Shiing-Shen Chern (1911–2004)

I was asked by Steve Krantz to collect essays in memory of my former teacher Shiing-Shen Chern, who was a great geometer. He passed away in December of 2004 in Tianjin in China.

This year would have been his 100th birthday. Many of his friends and his former students would like to take this occasion to express our admiration of our beloved mentor in mathematics. Some of the articles that we publish here have some overlap with the previous published account on Chern, including the essay by Singer, which was published by International Press about ten years ago.

One of the essays in this collection is written by Wen-Ling Huang and Karin Reich, who are not so closely related to Chern. It gives a good account of the unknown history when Chern was in Hamburg. I believe it is a worthwhile document to be read by the readers of the *Notices*.

—*Shing-Tung Yau*

Shing-Tung Yau

Chern's Work in Geometry

It is my honor to have had a great mathematician as my teacher; S.-S. Chern had a deep influence on my career, both mathematically and personally.

Looking back at the history of differential geometry, I would equate É. Cartan with the grandfather and S.-S. Chern the father of modern differential geometry. Together they had set the foundation of a beautiful subject that has reached out to every branch of mathematics and physics.

Immediately before his passing, Chern told others that he was going to see the Greek geometers. No doubt he had reached the same status as those giants.

The foundation of geometry had been laid by ancient geometers. Modern geometers have been standing on their shoulders and making spectacular achievements. This sentiment is reflected in an article written about thirty years ago by Weil. In the preface to *Selected Papers of S.-S. Chern*, he wrote: "The psychological aspects of true geometric intuition will perhaps never be cleared up. …Whatever the truth of the matter, mathematics in our century would not have made such impressive progress without the geometric sense of Élie Cartan, Heinz Hopf, Chern and a very few more. It seems safe to predict that such men will always be needed if mathematics is to go on as before."

Chern: Father of Global Intrinsic Geometry

Both Cartan and Chern saw the importance of fiber bundles to problems in differential geometry.

It is certainly true that global differential geometry was studied by other great mathematicians: Cohn-Vossen, Minkowski, Hilbert, and Weyl, among others. However, most of their works focused on global surfaces in three-dimensional Euclidean space.

Chern was the first who succeeded in building a bridge between intrinsic geometry and algebraic topology for manifolds of all dimensions, not just surface. (Cartan's work in differential geometry was more local in nature, except his investigation of Lie groups and symmetric spaces.) The theory of fiber bundles was his major tool.

Reflecting on intrinsic geometry, Chern commented: "Riemannian geometry and its generalization in differential geometry are local in character. It seems a mystery to me that we do need a whole

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This is the speech given at the Harvard Memorial Conference for S.-S. Chern.

space to piece the neighborhood together. This is achieved by topology."

Equivalence Problem

Most of the works of Chern were related to the problem of equivalence.

In 1869 E. Christoffel and R. Lipschitz solved the fundamental problem in Riemannian geometry—the form problem:

> To determine when two ds^2's differ by a change of coordinates, Christoffel introduced covariant differentiation, which is now called the Levi-Civita connection.

Cartan generalized this problem to a much more general setting; it is called the equivalence problem.

> Given a Lie group $G \in GL(n,\mathbb{R})$, given two sets of linearly independent linear differential forms θ^i and θ^{*j} in the coordinates x^k and x^{*l} respectively, $1 \le i,j,k,l \le n$, find conditions under which there are functions
>
> $$x^{*i} = x^{*i}(x^1, x^2, \ldots, x^n)$$
>
> such that θ^{*j}, after the substitution of these functions, differ from θ^i by a transformation of G.

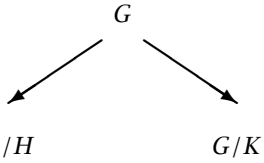
This problem involves local invariants. Cartan provided a procedure to generate such invariants. Almost all of the work of Chern is related to this problem.

Chern (1932–1943)

During the eleven-year period from 1932 to 1943, Chern studied web geometry, projective line geometry, invariants of the contact of pairs of submanifolds in projective space, and transformations of surfaces related to the Bäcklund transform in soliton theory. Chern continued this line of research in his collaborations with Griffiths and Terng later.

Between 1940 and 1942, Chern [5] started to generalize the theory of integral geometry, an area initially developed by Crofton and Blaschke. He observed that this theory can best be understood in terms of two homogeneous spaces with the same Lie group G (see also Weil's review [20]).

This leads to the diagram associated to two subgroups H and K of G:



Two cosets aH and bK are incident to each other if they intersect in G. This notion fits nicely in modern investigations of the Funk-Radon problem of determining a function on a manifold in terms of its integrals over certain submanifolds. Although the focus of this work is "classical" integral geometry, the double fibration diagrams depicted above prefigure in an uncanny way a different, more modern type of integral geometry: the theory of generalized Radon transforms developed by Gelfand and Graev in the late fifties and early sixties.

Using this approach, Chern generalized many important formulas of Crofton. In 1952 he [8] also generalized the kinematic formula of Poincaré, Santaló, and Blaschke.

On the impact of these generalizations by Chern, Weil commented: "It lifted the whole subject at one stroke to a higher plane than where Blaschke's school had lifted it. I was impressed by the unusual talent and depth of understanding that shone through it."

Chern's Visit to Princeton (1943–1945)

In 1943 Chern left Kunming for Princeton. He arrived at the moment when fiber bundle theory was beginning to evolve from the works of Cartan and Whitney.

When he arrived at Princeton, Weil had just published his work with Allendoerfer on the Gauss-Bonnet formula. Weil directed Chern toward the works of Todd and Eger on "canonical classes" in algebraic geometry. He pointed out to Chern that their work on characteristic classes for complex manifolds had only been completed in the spirit of Italian geometry and rested upon some unproved assumptions.

The first fundamental work Chern did on global intrinsic geometry was his intrinsic proof of the Gauss-Bonnet formula [6], re-proving the recent work of Allendoerfer and Weil [2] for general closed Riemannian manifolds. The proof of Allendoerfer-Weil rested on the use of local embeddings of the manifold in Euclidean space, isometric for an approximating real analytic metric. (The existence of local isometric embeddings for real analytic metrics had been established earlier by Burstin, Cartan, and Janet.)

Weil commented on Chern's proof of the Gauss-Bonnet formula: "Following the footsteps of H. Weyl and other writers, the latter proof, resting on the consideration of 'tubes', did depend (although this was not apparent at that time) on the construction of a sphere-bundle, but of a nonintrinsic one, viz. the transversal bundle for a given immersion. …Chern's proof operated explicitly for the first time with an intrinsic bundle, the bundle of tangent vectors of length one, thus clarifying the whole subject once and for all."

A century earlier, Gauss established the concept of intrinsic geometry. Chern's proof of Gauss-Bonnet opened up a new horizon. Global topology was now linked with intrinsic geometry through the concept of fiber bundle and transgression on

the intrinsic tangent sphere bundle. A new era of global intrinsic geometry arrived.

While Hopf's vector field theorem had clearly influenced Chern's thinking, the proof was a tour de force based on Chern's insights and powerful algebraic manipulation. Hopf declared that Chern's proof had brought differential geometry into a new era. In particular, it gave rise to the idea of transgression. Chern's proof remains one of the most admired proofs in the modern history of mathematics.

Chern Classes

Reflecting upon his early encounter with Chern classes, Chern said: "My introduction to the characteristic class was through the Gauss-Bonnet formula, known to every student of surface theory. Long before 1943, when I gave an intrinsic proof of the n-dimensional Gauss-Bonnet formula, I knew, by using orthonormal frames in surface theory, that the classical Gauss-Bonnet is but a global consequence of the Gauss formula which expresses the 'theorima egregium'. The algebraic aspect of this proof is the first instance of a construction later known as a transgression, which was destined to play a fundamental role in the homology theory of fiber bundles and in other problems."

Cartan's work on frame bundles and de Rham's theorem always stayed close to Chern's thinking. The concept of fiber bundles stands at the very heart of modern mathematics. It's a central unifying notion for many important objects in mathematics and physics. It is fitting to give a brief description of its history.

> In 1937 E. Stiefel (1936) and H. Whitney (1937) introduced Stiefel-Whitney classes; they were only defined modulo integer two.
>
> From 1939 to 1944 J. Feldbau (1939), C. Ehresmann (1941, 1942, 1943), Chern (1944, 1945) and N. Steenrod (1944) made a systematic study of the topology of fiber bundles.
>
> In 1942 L. Pontryagin introduced the Pontryagin class. In 1944 he associated topological invariants using the curvature of Riemannian manifolds. His papers were published in the *Doklady* journal. He was short in proving that these curvature invariants are the same as Pontryagin classes.

Let me describe what is needed to prove the equivalence of the curvature and topological definitions of characteristic classes.

Recall that, in the proof of the Gauss-Bonnet formula, one uses vector fields $s_1, \ldots, s_k$ in general position. In his thesis in 1936 Stiefel proved that the locus at which the vector fields are linearly dependent forms a $(k-1)$-dimensional cycle whose homology class is independent of the choice of s_i.

In 1937 Whitney [21] considered sections for more general sphere bundles from the point of view of obstruction theory. Whitney noticed the importance of the universal bundle over the Grassmannian $G(q,N)$ of q planes in $\mathbb{R}^N$. He showed that any rank q bundle over the manifold is isomorphic to the pullback of the universal bundle under a map $f : M \to G(q,N)$.

Pontryagin (1942) and Steenrod [19] observed that, when N is large, the map f is unique up to homotopy. The characteristic classes of the bundle are given by

$$f^*H^*(Gr(q,N)) \subset H^*(M)$$

In 1934 Ehresmann [17] studied the cohomology of $H^*(Gr(q,N))$ and showed that it is generated by Schubert cells.

At first, Chern tried to prove the equivalence of the Pontryagin classes defined using Schubert cells and classes defined using the curvature form. The difficulty was how to integrate these differential forms defined by the curvature form over the Schubert cells.

Chern quickly realized that it was easier to handle the case over complex numbers. Speaking of this discovery, Chern said: "It was a trivial observation and a stroke of luck, when I saw in 1944 that the situation for complex vector bundles is far simpler, because most of the classical complex spaces, such as the classical complex Grassmann manifolds, the complex Stiefel manifolds, etc., have no torsion."

For a complex vector bundle E, the Chern classes $c_i(E)$ belong to $H^{2i}(M,\mathbb{Z})$. By his discovery, Chern proved the equivalence of three definitions of characteristic classes: one using obstruction theory, one using Schubert cells, and the third using curvature forms of a connection on the bundle.

The Fundamental Paper of Chern (1946)

In the paper [7], Chern laid the foundation of Hermitian geometry on complex manifolds.

In this paper, Chern introduced the concept of Hermitian connections; using the curvature form Ω of the Hermitian connection, Chern defined

$$\det\left(I + \frac{\sqrt{-1}}{2\pi}\Omega\right) = 1 + c_1(\Omega) + \cdots + c_q(\Omega).$$

The importance of defining Chern classes by differential forms cannot be overstated in mathematics. It also provides fundamental tools in modern physics. An example is the concept of transgression introduced by Chern.

Let φ be the connection form defined on the frame bundle associated with the vector bundle.

The curvature form is

$$\Omega = d\varphi - \varphi \wedge \varphi$$

and

$$c_1(\Omega) = \frac{\sqrt{-1}}{2\pi}\mathrm{Tr}\,\Omega = \frac{\sqrt{-1}}{2\pi}d(\mathrm{Tr}\,\varphi)$$
$$\mathrm{Tr}\,(\Omega \wedge \Omega) = d(\mathrm{Tr}\,(d\varphi \wedge \varphi) + \frac{1}{3}\mathrm{Tr}\,(\varphi \wedge \varphi \wedge \varphi)) = d(CS(\varphi)).$$

The forms $\mathrm{Tr}\,\varphi$ and $CS(\varphi)$ are the transgressed forms for $c_1(\Omega)$ and $\mathrm{Tr}\,(\Omega \wedge \Omega)$, respectively.

