the Canadian Mathematical Society's 2010 G. de B. Robinson Award, given by the Canadian Mathematical Society, and the Israel Halperin Prize for outstanding work in operator algebras or operator theory by members of the Canadian mathematical community. Toms's mathematical interests include the classification of C*-algebras and points of contact between operator algebras, logic, and topology.

Photo courtesy of the Purdue University Department of Mathematics.

Andrew S. Toms

—Elaine Kehoe

Aschbacher and Putnam Awarded Rolf Schock Prizes

Michael Aschbacher of the California Institute of Technology and Hilary Putnam of Harvard University have been awarded Rolf Schock Prizes for 2011.

Aschbacher was awarded the 2011 Rolf Schock Prize in Mathematics "for his fundamental contributions to one of the largest mathematical projects ever, the classification of finite simple groups, notably his contribution to the quasi-thin case." He has made fundamental contributions to group theory, especially regarding the classification of finite simple groups. He received the AMS Cole Prize in 1980 and was elected a member of the U.S. National Academy of Sciences in 1990.

Putnam was awarded the 2011 Rolf Schock Prize in Logic and Philosophy "for his contribution to the understanding of semantics for theoretical and 'natural kind' terms and of the implications of this semantics for philosophy of language, theory of knowledge, philosophy of science and metaphysics." Putnam is best known among mathematicians for work that, together with work by Martin Davis, Julia Robinson, and Yuri Matiasevich, provided a solution to Hilbert's tenth problem.

The Rolf Schock Prizes are awarded every three years in the fields of logic and philosophy, mathematics, the visual arts, and the musical arts. The prize amount is US$75,000. They are awarded by the Royal Swedish Academy of Sciences, the Royal Swedish Academy of Fine Arts, and the Royal Swedish Academy of Music.

—The Rolf Schock Foundation

Clay Research Awards

The Clay Mathematics Institute has awarded its 2011 Research Awards to Yves Benoist, CNRS, Université de Paris Sud 11, and Jean-François Quint, CNRS, Université de Paris 13, for their work on stationary measures and orbit closures and to Jonathan Pila, Mathematical Institute, Oxford University, for his resolution of the André-Oort conjecture in the case of products of modular curves.

According to the citations, Benoist and Quint were honored "for their spectacular work on stationary measures and orbit closures for actions of nonabelian groups on homogeneous spaces. This work is a major breakthrough in homogeneous dynamics and related areas of mathematics. In particular, Benoist and Quint proved the following conjecture of Furstenberg: Let H be a Zariski dense semisimple subgroup of a Lie group which acts by left translations on the quotient of G by a discrete subgroup with finite covolume. Consider a probability measure m on H whose support generates H. Then any m-stationary probability measure for such an action is H-invariant." Pila was honored "for his resolution of the André-Oort conjecture in the case of products of modular curves. This work gives the first unconditional proof of fundamental cases of these general conjectures beyond the original theorem of André concerning the product of two such curves. The foundational techniques that Pila developed to achieve this breakthrough range from results in real analytic geometry which give sharp upper bounds for the number of rational points of bounded height on certain analytic sets to the use of O-minimal structures in mathematical logic."

—From a Clay Mathematics Institute announcement

Getz and Goresky Awarded 2011 Balaguer Prize

The Ferran Sunyer i Balaguer Foundation has awarded the Ferran Sunyer i Balaguer Prize for 2011 to JAYCE GETZ, McGill University, Montreal, and MARK GORESKY, School of Mathematics, Institute for Advanced Study, Princeton University, for their joint monograph *Hilbert Modular Forms with Coefficients in Intersection Homology and Quadratic Base Change*. According to the prize citation, the monograph explains "deep phenomena in number theory and algebraic geometry using geometric/topological methods, notably intersection homology. This builds on celebrated work by F. Hirzebruch and D. Zagier. It presents a pleasant equilibrium between the survey/monograph part and the research part: On the one hand it contains interesting results which appear here for the first time, but it also has several chapters which introduce the reader to the different subjects needed to understand the main results."

The Ferran Sunyer i Balaguer Foundation of the Institut d'Estudis Catalans (IEC) awards this international prize every year to honor the memory of Ferran Sunyer i Balaguer (1912–1967), a self-taught Catalan mathematician who gained international recognition for his research in mathematical analysis despite the serious physical disabilities with which he was born. The prize carries a cash award of 15,000 euros (approximately US$21,000); the winning monographs are published by Birkhäuser Verlag.

—From a Ferran Sunyer i Balaguer Foundation announcement

Sloan Fellowships Awarded

The Alfred P. Sloan Foundation has announced the names of the recipients of the 2011 Sloan Research Fellowships. Each year the foundation awards fellowships in the fields of mathematics, chemistry, computational and evolutionary molecular biology, computer science, economics, neuroscience, and physics. Grants of US$50,000 for a two-year period are administered by each fellow's institution. Once chosen, fellows are free to pursue whatever lines of inquiry that most interest them, and they are permitted to employ fellowship funds in a wide variety of ways to further their research aims.

Following are the names and institutions of the 2011 awardees in mathematics: SILAS D. ALBEN, Georgia Institute of Technology; MARIA CAMERON, University of Maryland, College Park; SABIN CAUTIS, Columbia University; CARINA P. CURTO, University of Nebraska, Lincoln; LAURENT DEMANET, Massachusetts Institute of Technology; VOLKER ELLING, University of Michigan; MIKHAIL ERSHOV, University of Virginia; TOBY S. GEE, Northwestern University; PHILIP T. GRESSMAN, University of Pennsylvania; MATTHEW HEDDEN, Michigan State University; MICHAEL A. HILL, University of Virginia; ROMAN HOLOWINSKY, Ohio State University; AARON D. LAUDA, Columbia University; XIAOQING LI, State University of New York at Buffalo; DRAGOS OPREA, University of California, San Diego; GRIGORIS PAOURIS, Texas A&M University; PER-OLOF PERSSON, University of California, Berkeley; JESSICA S. PURCELL, Brigham Young University; ROBERT M. STRAIN, University of Pennsylvania; IGNACIO URIARTE-TUERO, Michigan State University; DAPENG ZHAN, Michigan State University.

—From a Sloan Foundation announcement

Brenner Awarded Kovalevsky Lectureship

SUZANNE BRENNER of Louisiana State University has been chosen as the AWM-SIAM Sonia Kovalevsky Lecturer for 2011 by the Association for Women in Mathematics (AWM). She will deliver the AWM-SIAM Kovalevsky Lecture at the 2011 International Congress on Industrial and Applied Mathematics. Brenner was honored for significant research accomplishments in multigrid methods, domain decomposition methods, and finite element analysis.

—From an AWM announcement

Pelayo Receives Rubio de Francia Prize

ÁLVARO PELAYO of the Institute for Advanced Study and Washington University in St. Louis has been awarded the Rubio de Francia Prize for 2010 by the Royal Spanish Mathematical Society (RSME). The prize was awarded for contributions "at a very high level already at a very early stage of his career."

The prize honors the memory of J. L. Rubio de Francia (1949–1988), an internationally renowned Spanish analyst. It is awarded annually to a young mathematician from Spain, or residing in Spain, and it is the highest distinction given by the RSME.

The prize jury consisted of M. J. Carro, M. J. Esteban, M. L. Fernández, D. Nualart, J. M. Sanz, T. Tao, and E. Zelmanov.

—From a Royal Spanish Mathematical Society (RSME) announcement

Eisenbrand and Schröder Awarded Humboldt Professorships

FRIEDRICH EISENBRAND of École Polytechnique Fédérale de Lausanne and PETER SCHRÖDER of the California Institute of Technology have been awarded Alexander von Humboldt Professorships for 2011 by the Alexander von Humboldt Foundation.

Eisenbrand was awarded the professorship in mathematics. According to the prize citation, he "is a world leader in the field of algorithmics and discrete mathematics and works at the intersection between pure research and applications." He does research in integral optimization that "could be of use to both industry and telecommunications in capacity planning."

Peter Schröder was awarded the professorship in computer science. According to the prize citation, he "is regarded as one of the world's most eminent researchers in the field of computer graphics and the mathematics on which it is based. He thus acts as a bridge between numerical and geometry mathematics and computer science."

The Alexander von Humboldt Professorship honors researchers from outside of Germany who are internationally recognized leaders in their fields and allows them to spend five years conducting research at German universities. The award is valued at up to five million euros (approximately US$7,300,000) and is endowed by the Federal Ministry of Education and Research.

—From a Humboldt Foundation announcement

Glynn and Asmussen Awarded John von Neumann Theory Prize

The 2010 John von Neumann Theory Prize, the highest prize given in the field of operations research and management science, has been awarded to Peter Glynn of Stanford University and Søren Asmussen of Aarhus University "for their outstanding contributions in applied probability and the theory of stochastic simulation." According to the prize citation, Glynn "has made sustained and important contributions in stochastic simulation theory over the last thirty years." Asmussen "has made fundamental contributions in many areas of applied probability and stochastic operations research, including queueing systems, large deviations and rare events, heavy-tailed phenomena, insurance-risk models, matrix-analytic algorithms and the theory of stochastic simulation." The award, which is presented by the Institute for Operations Research and the Management Sciences (INFORMS) carries a cash prize of US$5,000.

—From an INFORMS announcement

Rollo Davidson Prizes Awarded

The Rollo Davidson Trust has awarded the 2011 Rollo Davidson Prize jointly to Christophe Garban of the École Normale Supérieure de Lyon and Gábor Pete of the University of Toronto "for striking and important new results for planar random processes, particularly in establishing a theory of noise sensitivity for critical percolation and the application of this theory to dynamical percolation."

The Rollo Davidson Trust was founded in 1975 and awards an annual prize to young mathematicians working in the field of probability.

—From a Rollo Davidson Trust announcement

Putnam Prizes Awarded

The winners of the seventy-first William Lowell Putnam Mathematical Competition have been announced. The Putnam Competition is administered by the Mathematical Association of America (MAA) and consists of an examination containing mathematical problems that are designed to test both originality and technical competence. Prizes are awarded to both individuals and teams.

The five highest ranking individuals, listed in alphabetical order, were Yu Deng, Massachusetts Institute of Technology; Brian R. Lawrence, California Institute of Technology; Seok Hyeong Lee, Stanford University; Colin P. Sandon, Massachusetts Institute of Technology; and Alex (Lin) Zhai, Harvard University. Each received a cash award of $2,500.

Institutions with at least three registered participants obtain a team ranking in the competition based on the rankings of three designated individual participants. The five top-ranked teams (with team members listed in alphabetical order) were: California Institute of Technology (Yakov Berchenko-Kogan, Jason C. Bland, Brian Lawrence); Massachusetts Institute of Technology (Sergei S. Bernstein, Whan Ghang, Jacob N. Steinhardt); Harvard University (Kevin Lee, Arnav Tripathy, Alex (Lin) Zhai); University of California, Berkeley (David D. Gee, Shiyu Li, Evan M. O'Dorney); and University of Waterloo (Steven N. Karp, Boyu Li, Malcolm A. Sharpe).

The first-place team receives an award of US$25,000, and each member of the team receives US$1,000. The awards for second place are US$20,000 and US$800; for third place, US$15,000 and US$600; for fourth place, US$10,000 and US$400; and for fifth place, US$5,000 and US$200.

The Elizabeth Lowell Putnam Prize, which goes to the outstanding woman in the competition, was awarded to Yinghui Wang of the Massachusetts Institute of Technology. She received a cash award of US$1,000.

—From a Putnam announcement

Intel Science Talent Search Winners Announced

Two students who work in the mathematical sciences have received scholarship awards in the 2011 Intel Science Talent Search. Evan O'Dorney, a seventeen-year-old student from Danville, California, was awarded the top prize of US$100,000 for his mathematics project "Continued fraction convergents and linear fractional transformations", in which he compared two ways to estimate the square root of an integer. Evan discovered precisely when the faster way would work. As a by-product of his research, he solved other equations useful for encrypting data. O'Dorney has been a member of the U.S. International Mathematical Olympiad Team and also won the National Spelling Bee in 2007 and the national Who Wants to Be a Mathematician competition in 2010 and 2011.

Keenan Monks, a seventeen-year-old student from Hazelton, Pennsylvania, was awarded sixth place and a prize of US$25,000 for his project "On supersingular elliptic curves and hypergeometric functions", detailing his research on a math equation that can help improve Internet security and cryptography.

—From an Intel Corporation announcement

NSF Graduate Research Fellowships Announced

The National Science Foundation (NSF) has awarded a number of Graduate Research Fellowships for fiscal year 2011. Further awards may be announced later in the year. This program supports students pursuing doctoral study in all areas of science and engineering and provides a stipend of US$30,000 per year for a maximum of three years of full-time graduate study. Following are the names of the awardees in the mathematical sciences selected so far in 2011, followed by their undergraduate institutions (in parentheses) and the institutions at which they plan to pursue graduate work.

Hannah Alpert (University of Chicago), Massachusetts Institute of Technology; Theresa Anderson (Brown University), Brown University; David Appelhans (University of Colorado at Boulder), University of Colorado at Boulder; Dena M. Asta (Carnegie-Mellon University), Carnegie-Mellon University; Kerstin Baer (Bryn Mawr College), Columbia University; Yakov I. Berchenko-Kogan (California Institute of Technology), Massachusetts Institute of Technology; Emily R. Berger (Massachusetts Institute of Technology), Massachusetts Institute of Technology; Sarah B. Brodsky (University of California Berkeley), University of California Berkeley; Charles D. Brummitt (University of California Davis), University of California Davis; Yaniel Cabrera (Texas A&M University), Texas A&M University; Carey Caginalp (University of Pittsburgh), Princeton University; Natasha A. Cayco Gajic (University of Washington), University of Washington; Otis A. Chodosh (Cambridge University), Stanford University; Zachary Clawson (North Carolina State University), Cornell University; Daniel Collins (Princeton University), Princeton University; Kathleen Curtius (University of Washington), University of Washington; Anil Damle (University of Colorado at Boulder), Yale University; James M. Davis (Cornell University), Cornell University; Michelle Delcourt (Georgia Institute of Technology), Rutgers University; Sarah A. Fletcher (Georgia Institute of Technology), Georgia Tech Research Corporation, Georgia Institute of Technology; Leilani Gilpin (University of California, San Diego), California Institute of Technology; Karsten Gimre (University of Oregon, Eugene), Stanford University; Sherry Gong (Harvard University), Princeton University; Alan Guo (Duke University), Massachusetts Institute of Technology; Melissa A. Gymrek (Massachusetts Institute of Technology), Harvard University; Daniel M. Harris, (Massachusetts Institute of Technology), Massachusetts Institute of Technology; Vivian O. Healey (Brown University), Brown University; Steven M. Heilman (New York University), New York University; Aukosh S. Jagannath (New York University), New York University; Carlee Joe-Wong (Princeton University), Stanford University; William A. Johnson (University of Washington), Massachusetts Institute of Technology; Tia Lee Lerud (University of Washington), Colorado State University; Kathleen Li (Rice University), University of California Berkeley; Katherine McLaughlin (University of California Berkeley), University of California Berkeley; Ekaterina Merkurjev (University of California Los Angeles), University of California Los Angeles; Ariana S. Minot (Duke University), University of California Berkeley; Alexander C. Moll (Columbia University), Massachusetts Institute of Technology; David W. Montague (University of Michigan Ann Arbor), Princeton University; Cris Negron (University of Washington), University of Washington; Kiva L. Oken (Carleton College), North Carolina State University; Vivek Pal (Florida State University), University of Michigan Ann Arbor; Aaron Palmer (University of California Santa Cruz), University of California Berkeley; John V. Pardon (Princeton University), University of California Berkeley; Helen F. Parks (University of California San Diego), University of California San Diego; Arthur J. Parzygnat (City University of New York Graduate School, University Center), City University of New York Graduate School, University Center; Oliver Pechenik (University of Illinois), University of Illinois at Urbana-Champaign; Alexander Perry (Columbia University), Massachusetts Institute of Technology; Scott Powers (University of North Carolina at Chapel Hill), Harvard University; Benjamin D. Preskill (University of California Berkeley), University of California Berkeley; Claudia C. Raithel (University of Michigan), New York University; Rebecca I. Rebhuhn-Glanz (Bryn Mawr College), University of Michigan Ann Arbor; Richard Z. Robinson (University of Washington), University of Washington; Rebecca S. Rothwell (University of North Carolina at Chapel Hill), University of North Carolina at Chapel Hill; Arman Sabbaghi (Harvard University), Harvard University; Geoffrey Schiebinger (Stanford University), Stanford University; Kimberly M. Short (University of Arizona), University of California Los Angeles; Robert A. Silversmith (Williams College), University of California Berkeley; Sean K. Simmons (University of Texas at Austin), University of Michigan Ann Arbor; Peter Smillie (Stanford University), Princeton University; Joel B. Specter (Wesleyan University), Columbia University; Melanie I. Stam (Georgia Institute of Technology), State University of New York at Stony Brook; Yi An Sun (University of Maryland College Park), Massachusetts Institute of Technology; Andrei Tarfulea (University of Chicago), New York University; Amelia N. Tebbe (University of Illinois at Urbana-Champaign), University of Illinois at Urbana-Champaign; Samantha M. Tracht (University of Tennessee Knoxville), University of Tennessee Knoxville; Arnav Tripathy (Harvard College), Princeton University; Bena M. Tshishiku (University of Chicago), University of Chicago; Dmitry Vaintrob (Harvard University), Massachusetts Institute of Technology; Martin R. Valdez-Vivas (Stanford University), Stanford University; Robert A. Van Gorder (University of Central Florida), Cornell University; Michael Viscardi (Harvard

University) Massachusetts Institute of Technology; ALEXANDRIA V. VOLKENING (University of Maryland Baltimore County), Johns Hopkins University; JONATHAN P. WANG (Harvard University), Massachusetts Institute of Technology; DANIEL K. WELLS (Northwestern University), Northwestern University; KATELYN R. WHITE (University of California Santa Cruz), University of California Santa Cruz; JOHN D. WILTSHIRE-GORDON (University of Chicago), University of Michigan Ann Arbor; SARAH WOLFF (Dartmouth College), Dartmouth College; CYNTHIA I. WOOD (Rice University), Rice University; VICTORIA Y. H. WOOD (University of California Berkeley), University of California Berkeley; JOSEPH WOODWORTH (University of Maryland), University of California Los Angeles.

—From an NSF announcement

Jungic Awarded 2011 PIMS Education Prize

VESELIN JUNGIC of Simon Fraser University has been awarded the 2011 PIMS Education Prize of the Pacific Institute for the Mathematical Sciences. The prize recognizes individuals who have played a major role in encouraging activities that have enhanced public awareness and appreciation of mathematics, as well as those who foster communication among various groups concerned with mathematical education at all levels.

Jungic has been involved in an enrichment program for high school students and the codevelopment of a distance-education version of introductory calculus in which lectures are available as video streams. He has been a leader in Simon Fraser University's Aboriginal university preparation programs and in creating mentorship programs for Aboriginal students at the Vancouver Friendship Center and at the Native Education College. He produced and coauthored an animated film in both English and Blackfoot as part of an initiative to develop curricular materials in an Aboriginal context.

—From a PIMS announcement

Guggenheim Fellowships Awarded

The John Simon Guggenheim Memorial Foundation has announced the names of 180 artists, scholars, and scientists from the United States, Canada, and the United Kingdom who were selected as Guggenheim Fellows for 2011. Guggenheim Fellows are appointed on the basis of distinguished achievement in the past and exceptional promise for future accomplishment. The mathematicians selected to receive the 2011 fellowships are BJORN POONEN, Massachusetts Institute of Technology, mathematics; DIMITRIS N. POLITIS, University of California San Diego, statistics; VIJAY VAZIRANI, Georgia Institute of Technology, computer science; and VAHID TAROKH, Harvard University, applied mathematics.

—From a Guggenheim Foundation news release

SIAM Fellows Elected

The Society for Industrial and Applied Mathematics (SIAM) has elected its new fellows for 2011. Their names and institutions follow.

MARK J. ABLOWITZ, University of Colorado Boulder; KENDALL E. ATKINSON, University of Iowa; CLAUDE W. BARDOS, Institut de Mathématiques de Jussieu; JOHN T. BETTS, Boeing; CHARLES R. DOERING, University of Michigan; JIM DOUGLAS JR., University of Chicago/Purdue University; ALAN S. EDELMAN, Massachusetts Institute of Technology; CHARBEL FARHAT, Stanford University; JEAN-PIERRE FOUQUE, University of California Santa Barbara; ALAN M. FRIEZE, Carnegie Mellon University; KENNETH M. GOLDEN, University of Utah; THOMAS A. GRANDINE, Boeing; WILLIAM D. GROPP, University of Illinois Urbana-Champaign; PHILIP HOLMES, Princeton University; ILSE C. F. IPSE, North Carolina State University; CHRISTOPHER K. R. T. JONES, University of North Carolina Chapel Hill; DAVID E. KEYES, Columbia University/King Abdullah University of Science and Technology; SUZANNE M. LENHART, University of Tennessee Knoxville; JOHN G. LEWIS, Cray Incorporated; ZHI-QUAN (TOM) LUO, University of Minnesota; OLVI L. MANGASARIAN, University of California San Diego/University of Wisconsin Madison; BERNARD J. MATKOWSKY, Northwestern University; JAMES MCKENNA, Bell Laboratories; VOLKER L. MEHRMANN, Technische Universität Berlin; BORIS MORDUKHOVICH, Wayne State University; K. W. (BILL) MORTON, University of Oxford Computing Laboratory; RICARDO H. NOCHETTO, University of Maryland College Park; BERESFORD N. PARLETT, University of California Berkeley; AHMED H. SAMEH, Purdue University; ROBERT D. SKEEL, Purdue University; CRAIG A. TRACY, University of California Davis; STEPHEN J. WRIGHT, University of Wisconsin Madison; JINCHAO XU, Pennsylvania State University; YA-XIANG YU, Chinese Academy of Sciences.

—From a SIAM announcement

Lalonde Appointed CRM Director

FRANÇOIS LALONDE of the University of Montreal has been appointed director of the Centre de Recherches Mathématiques (CRM) beginning June 1, 2011. Lalonde previously served as director from 2004 to 2008. He holds a Canada Research Chair in Differential Geometry and Topology and is a Fellow of the Royal Society of Canada and of the Fields Institute. His research focuses on fundamental problems in symplectic topology, including the classification of symplectic spaces and their mathematical structures, the study of their transformations and behavior under deformation, and their connections to the quantum domain.

—From a CRM announcement

Mathematics Opportunities

NSF Research Training Groups in the Mathematical Sciences

The National Science Foundation's (NSF) Research Training Groups in the Mathematical Sciences program provides funds for the training of U.S. students and postdoctoral researchers in the mathematical sciences. Proposals are solicited from groups of researchers based in a subarea of the mathematical sciences or linked by a multidisciplinary theme to support training at educational levels from undergraduate to postdoctoral within that focus. RTG awards are intended to support training programs that have strong potential to increase the number of well-prepared U.S. citizens, nationals, and permanent residents who pursue careers in the mathematical sciences and in other NSF-supported disciplines.

The deadline for full proposals is **July 19, 2011**. For more information, see `http://tinyurl.com/3v9urpd`.

—From an NSF announcement

Call for Nominations for SASTRA Ramanujan Prize

The Shanmugha Arts, Science, Technology Research Academy (SASTRA) invites nominations for the 2011 SASTRA Ramanujan Prize. The prize carries a cash award of US$10,000, and the winner will be invited to give a talk at the SASTRA conference in December 2011. The deadline for nominations is **August 15, 2011**. For more information, email `sastraprize@math.ufl.edu` or see the website `http://www.math.ufl.edu/sastra-prize/nominations-2011.html`.

—Krishnaswami Alladi
University of Florida

Call for Nominations for ICTP Ramanujan Prize

The Abdus Salam International Centre for Theoretical Physics (ICTP) invites nominations for the 2010 Ramanujan Prize for young mathematicians from developing countries. The prize, funded by the Niels Henrik Abel Memorial Fund, carries a cash award of US$15,000 and an allowance to visit ICTP to deliver a lecture.

The prize is awarded annually to a researcher from a developing country who is under forty-five years of age on December 31 of the year of the award and who has conducted outstanding research in a developing country. Researchers working in any branch of the mathematical sciences are eligible. The deadline for receipt of nominations is **October 30, 2011**. For further information, see the website `http://prizes.ictp.it/Ramanujan/`.

—From an ICTP announcement

Call for Nominations for Sloan Fellowships

Nominations for candidates for Sloan Research Fellowships, sponsored by the Alfred P. Sloan Foundation, are due by **September 15, 2011**. A candidate must be a member of the regular faculty at a college or university in the United States or Canada and must have received the Ph.D. or equivalent within the six years prior to the nomination. For information, write to: Sloan Research Fellowships, Alfred P. Sloan Foundation, 630 Fifth Avenue, Suite 2550, New York, New York 10111-0242; or consult the foundation's website: `http://www.sloan.org/fellowships`.

—From a Sloan Foundation announcement

Call for Nominations for Heineman Prize

The American Physical Society (APS) and the American Institute of Physics (AIP) are seeking nominations for the 2012 Dannie Heineman Prize for Mathematical Physics. The prize recognizes outstanding publications in the field of mathematical physics. The prize carries a cash award of US$10,000, an award certificate, and travel expenses to the meeting at which the prize is given. The deadline for nominations is **July 1, 2011**. For more information, see the APS website at `http://www.aps.org/programs/honors/prizes/heineman.cfm`.

—From an APS announcement

Call for Nominations for Parzen Prize

To promote the dissemination of statistical innovation, the Emanuel and Carol Parzen Prize for Statistical Innovation is awarded in even-numbered years to North American statisticians who have made outstanding and influential contributions by developing innovative statistical methods. Candidates must have received the Ph.D. degree more than twenty-five years before the nomination. The prize consists of an honorarium of US$1,000 and travel expenses to College Station, Texas, to present a lecture at the prize ceremony. Nominations for the 2012 Parzen Prize should be submitted by **October 1, 2011**, to Thomas Wehrly, Department of Statistics, 3143 TAMU, Texas A&M University, College Station, Texas 77843-3143.

—From a Texas A&M announcement

NSF Mentoring through Critical Transition Points in the Mathematical Sciences

The Mentoring through Critical Transition Points (MCTP) program of the National Science Foundation (NSF) provides funds for the training of U.S. students and postdoctoral researchers in the mathematical sciences. Proposals are solicited from departments of the mathematical sciences to support projects intended to improve training at critical transition points in the educational careers of students and junior researchers. MCTP awards are intended to support training programs that have strong potential to increase the number of well-prepared U.S. citizens, nationals, and permanent residents who pursue careers in the mathematical sciences and in other NSF-supported disciplines.

The deadline for full proposals for the program is **July 20, 2011**. For more information see `http://tinyurl.com/3eh8m73`.

—From an NSF announcement

NSF Integrative Graduate Education and Research Training

The Integrative Graduate Education and Research Training (IGERT) program was initiated by the National Science Foundation (NSF) to meet the challenges of educating Ph.D. scientists and engineers with the interdisciplinary backgrounds and the technical, professional, and personal skills needed for the career demands of the future. The program is intended to catalyze a cultural change in graduate education for students, faculty, and universities by establishing innovative models for graduate education in a fertile environment for collaborative research that transcends traditional disciplinary boundaries. It is also intended to facilitate greater diversity in student participation and to contribute to the development of a diverse, globally aware science and engineering workforce. Supported projects must be based on a multidisciplinary research theme and administered by a diverse group of investigators from U.S. Ph.D.-granting institutions with appropriate research and teaching interests and expertise.

The deadline for full proposals is **July 1, 2011**; full proposals may be sent by invitation only. Further information may be found at the website `http://www.nsf.gov/pubs/2011/nsf11533/nsf11533.htm`.

—From an NSF announcement

Research Programs at the Mathematical Sciences Research Institute

The Mathematical Sciences Research Institute (MSRI) in Berkeley, California, awards research professorships, research memberships, and NSF postdoctoral fellowships in connection with one or more of MSRI's 2012–2013 research programs.

The research programs for the academic year 2012–2013 are:

August 20, 2012–December 21, 2012: Cluster Algebras

August 20, 2012–May 24, 2013: Commutative Algebra

January 14, 2013–May 24, 2013: Noncommutative Algebraic Geometry and Representation Theory

For more information on eligibility, the application process, and deadlines, see `http://www.msri.org/web/msri/scientific/member-application`. The application forms will be available at `www.mathjobs.org` after August 1, 2011.

—From an MSRI announcement

Call for Proposals for the Mathematical Sciences Research Institute

The Mathematical Sciences Research Institute (MSRI) is now accepting proposals for scientific programs, workshops, and summer graduate schools.

Thematic Programs. MSRI is accepting proposals for semester-long or year-long scientific programs to be held at MSRI starting in spring 2015. Organizers are encouraged to submit a letter of intent prior to preparing a preproposal. Full proposals are considered in the fall and winter of each year and should be submitted by October 1 of the preceding year or January 1 of the year of the proposed program. For complete details, see http://www.msri.org/msri-progprop.

Hot Topics Workshops. Each year, MSRI hosts a week-long workshop to showcase what is new, innovative, and interesting to the mathematical sciences community. Proposals for Hot Topics Workshops to be held in 2012 should be submitted by **October 1, 2011**, or **January 1, 2012**. See http://www.msri.org/msri-htw.

Summer Graduate Schools. Each summer, MSRI organizes four 2-week-long summer graduate workshops, the majority of which are held at MSRI. To be considered for the summer of 2012, proposals should be submitted by **October 1, 2011**, or **January 1, 2012**. See http://www.msri.org/msri-sgw.

All proposals should be submitted to the director, deputy director, any member of the Scientific Advisory Committee, or by email to proposals@msri.org. For detailed information, see the website www.msri.org .

—From an MSRI announcement

For Your Information

Board on Mathematical Sciences Study on the Future of Mathematics

The Board on Mathematical Sciences and Their Applications of the National Academy of Science is conducting a study of the health of the mathematical sciences and prospects for the future. This NSF-sponsored study—called “The Mathematical Sciences in 2025”—is seeking input from members of the community with respect to any aspect of the study’s charge, which reads as follows:

“The study will produce a forward-looking assessment of the current state of the mathematical sciences and of emerging trends that will affect the discipline and its stakeholders as they look ahead to the quarter century mark. Specifically, the study will assess:

• The vitality of research in the mathematical sciences, looking at such aspects as the unity and coherence of research, significance of recent developments, rate of progress at the frontiers, and emerging trends.

• The impact of research and training in the mathematical sciences on science and engineering, on industry and technology, on innovation and economic competitiveness, on national security, and other areas of national interest.”

