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of the American Mathematical Society

November 2011

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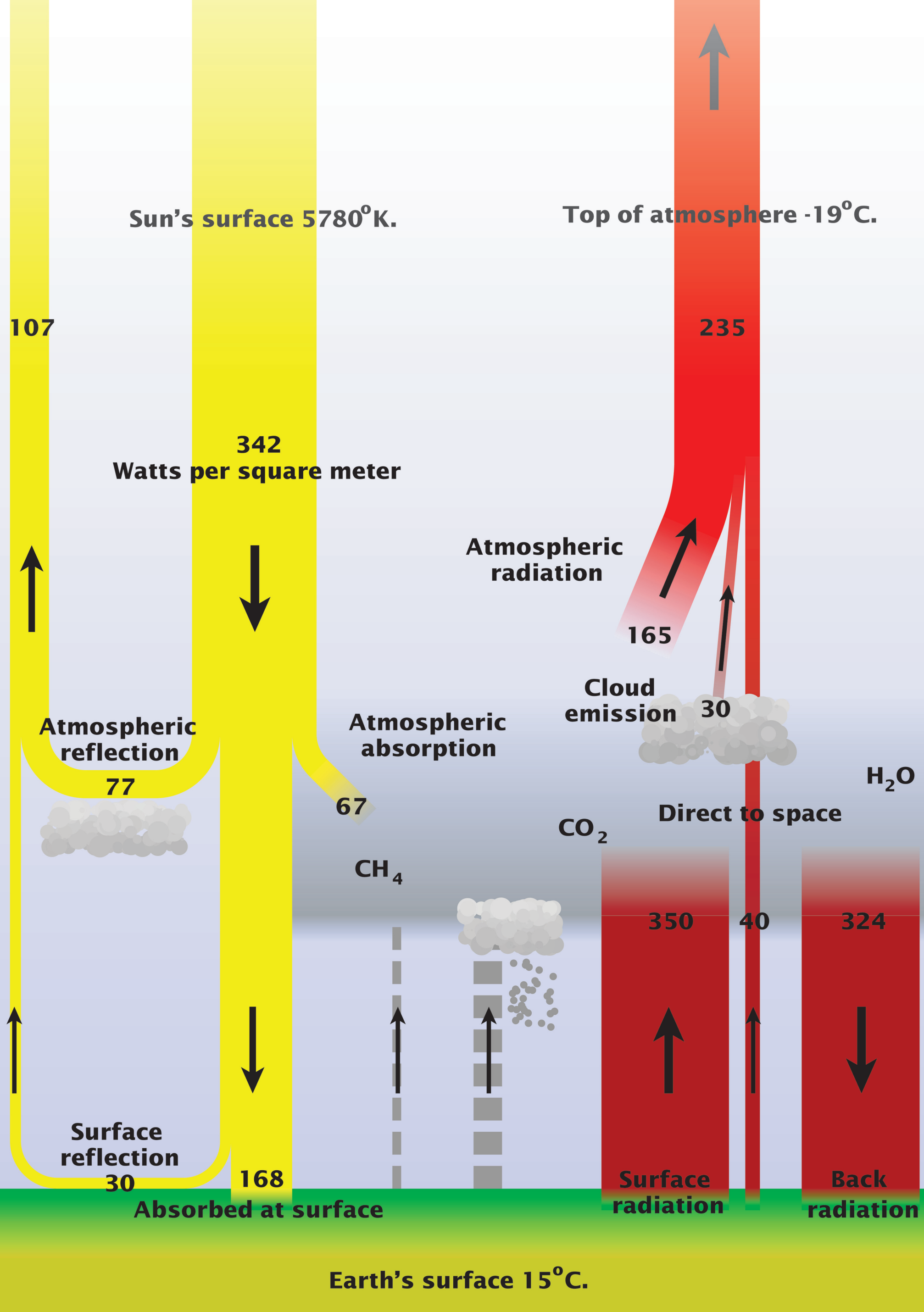

About the Cover:
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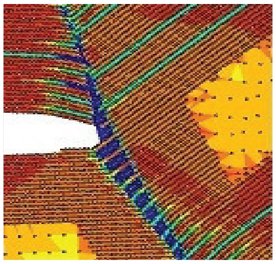
INSTITUTE FOR PURE AND APPLIED MATHEMATICS

Los Angeles, California



Materials Defects: Mathematics, Computation, and Engineering

September 10 – December 14, 2012

ORGANIZING COMMITTEE: Jiun-Shyan Chen, Co-Chair (UCLA), **Tim Schulze**, Co-Chair (Univ of Tennessee), **Vasily V. Bulatov** (Lawrence Livermore National Laboratory), **Kristen Fichthorn** (Penn State), **Nasr Ghoniem** (UCLA), **Mitchell Luskin** (Univ of Minnesota), **Michael Ortiz** (Caltech), **Vivek Shenoy** (Brown Univ), **Axel Voigt** (Technishche Universtitat Dresden)

Scientific Overview

Mathematics and computation have long played a significant role in materials science. Material defects present a huge challenge for mathematical modeling and simulation, as anything that breaks up the regular, homogeneous structure of a calculation requires special consideration. Examples include crack-propagation, dislocations, grain boundaries, impurities, shear bands and strain localization. In recent years, there has been particular focus on the multiscale nature of materials research -- how computational methods and mathematical models for describing materials vary from the atomistic to the continuum scale. Our long program will continue this trend, but with a new emphasis on defects. The science of material defects remains one of the most challenging subjects owing to its interdisciplinary nature that spans mechanics, mathematics, materials science, physics, computer science, and other scientific disciplines. This program will assemble a similarly diverse group to assess the current status of defect modeling, promote the development of new computational techniques, and stimulate new applications.

Workshop Schedule

- Tutorials: September 11-14, 2012
- Workshop 1: Quantum and Atomistic Modeling of Materials Defects, October 1-5, 2012
- Workshop 2: Atomistic and Mesoscale Modeling of Materials Defects, October 22-26, 2012
- Workshop 3: Mesoscale and Continuum Scale Modeling of Materials Defects, November 13-16, 2012
- Workshop 4: Computational Methods for Multiscale Modeling of Materials Defects, December 3 -7, 2012
- Culminating Workshop at Lake Arrowhead Conference Center, December 9-14, 2012

Participation

This long program will involve a community of senior and junior researchers. Full and partial support for long-term participants is available. We are especially interested in applicants who intend to participate in the entire program, but will consider applications for shorter periods. We have funding especially to support the participation of recent PhDs, graduate students, and other researchers in the early stages of their career, but we will consider applications from mathematicians and scientists at all levels. Encouraging the careers of women and minority mathematicians and scientists is an important component of IPAM's mission and we welcome their applications. More information and an application is available online.

www.ipam.ucla.edu/programs/md2012

NSF UCLA *IPAM is an NSF funded institute*

Institute for Pure & Applied Mathematics IPAM University of California, Los Angeles

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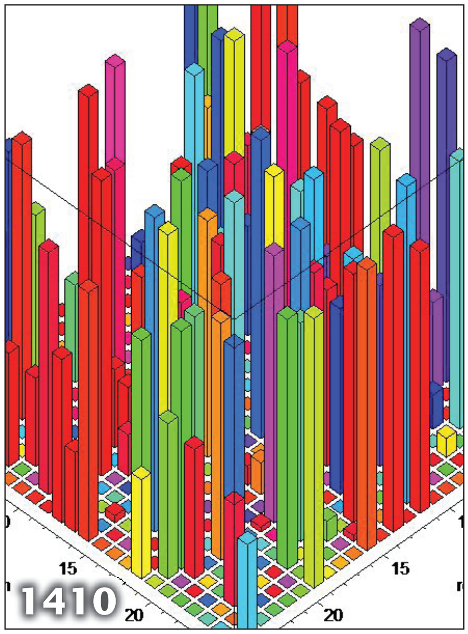
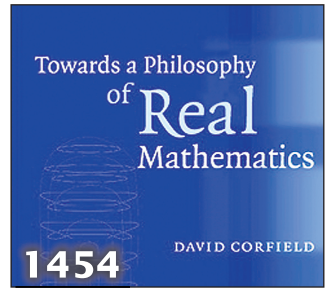
Communications

Commentary





Features

This month offers some new perspectives in mathematical thought. Goong (Gordon) Chen et al. explore mathematical models for global warming. Paulo Ferreira and Rowland Higgins discuss sampling as a scientific technique. David Bailey and Jon Borwein investigate exploratory experimentation and computation as they apply to the mathematical sciences. And Chris Sangwin discusses the evolution from problem solving in school to mathematical research. All of these provide new ways to view our subject, and new grist for our mill.

—Steven G. Krantz, Editor

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Departments

I thank Randi D. Ruden for her splendid editorial work, and for helping to assemble this issue. She is essential to everything that I do.

—Steven G. Krantz, Editor

Recommended Reading

Readers of the AMS *Notices* are kept apprised of many mathematical news items, such as Ingrid Daubechies being awarded the Benjamin Franklin Medal, the changing nature of mathematical publication, and the success of the University of Arizona's Math Center. However, they would not have read here about Nigel Kline's shocking new theory of vertical time, the impassioned public defense of her work on Newtonian Mechanics recently offered by Gabrielle Émilie Le Tonnelier de Breteuil, or the kidnapping of an innocent woman to test a quantitative theory of romantic compatibility by a Harvard graduate with a Ph.D. in mathematics and a Master's degree in human development.

Perhaps that is because these last three are not real events; each is from a recent work of fiction. (In particular, I am referring to *The Ah of Life!* by Banks Helfrich, *Emilie: La Marquise Du Châtelet Defends Her Life Tonight* by Lauren Gunderson, and *Twisted Seduction* by Dominique Adams.)

When I first announced in this publication in 1999 that I planned to compile a list of such works of mathematical fiction, I greatly underestimated the eventual size of this list. The number of entries on my "Mathematical Fiction Homepage" (`http://kasmana.people.cofc.edu/MATHFICT/`) recently reached *one thousand*, and it continues to grow rapidly. Please allow me to use this milestone as an excuse to explain why I think *Notices* readers *should* care about mathematical fiction.

The most obvious reason you should be interested in mathematical fiction is that you are likely to *enjoy* some of it, from the realistic portrayal of math research in Robert Carr's *Continuums* to the completely surrealistic self-referentiality of *How to Live Safely in a Science Fictional Universe* by Charles Yu.

A more serious reason is that fiction can be used as a *tool* for conveying complicated mathematical ideas to nonexperts. Think about Edwin Abbott Abbott's 1884 masterpiece *Flatland*, which continues to be used as a way to get people to understand the concept of higher-dimensional geometry. That is just one of many works in which an abstract mathematical concept is objectified into a (fictional) existence. Another technique for making math more accessible through fiction is to simply use the reader's interest in characters and plot to pull them through an otherwise traditional mathematics lesson. This is used quite effectively, for example, in *The Parrot's Theorem* by Denis Guedj.

Representations of math and mathematicians in fiction also offer us a glimpse of how we and our discipline are perceived by others. Occasionally, the image is flattering, such as the brilliant and entertainingly quirky mathematician in *Jurassic Park*. However, a large number of fictional mathematicians are dangerously anti-social and pathetically obsessed with trivial puzzles. To me, the least appealing image of our profession appears in Sue Woolfe's *Leaning Towards Infinity*. In that award-winning novel from 1996, participants at a math conference interrupt with meaningless objections to prevent speakers from stating their main results, taunt the only female mathematician present with cries of "I see her nipple!", and casually admit that all of the deep theorems of math are actually false.

Finally, the most *important* reason for us to be aware of mathematical fiction is that it not only reflects but actually *shapes* public opinion. To those who doubt this statement and imagine that people are too smart to let their view of reality be influenced by fiction, I offer the following evidence to the contrary:

- Much has been written about the tremendous influence that *Uncle Tom's Cabin* had on the abolitionist movement.
- One can find many biographies of modern astronomers and astronauts which cite "Star Trek" as a factor in their choice of career.
- Having little personal experience with hospitals or the legal system, I admit that my opinions about them are unduly influenced by what I saw on "E.R." and "Law and Order".

Now, consider a talented student choosing a major and a senior legislator voting on funding for math research. These hypothetical individuals are more likely to choose in favor of mathematics if they have taken to heart the TV show "NUMB3RS", whose protagonist successfully applies his remarkable knowledge of mathematics to the capture of criminals, than if their primary source of information on advanced mathematics was *Leaning Towards Infinity*. Since the health of our profession depends upon the decisions of such individuals, we ought to be interested in those things that will affect the outcome.

Some people may simply enjoy reading those misrepresentations of mathematicians that I find so frustrating, and it is not my intention to deny them that pleasure. However, I believe mathematical fiction also provides information about and even influences public opinion, which makes it a useful tool even for those who are not interested in it for their own pleasure. So, I urge you to use my website to familiarize yourself with this body of literature, suggest works to add to the database, consider incorporating some fiction into your courses, and perhaps even write some mathematical fiction yourself. We cannot afford to ignore this important resource, and while you are at it you just might also find something fun to read.

—Alex Kasman
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Letters to the Editor

On the Article of Schneiderman

In a recent article, Rob Schneiderman misjudges his music-theoretical competence, making several false statements about music in general and my own work in particular.

First, my orbifolds model inefficient as well as efficient voice leading. What matters is just that voice-leading distance is relevant to the creation or perception of music, a point almost nobody disputes.

Second, I have never asserted that efficient voice leading is more than a norm or "rule of thumb". My project is to describe the circumstances under which this important norm can be satisfied.

Third, Schneiderman claims, without evidence, that the "rule of thumb" is relevant only for beginners. But this is easily testable: we can ask how often composers like Bach use efficient voice leading. (Very frequently, it turns out.) My book, *A Geometry of Music,* is filled with statistical arguments demonstrating that the norm is robust throughout Western music.

Fourth, Schneiderman seems not to realize that efficient voice leading is used to connect scales in the process of modulation. In *A Geometry of Music*, as well as in several earlier papers, I have shown that this idea helps us understand classical music, twentieth-century music, and jazz.

Fifth, Schneiderman asserts, without evidence, that efficient voice leading is relevant only when an unremarkable (or "benign") accompaniment is desired. But Part II of my book presents dozens of examples where voice-leading distance, as modeled by geometry, reveals a logic not obvious on the musical surface. There is nothing particularly unremarkable or "benign" about these passages.

Sixth, Schneiderman makes the manifestly implausible claim that "the experienced creator of music certainly hears every voice". This flies in the face of perceptual studies of "voice denumerability". Nobody follows all the voices in Ligeti's *Requiem* or Tallis's *Spem in Alium*, and very few can follow all the voices in a complex baroque fugue.

Seventh, Schneiderman contrasts "what sounds good" with the norm of efficient voice leading. But psychological studies suggest that efficient voice leading contributes to the sense of musical coherence.

I want to encourage readers to take a look at *A Geometry of Music*. The book is meant to be accessible to mathematicians and musicians both, and it is written by a practical, committed composer—someone who spends most of his life writing music, and who is constitutionally skeptical of theorizing as an end in itself.