The term $CS(\varphi)$ is called the Chern-Simons form. It was introduced by Chern and Simons [15] in the early seventies. It has played a fundamental role in three-dimensional manifolds, in anomaly cancellation in string theory, and in solid state physics.

Performing transgression on the form level also gives rise to secondary operations on homology, like the Massey product, which was used in K. T. Chen's work on iterated integrals.

For a complex manifold, we can write

$$d = \partial + \bar{\partial}.$$

In their fundamental paper, Bott-Chern (1965) [4] found a double transgression form, which was a canonically constructed (local) $(i-1, i-1)$-form $\tilde{T}c_i(\Omega)$, so that $c_i(\Omega) = \bar{\partial}\partial\,\tilde{T}(c_i(\Omega))$.

Chern made use of this theorem to generalize Nevanlinna's theory of value distribution to holomorphic maps between higher-dimensional complex manifolds. The form $\tilde{T}c_i(\Omega)$ also plays a fundamental role in Arakelov theory developed later.

Donaldson used the case of $i = 2$ to prove the Donaldson-Uhlenbeck-Yau theorem for the existence of the Hermitian Yang-Mills connection on algebraic surfaces.

For a complex manifold X with Hermitian $h_{i\bar{j}}$,

$$c_1(T_X) = -\frac{\sqrt{-1}}{2\pi}\partial\bar{\partial}\log\det(h_{i\bar{j}}).$$

The right-hand side is the Ricci tensor of the metric. The simplicity of the first Chern form motivated the Calabi conjecture, which was eventually solved by me. Chern was excited by this development.

The curvature representation of Chern class means that all the Chern numbers are obtained by integrating some densities defined by curvatures alone. This allows Hirzebruch to derive the proportionality principle for locally symmetric space: that the ratio of the Chern numbers of covering spaces is proportional to the ratio of the volume. Similarly, it motivated me [22] to give a proof of the Miyaoka-Yau inequality based on Kähler-Einstein metrics. All these theorems would have been impossible without Chern's work on representing Chern classes by curvature forms.

As Chern himself observed many times, the simplicity and beauty of geometry over complex numbers can never be exaggerated.

Return to China after the War

After Chern established his two spectacular works in Princeton, he returned to China in April 1946. He was recruited by the national government to help one of his former teachers, Chiang, who taught him in Nankai University. While Chiang was the formal director, Chern was the one who ran the day-to-day business in the Mathematical Institute in the Academic Sinica. He lectured on contemporary research works in topology. He had many students, and postdoctoral fellows participated in his seminars. They include K. T. Chen, H. C. Wang, W. Wu, C. T. Yang, Z. D. Yan, and many others. This group of people later became leading mathematicians in China.

Chicago Days

On December 31, 1948, at the invitation of Veblen and Weyl, Chern left Shanghai for the Institute for Advanced Study in Princeton and spent the next winter term there. He was then offered a professorship by Marshall Stone in Chicago. His friend Weil played an important role in this offer. He settled down in Chicago and ran seminars with Weil, Singer, Sternberg, Kadison, and others in his class. His influence on geometry in America is deep. Singer respected Chern and called him his teacher. During this period of time, he trained several outstanding students, such as S. D. Liao, Wolf, and Nomizu.

After his fundamental paper on Chern classes in 1946, Chern began to explore the multiplicative structure of the characteristic classes in even greater detail. In 1951 he wrote a paper with E. Spanier on the Gysin sequence on a fiber bundle. They proved the Thom isomorphism, independent of the work of Thom.

In his 1953 paper [9], Chern showed that by considering an associated bundle with the flag manifold as fibers, the characteristic classes can be defined in terms of line bundles. As a consequence, the dual homology class of a characteristic class of an algebraic manifold can be represented by algebraic cycles. This paper gives rise to Hirzebruch's splitting principle in K-theory.

Subsequently, Washnitzer, and later Grothendieck in a more general setting, combined the splitting principle with the Thom isomorphism to define Chern classes of the associated bundles.

Hodge had studied the problem of representing homology classes using algebraic cycles. He also tried to prove this theorem of Chern. However, he proved only the case in which the manifold is a complete intersection of nonsingular hypersurface in a projective space.

This theorem of Chern is the only known general statement of the "Hodge conjecture" besides the case of dimension one and codimension one cycles, which is a relatively easy consequence of the Lefschetz $(1,1)$ theorem. Chern's theorem also provided a direct link between holomorphic K-theory and algebraic cycles.

In Chicago, Chern and Lashof [12] studied the concept of tight embedding of hypersurfaces in Euclidean spaces. This work was generalized and continued by Kuiper and Banchoff.

Berkeley Days and Return to China

Chern moved to Berkeley in 1961. He spent all his years in Berkeley until his retirement in 1979; he served at the math department for three more years after his retirement. The arrival of Chern and Smale in Berkeley coincided with the period when the mathematics department at Berkeley rose to become a major leading department in the world. Building on the strength of the existing faculties hired by Evans, Tarski, Morrey, Kelly, and others, Chern had hired many outstanding geometers and topologists who set up Berkeley to be the center in geometry and topology. Berkeley in the 1960s was an extraordinarily exciting place for people interested in geometry: students, faculty, and visiting mathematicians alike. The graduate students felt as if they were in the center of the universe of geometry. Everyone else in the world of geometry came to visit.

Chern trained many outstanding students in the period when he was in Berkeley. This group includes Garland, do Carmo, Shiffman, Weinstein, Banchoff, Millson, S. Y. Cheng, Peter Li, Webster, Donnelly, and Wolfson, not counting myself. Chern organized the training of his Ph.D. students through the help of some of his older students and his friends. For example, Garland got advice from H. C. Wang, Millson from Jim Simons. The charm of Chern was essential to keeping this large group of outstanding geometers working together in Campbell Hall and in Evans Hall in Berkeley. The geometry seminar and the colloquium in Berkeley were always packed with students, faculty members, and visitors. It is well known that Chern treated every visitor with a splendid dinner in a Chinese restaurant or else an elegant party in his house. His wife was able to entertain everybody with grace and nice Chinese food. This period of Berkeley days is unforgettable to two generations of geometers.

In Berkeley, Chern studied minimal surface theory along with Calabi and Osserman [14]. He also tried to generalize Nevanlinna theory to a broader setting; his effort led to the discovery of the Bott-Chern form and the Chern-Levine-Nirenberg intrinsic norms; all of these have had much impact in complex geometry beyond the original purpose. His work with Simons has had deep influence in geometry and physics, including knot theory. The work with Moser [13] on local invariants of real hypersurfaces in complex Euclidean spaces is fundamental in several complex variables. He and Griffiths [11] generalized his old work on web geometry.

In the early eighties, along with Singer and Moore, Chern founded MSRI at Berkeley. Later, he retired and returned to China. In China he formed a mathematics center in Nankai. That center is considered to be successful and influential.

Conclusion

Chern's ability to create invariants for important geometric structures was unsurpassed by any mathematician whom I have ever known. His works on the Gauss-Bonnet formula, on Chern classes, on projective differential geometry, on affine geometry, and on Chern-Moser invariants for pseudo-convex domains demonstrate his strength. Before he died, he had embarked on the major undertaking of applying the Cartan-Kähler system to a more general geometric setting.

Chern once said: "The importance of complex numbers in geometry is a mystery to me. It is well organized and complete."

Chern always regretted that ancient Chinese mathematicians never discovered complex numbers. Chern's everlasting works in complex geometry made up for the earlier losses by Chinese mathematics over the last two thousand years.

The Chinese astronomers named a star after Chern. May his accomplishments always shine on the future generations of mathematicians.

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F. Hirzebruch

Why Do I Like Chern, and Why Do I Like Chern Classes?

In 1949–50 I studied for three semesters at the ETH in Zurich and learned a lot from Heinz Hopf and Beno Eckmann [1], also about Chern classes, their applications, and their relations to Stiefel-Whitney classes ([2], [3], [4], [5]). Chern classes are defined for a complex vector bundle E over a reasonable space X with fiber $\mathbb{C}^n$. They are elements of the cohomology ring of X. The ith Chern class of E is an element of $H^{2i}(X,\mathbb{Z})$ where $0 \le i \le n$ and $c_0 = 1$. They are used for the investigation of fields of r-tuples of sections of the vector bundle, in particular if X is a compact complex manifold and E the tangent bundle of X. Then we have the basic fact: If there exists an r-tuple of sections which are linearly independent in every point of X, then $c_i = 0$ for $i \ge n - r + 1$. For real differentiable manifolds such questions are treated in the dissertation of Hopf's student Stiefel [4], later a well-known computer scientist. For a compact complex manifold X of dimension n, the n-dimensional products of the Chern classes of the tangent bundle (all dimensions complex) give the Chern numbers, when integrated over X, for example $c_n[X]$ is the Euler-Poincaré characteristic (Poincaré-Hopf theorem).

From 1950 to 1952 I was scientific assistant in Erlangen and wrote the paper [6] where ideas of Hopf entered [2]. Some of the results could have been generalized to higher dimensions. But the so-called "duality formula" was not yet proved. This formula says that the total Chern class $1 + c_1 + c_2 + \cdots$ of the direct sum of two complex vector bundles equals the product of the total Chern classes of the summands. The paper [6] has a remark written during proofreading that Chern and Kodaira told me that the "duality formula" is proved in a forthcoming paper of Chern [7]. In the commentary to my paper [6] in volume 1 of my *Collected Papers* (Springer 1987), I write that my knowledge about Chern classes increased with the speed of a flash when I came to Princeton in August 1952 as a member of the Institute for Advanced Study and talked with K. Kodaira, D. C. Spencer, and, a little later, with A. Borel, who told me about his thesis containing his theory about the cohomology of the classifying spaces of compact Lie groups. For the unitary group $U(n)$, this implies that the Chern class c_i can be considered in a natural way as the ith elementary symmetric function in certain variables $x_1, x_2, \ldots, x_n$.

My two years (1952–54) at the Institute for Advanced Study were formative for my mathematical career ([8],[9]). I had to study and develop fundamental properties of Chern classes, introduced the Chern character, which later (joint work with M. F. Atiyah) became a functor from K-theory to rational cohomology. I began to publish my results in 1953. The main theorem is announced in [10]. It concerns the Euler number of a projective algebraic variety V with coefficients in the sheaf of holomorphic sections of a complex analytic vector bundle W over V. Chern classes everywhere! I quote from [10]: "The main theorem expresses this Euler-Poincaré characteristic as a polynomial in the Chern classes of the tangential bundle of V and in the Chern classes of the bundle W."

The Chern classes accompanied me throughout all my mathematical life; for example: In 2009 I gave the annual Oberwolfach lecture about Chern classes [11].

My fiancée joined me in Princeton in November 1952. We married. A "marriage tour" was organized, for which Spencer gave me some support from his Air Force project. I lectured in seven places during this trip, including Chicago, where we met the great master Shiing-Shen Chern and his

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charming wife. He was forty-one, I was twenty-five. For me he was a gentleman advanced in age. But all shyness disappeared. He was interested in my progress in Princeton about which I also talked in my lecture. We must have spoken about his papers [3] and [7]. Chern begins in [3] with a study of the Grassmannian $H(n,N)$ of linear subspaces of dimension n in the complex vector space of dimension $n+N$. He defines the Chern classes of the n-dimension tautological bundle over the Grassmannian in terms of Schubert calculus. From here Chern comes to the definition using r-tuples of sections. For $N \to \infty$, the Grassmannian becomes the classifying space of $U(n)$, and we are close to what I learned from Borel. For Hermitian manifolds Chern shows how to represent the Chern classes by differential forms.

