Book Review

Mathematics Emerging: A Sourcebook 1540–1900

Reviewed by David Pengelley

Mathematics Emerging: A Sourcebook 1540–1900
Jacqueline Stedall
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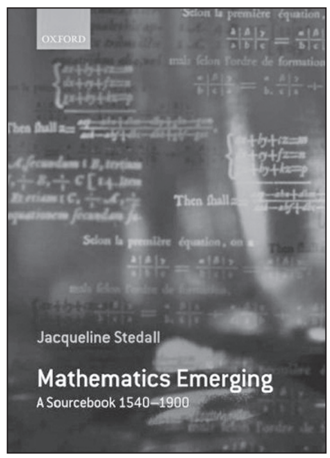
What is a mathematics sourcebook, and why should we welcome a new one? A sourcebook collects together excerpts from primary historical sources, each book with a particular emphasis, point of view, and intended audience. Over the past several decades a number of sourcebooks have appeared, and while some are specific to a particular subject area, a goodly number, like the one under review, are quite general [13]. Many also include commentary on the sources, providing context and direction.

Why should primary sources be important to today's mathematician or student of mathematics? And does Jacqueline Stedall's new sourcebook offer something especially different from the others?

Earlier mathematics is often as valid and valuable as the new, while perspective, conceptualization, motivation, notation, and ways of validating may have been dramatically different. Primary sources provide our main window into how mathematics was practiced in the past and often display key insights leading to important developments. In fact, today's top mathematicians are often quite familiar with the primary sources of their field, obtaining insight and inspiration from these sources



for their research. And primary sources seem to be chock full of phenomena that raise fascinating questions, leading us to why primary sources are also good for students.

Learning mathematics from primary sources offers students the benefits that the humanities have always possessed from this focus. Studying primary sources can foster motivation, broaden perspective, reveal context, draw attention to subtleties, hone deductive skills and conversion between verbal descriptions and modern mathematical formulations, provide excitement, bring students closer to the practice of research, show the genesis and progression of ideas, and display the human face of mathematics. Perhaps most importantly, the challenges of comprehending a primary source raise students from the role of spectator to practitioner. And students gain a more profound comprehension, because the initial simplicity of a theory, along with the questions that arise naturally from studying primary sources and that engender cognitive dissonance requiring conceptual understanding to resolve, can be a tremendous help in learning.

For these reasons, among others, increasingly many instructors are having students engage primary sources directly [5, 6, 7, 11, 12]. And, fortunately, a community of dedicated scholars and teachers has been working worldwide for

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many years to make primary sources accessible and usable [7, 11], so that now for school and undergraduate teaching levels, a meaningful fraction of the most useful sources has been and is being translated and collected in sourcebooks and in other organized ways [13]. These range from compiled raw resources for individual instructor selection all the way to coherent sequences of primary sources with commentary and exercises designed for teaching entire courses [9, 10]. Materials have also been organized into modular student project formats suitable for insertion into existing courses to supplement or replace parts or all of a textbook [1, 2, 3].

How does *Mathematics Emerging: A Sourcebook 1540–1900* enhance what we already have from other sourcebooks? The author, Jacqueline Stedall, explains that the book emerged from a course she designed with Raymond Flood, Peter Neumann, and Robin Wilson "to provide [Oxford University] mathematics undergraduates with some historical background to the material that is now taught universally to students in their final years at school and the first years at college or university: the core subjects of calculus, analysis, and abstract algebra, along with others such as mechanics, probability, and number theory. All of these evolved into their present form in a relatively limited area of western Europe from the mid-sixteenth century onward, and it is there that we find the major writings that relate in a recognizable way to contemporary mathematics. Hence the relatively narrow focus of this book."

Indeed, this aim, utilizing sources encompassing a decent breadth of typical undergraduate mathematics at both lower and upper levels, distinguishes this book clearly from all other sourcebooks. Probably three quarters of the sources featured are from the eighteenth and nineteenth centuries, whereas other sourcebooks tend more to emphasize mathematics from earlier periods, largely for audiences at school or beginning university level. And even from the centuries of possible overlap, the author has chosen many fresh materials to emphasize, often very nice lesser-known alternatives to now-standard sources in other books, so all told there is relatively little duplication of sources. I will analyze the detailed coverage below.

