Is Moore Better (in Precalculus)?

Karen S. Briggs, Brad Bailey, and Thomas E. Cooper

Have you ever thought, "The Moore Method is appropriate for a class like topology or real analysis" but wondered, "Would it ever work in a class like precalculus or college algebra?" The advantages of the Moore Method and other "minimal-guidance" teaching techniques over traditional lecture-style teaching is much debated among mathematics education experts, especially with regard to introductory-level mathematics courses. Intrigued by these debates, the authors of this paper considered using a modification of the Moore Method (commonly known as Modified Moore Method or MMM)[1] for an honors section of precalculus. We searched for resources to use in an MMM precalculus class and discovered Mahavier's "Trigonometry" notes in the *Journal of Inquiry-Based Learning* [9]. However, we came up empty-handed in our search for the advanced algebra portion of our precalculus course. We also found no empirical studies on the effectiveness of MMM in introductory mathematics classes. Without the appropriate resources or research, we decided not to use MMM in our honors precalculus. Nevertheless, we continued to wonder if an MMM could work for a class like precalculus.

With the approval of our department, we designed a quasi experiment [4] using three sections of precalculus (none of which was an honors section). One treatment section would be taught using an MMM, while the two control sections would receive traditional lectures. It should be noted that while all three instructors involved in the study were experienced lecturers, none had experience teaching precalculus with an MMM. In undertaking this study, we wanted not only to satisfy our own curiosities but also to join an existing debate on the Moore Method and perhaps provide a perspective on the use of an MMM in a type of course that had not yet been formally studied. In particular, we wanted to investigate the following:

1. Would there be a significant difference in scores on a common assessment between the MMM students and the traditional lecture students?
2. Would MMM be better (or worse) at teaching particular topics?
3. How would the self-efficacy of MMM students compare to that of the students taught by traditional lectures?
4. Would an MMM trigger changes in the students' attitudes about mathematics and the teaching and learning of mathematics?

Karen S. Briggs is associate professor of mathematics at North Georgia College & State University. Her email address is kbriggs@northgeorgia.edu.

Brad Bailey is associate professor of mathematics at North Georgia College & State University. His email address is brad.bailey@northgeorgia.edu.

Thomas E. Cooper is assistant professor of mathematics education at North Georgia College & State University. His email address is tecooper@northgeorgia.edu.

This research was supported by a grant from the Educational Advancement Foundation.

[1] *An MMM is any teaching style that is driven by students' deriving and presenting solutions to a carefully crafted sequence of problems. Several such Modified Moore Methods have been described in the literature, including by Chalice [2], Cohen [3], and Mahavier [8].*

To assess students' grade self-efficacy, task-specific self-efficacy, and intent to take and feelings of preparedness for calculus, we developed an Attitudes, Beliefs and Self-Efficacy (ABSE) survey that included twenty questions from Schoenfeld [11]. The ABSE survey was administered in all three sections the first class day after the drop/add period had ended and again during the last full week of classes. At midsemester the instructors coauthored a common final exam and developed a detailed grading rubric. The common final exam was administered to all three sections at the end of the semester, and the students' scores for each of the thirty-six items on the final exam were recorded.

In our MMM section, the instructor did not use a textbook but instead a set of course notes that included basic definitions and axioms together with a problem sequence through which the students discovered the course content. These notes, written by the treatment instructor throughout the course of the semester, included problems on advanced algebra topics together with problems from Mahavier's "Trigonometry" [9]. Each day the instructor assigned a set of problems from the course notes for the students to complete outside of class. Students were assigned to groups of three and were allowed to discuss the problems only with their group members. They were not allowed to seek assistance from friends, family, tutors, other instructors, the Internet, or a textbook. Each class period began with the instructor calling upon students to present solutions to one of the assigned problems at the board, while the remainder of the class evaluated the accuracy of the solutions and discussed any differences in their own solutions. Because these presentations affected the students' final grades, the instructor kept an up-to-date record of the number of times a student presented a problem and the quality of the student's solutions. Students who had the least number of "quality" presentations to date were given the first opportunity to present. The instructor would occasionally present a required definition or axiom, but the students presented the majority of the content.

The MMM students outperformed the traditional lecture students by about 10 percent on the 200-point common final exam. The mean, standard deviation, and sample sizes were, respectively, 136.5, 27.74, and 32 for the treatment section and 116.5, 35.68, and 67 for the control sections. When a two-sample t-test was performed, the difference in average final exam scores was statistically significant at the 0.005 level. We also compared the performances of the three sections on all thirty-six final exam items using ANOVA and post-hoc tests. Neither of the control sections did significantly better than the treatment section on any of the thirty-six final exam items. On the other hand, the treatment section did score significantly higher ($p < .05$) or marginally significantly higher ($p < .10$) than did at least one control section on twelve items.