—Dmitri Tymoczko
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(Received July 25, 2011)

Response to Tymoczko

Although I would much rather engage in a more philosophical musico-mathematical dialogue, Mr. Tymoczko's letter does present some clear examples of how our viewpoints fundamentally differ. For me, a geometric "model" would relate some mathematical and musical properties in a way that provides some transferable insight. From his work I do not see how our understanding of orbifolds significantly enhances our "understanding" of music (and the converse is certainly not true). As my essay mentions, parameterizations of possible musical states can stimulate experimentation, but the creator of music ultimately chooses what sounds right—yes, I believe that Bach, for instance, heard "every voice" of his beautiful and often complex compositions—and any musical value resulting from these choices does not have a well-defined correspondence to "logic" living in any parameter space.

I also do not share Mr. Tymoczko's belief that the notion of efficient voice leading is "important": Any statistical prevalence is explained by the elementary observation that musical tones tend to move incrementally more frequently than in large jumps; and examples of high prevalence in chorales and left-hand jazz piano accompaniments illustrate my use of the word "benign" to describe efficient voice leading, as the choral music bows to the religious lyric while the pianist's left hand provides subtle harmonic support for the featured melodic development in the right hand. My unwillingness to accept the importance of efficient voice leading is related to my general dissatisfaction with theories that parade lists of carefully chosen examples while ignoring the existence of multitudes of counterexamples: Sounds that may or may not have any amount of efficient voice leading, also may or may not be good/bad, happy/sad, soulful/trite, coherent/incoherent, interesting/boring,…, etc.

—Rob Schneiderman
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City University of New York
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(Received July 27, 2011)

On the Article of Schneiderman

In a recent *Notices* article [1], Rob Schneiderman wrote an extremely negative review of several works in the field of mathematics and music. In regard to [2] and [3], of which I was the principal author, he provides some important corrections. My descriptions of percussion scalograms and spectrograms as "objective" representations of musical data was an unfortunate choice of words. In the future, I will try to choose my words more accurately. It is possible that percussion scalograms will not end up providing an effective tool for analyzing musical rhythm. As pointed out in [2] and [3], however, our method is related to research by other workers. Their research, which was not discussed in Schneiderman's critique, may hold up even if ours does not.

For Gabor transforms, however, the probability is near zero that such a widely applied technique—in audio engineering, computer music, and even singing pedagogy (e.g., [4] and [5])—would prove to be ill-founded. Of course, it will undoubtedly be enhanced by new developments in time-frequency analysis. Some of these new developments were described in [3]. The work of the Feichtinger school, centered at NuHAG, is particularly profound both in applied and pure mathematical approaches and was extensively referenced in [3]. Much of this work is motivated by musical applications (e.g., [6]). The wide-ranging use and theoretical foundations for time-frequency methods was surveyed in [3], with over fifty references to its literature. As to the limited range of musical examples that Schneiderman faults [3] for, all I can say is that the examples were chosen to illustrate the points we wanted to make—the points that were relevant to our goal of briefly surveying over three decades of interdisciplinary work by scientists, engineers, mathematicians, and musicians, with a total of one hundred references. We leave it to *Notices* readers to judge whether Schneiderman's critique of [3] is fair and accurate.

References

[1] R. Schneiderman, Can one hear the sound of a theorem?, *AMS Notices*, August 2011.

[2] X. Cheng, J. V. Hart, and J. S. Walker, Time-frequency analysis of musical rhythm, *AMS Notices*, March 2009.

[3] G. W. Don, K. K. Muir, G. B. Volk, and J. S. Walker, Music: Broken symmetry, geometry, and complexity, *AMS Notices*, January 2010.

[4] R. Miller, *The Art of Singing*, Oxford Univ. Press, 1996.

[5] S. McCoy, *Your Voice: An Inside View. Multimedia Voice Science and Pedagogy,* Inside View Press, 2006.

[6] M. Dörfler (P.I.), AUDIOMINER project, available at `http://www.ofai.at/research/impml/projects/audiominer.html`.

—James S. Walker
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(August 10, 2011)

On Hearing the Sound of a Theorem

Rob Schneiderman's essay "Can one hear the sound of a theorem?" was an article waiting to be written. As a mathematician for many years—now retired—and a composer starting a decade before my retirement, I have always bridled at the common observation that mathematics and music are intimately related. (My answer has always been that they're related in one direction only. Mathematicians are much more likely to enjoy, and even to be talented in, music than musicians are to enjoy mathematics: anyone with talent in both areas is bound to realize at some point that mathematics is likely to afford him a better living than music….)

My musical compositions draw on no mathematics. I doubt very much that anyone hearing them, who doesn't know me, would guess that I spent most of my life as a mathematician. When I compose, I am guided by melodic, harmonic, and rhythmic considerations, and above all by the emotional content of what I write. Intuition plays a key role, of course, as it did in my mathematical research, but this would be true in any creative pursuit, whether in the arts or in the sciences. As Schneiderman's article shows, any attempt to analyze music using mathematical tools is bound to produce only trivial observations, and any attempt to create worthwhile music using mathematical tools in place of musical ones is bound to fail.

—Jacob E. Goodman
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(August 15, 2011)

Mathematical Intimidation

John Ewing's article, "Mathematical Intimidation" [May 2011 *Notices*] is right on the money. Indeed, "value added" is a more slippery notion than even his article suggests. There is no way to know just how much additional proficiency corresponds to raising a test score a given amount, such as ten points, and there is certainly no way to compare the gain in proficiency of a student whose score increases from, say, 40 to 50 with that of a student whose score increases from 70 to 80. Thus it can be totally misleading to base a comparison of teachers on the average score changes of their classes.

—Bob Stein
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(August 10, 2011)

Corrections

The email address given for author James Schwartz, "Gerhard Hochschild (1915–2010)", *Notices*, September 2010 issue, page 1082, was incorrect. The correct address is `jamesschwartz1@gmail.com`.

The Bôcher Prize citation for Gunther Uhlmann (reprinted in the April 2011 issue of the *Notices*, page 604), states that "The prize also recognizes Uhlmann's incisive work on boundary rigidity with L. Pestov and with P. Stepanov…". The name should be P. Stefanov.

—Sandy Frost

The *Notices* invites readers to submit letters and opinion pieces on topics related to mathematics. Electronic submissions are preferred (`notices-letters@ams.org`); see the masthead for postal mail addresses. Opinion pieces are usually one printed page in length (about 800 words). Letters are normally less than one page long, and shorter letters are preferred.

Exploratory Experimentation and Computation

David H. Bailey and Jonathan M. Borwein

> The authors' thesis—once controversial, but now a commonplace—is that computers can be a useful, even essential, aid to mathematical research.
>
> —Jeff Shallit

Jeff Shallit wrote this in his recent review (MR2427663) of [10]. As we hope to make clear, Shallit was entirely right in that many, if not most, research mathematicians now use the computer in a variety of ways to draw pictures, inspect numerical data, manipulate expressions symbolically, and run simulations. However, it seems to us that there has not yet been substantial and intellectually rigorous progress in the way mathematics is presented in research papers, textbooks, and classroom instruction or in how the mathematical discovery process is organized.

Mathematicians Are Humans

We share with George Pólya (1887–1985) the view [25, vol. 2, p. 128] that, while learned,

> intuition comes to us much earlier and with much less outside influence than formal arguments.

David H. Bailey is Chief Technologist of the Computational Research Department at Lawrence Berkeley National Laboratory. His email is dhbailey@lbl.gov. *This work was supported by the director, Office of Computational and Technology Research, Division of Mathematical, Information, and Computational Sciences of the U.S. Department of Energy, under contract number DE-AC02-05CH11231.*

Jonathan M. Borwein is Laureate Professor at the Centre for Computer Assisted Research Mathematics and its Applications (CARMA) at the University of Newcastle, Australia. His email address is jonathan.borwein@newcastle.edu.au.

Pólya went on to reaffirm, nonetheless, that proof should certainly be taught in school.

We turn to observations, many of which have been fleshed out in coauthored books such as *Mathematics by Experiment* [10] and *Experimental Mathematics in Action* [3], in which we have noted the changing nature of mathematical knowledge and in consequence ask questions such as "How do we teach what and why to students?", "How do we come to believe and trust pieces of mathematics?", and "Why do we wish to prove things?" An answer to the last question is "That depends." Sometimes we wish insight and sometimes, especially with subsidiary results, we are more than happy with a certificate. The computer has significant capacities to assist with both.

Smail [27, p. 113] writes:

> the large human brain evolved over the past 1.7 million years to allow individuals to negotiate the growing complexities posed by human social living.

As a result, humans find various modes of argument more palatable than others and are more prone to make certain kinds of errors than others. Likewise, the well-known evolutionary psychologist Steve Pinker observes that language [24, p. 83] is founded on

> the ethereal notions of space, time, causation, possession, and goals that appear to make up a language of thought.

This remains so within mathematics. The computer offers scaffolding both to enhance mathematical reasoning, as with the recent computation connected to the Lie group E_8 (see http://www.aimath.org/E8/computerdetails.html), and to restrain mathematical error.

Experimental Mathodology

Justice Potter Stewart's famous 1964 comment, "I know it when I see it," is the quote with which

The Computer as Crucible [13] starts. A bit less informally, by *experimental mathematics* we intend [10]:

(a) gaining insight and *intuition*;
(b) *visualizing* math principles;
(c) *discovering* new relationships;
(d) *testing* and especially *falsifying* conjectures;
(e) *exploring* a possible result to see if it *merits* formal proof;
(f) *suggesting* approaches for formal proof;
(g) *computing* replacing lengthy hand derivations;
(h) *confirming* analytically derived results.

Of these items, (a) through (e) play a central role, and (f) also plays a significant role for us but connotes computer-assisted or computer-directed proof and thus is quite distinct from *formal proof* as the topic of a special issue of the *Notices* in December 2008; see, e.g., [20].

Digital Integrity: I. For us, (g) has become ubiquitous, and we have found (h) to be particularly effective in ensuring the integrity of published mathematics. For example, we frequently check and correct identities in mathematical manuscripts by computing particular values on the LHS and RHS to high precision and comparing results—and then if necessary use software to repair defects.

As a first example, in a current study of "character sums" we wished to use the following result derived in [14]:

$$(1)\quad \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{(-1)^{m+n-1}}{(2m-1)(m+n-1)^3} \stackrel{?}{=} 4\,\mathrm{Li}_4\left(\frac{1}{2}\right)-\frac{51}{2880}\pi^4-\frac{1}{6}\pi^2\log^2(2) + \frac{1}{6}\log^4(2)+\frac{7}{2}\log(2)\zeta(3).$$

Here $\mathrm{Li}_4(1/2)$ is a polylogarithmic value. However, a subsequent computation to check results disclosed that, whereas the LHS evaluates to $-0.872929289\ldots$, the RHS evaluates to $2.509330815\ldots$. Puzzled, we computed the sum, as well as each of the terms on the RHS (sans their coefficients), to 500-digit precision, then applied the "PSLQ" algorithm, which searches for integer relations among a set of constants [16]. PSLQ quickly found the following:

$$(2)\quad \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{(-1)^{m+n-1}}{(2m-1)(m+n-1)^3} = 4\,\mathrm{Li}_4\left(\frac{1}{2}\right)-\frac{151}{2880}\pi^4-\frac{1}{6}\pi^2\log^2(2) + \frac{1}{6}\log^4(2)+\frac{7}{2}\log(2)\zeta(3).$$

In other words, in the process of transcribing (1) into the original manuscript, "151" had become "51". It is quite possible that this error would have gone undetected and uncorrected had we not been able to computationally check and correct such results. This may not always matter, but it can be crucial.

With a current research assistant, Alex Kaiser at Berkeley, we have started to design software to refine and automate this process and to run it before submission of any equation-rich paper. This semiautomated integrity checking becomes pressing when verifiable output from a symbolic manipulation might be the length of a Salinger novel. For instance, recently while studying expected radii of points in a hypercube [12], it was necessary to show the existence of a "closed form" for

$$(3)\quad J(t) := \int_{[0,1]^2}\frac{\log(t+x^2+y^2)}{(1+x^2)(1+y^2)}\,dx\,dy.$$

The computer verification of [12, Thm. 5.1] quickly returned a 100,000-character "answer" that could be numerically validated very rapidly to hundreds of places. A highly interactive process stunningly reduced a basic instance of this expression to the concise formula

$$(4)\quad J(2) = \frac{\pi^2}{8}\log 2-\frac{7}{48}\zeta(3)+\frac{11}{24}\pi\,\mathrm{Cl}_2\left(\frac{\pi}{6}\right) - \frac{29}{24}\pi\,\mathrm{Cl}_2\left(\frac{5\pi}{6}\right),$$

where Cl_2 is the *Clausen function* $\mathrm{Cl}_2(\theta) := \sum_{n\ge 1}\sin(n\theta)/n^2$ (Cl_2 is the simplest nonelementary Fourier series). Automating such reductions will require a sophisticated simplification scheme with a very large and extensible knowledge base.

Discovering a Truth

Giaquinto's [18, p. 50] attractive encapsulation—"In short, discovering a truth is coming to believe it in an independent, reliable, and rational way"—has the satisfactory consequence that a student can legitimately discover things already "known" to the teacher. Nor is it necessary to demand that each dissertation be absolutely original—only that it be independently discovered. For instance, a differential equation thesis is no less meritorious if the main results are subsequently found to have been accepted, unbeknownst to the student, in a control theory journal a month earlier—provided they were independently discovered. Near-simultaneous independent discovery has occurred frequently in science, and such instances are likely to occur more and more frequently as the earth's "new nervous system" (Hillary Clinton's term in a recent policy address) continues to pervade research.

Despite the conventional identification of mathematics with deductive reasoning, Kurt Gödel (1906–1978) in his 1951 Gibbs lecture said:

> If mathematics describes an objective world just like physics, there is no reason why inductive methods should not be applied in mathematics just the same as in physics.

He held this view until the end of his life despite—or perhaps because of—the epochal deductive achievement of his incompleteness results.

Also, we emphasize that many great mathematicians from Archimedes and Galileo—who reputedly said "All truths are easy to understand once they are discovered; the point is to discover them"—to Gauss, Poincaré, and Carleson have emphasized how much it helps to "know" the answer beforehand. Two millennia ago, Archimedes wrote, in the introduction to his long-lost and recently reconstituted *Method* manuscript:

> For it is easier to supply the proof when we have previously acquired, by the method, some knowledge of the questions than it is to find it without any previous knowledge.

Archimedes' *Method* can be thought of as an uber-precursor to today's interactive geometry software, with the caveat that, for example, the software package Cinderella actually does provide proof certificates for much of Euclidean geometry.

