Why Do So Many Students Take Calculus?

Keith Stroyan

Calculus is one of the great achievements of the human intellect. It has served as the language of change in the development of scientific thought for more than three centuries. The contemporary importance of calculus includes applications in economics, psychology, and the social sciences and continues to play a key role in its traditional areas of application. Our students' interests and preparation are changing—see [B-Launch], [B-Focus], [S-Focus]—but calculus deserves a place in the curriculum of educated people in many walks of life, not only as technical preparation for careers in math and the physical sciences. Here I suggest a method to improve reasoning skills, promote teamwork, and capture the interest of a broad spectrum of college students. Student projects can engage students in realistic problems they find interesting but, more importantly, they can help students synthesize and apply the knowledge gained by working template exercises and can send a message that the subject can solve real problems.

My favorite calculus question is: Why did we eradicate polio by vaccination, but not measles? I like this question because it has real meaning and because effective use of computing can make it accessible at a beginning level. Under some reasonable assumptions [C:TLC, Ch.2], one can begin to describe the changes in the susceptible (s) and infectious (i) fractions of a large population undergoing the outbreak of a disease that confers immunity. If a person is infectious, say for eleven days, and the population is large and asynchronous, about 1/11th of the infectious recover daily. The model assumes that there is an average number of "close" contacts (c) each infectious person makes that could transfer the disease. Combining the daily contacts with the susceptible fraction gives an expression for daily new cases (contacts with immune people don't transfer the disease).

At first students use these expressions recursively to estimate the course of the disease in one-day steps, subtracting the recoveries and adding the new cases to the infectious group. Next we talk about recursively updating the model in shorter time periods and then talk about the limiting case of small time steps as a derivative. Machine computing allows us to calculate enough recursive steps to produce interesting graphs of a model epidemic lasting months. Once this is available we shift the class emphasis to trying to understand what the graphs tell us.

Students think about the measles-versus-polio problem and discover that the question boils down to a negative slope of the graph of infectious people. They need to rewrite the condition in terms of the model as "$s<1/c$" and then compare the contact numbers "c" for different diseases ([S-Projects] and [S-D]). (A simple integral gives a "first order invariant" of the model, and this can be used

Keith Stroyan is professor of mathematics at the University of Iowa. His email address is `keith-stroyan@uiowa.edu`.

to measure the contact number c for real diseases.) When I teach calculus I have teams of students write a "lab report" or term paper describing their solution to the measles-versus-polio question beginning with a description of the model written in their own words. It is important that they recall how the model was built from simple ideas. I use a week of the course to show students how to write a technical report on a realistic problem and have a more independent project later.

Projects such as the herd immunity question allow students to think for themselves and write complete reasoned solutions. In my experience, most beginning calculus students are not ready to give technical proofs in the style of most calculus texts, but they can be encouraged to reason mathematically well beyond the level of template exercises by working on a larger problem that they find interesting. (The usual proofs most teachers know by heart are only "simple" and convincing once you understand the formulation in abstract function notation. See [Bos].) If students don't reason in some higher-level way about the material, the course tends to be reduced to template exercises and contrived "Story Problems". I believe a college course in math should move beyond this. (*The Far Side* captures the popular view of math in the "Hell's Library" cartoon in which all the books are "Story Problems".)

Most students at the University of Iowa take calculus for one semester or less (with AP credit), so I believe we should strive—in the first course—to really convince students that the subject speaks to their interests. A number of texts do this in different ways, such as [CinC], [C&M], [Smith-Moore], [Hilbert], [HarvardS and HarvardM], [Pengelley], and [Cohen], but I believe courses often fall short of showing students how calculus might affect their lives. It is easy to get sidetracked by algebra or trig skills and boil the course down to template exercises. That ends up reinforcing students' impression that math doesn't solve real problems.

Projects can have a profound impact on students because they "take intellectual ownership" of the problem. Most of my students can describe their projects a decade after they took the course. I wrote a book [S-Projects] with many projects (including some on pure math) to try to give every student something they find interesting. In these projects I try to keep the prerequisites simple, but always correct, science. I try to build on technical calculus topics developed in the course, and the assignments are timed to let students apply that new knowledge soon after it is encountered. We still have to do lots of "drill" and template exercises, but we don't stop there. My projects are less defined than some approaches that give more of a "road map". This takes more time but gives the students more responsibility. My goal for projects is to have students show themselves how calculus might affect their lives.