The author is quite clear that this sourcebook is also distinguished from others by the philosophy that "almost every source is given in its original form, not just in the language in which it was first written, but as far as practicable in the layout and typeface in which it was read by contemporaries." This welcome departure from previous sourcebooks is made not just for strong aesthetic reasons nor merely to entice readers to try their own hand at reading the originals, but reflects also the author's stated aim of encouraging readers to interpret historical materials directly for themselves, including emphasizing what one learns from the notation and layout of an original. Supplementing this admirable exhortation, English translations are nonetheless provided alongside all excerpts in other languages. The appearance of almost every excerpt as an image from an original publication, accompanied by a modern typeset English translation, indeed creates a consistently thrilling historical and emotional experience for the reader and particularly commends this sourcebook.

Stedall acknowledges the "relatively brief" nature of her commentary around each source extract and also any feeling on the reader's part that a source should have been included but wasn't, by encouraging the reader to "become not just a reader but a historian," seeking out other sources or opinions for study and comparative judgments. With this I heartily agree in principle, but I will discuss some places where I feel either that some commentary is misleading or confusing or that entire subjects have been given much too short a shrift for comfort. Nonetheless, the overall scope of the book's coverage is prodigiously valuable at over 600 pages of mostly primary source material. And the presource commentaries are, although sometimes brief, generally informative and helpful; the reader feels the confidence of having a guide.

As to Stedall's detailed choices of actual excerpt passages from within her chosen sources, I have two thoughts, related to her introductory remarks. First, she comments that some may object that "brief extracts" give only a partial picture, but she believes that "for beginners a complete picture would be overwhelming." Actually, I do not on balance find her extracts too brief at all for her aims; in fact, often they are refreshingly generous. However, I do not agree that beginners cannot benefit from a complete in-depth picture of a topic from primary sources. That is exactly how I teach with them myself, often having students learn a prescribed subject (not an overview of all of modern mathematics) in great depth from extensive study of primary texts [9, 10, 11]. In fact, I would like to see all our specific subject courses built around in-depth study of primary sources [12].

Second, Stedall comments that her extracts will "find mathematicians groping in the dark, experimenting with new ideas, making hypotheses and guesses, proving correct theorems wrongly, and even on occasion proving incorrect theorems wrongly too" and that the excerpts display the "process of discovery". I think her extracts succeed admirably in achieving this worthy aim, even including some sources of criticism and controversy. But the choice of passages with this aim sometimes makes me uncomfortable that the balance thereby shifts a bit more away from concrete important results and proofs than I would personally prefer; I sometimes feel that, while the chosen excerpt introduced new thinking, new concepts, and new notation, I and my students got no new mathematical results out of it. And I find the excerpts vary from appetizers to the impenetrable, although this may have been intentional to display full variety. Perhaps in the end my discomfort reflects nothing more than the richly different ways in which primary sources are valuable: I tend to teach with them to see how concrete results were first discovered, formulated, and proven, and I incline to provide more commentary, while Stedall's focus in the course underlying this book is to provide students with historical background for a great breadth of mathematics already studied in other courses and to encourage them to go further on their own via other materials to fill out the picture for themselves.

The eighteen chapters of Stedall's book arrange her source excerpts roughly chronologically, but beyond the first two, each chapter focuses on a sequence of sources from just one particular topic in a particular time period. This method of organization works well. I will address her chosen materials largely by topic.

• Stedall wisely sets her stage with the actors arithmetic, geometry, numbers, and algebra in a *Beginnings* chapter of a few sources from prior to her formal starting date of 1540. These sources span Babylonian times through Sacrobosco's thirteenth-century *Algorismus*, giving a very brief overview of the mathematical "common knowledge" of western Europe prior to the beginning of her story in the Renaissance.

• The second chapter, "Fresh Ideas", continues with European sources in four arenas bridging the sixteenth and seventeenth centuries: calculation, notation, analytic geometry, and indivisibles. Improvements in calculation are highlighted by reading Stevin on decimal fractions and Napier on logarithms. Unfortunately, the commentary introducing Stevin leaves the inexpert reader with the impression that Stevin was the first to invent decimal fractions around 1585. While I understand the author's reasons for focusing on European sources in this book, it is then particularly incumbent at least to mention earlier developments in other places: in this case, even outside China, decimal fractions had an extensive well-known history in the Arabic world [4, Ch. 2.3], [8, pp. 226–268, 278, 345–346], beginning with the text of al-Uqlīdīsī in the year 952 [14]. The translation of Stevin is the only one in the book in which I am a bit disappointed, seeming both stilted and a little inaccurate.