As 2006 Abel Prize Laureate Lennart Carleson describes in his 1966 ICM speech on his positive resolution of Luzin's 1913 conjecture (that the Fourier series of square-summable functions converge pointwise a.e. to the function), after many years of seeking a counterexample, he finally decided none could exist. He expressed the importance of this confidence as follows:

> The most important aspect in solving a mathematical problem is the conviction of what is the true result. Then it took 2 or 3 years using the techniques that had been developed during the past 20 years or so.

Digital Assistance

By *digital assistance*, we mean the use of:

(a) *integrated mathematical software* such as Maple and Mathematica, or indeed MATLAB and their open-source variants.

(b) *specialized packages* such as CPLEX, PARI, SnapPea, Cinderella, and MAGMA.

(c) *general-purpose programming languages* such as C, C++, and Fortran-2000.

(d) *Internet-based applications* such as Sloane's Encyclopedia of Integer Sequences, the Inverse Symbolic Calculator,[1] Fractal Explorer, Jeff Weeks's Topological Games, or Euclid in Java.[2]

(e) *Internet databases and facilities*, including Google, MathSciNet, arXiv, Wikipedia, MathWorld, MacTutor, Amazon, Amazon Kindle, and many more that are not always so viewed.

All entail data mining in various forms. The capacity to consult the Oxford dictionary and Wikipedia instantly within Kindle dramatically changes the nature of the reading process. Franklin [17] argues that Steinle's "exploratory experimentation" facilitated by "widening technology" and "wide instrumentation", as routinely done in fields such as pharmacology, astrophysics, medicine, and biotechnology, is leading to a reassessment of what legitimates experiment, in that a "local model" is not now a prerequisite. Thus a pharmaceutical company can rapidly examine and discard tens of thousands of potentially active agents and then focus resources on the ones that survive, rather than needing to determine in advance which are likely to work well. Similarly, aeronautical engineers can, by means of computer simulations, discard thousands of potential designs and submit only the best prospects to full-fledged development and testing.

Hendrik Sørenson [28] concisely asserts that experimental mathematics—as defined above—is following similar tracks with software such as Mathematica, Maple, and MATLAB playing the role of wide instrumentation:

> These aspects of exploratory experimentation and wide instrumentation originate from the philosophy of (natural) science and have not been much developed in the context of experimental mathematics. However, I claim that, e.g., the importance of wide instrumentation for an exploratory approach to experiments that includes concept formation also pertains to mathematics.

In consequence, boundaries between mathematics and the natural sciences and between inductive and deductive reasoning are blurred and becoming more so. (See also [2].) This convergence also promises some relief from the frustration many mathematicians experience when attempting to describe their proposed methodology on grant applications to the satisfaction of traditional hard scientists. We leave unanswered the philosophically vexing if mathematically minor question as to whether genuine mathematical experiments (as discussed in [10]) truly exist, even if one embraces a fully idealist notion of mathematical existence. It surely seems to us that they do.

[1] *Most of the functionality of the ISC, which is now housed at* `http://isc.carma.newcastle.edu.au/`, *is now built into the "identify" function of* Maple *starting with version 9.5. For example, the* Maple *command* `identify(4.45033263602792)` *returns* $\sqrt{3}+e$, *meaning that the decimal value given is simply approximated by* $\sqrt{3}+e$.

[2] *A cross-section of Internet-based mathematical resources is available at* `http://carma.newcastle.edu.au/portal/` *and* `http://www.experimentalmath.info`.

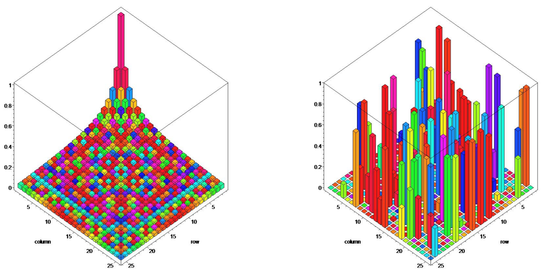

Figure 1. Plots of a 25 x 25 Hilbert matrix (L) and a matrix with 50% sparsity and random [0,1] entries (R).

Pi, Partitions, and Primes

The present authors cannot now imagine doing mathematics without a computer nearby. For example, characteristic and minimal polynomials, which were entirely abstract for us as students, now are members of a rapidly growing box of concrete symbolic tools. One's eyes may glaze over trying to determine structure in an infinite family of matrices, including

$$M_4 = \begin{bmatrix} 2 & -21 & 63 & -105 \\ 1 & -12 & 36 & -55 \\ 1 & -8 & 20 & -25 \\ 1 & -5 & 9 & -8 \end{bmatrix}$$

$$M_6 = \begin{bmatrix} 2 & -33 & 165 & -495 & 990 & -1386 \\ 1 & -20 & 100 & -285 & 540 & -714 \\ 1 & -16 & 72 & -177 & 288 & -336 \\ 1 & -13 & 53 & -112 & 148 & -140 \\ 1 & -10 & 36 & -66 & 70 & -49 \\ 1 & -7 & 20 & -30 & 25 & -12 \end{bmatrix},$$

but a command-line instruction in a computer algebra system will reveal that both $M_4^3 - 3M_4 - 2I = 0$ and $M_6^3 - 3M_6 - 2I = 0$. Likewise, more and more matrix manipulations are profitably, even necessarily, viewed graphically. As is now well known in numerical linear algebra, graphical tools are essential when trying to discern qualitative information such as the block structure of very large matrices. See, for instance, Figure 1.

Equally accessible are many matrix decompositions, the use of Groebner bases, Risch's decision algorithm (to decide when an elementary function has an elementary indefinite integral), graph and group catalogues, and others. Many algorithmic components of a *computer algebra system* are today extraordinarily effective compared with two decades ago, when they were more like toys. This is equally true of extreme-precision calculation—a prerequisite for much of our own work [8, 11, 9]. As we will illustrate, during the three decades that we have seriously tried to integrate computational experiments into research, we have experienced at least twelve Moore's law doublings of computer power and memory capacity [10, 13], which, when combined with the utilization of highly parallel clusters (with thousands of processing cores) and fiber-optic networking, has resulted in six to seven orders of magnitude speedup for many operations.

The Partition Function

Consider the number of additive partitions, $p(n)$, of a natural number, where we ignore order and zeroes. For instance, $5 = 4+1 = 3+2 = 3+1+1 = 2+2+1 = 2+1+1+1 = 1+1+1+1+1$, so $p(5) = 7$. The ordinary generating function (5) discovered by Euler is

$$\sum_{n=0}^{\infty} p(n) q^n = \prod_{k=1}^{\infty} \left(1 - q^k\right)^{-1}. \tag{5}$$

(This can be proven by using the geometric formula for $1/(1-q^k)$ to expand each term and observing how powers of q^n occur.)

The famous computation by MacMahon of $p(200) = 3972999029388$ at the beginning of the twentieth century, done symbolically and entirely naively from (5) on a reasonable laptop, took 20 minutes in 1991 but only 0.17 seconds today, while the many times more demanding computation

$$p(2000) = 4720819175619413888601432406799959512200344166$$

took just two minutes in 2009. Moreover, in December 2008, Crandall was able to calculate $p(10^9)$ in three seconds on his laptop, using the Hardy-Ramanujan-Rademacher "finite" series for $p(n)$ along with FFT methods. Using these techniques, Crandall was also able to calculate the probable primes $p(1000046356)$ and $p(1000007396)$, each of which has roughly $35{,}000$ decimal digits.

Such results make one wonder when easy access to computation discourages innovation: Would Hardy and Ramanujan have still discovered their marvelous formula for $p(n)$ if they had powerful computers at hand?

Quartic Algorithm for π

Likewise, the record for computation of π has gone from 29.37 *million* decimal digits in 1986 to over 5 *trillion* digits in 2010. Since the algorithm below was used as part of each computation, it is interesting to compare the performance in each case: Set $a_0 := 6 - 4\sqrt{2}$ and $y_0 := \sqrt{2} - 1$, then iterate

$$y_{k+1} = \frac{1 - (1 - y_k^4)^{1/4}}{1 + (1 - y_k^4)^{1/4}},$$

$$a_{k+1} = a_k(1 + y_{k+1})^4 - 2^{2k+3} y_{k+1}(1 + y_{k+1} + y_{k+1}^2). \tag{6}$$

Then a_k converges *quartically* to $1/\pi$—each iteration approximately quadruples the number of correct digits. Twenty-one full-precision iterations of (6), which were discovered on a 16K Radio Shack portable in 1983, produce an algebraic number that coincides with π to well

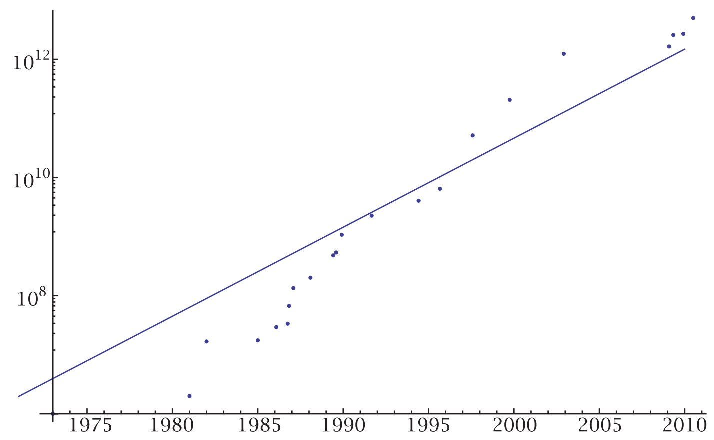

Figure 2. Plot of π calculations, in digits (dots), compared with the long-term slope of Moore's law (line).

more than 6 trillion places. This scheme and the 1976 Salamin-Brent scheme [10, Ch. 3] have been employed frequently over the past quarter century. Here is a highly abbreviated chronology (based on `http://en.wikipedia.org/wiki/Chronology_of_computation_of_pi`):

- 1986: Computing 29.4 million digits required 28 hours on one CPU of the new Cray-2 at NASA Ames Research Center, using (6). Confirmation using another algorithm took 40 hours. This computation uncovered hardware and software errors on the Cray-2. Success required developing faster FFTs [10, Ch. 3].
- January 2009: Computing 1.649 trillion digits using (6) required 73.5 hours on 1024 cores (and 6.348 Tbyte memory) of a Appro Xtreme-X3 system. This was checked with a computation via the Salamin-Brent scheme that took 64.2 hours and 6.732 Tbyte of main memory. The two computations differed only in the last 139 places.
- April 2009: Takahashi increased his record to an amazing 2.576 trillion digits.
- December 2009: Bellard computed nearly 2.7 trillion decimal digits of π (first in binary), using the Chudnovsky series given below. This took 131 days, but he later used only a single four-core workstation with lots of disk storage and even more human intelligence!
- August 2010: Kondo and Yee computed 5 trillion decimal digits using the same formula (14) due to the Chudnovskys. This was first done in binary, then converted to decimal. The binary digits were confirmed by computing 32 hexadecimal digits of π ending with position 4,152,410,118,610, using BBP-type formulas for π due to Bellard and Plouffe. Additional details are given at `http://www.numberworld.org/misc_runs/pi-5t/announce_en.html`. See also [6]. These digits appear to be "very normal".

Daniel Shanks, who in 1961 computed π to over 100,000 digits, once told Phil Davis that a billion-digit computation would be "forever impossible". But both Kanada and the Chudnovskys achieved that in 1989. Similarly, the intuitionists Brouwer and Heyting asserted the "impossibility" of ever knowing whether the sequence 0123456789 appears in the decimal expansion of π, yet it was found in 1997 by Kanada, beginning at position 17387594880. As late as 1989, Roger Penrose ventured in the first edition of his book *The Emperor's New Mind* that we likely will never know if a string of ten consecutive sevens occurs in the decimal expansion of π. This string was found in 1997 by Kanada, beginning at position 22869046249.

Figure 2 shows the progress of π calculations since 1970, superimposed with a line that charts the long-term trend of Moore's law. It is worth noting that whereas progress in computing π exceeded Moore's law in the 1990s, it has lagged behind Moore's law in the past decade. This may be due in part to the fact that π programs can no longer employ system-wide fast Fourier transforms for multiplication (since most state-of-the-art supercomputers have insufficient network bandwidth), and so less efficient hybrid schemes must be used instead.

Digital Integrity: II. There are many possible sources of errors in these and other large-scale computations:

- The underlying formulas used might conceivably be in error.
- Computer programs implementing these algorithms, which employ sophisticated algorithms such as fast Fourier transforms to accelerate multiplication, are prone to human programming errors.
- These computations usually are performed on highly parallel computer systems, which require error-prone programming constructs to control parallel processing.
- Hardware errors may occur. This was a factor in the 1986 computation of π, as noted above.

So why would anyone believe the results of such calculations? The answer is that such calculations are always double-checked with an independent calculation done using some other algorithm, sometimes in more than one way. For instance, Kanada's 2002 computation of π to 1.3 trillion decimal digits involved first computing slightly over one trillion hexadecimal (base-16) digits. He found that the 20 hex digits of π beginning at position $10^{12}+1$ are `B4466E8D21 5388C4E014`.

Kanada then calculated these hex digits using the "BBP" algorithm [7]. The BBP algorithm for π is based on the formula

$$(7)\quad \pi = \sum_{i=0}^{\infty} \frac{1}{16^i}\left(\frac{4}{8i+1} - \frac{2}{8i+4} - \frac{1}{8i+5} - \frac{1}{8i+6}\right),$$

which was discovered using the "PSLQ" integer relation algorithm [16]. Integer relation methods find or exclude potential rational relations between vectors of real numbers. At the start of this millennium, they were named one of the top ten algorithms of the twentieth century by *Computing in Science and Engineering*. The most effective is Helaman Ferguson's PSLQ algorithm [10, 3].

Eventually PSLQ produced the formula

$$(8)\quad \pi = 4\,{}_2F_1\left(\begin{matrix}1, \frac{1}{4} \\ \frac{5}{4}\end{matrix} \middle| -\frac{1}{4}\right) + 2\tan^{-1}\left(\frac{1}{2}\right) - \log 5,$$

where ${}_2F_1\left(\begin{matrix}1, \frac{1}{4} \\ \frac{5}{4}\end{matrix} \middle| -\frac{1}{4}\right) = 0.955933837\ldots$ is a Gaussian hypergeometric function.

From (8), the series (7) almost immediately follows. The BBP algorithm, which is based on (7), permits one to calculate binary or hexadecimal digits of π beginning at an arbitrary starting point, without needing to calculate any of the preceding digits, by means of a simple scheme that does not require very high precision arithmetic.

The result of the BBP calculation was `B4466E8D21 5388C4E014`. Needless to say, in spite of the many potential sources of error in both computations, the final results dramatically agree, thus confirming (in a convincing but heuristic sense) that both results are almost certainly correct. Although one cannot rigorously assign a "probability" to this event, note that the chances that two random strings of 20 hex digits perfectly agree is one in $16^{20} \approx 1.2089 \times 10^{24}$.