The paper [7] has the following definition of Chern classes: Let E be a complex vector bundle of dimension n over the base B. Let P be the associated projective bundle with fiber $\mathbb{P}_{n-1}(\mathbb{C})$. Let L be the tautological line bundle over P and $g = -c_1(L)$. Then g restricted to the fiber of P is the positive generator of $H^2(P_{n-1}(\mathbb{C}),\mathbb{Z})$. Integration of g^{n-1+m} over the fiber in P gives $\bar{c}_m$, the mth "dual" Chern class of E. The total "dual" Chern class $\bar{c} = 1 + \bar{c}_1 + \bar{c}_2 + \cdots$ is defined by

$$c \cdot \bar{c} = 1.$$

If $B = H(n,N)$, then $\bar{c}$ is the total Chern class of the complementary N-dimensional tautological bundle over B.

Chern uses this to prove that the Chern classes are represented by algebraic cycles if everything happens in the projective algebraic category.

The Cherns invited my wife and me for dinner in their home. For the first time we enjoyed the cooking of Mrs. Chern. Many meals in Berkeley would follow. The Chern family, with their two children in 1950, can be seen in the photograph on page XX of his *Selected Papers* (Springer 1978). Chern presented me a copy of this book with the dedication "To Fritz. Warmest regards. June 1979". The signature is in Chinese characters.

During 1955–56 I was an assistant professor at Princeton University. I gave a course on my book [12]. Chern and Serre attended at least occasionally. Chern, Serre, and I wrote a paper, "On the index of a fibered manifold", which was submitted in September 1956 [13]. There the multiplicativity of the signature [= index] is proved for fibrations of compact connected oriented manifolds provided the fundamental group of the base acts trivially on the rational cohomology of the fiber.

In 1960 Chern became a professor in Berkeley. I visited him there in 1962, 1963, 1967, 1968, 1973, 1974, 1979, 1983, 1986, and 1998, always with part of my family. Chern inspired an official offer to me by the University of California (November 1968). He wrote to me: "We all hope that you will find Berkeley sufficiently attractive to deserve your serious consideration. Some disturbances are expected but they need not concern you. I am going to submit to the NSF a new proposal for research support and will be glad to include you in the proposal." In Bonn I was very involved in discussions with the protesting students and expected to have a quieter life in Berkeley as a new faculty member with more time for mathematics. Finally I decided to stay in Bonn. Chern was very disappointed. But the invitations to Berkeley continued. The Cherns were always very helpful in many practical problems: picking us up at the airport, finding a house, lending us things useful for housekeeping, even lending us a car, depositing items in their house we had bought to be used during the next visit….We enjoyed the Cherns' hospitality in their beautiful home in El Cerrito, overlooking the Bay with the famous Bay Bridge, or in excellent Chinese restaurants in Berkeley and Oakland where the Cherns were highly respected guests. There were always interesting conversations with the Cherns and the other dinner guests.

In 1979 there was a conference, "The Chern Symposium", on the occasion of Chern's retirement as a professor of the university. In the *Proceedings* [14] I. M. Singer writes: "The conference also reflected Professor Chern's personality, active yet relaxed, mixed with gentleness and good humor. We wish him good health, a long life, happiness, and a continuation of his extraordinary deep and original contributions to mathematics." This came also from my heart.

Chern did not really retire. In 1981 he became the first director of the Mathematical Sciences Research Institute in Berkeley. When the MSRI building was ready, I sometimes used Chern's beautiful office with a wonderful view.

In 1981 I nominated Chern for the "Alexander von Humboldt-Preis". He received it and spent part of the summers of 1982 and 1984 in Bonn. He talked at the Arbeitstagungen of these years on the topics "web geometry" and "some applications of the method of moving frames".

In 1998 I was invited to be one of the first Chern professors in Berkeley. These visiting professorships are financed by Robert G. Uomini, a former student of Chern, who had won an enormous sum in the lottery. In my case a one-day Chern symposium was held, followed by a four-week course. The title of my Chern lecture in the symposium was "Why do I like Chern classes?" I gave four answers:

(1) *The Chern classes remind me of my youth.* I hope this became clear in the beginning of this contribution.

(2) *The Chern classes have so many different definitions.* As a joke I added: *I especially like that all these definitions are equivalent.* There are the definitions in Chern's papers [3] and [7]. The statement in the joke

needed some work, which was carried out by Borel and me and perhaps by others, too. The difficulty consisted in sign questions: Are we dealing with a complex vector bundle V or its dual V^*?

(3) *"Chern has a beautiful character."* There was the story that during a lecture about K-theory and its functor ch to rational cohomology I cried out, "Chern has a beautiful character!" Chern was present and smiled.

(4) *Chern classes have so many applications.*

In 1998 Chern was eighty-seven years old. He did not appear so old to me. He came to my Chern lecture and also to some lectures in my four-week course. The Cherns came to an official dinner. They invited us to a Chinese restaurant.

Berkeley 1998! The last time I saw Chern. The Cherns gradually moved to China.

But we stayed in contact. We edited the two volumes on the Wolf Prize winners in mathematics (published by World Scientific in 2000 and 2001).

My retirement as director of the Max Planck Institute for Mathematics in Bonn in 1995 was celebrated by a "party" with informal lectures, performances, music, lunches and dinners organized by Don Zagier. It lasted two or three days. Zagier had the idea to produce a book with essays or short statements by the participants and by some other people who could not attend. Chern did not come. But one page is by him (see Figure 1).

In 2005 the School of Mathematics of the Institute for Advanced Study in Princeton had its seventy-fifth anniversary. Of the older members Chern, Bott, Hirzebruch, and Atiyah were invited to present to the inner circle how the time at the Institute was formative for their careers, Chern by television. But he died in 2004. I also gave a mathematical lecture in which Borel and Chern figured prominently. Chern classes everywhere! Borel and I had shown in the 1950s how to calculate the Chern classes and the Chern numbers of compact complex homogeneous spaces. An example (in a formulation by E. Calabi):

> Let X be the projective contravariant tangent bundle of $\mathbb{P}_3(\mathbb{C})$ and Y the projective covariant tangent bundle. Then the Chern number c_1^5 of these five-dimensional complex homogeneous spaces X and Y, respectively, equals 4500 and 4860. This is interesting because X and Y are diffeomorphic (compare [11] and the work of D. Kotschick mentioned there).

Remark. It is unavoidable that this contribution has some overlap with [15] and with my interview about Chern of December 6, 2010, here in Bonn [Zala Films with George Csicsery for MSRI].

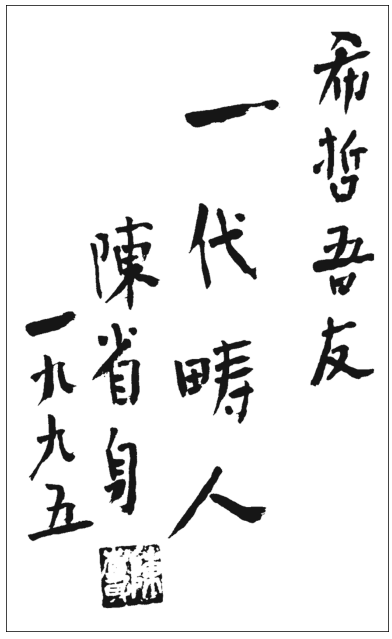

Figure 1.

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Michael Atiyah

Memories of S.-S. Chern

I first met Chern in Chicago in 1956. I had gone to the Institute for Advanced Study in Princeton after my Ph.D., and Chern invited me to give a seminar. He was a senior professor and I a raw Ph.D., but he took good care of my wife and me for our week in Chicago. We remained in frequent touch over the subsequent years, and the last time we saw him was as his houseguests on the campus of Nankai University, shortly before his death. One clear memory I have of him is at a conference in Durham, England, where, despite advancing years, he valiantly walked along Hadrian's Wall with the younger generation.

Chern was a geometer of the old school. His work had none of the polish of the postwar generation, his methods were direct and intuitive and at times cumbersome. For this reason I and others of my generation underestimated him. What he may have lacked in elegance he made up for by his breadth of interest and his deep geometrical insight. This took him in many pioneering directions and led to his extensive collaborations with diverse mathematicians such as Moser, Bott, Simons, and Griffiths. His connection with physicists such as C. N. Yang and T. D. Lee paved the way for the remarkable interaction between geometry and physics of the past few decades.

He was of course a legendary figure in China (and in Chinese restaurants in Berkeley), and it was through him that I and many other mathematicians were introduced to China. The Chern Institute at Nankai is a lasting tribute to his role in revitalizing Chinese mathematics.

I also owe Chern a debt of gratitude for persuading me, at an early stage, to publish my collected works and to make them available in China. As someone who has not fully adapted to modern technology, I find books more friendly and accessible than the electronic media.

Chern's influence, and the widespread affection felt for him by colleagues of all ages, is due in no small part to his personality. Despite becoming the grand old man of Chinese mathematics, he remained modest and unassuming, always willing to listen and to encourage the young. His photograph is one of the few in my study. Alongside it is a framed Chinese poem in beautiful calligraphy that Chern composed on the plane that flew him to England in 1976 for the joint LMS/AMS bicentennial meeting. Since I was LMS president at the time he presented it to me, together with an English translation, discreetly placed on the back.

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Manfredo do Carmo

On Collaborating with Chern

Chern was probably the most important influence of my life as a mathematician. As time goes by, I find myself using more and more in my work what I learned from him during the times I stayed in Berkeley, first as his student and later as a postdoctoral fellow. He was not a forceful person, and his teachings had to be found in his almost casual remarks and mostly in his personality that was, in a mysterious way, very kind but very firm.

I have already written somewhere else [1] my reminiscences as Chern's student; I now want to make some comments on the experience of collaborating with Chern.

In the winter of 1968 Chern gave a course on a preprint by Jim Simons, "Minimal varieties in Riemannian manifolds", later published in [2]. The paper was a breakthrough in the theory of minimal surfaces, and Chern decided to present the subject from the beginning using the method of moving frames; he worked miracles with this method, and it was beautiful to see how things would develop in a natural way through his treatment. For me, the course was an important opening. I had a secret love for the theory of minimal surfaces, but I had not been able to form a clear view of the subject. But then, sometime along the course, I began to feel at home with the beauty of the topic. One characteristic of any Chern course was the presence of interesting open problems, and this course was no exception. Implicit in Simons's paper was a question that Chern made explicit and proposed as a problem in one of the lectures of the course. I had followed the course closely, and that particular problem attracted me. I worked hard and found a solution that I sketched before the following class. After the class, I approached Chern to show my solution. From the other side,

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there came Kobayashi with a solution in the form of a paper already typed. After hearing both sides, Chern suggested, generously, that we should join our efforts in a common project. The final version of the Chern–do-Carmo–Kobayashi paper [3] was written by Kobayashi and became a much-quoted paper. This paper and Chern's course were the main sources of my interest in minimal submanifolds, in which I did a large portion of my research work.

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Robert E. Greene

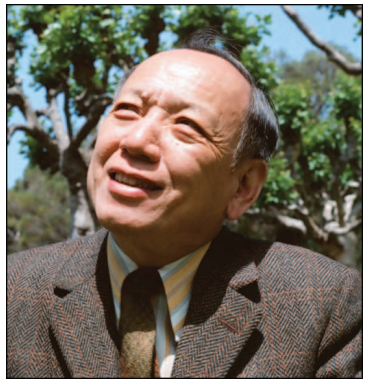
Recollections of S.-S. Chern from Berkeley in the 1960s and 1970s

I was happy to oblige when S. T. Yau suggested that I might write down a few recollections of my years as a graduate student at Berkeley and in particular my memories of Professor Chern, as we all called him, even in private. (Everybody else in their absence was just called by their last name or, if we knew them well, their first names. But Professor Chern was always called just that, title included.)