The study will make recommendations to the National Science Foundation’s Division of Mathematical Sciences on how to adjust its portfolio of activities to improve the vitality and impact of the discipline.

The committee would particularly welcome input about (a) major research trends and opportunities over the next fifteen years, (b) what our profession should do to make the most of these opportunities, and (c) stresses affecting the mathematical sciences.

To contribute your perspectives, please visit www.nas.edu/mathsci2025, where you can upload a .pdf or Word document or directly enter shorter input. The last date for submissions is **August 31, 2011**. This site also contains more detail about the study committee and its meetings to date. Questions should be directed to the study director, Scott Weidman, at mathsci2025@nas.edu.

Members of the Committee on the Mathematical Sciences in 2025 are: Thomas Everhart, California Institute of Technology, chair; Mark L. Green, University of California Los Angeles, vice-chair; Tanya Styblo Beder, SBCC Group, Inc.; James Berger, Duke University; Luis Caffarelli, University of Texas; Emmanuel Candès, Stanford University; Phillip Colella, E. O. Lawrence Berkeley National Laboratory; David Eisenbud, University of California Berkeley and the Simons Foundation; Peter Wilcox Jones, Yale University; Ju-Lee Kim, Massachusetts Institute of Technology; Yann LeCun, New York University; Jun Liu, Harvard University; Juan M. Maldacena, Institute for Advanced Study; John W. Morgan, Stony Brook University; Yuval Peres, Microsoft Research; Eva Tardos, Cornell University; Margaret H. Wright, New York University; and Joe B. Wyatt, Vanderbilt University.

—BMS announcement

Inside the AMS

Fan China Exchange Program

The Society's Fan China Exchange Program awards grants to support collaborations between Chinese and U.S. or Canadian researchers. Institutions in the United States or Canada apply for the funds to support a visitor from China or vice versa. This funding is made possible through a generous gift made to the AMS by Ky and Yu-Fen Fan in 1999.

RUTGERS UNIVERSITY received a grant of US$4,000 to support a visit by Haigang Li of Beijing Normal University.

For information about the Fan China Exchange Program, visit the website `http://www.ams.org/programs/travel-grants/china-exchange/china-exchange` or contact the AMS Membership and Programs Department, email: `chinaexchange@ams.org`., telephone 401-455-4170 (within the U.S. call 800-321-4267, ext. 4170).

—*AMS Membership and Programs Department*

Erdős Memorial Lecture

The Erdős Memorial Lecture is an annual invited address named for the prolific mathematician Paul Erdős (1913–96). The lectures are supported by a fund created by Andrew Beal, a Dallas banker and mathematics enthusiast. The Beal Prize Fund, now US$100,000, is being held by the AMS until it is awarded for a correct solution to the Beal Conjecture (see `www.math.unt.edu/~mauldin/beal.html`). At Mr. Beal's request, the interest from the fund is used to support the Erdős Memorial Lecture.

The Erdős Memorial Lecturer for 2011 is Emmanuel Candès of Stanford University. He will deliver the Erdős Lecture on October 15, 2011, at the Fall Central Section Meeting at the University of Nebraska, Lincoln.

—*AMS announcement*

AMS Announces Congressional Fellow

The American Mathematical Society (AMS) is pleased to announce that RICHARD YAMADA has been chosen as its Congressional Fellow for 2011–2012.

The AMS will sponsor Yamada's fellowship through the Congressional Fellowship program administered by the American Association for the Advancement of Science (AAAS). The fellowship is designed to provide a unique public policy learning experience, to demonstrate the value of science-government interaction, and to bring a technical background and external perspective to the decision-making process in Congress.

Richard Yamada

Fellows spend a year working on the staff of a member of Congress or a congressional committee, working as a special legislative assistant in legislative and policy areas requiring scientific and technical input. The fellowship program includes an orientation on congressional and executive branch operations and a year-long seminar series on issues involving science, technology, and public policy.

Richard Yamada earned his Ph.D. in applied mathematics from Cornell University and has most recently worked as a term assistant professor in the mathematics department at the University of Michigan.

For more information on the AMS Congressional Fellowship program, visit the website `http://www.fellowships.aaas.org`.

—*AMS Washington Office*

AMS Announces Mass Media Fellowship Award

The American Mathematical Society (AMS) has awarded its 2011 Mass Media fellowship to MELANIE DEVRIES, a Ph.D. student in mathematics at the University of Nebraska-Lincoln. She will work at KUNC-FM radio in Northern Colorado for ten weeks this summer.

The Mass Media Science and Engineering Fellowship program is organized by the American Association for the Advancement of Science (AAAS). It is a highly competitive program designed to improve public understanding of science and technology by placing advanced science, mathematics, and engineering students in newsrooms nationwide. Fellows work with media professionals to improve their communication skills and increase their understanding of the editorial process by which events and ideas become news.

Melanie DeVries

The program is available to college or university students (in their senior years or on any graduate or postgraduate level) in the natural, physical, health, engineering, computer, or social sciences or in mathematics with outstanding written and oral communication skills and a strong interest in learning about the media. The program has supported more than 500 fellows in its thirty-seven years.

For more information on the AAAS Mass Media Science and Engineering Fellowship Program, visit the website `http://www.aaas.org/programs/education/MassMedia/`.

—AMS Washington Office

AMS Holds Workshop for Department Chairs

The AMS held its annual workshop for department chairs prior to the Joint Mathematics Meetings in New Orleans, Louisiana. This one-day session is designed in a workshop format to stimulate discussion and allow the sharing of ideas and experiences among attending department chairs and to create an environment that allows attendees to address departmental challenges from new perspectives.

The 2011 workshop was led by Timothy Hodges, University of Cincinnati; John Meakin, University of Nebraska-Lincoln; and Stephen Robinson, Wake Forest University.

Workshop sessions have included a range of issues facing departments. This year's workshop focused on leading a department in challenging times, assessment, new approaches to nonmajor mathematics classes, and faculty evaluation as an opportunity.

The 2011 workshop was the largest ever for the AMS, with sixty department chairs and leaders attending from across the country.

—AMS Washington Office

Deaths of AMS Members

Janie L. Bell, of Monument, Colorado, died on November 24, 2010. Born on January 27, 1913, she was a member of the Society for 68 years.

Patrick P. Billingsley, of Chicago, Illinois, died on April 22, 2011. Born on May 1, 1925, he was a member of the Society for 59 years.

Robert C. Carson, of Barberton, Ohio, died on October 14, 2010. Born on March 11, 1924, he was a member of the Society for 58 years.

Charles A. Desoer, of Berkeley, California, died on November 1, 2010. Born on January 11, 1926, he was a member of the Society for 46 years.

Don Gilbert Dunmeyer, of Logan, Utah, died on February 26, 2011. Born on December 12, 1936, he was a member of the Society for 17 years.

Edward S. Eby, of Mystic, Connecticut, died on February 14, 2008. Born on October 3, 1934, he was a member of the Society for 50 years.

Hin-See H. Lee, of Toronto, Ontario, died on July 20, 2010. Born on January 30, 1932, he was a member of the Society for 30 years.

Jacob J. Levin, professor, University of Wisconsin, died on March 16, 2008. Born on December 21, 1926, he was a member of the Society for 55 years.

Benoit B. Mandelbrot, of Cambridge, Massachusetts, died on October 14, 2010. Born on November 20, 1924, he was a member of the Society for 55 years.

Lowell J. Paige, of Carmichael, California, died on December 10, 2010. Born on December 10, 1919, he was a member of the Society for 69 years.

Justin J. Price, of West Lafayette, Indiana, died on March 12, 2011. Born on June 18, 1930, he was a member of the Society for 58 years.

Richard H. Schelp, professor, University of Memphis, died on November 29, 2010. Born on April 21, 1936, he was a member of the Society for 43 years.

Harold T. Slaby, of Royal Oak, Michigan, died on October 6, 2010. Born on October 11, 1920, he was a member of the Society for 61 years.

James Laurie Snell, professor, Dartmouth College, died on March 19, 2011. Born on January 15, 1925, he was a member of the Society for 64 years.

William H. Spragens Jr., of Sewickley, Pennsylvania, died on March 23, 2011. Born on September 20, 1914, he was a member of the Society for 63 years.

Pit-Mann Wong, professor, University of Notre Dame, died on July 3, 2010. Born on September 7, 1949, he was a member of the Society for 27 years.

Aurel J. Zajta, of Houston, Texas, died on July 10, 2006. Born on March 17, 1926, he was a member of the Society for 35 years.

Reference and Book List

The ***Reference*** *section of the* Notices *is intended to provide the reader with frequently sought information in an easily accessible manner. New information is printed as it becomes available and is referenced after the first printing. As soon as information is updated or otherwise changed, it will be noted in this section.*

Contacting the *Notices*

The preferred method for contacting the *Notices* is electronic mail. The editor is the person to whom to send articles and letters for consideration. Articles include feature articles, memorial articles, communications, opinion pieces, and book reviews. The editor is also the person to whom to send news of unusual interest about other people's mathematics research.

The managing editor is the person to whom to send items for "Mathematics People", "Mathematics Opportunities", "For Your Information", "Reference and Book List", and "Mathematics Calendar". Requests for permissions, as well as all other inquiries, go to the managing editor.

The electronic-mail addresses are `notices@math.wustl.edu` in the case of the editor and `notices@ams.org` in the case of the managing editor. The fax numbers are 314-935-6839 for the editor and 401-331-3842 for the managing editor. Postal addresses may be found in the masthead.

Information for *Notices* Authors

The *Notices* welcomes unsolicited articles for consideration for publication, as well as proposals for such articles. The following provides general guidelines for writing *Notices* articles and preparing them for submission. Contact information for *Notices* editors and staff may be found on the *Notices* website, `http://www.ams.org/notices`.

Notices **readership**. The *Notices* publishes articles that have broad appeal for a diverse audience with many different types of readers: graduate students, academic mathematicians, industrial mathematicians, researchers in mathematically based fields, and amateur enthusiasts. The paper edition of the *Notices* is sent to the approximately 33,000 members of the AMS, most of whom are professional mathematicians; about 25,000 of them reside in North America. Because the *Notices* is accessible for free over the Internet, the number of readers is much larger than the AMS membership. All readers may be assumed to be interested in mathematics research, but they are not all active researchers.

Notices Feature Articles

Topics. The *Notices* seeks exceptional articles that report on major new developments in mathematics or that describe episodes from math-

Where to Find It

A brief index to information that appears in this and previous issues of the *Notices*.

ematics history that have connection to current research in the field. We also welcome articles discussing aspects of the mathematics profession, such as grant programs, the job market, professional opportunities for mathematicians, publishing, electronic communications, etc. We are also interested in articles about mathematics education at all levels. We publish reviews of books, films, plays, software, and mathematical tools.

Reaching the audience. Our goal is to educate the readership about new developments in mathematics and in the mathematics profession, as well as other matters of interest to the working mathematician. Each article is expected to have a large target audience of readers, perhaps 5,000 of the 33,000 subscribers. Authors must therefore write their articles for nonexperts rather than for experts or would-be experts. In particular, the mathematics articles in the *Notices* are expository. A *Notices* article should have an introduction that anyone can understand, and almost all readers should be able to understand the key points of the article.

Structure of articles. Most feature articles, including those on mathematics, are expected to be of long-term value and should be written as such. Ideally each article should put its topic in a context, providing some history and other orientation for the reader, and, as necessary, relating the subject matter to things that readers are likely to understand. In most cases, articles should progress to dealing with contemporary matters, not giving only historical material. The articles that are received the best by readers tend to relate different areas of mathematics to each other.

By design the *Notices* is partly magazine and partly journal, and authors' expository styles should take this into account. For example, many readers want to understand the mathematics articles without undue effort and without consulting other sources.

Format and length. Mathematics feature articles in the *Notices* are normally six to nine pages, sometimes a little longer. Shorter articles are more likely to be read fully than are longer articles. The first page is 400 or 500 words, and subsequent pages are about 800 words. From this one should subtract an allowance for figures, photos, and other illustrations and an appropriate allowance for any displayed equations and bibliography. The *Notices* is especially interested in the creative use of graphics and color and encourages illustrations. Articles on professional topics are typically 3 to 5 pages, as are book reviews.

Editorial process. The *Notices* aims to publish exceptionally well-written articles that appeal to a broad audience of mathematicians. Highly technical, specialized articles with a great deal of notation, insider jargon, and a long list of references are not suitable for the *Notices*. Some articles will be rejected by the editors without any external review. Other articles will be carefully refereed, and then a detailed editorial process will be used to bring the article up to the *Notices* standard. There will be considerable give and take between the author(s) and the editor, and it may take several drafts to get the article right.

The "WHAT IS...?" Column

Nearly every issue of the *Notices* carries an installment of the "WHAT IS...?" column. The purpose of the column is to provide brief, nontechnical descriptions of mathematical objects in use in current research. The target audience for the columns is first-year graduate students.

Each "WHAT IS...?" column provides an expository description of a single mathematical object being used in contemporary research. Thus "WHAT IS *M*-Theory?" would be too broad, but "WHAT IS a Brane?" would be appropriate; ideally "WHAT IS a Brane?" would give a flavor of what *M*-theory is.

The writing should be nontechnical and informal. Narrative description conveying main ideas should be favored over notation-heavy precision.

There is a strict limit of two *Notices* pages (1,400 words with no picture or 1,200 words with one picture). A list of "Further Reading" should contain no more than three references. Inquiries and comments about the "WHAT IS...?" column are welcome and may be sent to `notices-whatis@ams.org`.

Upcoming Deadlines

July 1, 2011: Nominations for 2012 Dannie Heineman Prize for Mathematical Physics. See "Mathematics Opportunities" in this issue.

July 1, 2011: Full proposals for NSF Integrative Graduate Education and Research Training (IGERT); by invitation only. See "Mathematics Opportunities" in this issue.

July 19, 2011: Full proposals for NSF Research Training Groups in the Mathematical Sciences. See "Mathematics Opportunities" in this issue.

July 20, 2011: Full proposals for NSF Mentoring through Critical Transition Points in the Mathematical Sciences program. See "Mathematics Opportunities" in this issue.

August 1, 2011: Applications for August review for National Academies Research Associateship Programs. See the National Academies website at `http://sites.nationalacademies.org/PGA/RAP/PGA_050491` or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email `rap@nas.edu`.

August 15, 2011: Nominations for SASTRA Ramanujan Prize. See "Mathematics Opportunities" in this issue.

August 31, 2011: Contributions to BMS study, "The Mathematical Sciences in 2025". See "For Your Information" in this issue.

September 15, 2011: Nomination for Sloan Research Fellowships. See "Mathematics Opportunities" in this issue.

October 1, 2011: Nominations for Emanuel and Carol Parzen Prize for Statistical Innovation. See "Mathematics Opportunities" in this issue.

October 1, 2011: Proposals for MSRI Hot Topic Workshops for 2012. See "Mathematics Opportunities" in this issue.

October 1, 2011: Proposals for MSRI Summer Graduate Schools for 2012. See "Mathematics Opportunities" in this issue.

October 1, 2011: Applications for AWM Travel Grants. See `http://www.awm-math.org/travelgrants.html#standard`.

October 1, 2011: Nominations for the 2012 Emanuel and Carol Parzen Prize. Contact Thomas Wehrly, Department of Statistics, 3143 TAMU, Texas A&M University, College Station, Texas 77843-314.

October 19, 2011: Proposals for NSF Postdoctoral Research Fellowships. See `http://www.nsf.gov/pubs/2008/nsf08582/nsf08582.htm`.

October 30, 2011: Nominations for ICTP Ramanujan Prize. See "Mathematics Opportunities" in this issue.

November 1, 2011: Applications for November review for National Academies Research Associateship Programs. See the National Academies website at `http://sites.nationalacademies.org/PGA/RAP/PGA_050491` or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email `rap@nas.edu`.

December 21, 2011: Nominations for the Schauder Medal. Contact Lech Gorniewicz, manager of Schauder Center, `tmna@mat.uni.torun.pl`.

January 1, 2012: Proposals for MSRI Hot Topic Workshops for 2012. See "Mathematics Opportunities" in this issue.

January 1, 2012: Proposals for MSRI Summer Graduate Schools for 2012. See "Mathematics Opportunities" in this issue.

Book List

The Book List highlights books that have mathematical themes and are aimed at a broad audience potentially including mathematicians, students, and the general public. When a book has been reviewed in the Notices*, a reference is given to the review. Generally the list will contain only books published within the last two years, though exceptions may be made in cases where current events (e.g., the death of a prominent mathematician, coverage of a certain piece of mathematics in the news) warrant drawing readers' attention to older books. Suggestions for books to include on the list may be sent to* `notices-booklist@ams.org`.

*Added to "Book List" since the list's last appearance.

Apocalypse When?: Calculating How Long the Human Race Will Survive, by Willard Wells. Springer Praxis, June 2009. ISBN-13: 978-03870-983-64.

At Home with Andre and Simone Weil, by Sylvie Weil. (Translation of *Chez les Weils*, translated by Benjamin Ivry.) Northwestern University Press, October 2010. ISBN 978-08101-270-43. (Reviewed May 2011.)

The Best Writing on Mathematics: 2010, edited by Mircea Pitici. Princeton University Press, December 2010. ISBN-13: 978-06911-484-10.

The Big Questions, Mathematics, by Tony Crilly. Quercus, April 2011. ISBN: 978-18491-624-01.

The Black Swan: The Impact of the Highly Improbable, by Nassim Nicholas Taleb. Random House Trade Paperbacks, second edition, May 2010. ISBN-13: 978-08129-738-15. (First edition reviewed March 2011.)

The Calculus Diaries: How Math Can Help You Lose Weight, Win in Vegas, and Survive a Zombie Apocalypse, by Jennifer Ouellette. Penguin, reprint edition, August 2010. ISBN 978-01431-173-77.

The Calculus of Selfishness, by Karl Sigmund. Princeton University Press, January 2010. ISBN-13: 978-06911-427-53.

**Chasing Shadows: Mathematics, Astronomy, and the Early History of Eclipse Reckoning,* by Clemency Montelle. Johns Hopkins University Press, April 2011. ISBN-13: 978-08018-969-10.

The Clockwork Universe: Isaac Newton, the Royal Society, and the Birth of the Modern World, by Edward Dolnick. Harper, February 2011. ISBN-13: 978-00617-195-16. (Reviewed April 2011.)

Complexity: A Guided Tour, by Melanie Mitchell. Oxford University Press, April 2009. ISBN-13: 978-01951-244-15. (Reviewed April 2011.)

**Crafting by Concepts: Fiber Arts and Mathematics,* by Sarah-Marie Belcastro and Carolyn Yackel. A K Peters/CRC Press, March 2011. ISBN-13: 978-15688-143-53.

The Cult of Statistical Significance: How the Standard Error Costs Us Jobs, Justice, and Lives, by Stephen T. Ziliak and Deirdre N. McCloskey, University of Michigan Press, February 2008. ISBN-13: 978-04720-500-79. (Reviewed October 2010.)

Duel at Dawn: Heroes, Martyrs, and the Rise of Modern Mathematics, by Amir Alexander. Harvard University Press, April 2010. ISBN-13: 978-06740-466-10. (Reviewed November 2010.)

Euler's Gem: The Polyhedron Formula and the Birth of Topology, by David S. Richeson. Princeton University Press, September 2008. ISBN-13: 978-06911-267-77. (Reviewed December 2010.)

The Grand Design, by Stephen Hawking and Leonard Mlodinow. Bantam, September 2010. ISBN-13: 978-05538-053-76.

Here's Looking at Euclid: A Surprising Excursion through the Astonishing World of Math, by Alex Bellos. Free Press, June 2010. ISBN-13: 978-14165-882-52.

Hidden Harmonies (The Lives and Times of the Pythagorean Theorem), by Robert and Ellen Kaplan. Bloomsbury Press, January 2011. ISBN-13:978-15969-152-20.

Hot X: Algebra Exposed, by Danica McKellar. Hudson Street Press, August 2010. ISBN 978-15946-307-05.

**I Want to Be a Mathematician: A Conversation with Paul Halmos.* A film by George Csicsery. Mathematical Association of America, March 2009. ISBN: 978-08838-590-94. (Reviewed in this issue.)

Le Operazioni del Calcolo Logico, by Ernst Schröder. Original German version of *Operationskreis des Logikkalkuls* and Italian translation with commentary and annotations by Davide Bondoni. LED Online, 2010. ISBN 978-88-7916-474-0.

Logicomix: An Epic Search for Truth, by Apostolos Doxiadis and Christos Papadimitriou. Bloomsbury USA, September 2009. ISBN-13: 978-15969-145-20. (Reviewed December 2010.)

Loving + Hating Mathematics: Challenging the Myths of Mathematical Life, by Reuben Hersh and Vera John-Steiner. Princeton University Press, January 2011. ISBN-13: 978-06911-424-70.

The Math Book: From Pythagoras to the 57th Dimension, 250 Milestones in the History of Mathematics, by Clifford A. Pickover. Sterling, September 2009. ISBN-13: 978-14027-579-69.

A Mathematician's Lament: How School Cheats Us Out of Our Most Fascinating and Imaginative Art Form, by Paul Lockhart. Bellevue Literary Press, April 2009. ISBN-13:978-1-934137-17-8.

Mathematicians: An Outer View of the Inner World, by Mariana Cook. Princeton University Press, June 2009. ISBN-13: 978-06911-3951-7. (Reviewed August 2010.)

Mathematicians Fleeing from Nazi Germany: Individual Fates and Global Impact, by Reinhard Siegmund-Schultze. Princeton University Press, July 2009. ISBN-13: 978-06911-4041-4. (Reviewed November 2010.)

**Mathematics Education for a New Era: Video Games as a Medium for Learning*, by Keith Devlin. CRC Press/A K Peters, February 2011. ISBN: 978-1-56881-431-5.

**Mathematics Emerging: A Sourcebook 1540–1900,* by Jacqueline Stedall. Oxford University Press, November 2008. ISBN-13: 978-01992-269-00. (Reviewed in this issue.)

A Motif of Mathematics: History and Application of the Mediant and the Farey Sequence, by Scott B. Guthery. Docent Press, September 2010. ISBN-13:978-4538-105-76.

**Mysteries of the Equilateral Triangle,* by Brian J. McCartin. Hikari, August 2010. ISBN-13: 978-954-91999-5-6. Electronic copies available for free at `http://www.m-hikari.com/mccartin-2.pdf`.

Naming Infinity: A True Story of Religious Mysticism and Mathematical Creativity, by Loren Graham and Jean-Michel Kantor. Belknap Press of Harvard University Press, March 2009. ISBN-13: 978-06740-329-34.

Nonsense on Stilts: How to Tell Science from Bunk, by Massimo Pigliucci. University of Chicago Press, May 2010. ISBN-13: 978-02266-678-67. (Reviewed April 2011.)

Number Freak: From 1 to 200—The Hidden Language of Numbers Revealed, by Derrick Niederman. Perigee Trade, August 2009. ISBN-10: 03995-345-98.

Numbers: A Very Short Introduction, by Peter M. Higgins. Oxford University Press, February 2011. ISBN 978-0-19-958405-5.

Numbers Rule: The Vexing Mathematics of Democracy, from Plato to the Present, by George G. Szpiro. Princeton University Press, April 2010. ISBN-13: 978-06911-399-44. (Reviewed January 2011.)

The Numerati, by Stephen Baker. Houghton Mifflin, August 2008. ISBN-13: 978-06187-846-08. (Reviewed October 2009.)

**Origami Inspirations,* by Meenakshi Mukerji. A K Peters, September 2010. ISBN-13: 978-1568815848.

Our Days Are Numbered: How Mathematics Orders Our Lives, by Jason Brown. Emblem Editions, April 2010. ISBN-13: 978-07710-169-74.

Perfect Rigor: A Genius and the Mathematical Breakthrough of the Century, by Masha Gessen. Houghton Mifflin Harcourt, November 2009. ISBN-13: 978-01510-140-64. (Reviewed January 2011.)

Pioneering Women in American Mathematics: The Pre-1940 Ph.D.'s, by Judy Green and Jeanne LaDuke. AMS, December 2008. ISBN-13: 978-08218-4376-5.

Plato's Ghost: The Modernist Transformation of Mathematics, by Jeremy Gray. Princeton University Press, September 2008. ISBN-13: 978-06911-361-03. (Reviewed February 2010.)

Probabilities: The Little Numbers That Rule Our Lives, by Peter Olofsson. Wiley, March 2010. ISBN-13: 978-04706-244-56.

Problem-Solving and Selected Topics in Number Theory in the Spirit of the Mathematical Olympiads, by Michael Th. Rassias. Springer, 2011. ISBN-13: 978-1-4419-0494-2.

Proofiness: The Dark Arts of Mathematical Deception, by Charles Seife. Viking, September 2010. ISBN 978-06700-221-68.

Proofs from THE BOOK, by Martin Aigner and Günter Ziegler. Expanded fourth edition, Springer, October 2009. ISBN-13: 978-3-642-00855-9.

The Quants: How a New Breed of Math Whizzes Conquered Wall Street and Nearly Destroyed It, by Scott Patterson. Crown Business, January 2011. ISBN 978-03074-533-89. (Reviewed May 2011.)

Recountings: Conversations with MIT Mathematicians, edited by Joel Segel. A K Peters, January 2009. ISBN-13: 978-15688-144-90.

Roger Boscovich, by Radoslav Dimitric (Serbian). Helios Publishing Company, September 2006. ISBN-13: 978-09788-256-21.

The Shape of Inner Space: String Theory and the Geometry of the Universe's Hidden Dimensions, by Shing-Tung Yau (with Steve Nadis). Basic Books, September 2010. ISBN-13: 978-04650-202-32. (Reviewed February 2011.)

The Solitude of Prime Numbers, by Paolo Giordano. Pamela Dorman Books, March 2010. ISBN-13: 978-06700-214-82. (Reviewed September 2010.)

Solving Mathematical Problems: A Personal Perspective, by Terence Tao. Oxford University Press, September 2006. ISBN-13: 978-0199-20560-8. (Reviewed February 2010.)

The Strangest Man, by Graham Farmelo. Basic Books, August 2009. ISBN-13: 978-04650-182-77.

Survival Guide for Outsiders: How to Protect Yourself from Politicians, Experts, and Other Insiders, by Sherman Stein. BookSurge Publishing, February 2010. ISBN-13: 978-14392-532-74.

Symmetry: A Journey into the Patterns of Nature, by Marcus du Sautoy. Harper, March 2008. ISBN: 978-00607-8940-4. (Reviewed February 2011.)

Symmetry in Chaos: A Search for Pattern in Mathematics, Art, and Nature, by Michael Field and Martin Golubitsky. Society for Industrial and Applied Mathematics, second revised edition, May 2009. ISBN-13: 978-08987-167-26.

Teaching Statistics Using Baseball, by James Albert. Mathematical Association of America, July 2003. ISBN-13: 978-08838-572-74. (Reviewed April 2010.)

**Train Your Brain: A Year's Worth of Puzzles,* by George Grätzer. A K Peters/CRC Press, April 2011. ISBN-13: 978-15688-171-01.

What's Luck Got to Do with It? The History, Mathematics and Psychology of the Gambler's Illusion, by Joseph Mazur. Princeton University Press, July 2010. ISBN: 978-069-113890-9.

AMERICAN MATHEMATICAL SOCIETY

AMS EXEMPLARY PROGRAM PRIZE

At its meeting in January 2004, the AMS Council approved the establishment of a new award called the AMS Award for an Exemplary Program or Achievement in a Mathematics Department. It is to be presented annually to a department that has distinguished itself by undertaking an unusual or particularly effective program of value to the mathematics community, internally or in relation to the rest of society. Examples might include a department that runs a notable minority outreach program, a department that has instituted an unusually effective industrial mathematics internship program, a department that has promoted mathematics so successfully that a large fraction of its university's undergraduate population majors in mathematics, or a department that has made some form of innovation in its research support to faculty and/or graduate students, or which has created a special and innovative environment for some aspect of mathematics research.

The prize amount is $5,000. All departments in North America that offer at least a bachelor's degree in the mathematical sciences are eligible.

The Prize Selection Committee requests nominations for this award, which will be announced in Spring 2012. Letters of nomination may be submitted by one or more individuals. Nomination of the writer's own institution is permitted. The letter should describe the specific program(s) for which the department is being nominated as well as the achievements that make the program(s) an outstanding success, and may include any ancillary documents which support the success of the program(s). The letter should not exceed two pages, with supporting documentation not to exceed an additional three pages.

All nominations should be submitted to the AMS Secretary, Robert J. Daverman, American Mathematical Society, 238 Ayres Hall, University of Tennessee, Knoxville TN 37996-1320. Include a short description of the work that is the basis of the nomination, with complete bibliographic citations when appropriate. The nominations will be forwarded by the Secretary to the Prize Selection Committee, which will make the final decision on the award.

Deadline for nominations is September 15, 2011.

Mathematics Calendar

Please submit conference information for the Mathematics Calendar through the Mathematics Calendar submission form at `http://www.ams.org/cgi-bin/mathcal-submit.pl`.
The most comprehensive and up-to-date Mathematics Calendar information is available on the AMS website at `http://www.ams.org/mathcal/`.

June 2011

1–3 **5th Global Conference on Power Control and Optimization**, Dubai, United Arab Emirates. (Mar. 2011, p. 491)

2–4 **IMA Hot Topics Workshop: Uncertainty Quantification in Industrial and Energy Applications: Experiences and Challenges**, Institute for Mathematics and its Applications (IMA), University of Minnesota, Minneapolis, Minnesota. (Apr. 2011, p. 625)

* 3–5 **2011 CMS Summer Meeting**, University of Alberta, Edmonton, Alberta, Canada.
Description: The meeting includes 17 scientific sessions in a wide variety of topics, plenary lectures presented by Leah Edelstein-Keshet (UBC), Olga Holtz (UC Berkeley; TU Berlin), François Lalonde (Montreal), Bjorn Poonen (MIT), and Roman Vershynin (Michigan); a public lecture presented by Gerda de Vries (Alberta), prize lectures presented by Rachel Kuske (UBC), Yvan Saint-Aubin (Montreal) and Kai Behrend (UBC); various panel discussions and a student poster session.
Information: `http://cms.math.ca/events/summer11`.