This raises the following question: What is more securely established, the assertion that the hex digits of π in positions $10^{12}+1$ through $10^{12}+20$ are `B4466E8D21 5388C4E014`, or the final result of some very difficult work of mathematics that required hundreds or thousands of pages, that relied on many results quoted from other sources, and that (as is frequently the case) only a relative handful of mathematicians besides the author can or have carefully read in detail?

In the most recent computation using the BBP formula, Tse-Wo Zse of Yahoo! Cloud Computing calculated 256 binary digits of π starting at the *two quadrillionth* bit [30]. He then checked his result using the following variant of the BBP formula due to Bellard:

(9)

$$\pi = \frac{1}{64}\sum_{k=0}^{\infty}\frac{(-1)^k}{1024^k}\left(\frac{256}{10k+1} + \frac{1}{10k+1} - \frac{64}{10k+3} - \frac{4}{10k+5} - \frac{4}{10k+7} - \frac{32}{4k+1} - \frac{1}{4k+3}\right).$$

In this case, both computations verified that the 24 hex digits beginning immediately after the 500 trillionth hex digit (i.e., after the two quadrillionth binary bit) are: `E6C1294A ED40403F 56D2D764`. More recent related computations are also described in [6].

Euler's Totient Function ϕ

As another measure of what changes over time and what does not, consider two conjectures regarding $\phi(n)$, which counts the number of positive numbers less than and relatively prime to n:

Giuga's Conjecture (1950). An integer $n > 1$ is a prime if and only if $\mathcal{G}_n := \sum_{k=1}^{n-1} k^{n-1} \equiv n-1$ mod n.

Counterexamples are necessarily *Carmichael numbers*—rare birds only proven infinite in 1994—and much more. In [11, p. 227] we exploited the fact that if a number $n = p_1 \cdots p_m$ with $m > 1$ prime factors p_i is a counterexample to Giuga's conjecture (that is, satisfies $s_n \equiv n-1 \bmod n$), then for $i \neq j$ we have $p_i \neq p_j$,

$$\sum_{i=1}^{m} \frac{1}{p_i} \quad > \quad 1,$$

and the p_i form a *normal sequence*: $p_i \not\equiv 1 \mod p_j$ for $i \neq j$. Thus the presence of 3 excludes $7, 13, 19, 31, 37, \ldots$, and of 5 excludes $11, 31, 41, \ldots$.

This theorem yielded enough structure, using some predictive experimentally discovered heuristics, to build an efficient algorithm to show—over several months in 1995—that any counterexample had at least 3459 prime factors and so exceeded 10^{13886}, extended a few years later to 10^{14164}, in a five-day desktop computation. The heuristic is self-validating every time that the program runs successfully. But this method necessarily fails after 8135 primes; someday we hope to exhaust its use.

While writing this piece, one of us was able to obtain almost as good a bound of 3050 primes in under 110 minutes on a laptop computer and a bound of 3486 primes and 14000 digits in less than fourteen hours; this was extended to 3678 primes and 17168 digits in ninety-three CPU-hours on a Macintosh Pro, using Maple rather than C++, which is often orders of magnitude faster but requires much more arduous coding.

An equally hard related conjecture for which much less progress can be recorded is:

Lehmer's Conjecture (1932). $\phi(n) \mid (n-1)$ *if and only if n is prime.* He called this "as hard as the existence of odd perfect numbers."

Again, prime factors of counterexamples form a normal sequence, but now there is little extra structure. In a 1997 Simon Fraser M.Sc. thesis, Erick Wong verified the conjecture for fourteen primes, using normality and a mix of PARI, C++, and Maple to press the bounds of the "curse of exponentiality". This very clever computation subsumed the entire scattered literature in one computation but could extend the prior bound only from thirteen primes to fourteen.

For Lehmer's related 1932 question *when does* $\phi(n) \mid (n+1)$*?*, Wong showed that there are eight solutions with no more than seven factors (six-factor solutions are due to Lehmer). Let

$$\mathcal{L}_m := \prod_{k=0}^{m-1} F_k$$

with $F_n := 2^{2^n} + 1$ denoting the *Fermat primes*. The solutions are

$$2, \mathcal{L}_1, \mathcal{L}_2, \ldots, \mathcal{L}_5,$$

and the rogue pair 4919055 and 6992962672132095, but analyzing just eight factors seems out of sight. Thus in seventy years the computer allowed the exclusion bound to grow by only one prime.

Lehmer could not factor 6992962672132097 in 1932. If it had been prime, a ninth solution would exist: since $\phi(n) \mid (n+1)$ with $n+2$ prime implies that $N := n(n+2)$ satisfies $\phi(N) \mid (N+1)$. We say *could not* because the number is divisible by 73, which Lehmer—a father of much factorization literature—could certainly have discovered had he anticipated a small factor. Today discovering that

$$6992962672132097 = 73 \cdot 95794009207289$$

is nearly instantaneous, while fully resolving Lehmer's original question remains as hard as ever.

Inverse Computation and Apéry-like Series

Three intriguing formulae for the Riemann zeta function are

(10)

$$(a)\ \zeta(2) = 3\sum_{k=1}^{\infty} \frac{1}{k^2\binom{2k}{k}},\ (b)\ \zeta(3) = \frac{5}{2}\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^3\binom{2k}{k}},$$

$$(c)\ \zeta(4) = \frac{36}{17}\sum_{k=1}^{\infty} \frac{1}{k^4\binom{2k}{k}}.$$

Binomial identity (10)(a) has been known for two centuries, whereas (b)—exploited by Apéry in his 1978 proof of the irrationality of $\zeta(3)$—was discovered as early as 1890 by Markov, and (c) was noted by Comtet [3].

Using integer relation algorithms, bootstrapping, and the "Pade" function (Mathematica and Maple both produce rational approximations well), in 1996 David Bradley and one of us [3, 11] found the following unanticipated generating function for $\zeta(4n+3)$:

(11)

$$\sum_{k=0}^{\infty} \zeta(4k+3)\, x^{4k} = \frac{5}{2}\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^3\binom{2k}{k}(1-x^4/k^4)} \prod_{m=1}^{k-1}\left(\frac{1+4x^4/m^4}{1-x^4/m^4}\right).$$

Note that this formula permits one to read off an infinity of formulas for $\zeta(4n+3)$, $n > 0$, beginning with (10)(b), by comparing coefficients of x^{4k} on the LHS and the RHS.

A decade later, following a quite analogous but much more deliberate experimental procedure, as detailed in [3], we were able to discover a similar general formula for $\zeta(2n+2)$ that is pleasingly parallel to (11):

(12)

$$\sum_{k=0}^{\infty} \zeta(2k+2)\, x^{2k} = 3\sum_{k=1}^{\infty} \frac{1}{k^2\binom{2k}{k}(1-x^2/k^2)} \prod_{m=1}^{k-1}\left(\frac{1-4\,x^2/m^2}{1-x^2/m^2}\right).$$

As with (11), one can now read off an infinity of formulas, beginning with (10)(a). In 1996 the authors could reduce (11) to a finite form that they could not prove, but Almquist and Granville did a year later. A decade later, the Wilf-Zeilberger algorithm [29, 23]—for which the inventors were awarded the Steele Prize—directly (as implemented in Maple) certified (12) [10, 3]. In other words, (12) was both discovered and proven by computer.

We found a comparable generating function for $\zeta(2n+4)$, giving (10) (c) when $x = 0$, but one for $\zeta(4n+1)$ still eludes us.

Reciprocal Series for π

Truly novel series for $1/\pi$, based on elliptic integrals, were discovered by Ramanujan around 1910 [3, 10, 31]. One is:

$$(13)\quad \frac{1}{\pi} = \frac{2\sqrt{2}}{9801}\sum_{k=0}^{\infty} \frac{(4k)!\,(1103+26390k)}{(k!)^4 396^{4k}}.$$

Each term of (13) adds eight correct digits. Gosper used (13) for the computation of a then-record 17 million digits of π in 1985—thereby completing the first proof of (13) [10, Ch. 3]. Shortly thereafter, David and Gregory Chudnovsky found the following variant, which lies in the quadratic number field $Q(\sqrt{-163})$ rather than $Q(\sqrt{58})$:

(14)

$$\frac{1}{\pi} = 12\sum_{k=0}^{\infty} \frac{(-1)^k\,(6k)!\,(13591409+545140134k)}{(3k)!\,(k!)^3\, 640320^{3k+3/2}}.$$

Each term of (14) adds fourteen correct digits. The brothers used this formula several times, culminating in a 1994 calculation of π to over four billion decimal digits. Their remarkable story was told in a prizewinning *New Yorker* article [26]. Remarkably, as we already noted earlier, (14) was used again in late 2009 for the current record computation of π.

Wilf-Zeilberger at Work. A few years ago Jesús Guillera found various Ramanujan-like identities for π, using integer relation methods. The three most basic—and entirely rational—identities are:

(15)
$$\frac{4}{\pi^2} = \sum_{n=0}^{\infty}(-1)^n r(n)^5(13+180n+820n^2)\left(\frac{1}{32}\right)^{2n+1}$$

(16)
$$\frac{2}{\pi^2} = \sum_{n=0}^{\infty}(-1)^n r(n)^5(1+8n+20n^2)\left(\frac{1}{2}\right)^{2n+1}$$

(17)
$$\frac{4}{\pi^3} \overset{?}{=} \sum_{n=0}^{\infty} r(n)^7(1+14n+76n^2+168n^3)\left(\frac{1}{8}\right)^{2n+1},$$

where $r(n) := (1/2 \cdot 3/2 \cdot \cdots \cdot (2n-1)/2)/n!$.

Guillera proved (15) and (16) in tandem, by very ingeniously using the Wilf-Zeilberger algorithm [29, 23] for formally proving hypergeometric-like identities [10, 3, 19, 31]. No other proof is known, and there seem to be no like formulae for $1/\pi^N$ with $N \geq 4$. The third, (17), is almost certainly true. Guillera ascribes (17) to Gourevich, who used integer relation methods to find it.

We were able to "discover" (17) using thirty-digit arithmetic, and we checked it to five hundred digits in 10 seconds, to twelve hundred digits in 6.25 minutes, and to fifteen hundred digits in 25 minutes, all with naive command-line instructions in Maple. But it has no proof, nor does anyone have an inkling of how to prove it; especially, as experiment suggests, since it has no "mate" in analogy to (15) and (16) [3]. Our intuition is that if a proof exists, it is more a verification than an explication, and so we stopped looking. We are happy just to "know" that the beautiful identity is true (although it would be more remarkable were it eventually to fail). It may be true for no good reason—it might just have no proof and be a very concrete Gödel-like statement.

In 2008 Guillera [19] produced another lovely pair of third-millennium identities—discovered with integer relation methods and proved with creative telescoping—this time for π^2 rather than its reciprocal. They are

(18)
$$\sum_{n=0}^{\infty}\frac{1}{2^{2n}}\frac{\left(x+\frac{1}{2}\right)_n^3}{(x+1)_n^3}\,(6(n+x)+1) = 8x\sum_{n=0}^{\infty}\frac{\left(\frac{1}{2}\right)_n^2}{(x+1)_n^2},$$

and

(19)
$$\sum_{n=0}^{\infty}\frac{1}{2^{6n}}\frac{\left(x+\frac{1}{2}\right)_n^3}{(x+1)_n^3}\,(42(n+x)+5) = 32x\sum_{n=0}^{\infty}\frac{\left(x+\frac{1}{2}\right)_n^2}{(2x+1)_n^2}.$$

Here $(a)_n = a(a+1)\cdot\cdot(a+n-1)$ is the *rising factorial*. Substituting $x = 1/2$ in (18) and (19), he obtained, respectively, the formulae

$$\sum_{n=0}^{\infty}\frac{1}{2^{2n}}\frac{(1)_n^3}{\left(\frac{3}{2}\right)_n^3}\,(3n+2) = \frac{\pi^2}{4},$$

$$\sum_{n=0}^{\infty}\frac{1}{2^{6n}}\frac{(1)_n^3}{\left(\frac{3}{2}\right)_n^3}\,(21n+13) = 4\,\frac{\pi^2}{3}.$$

Formal Verification of Proof

In 1611 Kepler described the stacking of equal-sized spheres into the familiar arrangement we see for oranges in the grocery store. He asserted that this packing is the tightest possible. This assertion is now known as the Kepler conjecture and has persisted for centuries without rigorous proof. Hilbert implicitly included the irregular case of the Kepler conjecture in problem 18 of his famous list of unsolved problems in 1900—*whether there exist nonregular space-filling polyhedra?*—the regular case having been disposed of by Gauss in 1831.

In 1994 Thomas Hales, now at the University of Pittsburgh, proposed a five-step program that would result in a proof: (a) treat maps that only have triangular faces; (b) show that the face-centered cubic and hexagonal-close packings are local maxima in the strong sense that they have a higher score than any Delaunay star with the same graph; (c) treat maps that contain only triangular and quadrilateral faces (except the pentagonal prism); (d) treat maps that contain something other than a triangular or quadrilateral face; and (e) treat pentagonal prisms.

In 1998 Hales announced that the program was now complete, with Samuel Ferguson (son of mathematician-sculptor Helaman Ferguson) completing the crucial fifth step. This project involved extensive computation, using an interval arithmetic package, a graph generator, and Mathematica. The computer files containing the source code and computational results occupy more than three Gbytes of disk space. Additional details, including papers, are available at `http://www.math.pitt.edu/~thales/kepler98`. For a mixture of reasons—some more defensible than others—the *Annals of Mathematics* initially decided to publish Hales's paper with a cautionary note, but this disclaimer was deleted before final publication.

Hales [20] has now embarked on a multiyear program to certify the proof by means of computer-based formal methods, a project he has named the "Flyspeck" project. As these techniques become better understood, we can envision a large number of mathematical results eventually being confirmed by computer, as instanced by other articles in the same issue of the *Notices* as Hales's article.

Limits of Computation

A remarkable example is the following:

(20)
$$\int_0^\infty \cos(2x) \prod_{n=1}^\infty \cos(x/n)\, \mathrm{d}x = 0.392699081698724154807830422909937860524646\underline{5434187231595926}\ldots$$

The computation of this integral to high precision can be performed using a scheme described in [5]. When we first did this computation, we thought that the result was $\pi/8$, but upon careful checking with the numerical value

$$0.392699081698724154807830422909937860524646\underline{174921888227621}\ldots,$$

it is clear that the two values disagree beginning with the forty-third digit!

Richard Crandall [15, §7.3] later explained this mystery. Via a physically motivated analysis of *running out of fuel* random walks, he showed that $\pi/8$ is given by the following very rapidly convergent series expansion, of which formula (20) above is merely the first term:

$$\text{(21)}\quad \frac{\pi}{8} = \sum_{m=0}^\infty \int_0^\infty \cos[2(2m+1)x] \prod_{n=1}^\infty \cos(x/n)\, \mathrm{d}x.$$

Two terms of the series above suffice for 500-digit agreement.