While from the viewpoint of history in its broader-brush aspects, to arrive in Berkeley in early 1965, as I did as a beginning graduate student, was to descend into a cauldron of political ferment, for me it was to arrive in a world of differential geometric activity beyond anything that a graduate student of geometry had any right to expect. There can have been few moments or places that offered grander opportunities to be present at the creation of everything that was being created at that active time in geometry. And, like all the graduate students with an interest in geometry, I awaited with eagerness the chance to learn from Professor Chern.

This chance came immediately, when Professor Chern was presenting a course in geometry, naturally to a packed house. Excitement was at a high level. Whatever the attitude of those around us in the outside world of political Berkeley ("Don't trust anyone over thirty", "Tune in, turn on, drop out"), we were all on the contrary most anxious to benefit to the full from sitting at the feet of one of the greatest of great masters.

The experience was startling. After a few preliminaries, Chern began to talk about his own work on characteristic classes. He was the soul of modesty and always referred to Chern classes as "so-called Chern classes", for example. But at the same time, the experience was awe-inspiring, because he would come in every day and fill the blackboards with the long calculations that are needed in treating the subject in terms of differential forms. He never brought any notes, never paused to pursue any elusive recollection, and never made mistakes. The whole subject unfolded as smoothly and gracefully as if he were reading from a perfectly written book.



Berkeley: top–1968, middle–1974, bottom–1977.

Finally, one of the more courageous students asked him after class how this was possible. He replied quietly that in fact he had developed the whole subject to begin with without writing anything down. He said it was as if he had a blackboard in his mind on which things could be written and stayed forever. This was said so gently that it did not come across as immodest in the least, just a simple explanation of his inimitable lecture style. Then he added with a tone of slight regret, "Nowadays I sometimes have to use a pencil and paper a little when I am thinking of something new."

To say that this overawed the students would understate the case. At some point soon after, I asked H. H. Wu, with whom I would later write my Ph.D. dissertation, whether such computational power was needed to work in differential

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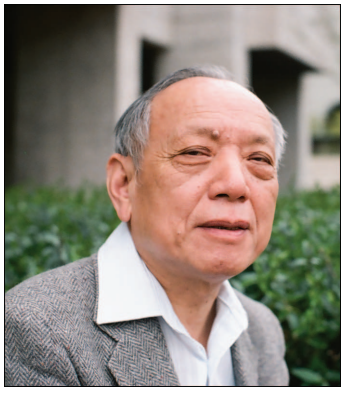
geometry, and I expressed the feeling that if so, I should look for some other field. He chuckled and said "If one had to be like Professor Chern to be a differential geometer, there would be very few differential geometers," and assured me that this kind of computational power was not required.



Top–1984, middle–1998, bottom–1999.

While I was more than happy, as who would not have been, to hear Professor Chern's ideas about complex manifold matters and anything else he might care to present in courses, he had at that time a great many students—eight, as I recall—and I felt diffident about being one of so many. And I felt equally in sympathy, perhaps even more so, with Wu's approach to complex manifolds. My estimate that Wu and I would be in sympathy mathematically proved to be an accurate one. He and I wrote a long sequence of papers together during the late 1960s, 1970s, and on into the 1980s, and we have remained good friends, personally and mathematically, ever since.

While I was not Professor Chern's Ph.D. student as such, I, like everyone in geometry, not just at Berkeley but everywhere, remained under his influence and guiding spirit. And around him, the whole world of geometry at Berkeley orbited. Part of the excitement of the scene was that it seemed that everyone came to Berkeley to present their results, whether or not they were directly related to Chern's own explicit research interests. I recall, for example, Detlef Gromoll coming as a Miller Fellow (in 1966) and introducing us all to his approach to the differentiable pinching problem for spheres. What is intriguing to note here is that even though the kind of geodesic geometry that was Gromoll's specialty, then and later, was quite remote from Professor Chern's own direct interests, it seemed perfectly natural for Gromoll, like any other geometer, to come to Berkeley. Professor Chern had created a situation in which geometry in all directions flourished, geometry of all kinds.

One surprising aspect of how much in touch with all developments Chern was is his extraordinary feeling for what would happen next, for where mathematics would go. For example, the idea goes back in effect to Poincaré that the number of parameters in local holomorphic diffeomorphisms of real analytic hypersurfaces up to a given order is smaller than the number of parameters for the hypersurfaces themselves so that not all (strongly pseudo-convex) hypersurfaces can be locally biholomorphically equivalent. But this observation acquires extra significance from Fefferman's result that biholomorphic maps of C^∞ strongly pseudoconvex domains are smooth to the boundary. Fefferman's result was published in 1974, almost simultaneously with the work of Chern and Moser on biholomorphic boundary invariants. This sort of anticipation of history, as it were, could seem almost uncanny. Without Fefferman's result, Chern-Moser invariants would have had much less significance in complex analysis.

Rutherford is supposed to have remarked in answer to the question of how he was always ahead of the wave in nuclear physics, "Well, I made the wave, didn't I?" Professor Chern would never have been so immodest as to have said a thing like that. But the comparison did come to mind. Chern was not only a great initiator himself, but he seemed to see the shape of things to come in a surprising way.

To return to the personal level, I left Berkeley for a Courant Institute postdoctoral instructorship in 1969. By that time, Wu and I were working regularly together, so I kept in close touch with Professor Chern at one remove, since Wu talked to him frequently about our work together. Chern was particularly intrigued by Wu's and my work together on the rigidity of punctured surfaces of positive curvature. He was so kind as to mention it in his survey article on differential geometry for the 1974 edition of the *Encyclopaedia Britannica*. This gave a real lift to my confidence in my youth. (My nine-year-old niece brought me down to earth by remarking that it did not amount to much, since I did not have a personal entry.)

After my two years at the Courant Institute, I joined the UCLA faculty and thus had occasions often to visit Berkeley again, especially since Wu and I were continuing our joint work apace. Thus I once again had more direct contact with Professor Chern. And the geometric activity at Berkeley was as extensive as ever. The abundance of visitors showed that still "all roads lead to Berkeley" for geometers.

I remember the arrival of Mikhail Gromov, who had just left the Soviet Union and was making one of his earliest public appearances in the United States. Gromov gave a brilliant and inspiring lecture, but it was entirely spoken. At the end of the lecture, only a single symbol had been written on

the blackboard, a large script V (for manifold, in the French style [*variete*]). Professor Chern rose—he was master of ceremonies for the occasion—after the vigorous applause and thanked Gromov for his remarkable lecture and then said, with the tact and gentleness of which he was a master, "Would you mind writing down just one theorem for us?"

While I had been at the Courant Institute, in the fall of 1969, another mathematician had arrived at Berkeley, not as a visitor but as a new graduate student. Berkeley had many graduate students, and new arrivals were not always much noticed. But Yau was an exception. I hope it will embarrass neither Wu nor Yau to quote from a letter that Wu sent me in New York soon after Yau arrived as a student. Wu wrote "A young man has arrived from China who I believe will change the face of differential geometry."

Few prophecies can have been more farsighted. I was surprised at the time, but Wu's remarkable words turned out to be simple truth. Only six years later, while Yau was visiting UCLA, in the autumn of 1976, occupying the office next to my own, he met me one day in the corridor next to our offices and said "I have finished the proof of the Calabi conjecture. Would you like to look at it?" I still have the same office, and I think of this often as I come to work in the morning, of being on a landmark location for mathematics, the place where the whole direction of a subject suddenly changed and a new set of possibilities for mathematics was revealed. (If California were Germany, there would be a plaque or perhaps a statue!)

The new generation for geometry had indeed arrived. And yet, one thinks still of Professor Chern. And after all, the Calabi conjecture is about Chern classes. Generations pass and new ones arrive, but great mathematics is eternal.

Wen-ling Huang and Karin Reich

Shiing-Shen Chern in Hamburg

Hamburg University and Wilhelm Blaschke

Hamburg University was founded in 1919. Its initial faculty consisted of Wilhelm Blaschke (1885–1962, geometry), Erich Hecke (1887–1947, analysis), and Johann Radon (1887–1956, applied mathematics). In 1922 Emil Artin (1898-1962) joined the Mathematical Seminar. With these four extraordinary mathematicians on staff, Hamburg University became an excellent center of mathematics [5].

Wilhelm Blaschke enjoyed travel. In 1932 he traveled around the world and visited China. In Peking, Blaschke gave lectures on "Topological questions in differential geometry", and Shiing-Shen Chern (1911–2004) was in the audience. At the same time, Blaschke recommended Emanuel Sperner (1905–1980) as a guest professor in Peking. As a result, Emmanuel Sperner became a professor at National Peking University, where he stayed for two years.

Chern later recalled these first meetings with Blaschke, that he was immediately impressed by Blaschke's insistence that mathematics be a lively and intelligible subject [1, 2]. When a fellowship was offered to him in 1934 to study abroad, Chern decided to go to Hamburg.

Chern as a Student in Hamburg 1934–1936

S. S. Chern, 1934, in Hamburg (source: Chern Institute of Mathematics at Nankai University).

Chern enrolled in Hamburg University on October 19, 1934. He wanted to earn his doctorate and to become a teacher. He stayed in Hamburg until 1936. Chern was not the only Chinese student studying mathematics in Germany at that time. Chiuntze C. Tsen (1898–1940), for example, was a student of Emmy Noether (1882–1935) in Göttingen.

Chern at the Mathematical Seminar in Hamburg and His Friendship with Erich Kähler. When the Third Reich began, many things changed. Famous universities such as Göttingen, Berlin, and Frankfurt lost many of their best mathematicians. And the young Hamburg University became the leading mathematical center in Germany. Blaschke, Hecke, and Artin got support from Erich Kähler

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We sincerely thank Udo Simon for suggesting that we write this essay for the Notices of the AMS *on Chern's studies in Hamburg. A long version in the Chinese language has appeared in* Mathmedia ***35, no. 2 (2001), Taipei.***

(1906–2000). Blaschke and Kähler traveled together to Moscow in 1934, where Kähler met Élie Cartan (1869–1951). During this journey, Kähler finished his book *Introduction to the Theory of Systems of Differential Equations*, which contained the later-named "Cartan-Kähler theorem". Chern participated in the celebration of the publication of Kähler's book. All the mathematicians present received a free copy of the work. Chern developed a good relationship with Kähler, and they met frequently for discussions. Blaschke sometimes joined them.[1] Chern also attended lectures by Artin, Blaschke, Hecke, the astronomer Richard Schorr (1867–1951), and the sinologist Fritz Jäger (1886–1957).

Chern recalled later that the contents of Kähler's lectures were very difficult and that student attendance declined constantly. At the end, Chern was the only remaining participant. When Kähler was a prisoner of war in France in the years 1945–1947, he wrote to Chern asking him for books and tea.[2] Chern sent him many books, so Kähler was able to continue his work. Chern stated later that Kähler was like a teacher to him. During a meeting in 1999 Kähler said that Chern was like a brother to him [3, p. 17]. In October of 1979 Chern sent his *Selected Papers* to Kähler with the personal dedication: *Herrn Professor Kähler, mit lieblicher Verehrung, Chern Oktober 1979.*

Chern's Thesis. On November 7, 1935, Chern submitted his graduation proposal. The thesis title was "Eine Invariantentheorie der Dreigewebe aus r-dimensionalen Mannigfaltigkeiten im R_{2r}". Blaschke reported the following [6]:

> Several geometers like Reidemeister and Thomsen have treated the 3-webs of curves and the figures connected with these. I had found out that these figures may be transferred to surface webs in the four-dimensional space. It was Mr. Chern's task to investigate these webs, which are even attractive when they are considered from the axiomatic point of view. At first Mr. Chern had studied the difficult theory of partial differential equations due to Élie Cartan. By means of this theory he was able to solve a broad range of problems, achieving very beautiful results. I hold this work to be *very good*.