But using projects in a calculus course has drawbacks. Students don't write very well, and an instructor needs to help with both the math and students' explanations. I use "dual submission". The students submit a first draft that I mark up with comments and questions. They correct it and submit a final draft. I believe that process is an important part of helping them make clear arguments. It's hard work for all, but I find that it improves students' reasoning skills.

You need to give students a week or two from the shopping list of calculus topics to think about *their* problem. I reduced some of the usual topics to find the time. We followed 2,286 students in seven later courses that use calculus and found no harm in technical preparation [BYU on C:TLC page]. It has worked very well for faculty who are willing to use this method of helping students discover for themselves, "What good is it?"

I love the subject in many ways—it is hard not to admire Gauss's, "General investigations of curved surfaces" [Gauss] or the many other mathematical advances of calculus, but I believe the primary importance to most students is as a language of science—in their chosen discipline. (I include math projects for a tiny minority.) A beginning course in calculus should make this point in clear terms the students understand and develop themselves. I find that the projects of [S-Projects] or [Smith-Moore] begin to get students to use calculus to think for themselves and hope this will serve many students in their manifold careers. This is a good reason for so many students to take calculus. (See [NAA], [NS], [NNAA], [SN] for a recent contribution of basic calculus.)

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SCRIPTA MANENT

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Journals in Flux

Peter J. Olver

Whither the academic journal? Publishers are scrambling to adapt to the new and rapidly evolving digital world. Libraries must balance declining resources with soaring prices and new bundled models of journal subscriptions. Meanwhile, management and investors are ever more nervous as tried-and-true economic models become obsolete. The mathematics community has reached a crossroads, requiring a full and frank discussion of the future role of journals in our profession.

The traditional published journal offered four primary benefits to the scientific community (see below for definitions): *enhancement*, *dissemination*, *archiving*, and *validation*. These formed the lure that, in the past, enabled publishers to sign on researchers to work pro bono (or even pay page charges) as authors, referees, and editors, while readers and libraries paid for material that the community freely supplied and evaluated.

By *enhancement* I mean the process of turning handwritten or, subsequently, typed manuscripts into polished, professionally typeset articles. With the ascendancy of TeX, the onus of enhancement has shifted to the author. Most journals now expect a LaTeX source file, adapted to their own peculiarities, with little or no editorial involvement. Some journals continue to copyedit papers, but they are now the exception. Thus, remarkably, publishers have managed to extend the free labor model to include most of their traditional enhancement functions.

Similarly, in the days of handwritten and typed manuscripts, *dissemination* of research to the

Peter J. Olver is professor of mathematics at the University of Minnesota, Minneapolis. His email address is `olver@math.umn.edu`.

wider community was an essential function of journals. Nowadays, preprints first appear on the arXiv, on university preprint servers, or on authors' personal websites, freely available to anyone with a sufficiently rapid and suitably uncensored Internet connection. In many fields, papers appear in journals, either electronic or print, as an afterthought, and those at the forefront of the subject have long since absorbed their contents. Fortunately, many publishers do allow authors to post at least the preprint versions of their works online, and authors and editorial board members should favor such journals. (And make sure to read the fine print: one publishing agreement I recently saw contains an unacceptable clause that appears to prevent authors from altering their personal or arXiv online version once the paper is submitted.)

Archiving of older material presents a different challenge, as many commercial publishers seek to profit from their past, limiting access to their archives to subscribers and thereby disadvantaging those without means or institutional support from being full members of the worldwide research community. Publishers may offer some level of near-term archival stability, but organizations and businesses can disappear, and previously accessible material, either paid or free, can be lost unless properly backed up by third parties. The rapid evolution and eventual obsolescence of hardware, software, and file formats compound the problem, and archivists face as yet unresolved difficulties with long-term electronic preservation of large volumes of material and prevention of data loss.

So, while enhancement, dissemination, and archiving all underlie the traditional journal system, to a large extent only validation persists as the reason for its continuation. By *validation*, I mean the confirmation of the correctness, originality, and status of a paper by its appearance in a refereed journal, which in turn helps validate its author's status insofar as hiring, promotion, grant funding, and salary rely (at least in part) on an individual's accumulated research output. But cracks in the system are starting to appear. In certain areas of applied mathematics, including computer vision, cryptography, and computer algebra, journals have been mostly superseded by prestigious conference proceedings, which are rigorously refereed and accept only a small percentage of contributions.