As a final sobering example, we offer the following "sophomore's dream" identity

$$\text{(22)}\quad \sigma_{29} := \sum_{n=-\infty}^{\infty} \operatorname{sinc}(n)\operatorname{sinc}(n/3)\operatorname{sinc}(n/5) \cdots \operatorname{sinc}(n/23)\operatorname{sinc}(n/29)$$
$$= \int_{-\infty}^{\infty} \operatorname{sinc}(x)\operatorname{sinc}(x/3)\operatorname{sinc}(x/5) \cdots \operatorname{sinc}(x/23)\operatorname{sinc}(x/29)\, dx,$$

(23)

where the denominators range over the odd primes, which was first discovered empirically. More generally, consider

(24)
$$\sigma_p := \sum_{n=-\infty}^{\infty} \operatorname{sinc}(n)\operatorname{sinc}(n/3)\operatorname{sinc}(n/5)\operatorname{sinc}(n/7) \cdots \operatorname{sinc}(n/p)$$
$$\stackrel{?}{=} \int_{-\infty}^{\infty} \operatorname{sinc}(x)\operatorname{sinc}(x/3)\operatorname{sinc}(x/5)\operatorname{sinc}(x/7) \cdots \operatorname{sinc}(x/p)\, dx.$$

Provably, the following is true: The "sum equals integral" identity for σ_p remains valid at least for p among the first 10176 primes but stops holding after some larger prime, and thereafter the "sum less the integral" is strictly positive, but *they always differ by much less than one part in a googolplex* = 10^{100}. An even stronger estimate is possible assuming the generalized Riemann hypothesis (see [15, §7] and [8]).

Concluding Remarks

The central issues of how to view experimentally discovered results have been discussed before. In 1993 Arthur Jaffe and Frank Quinn warned of the proliferation of not-fully-rigorous mathematical results and proposed a framework for a "healthy and positive" role for "speculative" mathematics [21]. Numerous well-known mathematicians responded [1]. Morris Hirsch, for instance, countered that even Gauss published incomplete proofs, and the fifteen thousand combined pages of the proof of the classification of finite groups raises questions as to when we should certify a result. He suggested that we attach a label to each proof—e.g., "computer-aided", "mass collaboration", "constructive", etc. Saunders Mac Lane quipped that "we are not saved by faith alone, but by faith and works," meaning that we need both intuitive work and precision.

At the same time, computational tools now offer remarkable facilities to confirm analytically established results, as in the tools in development to check identities in equation-rich manuscripts, and in Hales's project to establish the Kepler conjecture by formal methods.

The flood of information and tools in our information-soaked world is unlikely to abate. We have to learn and teach judgment when it comes to using what is possible digitally. This means mastering the sorts of techniques we have illustrated and having some idea why a software system does what it does. It requires knowing when a computation is or can—in principle or practice—be made into a rigorous proof and when it is only compelling evidence or is entirely misleading. For instance, even the best commercial linear programming packages of the sort used by Hales will not certify any solution, though the codes are almost assuredly correct. It requires rearranging hierarchies of what we view as hard and as easy.

It also requires developing a curriculum that carefully teaches experimental computer-assisted mathematics. Some efforts along this line are already under way by individuals including Marc Chamberland at Grinnell (`http://www.math.grin.edu/~chamberl/courses/MAT444/syllabus.html`), Victor Moll at Tulane, Jan de Gier in Melbourne, and Ole Warnaar at the University of Queensland.

Judith Grabiner has noted that a large impetus for the development of modern rigor in mathematics came with the Napoleonic introduction of regular courses: lectures and textbooks force a precision and a codification that apprenticeship obviates. But it will never be the case that quasi-inductive mathematics supplants proof. We need to find a new equilibrium. That said, we are only beginning to tap new ways to enrich mathematics. As Jacques Hadamard said [25]:

> The object of mathematical rigor is to sanction and legitimize the conquests of intuition, and there was never any other object for it.

Never have we had such a cornucopia of ways to generate intuition. The challenge is to learn how to harness them, how to develop and how to transmit the necessary theory and practice. The Priority Research Centre for Computer Assisted Research Mathematics and its Applications (CARMA), `http://carma.newcastle.edu.au/`, which one of us directs, hopes to play a lead role in this endeavor: an endeavor which in our view encompasses an exciting mix of exploratory experimentation and rigorous proof.

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Greenhouse Gas Molecules: A Mathematical Perspective

Goong Chen, Jaan Laane, Steven E. Wheeler, Zhigang Zhang

Introduction

Climate change and global warming have drawn worldwide attention in the new century. Concern about possible global warming and the controversy over the claim that it is mainly caused by the greenhouse effect provide motivation for better understanding the mathematics, physics, and chemistry of greenhouse gases. The molecules of these gases *trap heat in the form of infrared radiation*, causing the atmospheric temperature to rise. But which molecules are the greenhouse gases, and just how do they trap heat? In this article, we wish to give readers of the *Notices* some flavor of the mathematics involved so that we can understand and appreciate why mathematics is important and useful in chemistry and physics in the understanding of greenhouse gas molecules.

The greenhouse effect was first suggested by an unlikely hero, the mathematician Joseph Fourier (1768–1830) in 1824 [13, 27, 28]. He observed from the then available experimental data that the Earth gets most of its energy from solar radiation. He asked a simple question: What determines the average temperature of a planet like the Earth? From his calculations, he recognized that gases in the atmosphere were responsible for absorbing the radiation and warming the Earth and that, without them, the Earth would be much colder. He called the phenomenon "*un effet de verre*", which means "an effect of glass". Read more in Box 1.

The effect was reliably experimentally studied by the British physicist John Tyndall (1820–1893) in 1858 and then published quantitatively by the Swedish chemist and Nobel laureate Svante Arrhenius (1859–1927) in 1896. Any gas in the Earth's atmosphere capable of trapping heat is defined as a greenhouse gas. Major greenhouse gases include water vapor (H_2O), carbon dioxide (CO_2), methane (CH_4), nitrous oxide (N_2O), ozone (O_3), chlorofluorocarbons (CFC), and others. A more detailed list of greenhouse gases may be found in the Intergovernmental Panel on Climate Change (IPCC)'s list [17].

It is now well known that atoms and molecules interact with electromagnetic radiation according to quantum mechanics. This history has one of its beginnings in 1802, more than a century before the advent of quantum mechanics, when the English chemist William Hyde Wollaston (1766–1828) first observed the appearance of a number of dark features in the solar spectrum. This study was subsequently continued by great names that all enter the pantheon of science. In 1814 Joseph von Fraunhofer (1787–1826) independently rediscovered those dark lines and began a systematic study and careful measurement of the wavelength of those features. Gustav R. Kirchhoff (1824–1887) and Robert W. E. Bunsen (1811–1899) discovered that each chemical element was associated with a set of spectral lines and deduced that the dark lines in the solar spectrum were caused by absorption of those elements in the upper layers of the sun. Max

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Box 1. Joseph Fourier and the Accreditation of the Greenhouse Effect ([12])

J. R. Fleming, a historian of atmospheric science and geophysics, wrote an interesting article [12] concerning Fourier's 1827 work [13] and how Fourier got credited with the discovery of the greenhouse effect.

Fourier had lived a fascinating life overlapping the brisk period of the French Revolution and the Napoleonic wars. Fleming noted that Fourier

"…was known by his contemporaries as a friend of Napoleon, administrator, Egyptologist, mathematician and scientist whose fortunes rose and fell with the political tides. He was a mathematics teacher, a secret policeman, a political prisoner (twice), governor of Egypt, prefect of Isère and Rhône, baron, outcast, and perpetual member and secretary of the French Academy of Sciences. His reputation is largely based on his 'Fourier series'…."

Fourier's essay [13] was published in 1824 and reprinted in 1827. This work, regarded as the "first" allusion to the greenhouse effect, was merely a reprint of a descriptive memoir published three years earlier. The main emphasis of that memoir is on terrestrial temperatures rather than the atmospheric "greenhouse effect". Fleming [12] commented that "…While there are indeed greenhouse analogies in [[13] and in] Fourier's [other earlier] writings, they are not central to his theory of terrestrial temperatures, nor are they unambiguous precursors of today's theory of the greenhouse effect."

Fourier's understanding of the atmospheric absorption of solar radiation was cited by C. S. M. Pouillet as an "effect of diathermanous envelopes" in 1836 and referred to by J. Tyndall again in 1861. But S. Arrhenius was the first scientist to "officially" begin the practice of citing Fourier's 1827 reprint as the *first* mention of the greenhouse effect [1] in 1896, where he stated "Fourier maintained that the atmosphere acts like the glass of a hot-house, because it lets through the light rays of the Sun but retains the dark rays from the ground…". Many other scientists then followed this famous essay of Arrhenius by crediting the discovery of the greenhouse effect to Fourier, without going through Fourier's original work [13] themselves.

Fourier never actually used the term *serre* (French for greenhouse).

Planck (1858–1947) studied *black-body radiation* by using the idea of *quanta* and formulated his celebrated law of radiation for a black body in thermodynamic equilibrium. Wilhelm Wien (1864–1928) formulated the law that gives the frequency distribution of the emitted radiation, and the Stefan–Boltzmann law, due to Jožef Stefan (1835–1893) and Ludwig Boltzmann (1844–1906), gives the radiant intensity. Albert Einstein (1879–1955) in 1905 explained the photoelectric effect using *light quanta*, that is, *photons*.

When sunlight enters the atmosphere and strikes the Earth's surface, part of it is reflected back toward space as *infrared (IR) radiation* (heat). The sun's radiation heats the Earth. The Earth also emits thermal radiation as a black body following Planck's and Stefan–Boltzmann's laws, but at a much lower intensity because it is cooler. The Earth's overall temperature is primarily determined by the balance ([18, esp. Figure 7, p. 206]) between heating by incoming solar thermal radiation and cooling by its own outgoing thermal radiation. According to such a calculation, the Earth's surface would have an average temperature of $-18 \sim -19^\circ$C, far too cold for most living organisms to thrive. However, the greenhouse gases in the Earth's atmosphere can emit and absorb *infrared radiation*, trapping heat in the process and causing the Earth's temperature to rise by about 32°C (59°F) to the average of $14^\circ \sim 15^\circ$C, making it much more hospitable for human civilization to develop. Therefore the greenhouse effect is essential for mankind.

The Intergovernmental Panel on Climate Change (IPCC) has defined global warming to be the rise in global temperatures due to increased concentrations of greenhouse gases of anthropogenic origins since the time of the industrial revolution. There is concern that in its excess, the greenhouse effect can cause severe weather patterns, rapid melting of glaciers and polar ice caps, extinction of species, and so forth, and is therefore extremely harmful.

At a most fundamental level, the understanding of the greenhouse effect requires atomic and molecular physics and chemistry, as well as the associated mathematics: partial differential equations, group theory, and large-scale scientific computing. In fact, in order to fully understand the relevant properties of H_2O, CO_2, CH_4, and others, one needs at least a whole year's worth of graduate courses in *physical chemistry*. This will far exceed the scope of this article. Regardless of however small the amount of information we may be able to convey here is, we hope that we can arouse some interest in physical chemistry in the mathematics community.

The Schrödinger Equation with Coulomb Potentials as a Basic Model in Quantum Chemistry

Erwin Schrödinger (1887–1961) formulated his renowned equation for a single-particle system, that is, an electron, in 1926, chronicling the discovery of the wave quantum mechanics. The Schrödinger equation for (the steady states of) a single particle in a central force field may be stated as follows:

$$
\begin{aligned}
&H\psi(\boldsymbol{r}) = E\psi(\boldsymbol{r}), \boldsymbol{r} = (x_1, x_2, x_3) \in \mathbb{R}^3,\\
&H = \text{the Hamiltonian}\\
&\quad = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x_1} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2}\right) + V(\boldsymbol{r}),\\
&\hbar = h/2\pi = 1.055 \times 10^{-34} J \cdot s,\\
&\qquad \text{where } h \text{ is Planck's constant},\\
&\psi \text{ is the wave function of the particle},\\
&E = \text{energy level.}
\end{aligned}
$$

In 1927, only one year later, Schrödinger's new equation was applied to the simplest molecular systems of the hydrogen molecular ion H_2^+ by Burrau [3] and to the hydrogen molecule H_2 by Heitler and London [16] and Condon [7]. Also, Born and Oppenheimer [4] published their seminal paper dealing with molecular nuclear motions. Such a Schrödinger model provides successful explanations of the chemical, electromagnetic, and spectroscopic properties of atoms and molecules and constitutes the underlying theoretical basis for problems involving a few particles in chemical physics. Molecular quantum chemistry began to flourish and became a fascinating success story in the annals of twentieth-century science.

Assume that the system under consideration has N_1 nuclei with masses M_K and charges eZ_K, e being the electron charge, for $K = 1, 2, \ldots, N_1$, and $N_2 = \sum_{K=1}^{N_1} Z_K$ is the number of electrons. The position vector of the Kth nucleus will be denoted as $\boldsymbol{R}_K \in \mathbb{R}^3$, while that of the kth electron will be $\boldsymbol{r}_k \in \mathbb{R}^3$, for $k = 1, 2, \ldots, N_2$. Let m be the mass of the electron. See Figure 1. The kinetic energy operator of each particle is

$$
(1)\qquad
\begin{aligned}
&-\frac{\hbar^2}{2M_K}\nabla_K^2:\ \text{the } K\text{th nucleus};\\
&-\frac{\hbar^2}{2m}\nabla_k^2:\ \text{the } k\text{th electron},
\end{aligned}
$$

where

$$
\nabla_\alpha^2 = \frac{\partial^2}{\partial x_{1,\alpha}^2} + \frac{\partial^2}{\partial x_{2,\alpha}^2} + \frac{\partial^2}{\partial x_{3,\alpha}^2}, \text{ for } \alpha = K \text{ or } k,
$$

while the potential energy is the sum of the attracting and repelling Coulomb potentials between

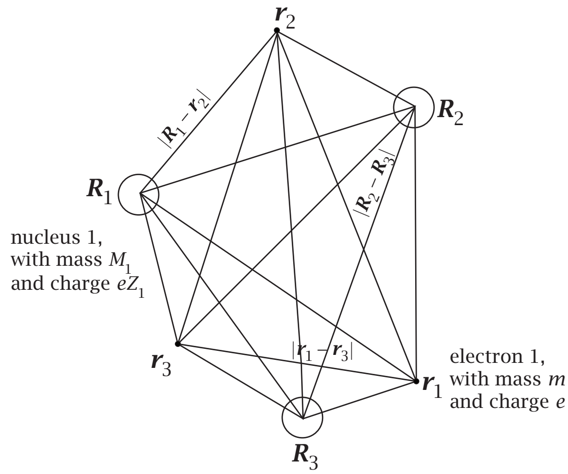


Figure 1. A possible molecular configuration. There are three nuclei, located at R_1, R_2, and R_3, and three valence electrons, located at r_1, r_2, and r_3. The interelectronic distances are $|r_i - r_j|$, $i \neq j$, such as $|r_1 - r_3|$ in the figure. The internuclear distances are $|R_i - R_j|$, $i \neq j$, such as $|R_2 - R_3|$ in the figure. The electron-nuclear distances are $|R_i - r_j|$, such as $|R_1 - r_2|$ in the figure.

electrons and nuclei, and between themselves:

$$
(2)\qquad V_{en} \equiv -\sum_{K=1}^{N_1}\sum_{k=1}^{N_2} \frac{Z_K e^2}{|\boldsymbol{R}_K - \boldsymbol{r}_k|}:
$$

electron-nucleus attracting potential;

$$
(3)\qquad V_e \equiv \frac{1}{2}\sum_{\substack{k \neq k'\\ k,k'=1}}^{N_2} \frac{e^2}{|\boldsymbol{r}_k - \boldsymbol{r}_{k'}|}:
$$

interelectronic repelling potential;

$$
(4)\qquad V_n \equiv \frac{1}{2}\sum_{\substack{K \neq K'\\ K,K'=1}}^{N_1} \frac{Z_K Z_{K'} e^2}{|\boldsymbol{R}_K - \boldsymbol{R}_{K'}|}:
$$

internuclear repelling potential.