[1][4, p. 861], [3, p. 17], *conversation with Charlotte Kähler in 2010.*

[2]*Charlotte Kähler retains a copy of this letter.*

Chern's Relationship to Hamburg after Graduation

For Chern's postdoctoral work Blaschke suggested to him either to stay in Hamburg and work with Artin on number theory or to go to Paris and work with Élie Cartan. He decided on Paris [2, p. 6]. Even after Chern had left Hamburg, he stayed in close contact with Blaschke by letter. One letter, still in Blaschke's estate, is dated July 20, 1949:

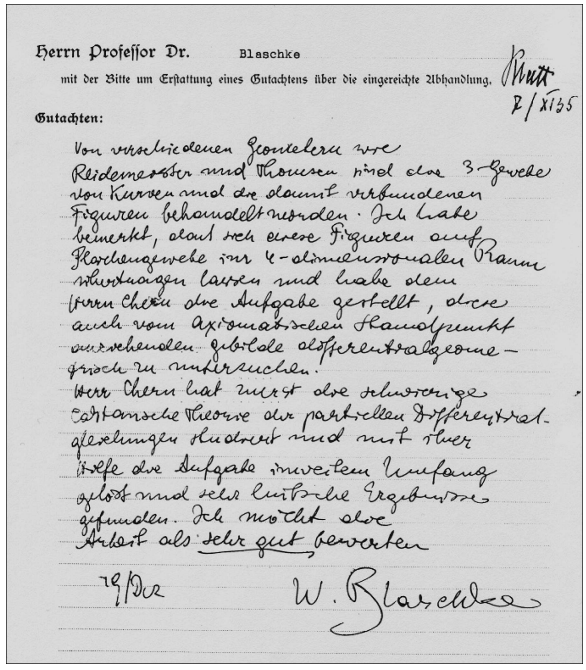
Herrn Professor Dr. Blaschke
mit der Bitte um Erstattung eines Gutachtens über die eingereichte Abhandlung.
R/X135
Gutachten:
Von verschiedenen Geometern wie Reidemeister und Thomsen sind also 3-Gewebe von Kurven und die damit verbundenen Figuren behandelt worden. Ich habe bemerkt, daß sich diese Figuren auf Flächengewebe im 4-dimensionalen Raum übertragen lassen und habe dem Herrn Chern die Aufgabe gestellt, diese auch vom axiomatischen Standpunkt aus wohl interessanten Gebilde differentialgeometrisch zu untersuchen.
Herr Chern hat zuerst die schwierige cartansche Theorie der partiellen Differentialgleichungen studiert und mit ihrer Hilfe die gestellte Aufgabe in großem Umfang gelöst und sehr schöne Ergebnisse gefunden. Ich möchte diese Arbeit als sehr gut bewerten
19/IX
W. Blaschke

Figure 2. Blaschke's report on Chern's thesis [6]**.**

> Dear Mr. Chern,
>
> I was very pleased by your letter of July 15 and by the news that you became a professor in Chicago at the Eckhart Hall, which I know. So you still have the possibility to continue successfully your work in mathematics. […]

In 1971 Hamburg University awarded Chern an honorary doctorate. On July 14, 1972, a Festkolloquium in honor of Chern took place in Hamburg, where Chern himself, Wilhelm Klingenberg from Bonn, and Kurt Leichtweiss from Stuttgart delivered talks. Chern's talk was "The mathematical works of Wilhelm Blaschke" [1].

In the 1980s Chern carried on intensive correspondence with Walter Benz (1931) regarding editing Blaschke's collected works. Benz was Sperner's successor at the Mathematical Seminar and the chairman of the Wilhelm Blaschke Memorial Foundation. On May 17, 1981, Chern traveled to Hamburg to aid in the work. He also delivered a talk at his "old university".

On September 6 and 7, 1985, one week before Blaschke's 100th birthday on September 13, 1885, the 100th anniversary colloquium for Blaschke took place in Hamburg. Mrs. Blaschke and the members of Blaschke's family, Chern and his wife, Kähler and his wife, and many others took part in the ceremony. Chern's talk was titled "On the geometry of webs". Chern visited Erich Kähler and his wife Charlotte in Hamburg during his two visits to that city in 1981 and 1985.

On April 30, 2001, the Wilhelm Blaschke Memorial Foundation of Hamburg decided to award the Blaschke Medal to Chern. In October 2001 Chern also received an honorary doctorate from the Technical University of Berlin in a ceremony in Tianjin.

Acknowledgments

We would like to thank the following people for their help: Christina Ahrens, Walter Benz, May Chu, Paul Chu, Roland Höfer, Charlotte Kähler, Eckart Krause, Alexander Kreuzer, Mike Lemke, Honghai Lv, Rainer Nicolaysen, Elena Roussanova, Sabine Simon, Udo Simon.

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Lizhen Ji

Seeking Roots: My Academic Grandfather Professor S.-S. Chern

My grandfather passed away when I was too young to speak, and I could not remember anything about him. I also never had the chance to talk to my academic grandfather Shiing-Shen Chern. It was only six years after he passed away that I first visited the Chern Institute of Mathematics at Nankai University,[1] established by my academic grandfather.

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[1] *The Chern Institute of Mathematics was originally called "Nankai Institute of Mathematics" and changed to the current name on December 3, 2005, in memory of Professor Chern.*

Certainly I have heard many things about Professor Chern: a star was named after him, there are three major awards for mathematicians named after him, and after he died, thousands of students and nonacademics mourned for him.

I have also heard of Chern classes, Chern-Simons theory, Chern-Moser theory, and his intrinsic proof of the Gauss-Bonnet formula. These are great achievements for a great mathematician. As my academic father, Professor Shing-Tung Yau, said in his summary *Chern's Work in Geometry*:

> It's fair to say that Élie Cartan is the grandfather of differential geometry and S.-S. Chern is the father of modern differential geometry. Together they have created a beautiful and rich subject that has reached out to every branch of mathematics and physics. Right before he died, Chern said that he is going to see the great Greek geometers. There is no doubt that he had reached the same status as these great geometers.

There must be something else that made him loved by many different people and allowed him to achieve so much in mathematics and within the mathematics community. I wanted to know more about my academic grandfather and, in doing so, discover more about my mathematical roots.

The great Benjamin Franklin wrote a classic autobiography so that his descendents could learn of his struggles and successes and acquire the keys to happiness. Unfortunately, I could not find an autobiography written by Professor Chern. Instead, I found an interview of his published in the *Notices of the AMS* in 1998. Two memorable quotes from this interview include "I don't think I have big views. I only have small problems" and "I have no difficulty in mathematics, so when I do mathematics, I enjoy it. And therefore I'm always doing mathematics, because the other things I cannot do. Like now, I am retired for many years, and people ask me if I still do mathematics. And I think my answer is, it's the only thing I can do. There is nothing else I can do. And this has been true throughout my life."

I asked myself, are these really candid and fair statements? I was intrigued by them. I wanted to find out more about Professor Chern. What I found is that he is second to none with respect to many things, and he had a grand vision for the development of Chinese mathematics and the confidence to carry out important initial steps for this purpose. For example, he started the Chern Institute of Mathematics in 1985 for the whole country of

China and secured the necessary funding when China was not as rich as now. Before he died, he was working furiously on a famous long-standing open problem in complex geometry.

He was also a very sentimental and caring person. He was a romantic, pragmatist, and idealist all wrapped into one. Yes, he lived for math and also died for math, but he led a very rich life that encompassed many different aspects.

I asked myself, how should I begin to seek my mathematical roots? In early June 2010, I went to UC Berkeley for a conference. During the first morning there, I crossed a small creek that separates the University faculty club and Evans Hall (the math department), and I wondered what I could find out about my academic grandfather. After all, this is where he helped build one of the best geometry groups in the world and spent over thirty years of his prime mathematical life. There must be things that remind one of his presence and glories during those years. I searched in and outside the building. Yes, the campus around Evans Hall is beautiful, but I could not find traces of him. Time has washed away many things. Maybe only mathematics lasts forever and is the best testimony to the achievements of mathematicians. I was disappointed, and people told me that MSRI should have some materials about him. Unfortunately, I was then reminded that MSRI was closed only during this period for yearly maintenance. This was bad luck and very disappointing.

Then it occurred to me that I should go and visit Nankai University. This is where Professor Chern's math career began and ended. He was an undergraduate in Nankai, returned later in life to work intensively on a famous open problem in geometry, and passed away while still struggling to resolve the problem. It was certainly a very special place for him. I wondered if there were things there that tell stories of his life. What contributed to the thoughts of his youth and his reflections and longings near the end of such a glorious life?

Among the three math institutes he established, the Nankai Institute of Math (or Chern Institute of Math) was the last and probably the most dear to Professor Chern.

I decided to visit Nankai on July 28 and talk with people who had worked with Professor Chern to develop the institute and ask them questions regarding all aspects of his life. Much has been mentioned of his mathematical achievements, but I wanted to learn more about him as a human being and about his nonmathematical life. What I found deepened my respect for him and opened my eyes to a lively and charming man.

Going to Nankai

I got up very early on the morning of July 28, hoping to maximize my time at Nankai University. After a slow taxi ride from Tsinghua University to the Beijing South train station and a train ride to Tianjing, I arrived at the Chern Institute of Mathematics quite early. The deputy manager of the office, Ms. Li Hongqin, was waiting for me at the entrance. I showed her the digital voice recorder and camera I bought for this trip and insisted that I was better prepared this time. She smiled. I had met her last year during a workshop in Nankai, and we also talked about Professor Chern at that time.

She took me directly to the director of the Chern Institute, Professor Long Yiming. When I entered his spacious office, what caught my eye immediately was an elegant plant on the coffee table. This is the first time I entered an office of a professor at the Chern Institute, and I was impressed.

A Great Mathematician and Diplomat

I asked Professor Long what he remembered most about Professor Chern. He said that one unique thing about Chern was that his abilities were not limited to mathematics but that he was also a great diplomat because of his charming and noble character. Establishing such an institute in 1985 and building a huge new building required dealing with people at many levels. For example, convincing top leaders in the central government of China to generously support the math institute was no small task.

He continued that the program of academic years Professor Chern started in 1985 has had a great impact on Chinese mathematics.[2] Most well-known Chinese mathematicians had been trained in these programs. This can be counted among Professor Chern's great contributions to Chinese math. It was one of the first attempts to include both Chinese and Western mathematicians. Convincing distinguished speakers to come and give lectures was not easy. For example, in 1985, some very distinguished Chinese mathematicians came and spent the whole year in Nankai preparing students for more advanced topics to be presented by experts from the West. Leaving home for one year is a nontrivial matter. Furthermore, everyone came and worked peacefully and willingly together for the same goal. This shows Professor Chern's great leadership. I asked him what Professor Chern's secret was. Professor Long said that since Professor Chern's achievements in math were admired by many people, naturally he had great influence in the mathematics community at large. More importantly, his charming and noble character and skill in dealing with people made him a truly exceptional character among great mathematicians. Yes, indeed there are many great

[2] *In each academic year, there is a major theme, with various activities supporting this theme. This is the reason for the name "academic year" program.*

mathematicians, but it is not easy to find a great mathematician who is also a great diplomat.

This point of view was also emphasized by Professor Chern's colleague at Berkeley, Hung-Hsi Wu: "He (Professor Chern) was a man of great leadership, notable for unique diplomatic skills that made him an outstanding administrator."

The current building for the Chern Institute is magnificent. It is one of the largest math institutes that I've seen. On the left side of the entrance, there is a couplet in honor of Professor Chern, composed by S. T. Yau, Professor Chern's academic son.

I wondered how they could secure the funding to build it. Professor Long said that Professor Hu Guoding probably knew more about the details. My next stop was to visit Professor Hu Guoding. Before I left, Professor Long warned me that Professor Hu may not have the energy to speak with me for more than half an hour, due to a serious car accident he had a few years ago.