3–8 **XIIIth International Conference on Geometry, Integrability and Quantization**, Sts. Constantine and Elena Resort near Varna, Bulgaria. (Apr. 2011, p. 625)

5–7 **National Conference On Nonlinear Analysis and Applications**, Department of Mathematics, H.N.B. Garhwal University, Campus Pauri, Pauri Garhwal, Uttarakhand. (Jun./Jul. 2010, p. 786)

5–10 **16th Workshop on Stochastic Geometry, Stereology and Image Analysis**, Sandbjerg Estate, Soenderborg, Denmark. (Mar. 2011, p. 492)

5–12 **Symmetry and Perturbation Theory 2011**, Otranto, near Lecce, Italy. (Feb. 2011, p. 335)

6–8 **Abelian Varieties & Galois Actions**, Faculty of Mathematics and Computer Sciences, the Adam Mickiewicz University, Poznań, Poland. (Apr. 2011, p. 626)

6–8 **Perspectives in Mathematics and Life Sciences**, Fundación Euroárabe (C/San Jerónimo 27), Granada, Spain. (May 2011, p. 740)

6–8 **The International Conference on Numerical Analysis and Optimization (IceMATH 2011)**, Universitas Ahmad Dahlan, Yogyakarta, Indonesia. (Feb. 2011, p. 335)

6–9 **Copula Models and Dependence**, Centre de recherches mathématiques, Université de Montréal, Pavillon André-Aisenstadt, Montréal, (Québec) H3T 1J4 Canada. (Aug. 2010, p. 905)

6–10 **Conference on Structure and Classification of C^*-Algebras**, Centre de Recerca Matemàtica, Bellaterra, Barcelona, Spain. (Mar. 2011, p. 492)

6–10 **Faces of Geometry: 3-manifolds, Groups and Singularities**, Columbia University/Barnard College, New York, New York. (Feb. 2011, p. 335)

6–10 **IMA Workshop: Large-scale Inverse Problems and Quantification of Uncertainty**, Institute for Mathematics and its Applications (IMA), University of Minnesota, Minneapolis, Minnesota. (Apr. 2010, p. 552)

6–10 **Low-dimensional manifolds and high-dimensional categories**, UC Berkeley, Berkeley, California. (Dec. 2010, p. 1496)

6–10 **Toric geometry and applications**, Catholic University of Leuven, Leuven (Heverlee), Belgium. (Feb. 2011, p. 335)

6–15 **11th Canadian Summer School on Quantum Information**, Centre de Villégiature de Jouvence, Québec, Canada. (Apr. 2011, p. 626)

This section contains announcements of meetings and conferences of interest to some segment of the mathematical public, including ad hoc, local, or regional meetings, and meetings and symposia devoted to specialized topics, as well as announcements of regularly scheduled meetings of national or international mathematical organizations. A complete list of meetings of the Society can be found on the last page of each issue.
An announcement will be published in the *Notices* if it contains a call for papers and specifies the place, date, subject (when applicable), and the speakers; a second announcement will be published only if there are changes or necessary additional information. Once an announcement has appeared, the event will be briefly noted in every third issue until it has been held and a reference will be given in parentheses to the month, year, and page of the issue in which the complete information appeared. Asterisks (*) mark those announcements containing new or revised information.
In general, announcements of meetings and conferences carry only the date, title of meeting, place of meeting, names of speakers (or sometimes a general statement on the program), deadlines for abstracts or contributed papers, and source of further information. If there is any application deadline with respect to participation in the meeting, this fact should be noted. All communications on meetings and conferences in the mathematical sciences should be sent to the Editor of the *Notices* in care of the American Mathematical Society in Providence or electronically to `notices@ams.org` or `mathcal@ams.org`.
In order to allow participants to arrange their travel plans, organizers of meetings are urged to submit information for these listings early enough to allow them to appear in more than one issue of the *Notices* prior to the meeting in question. To achieve this, listings should be received in Providence **eight months** prior to the scheduled date of the meeting.
The complete listing of the Mathematics Calendar will be published only in the September issue of the *Notices*. The March, June/July, and December issues will include, along with new announcements, references to any previously announced meetings and conferences occurring within the twelve-month period following the month of those issues. New information about meetings and conferences that will occur later than the twelve-month period will be announced once in full and will not be repeated until the date of the conference or meeting falls within the twelve-month period.
The Mathematics Calendar, as well as Meetings and Conferences of the AMS, is now available electronically through the AMS website on the World Wide Web. To access the AMS website, use the URL: `http://www.ams.org/`.

7–9 **4th International Workshop on Symbolic-Numeric Computation (SNC 2011)**, San Jose, California. (Mar. 2011, p. 492)

7–10 **9th International Conference on Applied Cryptography and Network Security (ACNS 2011)**, Nerja, Malaga, Spain. (Feb. 2011, p. 335)

7–11 **Finite Groups and Their Automorphisms**, Boğaziçi University, Istanbul, Turkey. (Apr. 2011, p. 626)

8–9 **Workshop on "Numerical knots: models & simulations"**, Centro di Ricerca Matematica "Ennio De Giorgi", Collegio Puteano, Piazza dei Cavalieri 3, 56126 Pisa, Italy. (May 2011, p. 740)

8–11 **2011 International Symposium on Symbolic and Algebraic Computation (ISSAC 2011)**, San Jose Convention Center, 150 West San Carlos St., San Jose, California. (Jan. 2011, p. 84)

* 11–17 **Optimization Theory on Inventory Model and Industry in the Context of India**, Dr. R. M. L. Avadh University, Faizabad, (U.P.) India.
Description: A research paper related to interdisciplinary research.
Information: http://www.rmlau.ac.in.

12–17 **Geometric and nonlinear analysis, meeting in Lorraine**, Université Henri Poincar'e, Nancy, France. (Jan. 2011, p. 84)

12–18 **International Conference on Waves and Stability in Continuous Media WASCOM XVI**, Brindisi, Italy. (Feb. 2011, p. 335)

* 12–18 **2011 MIT-RTG Geometry Workshop: Fukaya categories via microlocal sheaf theory mentored by Professor David Nadler**, Breckenridge, Colorado.
Description: The goal of this workshop is to study Fukaya categories (and questions about Lagrangians) via microlocal sheaf theory. The workshop will begin with the foundations of the subject (singularities, constructible sheaves, microlocal geometry of sheaves) and then branch out into more advanced topics to be determined by Prof. Nadler. The workshop discussions will have an expository nature and are aimed at graduate students and junior faculty interested in this area. No prior knowledge of microlocal sheaf theory will be assumed. Prof. Nadler will outline the program of talks to be given by the participants. Most participants will be expected to give one of the talks.
Deadline: Application deadline is May 1, 2011. Please note that the number of participants will be strictly limited for space reasons.
Information: For more detailed information, or to register for the workshop, please go to the website: http://math.mit.edu/conferences/geometryworkshop.

* 13–15 **2011 Mathematics and Engineering Hawaii University International Conferences**, Ala Moana Hotel, 410 Atkinson Drive, Honolulu, Hawaii.
Description: The conference welcomes and invites papers (in English only) from all areas of Mathematics, Statistics, and Engineering related fields. Submitted conference papers will be peer reviewed. The conference aim is to provide a platform for researchers, engineers, academicians as well as industrial professionals from all over the world to present their research results and development in their related fields.
Information: http://www.huichawaii.org.

13–15 **The Mathematics of Porous Media**, Radisson Blu Resort, Split, Croatia. (May 2011, p. 740)

13–16 **2011 International Conference on Applied Mathematics and Interdisciplinary Research**, Nankai University, Tianjin, China. (Sept. 2010, p. 1166)

13–16 **FPP6: Foundations of Probability and Physics-6**, Linnaeus University, Vaxjo, Sweden. (Jan. 2011, p. 84)

13–16 **Poisson Geometry and Applications**, Figueira da Foz, Portugal. (May 2011, p. 741)

13–17 **AIM Workshop: The Cohen-Lenstra heuristics for Class Groups**, American Institute of Mathematics, Palo Alto, California. (Nov. 2010, p. 1349)

13–17 **Cluster Algebras and Lusztig's Canonical Basis**, University of Oregon, Eugene, Oregon. (Apr. 2011, p. 626)

13–17 **Formal Power Series and Algebraic Combinatorics (conference)**, Reykjavik, Iceland. (Mar. 2011, p. 492)

* 13–17 **Mini-courses in Mathematical Analysis 2011**, University of Padova, Padova, Italy.
Description: Following a longstanding tradition, the Departments of Mathematics of the University of Padova are organizing the meeting "Mini-courses in Mathematical Analysis 2011". The meeting will take place at "Torre Archimede", a new building of the University of Padova in the city center. The program consists of four lecture courses delivered by invited speakers and a limited number of short communications. The meeting aims at introducing the participants to important current research fields in Mathematical Analysis. The meeting is particularly indicated not only to graduate students, postdocs and young researchers but also to well-established experts in Mathematical Analysis. In association with ISAAC–International Society for Analysis, its Applications and Computation.
Information: http://minicourses.dmsa.unipd.it/.

13–17 **Workshop on Moving in Geometry**, Centre de recherches mathématiques, Université de Montréal, Québec, Canada. (Apr. 2011, p. 626)

13–19 **Strobl2011 - From Abstract to Computational Harmonic Analysis**, Bifeb, Strobl, Salzburg, Austria. (Nov. 2010, p. 1349)

14–16 **CONIAPS-XIII: 13th Conference of International Academy of Physical Sciences**, University of Petroleum and Energy Studies, UPES campus, Dehradun. (Apr. 2011, p. 626)

14–17 **2011 World Conference on Natural Resource Modeling**, Ottowa, Canada. (Sept. 2010, p. 1035)

14–23 **Advanced Course on Dynamical Systems**, Centre de Recerca Matemàtica, Bellaterra, Barcelona, Spain. (May 2011, p. 741)

15–16 **Workshop on "DNA knots"**, Centro di Ricerca Matematica "Ennio De Giorgi", Collegio Puteano, Piazza dei Cavalieri 3, 56126 Pisa, Italy. (May 2011, p. 741)

15–17 **2011 Usenix Annual Technical Conference (USENIX ATC '11)**, Portland Marriott, Downtown Waterfront, 1401 SW Naito Parkway, Portland, Oregon. (Feb. 2011, p. 335)

15–18 **International Conference on Mathematical Methods and Models in Biosciences (BIOMATH 2011)**, Bulgarian Academy of Sciences, Sofia, Bulgaria. (May 2011, p. 741)

16–17 **8th Canadian Student Conference on Quantum Information**, Centre de Villégiature de Jouvence, Québec, Canada. (Apr. 2011, p. 626)

16–18 **Lorentz Geometry in Mathematics and in Physics**, Institut de Recherche Mathématique Avancée, University of Strasbourg, France. (Dec. 2010, p. 1496)

17–23 **The International Conference "Painleve Equations and related topics"**, Euler Institute, Saint Petersburg, Russia. (Feb. 2011, p. 335)

19–25 **49th International Symposium on Functional Equations**, Graz, Austria. (Dec. 2010, p. 1496)

* 20–22 **Eigth Advanced Course in Operator Theory and Complex Analysis**, El Puerto de Santa Maria, Spain.
Invited Speakers: M. A. (Rien) Kaashoek (Vrije Universiteit, Netherlands). Etienne Matheron (Universite d'Artois, France). Anders Olofsson (Lund University, Sweden). Sergei Treil (Brown Uversity, USA).
Information: http://congreso.us.es/ceacyto/2011.

20–23 **Complex Analysis and Potential Theory in honour of Paul M. Gauthier and Kohur Gowrisankaran**, Centre de recherches mathématiques, Université de Montréal, Montréal, Canada. (Apr. 2011, p. 626)

20–23 **2011 Workshop on Mixed Integer Programming (MIP2011)**, University of Waterloo, Waterloo, Ontario, Canada. (May 2011, p. 741)

20–24 **AIM Workshop: Careers in academia**, American Institute of Mathematics, Palo Alto, California. (Mar. 2011, p. 493)

20–24 **2011 Casablanca International Workshop on Mathematical Biology**, University Hassan II, Casablanca, Morocco. (May 2011, p. 741)

20–24 **Journées de Probabilités 2011**, Institut Elie Cartan Nancy, Nancy, France. (Mar. 2011, p. 493)

20–24 **NSF-CBMS Conference on Radial Basis Functions: Mathematical Developments and Applications**, University of Massachusetts Dartmouth, Dartmouth, Massachusetts. (Mar. 2011, p. 494)

20–24 **Permutation Patterns 2011**, California Polytechnic State University, San Luis Obispo, California. (Jan. 2011, p. 84)

20–25 **3rd Conference of the Euro-American Consortium for Promoting the Application of Mathematics in Technical and Natural Sciences**, Resort of Albena, Bulgaria. (Jan. 2011, p. 84)

20–July 1 **IMA New Directions Short Course: Invariant Objects in Dynamical Systems and their Applications**, Institute for Mathematics and its Applications (IMA), University of Minnesota, Minneapolis, Minnesota. (Mar. 2011, p. 493)

20–July 1 (NEW DATE) **Polyhedral Geometry and Algebraic Combinatorics**, University of Wyoming, Laramie, Wyoming. (Jan. 2011, p. 84)

20–July 2 **School on D-modules and applications in Singularity Theory**, Mathematical Research Institute, University of Sevilla, Spain (first week); Mathematical Sciences Institute, CSIC, Madrid, Spain (second week). (Mar. 2011, p. 493)

20–July 8 **RTG Summer School on Inverse Problems & Partial Differential Equations**, Seattle, Washington. (Apr. 2011, p. 626)

21–24 **Summer School Geodesic Flow, Negative Curvature and Isomorphism Conjectures**, Georg-August Universität, Göttingen, Germany. (May 2011, p. 741)

22–24 **3rd IMA International Conference Mathematics in Sport**, The Lowry, Salford Quays, United Kingdom. (Sept. 2010, p. 1035)

26–July 1 **II Jaen Conference on Approximation Theory, Computer Aided Geometric Design, Numerical Methods and Applications**, Ubeda, Jaen, Spain. (May 2011, p. 741)

26–July 2 **New developments in noncommutative algebra and its applications**, Sabhal Mòr Ostaig, a Gaelic college in Sleat, Isle of Skye, Scotland. (Mar. 2011, p. 493)

26–July 2 **XXX Workshop on Geometric Methods in Physics**, Bialowieza, Poland. (Mar. 2011, p. 493)

27–July 1 **3rd Workshop on Fluids and PDE**, University of Campinas, Campinas, SP Brazil. (May 2011, p. 741)

* 27–July 1 **JISD 2011: Workshop on Dynamical Systems and EDP's**, Universitat Politecnica de Catalunya, Barcelona, Spain.
Description: There will be four courses, taught by Luis Caffarelli, Lorenzo Diaz, Vadim Kaloshin, and Yanyan Li, within the Master of Science in Advanced Mathematics and Mathematical Engineering (MAMME) of the UPC Graduate School, as well as communications and posters. The Jornades are organized by Xavier Cabre, Maria del Mar Gonzalez and Tere M. Seara. Courses: Fully non-linear equations I: Basic theory for local and non-local elliptic equations, by Luis Caffarelli (Univ. of Texas Austin, USA); Fully non-linear equations II: Applications to conformal geometry, by Yanyan Li (Rutgers University, USA); Principle of minimal action in dynamics, by Vadim Kaloshin (University of Maryland); Rich dynamics in heterodimensional cycles, by Lorenzo Diaz (Catholic University of Rio de Janeiro). There will be some *financial support* available for this edition. Deadline to apply for financial support: April 15, 2011.
Information: `http://www.ma1.upc.edu/recerca/seminaris/JISD2011/indexjisd2011.html`.

27–July 8 **Metric Measure Spaces: Geometric and Analytic Aspects**, Université de Montréal, Pavillon André-Aisenstadt, Montréal, Québec, Canada. (Feb. 2011, p. 335)

* 28–30 **Eighth International Symposium on Voronoi Diagrams in Science and Engineering (ISVD 2011)**, Qing Dao, China.
Description: The 8th ISVD International Symposium on Voronoi Diagrams in Science and Engineering is devoted to the research on Voronoi diagrams and their applications to all fields. The conference provides a forum for scientists and practitioners working on Voronoi Diagrams and on their applications in science and engineering including, geo-spatial information sciences (GIS), visualization, urban planning, computer science, geology, robotics, bioinformatics, high-performance computing, bioinformatics, physics, chemistry and other applications. We solicit original research contributions and industrial papers. The conference proceedings with ISBN will be published by the IEEE Computer Society Press and selected papers will be published in the Springer LNCS Transactions on Computational Science and Mathematics Journal of Chinese Universities (SCI). All submitted papers will be reviewed by the Program Committee, and acceptance decisions will be based on their quality, originality and relevance.
Information: `http://i.cs.hku.hk/~isvd2011/index.html`.

29–July 2 **International Conference on Applied Analysis and Algebra-ICAAA2011**, Yildiz Technical University, Davutpasa Campus, Davutpasa, Istanbul, Turkey. (Mar. 2011, p. 494)

29–July 5 **The Seventh Congress of Romanian Mathematicians**, Transilvania University, Brasov, Romania. (Mar. 2011, p. 494)

30–July 2 **IDOTA—Integral and Differential Operators and their Applications**, Department of Mathematics, University of Aveiro, Aveiro, Portugal. (Feb. 2011, p. 335)

July 2011

1–3 (NEW DATE) **The 4th Congress of the Turkic World Mathematical Society (TWMS)**, Baku, Azerbaijan. (Apr. 2011, p. 626)

1–5 **8-International Conference on the Computer Analysis of Problems of a Science and Technology**, Dushanbe, Tajikistan. (Apr. 2011, p. 626)

1–29 **Non-equilibrium Statistical Mechanics**, Centre de recherches mathématiques, Université de Montréal, Pavillon André-Aisenstadt, Montréal, Québec, Canada. (Apr. 2011, p. 626)

* 2–8 **ESF-ERC-CRM-Pi Conference on "Knots and Links: from Form to Function"**, Centro di Ricerca Matematica "Ennio De Giorgi", Piazza dei Cavalieri 3, Pisa, Italy.
Description: In recent years there has been a growing interest in "applicable" aspects of classical knot theory in 3D, motivated by the discovery of new concepts and techniques with strong geometric flavour, both in pure and applied mathematics. Many of these new mathematical approaches have been stimulated by research interaction between mathematics, physics, chemistry and biology. This conference aims at bringing together world specialists in knot theory and applications by fostering a strong interaction between theoretical methods, numerical methods and applications in mathematical and theoretical physics, polymer physics and biology. Chaired by: Prof. Renzo L. Ricca, Univ. of Milano-Bicocca, Italy. Co-Chair: Prof. De Witt Sumners, Florida State Univ. Advisory Board: Prof. Kenneth Millett, Univ. of Californa, Santa Barbara, and Prof. Carlo Petronio, University of Pisa, Italy.

Grants are available for researchers to cover the conference fee and possibly part of the travel costs.
Information: http://www.esf.org/activities/esf-conferences/details/2011/confdetail379.html?conf=379&year=2011.

* 2–12 **International Mathematical Summer School for Students**, Jacobs University, Bremen, Germany.
Description: The International Mathematical Summer School for Students offers 10 days of intense learning and interaction with some of the world's leading mathematicians. For talented students in their last two years of high school or in their first two years at university. Open to international students from all countries. The language of the school is English. The program features plenary talks and mini-courses by leading international mathematicians. Students and instructors live on the park-like campus with ample opportunities for informal interaction. Hosted by Jacobs University in the Northern German city of Bremen.
Information: http://math.jacobs-university.de/summerschool/.

* 3–8 **23rd British Combinatorial Conference**, University of Exeter, Exeter, United Kingdom.
Description: The British Combinatorial Conference is held every two years and is the major regular European combinatorics conference. It attracts around 200 delegates, these being a mixture of established academics, young researchers, and postgraduates. It provides excellent opportunities for the exchange of ideas and for networking between research groups and individuals. In the past, many successful collaborations have arisen from talks given and contacts made at the BCC. The areas covered are extensive and range from the highly theoretical to very practical issues in aspects such as coding and cryptography.
Information: http://empslocal.ex.ac.uk/people/staff/rjchapma/bcc23/.

3–8 **Completely Integrable Systems and Applications - ESF-EMS-ERCOM Conference**, Erwin Schrödinger Institute, Vienna, Austria. (Apr. 2011, p. 627)

3–9 **22nd International Workshop on Operator Theory and Applications (IWOTA 2011)**, Universidad de Sevilla, Spain. (Dec. 2010, p. 1497)

3–16 **41st Probability Summer School**, Saint-Flour, France. (Apr. 2011, p. 627)

4–8 **International Conference on Differential & Difference Equations and Applications: Conference in honour of Professor Ravi P. Agarwal**, Azores University, Ponta Delgada, Portugal. (Mar. 2011, p. 494)

* 4–8 **5th Summer School on Geometry, Mechanics and Control**, La Cristalera, Miraflores de la Sierra, Spain.
Description: This school is oriented to young researchers in Mathematics, Physics and Engineering. It is intended to present an up-to-date view of some fundamental issues in topics around three axes: Geometry, Mechanics and Control. Participants attention will be drawn to some open problems. This event is a post-graduate course of Consejo Superior de Investigaciones Cientificas on science and physical technologies within the activities of the Instituto de Ciencias Matematicas and the Geometry, Mechanics and Control Network.
Mini-courses: An introduction for Lie groupoids through Poisson Geometry, Rui Loja Fernandes, Instituto Superior Técnico de Lisboa, Portugal. Geometric Method for the N-body problem, Richard Montgomery, University of California, Santa Cruz, USA. Symplectic dynamics of completely integrable Hamiltonian Systems, Alvaro Pelayo, Institute for Advanced Study, Princeton, USA.
Deadlines: Scholarships deadline: April 15, 2011. Registration deadline: May 15, 2011.
Information: http://www.gmcnetwork.org/ssgmc11.

4–8 **2011 Taiwan International Conference on Geometry: Special Lagrangians and Related Topics**, Department of Mathematics, National Taiwan University, Taipei, Taiwan. (May 2011, p. 742)

4–10 **International Conference on Topology and its Applications (ICTA), 2011**, Department of Mathematics, COMSATS Institute of Information Technology (CIIT), Islamabad, Pakistan. (Oct. 2010, p. 1166)

4–15 **The 2011 Gene Golub SIAM Summer School on Waves and Imaging**, University of British Columbia, Vancouver, Canada. (Mar. 2011, p. 494)

5–8 **International Conference on Nonlinear Operators, Differential Equations and Applications (ICNODEA-2011)**, Babeş-Bolyai University, Cluj-Napoca, Romania. (Dec. 2010, p. 1497)

6–8 **DIMACS Workshop on Competitive Algorithms for Packet Scheduling, Buffering and Routing in the Internet**, DIMACS Center, CoRE Building, Rutgers University Piscataway, New Jersey. (May 2011, p. 742)

6–8 **The 2011 International Conference of Applied and Engineering Mathematics**, Imperial College London, London, United Kingdom. (Nov. 2010, p. 1349)

6–8 **IMA Conference on Nonlinearity and Coherent Structures**, University of Reading, United Kingdom. (Sept. 2010, p. 1035)

6–16 **Graduate Summer School: Probabilistic Models of Cognition**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California. (Mar. 2011, p. 494)

* 7–9 **LEA MathMode French-Romanian Workshop on Complex Geometry**, Institute of Mathematics "Simion Stoilow" of the Romanian Academy, Bucharest, Romania.
Description: The workshop brings together French and Romanian experts in complex geometry. The participation of young mathematicians is encouraged.
Information: http://www.imar.ro/~aprodu/cg2011/CG2011.

9–10 **Directions in Matrix Theory 2011**, Department of Mathematics, University of Coimbra, Portugal. (May 2011, p. 742)

10–16 **International Conference on Rings and Algebras in Honor of Professor Pjek-Hwee Lee**, National Taiwan University, Taipei, Taiwan. (Sept. 2010, p. 1035)

* 11–12 **Workshop on Global Optimization**, Izmir University of Economics, Izmir, Turkey.
Description: In the workshop, the following speakers will be giving one hour lectures on special topics of Global Optimization: Shu-Cherng Fang (NC State University, USA), Emre Alper Yildirim (Koç University, Turkey), David Gao (Virginia Tech University, USA), Kok Lay Teo (Curtin University, Australia), Murat Adivar (Izmir University of Economics, Turkey), Ruey Lin Sheu (National Cheng-Kung Univ., Taiwan), Song Wang (The University of Western Australia), Ş. Ilker Birbil (Sabancí University, Turkey), Refail Kasimbeyli (Izmir University of Economics).
Information: http://dm.ieu.edu.tr/wgo2011/.

11–15 **Exploratory Workshop on Emerging Infectious Diseases and Mathematical Modelling**, Centre de Recerca Matemàtica, Bellaterra, Barcelona, Spain. (Mar. 2011, p. 494)

* 11–15 **Combinatorics Conference in Lisboa**, Faculdade de Ciencias e Tecnologia, UNL, Lisbon, Portugal.
Description: Combinatorics Conference in Lisboa (CCL 2011) aims to bring together researchers who are interested in any area of Combinatorics.
Topics: Include all aspects of combinatorics, graph theory, combinatorial number theory, and combinatorial geometry, from theoretical to computational. It will consist of two mini-courses, given by Béla Bollobás (University of Memphis; Trinity College, Cambridge, United Kingdom) and Jozsef Solymosi (University of British Columbia,

Canada). There will also be plenary talks given by Peter Keevash (Queen Mary, University of London, United Kingdom), Tomasz Luczak (Adam Mickiewicz University, Poznan, Poland; Emory University, Georgia, USA) and Anusch Taraz (Technische Universitä, München, Germany) and contributed talks of twenty five minutes.
Information: http://www.fct.unl.pt/ccl2011.

11–15 **The 10th International Conference on Finite Fields and their Applications**, Ghent, Belgium. (Jun./Jul. 2010, p. 786)

11–15 **Workshop in Mathematical Physics**, Agia Napa, Cyprus. (May 2011, p. 742)

12–13 **Workshop on "Topological dynamics in physics and biology"**, Centro di Ricerca Matematica "Ennio De Giorgi", Collegio Puteano, Piazza dei Cavalieri 3, 56126 Pisa, Italy. (May 2011, p. 742)

12–15 **The 6th SEAMS-GMU 2011 International Conference on Mathematics and Its Applications; Workshop on Financial Mathematics and Workshop on Dynamical System in Biology**, Gadjah Mada University, Yogyakarta, Indonesia. (Oct. 2010, p. 1166)

* 13–16 **IMA Special Workshop: Wavelets and Applications: A Multi-Disciplinary Undergraduate Course with an Emphasis on Scientific Computing**, Institute for Mathematics and its Applications (IMA), University of Minnesota, Minneapolis, Minnesota.
Description: In this workshop, we will discuss the basic topics covered in the course as well as ideas for presenting the material. We will begin with applications such as data compression, edge detection, and signal de-noising. Associated labs utilize a package (available in Mathematica or Matlab) of graphical and computational routines. Instructions are provided for developing the latter set of routines from scratch should the instructor so desire.
Information: http://www.ima.umn.edu/2010-2011/SW7.13-16.11/.

* 14 **NSV-2011: Fourth International Workshop on Numerical Software Verification**, Cliff Lodge, Snowbird, Utah.
Description: The NSV-2011 workshop is associated to the Conference on Computer Aided Verification CAV 2011, and dedicated to the current development and future prospects on applying logical and mathematical techniques for reasoning about numerical aspects of software. It will also be an occasion to discuss robustness of software and systems under uncertainty of values and numerical perturbations, which is a central issue in numerical methods and robust control.
Scope: The scope of the workshop includes the following topics: Models and abstraction techniques, formal verification of numerical programs, quality of finite precision implementations, propagation of uncertainties, deterministic and probabilistic models, numerical functional properties, robustness of control software, hybrid systems verification, validation for avionics, automotive and real-time application, validation for scientific computing programs, benchmarks and tools for numerical software verification.
Information: http://www.lix.polytechnique.fr/Labo/Sylvie.Putot/NSV-2011/index.html.

15–30 **XIV Summer Diffiety School**, Levi-Civita Institute, Santo Stefano del Sole (AV), Italy. (Dec. 2010, p. 1497)

* 18–20 **Toric Methods in Homotopy Theory and Related Subjects**, Queen's University Belfast, Pure Mathematics Research Centre, Belfast, Northern Ireland, United Kingdom.
Description: The conference focuses on the interaction of toric methods with homotopy theory, topology and algebra.
Main speakers: Fred Cohen (University of Rochester, United States), Jack Morava (Johns Hopkins University, United States), Taras Panov (Moscow State University, Russia), Sam Payne (Yale University, United States), Oliver Röndigs (Universität Osnabrück, Germany), Alex Suciu (Northeastern University, United States).
Organizers: Thomas Hüttemann (Queen's University Belfast) and Nigel Ray (University of Manchester).
Support: The conference is supported by an LMS conference grant, and by EPSRC research project EP/H018743/1.
Information: http://toricmethodsbelfast.zzl.org/.

18–22 **7th International Congress on Industrial and Applied Mathematics - ICIAM 2011**, Vancouver, BC, Canada. (Jun./Jul. 2010, p. 786)

18–22 **AIM Workshop: Research experiences for undergraduate faculty**, American Institute of Mathematics, Palo Alto, California. (Mar. 2011, p. 494)

18–22 **Geometry & Topology Down Under – A Conference in Honour of Hyam Rubinstein**, The University of Melbourne, Australia. (Sept. 2010, p. 1035)

18–22 **Infinity Conference**, Centre de Recerca Matemàtica, Bellaterra, Barcelona, Spain. (May 2011, p. 742)

* 18–29 **Second International Conference and Workshop on Valuation Theory**, 1st week: "Palacio Mansilla", Segovia, Spain. 2nd week: Residence "San José", El Escorial. Spain.
Description: The Second International Conference and Workshop on Valuation Theory is an international meeting whose aim is to explore the connections among several topics in algebra and geometry, which are connected through the notion of valuation. It is a continuation, twelve years later, of the First International Conference and Workshop on Valuation Theory which took place in Saskatoon (Canada) in the summer of 1999. Apart from the lectures of the invited speakers, there will be a few number of specialized courses in these topics, mainly on the first week of the conference. In addition, a small number of talks will be selected by the Scientific Committee from the proposals made by the participants. Participants are also invited to submit posters.
Information: http://www.singacom.uva.es/oldsite/seminarios/ConfWorkVT.

19–22 **The SUMMER 9th International Conference on Computing, Communications and Control Technologies: CCCT 2011**, Orlando, Florida. (Jan. 2011, p. 84)

21–23 **The 7th IMT-GT International Conference on Mathematics, Statistics, and its Applications (ICMSA 2011)**, Bangkok, Thailand. (Feb. 2011, p. 335)

* 21–24 **International Conference on Theory and Applications in Mathematics and Informatics-ICTAMI 2011**, University of Alba Iulia, Alba Iulia, Romania.
Description: The aim of the conference is to bring together mathematicians and informaticians from all over the world and to attract original papers on the following topics: Algebra, analysis and complex analysis, topology and geometry, differential equations, probability and statistics, applied mathematics, computer science, intelligence computation, product and process modelling. embedded systems, knowledge engineering, E-education.
Information: http://www.uab.ro/ictami.