Thus the total Hamiltonian operator H is the sum of all (1)–(4) above, and the steady-state Schrödinger equation for the overall system is given by

$$
(5)\qquad
\begin{aligned}
H\Psi(\boldsymbol{R}, \boldsymbol{r}) = \Bigg[&-\sum_{K=1}^{N_1} \frac{\hbar^2}{2M_K}\nabla_K^2 - \sum_{k=1}^{N_2}\frac{\hbar^2}{2m}\nabla_k^2\\
&+ V_{en} + V_e + V_n\Bigg]\Psi(\boldsymbol{R}, \boldsymbol{r}) = E\Psi(\boldsymbol{R}, \boldsymbol{r}),
\end{aligned}
$$

where H is the Hamiltonian and

$$
\begin{aligned}
\boldsymbol{R} &= (\boldsymbol{R}_1, \boldsymbol{R}_2, \ldots, \boldsymbol{R}_{N_1}) \in \mathbb{R}^{3N_1},\\
\boldsymbol{r} &= (\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_{N_2}) \in \mathbb{R}^{3N_2},
\end{aligned}
$$

which constitutes an eigenvalue problem for the operator H in $L^2(\mathbb{R}^{3(N_1+N_2)})$, where E represents an eigenvalue. Equation (5) is too complex for the practical purpose of studying molecular problems. Born and Oppenheimer [4] provided a *reduced order model* by approximation and separation, permitting a particularly accurate *decoupling* of the motions of the electrons and the nuclei. The main idea is to assume that Ψ in (5) takes the form of a product

$$\Psi(\boldsymbol{R},\boldsymbol{r}) = \Psi_N(\boldsymbol{R})\Psi_e(\boldsymbol{R},\boldsymbol{r}). \tag{6}$$

Substituting (6) into (5), we obtain

$$\begin{aligned}\Psi_N(\boldsymbol{R})&\left\{\left[-\sum_{k=1}^{N_2}\frac{\hbar^2}{2m}\nabla_k^2 + V_{en} + V_e\right]\Psi_e(\boldsymbol{R},\boldsymbol{r})\right\}\\ &+\left\{\left[-\sum_{K=1}^{N_1}\frac{\hbar^2}{2M_K}\nabla_K^2 + V_n\right]\Psi_N(\boldsymbol{R})\right\}\Psi_e(\boldsymbol{R},\boldsymbol{r})\\ &+\mathcal{T}_1+\mathcal{T}_2 = E\Psi_N(\boldsymbol{R})\Psi_e(\boldsymbol{R},\boldsymbol{r}),\end{aligned} \tag{7}$$

where

$$\begin{aligned}\mathcal{T}_1 &\equiv -\Psi_N(\boldsymbol{R})\sum_{K=1}^{N_1}\frac{\hbar^2}{M_K}\nabla_K\Psi_N(\boldsymbol{R})\cdot\nabla_K\Psi_e(\boldsymbol{R},\boldsymbol{r}),\\ \mathcal{T}_2 &\equiv -\Psi_N(\boldsymbol{R})\sum_{K=1}^{N_1}\frac{\hbar^2}{2M_K}\nabla_K^2\Psi_e(\boldsymbol{R},\boldsymbol{r}).\end{aligned} \tag{8}$$

The essential step in the Born–Oppenheimer separation consists of dropping $\mathcal{T}_1$ and $\mathcal{T}_2$ in (7). This leads to the separation into an equation for the *electronic wave function*

$$\begin{aligned}&\left(-\frac{\hbar^2}{2m}\sum_{k=1}^{N_2}\nabla_k^2 + V_{en} + V_e + V_n\right)\Psi_e(\boldsymbol{R},\boldsymbol{r})\\ &\quad = E_e(\boldsymbol{R})\Psi_e(\boldsymbol{R},\boldsymbol{r}),\end{aligned} \tag{9}$$

and a second equation for the *nuclear wave function*

$$\left[-\sum_{K=1}^{N_1}\frac{\hbar^2}{2M_K}\nabla_K^2 + E_e(\boldsymbol{R})\right]\Psi_N(\boldsymbol{R}) = E\Psi_N(\boldsymbol{R}), \tag{10}$$

where $E_e(\boldsymbol{R})$ is the constant of separation.

In "typical" molecules, the time scale for the valence electrons to orbit about the nuclei is about once every 10^{-15} sec (and that of the inner-shell electrons is even shorter), that of the molecular vibration is about once every 10^{-14} sec, and that of the molecule rotation is every 10^{-11} sec. This difference of time scale is what makes $\mathcal{T}_1$ and $\mathcal{T}_2$ in (8) negligible, as the electrons move so fast that they can instantaneously adjust their motions with respect to the vibration and rotation movements of the slower and much heavier nuclei.

We resolve (9) first by implementing a *geometry optimization* program to find a stable nuclei placement $\boldsymbol{R}_{eq}$ ("eq" here means "equilibrium"), which makes $E_e(\boldsymbol{R})$ a minimum in the $\mathbb{R}^{3N_1}$ space. Solutions or approximations for $\Psi_e(\boldsymbol{R}_{eq},\boldsymbol{r})$ in (9) correspond to *electronic states* of the molecule, and the corresponding values of $E_e(\boldsymbol{R}_{eq})$ correspond to *electronic energy levels*. Transitions between electronic states are typically in the ultraviolet (UV)/visible light range of the electromagnetic spectrum. We refer the readers to [6] for the visualization of electron wave functions of some simple atoms and molecules.

Example 1. The Gaussian computational-chemistry package ([14]), which uses Gaussian functions (see (18) in the section Computational Modeling of Greenhouse Gas Molecules) as a basis set, contains subroutines for performing such geometry optimizations. Gaussian was founded by John Anthony Pople (1925–2004), who received a math Ph.D. from Cambridge University in 1951 and a Nobel Prize in chemistry in 1998. Gaussian was utilized to determine the cis- and trans-1,2 dichloroethylene $C_2H_2Cl_2$ configurations shown in Figure 10, in the section Molecular Spectroscopy and Group Theory. The calculation on a laptop takes but a few seconds. The numerical results show that the cis- configuration is only about 0.2 kcal/mol (70 cm^{-1}) lower in energy than trans-dichloroethylene.

Pople was one of the very few (perhaps only two) math Ph.D.s ever to receive Nobel Prizes in chemistry (the other one being Herbert Hauptman (1917–present). He considered himself more of a mathematician than a chemist, but his contributions to theoretical and computational chemistry are regarded as major and far reaching by his chemistry peers. The 1998 Nobel Prize in chemistry was shared by Pople and Walter Kohn, the Austrian-born American physicist who led the development of *density functional theory* (DFT). DFT is accompanied by its own set of mathematical problems, ranging from efficient numerical integration techniques to the N-representability problem. DFT has become an increasingly popular way of solving the electronic structure problem in molecular applications, as well as for problems in solid-state systems. □

In studying any molecules' greenhouse effects, *the nuclear wave equation* (10) *is the focus of attention.* The molecular motions involve translations, rotations, and vibrations. Generally, the PDE (10) is replaced by an approximate equation

$$\begin{aligned}&\widetilde{H}G(\boldsymbol{R}) = EG(\boldsymbol{R}),\\ &\widetilde{H} \equiv -\sum_{K=1}^{N_1}\frac{\hbar^2}{2M_K}\nabla_J^2 + \frac{1}{2}(\boldsymbol{R}-\boldsymbol{R}_{eq})^{Tr}V_0(\boldsymbol{R}-\boldsymbol{R}_{eq}),\end{aligned} \tag{11}$$

by expansion around $\boldsymbol{R}_{eq}$ and keeping only the *quadratic terms.* Normally, $3N_1$ eigenfrequencies and eigenmodes are calculated from (11). For example, for methane CH_4, there are five atoms, that is, $N_1 = 5$, so $3N_1 = 3\times 5 = 15$. *The IR active molecular motions are the ones most strongly related to the greenhouse effects.* These IR active vibrations can be determined through the application of tools from *mathematical group theory*, as described below.

Molecular Spectroscopy and Group Theory

We learned in the Introduction that the Earth is warmed by the electromagnetic radiation shining from the sun. The electromagnetic spectrum is shown in Figure 2. As can be seen, the frequency and wavelength of light can range over fifteen orders of magnitude in going from radio frequency waves to gamma rays.

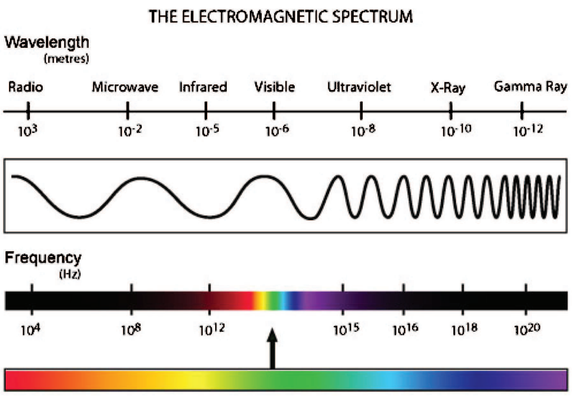


Figure 2. Electromagnetic spectrum.

The energy of each individual light particle (photon) hitting the Earth is given by Planck's relationship $E = h\nu$, where h is Planck's constant and ν is the frequency of light. Thus the higher frequency radiation (ultraviolet and visible) transmits much more energy to the Earth than does low-frequency radiation such as radio frequency. The principal components of the Earth's atmosphere (nitrogen at 78.1 percent, oxygen at 21.0 percent, and argon at 0.9 percent) *do not* absorb infrared, visible, or ultraviolet light except at very high frequencies. Thus solar radiation, which has a maximum intensity near 500 nm (nanometer, 10^{-9} meter) in the blue-green part of the visible spectrum, is diminished little by atmospheric absorption. About 20 percent is reflected by clouds and 6 percent is scattered by the atmosphere, but well over half transfers energy to the land and oceans of planet Earth. The temperature of the planet depends on how much energy in the form of electromagnetic radiation is released back into space. This released energy is to a large degree *infrared radiation*, which we commonly perceive of as heat. How much is released depends on whether the infrared light is absorbed by molecules of low abundance in the atmosphere. While nitrogen, oxygen, and atomic argon do not absorb infrared radiation, other molecular species in lower concentrations, such as water (~1 percent), carbon dioxide (0.04 percent), and methane, do absorb infrared light. Hence, their presence in air results in their absorption of emitted heat from the Earth's surface, and this produces the greenhouse effect. Consequently, it is important to understand the spectroscopic properties of these molecules.

The technique of *infrared (IR) absorption spectroscopy* is one of the most commonly used by chemists for understanding molecular properties, for determining molecular structures, for quantitative measurements, and for chemical identification. Each molecule has a unique infrared spectrum that can be regarded as the fingerprint or signature of the molecule. The spectroscopy technique in chemistry and physics is the equivalent of the *spectral theory of partial differential operators* and *inverse spectral problems* in mathematics. Figure 3 shows a diagram of a typical infrared instrument. This is usually referred to as FT-IR, where FT = Fourier transform.

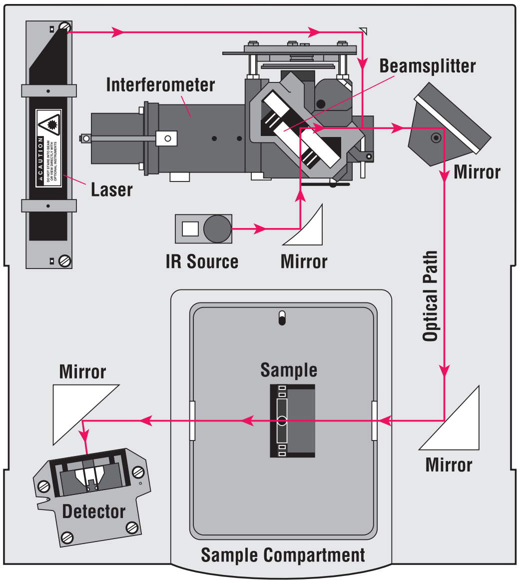


Figure 3. Layout of FT-infrared instrument excerpted from Thermo Nicolet [26]: Introduction to Fourier Transform Infrared FT-IR Spectrometry, courtesy of Thermo Fisher Scientific (Thermo Nicolet), 5225 Verona Road, Madison, WI 53711-4495. In this instrument the light from an infrared source is split into two beams by a beam splitter and then rejoined, creating an interference pattern called an interferogram, where the intensity at the detector is collected as a function of path difference between the two light beams. The interferogram can then be converted to a spectrum, intensity versus frequency, by carrying out a (digital) *Fourier transform*.

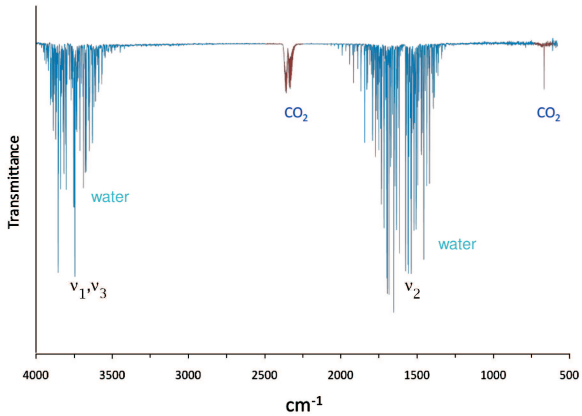


Figure 4. Infrared spectrum of air. Note the water molecular bands. By convention, the vibrations of a molecule are labeled sequentially starting with the vibrations of the first symmetry species in the character table (see Table 2), then followed by the next one, etc. For each symmetry species the highest frequency vibrations are labeled first. Thus the water symmetric stretching vibration of symmetry species A_1 at 3657 cm^{-1} is labeled as ν_1, the A_1 angle bending at 1595 cm^{-1} is labeled as ν_2, and the B_1 antisymmetric stretching at 3756 cm^{-1} is labeled ν_3. The A_1 vibrations have dipole changes along the z direction while the B_1 vibration creates a change in the charge distribution along the x direction. An A_2 vibration would not be infrared active since it does not induce any dipole change. As can be seen from the spectrum, there is not a single frequency associated with each vibration. Instead, there is a great deal of associated *fine structure*. When the vibrational quantum states of the water vibrations are excited, there is typically also a change in the rotational quantum state. These are also governed by *symmetry selection rules* (see Box 2) and give rise to the *fine structure* seen in the spectrum. The spectral absorption bands near 3600 cm^{-1} arise from an overlap of the two O-H stretching bands, ν_1 and ν_3, primarily from the latter. The band centered at 1595 cm^{-1} arises from ν_2, the angle bending. In addition to the fundamental vibrational frequencies of water (ν_1, ν_2, and ν_3), *overtones* and *combination bands* will also be present in the spectra. These will also play a role in absorption of infrared and near-infrared radiation trying to escape from the atmosphere. These will be approximately at frequencies $2\nu_1, 3\nu_1, \ldots, 2\nu_2, 3\nu_2, \ldots, 2\nu_3, 3\nu_3, \ldots, \nu_1 + \nu_2, \nu_2 + \nu_3$, etc., but these absorptions will typically be more than ten times weaker in intensity.