For Napier's logarithms, a particularly challenging text given how different his point of view was from today's, the commentary is hampered rather than aided by the attached diagram, whose notation seems quite disconnected from the commentary and source.

But the rest of the chapter is excellent, with commentary providing in-depth discussion and connections to larger context for sources highlighting improvements in notation by Harriot and Descartes; the invention of analytic geometry by Viète, Fermat, and Descartes; and the theory of indivisibles by Cavalieri and Wallis; along with a critique by Hobbes. One of the strengths of the book shows itself already here in the stimulating back and forth of controversy, with Hobbes's critique sounding much like Bishop George Berkeley's later attack on the calculus of Leibniz and Newton.

The succeeding sixteen topic-oriented chapters seem to me somewhat unbalanced, since fully ten of them fall within the breadth of what we today call analysis. Indeed, I will argue that some other areas of mathematics have been unfairly shortchanged, but it is also undeniably the case that the seventeenth through nineteenth centuries were largely dominated by the development of analysis, and the profusion of sources in this book allows us to revel in all its glorious twists and turns.

• *Analysis*: The scope of over seventy-five sources in these ten chapters is enormous, ranging from Fermat and Descartes finding tangents to curves in the 1630s to Lebesgue on integration more than two and a half centuries later. And along the way, these analysis sources, many of which are not the best-known ones, follow various branches and influences. For instance, Newton's *Principia* has a chapter to itself, showing the strong interplay between geometry, physics, and limiting processes, and provoking issues we are not used to today. And there are entire sequences of sources focusing on each of power series, the function concept, foundations of calculus, applications involving differential equations, limits and continuity, derivatives and integrals, complex analysis, and convergence and completeness, offering multiple perspectives on each. I cannot adequately emphasize the exuberance one feels being inside the grand developments of analysis through these admirably chosen source excerpts with very good introductory commentary (although I sometimes wished for more to help me through certain difficult parts), enhanced by the give and take of controversy included in the sources and by a focus on struggles, not just results, leaving the reader to make judgments. All told, these sources provide a wealth of understanding of much of the development of analysis, and students who have already studied some modern analysis will benefit enormously from them.

• *Algebra*: This is the second most extensively treated topic in the book, featured in three chapters. The first follows solving polynomial equations via sources from Cardano to Lagrange, while the second shows the nineteenth-century emergence of abstract algebra from Cauchy and Galois through Cayley to Kummer and Dedekind on ideals. Even the latter two sources will be somewhat accessible and enlightening to students who have already studied some modern algebra. The third algebra chapter is a tour of three centuries and eleven sources, today united under the umbrella "linear algebra", from determinants through eigenvalues and matrices to vector spaces. It is fascinating to see how these themes proceeded independently in the problems of each era, in contrast with our more unified understanding today. I only feel a little sorry for poor Arthur Cayley. He was disparaged in an earlier chapter for his work on groups having "never [come] anywhere near the depth or sophistication of Cauchy or Galois." Now he is belittled, unfairly in my view, by the book's statement that in his development of an algebraic theory of matrices, e.g., the Cayley-Hamilton theorem, he "succeeded in performing a few low level calculations, but failed to see the larger picture, or work with anything like the sophistication of his continental contemporaries."

• *Probability*: This chapter is a refreshing addition in a sourcebook. It does not just provide the early, discrete developments of Pascal and Bernoulli, but quickly moves to sources on more advanced analytical topics like DeMoivre on the normal distribution and confidence intervals, and Bayes's theorem. We end the chapter reading a sophisticated 1812 application by Laplace in which he analyzes differences in the ratios of boy to girl baptisms in London and Paris and speculates that his results suggest that French farmers choose preferentially to abandon their female infants to adoption. It is a fascinating source connecting advanced mathematics to social issues. This chapter would greatly enhance an advanced probability course.

• *Foundations*: Here we have sources by Cantor and Dedekind on the real and natural numbers and their cardinalities, and by Hilbert on the axiomatization of Euclidean geometry. But in this chapter one feels acutely the lack of one of the missing topics in the book, since non-Euclidean geometries are mentioned only once in the entire book, in passing here. One realizes that the book contains nothing on projective, algebraic, or hyperbolic geometries; only in this final chapter would the reader even learn that non-Euclidean geometries exist. And while mentioning geometry, a real disappointment is that its modern child topology is nowhere in the book, not for lack of wonderful sources from Leibniz's or Euler's time forward. Finally, a chapter titled "Foundations" also makes one realize that mathematical logic is not to be found, despite its underpinnings arising strongly from the nineteenth century.