21–27 **Loops' 11**, Trest, Czech Republic. (Dec. 2010, p. 1497)

24–29 **ICDEA2011: 17th International Conference on Difference Equations and Applications**, Université du Québec à Trois-Rivières, Québec, Canada. (May 2011, p. 742)

24–30 **Dynamical models in life sciences**, University of Evora, Evora, Portugal. (May 2011, p. 742)

25–27 **Euler Society 2011 Conference**, Carthage College, Kenosha, Wisconsin. (Apr. 2011, p. 627)

25–27 **SIAM Conference on Control and Its Applications (CT11)**, Hyatt Regency Baltimore, Baltimore, Maryland. (Sept. 2010, p. 1035)

25–29 **AIM Workshop: Branching Problems for Unitary Representations**, Max Planck Institute for Mathematics, Bonn, Germany. (Nov. 2010, p. 1349)

* 25–29 **Infinite Analysis 11 –Frontier of Integrability– (in honor of Professor Michio Jimbo's 60th birthday)**, Graduate School of Mathematical Sciences, University of Tokyo, Tokyo, Japan.
Description: This is the third session in a series of "Infinite Analysis" started in 2009. The purpose this year is to advance the frontiers of integrability. The workshop focuses on developments in integrable systems and also related disciplines, for example, representation theory, differential equations and mathematical physics. We hope that our workshop will stimulate researchers in both mathematics and theoretical physics, and encourage international cooperation. There will be an arranged Poster Session by young researchers in the afternoon of July 27 (Wed). The details will be announced in our web-page.
Information: `http://sites.google.com/site/infiniteanalysis2011/`.

25–29 **IMA Special Workshop: Macaulay2**, Institute for Mathematics and its Applications (IMA), University of Minnesota, Minneapolis, Minnesota. (Feb. 2011, p. 336)

25–29 **International Conference on Applied Mathematics, Modeling and Computational Science (Laurier Centennial Conference: AMMCS-2011)**, Waterloo, Ontario, Canada. (Mar. 2011, p. 495)

25–29 **International Conference on Groups and Semigroups: Interactions and Computations**, University of Lisbon, Portugal. (Mar. 2011, p. 495)

25–29 **Non-Associative Algebras and Related Topics**, Department of Mathematics, University of Coimbra, Coimbra, Portugal. (Feb. 2011, p. 336)

25–29 **NSF-CBMS Regional Research Conference, Mathematical Epidemiology and its Applications**, East Tennessee State University, Johnson City, Tennessee. (May 2011, p. 743)

25–August 12 **International Seminar and Workshop on Weak Chaos, Infinite Ergodic Theory, and Anomalous Dynamics**, Max Planck Institute for the Physics of Complex Systems, Dresden, Germany. (Apr. 2011, p. 627)

25–August 12 **IMA Participating Institution Summer Graduate Program Topological Methods in Complex Systems**, University of Pennsylvania, Philadelphia, Pennsylvania. (Apr. 2011, p. 627)

26–29 **Conference in Harmonic Analysis and Partial Differential Equations in honour of Eric Sawyer**, Fields Institute, Toronto, Canada. (Jun./Jul. 2010, p. 786)

28–30 **International Conference on Special Functions & their Applications (ICSFA 2011)**, Department of Mathematics & Statistics, J, N. Vyas University, Jodhpur (Rajasthan) 342 005, India. (Jan. 2011, p. 84)

August 2011

* 1–4 **International Conference on Nonlinear Partial Differential Equations and Applications (2011)**, University of Science and Technology of China, Hefei, China.
Description: The conference will focus on recent developments in nonlinear partial differential equations and their applications.
Topics: Nonlinear PDES related to conformal geometry, Curvature flow problems, Nonlinear PDES related to pattern formation, and Geometric properties of solutions to nonlinear PDEs among others.
Information: `http://math.ustc.edu.cn/Conference/2011PDE/index.html`.

1–5 **2011 CBMS-NSF Conference: 3-Manifolds, Artin Groups and Cubical Geometry**, CUNY Graduate Center, New York City, New York. (Apr. 2011, p. 627)

1–5 **Categories, Geometry and Physics**, Santa Marta, Colombia. (Jun./Jul. 2010, p. 786)

1–5 **Conference in Honour of Søren Asmussen—New Frontiers in Applied Probability**, Sandbjerg Estate, Sønderborg, Denmark. (Jun./Jul. 2010, p. 786)

1–5 **Equadiff 2011**, Loughborough University, Leicestershire, United Kingdom. (Mar. 2011, p. 495)

* 1–5 **ICERM Topical Workshop: Mathematical Aspects of P versus NP and its Variants**, Institute for Computational and Experimental Research in Mathematics (ICERM), Providence, Rhode Island.
Description: This workshop will bring together computer scientists and mathematicians to examine the P v. NP problem and its variants from the perspectives of algebra, geometry, and number theory, and to introduce the mathematical aspects of these questions to a larger audience. Diverse researchers working on different aspects of these problems will clarify connections between different approaches.
Information: `http://icerm.brown.edu/topical/tw11-1-pnp/`.

* 3–12 **IMA Workshop: Mathematical Modeling in Industry XV — A Workshop for Graduate Students**, Institute for Mathematics and its Applications (IMA), University of Minnesota, Minneapolis, Minnesota.
Description: The IMA is holding a 10-day workshop on Mathematical Modeling in Industry. The workshop is designed to provide graduate students and qualified advanced undergraduates with first hand experience in industrial research. Students will work in teams of up to 6 students under the guidance of a mentor from industry. The mentor will help guide the students in the modeling process, analysis and computational work associated with a real-world industrial problem. A progress report from each team will be scheduled during the period. In addition, each team will be expected to make an oral final presentation and submit a written report at the end of the 10-day period.
Information: `http://www.ima.umn.edu/2010-2011/MM8.3-12.11/`.

* 8–10 **Higher Structures in China II**, Jilin University, Changchun, China.
Description: Recently, higher categories have appeared in many aspects of mathematics. Higher Structures in China is a series of conferences aimed at bringing together international and Chinese mathematicians in various intersecting areas of research to facilitate the exchange of ideas and further the development of applications of higher structures in their respective fields.
Information: `http://www.crcg.de/wiki/Higher_Structures_in_China_II`.

8–12 **AIM Workshop: Relating test ideals and multiplier ideals**, American Institute of Mathematics, Palo Alto, California. (Apr. 2011, p. 627)

8–13 **Formal and analytic solutions of differential and difference equations**, Mathematical Research and Conference Center in Bedlewo, Poland. (Sept. 2010, p. 1036)

8–13 **Toposym 2011: 11th Prague Topological symposium**, Prague, Czech Republic. (Jan. 2011, p. 85)

10–12 **USENIX Security 2011: 20th USENIX Security Symposium**, The Westin St. Francis, 335 Powell Street, San Francisco, California. (Feb. 2011, p. 336)

14–20 **Special Functions and Orthogonal Polynomials of Lie Groups and their Applications**, Czech Technical University in Prague, Děčín, Czech Republic. (Feb. 2011, p. 336)

* 15–16 **2nd International Conference on Software Engineering , Management & Applications (ICSEMA 2011)**, Zurich, Switzerland.
Information: `http://www.icsema.com`.

15–19 **AIM Workshop: Graph and Hypergraph Limits**, American Institute of Mathematics, Palo Alto, California. (Sept. 2010, p. 1036)

* 15–19 **ICERM Topical Workshop: Cluster Algebras and Statistical Physics**, Institute for Computational and Experimental Research in Mathematics (ICERM), Providence, Rhode Island.
Description: Cluster algebras are commutative algebras with a distinguished set of generators grouped into overlapping subsets of fixed cardinality; the generators and the relations among them are not given from the outset, but are produced by an iterative process of successive mutations. Cluster algebras encode a surprisingly widespread range of phenomena in settings as diverse as quiver representations, Teichm¸ller theory, invariant theory, tropical calculus, Poisson geometry, and polyhedral combinatorics.
Information: `http://icerm.brown.edu/topical/tw11-2-cluster/`.

17–20 **2011 Shanghai International Conference on Social Sciences (SICSS 2011)**, Shanghai, China. (Feb. 2011, p. 335)

18–20 **The First Strathmore University Mathematics Conference. "Strengthening Mathematics Research, Applications and Education in Africa"**, Strathmore University, Nairobi, Kenya. (May 2011, p. 743)

22–24 **The 3rd International Conference on Control and Optimization with Industrial Applications: COIA 2011**, Bilkent University, Ankara, Turkey. (Jan. 2011, p. 85)

22–27 **10th International Symposium on Generalized Convexity and Monotonicity (GCM10)**, Babes-Bolyai University, Cluj-Napoca, Romania. (Jan. 2011, p. 85)

* 22–September 2 **School and Conference on Algebraic Methods in Geometry: Commutative and Homological Algebra in Foliations and Singularities**, CIMAT, Guanajuato, Mexico.
Description: In honour of Xavier Gómez Mont on the occasion of his 60th birthday. The school will take place from August 22 to August 26 of 2011 and it is an introduction to subjects of current interest related to foliations.
Aim: To introduce some of its major developments and tools for its study. There will be a mixture of courses, lectures, and posters. The Conference will take place from August 29 to September 2 of 2011.
Scientific Committee: Christian Bonatti, Leticia Brambila Paz, Dominique Cerveau, Ignacio Luengo, Rick Miranda, José A. Seade, Alberto Verjovsky, Jorge Vitório Pereira, Yulij S. Ilyashenko.
Organizing Committee: V. Castellanos Vargas, M. Cruz López, P. L. del Ángel Rodríguez, E. López González, P. E. Jimenez Gallegos, J.R. Muciño Raymundo, L. Ortiz Bobadilla, C. Reynoso Alcántara, E. Rosales González, F. Sánchez Bringas.
Information: `http://www.matem.unam.mx/gomezmont/`.

22–December 21 **Multiscale Numerics for the Atmosphere and Ocean**, Isaac Newton Institute for Mathematical Sciences, Cambridge, United Kingdom. (Apr. 2011, p. 628)

29–31 **14th International Conference on Computer Analysis of Images and Patterns (CAIP 2011)**, Escuela Tecnica Superior de Ingenieria Informatica, University of Seville, Seville, Spain. (Mar. 2011, p. 495)

* 29–September 1 **2011 MBI Workshop for Young Researchers in Mathematical Biology (WYRMB)**, Mathematical Biosciences Institute, The Ohio State University, Columbus, Ohio.
Description: The workshop is intended to broaden the scientific perspective of young researchers (primarily junior faculty, postdocs, and senior graduate students) in mathematical biology and to encourage interactions with other scientists. Workshop activities include plenary talks and poster sessions, as well as group discussions on issues relevant to mathematical biologists. We cordially invite young researchers to apply for participation in the workshop! All invitees will be expected to present a poster, and a select number will be invited to give short talks. Limited funding for travel expenses will be provided by the MBI on a competitive basis.
Deadline: Application: May 15, 2011.
Information: `http://mbi.osu.edu/wyrmb/wyrmb2011.html`.

29–September 2 **11th International Workshop on Orthogonal Polynomials, Special Functions, and Applications**, Universidad Carlos III de Madrid, c/Universidad, 30 28911 Leganes, Madrid, Spain. (Nov. 2010, p. 1349)

29–September 2 **European Conference on Combinatorics, Graph Theory and Applications 2011**, Alfred Renyi, Institute of Mathematics, Budapest, Hungary. (Jan. 2011, p. 85)

* 29–September 3 **Walkshop "Groups in Action in Harz"**, Sankt Andrasberg, Germany.
Description: The conference starts in Sankt Andreasberg on Monday evening; moves to Braunlage on Wednesday afternoon; and to Schierke on Thursday afternoon. An excursion to the "Brocken" can be made on Friday.
Topic: "Groups in action", and covers recent developments in group theory; in particular, embeddings and actions of discrete groups in Lie groups, l^p spaces, diffeomorphism groups of manifolds. The mornings are reserved for talks, and afternoons for free discussions, excursions (Wednesday and Thursday), hikes towards the next village. A minibus carries the whiteboards, luggage, and participants that do not wish to walk.
Main Speakers: Miklos Ábert, Yves de Cornulier, Martin Kassabov, Andres Navas, Yuval Peres.
Information: `http://sites.google.com/site/harz2011/`.

* 30–31 **International Conference on Communication Technology & Networking Services (ICCTNS 2011)**, Taipei, Taiwan.
Description: Due to the grand success of ICCTNS 2010 held in Kathmandu, Nepal, on December 6–7, 2010, The Open Learning Society (OLS) is purposed at bringing about the ICCTNS 2011 at Taipei, Taiwan. Through the creation and massive utilization of knowledge shared through interactive programs like conferences, seminars, it aims at implementing the recent development trends in places of need.
Information: `http://www.icccnt.com`.

31–September 3 **XX International Fall Workshop on Geometry and Physics**, Instituto de Ciencias Matemáticas, ICMAT (CSIC-UAM-UC3M-UCM), Madrid, Spain. (May 2011, p. 743)

September 2011

1–3 **Algebraic Representation Theory Conference**, Uppsala University, Uppsala, Sweden. (Mar. 2011, p. 496)

* 1–3 **Elementary Geometry from an Advanced Point of View**, University of Aveiro, Aveiro, Portugal.
Description: The general aim of the conference is to present several contemporary perspectives on Geometry including, among others, talks on visualization, applications and surveys, both at elementary and more advanced levels. The goal is to contribute to the current international reflection on the ICMI/IMU Klein Project concerning central topics on Geometry, its contents, interdisciplinary connections and approaches for the teaching of this mathematics discipline at senior secondary school and first years at University level. The conference includes the Workshop "EnsGeo I" to be held on September 3 (last day of the conference), focused on dissertation works in Geometry carried out on Graduate courses in Mathematics for Teaching.
Information: `http://c3.glocos.org/egapv2011/`.

3–9 **10th International Conference on Geometry and Applications**, Geometrical Society "Boyan Petkanchin", Sofia, Bulgaria. (Apr. 2011, p. 628)

5–9 **AGMP&MP2 Summer School: Mathematical Topics in Quantum Mechanics and Quantum Information**, Tjärnö, Sweden. (May 2011, p. 743)

* 5–9 **5th International Conference on Stochastic Analysis and Applications**, Bonn, Germany.

Description: The main topics of the conference will be (but not limited to) A. Dirichlet forms and stochastic analysis B. Jump processes C. Stochastic partial differential equations D. Stochastic analysis and geometry E. Optimal transport and allocation problems F. Functional analysis G. Random media, percolation clusters and fractals H. Stochastic models in physics and biology. These areas are strongly related to each other and have been very active in recent years. They occupy a central place in modern probability theory and analysis. The primary goal of the conference is to bring researchers in areas listed above, from all over the world, to survey the fields, exchange ideas and to foster future collaborations. Another important goal is to expose young researchers and Ph.D. students to the most recent developments in active areas of probability theory. Deadline for submitting a talk: May 1st. Conference fee of 90(euro) for non-students and 60 for students to be paid on arrival.
Information: `http://icsaa.iam.uni-bonn.de/`.

* 5–9 **Nonlinear Dynamics Conference in Memory of Alexei Pokrovskii**, University College Cork, Cork, Ireland.
Description: The aim of this meeting is to honour the memory and work of Alexei Pokrovskii, an eclectic mathematician and a pioneer in the mathematical theory of systems with hysteresis, who unexpectedly died on September 1, 2010, aged 62. For the last nine years Alexei was Professor and Head of Department of Applied Mathematics at University College Cork. Topics of this conference will reflect the diversity of Alexei's contribution to science and will include nonlinear dynamical systems, systems with hysteresis, chaos and complexity, stochastic systems, control theory, nonlinear functional analysis, singularly perturbed systems and mathematical modeling for applications in engineering, physics, biology and economics. Sponsors: KE Consulting Group (Cork), Tyndall National Institute (Cork).
Information: `http://www.ucc.ie/en/euclid/NonlinearDynamicsConferenceinMemoryofAlexeiPokrovskii/`.

* 5–9 **RSME2011 Congress of Young Researchers**, Campus Duques de Soria Universidad de Valladolid soria, Spain.
Description: Congress devoted to young researchers in mathematics in ocasion of the Spanish Royal Mathematical Society centenary.
Information: `http://www.jirsme.uva.es/`.

5–10 **Toric Topology and Automorphic Functions**, Pacific National University, Khabarovsk, Russia. (May 2011, p. 743)

* 7–9 **Fourth International Workshop on Analysis and Numerical Approximation of Singular Problems (IWANASP 2011)**, University of Chester, Chester, United Kingdom.
Description: The mathematical modelling of physical problems often leads to differential or integral equations whose coefficients have singularities. The numerical treatment of such models requires: a) the correct formulation of problems and the analysis of qualitative behavior of solutions; b) the use of efficient numerical algorithms which take into account the singularities of the problems. The main objective of the present workshop is to bring together mathematicians who deal with problems of this kind in different fields. The workshop will focus on the asymptotic properties of the solutions of equations and discretization methods. The themes that will be discussed include: mathematical modelling of physical phenomena involving singularities; numerical solution of singular boundary value problems for ordinary differential equations; numerical integration of functions with singularities; computational methods for integral equations with singular kernels.
Information: `http://www.stochasticdelay.org.uk/IWANASP`.

* 7–9 **ICERM Semester Program - Kinetic Theory: Analysis and Computation**, ICERM, Providence, Rhode Island.
Description: This semester-long program in kinetic theory and computation will provide the participants with an introduction to a broad range of analytical and computational aspects of kinetic theory. The program will be centered around three broad topics, for each of which an international workshop will be held.
Information: `http://icerm.brown.edu/sp-f11/`.

7–9 **IMA Hot Topics Workshop: Instantaneous Frequencies and Trends for Nonstationary Nonlinear Data**, Institute for Mathematics and its Applications (IMA), University of Minnesota, Minneapolis, Minnesota. (Oct. 2010, p. 1166)

* 8–9 **New York Workshop on the Symmetric Group and Related Topics**, CUNY Graduate Center, Manhattan, New York.
Description: This workshop will focus on current research and new developments in the study of the symmetric groups. Topics will include: ordinary and modular theory of the symmetric groups and related groups, graded and hecke algebras, connections with geometry, and combinatorics, number theory and partitions.
Information: `http://symmetricgroup.commons.gc.cuny.edu/2010/10/03/ny-workshop-on-the-symmetric-group-and-related-topics/`.

* 9–10 **IV International Conference named by academician Ivan I. Lyashko—Computational and Applied Mathematics**, Deparment of Cybernetics of the Kiev National Taras Shevchenko University, Kiev, Ukraine.
Description: Main areas of the conference: computational mathematics optimal control and theory of extremal problems mathematical modelling filtration theory.
Information: `http://om.univ.kiev.ua/conf/`.

10–11 **AMS Eastern Section Meeting**, Cornell University, Ithaca, New York. (Sept. 2010, p. 1036)

10–17 **International Conference "Harmonic Analysis and Approximations, V"**, Tsaghkadzor, Armenia. (Mar. 2011, p. 496)

11–15 **Israeli-Polish Mathematical Meeting**, University of Łódz´, Poland. (May 2011, p. 743)

11–17 **14th International Conference on Functional Equations and Inequalities**, Mathematical Research and Conference Center, Bedlewo (near Poznan), Poland. (Apr. 2011, p. 628)

11–17 (NEW DATE) **Turning Dreams into Reality: Transformations and Paradigm Shifts in Mathematics Education**, Rhodes University, Grahamstown, South Africa. (Feb. 2010, p. 307)

* 12–14 **Workshop on Fluid Dynamics in Porous Media**, Department of Mathematics of University of Coimbra, Coimbra, Portugal.
Description: The workshop on Fluid Dynamics in Porous Media is an initiative of the UT Austin–Portugal programme for Mathematics, in partnership with CMUC (Centre for Mathematics of University of Coimbra). The event focuses on mathematical models and numerical simulation in fluid dynamics in porous media bringing together mathematicians, engineers, geoscientists, and computing experts in a cooperative environment.
Invited speakers: Faruk Civan, University of Oklahoma, USA; Vivette Girault, Université Pierre et Marie Curie, France; Majid Hassanizaded, Utrecht University, Netherlands; Andro Mikelic, Université Claude Bernard Lyon 1, France; Roland Muggli, Galp E&P, Portugal; Paula de Oliveira, University of Coimbra, Portugal; Daniel Tartakovsky, University of California, USA.
Information: `http://www.mat.uc.pt/~wfdpm`.

12–16 **8th International Conference on Combinatorics on Words, WORDS 2011**, Czech Technical University in Prague, Prague, Czech Republic. (Dec. 2010, p. 1498)

12–16 **25th IFIP TC 7 Conference on System Modeling and Optimization**, University of Technology, Berlin, Germany. (Nov. 2010, p. 1349)

12–16 **AIM Workshop: L^2 invariants and their relatives for finitely generated groups**, American Institute of Mathematics, Palo Alto, California. (Aug. 2010, p. 905)

12–16 **Mathematical and Computational Approaches in High-Throughput Genomics**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California. (Sept. 2010, p. 1036)

12–16 **Satellite Meeting Unravelling and Controlling Discrete Dynamical Systems**, Vienna, Austria. (May 2011, p. 743)

* 17–18 **40 Years and Counting: AWM's Celebration of Women in Mathematics**, Brown University, Providence, Rhode Island.
Description: 40 Years and Counting: 2011 is the 40th anniversary of the Association for Women in Mathematics (AWM). With this conference, AWM continues to celebrate the progress of women in mathematical professions and to recognize individual achievements. Join us this fall on the Brown University campus in historic Providence, RI.
Organizing Committee: Georgia Benkart (University of Wisconsin-Madison), Kristin Lauter (Microsoft Research), Jill Pipher (Brown University/ICERM).
Plenary Speakers: Andrea Bertozzi (UCLA), Laura DeMarco (University of Illinois at Chicago), Barbara Keyfitz (The Ohio State University), Hee Oh (Brown University).
Information: http://icerm.brown.edu/events/awm-anniversary-2011/.

* 18–22 **Getting started with PDE**, Technion- I.I.T., Haifa, Israel.
Description: The workshop's aim is to introduce graduate and advanced undergraduate students to a variety of subjects of current research in Partial Differential Equations and Applied Mathematics. The required prerequisites are familiarity with the basic material taught in undergraduate courses in mathematics.
Information: http://www.math.technion.ac.il/cms/decade_2011-2020/year_2010-2011/summer-workshop/.

18–24 **8th International Conference on Function Spaces, Differential Operators, Nonlinear Analysis (FSDONA-2011)**, Tabarz/ Thuringia, Germany. (Apr. 2011, p. 628)

19–23 **The Sixteenth Asian Technology Conference in Mathematics (ATCM 2011)**, Abant izzet Baysal University, Bolu, Turkey. (Dec. 2010, p. 1498)

* 19–23 **Workshop: Vlasov Models in Kinetic Theory**, ICERM, Providence, Rhode Island.
Description: Vlasov-type models deal with continua of particles where the electric charges dominate the collisions, so that the collisions are ignored. They occur in physical plasmas, including astrophysical plasmas and fusion reactors. There are many examples of astrophysical plasmas of this type, such as the solar wind. When a fusion reactor is very hot, the relevant times scales are so short that collisions can be ignored. Vlasov theory also models systems where the dominant force is gravity, such as clusters of stars or galaxies.
Information: http://icerm.brown.edu/sp-f11/workshop-1.php.

19–25 **ICNAAM2011 Symposium: Semigroups of Linear Operators and Applications**, G-Hotels, Halkidiki, Greece. (Feb. 2011, p. 336)

19–26 **Conference on Geometric Structures in Mathematical Physics**, Albena, Bulgaria. (Apr. 2011, p. 628)

* 22–25 **16th annual cum 2nd International Conference of Gwalior Academy of Mathematical Sciences (GAMS) & 2nd International conference of bioinformatics**, S. S. Dempo College of Commerce and Economics, Altinho Panaji Goa, India.
Description: (GAMS) is a forum for activities on Mathematical Modeling of real life problems, launched in 1994 at School of Mathematics and Allied Sciences, Jiwaji University, Gwalior, India. Besides publishing "GAMS Journal of Mathematical Biosciences", it holds annual conferences every year, international conference every 4 years, organizes workshops and similar research level programs. Now GAMS is going to organize the Second International Conference besides the 16th Annual Conference. Original research papers are invited for presentation at the conference. Intended participants are invited to submit abstracts not exceeding 250 words on A-4 size paper in double spacing by April 30, 2011. Acceptance of the papers will be sent by June 6 2011. The abstracts may also be sent by email to: bioinfo2011@gmail.com.
Organizers: The event is jointly organized by GAMS and S. S. Dempo College of Commerce and Economics, Altinho, Panaji Goa.
Information: http://www.gamsinfo.com.

* 22–25 **The 19th conference on applied and industrial mathematics CAIM 2011**, "Ion Ionescu de la Brad" University, Mihail Sadoveanu Lane, 3, 700490, Iasi, Romania.
Description: The Romanian Society of Applied and Industrial Mathematics (ROMAI), "Ion Ionescu de la Brad" University of Agricultural Sciences and Veterinary Medicine of Iasi (UASVM), The Academy of Romanian Scientists (ARS), the Faculty of Mathematics of the "Al. I. Cuza" University of Ias (UAIC), "Gh. Asachi" Technical University of Iasi (TUI) and the Romanian Mathematical Society, Iasi branch announce the organization of the 19th Conference on Applied and Industrial Mathematics – CAIM 2011.
Information: http://www.uaiasi.ro/caim2011/.

24–25 **AMS Western Section Meeting**, Wake Forest University, Winston Salem, North Carolina. (Sept. 2010, p. 1036)

* 25–October 2 **II International Conference "Optimization and Applications" (OPTIMA-2011)**, Petrovac, Montenegro.
Call for participation: September 25–October 2, 2011, Montenegrian Academy of Sciences and Arts, University of Montenegro and Dorodnicyn Computing Center Russian Academy of Sciences will organize Second International conference "Optimization and Applications" (OPTIMA-2011). Abstracts of the talks are invited.
Conference themes: Mathematical programming, Global optimization, Nondifferential Optimization, Integer Programming and Combinatirial Optimization, Multicriteria Optimization, Equilibrium programming, Game Theory, Optimal Control, Applications in natural sciences, engineering, economics, biology, medicine, etc.
Information: http://www.ccas.ru/optima2011/.

26–30 (NEW DATE) **IMA Workshop: High Dimensional Phenomena**, Institute for Mathematics and its Applications (IMA), University of Minnesota, Minneapolis, Minnesota. (Oct. 2010, p. 1166)

* 26–30 **Mathematical Oncology: New Challenges for Systems Biomedicine**, "Ettore Majorana" Foundation and Centre for Scientific Culture Erice, Sicily, Italy.
Description: Tumors are dynamical diseases characterized by multiple scales. The complexity of clinical-genomic tumor-related data implies that biostatistics and bioinformatics analyses are no more sufficient to cope with such data in order to explain them and to produce predictions. Mechanistic models of biomedical phenomena with complex outputs must be built, to open the road for tailored therapies. This is a huge challenge for contemporary mathematics. Infos to submit contributed talks, registration etc.: http://www.dm.unipi.it/~erice2011/.
Invited Speakers: Z. Agur, R. Barbuti, N. Bellomo, F. Bonino, H. Byrne, V. Capasso, F. Castiglione, O. Demin, A. d'Onofrio, D. Drasdo, H. Enderling, A. Fasano, G. Finocchiaro, A. Friedman, K. Hicks, A. Iudice, B. Kholodenko, U. Ledzewicz, A. Palladini, L. Preziosi, A. Radunskaya, P. T. Ram, B. Ribba, A. Sigal, R. V. Solé, P. Ubezio.
Organizers: Z. Agur, P. Cerrai, A. d'Onofrio, A. Gandolfi.
Information: http://www.dm.unipi.it/~erice2011/.

* 28–30 **Balance, Boundaries and Mixing in the Climate Problem**, Centre de recherches mathématiques, Université de Montréal, Pavillon André-Aisenstadt, 2920, Chemin de la tour, 5th floor, Montréal, Québec, H3T 1J4 Canada.

Description: Turbulent mixing in the atmosphere and oceans will be the focus. Although crucial to coarse-resolution numerical modelling efforts, much of it really occurs below typical grid scales. Recent progress in geophysical fluid dynamics will be brought to the community that can benefit from it to improve their integrated studies of complex environmental systems.
Information: `http://www.crm.umontreal.ca/Mix11/index_e.php`.

October 2011

3 **Philosophy of Information**, American University, Washington, District of Columbia. (Mar. 2011, p. 496)

4–6 **Sixth International Workshop Meshfree Methods for Partial Differential Equations**, Universitätsclub Bonn, Bonn, Germany. (Apr. 2011, p. 628)

7–9 **ICMA2011: The Third Conference on Mathematical Modeling and Analysis of Populations in Biological Systems**, Trinity University, San Antonio, Texas. (May 2011, p. 743)

10–14 **AIM Workshop: Weighted singular integral operators and non-homogenous harmonic analysis**, American Institute of Mathematics, Palo Alto, California. (May 2011, p. 744)

* 10–14 **International conference "Kolmogorov Readings: General control problems and their applications (GCP - 2011)"**, Tambov State University named after G. R. Derzhavin, Tambov, Russia.
Description: The conference is the 5th one in the series "Kolmogorov readings" gathering international scientists in the city where the outstanding mathematician, A. N. Kolmogorov, was born. Traditionally the conference will mainly focus on general control problems and their applications in natural and human sciences, optimization theory, differential equations and inclusions. There are planned plenary (40 min.) and sectional (20 min.) talks, as well as a school on optimal control aimed to Ph.D. students and young researches.
Information: `http://www.tambovopu2011.narod.ru/`.

10–14 **International Conference on Scientific Computing 2011 (SC2011) dedicated to Claude Brezinski and Sebastiano Seatzu on the occasion of their 70th birthday**, S. Margherita di Pula, Sardinia, Italy. (Mar. 2011, p. 496)

11–14 **Mal'tsev Meeting**, Sobolev Institute of Mathematics, Novosibirsk, Russia. (Apr. 2011, p. 628)

14–16 **AMS Central Section Meeting**, University of Nebraska-Lincoln, Lincoln, Nebraska. (Sept. 2010, p. 1036)

* 17–21 **Workshop: Applications of Kinetic Theory and Computation**, ICERM, Providence, Rhode Island.
Description: There are several fundamental applications involving kinetic theory and computations. They range from semiconductor modeling involving kinetic and quantum charged transport, radiative transfer in cosmology, conservative and dissipative phenomena in rarefied gas dynamics in mixtures, and grain and polymer flows.
Information: `http://icerm.brown.edu/sp-f11/workshop-2.php`.