Box 2. The Selection Rule for Infrared Transitions

For an IR transition, i.e., absorption or emission of an IR photon, to occur, the molecule must have a change in *dipole moment* during its vibration. This can be determined by symmetry considerations. Let Ψ_i denote the initial quantum state and Ψ_f the final quantum state. The following transition moment integral R_{if} must be nonzero for an "admissible transition" to occur:

$$R_{if} \equiv \int \Psi_f^* M_\sigma \Psi_i \, d\boldsymbol{r} \equiv \langle f|\sigma|i\rangle \neq 0,$$

where "$*$" denotes the complex conjugate and $\langle \cdot|\cdot|\cdot\rangle$ is the Dirac bra-ket. The dipole moment M_σ in the σ-direction ($\sigma = x, y$, or z) has a Taylor expansion

$$M_\sigma = M_\sigma^0 + \left(\frac{\partial M_\sigma}{\partial Q}\right)\Big|_{Q=0} Q + \left(\frac{\partial^2 M_\sigma}{\partial Q^2}\right)\Big|_{Q=0} Q^2 + \cdots,$$

where Q is a (generalized) vibrational coordinate. Then

$$R_{if} = \langle f|\sigma|i\rangle = M_\sigma^0 \langle f|i\rangle + \left(\frac{\partial M_\sigma}{\partial Q}\right)\Big|_{Q=0} \langle f|Q|i\rangle + \cdots.$$

It can readily be shown that R_{if} is nonzero only if the product of the symmetries of Ψ_i, Q, and Ψ_f is totally symmetric where all the characters for the symmetry species are $+1$ (A_1 for C_{2v} symmetry, for example). For vibrations the initial state is totally symmetric and the symmetry species for Q is the same as the translation operators T_x, T_y, or T_z. Thus for R_{if} to be nonzero, Ψ_f must have the same symmetry species as T_x, T_y, or T_z. This is because the direct product of a symmetry species with itself is totally symmetric (e.g., $B_1 \times B_1 = A_1$ for C_{2v}). This provides a simple way to identify infrared active vibrations since they are those that have an x, y, z at the right-hand side of a character table. For C_{2v} (water vapor) these are A_1, B_1, and B_2.

Figure 4 shows the infrared spectrum of air at one atmosphere of pressure over a path length of about one meter. The transmittance is shown as a function of wavenumber $\bar{\nu}$, in cm^{-1}, where $\bar{\nu}$ is the reciprocal of the wavelength $\bar{\nu} = 1/\lambda = \nu/c$ where c is the speed of light. The wavenumber is directly proportional to the energy of light since $E = h\nu = h\bar{\nu}c$. The air spectrum shows only absorption by water and carbon dioxide, although over much longer path lengths of light travel, trace components such as *methane* can also absorb significant amounts of light.

A key tool in the analysis of molecular spectra is symmetry point groups. A list of common symmetry point groups with their symmetry elements may be found in Table 1. These can be used to determine properties of molecular vibrations.

Symmetry and the application of mathematical group theory allow us to understand the characteristics of molecular vibrations, and hence how the molecular motions of water, carbon dioxide, and methane result in the absorption of infrared light. We will first consider the molecular motions of water; see Figure 5. Since it has three atoms ($N = 3$), it requires $3N = 9$ coordinates to fully describe its motions. Of these, three will be translations (T_x, T_y, T_z) and three will be rotations (R_x, R_y, R_z). The remaining three will be molecular vibrations, of which two are IR active. The symmetry point group is C_{2v} [8], whose characters and irreducible representations are well understood; see Table 2. Next, the "reducible representation" must be determined and broken down to the irreducible representation which characterizes the nine molecular motions, including the vibrations with symmetry $2A_1 + B_1$; see Box 3. Technical details are omitted.

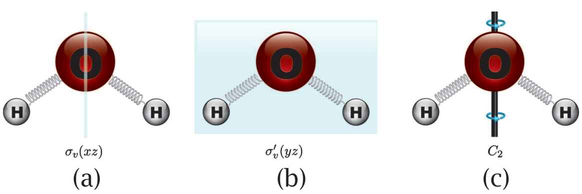


Figure 5. Symmetry elements of water. It has two reflection planes, identified with σ_v *operations*, each of which can cut the molecule in half, leaving two identical sides; see (a) and (b). One is the plane of the molecule, the other is a plane perpendicular to the molecule through the oxygen atom. The third symmetry operation is a C_2 *proper rotation* about an axis through the oxygen atom about which the molecule can be rotated by 180°, leaving it in an identical configuration; see (c).

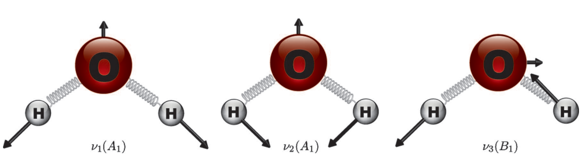


Figure 6. Vibrations of the water molecule.

The ozone molecule, O_3, is similar to water in that it is a bent triatomic molecule of C_{2v} symmetry. Hence its vibrations have the same

Box 3. Character Table and Molecular Symmetries

We use Table 2 to illustrate the construction and interpretation of the character table for the C_{2v} symmetry point group of the water molecule. According to the fundamental *Schur's lemma* in the representation theory of finite groups (see, e.g., classics by Curtis and Reiner [10] and Serre [21]), matrix elements of irreducible representations of a finite group satisfy certain orthogonality relations, called the *Great Orthogonality Theorem* ([8, Section 4.3]). The trace (i.e., sum of all diagonal entries of a matrix representation) is called a *character*. The number of irreducible representations of a finite group is equal to the number of conjugacy classes of that group. Each character takes a constant value on a conjugacy class.

In Table 2, there are a total of four conjugacy classes represented by four symmetry operations—E (identity), C_2 (proper rotation by 180°), $\sigma_v(xz)$ (reflection about the xz-plane) and $\sigma'_v(yz)$ (reflection about the yz-plane). In this case, each conjugacy class is a singleton. One can see that the column vectors (not including the very bottom row $(9, -1, 3, 1)$ in Table 2) are all orthogonal.

The symmetry species A_1, A_2, B_1, and B_2 each characterize symmetry properties of a specific molecular motion (A, when rotation about the principal axis is symmetrical; B, when antisymmetrical).

For the characters in the table, "1" indicates a symmetric motion with respect to a symmetry operation, while "-1" indicates an *antisymmetric* motion. "2" (from $+1+1$) (such as in Table 3), "-2" (from $-1-1$) and 0 (from $+1-1$) characters may exist for doubly degenerate (i.e., multiplicity 2) motions of equal energy.

The x, y, z symbols indicate symmetry species for *translations* along these directions. The R_x, R_y, R_z symbols indicate symmetry species for rotations about these axes.

The $x^2, y^2, z^2, xy, yz, xz$ symbols indicate symmetry species for tensor elements which provide Raman activity (spectroscopy) as the Raman effect is a *2-step process*.

The last row of Table 2, Γ (reducible representation) signifies *all* the motions of a molecule (translations, rotations, and vibrations). It can be broken down into the symmetry species of the individual motions. These calculations are somewhat more involved and hence omitted.

Table 1. Summary of Common Point Groups

Point Group	Important Symmetry Elements	Order of the Group
C_1	E (identity)	1
C_i	i (inversion center)	2
C_s	σ (reflection about a plane of symmetry)	2
C_n	C_n (rotation by an angle of $2\pi/n$)	n
C_{nv}	C_n, σ_v (σ_v is reflection about a vertical plane)	$2n$
C_{nh}	C_n, σ_h (σ_h is reflection about a horizontal plane)	$2n$
D_{nd}[†]	$C_n, \perp C_2, \sigma_d$ (also S_{2n})	$4n$
D_{nh}	$C_n, \perp C_2, \sigma_h$	$4n$
$C_{\infty v}$	linear molecules without center of inversion	∞
$D_{\infty h}$	linear molecules with center of inversion	∞
T_d	tetrahedral symmetry	24
O_h	octahedral symmetry	48

[†]The axis of rotation of $\perp C_2$ is perpendicular to that of C_n. Each S_{2n} operation is a C_{2n} operation followed by a σ_h operation. Note that here σ_d is reflection about a dihedral plane.

Table 2. The Character Table for the C_{2v} Symmetry Point Group (for H_2O) and the Reducible Representation

symmetry point group — symmetry operations — tensor elements (for Raman effects, etc.)

C_{2v}	E	C_2	$\sigma_v(xz)$	$\sigma'_v(yz)$		
A_1	1	1	1	1	z	x^2, y^2, z^2
A_2	1	1	−1	−1	R_z	xy
B_1	1	−1	1	−1	x, R_y	xz
B_2	1	−1	−1	1	y, R_x	yz
Γ	9	−1	3	1		

symmetry species; reducible representation (Γ); characters; translational and rotational symmetries

descriptions and the same symmetry species. Namely, its symmetric stretching ν_1 and its bending ν_2 have A_1 symmetry while its antisymmetric stretching ν_3 has B_1 symmetry. The vibrations are all infrared active and the corresponding wavenumbers are 1135, 716, and 1089 cm^{-1}, respectively.

We now turn to examining the vibrations of carbon dioxide, which is a *linear molecule* with all three nuclei in a straight line. Linear molecules have only two rotations since there is no rotational moment of inertia about the molecular axis. Hence linear molecules in general have $3N-5$ vibrations, and CO_2 has four (i.e., $3N-5 = 3\cdot 3-5 = 4$). The symmetry point group for this molecule is $D_{\infty h}$, and its four vibrations have symmetry species (whose symbols are borrowed from angular momentum theory, $\Sigma, \Pi, \Delta, \ldots$, with subscript g denoting the German "gerade" or even, and u "ungerade" or uneven):

$$\Sigma_g^+ + \Sigma_u^+ + \Pi_u.$$

The Π_u symmetry species is doubly degenerate (i.e., multiplicity 2), meaning that there are two vibrations of exactly the same frequency. These are the two bending vibrations of the O=C=O angle which can bend away from 180° in either the x or y direction (where z is the molecular axis). The Σ_g^+ vibration is the symmetric stretching vibration of the C=O bonds where both stretch simultaneously. This vibration is infrared inactive but is observed in Raman spectra. The other vibrations are Raman inactive but do give rise to infrared absorptions. The Σ_u^+ antisymmetric stretch is at 2349 cm^{-1} while the bending is at 667 cm^{-1}. Both can be seen in the spectrum of air in Figure 4, although rotational fine structure complicates the band structure. These vibrations are depicted in Figure 7.

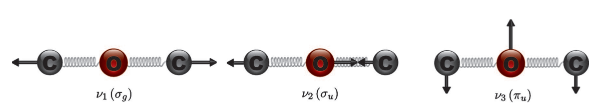


Figure 7. Vibrations of carbon dioxide.

The methane molecule CH_4 is *tetrahedral* and belongs to the T_d symmetry point group (in

Table 3. Character Table for T_d Tetrahedral Symmetry Point Group (for Methane CH_4) along with Reducible Representation

T_d	E	$8C_3$	$3C_2$	$6S_4$	$6\sigma_d$		
A_1	1	1	1	1	1		$x^2+y^2+z^2$
A_2	1	1	1	−1	−1		
E	2	−1	2	0	0		$(2z^2-x^2-y^2, x^2-y^2)$
T_1	3	0	−1	1	−1	(R_x, R_y, R_z)	
T_2	3	0	−1	−1	1	(x, y, z)	(xy, xz, yz)
Γ	15	0	−1	−1	3		

Table 4. Characters for the Reducible Representation for *cis*-1,2-dichloroethylene of C_{2v} Symmetry

C_{2v}	E	C_2	$\sigma_v(xz)$	$\sigma_v(yz)$
Γ	18	0	6	0

Table 5. Character Table and Reducible Representation for *trans*-1,2-dichloroethylene of C_{2h} Symmetry

C_{2h}	E	C_2	i	σ_h		
A_g	1	1	1	1	R_z	x^2, y^2, z^2, xy
B_g	1	−1	1	−1	R_x, R_y	xz, yz
A_u	1	1	−1	−1	z	
B_u	1	−1	−1	1	x, y	
Γ	18	0	0	6		

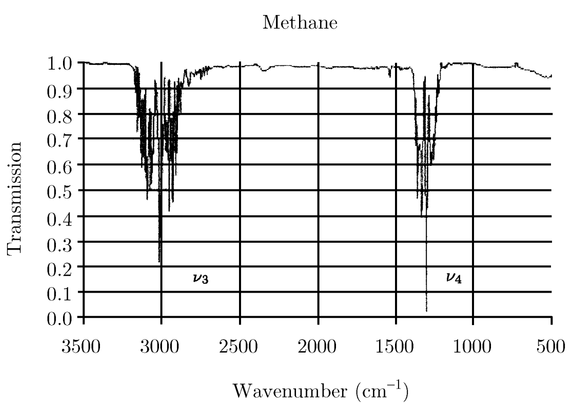


Figure 8. Infrared absorption spectrum of methane.

Table 1). All of the atoms are symmetrically equivalent, and the H-C-H angles are all tetrahedral at about 109.5°. The T_d character table is shown in Table 3. Thus the infrared spectrum of methane, Figure 8, only has infrared absorption for ν_3, the antisymmetric C-H stretching at 3020 cm^{-1}, and for ν_4, the $H-C-H$ angle bending at 1306 cm^{-1} as shown in Figure 9. Again, rotational fine structure can be observed for each vibrational band.

The power of symmetry and group theory can be demonstrated by considering the spectra of isomeric forms of the same molecule. These are molecules with the same formula and with the same bonds, but with different geometrical arrangements so that the symmetry point groups are typically different. A nice example of this (which, though, is not regarded as a greenhouse gas) involves *cis*- and *trans*-1,2-dichloroethylene (chemical formula $C_2H_2Cl_2$) shown in Figure 10.