Due to the survey's Likert-type scale,[2] we used a series of Mann-Whitney U tests[3] to compare the responses of the control and treatment sections with a 0.05 significance level. The analysis of the first round of the ABSE survey showed that after roughly one week of class, the students in the treatment section felt significantly less confident about their grades and mathematical abilities than the students in the control sections. When the students completed the survey again at the end of the semester, the differences between the responses of the treatment and control sections on the grade-efficacy items were no longer statistically significant.

At the beginning of the semester, the only significant difference between the treatment and control sections on the attitudes and beliefs items was that the control group was less likely than the treatment group to agree with the statement "In mathematics, something is either right or wrong." This particular difference did not exist at the end of the semester. For the treatment students, four items on the attitudes portion of the survey showed shifts over the semester that reflected an increase in mathematical maturity and awareness. Perhaps most interesting among these changes was that the treatment students were less likely than they were at the beginning of the semester to agree with the statement "Everything important in math is already known by mathematicians." Since many students have the misconception that nothing new is currently being discovered in mathematics, their unwillingness to agree with this statement was encouraging. Finally, the responses to the question about intent to take calculus showed through a sign test[4] that the treatment group was less likely to take calculus at the end of the semester than at the beginning. A statistical analysis of these results appears in [1] and [4].

The question remains, is Moore better? In our study, an MMM produced mathematically mature and aware students who performed nearly 10 percent better than the traditional students on a common assessment but who also reported that they were less likely to take calculus after experiencing an MMM. As with most things in life,

[2] *A scale used in survey research to measure the level at which a respondent agrees or disagrees with a statement [6].*

[3] *A nonparametric test used in statistics for assessing whether two independent samples of observations come from the same distribution* [7].

[4] *A nonparametric statistical test used to analyze the signs of differences of scores between matched pairs of subjects under two experimental conditions [5].*

any teaching method will have its advantages and disadvantages. Enhanced mathematical ability and increased maturity are wonderful advantages that the MMM appears to offer. However, the MMM seems a bit humbling to some students and may discourage them from taking further math classes. The purpose of reporting these results is twofold. First, knowing the effects that our implementation of the MMM had on our students might enable others to decide whether or not to use the method. Second, modifications to our method based on our results might help overcome the result that a smaller percentage were likely to take calculus. We hope that this article has inspired you the reader to consider if Moore would be better for your students in your introductory classes.

References

[1] BAILEY, B., COOPER, T., and BRIGGS, K., The impact of a Modified Moore Method on efficacy and performance in precalculus, submitted (2010).

[2] CHALICE, D. R., How to teach a class by the Modified Moore Method, *American Mathematical Monthly* **102**(4) (1995), 317–321.

[3] COHEN, D. W., A Modified Moore Method for teaching undergraduate mathematics, *American Mathematical Monthly* **89**(7) (1982), 473–474, 487–490.

[4] COOPER, T., BAILEY, B., and BRIGGS, K., The effect of a Modified Moore Method on attitudes and beliefs in precalculus, *School Science and Mathematics*, accepted (2011).

[5] KARAS, J. and SAVAGE, I. R., Publications of Frank Wilcoxon (1892–1965), *Biometrics* **23**(1) (1967), 1–10.

[6] LIKERT, RENSIS, A technique for the measurement of attitudes, *Archives of Psychology* **140** (1932), 1–55.

[7] MANN, H. B. and WHITNEY, D. R., On a test of whether one of two random variables is stochastically larger than the other, *Annals of Mathematical Statistics* **18** (1) (1947), 50–60.

[8] MAHAVIER, W. T., A gentle discovery method, *College Teaching* **45** (1997), 132–135.

[9] MAHAVIER, W. T., Trigonometry, *Journal of Inquiry-Based Learning in Mathematics* **1** (2007). Retrieved from `http://www.jiblm.org/downloads/jiblmjournal/V070101/V070101.pdf`.

[10] REICHARDT, C. S., Quasi-experimental design, *The SAGE Handbook of Quantitative Methods in Psychology*, R. E. Millsap and A. Maydeu-Olivares (eds.), Sage Publications Ltd., Thousand Oaks, CA, 2009.

[11] SCHOENFELD, A., Explorations of students' mathematical beliefs and behavior, *Journal for Research in Mathematics Education* **20**(4) (1989), 338–355.