22–23 **AMS Western Section Meeting**, University of Utah, Salt Lake City, Utah. (Sept. 2010, p. 1036)

24–26 **Algebra Geometry Mathematical Physics**, University of Haute Alsace, Mulhouse, France. (May 2011, p. 744)

* 24–27 **SIAM Conference on Geometric and Physical Modeling (GD/SPM11)**, Wyndham Orlando Resort, Orlando, Florida.
Description: The 2011 SIAM Conference on Geometric and Physical Modeling seeks high quality, original research contributions that strive to advance all aspects of geometric and physical modeling, and their application in design, analysis and manufacturing, as well as in biomedical, geophysical, digital entertainment, and other areas. A shared objective of both the SIAM GD and ACM SPM communities is a desire to highlight work of the highest quality on the problems of greatest relevance to industry and science. In ACM SPM tradition, the conference will include a track for submission of peer-reviewed technical papers for those wishing rigorous peer review and published proceedings. In addition, in the tradition of previous SIAM GD events, abstracts for minisymposia and contributed talks/posters are solicited.
Information: `http://www.siam.org/meetings/gdspm11/`.

24–28 **AIM Workshop: The Kardar-Parisi-Zhang equation and universality class**, American Institute of Mathematics, Palo Alto, California. (May 2011, p. 744)

24–28 **Galois Conference: Bicentennial of Evariste Galois's birth Colloque Galois—Bicentenaire de la naissance d'Évariste Galois**, Institut Henri Poincaré, Paris, France.

24–28 **IMA Workshop: Large Graphs: Modeling, Algorithms and Applications**, Institute for Mathematics and its Applications (IMA), University of Minnesota, Minneapolis, Minnesota. (Oct. 2010, p. 1166)

* 24–28 **Heritage of E. Galois's work**, Institut Henri Poincaré, Paris, France.
Description: Institut Henri Poincaré and Société Mathématique de France will organize a conference to illustrate the heritage of E. Galois's work, both from a mathematical and an historical point of view.
Information: `http://www.galois.ihp.fr/`.

* 24–28 **Optimization, Theory, Algorithms and Applications in Economics**, Centre de Recerca Matematica, Universitat Autonoma de Barcelona, Barcelona, Spain.
Description: This international conference is a tribute to Juan-Enrique Martinez-Legaz on the occasion of his 60th birthday. It has the aim of a top-level scientific meeting to promote a broad exchange of information and new developments in the areas of Functional and Variational Analysis, Control, Optimization and Applications in Economics. The event will include two half-day mini workshops (Nuclei) in Economics and a mini-workshop in applications of Optimization in Engineering.
Information: `http://mat.uab.cat/~opt2011`.

* 26–29 **Integers Conference 2011**, University of West Georgia, Carrollton, Georgia.
Description: The editors of Integers are pleased to announce the Integers Conference 2011. The Integers conferences are international conferences in combinatorial number theory, held for the purpose of bringing together mathematicians, students, and others interested in combinatorics and number theory.
Plenary Speakers: Ken Ono, Emory University; Carla Savage, North Carolina State University; Laszlo Szekely, University of South Carolina; Frank Thorne, University of South Carolina; Julia Wolf, Ecole Polytechnique. Organizers: Bruce Landman, Melvyn Nathanson, Jaroslav Nesetril, Richard Nowakowski, Carl Pomerance.
Information: `http://www.westga.edu/~math/IntegersConference2011/`.

31–November 4 **AIM Workshop: Geometry of large networks**, American Institute of Mathematics, Palo Alto, California. (May 2011, p. 744)

November 2011

* 1–3 **Central and Eastern European Software Engineering Conference in Russia (CEE-SECR 2011)**, Moscow, Russian Federation.
Description: CEE-SECR is the premier software engineering conference in Russia. Up to 1000 participants from over 20 countries are expected to attend the event in 2011, presenting and discussing innovations, trends, results, experiences and concerns in the field of software engineering. The conference will be composed of research presentations, experience reports, poster presentations, panel discussions, workshops, invited presentations, and keynote lectures. At previous conferences, keynote/invited/panel speakers have included: Bjarne Stroustrup, Thomas Erl, Richard Soley, Igor

Agamirzian, Grady Booch, Lars Bak, Yuri Gurevich, Alexander L. Wolf, Erich Gamma, Victor Ivannikov, Stephen Mellor, Rick Kazman, Larry L. Constantine, Ivar Jacobson, Mark Paulk, Michael Cusumano, and other distinguished figures.
Information: http://www.cee-secr.org.

* 1–3 **International Seminar on the Application of Science and Mathematics 2011**, Putra World Trade Centre, Kuala Lumpur, Malaysia.
Description: Introduction ISASM 2011 aims to provide an international forum for researchers to present and discuss recent advances and new techniques in science and mathematics and its applications. Seminar Objectives (i) To provide a platform for the exchange of new ideas and interaction between local and international participants, updating the latest research in application of science and mathematics. (ii) To bring together researcher and scientist in promoting and enhancing research collaboration among local and international participants. (iii) To encourage and stimulate publications in the areas of application of science and mathematics.
Official Languages: English and Malay.
Information: http://uhsb.uthm.edu.my/isasm2011/.

1–5 **International conference of Settat on Operator algebras and applications**, Faculty of Sciences and Techniques, University Hassan I. Settat, Morocco. (Mar. 2011, p. 496)

* 5–6 **Second International Conference on Biologically Inspired Cognitive Architectures (BICA 2011)**, Holiday Inn Arlington, Arlington, VA, next to Washington DC.
Description: The challenge of creating a real-life computational equivalent of the human mind calls for our joint efforts to better understand at a computational level how natural intelligent systems develop their cognitive and learning functions.The focus of BICA 2011 includes: BICA models of robust learning mechanisms; models of perception, cognition and action; emotional and social intelligence in artifacts; vital constraints informed by neuroscience, human-like episodic and semantic memory; metacognition, human-like self-regulated learning, bootstrapped and meta-learning; language acquisition and symbol grounding; the "critical mass" for cognitive growth in a learning environment, scalability of learning; the roadmap to solving the challenge.
Information: http://bicasociety.org/2011/.

* 5–8 **ICMIS2011: 2nd International Conference on Mathematics & Information Science**, Sohag University, Sohag, Egypt.
Description: The conference will feature advances in mathematical science and technology presented by leading African and international researchers and will provide the opportunity to showcase research in mathematics, theoretical physics and information science and technology to engender dialogue and collaboration between Egyptian and international researchers. The conference is a part of a series of conferences dedicated to bringing top scientists and technologists to Egypt thereby helping to raise Egyptian science and technology to the highest international standard, raise awareness of governments and industry in Egypt of the importance and excitement of new research and development in technologies, and engender collaborations and research exchanges. The first conference of this Conference Series was "International Conference on Mathematics and Information Security, Sohag, Egypt, November 2009".
Information: http://www.naturalspublishing.com/sohag2011/.

7–11 **AIM Workshop: The Klein project**, American Institute of Mathematics, Palo Alto, California. (May 2011, p. 744)

* 7–11 **Workshop: Boltzmann Models in Kinetic Theory**, ICERM, Providence, Rhode Island.
Description: The focus of the program is to bring computational and theoretical people together to investigate problems of fundamental importance.
Information: http://icerm.brown.edu/sp-f11/workshop-3.php.

* 7–11 **Waves in Science and Engineering WISE 2011**, Mexico City, Mexico.
Description: It is intended to bring together experts from different fields of the general area of classical wave theory and applications including acoustic, electromagnetic, and elastic wave propagation. The mathematical and numerical modeling procedures in these fields contribute to a considerable number of applied physical and engineering problems, over a large range of length scales. Among these are problems in sonar, radar, medical imaging, detection, materials, and wave interactions with surfaces and obstacles. The conference will cover many of the current mathematical and numerical techniques that are applied across disciplines. Mathematicians, physicists, and engineers of varying backgrounds and occupations will present recent developments in wave phenomena in science and engineering.
Information: http://www.wise.ipn.mx.

12 **Information Theory and Shrinkage Estimation**, American University, Washington, District of Columbia. (Mar. 2011, p. 496)

14–18 **IMA Workshop: Large Data Sets in Medical Informatics**, Institute for Mathematics and its Applications (IMA), University of Minnesota, Minneapolis, Minnesota. (Nov. 2010, p. 1349)

17–18 **Jornadas de Criptografía (Spanish Cryptography Days)**, Murcia, Spain. (May 2011, p. 744)

19–21 **International Conference on Analysis and its Applications**, Department of Mathematics, Aligarh Muslim University, Aligarh, India. (Dec. 2010, p. 1498)

* 28–December 2 **School of Applied Mathematics and Innovation 2010: Celestial Mechanics and Computing Orbits**, Campus Rodrigo Noguera Laborde of Universidad Sergio Arboleda, Carrera 29D 30 - 207 Troncal del Caribe Santa Marta, Colombia.
Description: This doctoral school will be devoted to study of techniques in Celestial Mechanics and Computer Orbits.
Lecturers: Alain Chenciner, Observatoire de Paris, Paris, France. Rafael Ortega, Universidad de Granada, Granada, Spain. Juan Ramón Pacha, Universitat Politécnica de Catalunya, Barcelona, Spain.
Organizing Committee: Luz Myriam Echeverry, Universidad Sergio Arboleda, Universidad de los Andes, Colombia. Andres Mauricio Rivera, Pontificia Universidad Javeriana, Cali, Colombia.
Scientific Committee: Primitivo Acosta-Humánez, IMA-Universidad Sergio Arboleda, Colombia. Amadeu Delshams, Universitat Politécnica de Catalunya, España. Juan Morales-Ruiz, Universidad Politécnica de Madrid, España.
Information: http://ima.usergioarboleda.edu.co/SAMI/SAMI2011.htm.

December 2011

1–April 1 **Call for papers: A special issue of *Symmetry* (ISSN 2073-8994) "Symmetry in Probability and Inference"**, *Symmetry Journal*, MDPI Publishing, Basel, Switzerland. (Mar. 2011, p. 497)

2–4 **Introduction to Neutrosophic Physics: Unmatter & Unparticle**, The University of New Mexico, Mathematics & Sciences Department, 200 College Rd., Gallup, New Mexico. (Mar. 2011, p. 497)

4–9 **LISA'11: 25th Large Installation System Administration Conference**, Sheraton Boston Hotel at 39 Dalton St., Boston, Massachusetts. (Feb. 2011, p. 336)

* 5–7 **International symposium on recurrence plots**, Hong Kong Polytechnic University, Hong Kong, China.
Description: The objective of the symposium is to encourage the exchange of knowledge among scientists working in the disciplines of time and spatial series analyses. Recurrence plots and recurrence quantifications are general methods for visualising and analysing

both linear and nonlinear time series data. We continue to witness many new technical developments related to recurrence plots. Some of these include: a framework to treat recurrence plots as a network from which one can obtain network-related statistics; inferring directional couplings; obtaining confidence intervals. In addition, applications of recurrence plots are increasing in many areas ranging from, e.g., physiology over climate to financial systems. This symposium will provide a unique forum to help combine the recent theoretical developments in recurrence science with applications from various fields. We welcome both theoretical and applied contributions that use recurrence related methodologies.
Information: http://symposium.recurrence-plot.tk.

5–9 **AIM Workshop: Stability, hyperbolicity, and zero localization of functions**, American Institute of Mathematics, Palo Alto, California. (May 2011, p. 744)

* 12–16 **AIM Workshop: Singular learning theory, connecting algebraic geometry and model selection in statistics**, American Institute of Mathematics, Palo Alto, California.
Description: This workshop, sponsored by AIM and the NSF, will be devoted to singular learning theory, the application of algebraic geometry to problems in statistical model selection and machine learning.
Information: http://www.aimath.org/ARCC/workshops/modelselection.html.

14–16 **5th Indian International Conference on Artificial Intelligence**, Tumkur (near Bangalore), India. (Mar. 2011, p. 497)

* 15–17 **Applied Mathematics & Stochastic Processes**, Sacred Heart College, Tamilnadu, India.
Description: International conference on stochastic processes was proposed by the department of mathematics.
Information: http://www.shctpt.edu.

16–18 **The International Congress on Science and Technology**, Allahabad, U.P., INDIA (Oct. 2009, p. 1148)

* 17–18 **1st International Conference on Mathematical Sciences and Applications**, India Habitat Centre, Lodhi Road, New Delhi, India.
Call for Papers: The International Conference on Mathematical Science and Applications (ICMSA-2011) is a premier forum for the presentation of new advances and research results in all areas of Mathematical Sciences and Applications. ICMSA-2011 will bring together leading researchers, engineers and scientists in the domain of interest from around the world. Leading Mathematicians around the world shall deliver Key Note Addresses and chair sessions.
Information: http://ijmsa.yolasite.com/conference-announcement.php.

17–18 **The International Symposium on Biomathematics and Ecology: Education and Research (BEER-2011)**, University of Portland, Portland, Oregon. (Apr. 2011, p. 628)

* 18–20 **The 5th International Conference of IMBIC on "Mathematical Sciences for Advancement of Science and Technology" (MSAST 2011)**, Institute for Mathematics, Bioinformatics, Information Technology and Computer Science (IMBIC), Salt Lake, Kolkata, India.
Description: All the authors are requested to submit the full original papers related to the Theme of the Conference: "Mathematical Sciences for Advancement of Science and Technology" to the Secretary of IMBIC: Dr Avishek Adhikari; email: avishek.adh@gmail.com, by September 1, 2011. All the papers are to be screened for presentation in the Conference.
Information: http://imbic.org/forthcoming.html.

January 2012

4–7 **Joint Mathematics Meetings**, Boston, Massachusetts. (May 2011, p. 744)

* 9–13 **AIM Workshop: Mapping theory in metric spaces**, American Institute of Mathematics, Palo Alto, California.
Description: This workshop, sponsored by AIM and the NSF, will be devoted to mappings between metric spaces and the recent advances on basic questions concerning uniqueness, extendability, embeddability, uniformization and extremality of mappings in a variety of regularity classes.
Information: http://aimath.org/ARCC/workshops/mappingmetric.html.

9–July 6 **Semantics and Syntax: A Legacy of Alan Turing**, Isaac Newton Institute for Mathematical Sciences, Cambridge, United Kingdom. (Apr. 2011, p. 629)

17–19 **ACM-SIAM Symposium on Discrete Algorithms (SODA12)**, The Westin Miyako, Kyoto, Japan. (Apr. 2011, p. 628)

23–27 **AIM Workshop: Set theory and C^*-algebras**, American Institute of Mathematics, Palo Alto, California. (May 2011, p. 744)

* 30–May 4 **ICERM Semester Program: Complex and Arithmetic Dynamics**, ICERM, Providence, Rhode Island.
Description: The goal of this program is to bring together researchers in complex dynamics, arithmetic dynamics, and related fields, with the purpose of stimulating interactions, promoting collaborations, making progress on fundamental problems, and developing theoretical and computational foundations on which future work will build.
Information: http://icerm.brown.edu/sp-s12/.

February 2012

* 13–17 **ICERM Workshop: Complex and p-adic Dynamics**, ICERM, Providence, Rhode Island.
Description: This workshop will bring together researchers working in classical complex dynamics and in the newer area of p-adic (nonarchimedean) dynamics. It will promote interactions between the two groups by highlighting the similarities and differences between complex and p-adic dynamics. In particular, it will address Berkovich space, whose introduction has greatly enhanced the exchange of ideas between complex and p-adic dynamics.
Information: http://icerm.brown.edu/sp-s12/workshop-1.php.

13–17 **The 10th International Conference on Monte Carlo and Quasi-Monte Carlo Methods in Scientific Computing (MCQMC 2012)**, The University of New South Wales, Sydney, NSW, Australia. (Apr. 2011, p. 629)

* 20–24 **AIM Workshop: Stochastic dynamics of small networks of neurons**, American Institute of Mathematics, Palo Alto, California.
Description: This workshop, sponsored by AIM and the NSF, will be devoted to the area between carefully crafted stochastic models of single neurons and large networks of simpler ones.
Information: http://www.aimath.org/ARCC/workshops/neuronnetwork.html.

27–March 2 **IMA Workshop: Network Links: Connecting Social, Communication and Biological Network Analysis**, Institute for Mathematics and its Applications (IMA), University of Minnesota, Minneapolis, Minnesota. (Jan. 2011, p. 85)

March 2012

3–4 **AMS Western Section Meeting**, University of Hawaii, Honolulu, Hawaii. (May 2011, p. 744)

5–9 **5th International Conference on High Performance Scientific Computing**, Institute of Mathematics, Vietnam Academy of Science and Technology, 18 Hoang Quoc Viet Road, Hanoi, Vietnam. (Dec. 2010, p. 1498)

10–11 **AMS Southeastern Section Meeting**, University of South Florida, Tampa, Florida. (May 2011, p. 744)

12–15 **Computational Methods in High Energy Density Plasmas**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California. (Mar. 2011, p. 497)

12–16 **AIM Workshop: Classifying fusion categories**, American Institute of Mathematics, Palo Alto, California. (May 2011, p. 744)

* 12–16 **ICERM Workshop: Global Arithmetic Dynamics**, ICERM, Providence, Rhode Island.
Description: The aim of this workshop is to bring together leading researchers in global arithmetic dynamics and related fields to discuss recent result. In particular, we hope to attract researchers who work in arithmetic geometry, algebraic geometry, model theory, and computational algebra and number theory, with the dual goals of introducing the field of arithmetic dynamics and encouraging interactions among people working in these varied fields.
Information: http://icerm.brown.edu/sp-s12/workshop-2.php.

17–18 **AMS Eastern Section Meeting**, George Washington University, Washington, District of Columbia. (May 2011, p. 744)

* 25–28 **Conference on Partial Differential Equations and Applications**, Vietnam National University, Hanoi, Vietnam.
Description: The conference on partial differential equations and applications aims to present a broad and interdisciplinary overview of the current, state-of-the-art methods and techniques for characterizing PDEs. Moreover, various application areas will be highlighted where such techniques play critical roles in solving fundamental problems of interest to the broader scientific community. The conference will be preceded by a workshop aimed at graduate students and postdocs in the mathematical sciences who have a fundamental interest in PDE methods and applications.
Plenary Speakers: Marsha Berger (Courant, NYU), Peter Constantin (Chicago), Craig Evans (Berkeley), Tom Hou (Caltech), Carlos Kenig (Chicago), Sergiu Klainerman (Princeton), Randall LeVeque (Washington), Louis Nirenberg (Courant, NYU), Atsushi Yagi (Osaka).
Information: http://www.amath.washington.edu/events/vietnam2012/.

26–30 (NEW DATE) **IMA Workshop: Machine Learning: Theory and Computation**, Institute for Mathematics and its Applications (IMA), University of Minnesota, Minneapolis, Minnesota. (Jan. 2011, p. 85)

30–31 **Information and Econometrics of Networks**, American University, Washington, District of Columbia. (Mar. 2011, p. 497)

30–April 1 **AMS Central Section Meeting**, University of Kansas, Lawrence, Kansas. (May 2011, p. 744)

April 2012

* 16–20 **ICERM Workshop: Moduli Spaces Associated to Dynamical Systems**, ICERM, Providence, Rhode Island.
Description: The set of rational self-maps of Pn of degree d, which is denoted Ratdn, has a natural structure as an affine variety. The group PGLn+1 acts by conjugation on Ratdn, and the quotient space is the dynamical moduli space Mdn.
Information: http://icerm.brown.edu/sp-s12/workshop-3.php.

May 2012

7–11 **IMA Workshop: User-Centered Modeling**, Institute for Mathematics and its Applications (IMA), University of Minnesota, Minneapolis, Minnesota.

20–22 **SIAM Conference on Imaging Science (IS12)**, Doubletree Hotel Philadelphia, Philadelphia, Pennsylvania.

20–25 **7th European Conference on Elliptic and Parabolic Problems**, Gaeta, Italy. (May 2011, p. 744)

* 28–June 3 **International Conference "Theory of Approximation of Functions and its Applications"**, Kamianets-Podilsky Ivan Ohienko National University, Kamianets-Podilsky, Ukraine.
Description: International Conference "Theory of Approximation of Functions and its Applications" dedicated to the 70th anniversary of corresponding member of National Academy of Sciences of Ukraine, Professor O. I. Stepanets (1942–2007).
Information: http://www.imath.kiev.ua/~funct/stepconf2012/en/.

The following new announcements will not be repeated until the criteria in the next to the last paragraph at the bottom of the first page of this section are met.

July 2012

* 9–15 **The 10th International Conference on Fixed Point Theory and its Applications (ICFPTA-2012)**, Faculty of Mathematics and Computer Science, Babes, Bolyai University, Cluj-Napoca, Romania.
Description: The purpose of the conference is to bring together leading experts and researchers in fixed point theory and to assess new developments, ideas and methods in this important and dynamic field. A special emphasis will be put on applications in related areas, as well as other sciences, such as the natural sciences, medicine, economics and engineering. The conference will continue the tradition of the previous fixed point theory meetings which were held in Marseille (1989), Halifax (1991), Seville (1995), Kazimierz Dolny (1997), Haifa (2001), Valencia (2003), Guanajuato (2005), Chiang Mai (2007) and Changhua (2009). The conference will honour Professor Kazimierz Goebel, on the occasion of his retirement, and Professors Ljubomir Ciric, William Art Kirk and Ioan A. Rus, on the occasion of their 75th birthday.
Information: http://www.cs.ubbcluj.ro/~fptac/.

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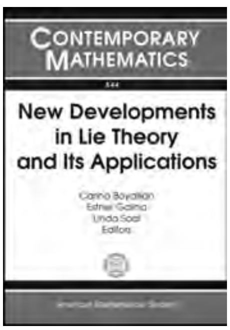
Algebra and Algebraic Geometry



New Developments in Lie Theory and Its Applications

Carina Boyallian, **Esther Galina**, and **Linda Saal**, *Universidad Nacional de Córdoba, Argentina*, Editors

This volume contains the proceedings of the Seventh Workshop in Lie Theory and Its Applications, which was held November 27–December 1, 2009 at the Universidad Nacional de Córdoba, in Córdoba, Argentina. The workshop was preceded by a special event, "Encuentro de teoria de Lie", held November 23–26, 2009, in honor of the sixtieth birthday of Jorge A. Vargas, who greatly contributed to the development of Lie theory in Córdoba.

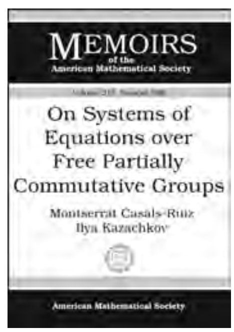
This volume focuses on representation theory, harmonic analysis in Lie groups, and mathematical physics related to Lie theory. The papers give a broad overview of these subjects and also of the recent developments of the authors' research.

This item will also be of interest to those working in analysis and mathematical physics.

Contents: F. Levstein, **C. Maldonado**, and **D. Penazzi**, Lattices, frames and Norton algebras of dual polar graphs; **J. Faraut**, Asymptotic spherical analysis; **J. Vargas**, Restriction of discrete series of a semisimple Lie group to reductive subgroups; **S. Dann** and **G. Ólafsson**, Paley-Wiener theorems with respect to the spectral parameter; **G. Ólafsson** and **J. A. Wolf**, Extension of symmetric spaces and restriction of Weyl groups and invariant polynomials; **A. H. Dooley**, Intertwining operators, the Cayley transform, and the contraction of K to NM; **L. Gutíerrez**, **J. Pantoja**, and **J. Soto-Andrade**, On generalized Weil representations over involutive rings; **N. Andruskiewitsch**, **I. Angiono**, and **H. Yamane**, On pointed Hopf superalgebras; **V. Serganova**, Quasireductive supergroups.

Contemporary Mathematics, Volume 544

June 2011, 159 pages, Softcover, ISBN: 978-0-8218-5259-0, LC 2011007612, 2010 *Mathematics Subject Classification:* 17A70, 16T05, 05C12, 43A90, 43A35, 43A75, 22E27, **AMS members US$47.20**, List US$59, Order code CONM/544



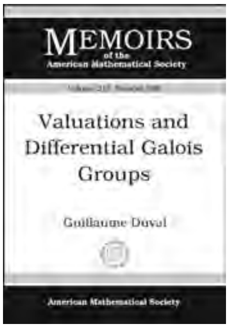
On Systems of Equations over Free Partially Commutative Groups

Montserrat Casals-Ruiz and **Ilya Kazachkov**, *McGill University, Montreal, QC, Canada*

Contents: Introduction; Preliminaries; Reducing systems of equations over $\mathbb{G}$ to constrained generalised equations over $\mathbb{F}$; The process: Construction of the tree T; Minimal solutions; Periodic structures; The finite tree $T_0(\Omega)$ and minimal solutions; From the coordinate group $\mathbb{G}_{R(\Omega^*)}$ to proper quotients: The decomposition tree $T_{\rm dec}$ and the extension tree $T_{\rm ext}$; The solution tree $T_{\rm sol}(\Omega)$ and the main theorem; Bibliography; Index; Glossary of notation.

Memoirs of the American Mathematical Society, Volume 212, Number 999

June 2011, 153 pages, Softcover, ISBN: 978-0-8218-5258-3, LC 2011011927, 2010 *Mathematics Subject Classification:* 20F70; 20F10, 20F36, **Individual member US$46.20**, List US$77, Institutional member US$61.60, Order code MEMO/212/999



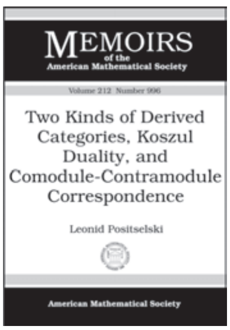
Valuations and Differential Galois Groups

Guillaume Duval

Contents: Introduction; Invariant valuations and solutions of l.d.e.; Examples and use of invariant valuations; Continuity of derivations, geometry and examples; Continuity and field extensions; Invariant valuations and singularities of l.d.e.; Existence and geometry of invariant valuations; Bibliography.

Memoirs of the American Mathematical Society, Volume 212, Number 998

June 2011, 68 pages, Softcover, ISBN: 978-0-8218-4906-4, LC 2011016003, 2010 *Mathematics Subject Classification:* 34M15; 12J20, **Individual member US$36**, List US$60, Institutional member US$48, Order code MEMO/212/998



Two Kinds of Derived Categories, Koszul Duality, and Comodule-Contramodule Correspondence

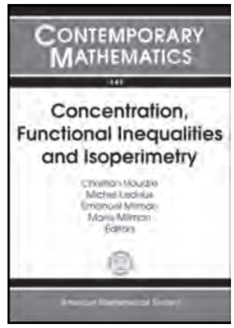
Leonid Positselski, *Institute for Information Transmission Problems, Moscow, Russia*

Contents: Introduction; Derived category of DG-modules; Derived categories of DG-comodules and DG-contramodules; Coderived and contraderived categories of CDG-modules; Coderived category of CDG-comodules and contraderived category of CDG-contramodules; Comodule-contramodule correspondence; Koszul duality: Conilpotent and nonconilpotent cases; A_∞-algebras and curved A_∞-coalgebras; Model categories of DG-modules, CDG-comodules, and CDG-contramodules; Model categories of DG-algebras and CDG-coalgebras; Appendix A. Homogeneous Koszul duality; Appendix B. $\mathcal{D}$–Ω duality; Bibliography.

Memoirs of the American Mathematical Society, Volume 212, Number 996

June 2011, 133 pages, Softcover, ISBN: 978-0-8218-5296-5, LC 2011016023, 2010 *Mathematics Subject Classification:* 18E30, 18G10, 16T15, 16S37, 14F10; 18G55, 17B55, 16E65, 18G15, 58J10, **Individual member US$42.60**, List US$71, Institutional member US$56.80, Order code MEMO/212/996

Analysis



Concentration, Functional Inequalities and Isoperimetry

Christian Houdré, *Georgia Institute of Technology, Atlanta, GA*, **Michel Ledoux**, *Université de Toulouse, France*, **Emanuel Milman**, *Technion-Israel Institute of Technology, Haifa, Israel*, and **Mario Milman**, *Florida Atlantic University, Boca Raton, FL*, Editors

The volume contains the proceedings of the international workshop on Concentration, Functional Inequalities and Isoperimetry, held at Florida Atlantic University in Boca Raton, Florida, from October 29–November 1, 2009.

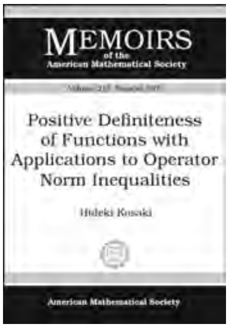
The interactions between concentration, isoperimetry and functional inequalities have led to many significant advances in functional analysis and probability theory. Important progress has also taken place in combinatorics, geometry, harmonic analysis and mathematical physics, to name but a few fields, with recent new applications in random matrices and information theory.

This book should appeal to graduate students and researchers interested in the fascinating interplay between analysis, probability, and geometry.

Contents: **S. Aida**, COH formula and Dirichlet Laplacians on small domains of pinned path spaces; **N. Badr** and **G. Dafni**, Maximal characterization of Hardy-Sobolev spaces on manifolds; **S. G. Bobkov**, On Milman's ellipsoids and *M*-position of convex bodies; **S. G. Bobkov**, **M. Madiman**, and **L. Wang**, Fractional generalizations of Young and Brunn-Minkowski inequalities; **R. Eldan** and **B. Klartag**, Approximately Gaussian marginals and the hyperplane conjecture; **O. N. Feldheim** and **S. Sodin**, One more proof of the Erdős-Turán inequality, and an error estimate in Wigner's law; **A. Figalli**, Quantitative isoperimetric inequalities with applications to the stability of liquid drops and crystals; **R. L. Frank** and **E. H. Lieb**, Spherical reflection positivity and the Hardy–Littlewood–Sobolev inequality; **A. Giannopoulos**, **G. Paouris**, and **P. Valettas**, On the existence of subgaussian directions for log-concave measures; **A. V. Kolesnikov** and **R. I. Zhdanov**, On isoperimetric sets of radially symmetric measures; **M. Ledoux**, From concentration to isoperimetry: Semigroup proofs; **J. Martín** and **M. Milman**, Sobolev inequalities, rearrangements, isoperimetry and interpolation spaces; **E. Milman**, Isoperimetric bounds on convex manifolds; **F. Morgan**, The log-convex density conjecture.