How the Chern Institute Was Started

Professor Hu was an old friend of Professor Chern's and was the second director of the Chern Institute. He was one of the key figures in developing the Chern Institute and also secured much of the funding necessary for the main buildings. I prepared a list of questions to ask during this trip, but for Professor Hu, my list was not necessary. He said "Let me start to tell you about the Chern Institute and Chern, and you can ask some questions later." Due to the car accident, he could not move much in his sofa, but his voice was strong. He was full of energy and talked for over an hour. Those exciting years brought back dear memories of his work with Professor Chern to organize the institute program of academic years.

I listened and listened, wishing to hear more. Ms. Lin Hongqin, who accompanied me, became worried that the continuous talking might tire Professor Hu, and the meeting soon ended.

At the beginning, Professor Hu said that it should be made clear that everything was done by Professor Chern. After the Cultural Revolution, the state of Chinese mathematics was in ruins. Professor Chern encouraged Hu and other Chinese mathematicians by saying that they should be on equal footing with Western mathematicians. There was no reason why they could not participate at the international level. There were two options to train young Chinese mathematicians: send them abroad to study in the West, or gather the best Chinese students in one location and invite leading experts from around the world to give lectures. Chern felt that the second option was more practical and a better deal. The idea of such an academic program at Nankai was proposed. Using his influence, Professor Chern invited speakers from the West, and the best Chinese mathematicians and students were selected to participate.

During this process, a problem was looming in the background. The foreign experts could stay in hotels, but they also needed office space. More importantly, housing was needed for the Chinese participants. They needed money for new buildings, but the funding was limited. Professor Chern told Professor Hu not to worry and these problems would find solutions.

In the early 1980s the ministry of education encouraged universities to reform their localized approaches, i.e., to open their resources and share with developing universities. Few places were actually doing this. Professor Hu mentioned one example. One physics department obtained funds for a lab by claiming that this lab would be shared with other universities. They then declared that the lab would be used by their department during the daytime and that other departments could use the lab at night.

During a dinner with the minister of education, Chern explained the vision and goals of the program of academic years to train young Chinese mathematicians from the entire country. The minister told Chern that he and his fellow administrators were really "reforming and opening up". The ministry of education gave special funds to construct a building used to house the participants during the first academic year, 1985.

After ten years of the successful program of academic years, the Chinese math community was much bigger and stronger than before. Professor Chern admitted to Professor Hu that Chinese mathematics still had a long way to go. In order to transform China into a "big" country of mathematics, the Math Institute should have an impressive building with better facilities. Professor Hu then talked about how through his personal connection with former President Jiang, he was able to convey Professor Chern's dream and secured funding for the current math building. Before the liberation, Hu was a Communist Party member and the leader of the student union of Shanghai Jiao-Tung University. President Jiang was also an active member of the union.

Professor Long invited me for lunch and was waiting for me at the restaurant. He did not expect my meeting with Professor Hu to last as long as it did.

Chern's Life at Nin Yuan (His House in Nankai)

After lunch Ms. Li took me to meet Mr. Hu Delin, Professor Chern's former driver and private secretary. Hu Delin was probably the person closest to Professor Chern in Nankai. In fact, after the death of Professor Chern's wife in 2000, Hu played a very special role in his daily life. So I put forward a general request: tell me about Professor Chern's everyday life.

We talked for nearly an hour and a half, and he told me many unforgettable things about Chern. Probably the most touching moment came when he started to cry. Hu said that Professor Chern was such a good person. His sobbing voice and hands trying to cover his tears will stay in my memory forever. I will describe several stories Hu told me.

a) Chern's Wife

Chern's wife died in January of 2000. Chern went back to California in April to settle her affairs and then moved to China permanently in August.

According to Hu, he could see visible changes in Chern after he came back. Chern told him that there was no one he could talk to anymore, and people could not understand him. Hu sometimes tried to comfort him and said, "You have experienced other difficult things in life and time will ease things." Professor Chern asked Hu how he could forget sixty years of life together, and there was no one that could replace her.

Sixty years is an important cycle and certainly a long time for a marriage. It certainly represents deep commitment and a lifetime love. I asked Hu if Professor Chern had ever talked about his marriage and how he met and courted his wife.

Hu told me that when Chern went to Tsinghua University, his future father-in-law was a professor of mathematics there. C. N. Yang's father was also a professor of mathematics at Tsinghua, and both faculty members recognized Chern's talent and potential. They often invited Chern to their house parties, allowing ample opportunity for Chern and his future wife to meet. After Chern came back from Europe and went to the Southwest United University, the marriage of the couple came to fruition.

Hu said that Professor Chern was a very devoted and caring person. In his heart, he had a place for everyone in his life. He always remembered people who had passed through his life. Even after seventy years, memories could instantly come back to him. Hu said that Professor Chern had probably never forgotten the people he met.

Hu shared the following story with me. I think it shows Professor Chern's sincere nature and his kind and thoughtful heart.

b) An Unexpected Visitor on a Cold Day

It was a very cold day in December 2002. There was a knock on the front door of Chern's house at Nankai. Hu opened the door and saw a stranger. The stranger told him that he wanted to see Professor Chern. Hu was surprised. According to the rules of the Math Institute and Nankai University, any person who wanted to see Professor Chern should either be invited or go through the Math Institute first for approval. Thus, Hu denied the visitor. The stranger said, "Wait! I have a picture of Professor Chern when he was young." In this picture, Professor Chern was surrounded by members of two families. The visitor explained the situation.

When Professor Chern was in Nankai attending college, his mother had a very good friend (a kind of sister) who lived nearby. The neighbor had two daughters. The older one was about Professor Chern's age. The two families were talking about marrying Professor Chern, the oldest son in his family, to the oldest daughter of the other family. After Professor Chern went to Tsinghua and then Europe, the potential for marriage was not discussed further. The picture showed the members of both families.

The stranger, Mr. Shen, was a son of the older sister. His mother had died six years earlier, but the younger sister never married and lived with her nephew. During and after the ICM 2002, there were many newspaper articles and television reports about Professor Chern. The younger sister learned that her old neighbor (Chern) worked in Nankai University. She asked her nephew to go and look for Professor Chern. Mr. Shen said that if Hu mentioned the name of his mother and aunt, Professor Chern would remember them.

After hearing this story, Hu thought that Mr. Shen should probably see Professor Chern. He told Mr. Shen to wait in the reception room and took the picture to Professor Chern. Hu repeated the story and showed Professor Chern the picture. Professor Chern said that he did remember the people in the picture and asked for the visitor's age. When Hu told him that he was about sixty, Professor Chern immediately said that Mr. Shen could not be in the picture since he was too young. Hu was amazed that after seventy years, Professor Chern could remember all these things clearly and instantly.

Professor Chern immediately asked Mr. Shen to come upstairs and inquired about his family. Later, he visited the home of Mr. Shen and the younger sister. They lived in a poor building. The heating was provided by a coal stove. Professor Chern immediately offered to buy them an electric heating unit. The little sister refused at first. Professor Chern said that even though he was not rich, he had more money than she and could help her, and then he elaborated on the dangers of using coal heat. Professor Chern also offered to pay for the electricity necessary to run the heater. From then on, Professor Chern invited the younger sister and her family to come to his house once a month to have a meal and chat. Near the end of each year, Professor Chern also gave them some money.

c) Professor Chern as a Kind Person

Hu commented that Professor Chern valued friendship. For example, one reason he came to Nankai to start a math institute was that he had several classmates from college working at Nankai.

Not only was Professor Chern nice to his relatives and friends, but he was also very kind to

everyone around him. Many bigshots may not bother with such things, but not Professor Chern.

Here are two stories that depict his good nature. After he came back to Nankai in August 2000, Professor Chern had to use a wheelchair. Two nurses were hired to take care of him. Before he died, Professor Chern said to the director of the Math Institute and other university officials that if it were possible, he hoped that the Math Institute could find jobs for the two nurses in the event they wanted to stay in Nankai. Hu said that they both currently work for the Chern Institute.

After Professor Chern died, his daughter May Chu came and told Hu that her father asked her to give some money to every staff member at Nin Yuan (his house at Nankai). Professor Chern arranging gifts for people who worked around him after he died; this really touched me. Hu said that Professor Chern always thought of others and was such a good person. As he said this, he started to cry. It was a very touching moment. Even now as I am writing this paragraph, my eyes moisten. It is difficult not to love such a caring person regardless of their achievements in math.

d) Professor Chern's Diplomatic Skills

Hu also said that Professor Chern knew how to handle people and situations in a very diplomatic and smooth manner. He mentioned the following incident.

The 2002 ICM was certainly an important event in China, and Professor Chern contributed much to its successful opening in Beijing. He played a prominent role throughout the congress. Many reporters requested a press conference with him. As a kind person, he could not say no, even though he had a severe cold. On the fourth day of the congress, he had an open hour before lunch but was booked with important meetings during the afternoon.

Many reporters asked questions, and Professor Chern answered them patiently, even though some were repeated. It was getting late and he had to leave, but the questions kept on coming. How could he solve this problem? Hu became worried. Professor Chern said that it was not a problem and he would handle it. He turned to the reporters and said "You have asked me many questions. Now it is my turn to ask you a question. If you can answer my question, then you can ask more. Here is my question. There have been many different theories about the cause of death of the famous historian, Sima Qian. Could you tell me a new theory about his death?" The reporters were stunned, and no one could answer. Chern then said, "After you find an answer, you can come to Nankai and ask me more questions."

Hu once told Professor Chern that he would make a good politician. He said, "Yes, I can do it, but I don't want to do it. I don't want to fight with people in those kind of places."

Professor Chern's diplomatic skills were also confirmed by Lv Honghai, the office manager at the Chern Institute. On the way back to the train station, Lv chatted with me. I asked him how he would best characterize Professor Chern. Lv said that Professor Chern was very patriotic, much more than many people who spent their entire lives in China. I asked Lv for a story about Professor Chern, and he told me the following.

During banquets in China, seating arrangements are important, especially determining who will sit at the main table. Professor Chern always disliked this. Once there was a major event with many important guests. They could not all sit at the main table. How can one solve this delicate problem? Professor and Mrs. Chern came up with a good solution. They assigned a number to each table. At the entrance, Mrs. Chern held a bag containing numbered pieces of paper and with a smiling face, told guests that they were to take a piece of paper and sit at the table with the corresponding number. What an original and interesting way to solve an otherwise embarrassing and difficult problem!

d) Professor Chern's Final Days at Nankai

Professor Chern told Hu that it was a good thing that the ICM was held in 2002. If it were scheduled for 2003, it would probably be postponed due to SARS. Nevertheless, the virus affected the final stage of Professor Chern's life.

Hu mentioned that during the SARS pandemic, Nankai University was very concerned about Professor Chern's health. Before the outbreak, the two nurses replaced each other after a full twenty-four-hour shift. During the pandemic, each nurse stayed with Professor Chern for one month. Before changing shifts, the nurses were subject to a thorough physical examination. The nurse on duty lived in the upstairs quarters and could come down only to fetch Professor Chern's meals. Hu could talk to Professor Chern only over the phone from downstairs.

After a while, Professor Chern became upset. He told Hu "You are putting me into a prison" and demanded "I want you to come upstairs right away and talk to me!"

Professor Chern was very busy at other times with visitors and invitations to conferences. Now he had some free time to work on a famous open problem in differential geometry: the nonexistence of complex structures on S^6. According to Hu, Professor Chern had already worked on this in the 1970s.

After much work, he wrote up a preliminary version of his paper. In a conference held in Nankai during August of 2004, some gaps were found. This was a blow to Professor Chern. Some tried to convince him to halt work on this problem. Several

mathematicians told Hu that such a problem was not for a person over ninety, or even fifty, and that only young people could try to work on it. Professor Chern was stubborn, and he told Hu that there were gaps in his methods, but he believed that he could solve it.