• *Number theory*: This is the one remaining topic represented with a chapter and promised in the introduction. But it is far and away the shortest chapter of all and ends around 1650 with Fermat. The author claims that "most of modern number theory is beyond the scope of this book," but I feel this is no more true of number theory than of the analysis and algebra to which thirteen chapters of sources were devoted. There is a plethora of wonderful number theory sources by Euler, Lagrange, Legendre, Gauss, Riemann, etc., which are actually more accessible than many of the challenging sources in the book and which could introduce the reader to the distribution of primes via the quadratic reciprocity law, prime number theorem, and zeta function, or to Fermat's last theorem [9, 10]. I think an opportunity has been missed here, with number theory being shortchanged as something of an afterthought despite being explicitly claimed as a subject of focus for the book.

In addition to sources and commentary, the book has another nice feature, a list of relevant mathematicians, their institutional affiliations and connections to other people, to help tie the big picture together, as well as a list of historical institutions and journals, a useful bibliography, a list of a few modern digital archives, and an index. But there are some things that could have benefited from more attention to detail in final editing. For instance, it is amusing to learn that Lagrange replaced himself at the Berlin Academy and a little disappointing that the likes of Legendre, Lobachevsky, and Roberval are not in the list of people. And it is frustrating in the text to have people sometimes introduced only by surname, as if we should already know them well. This can even lead to confusion; for instance, unless we look him up, we might wonder the first few times we read about Mercator whether he is the same Mercator who developed the map projection (he isn't). And there are a very few errors, e.g., Euler's *Institutiones Calculi Differentialis* is not volume II of his *Introductio in Analysin Infinitorum.* The *Introductio* contained no calculus and is a spectacular triumph for developing the properties, expansions, and identities of the important functions of analysis using only infinite algebra.

Finally, there is one disappointment in the otherwise beautiful production of this sourcebook, namely, the very poor visual quality of a number of the photographically reproduced sources. I know firsthand that there are real challenges associated with reproductions, but I simply do not think that so much faintness and distortion from bindings was necessary if the authors had devoted a bit more effort to finding good-quality original publications.

Despite these few weaknesses, the overall quality and magnitude of the endeavor, along with the distinctive provision of original source images, are outstanding, so *Mathematics Emerging* is a highly valuable and uniquely positioned new contribution, providing something extensive for upper-level undergraduates for the first time. I am extremely impressed by its coverage of analysis, for which it provides voluminous enrichment for instructors and students at many levels, and it accomplishes in large measure the same for students of modern algebra and perhaps also probability. The areas where it offers relatively little or nothing at all and could have offered so much more by choosing a better balance against analysis are number theory, modern geometries, topology, and logic.

Every scholarly library should own *Mathematics Emerging*. What about its usefulness for teachers or directly for students in the classroom? The Oxford course on which it was based is highly unusual, as are the courses I teach basing student learning of content directly on primary sources. And the sources chosen here were not selected to provide a primary learning path through the mathematics, but rather as a supplement to other study. If you engage undergraduates, the book by itself will greatly enhance your teaching, as it has for me, and you will be changed slowly by it as you read different parts at different times. Personally I would not hesitate to use *Mathematics Emerging* as the principal student text for an upper undergraduate history of mathematics course, supplemented by other sources in certain areas. And I will very confidently and happily assign targeted excerpts or chapters in many other regular courses in the curriculum. It all comes down to whether we will know and teach mathematics as a living, breathing subject with a bountiful and very human life history. This book is a huge enabling step.

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About the Cover

Forty-six years of topology from the differentiable viewpoint

This month's cover was stimulated by John Milnor's article in this issue. It is a photograph of the sketch by him that appeared on the cover of his classic text, *Topology from the Differentiable Viewpoint,** published in 1965 by the University Press of Virginia.

Milnor tells us, "The original sketch of the grouchy vector field picture may exist in some box, but the probability of finding it is near zero. The motivation for the picture is described in §6 (and the end of §5) of the book. I was just sketching a vector field in the disk, pointing out around the boundary, with one saddle point and one index +2 point. It came out looking decidedly cross."

We thank Milnor and the University Press of Virginia for permission to reproduce the cover image.

—Bill Casselman
Graphics editor
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