Contemporary Mathematics, Volume 545

July 2011, 211 pages, Softcover, ISBN: 978-0-8218-4971-2, LC 2011008188, 2010 *Mathematics Subject Classification:* 26D10, 32F32, 46E30, 46G12, 53C20, 53C21, 60B99, 60E15, **AMS members US$63.20**, List US$79, Order code CONM/545



Positive Definiteness of Functions with Applications to Operator Norm Inequalities

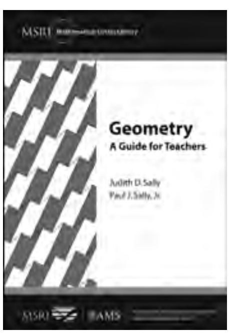
Hideki Kosaki, *Kyushu University, Fukuoka, Japan*

Contents: Introduction; Preliminaries; Fourier transforms and positive definiteness; A certain Heinz-type inequality and related commutator estimates; Norm comparison for various operator means; Norm inequalities for $H^{\frac{1}{2}+\beta}XK^{\frac{1}{2}-\beta} + H^{\frac{1}{2}-\beta}XK^{\frac{1}{2}+\beta} \pm H^{1/2}XK^{1/2}$; Norm comparison of Heron-type means and related topics; Operator Lehmer means and their properties; Appendix A. A direct proof for Proposition 7.3; Appendix B. Proof for Theorem 7.10; Bibliography; Index.

Memoirs of the American Mathematical Society, Volume 212, Number 997

June 2011, 80 pages, Softcover, ISBN: 978-0-8218-5307-8, LC 2011011926, 2010 *Mathematics Subject Classification:* 47A63, 47A64; 15A42, 15A60, 47A30, **Individual member US$39.60**, List US$66, Institutional member US$52.80, Order code MEMO/212/997

Geometry and Topology



Geometry

A Guide for Teachers

Judith D. Sally, *Northwestern University, Evanston, IL*, and **Paul J. Sally, Jr.**, *University of Chicago, IL*

This geometry book is written foremost for future and current middle school teachers, but is also designed for elementary and high school teachers. The book consists of ten seminars covering in a rigorous way the fundamental topics in school geometry, including all of the significant topics in high school geometry. The seminars are crafted to clarify and enhance understanding of the subject. Concepts in plane and solid geometry are carefully explained, and activities that teachers can use in their classrooms are emphasized. The book draws on the pictorial nature of geometry since that is what attracts students at every level to the subject. The book should give teachers a firm foundation on which to base their instruction in the elementary and middle grades. In addition, it should help teachers give their students a solid basis for the geometry that they will study in high school. The book is also intended to be a source for problems in geometry for enrichment programs such as Math Circles and Young Scholars.

Titles in this series are co-published with the Mathematical Sciences Research Institute (MSRI).

Contents: Seminar 1: Polygons in the plane; Seminar 2: More fundamentals of plane geometry; Seminar 3: Tessellation; Seminar 4: Regular polygons and regular polyhedra; Seminar 5: Symmetry; Seminar 6: Lattice polygons; Seminar 7: The area of polygonal regions; Seminar 8: The area of a disk and disk packing; Seminar 9: Dissection; Seminar 10: Geometry in three dimensions; Index.

MSRI Mathematical Circles Library, Volume 3

June 2011, 202 pages, Softcover, ISBN: 978-0-8218-5362-7, LC 2011010067, 2010 *Mathematics Subject Classification:* 51-01, 97Gxx, 97G30, 97G40, **AMS members US$31.20**, List US$39, Order code MCL/3

Mostly Surfaces

Richard Evan Schwartz, *Brown University, Providence, RI*

This book presents a number of topics related to surfaces, such as Euclidean, spherical and hyperbolic geometry, the fundamental group, universal covering surfaces, Riemannian manifolds, the Gauss-Bonnet Theorem, and the Riemann mapping theorem. The main idea is to get to some interesting mathematics without too much formality. The book also includes some material only tangentially related to surfaces, such as the Cauchy Rigidity Theorem, the Dehn Dissection Theorem, and the Banach–Tarski Theorem.

The goal of the book is to present a tapestry of ideas from various areas of mathematics in a clear and rigorous yet informal and friendly way. Prerequisites include undergraduate courses in real analysis and in linear algebra and some knowledge of complex analysis.

Contents: Book overview; Definition of a surface; The gluing construction; The fundamental group; Examples of fundamental groups; Covering spaces and the deck group; Existence of universal covers; Euclidean geometry; Spherical geometry; Hyperbolic geometry; Riemann metrics on surfaces; Hyperbolic surfaces; A primer on complex analysis; Disk and plane rigidity; The Schwarz-Christoffel transformation; Riemann surfaces and uniformization; Flat cone surfaces; Translation surfaces and the Veech group; Continued fractions; Teichmüller space and moduli space; Topology of Teichmüller space; The Banach Tarski theorem; Dehn's dissection theorem; The Cauchy rigidity theorem; Bibliography; Index.

Student Mathematical Library, Volume 60

July 2011, 314 pages, Softcover, ISBN: 978-0-8218-5368-9, LC 2011005544, 2010 *Mathematics Subject Classification:* 14Q10, 30F10, 32J15, 37E30, 53A05, 51M20, 32Q45, **AMS members US$37.60**, List US$47, Order code STML/60

Number Theory

On Certain L-Functions

James Arthur, *University of Toronto, ON, Canada*, **James W. Cogdell**, *Ohio State University, Columbus, OH*, **Steve Gelbart**, *Weizmann Institute of Science, Rehovot, Israel*, **David Goldberg**, *Purdue University, West Lafayette, IN*, **Dinakar Ramakrishnan**, *California Institute of Technology, Pasadena, CA*, and **Jiu-Kang Yu**, *Purdue University, West Lafayette, IN*, Editors

This volume constitutes the proceedings of a conference, "On Certain L-functions", held July 23–27, 2007 at Purdue University, West Lafayette, Indiana. The conference was organized in honor of the 60th birthday of Freydoon Shahidi, widely recognized as having made groundbreaking contributions to the Langlands program.

The articles in this volume represent a snapshot of the state of the field from several viewpoints. Contributions illuminate various areas of the study of geometric, analytic, and number theoretic aspects of automorphic forms and their L-functions, and both local and global theory are addressed.

Topics discussed in the articles include Langlands functoriality, the Rankin–Selberg method, the Langlands–Shahidi method, motivic Galois groups, Shimura varieties, orbital integrals, representations of p-adic groups, Plancherel formula and its consequences, the Gross–Prasad conjecture, and more. The volume also includes an expository article on Shahidi's contributions to the field, which serves as an introduction to the subject.

Experts will find this book a useful reference, and beginning researchers will be able to use it to survey major results in the Langlands program.

This item will also be of interest to those working in algebra and algebraic geometry.

Titles in this series are co-published with the Clay Mathematics Institute (Cambridge, MA).

Contents: S. Gelbart, Shahidi's work "On certain L-functions": A short history of Langlands-Shahidi theory; **J. Arthur**, The embedded eigenvalue problem for classical groups; **M. Asgari** and **A. Raghuram**, A cuspidality criterion for the exterior square transfer of cusp forms on GL(4); **C. J. Bushnell**, **G. Henniart**, and **P. C. Kutzko**, Types and explicit Plancherel formulae for reductive p-adic groups; **B. Casselman**, Jacquet modules and the asymptotic behaviour of matrix coefficients; **L. Clozel**, The *ABS* principle: Consequences for $L^2(G/H)$; **L. Clozel** and **J.-P. Labesse**, Orbital integrals and distributions; **J. W. Cogdell**, **I. I. Piatetski-Shapiro**, and **F. Shahidi**, Functoriality for the quasisplit classical groups; **D. Ginzburg**, **D. Jiang**, and **D. Soudry**, Poles of L-functions and theta liftings for orthogonal groups, II; **D. Goldberg**, On dual R-groups for classical groups; **H. Hida**, Irreducibility of the Igusa tower over unitary Shimura varieties; **H. Jacquet** and **S. Rallis**, On the Gross-Prasad conjecture for unitary groups; **H. H. Kim** and **W. Kim**, On local L-functions and normalized intertwining operators II; Quasi-split groups; **R. P. Langlands**, Reflexions on receiving the Shaw Prize; **E. Lapid**, On Arthur's asymptotic inner product formula of truncated Eisenstein series; **C. Moeglin**, Multiplicité 1 dans les paquets d'Arthur aux places p-adiques; **G. Muić** and **M. Tadić**, Unramified unitary duals for split classical p-adic groups; The topology and isolated representations; **F. Murnaghan**, Parametrization of tame supercuspidal representations; **V. K. Murty**, On the Sato-Tate conjecture, II; **D. Ramakrishnan**, Icosahedral fibres of the symmetric cube and algebraicity; **J. Rohlfs** and **B. Speh**, Pseudo Eisenstein forms and the cohomology of arithmetic groups III: Residual cohomology classes; **P. Schneider** and **M.-F. Vigneras**, A functor from smooth o-torsion representations to (φ, Γ)-modules; **H. Yoshida**, Motivic Galois groups and L-groups.

Clay Mathematics Proceedings, Volume 13

June 2011, 647 pages, Softcover, ISBN: 978-0-8218-5204-0, LC 2011003659, 2010 *Mathematics Subject Classification:* 11F30, 11F70, 11F80, 11S37, 20G05, 20G25, 22E35, 22E50, 22E55, **AMS members US$103.20**, List US$129, Order code CMIP/13

New AMS-Distributed Publications

Algebra and Algebraic Geometry

Multiplicative Properties of the Slice Filtration

Pablo Pelaez, *Universitaet Duisburg-Essen, Germany*

Let S be a Noetherian separated scheme of finite Krull dimension, and $\mathcal{SH}(S)$ be the motivic stable homotopy category of Morel-Voevodsky. In order to get a motivic analogue of the Postnikov tower, Voevodsky (MR 1977582) constructs the slice filtration by filtering $\mathcal{SH}(S)$ with respect to the smash powers of the multiplicative group $\mathbb{G}_m$.

The author shows that the slice filtration is compatible with the smash product in Jardine's category $\mathrm{Spt}_T^{\Sigma}\mathcal{M}_*$ of motivic symmetric T-spectra (MR 1787949) and describes several interesting consequences that follow from this compatibility. Among the consequences that follow from this compatibility is that over a perfect field all the slices s_q are in a canonical way modules in $\mathrm{Spt}_T^{\Sigma}\mathcal{M}_*$ over the motivic Eilenberg-MacLane spectrum $H\mathbb{Z}$, and if the field has characteristic zero it follows that the slices s_q are big motives in the sense of Voevodsky. This relies on the work of Levine (MR 2365658), Röndigs-Østvær (MR 2435654), and Voevodsky (MR

2101286). It also follows that the smash product in $\mathrm{Spt}_T^{\Sigma}\mathcal{M}_*$ induces pairings in the motivic Atiyah-Hirzebruch spectral sequence.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Contents: Preliminaries; Motivic unstable and stable homotopy theory; Model structures for the slice filtration; Bibliography; Index.

Astérisque, Number 335

March 2011, 291 pages, Softcover, ISBN: 978-2-85629-305-8, 2010 *Mathematics Subject Classification:* 14F42, 18D10, 18G40, 18G55, **Individual member US$81**, List US$90, Order code AST/335

Analysis

Neumann and Dirichlet Heat Kernels in Inner Uniform Domains

Pavel Gyrya and **Laurent Saloff-Coste**, *Cornell University, Ithaca, NY*

This monograph focuses on the heat equation with either the Neumann or the Dirichlet boundary condition in unbounded domains in Euclidean space, Riemannian manifolds, and in the more general context of certain regular local Dirichlet spaces. In works by A. Grigor'yan, L. Saloff-Coste, and K.-T. Sturm, the equivalence between the parabolic Harnack inequality, the two-sided Gaussian heat kernel estimate, the Poincaré inequality and the volume doubling property is established in a very general context.

The authors use this result to provide precise two-sided heat kernel estimates in a large class of domains described in terms of their inner intrinsic metric and called inner (or intrinsically) uniform domains. Perhaps surprisingly, they treat both the Neumann boundary condition and the Dirichlet boundary condition using essentially the same approach, albeit with the additional help of a Doob's h-transform in the case of Dirichlet boundary condition.

The main results are new even when applied to Euclidean domains with smooth boundary where they capture the global effect of the condition of inner uniformity as, for instance, in the case of domains that are the complement of a convex set in Euclidean space.

This item will also be of interest to those working in differential equations.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Contents: Introduction; Harnack-type Dirichlet spaces; The Neumann heat kernel in inner uniform domains; The harmonic profile of an unbounded inner uniform domain; The Dirichlet heat kernel in inner uniform domains; Examples; Bibliography.

Astérisque, Number 336

March 2011, 145 pages, Softcover, ISBN: 978-2-85629-306-5, 2010 *Mathematics Subject Classification:* 35K20, 58J35, 60J60, 31C12, 58J65, 60J45, **Individual member US$40.50**, List US$45, Order code AST/336

Differential Equations

Aspects Théoriques et Appliqués de Quelques EDP Issues de la Géométrie ou de la Physique

Ahmad El Soufi, *Université François Rabelais, Tours, France*, and **Mustapha Jazar**, *Université Libanaise, Tripoli, Liban*, Editors

This book is a result of several courses given at the CIMPA School, which was held in Damascus in May 2004. The aim of these courses was to present various topics of current research in analysis where the underlying problems come from physics. Thus, one finds courses on extended dynamical systems (P. Collet), the semi-classical analysis of Schrödinger operators in connection with superconductivity (B. Helffer), free-boundary problems (R. Monneau), and partial functional differential equations (M. Adimy and K. Ezzinbi).

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Contents: **P. Collet**, Introduction to the theory of extended dynamical systems; **B. Helffer**, Introduction to semi-classical methods for the Schrödinger operator with magnetic field; **R. Monneau**, Méthodes géométriques pour les problèmes à frontières libres; **M. Adimy** and **K. Ezzinbi**, Existence, regularity, stability and boundedness for some partial functional differential equations.

Séminaires et Congrès, Number 17

April 2011, 182 pages, Softcover, ISBN: 978-2-85629-239-6, 2010 *Mathematics Subject Classification:* 34K05, 34K13, 34K14, 34K20, 34K25, 34K50, 35B65, 35K55, 35P15, 35R35, 37L05, 37L30, 47D06, 47D62, **Individual member US$63**, List US$70, Order code SECO/17

General Interest

Stefan Banach

Remarkable Life, Brilliant Mathematics

Emilia Jakimowicz, *Gdańsk University, Gdańsk-Oliwa, Poland*, and **Adam Miranowicz**, *Adam Mickiewicz University, Poznań, Poland*, Editors

This meticulously researched and detailed account of the life of the Polish mathematician Stefan Banach presents previously unknown facts that shed new light on his accomplishments and chronicles the many dramatic events of his life.

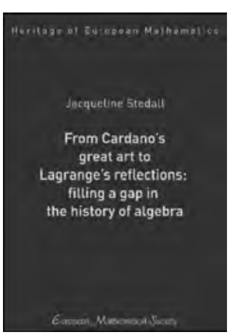
A self-taught prodigy and one of the great scientists of the twentieth century, Banach established modern functional analysis, an entirely new branch of mathematics with important applications. He also helped to develop the theory of topological vector spaces. Such notions as Banach space, Banach algebra, Banach manifold, Banach measure, Banach integral, Banach limit, and Banach bundle are widely used in today's mathematics.

The authors interviewed Banach's living family members, former students and acquaintances, unearthed old documents and records, and collected previously unpublished letters and photographs to compile this biography. They also added a concise overview of his pioneering work. Their research was motivated by a desire to provide an accurate and authoritative account of the life and achievements of one of Poland's most famous and celebrated mathematicians.

A publication of Gdańsk University Press. Distributed non-exclusively worldwide by the American Mathematical Society.

Contents: A remarkable life (by Emilia Jakimowicz); Letters; Recollections; Stefan Banach in the light of archives (by Stanisław Domoradzki, Zofia Pawlikowska-Brożek, and Mikhąilo Zarichny); Banach's opus scientificus (by Julian Musielak); Stefan Banach and Lvov Mathematical School (by Krzysztof Ciesielski and Zdzislaw Pogoda); The Scottish Book (by Marek Kordos); The New Scottish Book (by Roman Duda); Selected bibliography in English and French; Selected bibliography in Polish; Biographical notes; List of illustrations; Index of names.

April 2011, 186 pages, Hardcover, ISBN: 978-83-7326-798-5, 2010 *Mathematics Subject Classification:* 01-XX, **AMS members US$44**, List US$55, Order code BANACH



From Cardano's Great Art to Lagrange's Reflections: Filling a Gap in the History of Algebra

Jacqueline Stedall, *Queens College, Oxford, England*

This book is an exploration of a claim made by Lagrange in the autumn of 1771 as he embarked upon his lengthy "Réflexions sur la resolution algébrique des equations": that there had been few advances in the algebraic solution of equations since the time of Cardano in the mid sixteenth century. That opinion has been shared by many later historians. The present study attempts to redress that view and to examine the intertwined developments in the theory of equations from Cardano to Lagrange. A similar historical exploration led Lagrange himself to insights that were to transform the entire nature and scope of algebra.

Progress was not confined to any one country: at different times mathematicians in Italy, France, the Netherlands, England, Scotland, Russia, and Germany contributed to the discussion and to a gradual deepening of understanding. In particular, the national Academies of Berlin, St. Petersburg, and Paris in the eighteenth century were crucial in supporting informed mathematical communities and encouraging the wider dissemination of key ideas. This study therefore truly highlights the existence of a European mathematical heritage.

The book is written in three parts. Part I offers an overview of the period from Cardano to Newton (1545 to 1707) and is arranged chronologically. Part II covers the period from Newton to Lagrange (1707 to 1771) and treats the material according to key themes. Part III is a brief account of the aftermath of the discoveries made in the 1770s. The book attempts throughout to capture the reality of mathematical discovery by inviting the reader to follow in the footsteps of the authors themselves, with as few changes as possible to the original notation and style of presentation.

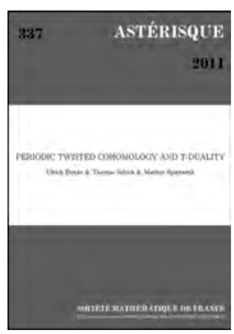
A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.

Contents: From Cardano to Newton: 1545 to 1707; From Newton to Lagrange: 1707 to 1771; After Lagrange; Bibliography; Index.

Heritage of European Mathematics, Volume 5

March 2011, 236 pages, Hardcover, ISBN: 978-3-03719-092-0, 2010 *Mathematics Subject Classification:* 01-02, 01A40, 01A45, 01A50, **AMS members US$70.40**, List US$88, Order code EMSHEM/5

Geometry and Topology



Periodic Twisted Cohomology and T-Duality

Ulrich Bunke, *Universität Regensburg, Germany*, **Thomas Schick**, *Georg-August-Universität Göttingen, Germany*, and **Markus Spitzweck**, *Universität Regensburg, Germany*

Using the differentiable structure, twisted 2-periodic de Rham cohomology is well known and shows up as the target of Chern characters for twisted K-theory. The main motivation of this work is a topological interpretation of two-periodic twisted de Rham cohomology, which is generalizable to arbitrary topological spaces and at the same time to arbitrary coefficients.

To this end the authors develop a sheaf theory in the context of locally compact topological stacks with emphasis on the construction of the sheaf theory operations in unbounded derived categories, elements of Verdier duality, and integration. The main result is the construction of a functorial periodization associated to a $U(1)$-gerbe.

As an application the authors verify the T-duality isomorphism in periodic twisted cohomology and in periodic twisted orbispace cohomology.

This item will also be of interest to those working in analysis.

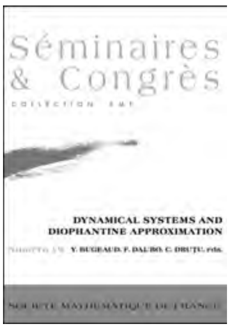
A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Contents: Introduction; Gerbes and periodization; Functorial periodization; T-duality; Orbispaces; Verdier duality for locally compact stacks; Bibliography.

Astérisque, Number 337

March 2011, 134 pages, Softcover, ISBN: 978-2-85629-307-2, 2010 *Mathematics Subject Classification:* 55N30, 46M20, 14A20, **Individual member US$40.50**, List US$45, Order code AST/337

Number Theory



Dynamical Systems and Diophantine Approximation

Yann Bugeaud, *Université de Strasbourg, France*, **Françoise Dal'Bo**, *Université Rennes 1, France*, and **Cornelia Druţu**, *Université des Sciences et Technologies de Lille I, Villeneuve d'Ascq, France*, Editors

On June 7-9, 2004, a conference on Dynamical Systems and Diophantine Approximation was held at the Institut Henri Poincaré. One of the aims of this conference was to give a survey of research tools at the interface between these two domains. The editors' goal was also to highlight methods and open questions. The proceedings the editors are presenting in this volume reflect the spirit of this conference. The reader will find surveys and articles on the convergence or divergence points between dynamical systems and Diophantine approximation. All the papers are accessible to a wide audience.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Contents: M. M. Dodson, Diophantine approximation, Khintchine's theorem, torus geometry and Hausdorff dimension; **V. Beresnevich** and **S. Velani**, Ubiquity and a general logarithm law for geodesics; **F. Maucourant**, Arithmetical and geometrical aspects of homogeneous diophantine approximation by algebraic numbers in a given number field; **C. Druţu**, Transference principles and locally symmetric spaces; **N. A. Shah**, Unipotent flows on products of $\mathrm{SL}(2, K)/\Gamma$'s; **Y. Bugeaud**, Multiplicative Diophantine approximation; **M. Queffélec**, An introduction to Littlewood's conjecture; **A. Broise-Alamichel**, On the characteristic exponents of the Jacobi-Perron algorithm; **S. Troubetzkoy**, Approximation and billiards.

Séminaires et Congrès, Number 19

April 2011, 185 pages, Softcover, ISBN: 978-2-85629-303-4, 2010 *Mathematics Subject Classification:* 11Gxx, 11H16, 11H46, 11J06, 11J13, 11J17, 11J25, 11J70, 11J83, 11K50, 11K55, 11K60, 22E40, 22E46, 28A80, 30F40, 37A15, 37A17, 37A30, 37D50, 37H15, 53C35, **Individual member US$54**, List US$60, Order code SECO/19

Crossword Puzzle Answers

Inspiring Dreams…(puzzle on page 822)

Answers

A	F	T	A	■	A	C	D	C	■	O	V	I	D	■
C	R	U	Z	■	D	A	R	E	■	N	I	N	A	■
N	E	S	T	E	D	R	A	D	I	C	A	L	S	■
E	T	H	E	L	■	■	K	E	N	O	■	O	H	M
■	■	■	C	L	O	S	E	D	C	U	R	V	E	S
D	A	H	■	E	S	P	■	■	■	R	E	E	D	S
E	M	A	G	■	H	U	N	■	T	S	P	■	■	■
C	O	H	E	R	E	N	T	S	H	E	A	V	E	S
■	■	■	N	E	A	■	H	U	E	■	Y	I	P	E
D	I	X	I	E	■	■	■	B	R	A	■	P	A	C
I	N	F	I	N	I	T	E	S	E	T	S	■	■	■
T	A	I	■	A	N	O	N	■	■	R	A	S	P	Y
■	B	L	O	C	K	A	N	D	T	A	C	K	L	E
■	I	E	S	T	■	D	U	S	K	■	R	E	E	L
■	T	S	O	S	■	S	I	L	O	■	E	G	A	L

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000034

MICHIGAN

MICHIGAN STATE UNIVERSITY
AssistantProfessor

The Department of Mathematics at Michigan State University will have a tenure-track position to begin fall 2011. It is expected that successful applicants will be appointed at the rank of assistant professor. Excellence is essential in both research and teaching, and it is expected that the successful candidate will have at least 3 years of experience beyond the Ph.D. Strong preference will be given to candidates in the area of computational mathematics with emphasis on Discontinuous Galerkin Method. The candidate should have an NSF grant. Application deadline: July 15, 2011. Applicants should send a vita and statement of research interests, and should arrange for at least 4 letters of recommendation to be sent, one of which must specifically address the applicant's ability to teach. Please apply by visiting `www.mathjobs.org`. MSU is an Affirmative Action, Equal Opportunity Employer. MSU is committed to achieving excellence through cultural diversity. The university actively encourages applications and/or nominations of women, persons of color, veterans and persons with disabilities.

000033

SOUTH CAROLINA

UNIVERSITY OF SOUTH CAROLINA
Department of Mathematics
Endowed Chair in Data Analysis, Simulation, Imaging and Visualization

The Department of Mathematics at the University of South Carolina invites applications for the Endowed Chair in Data Analysis, Simulation, Imaging, and Visualization (CDASIV) in conjunction with the Williams-Hedberg-Hedberg Endowed Chair of Mathematics. This position is sponsored in part by the South Carolina Centers of Economic Excellence (CoEE) program (`http://www.sccoee.org/dataanalysis.asp`) and carries a total endowment of $4M. Position requires a Ph.D. or equivalent and an outstanding record of research and teaching. For a complete description of qualifications, go to: `http://www.sccoee.org/documents/DataAnalysis.pdf`.

Applications should be submitted electronically to Professor Pencho Petrushev (`coee@math.sc.edu`). Applicants are encouraged to submit a letter describing their interests and credentials, curriculum vitae, a list of publications, a statement of research interests and accomplishments, and three letters of reference.

For full consideration by the search committee, applications should be received no later than August 1, 2011.

The University of South Carolina is an Affirmative Action/Equal Opportunity Employer. Minorities and women are especially encouraged to apply.

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Suggested uses for classified advertising are positions available, books or lecture notes for sale, books being sought, exchange or rental of houses, and typing services.

The 2011 rate is $3.25 per word. No discounts for multiple ads or the same ad in consecutive issues. For an additional $10 charge, announcements can be placed anonymously. Correspondence will be forwarded.

Advertisements in the "Positions Available" classified section will be set with a minimum one-line headline, consisting of the institution name above body copy, unless additional headline copy is specified by the advertiser. Headlines will be centered in boldface at no extra charge. Ads will appear in the language in which they are submitted.

There are no member discounts for classified ads. Dictation over the telephone will not be accepted for classified ads.

Upcoming deadlines for classified advertising are as follows: August 2011 issue–May 27, 2010; September 2011 issue–June 28, 2011; October 2011 issue–July 28, 2011; November 2011 issue–August 30, 2011; December 2011 issue–December 28, 2011.

U.S. laws prohibit discrimination in employment on the basis of color, age, sex, race, religion, or national origin. "Positions Available" advertisements from institutions outside the U.S. cannot be published unless they are accompanied by a statement that the institution does not discriminate on these grounds whether or not it is subject to U.S. laws. Details and specific wording may be found on page 667 (vol. 56).

Situations wanted advertisements from involuntarily unemployed mathematicians are accepted under certain conditions for free publication. Call toll-free 800-321-4AMS (321-4267) in the U.S. and Canada or 401-455-4084 worldwide for further information.

Submission: Promotions Department, AMS, P.O. Box 6248, Providence, Rhode Island 02940; or via fax: 401-331-3842; or send email to `classads@ams.org`. AMS location for express delivery packages is 201 Charles Street, Providence, Rhode Island 20904. Advertisers will be billed upon publication.

Meetings & Conferences of the AMS

IMPORTANT INFORMATION REGARDING MEETINGS PROGRAMS: AMS Sectional Meeting programs do not appear in the print version of the *Notices*. However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on the AMS website. See `http://www.ams.org/meetings/`. Final programs for Sectional Meetings will be archived on the AMS website accessible from the stated URL and in an electronic issue of the *Notices* as noted below for each meeting.

Ithaca, New York

Cornell University

September 10–11, 2011

Saturday – Sunday

Meeting #1072

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: June/July 2011
Program first available on AMS website: July 28, 2011
Program issue of electronic *Notices*: September 2011
Issue of *Abstracts*: Volume 32, Issue 4

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: June 29, 2011

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Mladen Bestvina, University of Utah, *Topology and geometry of out(F_n)*.

Nigel Higson, Pennsylvania State University, *C*-algebras and group representations*.

Gang Tian, Princeton University, *Title to be announced*.

Katrin Wehrheim, Massachusetts Institute of Technology, *How to construct topological invariants via decompositions and the symplectic category*.

Special Sessions

Analysis, Probability, and Mathematical Physics on Fractals (Code: SS 10A), **Luke Rogers**, University of Connecticut, **Robert Strichartz**, Cornell University, and **Alexander Teplyaev**, University of Connecticut.

Difference Equations and Applications (Code: SS 1A), **Michael Radin**, Rochester Institute of Technology.

Gauge Theory and Low-dimensional Topology (Code: SS 7A), **Weimin Chen**, University of Massachusetts-Amherst, and **Daniel Ruberman**, Brandeis University.

Geometric Aspects of Analysis and Measure Theory (Code: SS 9A), **Leonid Kovalev** and **Jani Onninen**, Syracuse University, and **Raanan Schul**, State University of New York at Stony Brook.

Geometric Structures on Manifolds with Special Holonomy, and Applications in Physics (Code: SS 16A), **Tamar Friedmann**, University of Rochester, **Colleen Robles**, Texas A&M University, and **Sema Salur**, University of Rochester.

Geometric and Algebraic Topology (Code: SS 15A), **Boris Goldfarb** and **Marco Varisco**, University at Albany, SUNY.

Geometry of Arithmetic Groups (Code: SS 11A), **Mladen Bestvina**, University of Utah, and **Ken Brown**, **Martin Kassabov**, and **Tim Riley**, Cornell University.

Kac-Moody Lie Algebras, Vertex Algebras, and Related Topics (Code: SS 14A), **Alex Feingold**, Binghamton University, and **Antun Milas**, State University of New York at Albany.

Mathematical Aspects of Cryptography and Cyber Security (Code: SS 8A), **Benjamin Fine**, Fairfield University, **Delaram Kahrobaei**, City University of New York, and **Gerhard Rosenberger**, Passau University and Hamburg University, Germany.

Multivariable Operator Theory (Code: SS 13A), **Ronald G. Douglas**, Texas A&M University, and **Rongwei Yang**, State University of New York at Albany.

Parabolic Evolution Equations of Geometric Type (Code: SS 4A), **Xiaodong Cao**, Cornell University, and **Bennett Chow**, University of California San Diego.

Partial Differential Equations of Mixed Elliptic-Hyperbolic Type and Applications (Code: SS 3A), **Marcus Khuri**, Stony Brook University, and **Dehua Wang**, University of Pittsburgh.