Hu was worried about Professor Chern's health. Even though there was a nurse present at all times, Hu also stayed and lived upstairs to make sure that everything went accordingly.

Professor Chern could not sleep well in those times. He needed sleeping pills to fall asleep. Every night Professor Chern would get up around one o'clock in the morning and ask the nurse to help position him in a sofa with a small table in front. Then he would do computations with pencil and paper.

Professor Chern's health deteriorated fast. Some discouraged his late-night hours, but he was persistent. Sometimes he lost his temper toward those who tried to convince him not to work. Professor Hu thought it possible that Professor Chern sensed his life coming to an end, and he felt the need to resolve this long-standing problem.

After a very successful and long life, Professor Chern died on December 3, 2004.

Hu also commented that Professor Chern led a very simple life. His oldest shirt was thirty years old, and his newest was over ten years old. When he received the Shaw Prize in September 2004, his daughter bought him a new shirt for the award ceremony. Hu said that Professor Chern donated three cars to the Chern Institute, and all these came out of his savings.

Concluding Remarks

Before I left Nankai, I wanted to take some pictures of the entrance of Nin Yuan, Professor Chern's house.

Standing there, I was thinking about what a great academic grandfather I had. He had achieved so much in math and in life. He was such a well-balanced person and had led a full and rich life. Until his last days, he was still trying to solve a famous problem: the nonexistence of complex structures on S^6 is not a small problem at all!

During the trip, I was hoping to find more about the roots of Professor Chern's mathematics and his secrets to success. Instead, I heard many stories about him as a normal human being: a caring, sentimental, considerate person well liked by many people. Maybe these are the qualities that made him stand out among the great mathematicians.

I wondered if he had some other dreams and longings in the last years of his life and how he thought of his life in his youth. I wished that I could have a chance to talk to such a charming, interesting, and kind person, as a grandson with his grandfather.

Acknowledgments

I would like to thank Professor S. T. Yau, Professor Yang Lo for suggestions on how to make this trip fruitful, Professor Long Yiming for giving me the chance to talk to various people and his hospitality during the trip, and Professor Hu Guoding for the unforgettable conversation in spite of his health condition. I would also like to thank Ms. Li Hongqin, Mr. Lv Honghai, and Mr. Hu Delin for their conversations and help with the trip. The detailed stories from Mr. Hu Delin were especially appreciated. Finally I would like to thank Patrick and Julee Boland for reading a preliminary version of this article carefully and making many helpful suggestions which greatly improve this article.

Jun Li

Read Classical, Chern Told Us

It was during my junior year that I first learned Chern classes. I consider this my first academic acquaintance with Chern. During this period of time, I studied extensively the characteristic classes, Chern's treatment of Chern classes, and Chern classes and Riemann surfaces. To a student who was anxious in setting a foot in research mathematics, I felt a great reverence for the name Chern.

I first met him when I was a graduate student at San Diego. He gave a lecture in the department, and he came to the welcome party hosted by Yau. With many students of Yau, I was present at the party. I still vividly recall that Chern asked the name of and chatted briefly with each of us students; his wife put down our names in her small notebook.

It was in the early 1990s during the year I visited MSRI that I began to meet and chat with him regularly. I was present at many lunches and dinners Chern hosted for his colleagues and friends; I visited him and his wife many times during my stay in MSRI. This tradition of visiting him whenever I got a chance continued even after he moved to China.

It is my privilege to have had many occasions, while my wife and his wife sat at the kitchen table, to have sat with Chern on his teak couch chatting. The topics usually began with my explaining to him my research interests, then his telling me the stories of many of his contemporaries. Once on hearing that I came back from Oberwolfach, he told me that the treasure of Oberwolfach was its library, where much classical mathematics literature was on open shelves. He told me several times, "it is important to read original writing of great mathematicians," followed by, "have you?"

Over the years, I began to regret that I had not followed more on his advice. The ideas of great

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minds are timeless. We may have moved to more elaborate and sophisticated mathematical tools and languages; however, the new developments usually are rooted in the classical mathematics, where simplicity and insights are most treasured.

An example is Chern's introduction and treatment of the Chern class, an idea of timeless beauty.

Ernest Kuh and Y. Ron Shen

Remembering Our Dear Friend Professor S.-S. Chern

On behalf of Chern's Chinese colleagues in Berkeley, we offer in this short article a tribute to our most respected elder and dear friend; his life in Berkeley; his impact on our lives. Besides his scientific prowess and his distinguished work in geometry, Chern was a wonderful individual of culture who always inspired people around him, especially the young. Yet he was a very modest person. Despite his status, he was never intimidating. He and Mrs. Chern always enjoyed getting together with acquaintances. Over the years we enjoyed our friendship and good times with them. We also learned a lot from his wisdom.

Chern came to Berkeley from Chicago in 1960. There were very few faculty of Chinese origin at Berkeley in the 1960s, and Chern soon became a leader and known to everyone in the local Chinese community. Because of his pleasant personality and vast knowledge of Chinese culture and history, he naturally attracted everyone around him. He served as a bridge between the young and the old generations (famous among them are Y. R. Chao in linguistics, S. H. Chen in Chinese literature, and T. Y. Lin in civil engineering and architecture). He had very broad interests outside mathematics, ranging from culture and literature to history and politics. Conversation with him never had a dull moment. He was a most welcomed person in any social function. Before long we discovered that he also enjoyed good food. He easily made friends with chefs, waiters/waitresses, and owners of restaurants, especially the Chinese restaurants. When we went out with him to any restaurants we always got special treatment.

Chern attracted many visitors to Berkeley. They were not necessarily his friends and students in mathematics. Among them were C. N. Yang and T. D. Lee, the famous theoretical physicists who won the Nobel Prize in 1957. Yang and Lee consider themselves former students (at the Southwest United University during World War II) and dear friends of Chern. They would always come to pay him respect whenever they happened to be in the San Francisco Bay area. Jin Yong, the famous writer of Chinese martial arts novels, was Chern's admirer and presented him with a complete set of autographed volumes of his novels that Chern had enjoyed reading.

In 1972 when President Nixon opened the door to China, almost immediately Chern followed. Many of us also visited China in the following year or two. The whole nation was crazed about China at that time. Conversation topics in our circle naturally also focused on topics on China. Chern had many old friends in China and was most informative about China. Again he was very much the center of our gathering. The Berkeley Chinese community in the meantime became more active and organized. A faculty group of Chinese origin started regular social dinner parties, and the Cherns were among the most enthusiastic members of the group. Toward the late 1970s, numerous Chinese delegations visited Berkeley. The Cherns often served as the local hosts, welcoming them at their beautiful El Cerrito home overlooking the San Francisco Bay and the Golden Gate Bridge. Meanwhile, his life also changed somewhat as the United States started to allow Chinese students and visitors to come for long-term stays. Chern became busier and naturally received many requests for research supervision. He was the first faculty member to welcome long-term researchers and students from China to Berkeley. Chern officially retired from Berkeley in 1979 but was as busy as ever after retirement. He co-founded the NSF Mathematics Science Research Institute (MSRI) in 1981 and served as its founding director. The famous MSRI located on top of the Berkeley hill has attracted generations of visitors in various fields of mathematics. After stepping down from the directorship in 1984, he immediately assumed the founding directorship of the Mathematics Institute at Nankai University in Tianjin, China. Ever since the opening of China, he had sensed his responsibility to help the development of mathematical science in China. He strongly believed that China could become a nation superb in mathematical science. Nankai University is his undergraduate alma mater, and he would use his influence to help Nankai form a world center in mathematics. His early proposal to establish a math institute in China at Nankai University was warmly received by the Chinese leaders, who provided the needed budget, and by the university with a new building. As expected, soon after the Institute was set up, many established mathematicians around the world flocked to Nankai, making Nankai

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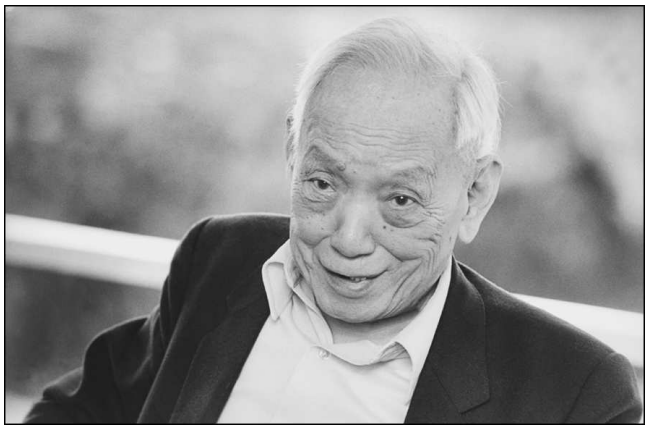

S.-S. Chern during an interview for the *Notices* in 1998.

instantly a mathematics center for scholarship and research.

Born and educated in China, Chern loved his native country. He was willing to do anything to serve the country. He remembered well the Japanese invasion and atrocities in China before and during World War II. So when the overseas Chinese organized the strong movement in 1971 protesting against the Japanese occupation of the Diao Yu Dai Islands, he was among the notables who signed the open letter in the *New York Times* calling on President Nixon "to take appropriate measures to ensure the Chinese sovereignty over these islands."

In the 1980s Chern had more time to pursue his other interests besides mathematics. During our social gatherings he talked about what he recently learned on the early history of the Tsing dynasty, how the queen grandmother, Xiao Zhuang, of the early emperor, Kang Xi, contributed through her influence on the emperors to the most prosperous period of the dynasty. He even wrote an article on this that was published in the Chinese *Journal of Biographical Literature*. He became more interested in physics and relations between physics and mathematics. He was very much intrigued by the surprisingly close connection between the Chern-Simons theory and the Yang-Mills theory. He borrowed books to learn about Feynman path integrals. He was extremely proud of the 1987 discovery of his son-in-law, Paul Chu, on high-temperature superconductors that created a huge splash in physics. He would certainly be very happy to learn that the well-known Chern class and Chern number have emerged as common words in theoretical physics in recent years. Chern also became more keenly interested in politics and in China.

As the years went by, more dinner parties were organized, some small and some large. We became closer to him and Mrs. Chern, having frequent meals together at each others' homes. The Cherns actively resumed contacts with their old friends in China and warmly welcomed them at their home, and they helped to take care of their siblings who visited the United States. As a matter of fact, one of us (YRS) is indebted to the Cherns for meeting his wife, Hsiaolin, at the Cherns' house for the first time. Hsiaolin is the daughter of one of his old friends in China and had come to the Berkeley area to study after the Cultural Revolution. The Cherns' house was her home away from home during holidays. We remember well the happiest party for everyone who attended the wonderful event of the Cherns' sixtieth wedding anniversary at the East Ocean Restaurant in Emeryville. The whole front room was packed by their families and friends. When Chern stood up to thank everyone, he was obviously very excited. He talked at length about his life but then forgot to acknowledge his wife, Shining, to whom he was always appreciative for having supported him so well throughout the years. Someone in the audience wanted to remind him, shouting "where is the bride?"

Starting in late 1970s, the Cherns visited China very often. He was a towering figure and commanded great respect, certainly no less than Yang and Lee, in China. He was a person the leaders of the Chinese government and universities were always anxious to consult. He was received by the top leaders, including Vice Premier Deng Xiaoping and President Jiang Zemin, every time he visited China.

With the families of Chern's son and daughter far away, independent living became more difficult for the old couple, despite the fact that all their friends were more than willing to help. On the other hand, life in China seemed attractive since it could be much easier with hired helpers readily available. In late 1999 they finally decided to move back and settle in Nankai University, Tianjin. The university built a house on campus for them and made sure that they received all the care they needed to enjoy the campus life. Chern was delighted to have again a life surrounded by students and young colleagues.