One increasingly contentious issue is journal ranking. While certainly not foolproof, the prestige of a journal is commonly regarded as confirmation of a paper's relative worth, especially by nonexperts. Publishers and administrators are consequently pushing citation-driven metrics such as the Impact Factor and its variants. Indeed, China now pays cash rewards for published papers, using a sliding scale based on the journal's Impact Factor [1]. But such metrics are known to be unreliable and, even worse, subject to abuse, motivating unscrupulous journal editors and publishers to artificially manipulate the Impact Factor and hence the supposed ranking of their journals [2]. While established researchers tend to have reasonably consistent estimates of journals' ratings (although this seems more true in pure than applied mathematics), those "not in the know" may easily be led astray, not to mention the administrators and bureaucrats who know nothing of the field.

Of course, while they are presumably correlated, a journal's ranking cannot unambiguously rank its individual contents, although reputation, selectivity, prestige of the editorial board, and stringency of the refereeing process do confer an implied status on the papers therein. On the other hand, too many high-profile researchers have been willing to lend their name to editorial boards without paying close attention to the journal's contents or operation; see [3] for a particularly egregious example. In 2010 the General Assembly of the International Mathematics Union (IMU) approved a document, *Best Current Practices for Journals* [4], prepared by its Committee for Electronic Information and Communication (CEIC). The document describes how well-run journals are managed through adherence to the fundamental principles of transparency, integrity, and professionalism, as well as detailing the rights and responsibilities of authors, referees, editors, and publishers.

Can the status of a paper be assessed, even without its appearance in a refereed journal? Quality evaluations in other contexts, say restaurants, might provide an answer. For a number of years, I've toyed with the idea of starting a "Michelin Guide" for math papers. Done right, with a rigorous refereeing process, this could completely supplant the validation provided by journals. Thus in one's vita one could point to having posted online, say in the arXiv, five one-star papers, three two-star papers, and one very rare three-star "Mathelin"-rated paper. Indeed, one can envision a variety of general-purpose guides with competing ratings, as well as specialized guides that would convey status within a particular field. The worth of each rating would depend on the reliability of the particular guide. Furthermore, unlike journals, these guides—like those for restaurants—could allow fluctuating ratings over time, as might happen when a once obscure, unrated paper suddenly provides the key to solving an important problem. Or, vice versa, a previously undetected error is found or the subject area falls out of favor. Less clear is how such a system could be practically instituted. (Michelin's original motivation was to sell more tires by encouraging car owners to drive to faraway restaurants, but eventually their guide and others became economically viable on their own merits.) *Math Reviews* was initially designed to play such a role, but most of its reviews nowadays are mere restatements of abstracts, and serious reviewing (except for the occasional book) has all but disappeared. The challenge is to devise a sound economic model for such scholarly guides, which ideally would include mechanisms for suitably compensating reviewers.

Finally, I feel compelled to say a brief word about the "dark side" of scientific publishing. I already noted journals artificially manipulating metrics such as the Impact Factor. More odious are predatory journals and conferences, which seek to profit from naïve and unscrupulous researchers through registration fees, pay-to-publish models, and the like [5]. Furthermore, the bane of plagiarism is more widespread than many of us acknowledge; see [6] for some recent cases in SIAM and my own website [7] for a personal experience that culminated in legal action and an official acknowledgment (as well as some remuneration) from the publisher. The community has been far too willing to overlook such abuses, which has only served to embolden the perpetrators. For instance, see [8] for an astonishing case of serial academic fraud in economics. Only full publicity, including naming names and, when appropriate, taking legal steps, will counter these insidious practices. Thus, while the electronic era has exacerbated older problems and created new ones such as citation-based metrics, it also provides a range of potentially powerful tools that can be employed to combat such nefarious influences on the profession.

The time is ripe for a radical rethinking of the traditional academic model for scholarly communication within mathematics. While many established researchers seem uninterested in or unwilling to fully come to terms with the rapidly shifting electronic publishing landscape, the community as a whole cannot afford to lull itself into a false sense of security. If we are not properly engaged, the future will be decided for us and, almost certainly, will not be to our liking.

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