Representations of Local and Global Groups (Code: SS 12A), **Mahdi Asgari**, Oklahoma State University, and **Birgit Speh**, Cornell University.

Set Theory (Code: SS 2A), **Paul Larson**, Miami University, Ohio, **Justin Moore**, Cornell University, and **Ernest Schimmerling**, Carnegie Mellon University.

Species and Hopf Algebraic Combinatorics (Code: SS 6A), **Marcelo Aguiar**, Texas A&M University, and **Samuel Hsiao**, Bard College.

Symplectic Geometry and Topology (Code: SS 5A), **Tara Holm**, Cornell University, and **Katrin Wehrheim**, Massachusetts Institute of Technology.

Accommodations

Participants should make their own arrangements directly with the hotel of their choice as early as possible. Special discounted rates have been negotiated with the hotels listed below. Rates quoted do not include the combined New York state sales (8%) and occupancy tax (5%). Participants must state that they are with the **American Mathematical Society (AMS) Meeting at Cornell University** to receive the discounted rate. The AMS is not responsible for rate changes or for the quality of the accommodations. **Hotels have varying cancellation and early checkout penalties; be sure to ask for details when making your reservation**.

Best Western University Inn, 1020 Ellis Hollow Rd., East Hill Plaza, Ithaca, NY 14850; 607-272-6100; 800-937-8376 (toll free); `http://www.BestWesternUniversityInnIthaca.com`. Rates are US$136 for single/double occupancy. Amenities include deluxe complimentary continental breakfast, free shuttle service to and from Cornell University, downtown Ithaca Commons, and Tompkins County Airport (7 a.m.–11 p.m.); in-room refrigerator, coffee maker, microwave oven; complimentary parking; free wired and wireless Internet in guest rooms; and fitness room. This hotel is pet friendly. This property is located approximately one mile from the campus. **Cancellation and early check-out policies vary**; be sure to check when you make your reservation. The deadline for reservations at this rate is **August 1, 2011**.

Econo Lodge 2303 N. Triphammer Rd., Ithaca, NY 14850; 607-257-1400; `http://www.econolodge.com/hotel/ny127`. Rates are US$119 for single /double occupancy. Amenities include complimentary deluxe continental breakfast, limited free shuttle service within five miles of the hotel (9 a.m.–5 p.m.), in-room coffee maker, 24-hour complimentary coffee and tea in lobby, complimentary parking, and free in-room wireless Internet. This hotel is pet friendly. This property is located approximately three and one half miles from the campus. **Cancellation and early check-out policies vary**; be sure to check when you make your reservation. The deadline for reservations at this rate is **August 8, 2011**.

Holiday Inn–Downtown Ithaca, 222 S. Cayuga St., Ithaca, NY 14850; 607-272-1000; 800-HOLIDAY (toll free); `http://www.HiIthaca.com`. Rates are US$149 per night for single/double occupancy for the dates of this meeting. Amenities include cable TV in guest rooms, free shuttle service to Tompkins County Airport (24 hour), in-room coffee maker, limited complimentary parking, free wireless Internet throughout the hotel, indoor heated swimming pool, fitness center and restaurant on property. This property is located approximately one mile from the campus. **Cancellation and early check-out policies vary**; be sure to check when you make your reservation. The deadline for reservations at this rate is **August 10, 2011**.

Homewood Suites by Hilton–Ithaca, 36 Cinema Drive, Ithaca, NY 14850; 607-266-0000; 800-CALL-HOME (toll free); `http://www.Ithaca.HomewoodSuites.com`. Rates are US$159 for single occupancy. This is an all-suite property and each room has a fully-equipped kitchen with full-size refrigerator, microwave, two-burner stove, and dishwasher. Amenities include a complimentary hot breakfast, free in-room wireless Internet, complimentary beverage service, snack shop, fitness center, indoor heated pool, and basketball and tennis court. This property is pet friendly. This property is located approximately four miles from the campus. **Cancellation and early check-out policies vary**; be sure to check when you make your reservation. The deadline for reservations at this rate is **July 29, 2011**.

Hilton Garden Inn Downtown, 130 E. Seneca Drive, Ithaca, NY 14850; 607-277-8900; 877-STAY-HGI (toll free); `http://www.Ithaca.StayHGI.com`. Rates are US$169 per night for single/double occupancy. Amenities include in-room refrigerator, coffee maker, and microwave oven; complimentary parking; free wired and wireless Internet in guest rooms; fitness room; indoor heated pool; and a 24-hour business center with remote printing capabilities. For dining options this hotel offers a breakfast restaurant, on-site Irish pub, on-site Starbucks, convenience store, and room service. This property is located approximately one mile from the campus. **Cancellation and early check-out policies vary**; be sure to check when you make your reservation. The deadline for reservations at this rate is **August 10, 2011**.

Ramada, 2310 N. Triphammer Rd., Ithaca, NY 14850; 607-257-3100; 800-THE-RAMADA (toll free); `http://www.RamadaInnIthaca.com`. Rates are US$99.95 per night for single/double occupancy. Amenities include, limited free shuttle service to and from Cornell University and Tompkins County Airport; in-room refrigerator, coffee maker, and microwave oven; complimentary parking; high-speed Internet in select guest rooms (be sure to request it, if needed); a heated indoor pool; fitness room; game room;and a business center. For dining options this hotel offers an in-hotel restaurant, lounge, and room service. This hotel is pet friendly. This property is located approximately three miles from the campus. **Cancellation and early check-out policies vary**; be sure to check when you make your reservation. The deadline for reservations at this rate is **August 12, 2011**.

Local Information and Maps

A campus map can be found at `www.cornell.edu/maps/` Information about the Cornell University Department of Mathematics may be found at `www.math.cornell.edu`. Please watch the website available at `http://www.ams.org/meetings/sectional/sectional.html` for additional information on this meeting. Please visit the Cornell University website at `www.cornell.edu` for additional information on the campus.

Other Activities

Book Sales: Stop by the on-site AMS bookstore and review the newest titles from the AMS, enjoy up to 25% off all AMS publications, or take home an AMS t-shirt! Complimentary coffee will be served courtesy of AMS Membership Services.

AMS Editorial Activity: An acquisitions editor from the AMS book program will be present to speak with prospective authors. If you have a book project that you would like to discuss with the AMS, please stop by the book exhibit.

Parking

Hotels are all approximately between one and four miles driving distance from the Cornell Campus. The primary parking garage is on Hoy Road, adjacent to Schoellkopf Field. There is also limited parking available on the weekend adjacent to both Malott Hall and Rockefeller Hall. Parking in many areas of the campus, including the parking garage on Hoy Road and all metered lots, is open after 5:00 p.m., Monday through Friday, and all day Saturday and Sunday. Please read all signs when entering a lot to be sure of the restrictions. All metered parking on campus is free on weekends. Drivers should carefully read signage as some parking spaces require a permit on the weekend.

Registration and Meeting Information

Registration, the AMS book exhibit, and all Invited Addresses will be held in the fifth floor lounge of Malott Hall. Special Sessions will be held in Malott Hall, Phillips Hall, and Rockefeller Hall. Please refer to the campus map at `www.cornell.edu/maps/` for specific locations. The registration desk will be open on Saturday, September 10, 7:30 a.m.-4:00 p.m. and Sunday, September 11, 8:00 a.m.-12:00 p.m. Fees are US$52 for AMS members, US$72 for nonmembers; and US$5 for students, unemployed mathematicians, and emeritus members. Fees are payable on-site via cash, check, or credit card.

For those with mobility issues, the entrance with the most direct access to the elevator in Malott Hall, is the NW entrance. Please look for signs directing you to this entrance. The only use of this entrance on the weekend is to access the elevator.

There will be a **reception** sponsored by the Cornell Mathematics Department. It will take place between 6:00 p.m. and 8:00 p.m. on Saturday, September 10th, in the Big Red Barn, which is adjacent to Malott Hall. The AMS thanks our hosts for their gracious hospitality.

Travel

Cornell is approximately a 10-minute drive from Ithaca Tomkins Regional Airport (ITH) and 90-minute drive from Syracuse Hancock International Airport (SYR). To view several other airport options please reference the Cornell University website at `www.cornell.edu/visiting/ithaca/directions_air.cfm`.

By Air: The Ithaca Tompkins Regional Airport (ITH) is located on Brown Road, Ithaca, NY, a 10-minute drive from campus and downtown areas. Ithaca is serviced by Delta, Continental Airlines, US Airways, and United.

The exclusive provider for ground transportation for Tompkins Airport is Ithaca Dispatch 607-277-7777; 888-321-1149 (toll free). Reservations are not normally required for ground transportation and there is a desk for this vendor located at the baggage claim at the airport. Fares can range between US$10–US$18 one way.

Syracuse Hancock International Airport (SYR) is located in Syracuse, NY, approximately 60 miles from Cornell and 90 minutes driving time. Door-to-door taxi and van service (only economical for groups) is available through the following vendors: Ithaca Airline Limousine 607- 273-3030; 800-273-9197, Executive Limousine and Sedan 607-753- 9180, Superior Limousine 607-758-3333, and IthaCar Service 607-229-3500.

By Train: The nearest train stop is located in Syracuse, NY, approximately 90 minutes from Ithaca. Syracuse is connected to the Empire (N.Y.C.-Niagara Falls), Lake Shore Limited (Chicago-Boston), and Maple Leaf (N.Y.C.-Toronto) routes. The train station is located at 131 P&C Parkway, Syracuse, NY, 800-872-7245. Bus service is offered by New York Trailways to Ithaca from Syracuse (800) 225-6815. In addition to bus service from Syracuse to Ithaca, door-to-door taxi and van service is available to and from the Syracuse Amtrak Station. Connections should be reserved ahead of time with the Ithaca Airline Limousine 607- 273-3030; 800-273-9197 (toll free). Taxi service from Syracuse is less economical.

By Car: Travel on Rte. 79 East. Turn left onto Mitchell St. /Rte. 366. Follow Mitchell up the hill and bear left onto Ithaca Road. Ithaca Road will become Dryden Road at the stoplight. From Dryden Road, turn left onto Hoy Road at the flashing light. Follow Hoy Road to campus. For more detailed directions from all primary directions or for door-to-door directions please visit `www.cornell.edu/visiting/ithaca/directions_car.cfm#doortoinfo`.

Car Rental: There are two car rental agencies with offices at Ithaca Tompkins Regional Airport. For reservations with AVIS please dial toll free 800-831-2847 or visit their website at `www.avis.com`. For reservations with Hertz please dial toll free 800-654-3131 or visit their website at `www.hertz.com`.

Local Transportation

Taxi Service: Three Ithaca taxi companies operate under the same ownership, Ithaca Dispatch, 888-321-1149. These are the individual companies operating under Ithaca Dispatch: Blue Light Transportation 607-277-CABS (no relation to Cornell's Blue Light Services), Cayuga Taxi 607-277-TAXI, and University Taxi 607-277-7777.

Bus Service within Ithaca: Tompkins Consolidated Area Transit (TCAT) buses service Ithaca and the surrounding areas. Routes 14, 20, and 21 connect campus with the Ithaca bus terminal. TCAT also serves as Cornell's primary form of on-campus transportation. Route 31 serves the Ithaca airport.

Weather

The average high temperature for September is approximately 72 degrees and the average low is approximately 52 degrees. Rain is common for this time of year. Visitors should be prepared for inclement weather and check weather forecasts in advance of their arrival.

Information for International Participants

Visa regulations are continually changing for travel to the United States. Visa applications may take from three to four months to process and require a personal interview, as well as specific personal information. International participants should view the important information about traveling to the U.S. found at `http://sites.nationalacademies.org/pga/biso/visas/` and `http://travel.state.gov/visa/visa_1750.html`. If you need a preliminary conference invitation in order to secure a visa, please send your request to `mac@ams.org`. If you discover you do need a visa, the National Academies website (see above) provides these tips for successful visa applications:

* Visa applicants are expected to provide evidence that they are intending to return to their country of residence. Therefore, applicants should provide proof of "binding" or sufficient ties to their home country or permanent residence abroad. This may include documentation of the following:

- family ties in home country or country of legal permanent residence

- property ownership

- bank accounts

- employment contract or statement from employer stating that the position will continue when the employee returns;

* Visa applications are more likely to be successful if done in a visitor's home country than in a third country;

* Applicants should present their entire trip itinerary, including travel to any countries other than the United States, at the time of their visa application;

* Include a letter of invitation from the meeting organizer or the U.S. host, specifying the subject, location and dates of the activity, and how travel and local expenses will be covered;

* If travel plans will depend on early approval of the visa application, specify this at the time of the application;

* Provide proof of professional scientific and/or educational status (students should provide a university transcript).

This list is not to be considered complete. Please visit the websites above for the most up-to-date information.

Winston-Salem, North Carolina

Wake Forest University

September 24–25, 2011

Saturday – Sunday

Meeting #1073

Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: June/July 2011
Program first available on AMS website: August 11, 2011
Program issue of electronic *Notices*: September 2011
Issue of *Abstracts*: Volume 32, Issue 4

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: August 2, 2011

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Benjamin B. Brubaker, Massachusetts Institute of Technology, *Square ice, symmetric functions, and their connections to automorphic forms*.

Shelly Harvey, Rice University, *4-dimensional equivalence relations on knots*.

Allen Knutson, Cornell University, *Modern developments in Schubert calculus*.

Seth M. Sullivant, North Carolina State University, *Algebraic statistics*.

Special Sessions

Algebraic and Geometric Aspects of Matroids (Code: SS 1A), **Hoda Bidkhori**, **Alex Fink**, and **Seth Sullivant**, North Carolina State University.

Applications of Difference and Differential Equations to Biology (Code: SS 2A), **Anna Mummert**, Marshall University, and **Richard C. Schugart**, Western Kentucky University.

Combinatorial Algebraic Geometry (Code: SS 6A), **W. Frank Moore**, Wake Forest University and Cornell University, and **Allen Knutson**, Cornell University.

Extremal Combinatorics (Code: SS 7A), **Tao Jiang**, Miami University, and **Linyuan Lu**, University of South Carolina.

Geometric Knot Theory and its Applications (Code: SS 12A), **Yuanan Diao**, University of North Carolina at

Charlotte, **Jason Parsley**, Wake Forest University, and **Eric Rawdon**, University of St. Thomas.

Low-Dimensional Topology and Geometry (Code: SS 13A), **Shelly Harvey**, Rice University, and **John Etnyre**, Georgia Institute of Technology.

Modular Forms, Elliptic Curves, and Related Topics (Code: SS 11A), **Matthew Boylan**, University of South Carolina, and **Jeremy Rouse**, Wake Forest University.

New Developments in Graph Theory (Code: SS 10A), **Joshua Cooper** and **Kevin Milans**, University of South Carolina, and **Carlos Nicolas** and **Clifford Smyth**, University of North Carolina at Greensboro.

Noncommutative Algebra (Code: SS 5A), **Ellen E. Kirkman** and **James J. Kuzmanovich**, Wake Forest University.

Nonlinear Boundary Value Problems (Code: SS 9A), **Maya Chhetri**, University of North Carolina at Greensboro, and **Stephen B. Robinson**, Wake Forest University.

Nonlinear Dispersive Equations (Code: SS 4A), **Sarah Raynor**, Wake Forest University, **Jeremy Marzuola**, University of North Carolina at Chapel Hill, and **Gideon Simpson**, University of Toronto.

Recent Advances in Infectious Disease Modeling (Code: SS 8A), **Fred Chen** and **Miaohua Jiang**, Wake Forest University.

Set Theoretic Topology (Code: SS 14A), **Peter Nyikos**, University of South Carolina.

Symmetric Functions, Symmetric Group Characters, and Their Generalizations (Code: SS 3A), **Sarah Mason**, Wake Forest University, **Aaron Lauve**, Loyola University-Chicago, and **Ed Allen**, Wake Forest University.

Accommodations

Participants should make their own arrangements directly with a hotel of their choice as early as possible. Special rates have been negotiated with the hotels listed below. Rates quoted do not include sales tax of 13.75%. The AMS is not responsible for rate changes or for the quality of the accommodations. When making a reservation, **participants should state that they are with the American Mathematical Society (AMS) Meeting at Wake Forest University**. Cancellation and early checkout policies vary; be sure to check when you make your reservation.

Residence Inn, 7835 North Point Boulevard, Winston-Salem, NC 27106; phone: 336-759-0777; fax: 336-759-9671. Rates are US$90/night featuring a studio suite with king bed and includes full kitchen with utensils, wireless Internet, full hot breakfast buffet, and evening socials. Ask at the front desk for details regarding the shuttle service to the Wake Forest Campus (US$7 each way; price will be lower per person if there are multiple passengers on one trip). This is a smoke-free hotel. The hotel is located 1.38 miles from the meeting site on campus. Cancellation must be made 72 hours prior to arrival or penalties may apply. Early checkout policies vary; be sure to check when you make your reservation. You may make your reservation online at `http://www.marriott.com/hotels/travel/intup-residence-inn-winston-salem/?toDate=9/25/11&groupCode=WMDWMDA&fromDate=9/23/11&app=resvlink`. The **deadline for reservations is August 19, 2011**; if rooms are still available after the deadline, the special meeting rate will be honored.

Courtyard Winston-Salem University, 3111 University Parkway, Winston-Salem, North Carolina 27105; phone: 336-727-1277; fax: 336-722-8219. Rates are US$89/single or double, featuring wired/wireless Internet access, in-room coffee maker; complimentary coffee in hotel lobby; the Bistro is open for breakfast and also serves light meals with alcoholic and nonalcoholic beverages in the evenings; fitness center and outdoor pool on premises. The hotel is located .86 miles from the meeting site on campus. You may make your reservation on line at `http://www.marriott.com/hotels/travel/Intcy?groupCode=AMSAMSA&app=resvlink&fromDate=9/23/11&toDate=9/26/11`. Cancellation and early check-out policies vary; be sure to check when you make your reservation. The **deadline for reservations is August 19, 2011**; if rooms are still available after the deadline, the special meeting rate will be honored.

Holiday Inn Winston-Salem, 5790 University Parkway, Winston-Salem, NC; phone: 336-767-9595; fax: 336-744-1888. See `www.holidayinn.com/hotels/us/en/winston-salem/intup/hoteldetail` for more information. Rates are US$89/night for a double/king standard room, or US$99/night for a king/double executive room. Both room types include high-speed wireless access and a Grab-n-Go breakfast (coffee, juice, fresh fruit, and muffin). A cocktail lounge and a restaurant offering room service are on premises, along with a fitness center and outdoor pool. The hotel is located 3.18 miles from the meeting site on campus. Cancellation and early checkout policies vary; be sure to check when you make your reservation. The **deadline for reservations is August 19, 2011**; if rooms are still available after the deadline, the special meeting rate will be honored.

Graylyn International Conference Center, 1900 Reynolda Rd., Winston-Salem, North Carolina 27106; phone: 800-472-9596, 336-758-2600; fax: 336-725-5180; see `http://www.graylyn.com/guest_rooms/gr_overview.htm` for more information. The special WFU rate of US$149/ person extended to meeting participants includes a sumptuous Graylyn buffet breakfast. **Rooms may be reserved on a space-available basis only**. This grand estate on 55 acres is now operated as a conference center by WFU. Its magnificent interior completed in 1932 has been meticulously maintained and is surrounded by park-like grounds. In keeping with our residential estate ambience, we have created a nontipping environment by implementing a daily estate fee of US$15 per day, which includes all bell, valet, and housekeeping gratuities. Additional services and amenities included are fitness center access, daily newspaper, valet parking, individual high-speed wireless Internet access, all local calls, and twice-daily housekeeping service. The hotel is located 2.78 miles from the meeting site on campus. Cancellation and early checkout policies vary; be sure to check when you make your reservation.

Food Services

On Campus:

The Fresh Food Company, Reynolda Hall: Also known as "The Pit" (not for its food, but for its location on the lowest level of Reynolda Hall), enjoy all-you-care-to-eat dining for breakfast, lunch, and dinner. Choose from home-style favorites, rotisserie items, sandwiches, fresh salads, pastas, Italian dishes, international meals, home-made desserts, and more.

The Magnolia Room, Reynolda Hall: Enjoy an all-you-care-to-eat lunch buffet prepared by the Executive Catering Chef in a more formal atmosphere.

Benson Center Food Court: Benson's retail food court includes Chick-Fil-A; Mexican dishes from Zoca; World's Fare, featuring international menus and home-style meals; Freshens Energy Zone, offering smoothies, milkshakes, micro blasts, and energy snacks; and grab-and-go options.

Shorty's, Benson Center: Shorty's is a Wake Forest tradition since Shorty Joyner's restaurant started serving students on the Old Campus starting in 1916. This newly remodeled version offers a restaurant experience without leaving campus. Enjoy table-service dining and new menu offerings.

Subway, Hearn Plaza (The Quad): Eat fresh and enjoy the convenience of all your favorite Subway foods right here on campus.

Off Campus:

Simplyummy, 122 Reynolda Village, 336-724-9779;
Golden India Restaurant, 2837 Fairlawn Dr., 336-777-0004;
Vincenzo's Italian Restaurant, 3449 Robinhood Rd., 336-765-3176;
River Birch Lodge, 3324 Robinhood Rd., 336-768-1111;
Village Tavern, 221 Reynolda Village, 336-748-0221;
Red, Hot and Blue BBQ, 613 Deacon Boulevard, 336-770-4227.

Meeting and Registration Information

Invited Addresses and all other sessions will be held in Manchester Hall, Greene Hall, and Carswell Hall. Registration will be held in The Benson Center, 3rd Floor Rotunda, 7:30 a.m. to 4:00 p.m. on Saturday and 8:00 a.m. to noon on Sunday. Fees are US$52 for AMS members; US$72 for nonmembers; and US$5 for students, unemployed mathematicians, and emeritus members. **Fees are payable on-site via cash, check, or credit card**.

Other Activites

Book Sales: Stop by the AMS onsite bookstore located in Benson, 401-B, and review the newest titles, enjoy up to 25% off all AMS publications, or take home an AMS t-shirt! Complimentary coffee will be served courtesy of AMS Membership Services.

AMS Editorial Activity: An acquisitions editor from the AMS book program will be present to speak with prospective authors. If you have a book project that you would like to discuss with the AMS, please stop by the book exhibit.

Parking and Local Information

The university's main webpage is found at `www.wfu.edu`; the Department of Mathematics website is `www.math.wfu.edu`. Maps of campus and the surrounding areas (a self-guided walking tour; Reynolda Historic District featuring Reynolda Village, gardens, and museums; and the areas surrounding the campus) are located at `www.wfu.edu/visitors/maps/`. On the weekend of the meeting, participants may park in any lot for faculty, staff, or visitors. A campus parking map is at `www.wfu.edu/facilities/parking/ParkingMap_082010.pdf`.

The GPS coordinates for Wake Forest University, Reynolda Campus, Undergraduate Admissions Office, 1834 Wake Forest Road, Winston-Salem, NC 27106 are 36.131378, -80.283661.

Travel

By Plane: The Piedmont Triad International Airport (GSO) in Greensboro, often called PTI for short, is the closest airport to campus; about a 42-minute ride (32.6 miles) from the airport to campus/hotels. Charlotte Douglas International Airport (CLT) is about a 90-minute drive to campus. Raleigh-Durham Airport (RDU) is about a 2-hour drive to campus. Smith Reynolds Airport in Winston-Salem provides services for private aircraft, charters and non-commercial flights. There are shuttles to and from GSO, CLT, and RDU. ABC Door 2 Door Transportation `www.abcdoor2door.com` (US$49 one way, 336-721-9921), in Winston-Salem provides transportation to and from PTI. You may also try Yellow Cabs, 336-722-7121, where the cost is US$3.90 for the first mile plus US$2.10/mile for the remaining trip. Prices were accurate at the time this announcement was published. Reservations are highly recommended.

By Bus: Greyhound Bus Lines, 250 Greyhound Ct., Winston Salem, NC 27101, 336-724-1429.

By Car: From Washington, D.C. (and points north): Take I-95 south to I-85 south to I-40 west. In Winston-Salem take Business 40 to the Silas Creek Parkway North exit and follow the signs.

From Raleigh (and points east): Take I-40 west to Winston-Salem. Take Business 40 to the Silas Creek Parkway North exit and follow the signs.

From Asheville (and points west): Take I-40 east to Winston-Salem. Take Business 40 to the Silas Creek Parkway North exit and follow the signs.

Car Rental

Hertz is the official car rental company for the meeting. To make a reservation using our special meeting rates, please provide our convention number: CV#, **04N30001** when making reservations. Details on rates and general instructions can be found at `http://www.ams.org/amsmtgs/04N30001.win11.pdf`. You can make reservations online at `http://link.hertz.com/link.html?id=23051&LinkType=HZLK&TargetType=Homepage&ret_url=www.ams.org` or call Hertz directly at 800-654-2240 (U.S. and Canada) or

405-749-4434 (other countries). At the time of reservation, the meeting rates will be automatically compared to other Hertz rates and you will be quoted the best comparable rate available.

Weather

Average daytime temperatures in September are around 79°F dropping to about 60°F at night. Precipitation is around 4.3 inches.

Information for International Participants

Visa regulations are continually changing for travel to the United States. Visa applications may take from three to four months to process and require a personal interview, as well as specific personal information. International participants should view the important information about traveling to the U.S. found at `http://sites.nationalacademies.org/pga/biso/visas/` and `http://travel.state.gov/visa/visa_1750.html`. If you need a preliminary conference invitation in order to secure a visa, please send your request to `dls@ams.org`.

If you discover you do need a visa, the National Academies website (see above) provides these tips for successful visa applications:

* Visa applicants are expected to provide evidence that they are intending to return to their country of residence. Therefore, applicants should provide proof of "binding" or sufficient ties to their home country or permanent residence abroad. This may include documentation of the following:

- family ties in home country or country of legal permanent residence
- property ownership
- bank accounts
- employment contract or statement from employer stating that the position will continue when the employee returns;

* Visa applications are more likely to be successful if done in a visitor's home country than in a third country;

* Applicants should present their entire trip itinerary, including travel to any countries other than the United States, at the time of their visa application;

* Include a letter of invitation from the meeting organizer or the U.S. host, specifying the subject, location and dates of the activity, and how travel and local expenses will be covered;

* If travel plans will depend on early approval of the visa application, specify this at the time of the application;

* Provide proof of professional scientific and/or educational status (students should provide a university transcript).

This list is not to be considered complete. Please visit the websites above for the most up-to-date information.

Lincoln, Nebraska

University of Nebraska-Lincoln

October 14–16, 2011

Friday – Sunday

Meeting #1074

Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: August 2011
Program first available on AMS website: September 1, 2011
Program issue of electronic *Notices*: October 2011
Issue of *Abstracts*: Volume 32, Issue 4

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: June 28, 2011
For abstracts: August 23, 2011

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Lewis Bowen, Texas A&M University, *Title to be announced*.

Emmanuel Candes, Stanford University, *Title to be announced* (Erdős Memorial Lecture).

Alina Cojocaru, University of Illinois at Chicago, *Title to be announced*.

Michael Zieve, University of Michigan, *Title to be announced*.

Special Sessions

Algebraic Geometry and Graded Commutative Algebra (Code: SS 8A), **Susan Cooper** and **Brian Harbourne**, University of Nebraska Lincoln.

Algorithmic and Geometric Properties of Groups and Semigroups (Code: SS 10A), **Susan Hermiller** and **John Meakin**, University of Nebraska Lincoln.

Association Schemes and Related Topics (Code: SS 1A), **Sung Y. Song**, Iowa State University, and **Paul Terwilliger**, University of Wisconsin Madison.

Asymptotic Behavior and Regularity for Nonlinear Evolution Equations (Code: SS 4A), **Petronela Radu** and **Lorena Bociu**, University of Nebraska Lincoln.

Coding Theory (Code: SS 7A), **Christine Kelley** and **Judy Walker**, University of Nebraska Lincoln.

Commutative Algebra (Code: SS 16A), **Christina Eubanks-Turner**, University of Louisiana at Lafayette, and **Aihua Li**, Montclair State University.

Computational and Applied Mathematics (Code: SS 13A), **Ludwig Kohaupt**, Beuth University of Technology Berlin, Germany, and **Yan Wu**, Georgia Southern University.

Continuous and Numerical Analysis in the Control of PDE's (Code: SS 9A), **George Avalos**, **Mohammad**

Rammaha, and **Daniel Toundykov**, University of Nebraska Lincoln.

Discrete Methods and Models in Biomathematics (Code: SS 18A), **Dora Matache** and **Jim Rogers**, University of Nebraska Omaha, and **Alan Veliz-Cuba**, University of Nebraska Lincoln.

Dynamic Systems on Time Scales with Applications (Code: SS 3A), **Lynn Erbe** and **Allan Peterson**, University of Nebraska Lincoln.

Dynamical Systems and Operator Algebras (Code: SS 15A), **Lewis Bowen**, Texas A&M University, and **David Kerr**, Texas A&M University at Galveston.

Extremal and Probabilistic Combinatorics (Code: SS 5A), **Stephen Hartke** and **Jamie Radcliffe**, University of Nebraska Lincoln.

Invariants in Knot Theory and Low-dimensional Topology (Code: SS 14A), **Mark Brittenham**, University of Nebraska Lincoln, and **Robert Todd**, University of Nebraska Omaha.

Local Commutative Algebra (Code: SS 11A), **H. Ananthnarayan**, University of Nebraska Lincoln, **Ines B. Henriques**, University of California Riverside, and **Hamid Rahmati**, Syracuse University.

Matrices and Graphs (Code: SS 12A), **In-Jae Kim**, Minnesota State University, **Adam Berliner**, St. Olaf College, **Leslie Hogben**, Iowa State University, and **Bryan Shader**, University of Wyoming.

Quantum Groups and Representation Theory (Code: SS 2A), **Jonathan Kujawa**, University of Oklahoma, and **Natasha Rozhkovskaya**, Kansas State University.

Recent Directions in Number Theory (Code: SS 17A), **Alina Cojocaru**, University of Illinois at Chicago, and **Michael Zieve**, University of Michigan.

Recent Progress in Operator Algebras (Code: SS 6A), **Allan P. Donsig** and **David R. Pitts**, University of Nebraska Lincoln.