One of Chern's great assets was being able to easily interact with others. He was particularly helpful to the young faculty through such frequent interactions. This happened in Berkeley and in Nankai. He was an extremely modest person in spite of his towering reputation. He could inspire others in a casual conversation and in a seemingly very natural manner. We all learned a great deal from him, especially on being a person and on the philosophy of life.

We visited him and Mrs. Chern a few times at Nankai. They were apparently happy with their life in China, having more occasions to meet with old acquaintances and more opportunities to interact with young students and colleagues. Almost immediately, Chern became one of the most famous persons in Tianjin. The mayor and party secretary of the city would come to his house to pay him respect every year. Once during our visit, he happily mentioned that his bronze head figure would soon be placed in the city park with the other famous Tianjin citizens. He clearly enjoyed his status in China. Our last visits were shortly before he passed away (Kuh's in the summer of 2000 and Shen's in October 2004). We had dinners at his house, where his chef and servant served a wonderful meal, and we had a very pleasant time. Even in October 2004, he looked very healthy and was in a very good mood to talk. He mentioned he was looking forward to the visit of C. N. Yang the following week. He talked about his recent happy reunion with his friend of seventy years ago and showed us the poems they had written on the occasion. There was no sign that his body would soon give up.

We have lost a giant in science and will miss a great friend in life.

I. M. Singer

Reprinted from *S. S. Chern: A Great Geometer of the Twentieth Century*, International Press, Boston 1998.

Last term I gave some extra evening lectures to my class in geometry and quantum field theory. The younger graduate students needed the geometric background for gauge theories. I happily talked about connections on fiber bundles, curvature, characteristic and secondary characteristic classes. I reviewed the many excellent treatments but finally followed Chern's classic "Geometry of characteristic classes". I urged the class to study this short, concrete, elegant, and deep paper.

Rereading it brought back vivid memories of Chern's course at the University of Chicago in 1949–50. Differential forms were not well known then. In fact, a year earlier, André Weil had introduced us to differential forms in a seminar (for graduate students and faculty) in which he recast the calculus of several real variables in the language of forms. Barely familiar with Grassman algebras, students in Chern's course were amazed and intrigued at Chern's effective use of forms in local and global geometry.

Because I was writing a dissertation at the time, I was a passive participant in the many student discussions, trying to understand and absorb the material in Chern's lectures. However, a year later at M.I.T., Warren Ambrose was eager to learn differential geometry. He organized a seminar in which I lectured on my notes of Chern's course. Like many others, I struggled with Chern's "Let p be a point and dp its differential" and finally understood how convenient a notation it was for the identity map and its differential.

My recent evening lectures reminded me of our seminar over forty years ago held in the same room—with blackboards on three walls. We filled all the blackboards with the definition of a connection. We hadn't yet learned to first expose the properties of principle bundles and of Lie groups. Chern does that very neatly in the paper cited above. Only long experience has taught me how much goes into making deep ideas simple, as Chern did. To quote him, "This train of ideas is so simple and natural that its importance can hardly be exaggerated." Under Chern's influence, both Ambrose and I taught courses in differential geometry at M.I.T. Others did the same elsewhere. Books were written and the subject flourished. I need hardly dwell on what we all know: Chern introduced global differential geometry to American mathematics, as well as the use of differential forms in a host of subjects. Geometry is harder to define than most mathematical disciplines. Whatever it is, Chern showed us for half a century how to do differential geometry. Each time we met, he showed me something new—and I wish I had been a more attentive student. Why didn't I listen more carefully to his discussion of Bäcklund transformations, or his exposition of Finsler geometry?

In time, Chern's course and Ambrose's seminar had their effect. I switched from functional analysis to global geometry. My present interest is in quantum field theory. Because the ideas of interest in physics are geometric, I continue to be immersed in global geometry. In fact, what's missing in physics is some quantized version of geometry that we only dimly perceive in string theory.

I have emphasized Chern's scientific influence. His personal influence has been just as strong. He was very encouraging and gentle with students and younger colleagues. And he ran conferences or institutes with the same directness and elegant simplicity as in his papers.

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It was a privilege to know Chern and his wife, Shih Ning, as friends. In fact it was fun—I can recall the many Chinese restaurants, famous or fancy, in which we discussed mathematics, politics, family, and life.

In my view, Chern was one of the great figures in twentieth-century mathematics.

Alan Weinstein

Almost every differential geometer is in some way a student of Chern. I had the honor and pleasure of being one of his "official" students, as well as his colleague. Chern's role in creating a world-famous department at Berkeley is amply documented in Calvin Moore's book, *Mathematics at Berkeley: A History* (A K Peters, 2007), so I will confine myself here to some personal remarks.

As his student, I learned, of course, much about differential geometry, though I realized only many years later how much I had learned. But I also learned, and gained, many other things from having been Chern's student.

I learned how to listen. I was always amazed by how much time Chern seemed to find for me across a desk covered with papers. I would go in and tell him what I was doing. He listened, asked me questions, and gave some advice and lots of encouragement.

A degree from Chern was a ticket into the wider mathematical world. In 1967 I finished my thesis a few months before the end of the spring semester. Chern was a visitor at IHES at the time, and he arranged for me to be invited there, though I was officially still a student. This was the beginning of two lifelong relationships—with France, particularly the mathematical life in Paris, and with my wife Margo, to whom I was married in Orsay. Shortly before our wedding that May, Professor and Mrs. Chern invited us to dinner in Paris at la Méditerranée, a favorite restaurant of Jean Cocteau, where we had a wonderful meal. As with Chinese meals with Chern, when we went back there later without him, it was not the same.

As a student of Chern, I got to ride on his coattails as a descendant of everyone from whom he was descended. Nowadays, we can find all the names at the website of the Mathematics Genealogy Project. Chern was a student of Wilhelm Blaschke, who was a student of Wirtinger. Skipping back for a few generations, we find Möbius, before him Pfaff; going forward from Pfaff, we come to a mathematical great, great, …uncle: Gauss.

Genealogy goes forward as well as backward, and it is again to Chern that I owe some of my own best students, including a series of graduates of Peking University. In 1983 I went to Beijing for the fourth in a series of meetings on differential geometry and differential equations, initiated by Chern to improve relations between Chinese and foreign mathematicians as China opened up to the outside world after the Cultural Revolution. There I met Jiang-Hua Lu, an undergraduate student of Qian Min. Qian had earlier come to Berkeley as Chern's guest; since I was Chern's student, he felt quite confident in sending to me not just Lu but eventually five students in all.

Despite the widespread use of genealogical terminology in academia, it is only some advisors who provide a real feeling of "family" among their students. With Chern it was a privilege to feel like part not only of his mathematical family but of his immediate family as well. His wife Shi-Ning's hospitality at home was renowned, as was Chern's reputation at the Chinese restaurants of the San Francisco Bay Area, where he seemed to know all the best chefs.

My wife, our daughter Asha, and I had the privilege in 1999 of being the Cherns' guests at a fine restaurant near the West Lake in China. Chern described it as one of Chairman Mao's favorite resorts; although he was no fan of Mao, he was not immune to the importance of such a designation. But, like the ancient Master,[1] who "acts without doing anything and teaches without saying anything", Chern had a very different style. His kind and subtle but firm leadership set an example which remains as an essential part of his legacy.

Joseph A. Wolf

Recollections of Professor S.-S. Chern

I met Chern when I was a fourth-year undergraduate at the University of Chicago. In 1955 I took Chern's course on differentiable manifolds and Riemannian geometry. It was tough going because the course was rigorous and my background was weak. At one point he called me into his office, asked me how old I was (I had just turned nineteen), asked me what I'd published (nothing), showed slight but definite disapproval, and told me that it was better to start publishing when you are young. Then he told me to read Whitney's paper on real analytic embeddings and explain it to the class. Doing that, I had my first hint of what it was to understand mathematics in an active way. I continued on as a grad student at Chicago and took courses in geometry and Lie

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[1] *From the* Tao te Ching*, translation by Stephen Mitchell, Harper and Row, 1988.*

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theory from Chern, Spanier, Kaplansky, Reinhart, Palais, and Helgason. In the meantime Chern gave me informal "homework" problems, e.g., work out the structure equations and sectional curvature for hypersurfaces obtained by rotating graphs. Then, with Chern's active encouragement, I helped organize a seminar on Riemannian symmetric spaces.

At that time the math department at Chicago was a fairly small place. There was a lot of social interaction between grad students, visitors, and faculty. In my second year as a grad student I started thesis work with Chern, and my wife Lois and I were often invited to social events at the Cherns'. I remember one such event—a buffet dinner—where everyone took chopsticks, except that Chern took a fork, explaining that it was more efficient.

Chern directed me to the papers of Preissmann and Hadamard, and soon I was working on Riemannian curvature and the structure of manifolds that satisfy various curvature conditions. I decided to solve the Clifford-Klein spherical space form problem for my thesis, but Chern gently let me know that I didn't know enough finite group theory to do that. So instead I pushed some of the then-available information about spherical space forms from spheres to some other Riemannian homogeneous spaces. This involved some Lie group theory, and at the time Chern was not fond of Lie groups, so he sent me to Princeton for a few days to talk to Borel—who, to my relief, approved of my efforts and made some useful suggestions.

Early in the 1958–59 academic year, Chern told me that he would spend the following year in Paris and would not return to Chicago. Instead, he would go to Berkeley starting in fall 1960, and I should go to Berkeley as a grad student in fall 1959 to redo language exams, qualifying exams, residency requirements, and so forth. It seemed to me that it would be more interesting to finish my thesis and follow Chern to Paris, and Lois agreed, so that worked out. The gracious hospitality of the Cherns, and Chern's suggestions for getting around in the Sorbonne and the École Polytechnique, were very helpful. Also, and this certainly smoothed the way for me, Chern introduced me to André Weil, Henri Cartan, Jean Dieudonné, Charles Ehresmann, Armand Borel, Jacques Tits, Friedrich Hirzebruch, Georges de Rham, and André Lichnerowicz as a colleague. (Weil and Dieudonné already knew me as a student, and it was quite something to be reintroduced to them as a colleague.)

I spent 1960–61 and 1961–62 at the Institute in Princeton. There most of my work was on Riemannian and pseudo-Riemannian homogeneous spaces, and I also had a lot of correspondence with Chern. In 1962, with Chern's encouragement and support, I came to Berkeley. Here in Berkeley the geometry-topology group was extremely active and cohesive, both mathematically and socially, and everyone gave parties and dinners. In 1964 I followed Chern's indirect advice to learn enough finite group theory to settle the Clifford-Klein spherical space form problem. My calculations didn't all agree with some results in Zassenhaus's thesis, so I first thought that I had it wrong, but Chern had been in Hamburg when the errors were found in Zassenhaus's thesis, and Zassenhaus had promised (but never published) a correction. With that information and encouragement from Chern, I completed Zassenhaus's thesis and settled the space form problem.

In the late 1960s and 1970s Berkeley was very political, and at the same time the field of differential geometry was fragmenting, so Chern's geometry group became less cohesive. Chern moved toward PDE, Hsiang toward transformation groups, and Wu toward potential theory. I was moving more and more into the representation theory of semisimple Lie groups. Then in the 1980s and 1990s Chern's interests were focused more and more on his Nankai Institute. As a result we saw each other less often, and then mostly in the math department. My last mathematical contact with Chern was when he and David Bao had just finished their book on Finsler spaces. Chern suggested that I look at the structure of Finsler symmetric spaces, starting with Berwald symmetric spaces. This was a remarkable insight because it turned out that, as homogeneous spaces, they were the same as Riemannian symmetric spaces. Unfortunately Chern passed away before I could tell him the result of his suggestion. To me, Chern's passing was the loss of a great colleague, friend, and mentor.

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