Salt Lake City, Utah

University of Utah

October 22–23, 2011

Saturday – Sunday

Meeting #1075

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: August 2011
Program first available on AMS website: September 8, 2011
Program issue of electronic *Notices*: October 2011
Issue of *Abstracts*: Volume 32, Issue 4

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: July 5, 2011
For abstracts: August 30, 2011

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Graeme Milton, University of Utah, *Title to be announced.*

Lei Ni, University of California San Diego, *Title to be announced.*

Igor Pak, University of California Los Angeles, *Title to be announced.*

Monica Visan, University of California Los Angeles, *Title to be announced.*

Special Sessions

Algebraic Geometry (Code: SS 8A), **Tommaso de Fernex** and **Christopher Hacon**, University of Utah.

Applied Analysis (Code: SS 15A), **Marian Bocea**, North Dakota State University, and **Mihai Mihailescu**, University of Craiova Romania.

Category Theory in Graphs, Geometry and Inverse Problems (Code: SS 12A), **Robert Owczarek**, Enfitec. Inc., and **Hanna Makaruk**, Los Alamos National Laboratory NM.

Celestial and Geometric Mechanics (Code: SS 5A), **Lennard Bakker** and **Tiancheng Ouyang**, Brigham Young University.

Commutative Algebra (Code: SS 3A), **Chin-Yi Jean Chan**, Central Michigan University, and **Lance E. Miller** and **Anurag K. Singh**, University of Utah.

Computational and Algorithmic Algebraic Geometry (Code: SS 17A), **Zach Teitler**, Boise State University, and **Jim Wolper**, Idaho State University.

Electromagnetic Wave Propagation in Complex and Random Environments (Code: SS 4A), **David Dobson**, University of Utah, and **Peijun Li**, Purdue University.

Geometric Evolution Equations and Related Topics. (Code: SS 2A), **Andrejs Treibergs**, University of Utah Salt Lake City, **Lei Ni**, University of California San Diego, and **Brett Kotschwar**, Arizona State University.

Geometric, Combinatorial, and Computational Group Theory (Code: SS 1A), **Eric Freden**, Southern Utah University, and **Eric Swenson**, Brigham Young University.

Harmonic Analysis and Dispersive PDE (Code: SS 6A), **Xiaoyi Zhang**, University of Iowa, and **Monica Visan** and **Betsy Stovall**, University of California Los Angeles.

Hypergeometric Functions and Differential Equations (Code: SS 13A), **Laura F. Matusevich**, Texas A&M University, and **Christine Berkesch**, Stockholm University.

Inverse Problems and Homogenization (Code: SS 10A), **Elena Cherkaev** and **Fernando Guevara Vasquez**, University of Utah.

Noncommutative Geometry and Algebra (Code: SS 11A), **Kenneth R. Goodearl**, University of California Santa Barbara, and **Milen Yakimov**, Louisiana State University.

Nonlinear Waves (Code: SS 7A), **Zhi-Qiang Wang** and **Nghiem Nguyen**, Utah State University.

Recent Progress in Numerical PDEs (Code: SS 9A), **Jichun Li**, University of Nevada Las Vegas, and **Shue-Sum Chow**, Brigham Young University.

Reductive Groups and Hecke Algebras (Code: SS 14A), **Dan Ciubotaru**, University of Utah, **Cathy Kriloff**, Idaho State University, and **Peter Trapa**, University of Utah.

Understanding Bio-fluids via Modeling, Simulation and Analysis (Code: SS 16A), **Christel Hohenegger**, University of Utah.

Port Elizabeth, Republic of South Africa

Nelson Mandela Metropolitan University

November 29 – December 3, 2011

Tuesday – Saturday

Meeting #1076

First Joint International Meeting between the AMS and the South African Mathematical Society.
Associate secretary: Matthew Miller
Announcement issue of *Notices*: June/July 2011
Program first available on AMS website: Not applicable
Program issue of electronic *Notices*: Not applicable
Issue of *Abstracts*: Not applicable

Deadlines

For organizers: Please see the website.
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/internmtgs.html`.

Invited Addresses

Mark J. Ablowitz, University of Colorado, *Title to be announced.*

Mikhail Petrov, University of Swaziland, *Title to be announced.*

James Raftery, University of Kwazulu Natal, *Title to be announced.*

Daya Reddy, University of Cape Town, *Title to be announced.*

Peter Sarnak, Princeton University, *Title to be announced.*

Robin Thomas, Georgia Institute of Technology, *Title to be announced.*

Amanda Weltman, University of Cape Town, *Title to be announced.*

Special Sessions

Combinatorial and Computational Group Theory with Applications, **Gilbert Baumslag**, City College of New York, **Mark Berman**, University of Cape Town, and **Vladimir Shpilrain**, City College of New York.

Combinatorics and Graph Theory, **Michael Henning**, University of Johannesburg, **Robin Thomas**, Georgia Institute of Technology, and **Jacques Verstraete**, University of California San Diego.

Finite Groups and Combinatorial Structures, **Jashmid Moori**, North-West University, Mafikeng, and **B. Rodrigues**, University of Kwazulu-Natal, Westville.

Geometry and Differential Equations, **Jesse Ratzkin**, University of Cape Town.

High Performance Computing and Imaging, **Steven B. Damelin**, Georgia Southern University and University of the Witswatersrand, and **Hari Kumar**, University of the Witswatersrand.

Nonlinear Waves and Integrable Systems, **Mark Ablowitz**, University of Colorado at Boulder, and **Barbara Prinari**, University of Colorado at Colorado Springs.

Recent Advances in Computational Methods for Partial Differential Equations, **Kailash C. Patidar**, University of the Western Cape.

Operator and Banach Algebras, and Noncommutative Analysis, **David Blecher**, University of Houston, **Garth Dales**, University of Leeds, **Louis Labuschagne**, North-West University, Potchefstroom Campus, and **Anton Stroh**, University of Pretoria.

Topology and Categories, **Hans-Peter Kuenzi**, University of Cape Town.

This announcement was composed with information taken from the website maintained by the local organizers at `http://www.nmmu.ac.za/sams-ams2011/index.htm`. *Please watch this website for the most up-to-date information.*

Abstract Submission

Talks in Special Sessions are generally by invitation of the organizers. It is recommended that you contact an organizer before submitting an abstract for a Special Session if you have not been specifically invited to speak in the session. Please watch the website cited above for more information on abstract submission and associated deadlines.

Accommodations

For free assistance with making reservations and inquiries about tourist information, please contact Nelson Mandela Bay Tourism's call center at 011 27 41 582 2575 (from the U.S.), send email to `info@nmbt.co.za` or visit `www.nmbt.co.za`. All prices are quoted in Rands; at the time this announcement was published, 1R = US$0.15. Cancellation and early checkout policies vary; be sure to check when you make your reservation.

The local host organizers plan to contract for shuttle services between the conference, the hotels listed below, and the Langerry Group apartments; there will be an option to purchase this service when you register online or at the conference.

Summerstrand Hotel, Marine Drive, Summerstrand; telephone 011 27 41 583 3131 (from the U.S.); email: `info@hotsummer.co.za`; website: `http://www.hotsummer.co.za.` Nestled on the shore of Nelson Mandela Bay and the warm Indian Ocean, in very close proximity to NMMU, we welcome you to experience our hospitality from the heart. We are here to make your stay for the conference a pleasurable one. Individuals should identify themselves as delegates of the AMS/SAMS conference and quote the following reference: **SAMS Conference**. All rates include breakfast. If reservations are made by June 30, the following rates apply: R789/single or R893/double. From July 1 through October 31: R815/per single or R945/double. From November 1 until the beginning of the conference: R861/single or R984/double. Check-in is at 2:00 p.m.; check-out is 11:00 a.m. It is about 1.5 km (one mile) to the meeting venues.

Pine Lodge, 1 Marine Drive, Port Elizabeth; telephone 011 27 41 583-4004 (from the U.S.); email: enquiries@pinelodge.co.za (Keri Saunders); website: http://www.pinelodge.co.za. Nestled among the picturesque dunes of the Cape Recife Nature Reserve are the three-star timber chalets of our resort. Please reference **MATHS** when making your reservation to take advantage of the special rates. Rates including breakfast are R755/single or R942/double. Chalets with kitchens (no breakfast included) are R660 per person. A full buffet breakfast is available for R95. It is about 1.3 km (.8 mile) to the meeting venues.

The Beach Hotel, Marine Drive, Port Elizabeth; email: info@thebeachhotel.co.za; website: http://www.thebeachhotel.co.za. Rates for rooms with courtyard views are R975/single or R1280/double. Rates for rooms with sea views are R1175/single or R1540/double. These rates are net per day and include a VAT at 14% and a full English breakfast, but do not include a 1% tourism tax charged per day. The hotel offers three restaurants for your dining pleasure. It is about 1.5 km (one mile) to the meeting venues.

For a full array of apartment-style accommodations, many by the seaside, contact the Langerry Group of Holiday Apartments through http://www.langerry.co.za/default.aspx. These may be a more economical option than the hotels listed above, although they may be a bit further away from the conference venues. They will be included on the conference shuttle route for which our hosts are making arrangements.

For links to other types of accommodations in the Summerstrand, see http://www.wheretostay.co.za/ec/cs/accommodation/summerstrand.php. For links to bed and breakfast accommodations in Humewood, see http://www.wheretostay.co.za/towns/humewood/bed-breakfast.

Local Information/Tourism

Local and area maps are posted on the website cited at the beginning of this announcement. The local organizers are planning two excursions for participants and guests so that you may experience so much more of what South Africa has to offer (extra cost). Watch the website for details.

South Africans drive on the left hand side of the road, so all rental cars will have steering wheels on the right-hand side of the cars. Most cars have manual transmissions. Maps of the area are found at www.nmmu.ac.za/sams-ams2011/maps.htm. If you plan to rent a car, you must have an international driving permit. Your nearest AAA office (http://www.aaa.com) can help you obtain this document.

It is advisable to rent a cell phone at the airport (there are three national cell phone companies). This is a more affordable option for local calls than using your cell phone from home.

It is easiest to exchange currency at the Port Elizabeth airport; however, there are some banks in the Summerstrand near campus that have foreign exchange facilities.

Electrical plugs in South Africa are type M. You may also use type C; however, you will need an adapter to type M to be able to plug appliances into the wall. The current is 220/230V, 50Hz.

Registration, Opening Reception, and Conference Dinner

The conference will take place at Nelson Mandela Metropolitan University, Summerstrand South Campus, in Building 123, Building 35, and the South Campus Auditorium. Watch the local organizers' website for registration fees, online registration procedures, and specific onsite locations. The registration fee will include an Opening Reception held on Tuesday, November 29, and the Conference Dinner, Friday, December 2.

Travel

Participants should plan to fly into Port Elizabeth International Airport (PLZ); however, you must fly into either the Johannesburg or Cape Town International airports first. Make sure that you have at least two blank pages available in your passport. If you are a U.S. national traveling from the U.S. and plan to stay for fewer than 90 days, no visa is required. Other travelers should check the information at http://www.southafrica.info/travel/documents/visas.htm. Portions of northern part of the country report malaria; however, the Eastern Cape and Nelson Mandela Bay do not. If you have concerns, please visit the Centers for Disease Control website at http://wwwnc.cdc.gov/travel/destinations/south-africa.htm, or consult a health-care provider who specializes in travel medicine.

Driving to the campus from the airport: Upon exiting the airport area, turn right onto Allister Miller Drive and proceed for approximately 1.5 km (one mile); turn right at the first traffic light into La Roche Road and proceed for approximately 1.5 km (one mile); turn right at the second traffic light into Strandfontein Rd. and proceed for approximately three km (two miles); turn left at the SPAR supermarket into Admiralty Rd. and proceed for approximately two km (1.3 miles); turn right at the first traffic light onto University Way and proceed straight along past the North Campus on the left into the South Campus. Once past the security gate, clear signage will be visible to direct you to the registration area and the lecture halls.

The recommended car rental agency for the conference is Budget; see http://www.budget.com.

Weather

Early December weather is generally warm to hot, 75° F (24° C), during the day, and falling to 55° F (14° C) at night. Please prepare accordingly.

Boston, Massachusetts

John B. Hynes Veterans Memorial Convention Center, Boston Marriott Hotel, and Boston Sheraton Hotel

January 4–7, 2012

Wednesday – Saturday

Meeting #1077

Joint Mathematics Meetings, including the 118th Annual Meeting of the AMS, 95th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: October 2011
Program first available on AMS website: November 1, 2011
Program issue of electronic *Notices*: January 2012
Issue of *Abstracts*: Volume 33, Issue 1

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: September 22, 2011

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/national.html.

Joint Invited Addresses

Erik Demaine, Massachusetts Institute of Technology, *Title to be announced* (AMS-MAA-SIAM Gerald and Judith Porter Public Lecture).

AMS Invited Addresses

George E. Andrews, Penn State University, *Title to be announced* (AMS Retiring Presidential Address).

Bradley Efron, Stanford University, *Title to be announced* (AMS Josiah Willard Gibbs Lecture).

Edward Frenkel, University of California Berkeley, *Langlands program, trace formulas, and their geometrization* (AMS Colloquium Lectures).

Larry Guth, University of Toronto, *The polynomial method in combinatorial geometry*.

Assaf Naor, Courant Institute of Mathematical Sciences, *Title to be announced*.

Eric Rains, California Institute of Technology, *Beyond q: Special functions on elliptic curves*.

Wilhelm Schlag, University of Chicago, *Invariant manifolds and dispersive Hamiltonian evolution equations*.

Honolulu, Hawaii

University of Hawaii

March 3–4, 2012

Saturday – Sunday

Meeting #1078

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: October 2012
Program first available on AMS website: January 26, 2012
Program issue of electronic *Notices*: March 2012
Issue of *Abstracts*: Volume 33, Issue 2

Deadlines

For organizers: August 3, 2011
For consideration of contributed papers in Special Sessions: November 22, 2011
For abstracts: December 13, 2011

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Zhiqin Lu, University of California Irvine, *Title to be announced*.

Peter Schroder, California Institute of Technology, *Title to be announced*.

Pham Tiep, University of Arizona, Tucson, *Title to be announced*.

Lauren Williams, University of California Berkeley, *Title to be announced*.

Special Sessions

Kaehler Geometry and Its Applications (Code: SS 1A), **Zhiqin Lu**, University of California Irvine, **Jeff Streets**, Princeton University, **Li-Sheng Tseng**, Harvard University, and **Ben Weinkove**, University of California San Diego.

Linear and Permutation Representations (Code: SS 2A), **Robert Guralnick**, University of Southern California, and **Pham Huu Tiep**, University of Arizona.

Tampa, Florida

University of South Florida

March 10–11, 2012

Saturday – Sunday

Meeting #1079

Southeastern Section

Associate secretary: Matthew Miller
Announcement issue of *Notices*: January
Program first available on AMS website: February 2, 2012
Program issue of electronic *Notices*: March 2012
Issue of *Abstracts*: Volume 33, Issue 2

Deadlines

For organizers: August 10, 2011
For consideration of contributed papers in Special Sessions: November 29, 2011
For abstracts: January 18, 2012

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Anne Condon, University of British Columbia, *Title to be announced*.

Mark Ellingham, Vanderbilt University, *Title to be announced*.

Mauro Maggioni, Duke University, *Digital data sets: Geometry, random walks, multiscale analysis, and applications*.

Weiqiang Wang, University of Virginia, *Title to be announced*.

Special Sessions

Algebraic and Combinatorial Structures in Knot Theory (Code: SS 2A), **J. Scott Carter**, University of South Alabama, and **Mohamed Elhamdadi** and **Masahico Saito**, University of South Florida.

Analysis in Metric Spaces (Code: SS 3A), **Thomas Bieske**, University of South Florida, and **Jason Gong**, University of Pittsburgh.

Discrete Models in Molecular Biology (Code: SS 1A), **Alessandra Carbone**, Université Pierre et Marie Curie and Laboratory of Microorganisms Genomics, **Natasha Jonoska**, University of South Florida, and **Reidun Twarock**, University of York.

Washington, District of Columbia

George Washington University

March 17–18, 2012

Saturday – Sunday

Meeting #1080

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: January 2012
Program first available on AMS website: February 9, 2012
Program issue of electronic *Notices*: March 2012
Issue of *Abstracts*: Volume 33, Issue 2

Deadlines

For organizers: August 17, 2011
For consideration of contributed papers in Special Sessions: December 6, 2011
For abstracts: January 31, 2012

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Jim Geelen, University of Waterloo, *Title to be announced*.

Boris Solomyak, University of Washington, *Title to be announced*.

Gunther Uhlmann, University of Washington, *Title to be announced* (Einstein Public Lecture in Mathematics).

Anna Wienhard, Princeton University, *Title to be announced*.

Special Sessions

Homology Theories Motivated by Knot Theory (Code: SS 3A), **Jozef H. Przytycki**, George Washington University, **Radmila Sazdanovic**, University of Pennsylvania, and **Alexander N. Shumakovitch** and **Hao Wu**, George Washington University.

Matroid Theory (Code: SS 1A), **Joseph E. Bonin**, George Washington University, and **Sandra Kingan**, Brooklyn College.

Optimization: Theory and Applications (Code: SS 2A), **Roman Sznajder**, Bowie State University.

Structural and Extremal Problems in Graph Theory (Code: SS 4A), **Daniel Cranston**, Virginia Commonwealth University, and **Gexin Yu**, College of William & Mary.

Lawrence, Kansas

University of Kansas

March 30 – April 1, 2012

Friday – Sunday

Meeting #1081

Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: February 2012
Program first available on AMS website: March 8, 2012
Program issue of electronic *Notices*: March 2012
Issue of *Abstracts*: Volume 33, Issue 2

Deadlines

For organizers: August 31, 2011
For consideration of contributed papers in Special Sessions: December 20, 2011
For abstracts: February 14, 2012

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Frank Calegari, Northwestern University, *Title to be announced*.

Christopher Leininger, University of Illinois at Urbana-Champaign, *Title to be announced*.

Alina Marian, University of Illinois at Chicago, *Title to be announced*.

Catherine Yan, Texas A&M University, *Title to be announced*.

Special Sessions

Combinatorial Commutative Algebra (Code: SS 1A), **Christopher Francisco** and **Jeffrey Mermin**, Oklahoma State University, and **Jay Schweig**, University of Kansas.

Rochester, New York

Rochester Institute of Technology

September 22–23, 2012

Saturday – Sunday

Meeting #1082

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: May 2012
Program first available on AMS website: July 19, 2012
Program issue of electronic *Notices*: September 2012
Issue of *Abstracts*: Volume 33, Issue 3

Deadlines

For organizers: February 22, 2012
For consideration of contributed papers in Special Sessions: May 15, 2012
For abstracts: July 10, 2012

New Orleans, Louisiana

Tulane University

October 13–14, 2012

Saturday – Sunday

Meeting #1083

Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: June 2012
Program first available on AMS website: September 6, 2012
Program issue of electronic *Notices*: October 2012
Issue of *Abstracts*: Volume 33, Issue 3

Deadlines

For organizers: March 13, 2012
For consideration of contributed papers in Special Sessions: July 3, 2012
For abstracts: August 28, 2012

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Anita Layton, Duke University, *Title to be announced*.

Lenhard Ng, Duke University, *Title to be announced*.

Henry K. Schenck, University of Illinois at Urbana-Champaign, *From approximation theory to algebraic geometry: The ubiquity of splines*.

Milen Yakimov, Louisiana State University, *Title to be announced*.

Akron, Ohio

University of Akron

October 20–21, 2012

Saturday – Sunday

Meeting #1084

Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: August 2012
Program first available on AMS website: September 27, 2012
Program issue of electronic *Notices*: October 2012
Issue of *Abstracts*: Volume 33, Issue 4

Deadlines

For organizers: March 22, 2012
For consideration of contributed papers in Special Sessions: July 10, 2012
For abstracts: September 4, 2012

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Tanya Christiansen, University of Missouri, *Title to be announced*.

Tim Cochran, Rice University, *Title to be announced*.

Ronald Solomon, Ohio State University, *Title to be announced*.

Ben Weinkove, University of California San Diego, *Title to be announced*.

Tucson, Arizona

University of Arizona, Tucson

October 27–28, 2012

Saturday – Sunday

Meeting #1085

Western Section
Associate secretary: Michel L. Lapidus

Announcement issue of *Notices*: August 2012
Program first available on AMS website: October 4, 2012
Program issue of electronic *Notices*: October 2012
Issue of *Abstracts*: Volume 33, Issue 4

Deadlines

For organizers: March 27, 2012
For consideration of contributed papers in Special Sessions: July 17, 2012
For abstracts: September 11, 2012

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Michael Hutchings, University of California Berkeley, *Title to be announced.*

Kenneth McLaughlin, University of Arizona, Tucson, *Title to be announced.*

Ken Ono, Emory University, *Title to be announced* (Erdős Memorial Lecture).

Jacob Sterbenz, University of California San Diego, *Title to be announced.*

Goufang Wei, University of California Santa Barbara, *Title to be announced.*

Special Sessions

Harmonic Maass Forms and q-series (Code: SS 1A), **Ken Ono**, Emory University, **Amanda Folsom**, Yale University, and **Zachary Kent**, Emory University.

San Diego, California

San Diego Convention Center and San Diego Marriott Hotel and Marina

January 9–12, 2013

Wednesday – Saturday

Meeting #1086

Joint Mathematics Meetings, including the 119th Annual Meeting of the AMS, 96th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: October 2012
Program first available on AMS website: November 1, 2012
Program issue of electronic *Notices*: January 2012
Issue of *Abstracts*: Volume 34, Issue 1

Deadlines

For organizers: April 1, 2012
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Chestnut Hill, Massachusetts

Boston College

April 6–7, 2013

Saturday – Sunday
Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 6, 2012
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Ames, Iowa

Iowa State University

April 27–28, 2013

Saturday – Sunday
Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: April 2013
Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 27, 2012
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Special Sessions

Operator Algebras and Topological Dynamics (Code: SS 1A), **Ken Ono**, Emory University, **Amanda Folsom**, Yale University, and **Zachary Kent**, Emory University.

Alba Iulia, Romania

June 27–30, 2013

Thursday – Sunday

First Joint International Meeting of the AMS and the Romanian Mathematical Society, in partnership with the "Simion Stoilow" Institute of Mathematics of the Romanian Academy.

Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: Not applicable
Program issue of electronic *Notices*: Not applicable
Issue of *Abstracts*: Not applicable

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

St. Louis, Missouri

Washington University

October 18–20, 2013

Friday – Sunday

Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 20, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Riverside, California

University of California Riverside

November 2–3, 2013

Saturday – Sunday

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 2, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Baltimore, Maryland

Baltimore Convention Center, Baltimore Hilton, and Marriott Inner Harbor

January 15–18, 2014

Wednesday – Saturday

Joint Mathematics Meetings, including the 120th Annual Meeting of the AMS, 97th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Matthew Miller
Announcement issue of *Notices*: October 2013
Program first available on AMS website: November 1, 2013
Program issue of electronic *Notices*: January 2013
Issue of *Abstracts*: Volume 35, Issue 1

Deadlines

For organizers: April 1, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Tel Aviv, Israel

Bar-Ilan University, Ramat-Gan and Tel-Aviv University, Ramat-Aviv

June 16–19, 2014

Monday – Thursday

The 2nd Joint International Meeting with Israel is co-hosted by the Bar-Ilan University and the Tel-Aviv University, both in the Tel-Aviv Area. Contact person for this meeting is Louis Rowen, Mathematics Department, Bar-Ilan University, Rama-Gan, Israel; email: `rowen@math.biu.ac.il`. *This meeting is co-sponsored by the Israel Mathematical Union.*

Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

San Antonio, Texas

Henry B. Gonzalez Convention Center and Grand Hyatt San Antonio

January 10–13, 2015

Saturday – Tuesday

Joint Mathematics Meetings, including the 121st Annual Meeting of the AMS, 98th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: October 2014
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2015
Issue of *Abstracts*: Volume 36, Issue 1

Deadlines

For organizers: April 1, 2014
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Seattle, Washington

Washington State Convention Center and the Sheraton Seattle Hotel

January 6–9, 2016

Wednesday – Saturday

Joint Mathematics Meetings, including the 122nd Annual Meeting of the AMS, 99th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: October 2015
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2016
Issue of *Abstracts*: Volume 37, Issue 1

Deadlines

For organizers: April 1, 2015
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Atlanta, Georgia

Hyatt Regency Atlanta and Marriott Atlanta Marquis

January 4–7, 2017

Wednesday – Saturday

Joint Mathematics Meetings, including the 123rd Annual Meeting of the AMS, 100th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Georgia Benkart
Announcement issue of *Notices*: October 2016
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2017
Issue of *Abstracts*: Volume 38, Issue 1

Deadlines

For organizers: April 1, 2016
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

San Diego, California

San Diego Convention Center

January 10–13, 2018

Wednesday – Saturday

Joint Mathematics Meetings, including the 124th Annual Meeting of the AMS, 101st Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Matthew Miller
Announcement issue of *Notices*: October 2017
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 1, 2017
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Meetings and Conferences of the AMS

Associate Secretaries of the AMS

Western Section: Michel L. Lapidus, Department of Mathematics, University of California, Surge Bldg., Riverside, CA 92521-0135; e-mail: `lapidus@math.ucr.edu`; telephone: 951-827-5910.

Central Section: Georgia Benkart, University of Wisconsin-Madison, Department of Mathematics, 480 Lincoln Drive, Madison, WI 53706-1388; e-mail: `benkart@math.wisc.edu`; telephone: 608-263-4283.

Eastern Section: Steven H. Weintraub, Department of Mathematics, Lehigh University, Bethlehem, PA 18105-3174; e-mail: `steve.weintraub@lehigh.edu`; telephone: 610-758-3717.

Southeastern Section: Matthew Miller, Department of Mathematics, University of South Carolina, Columbia, SC 29208-0001, e-mail: `miller@math.sc.edu`; telephone: 803-777-3690.

The Meetings and Conferences section of the *Notices* gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited in the table of contents on this page for more detailed information on each event. Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. **Information in this issue may be dated. Up-to-date meeting and conference information can be found at** `www.ams.org/meetings/`**.**

Meetings:

Important Information Regarding AMS Meetings

Potential organizers, speakers, and hosts should refer to page 100 in the January 2011 issue of the *Notices* for general information regarding participation in AMS meetings and conferences.

Abstracts

Speakers should submit abstracts on the easy-to-use interactive Web form. No knowledge of LaTeX is necessary to submit an electronic form, although those who use LaTeX may submit abstracts with such coding, and all math displays and similarily coded material (such as accent marks in text) must be typeset in LaTeX. Visit `http://www.ams.org/cgi-bin/abstracts/abstract.pl`. Questions about abstracts may be sent to `abs-info@ams.org`. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

Conferences: (see `http://www.ams.org/meetings/` for the most up-to-date information on these conferences.)

June 12–July 2, 2011: Mathematics Research Research Communities, Snowbird, Utah. (Please see `http://www.ams.org/amsmtgs/mrc.html` for more information.)

July 4–7, 2011: von Neumann Symposium on Multimodel and Multialgorithm Coupling for Multiscale Problems, Snowbird, Utah. (Please see `http://www.ams.org/meetings/amsconf/symposia/symposia-2011` for more information.)

July 24–29, 2011: Conference on Applied Mathematics, Modeling, and Computational Science, Waterloo, Canada (held in cooperation with the AMS).

AMERICAN MATHEMATICAL SOCIETY

For the Avid Reader

Computational Topology

An Introduction

Herbert Edelsbrunner, *Duke University, Durham, NC, and Geomagic, Research Triangle Park, NC*, and **John L. Harer**, *Duke University, Durham, NC*

An introduction to the often difficult concepts of algebraic topology in a way motivated by applications

2010; 241 pages; Hardcover; ISBN: 978-0-8218-4925-5; List US$59; AMS members US$47.20; Order code MBK/69

The Knot Book

An Elementary Introduction to the Mathematical Theory of Knots

Colin C. Adams, *Williams College, Williamstown, MA*

A compelling look at the marvelous world of knot theory and its connection to results in topology, geometry, and scientific fields

2004; 307 pages; Softcover; ISBN: 978-0-8218-3678-1; List US$30; AMS members US$24; Order code KNOT

Famous Puzzles of Great Mathematicians

Miodrag S. Petković, *University of Nis, Serbia*

Recreational mathematics used as a framework for communicating brilliant ideas in mathematical thought

2009; 325 pages; Softcover; ISBN: 978-0-8218-4814-2; List US$36; AMS members US$28.80; Order code MBK/63

Mathematics Everywhere

Martin Aigner and **Ehrhard Behrends**, *Freie Universität Berlin, Germany*, Editors

Translated by Philip G. Spain

Everyday examples of how mathematics permeates the real world, in far-ranging areas from compact discs to climate change

2010; 330 pages; Softcover; ISBN: 978-0-8218-4349-9; List US$49; AMS members US$39.20; Order code MBK/72

A Mathematical Medley

Fifty Easy Pieces on Mathematics

George G. Szpiro, *Neue Zürcher Zeitung, Zurich, Switzerland*

Easy-to-read articles that explain mathematical problems and research for an audience with little specialized knowledge of the subject

2010; 236 pages; Softcover; ISBN: 978-0-8218-4928-6; List US$35; AMS members US$28; Order code MBK/73

Mathematical Connections

A Capstone Course

John B. Conway, *George Washington University, District of Columbia*

Descriptions of the interrelations between various parts of mathematics, designed to help the undergraduate see an integrated body of knowledge

2010; 243 pages; Softcover; ISBN: 978-0-8218-4979-8; List US$55; AMS members US$44; Order code MBK/75

Markov Chains and Mixing Times

David A. Levin, *University of Oregon, Eugene, OR*, **Yuval Peres**, *Microsoft Research, Redmond, WA, and University of California, Berkeley, CA*, and **Elizabeth L. Wilmer**, *Oberlin College, OH*

An introduction to the modern approach to the theory of Markov chains, emphasizing probabilistic methods

2009; 371 pages; Hardcover; ISBN: 978-0-8218-4739-8; List US$65; AMS members US$52; Order code MBK/58

The Game of Cops and Robbers on Graphs

Anthony Bonato, *Ryerson University, Toronto, ON, Canada*, and **Richard J. Nowakowski**, *Dalhousie University, Halifax, NS, Canada*

Student Mathematical Library, Volume 61; 2011; approximately 267 pages; Softcover; ISBN: 978-0-8218-5347-4; List US$45; AMS members US$36; Order code STML/61

Stefan Banach

Remarkable Life, Brilliant Mathematics

Emilia Jakimowicz, *Gdańsk University, Gdańsk-Oliwa, Poland*, and **Adam Miranowicz**, *Adam Mickiewicz University, Poznań, Poland*, Editors

A publication of Gdańsk University Press. Distributed non-exclusively worldwide by the American Mathematical Society.

2010; 186 pages; Hardcover; ISBN: 978-83-7326-798-5; List US$55; AMS members US$44; Order code BANACH

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