The C_{2v} character table for the *cis* molecule is shown in Table 2, and its reducible representation is shown in Table 4. The C_{2h} character table for the *trans* molecule is shown in Table 5, along with its reducible representations Γ.

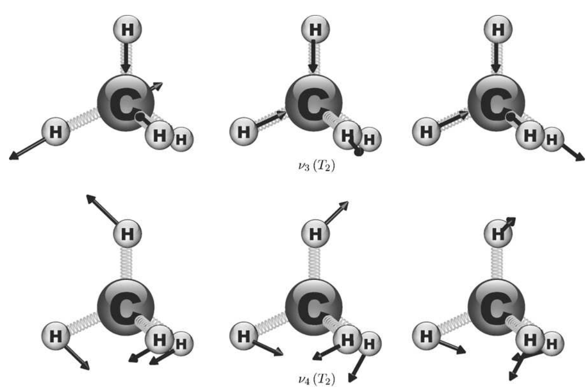


Figure 9. The infrared active vibrations of methane, ν_3 and ν_4.

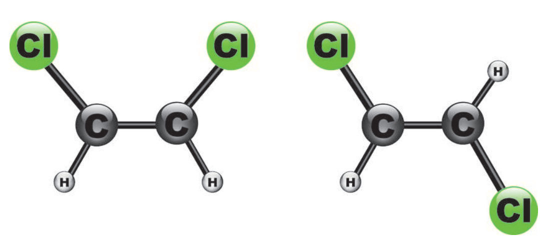


Figure 10. *cis*-1,2-dichloroethylene (left) and *trans*-1,2-dichloroethylene (right).

When the reducible representations are broken down to the irreducible ones and the rotational

Table 6. Expected numbers of spectral bands of each type for isomers of 1,2-dichloroethylene

Isomer	Symmetry	Raman bands	Raman polarized	Infrared bands
cis	C_{2v}	12	5	10
trans	C_{2h}	6	5	6

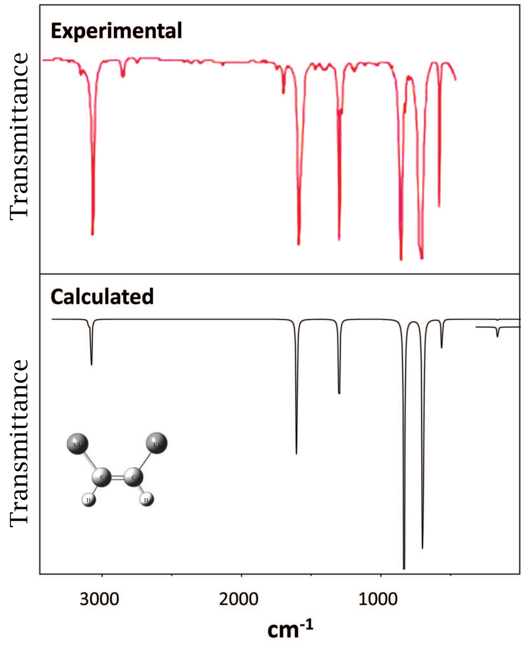
and translational motions are subtracted out, we have for the *cis* isomer

$$\Gamma_{cis}(C_{2v}) = \underset{IR,R(P)}{5A_1} + \underset{R}{2A_1} + \underset{IR,R}{4B_1} + \underset{IR,R}{B_2},$$

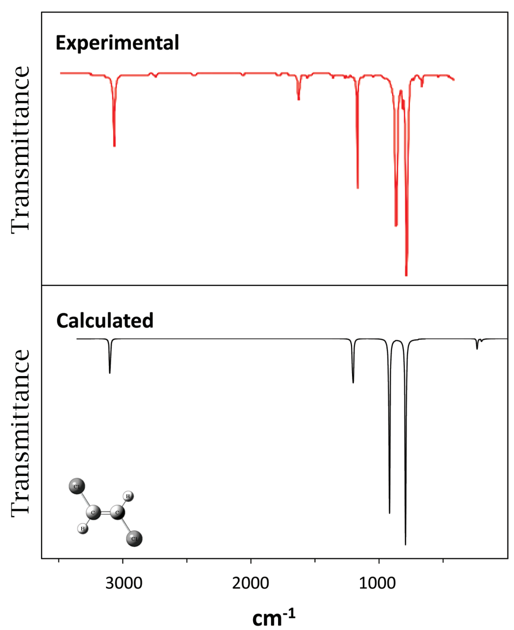
where the IR and R below each symmetry species indicate which ones are infrared and Raman active, respectively. $R(P)$ indicates symmetry species which are "Raman polarized" and can be experimentally distinguished from those Raman bands that are not polarized. For the *trans* isomer, we have

$$\Gamma_{trans}(C_{2h}) = \underset{R(P)}{5A_g} + \underset{R}{B_g} + \underset{IR}{2A_u} + \underset{IR}{4B_u}.$$

Note that for C_{2h} symmetry the molecule follows the "mutual exclusion rule" for molecules with an inversion center—namely, no vibration is both infrared and Raman active. Table 6 summarizes the expected spectral features for the two isomers, and these can be seen to be very different from each other. We have computed the infrared and Raman spectra for each and compared these to the experimental spectra using the computational approaches to be discussed in the next section. Figure 11(a) compares the experimental infrared spectrum [19] to the one that we have calculated for the *cis* isomer, and Figure 11(b) does the same for the *trans* isomer. There appears to be an extra band in the experimental spectrum of the *trans* isomer near 1700 cm^{-1}, but this is a combination band of the type not predicted by the computation of fundamental vibrations. The intensities of the spectral bands are quite different from each other, and in the figures some of the bands are shown to go off scale so that the weaker bands can be seen. As can be seen from Figure 11, the infrared spectra are dramatically different from each other and reflect the importance of symmetry in determining the spectral properties.



(a) *cis*-1,2-dichloroethylene



(b) *trans*-1,2-dichloroethylene

Figure 11. Comparison of the experimental and calculated infrared spectra.

Remark 1. As a conclusion of this section, we can now explain, in a simplistic way, how greenhouse gases trap heat in the atmosphere [28]:

(1) About 50% of the sun's incoming radiation is absorbed by the Earth's surface. This absorbed energy warms Earth's surface.
(2) The earth re-radiates outwardly a bulk of the energy as a *black body*, by way of infrared photons with wavelength $4 \sim 100\mu m$.
(3) Greenhouse gases in various layers of the atmosphere absorb (or, "intercept") the outward infrared radiation. The absorbed infrared photons can also be emitted through a spontaneous

absorption/emission process, in *all* directions, both upwards and downwards. Thus, greenhouse gases serve as a "heat reservoir". They hinder the escape of thermal radiation into the outer space.

(4) Intermolecular collisions between all gas molecules pass on the energy absorbed and emitted by the greenhouse gases to the other gases, further increasing the kinetic energy of all gases and raising the temperature of the atmosphere. □

Computational Modeling of Greenhouse Gas Molecules

Computational chemistry is an active, highly intensive area of large-scale modern computational science. Tools of *computational quantum chemistry*, which comprise families of methods to approximately solve (9) and (10), have advanced to the point that vibrational frequencies can be predicted with high accuracy for many small molecules, including greenhouse gas molecules. This can be seen in Figure 11 for two isomers of dichloroethylene. Computational chemistry can thus provide complementary information to molecular spectroscopy through studies of species that are difficult to isolate in the lab or by aiding in the assignment and interpretation of experimentally measured molecular spectra.

In computational quantum chemistry there are two broad families of methods: *ab initio* molecular orbital theory, in which approximate wave functions and energies satisfying (9) are constructed, and *density functional theory (DFT)* (cf. Example 1 in the section The Schrödinger Equation with Coulomb Potentials as a Basic Model in Quantum Chemistry, and see, e.g., [11] for DFT), in which $E_e(R)$ is computed indirectly from the electron density rather than the electronic wave function. Although DFT has become increasingly popular in molecular applications of quantum chemistry, for lack of space we will discuss only the former. In ab initio molecular orbital theory, one starts from a *Hartree–Fock (HF)* wave function (discussed below) and then builds increasingly sophisticated wave functions. These methods provide a means of systematically approaching the exact solution to (9) and include *configuration interaction (CI)* [23], *coupled cluster theory (CC)* [9] and *Moller–Plesset perturbation theory (MPn)*. Each of these methods has its advantages, although for high-accuracy predictions coupled-cluster methods are the most robust theoretical approach.

The simplest physically meaningful way of solving (9) is via Hartree–Fock (HF) theory (see, e.g., [24]), due to Douglas R. Hartree (1897–1958), Vladimir Fock (1898–1974), and John C. Slater (1900–1976). HF theory forms the basis of a large family of theoretical methods. In Hartree–Fock theory the wave function is written as a single antisymmetrized product of one-electron functions (*molecular orbitals*, MOs, ϕ_i) [24]. Each of these molecular orbitals holds two electrons with opposite spins. In general, these orbitals will not only be functions of the spatial coordinates of the electron but will also depend on spin, denoted collectively by $\boldsymbol{x}_i$. For example, for a two-electron system, this antisymmetric product takes the form

$$\Psi(\boldsymbol{x}_1,\boldsymbol{x}_2) = \frac{1}{\sqrt{2}}\left[\phi_1(\boldsymbol{x}_1)\,\phi_2(\boldsymbol{x}_2) - \phi_1(\boldsymbol{x}_1)\,\phi_2(\boldsymbol{x}_2)\right] \quad (12)$$

$$= -\Psi(\boldsymbol{x}_2,\boldsymbol{x}_1). \quad (13)$$

Slater noted that the antisymmetric product for a *many-fermion* system can be compactly written as a determinant,

$$(14)\quad \Psi(\boldsymbol{r}_1,\boldsymbol{r}_2,\cdots,\boldsymbol{r}_{N_2}) = \frac{1}{\sqrt{N_2!}}\begin{vmatrix} \phi_1(x_1) & \phi_2(x_1) & \cdots & \phi_{N_2}(x_1) \\ \phi_1(x_2) & \phi_2(x_2) & \cdots & \phi_{N_2}(x_2) \\ \vdots & \vdots & & \vdots \\ \phi_1(x_{N_2}) & \phi_2(x_{N_2}) & \cdots & \phi_{N_2}(x_{N_2}) \end{vmatrix}.$$

The exchange of two electrons, which corresponds to swapping two rows in the determinant, naturally introduces a sign change and thus makes (14) antisymmetric. The energy of such a determinant will be an *upper bound* to the exact energy, due to the variation principle. The HF energy and orbitals are obtained by requiring E_{HF} to be stationary with respect to variations in the orbitals, under the constraint that the orbitals remain orthonormal. This yields an eigenvalue equation for each orbital,

$$\hat{F}(\boldsymbol{r}_1)\,\phi_i(\boldsymbol{r}_1) = \epsilon_i\phi_i(\boldsymbol{r}_1), \quad (15)$$

where $\{\epsilon_i\}$ are orbital energies and the Fock operator $\hat{F}$ is a one-electron operator that includes the average Coulombic interaction with the $N_2 - 1$ other electrons. These eigenvalue equations (15) must be solved iteratively, because the Fock operator depends on the set of molecular orbitals. To enable efficient solution via computer, the molecular orbitals are typically expanded in a basis set of M functions, χ_μ,

$$\phi_i(\boldsymbol{r}_1) = \sum_{\mu=1}^{M} C_\mu^i \chi_\mu(\boldsymbol{r}_1). \quad (16)$$

The expansion coefficients $\left\{C_\mu^i\right\}$ are referred to as MO coefficients and provide a set of parameters that are varied to minimize the energy. Roothaan (1918–) showed that the introduction of this basis allows the HF equations to be recast as a matrix equation, leading to the self-consistent field (SCF) method

$$\boldsymbol{FC} = \boldsymbol{SC\epsilon}, \quad (17)$$

where $\boldsymbol{F}$ is a matrix representation of the Fock operator with entries formed by the inner products

$F_{\mu\nu} = \langle \chi_\mu | \hat{F} | \chi_\nu \rangle$ (in the Dirac bra-ket notation) and $\boldsymbol{C}$ is a matrix containing the MO coefficients, C^i_μ. The overlap matrix $\boldsymbol{S}$ deviates from unity because the basis functions are in general nonorthogonal. For a finite basis of size M, this pseudoeigenvalue equation can be solved iteratively to yield M molecular orbitals and corresponding orbital energies, of which the $N_2/2$ lowest energy orbitals will be occupied by a pair of electrons.

In practice, basis sets comprise atom-centered *Gaussian functions* of the form

$$\chi_\mu\,(x_A, y_A, z_A) = N x^l y^m z^n e^{-\alpha_\mu r_A^2}, \tag{18}$$

where N is a normalization factor, (x_A, y_A, z_A) are Cartesian coordinates centered on nucleus A, and $r_A = \sqrt{x_A^2 + y_A^2 + z_A^2}$. Standard sets of exponents, α_μ, have been optimized for each atomic element. The required integrals for the inner product in the appropriate Hilbert spaces (such as (21) later) can be readily computed.

SCF theory requires the diagonalization of the Fock matrix for each iteration, with the dimension equal to the number of basis functions, M. For H_2O, a typical medium-sized Gaussian basis set will contain fifty-eight functions. By considering the point group symmetry of H_2O as discussed above, the Fock matrix can be block diagonalized and the problem reduced to the diagonalization of four submatrices, corresponding to the four irreducible representations of the C_{2v} point group. For a larger molecule, say the cancer drug paclitaxel (which has 113 atoms, 452 electrons, and no point-group symmetry), there will be 2,516 basis functions if this same basis set is employed. Solution of the SCF equations for systems of this size can be carried out on a modern workstation.

The traditional means of improving the Hartree–Fock wave function is via *configuration interaction (CI)* (see, e.g., [23]), in which the electronic wave function is expanded in terms of a set of Slater determinants constructed by considering "excitations" of electrons from the $N_2/2$ occupied orbitals to the $M - N_2/2$ unoccupied MOs,

$$\Psi_{CI} = \sum_I^{CI} C_I \,|\Psi_I\rangle\,, \tag{19}$$

where the sum runs over Slater determinants built from a set of SCF orbitals and the expansion coefficients, C_I, are referred to as CI coefficients. In the case that all possible determinants constructed by exciting electrons from the Hartree–Fock reference are considered, this approach is referred to as Full CI (or FCI) and delivers the exact solution to (9) within the space spanned by the orbitals. By introducing this determinantal basis and again relying on the variational principle, (9) can be recast as a matrix eigenvalue problem,

$$\boldsymbol{Hc} = E\boldsymbol{c}, \tag{20}$$

where the Hamiltonian matrix elements are of the form

$$\left\langle \Psi_{ij\cdots}^{ab\cdots} \middle| \hat{H} \middle| \Psi_{i'j'\cdots}^{a'b'\cdots} \right\rangle \tag{21}$$

and $\boldsymbol{c}$ contains the CI coefficients. The notation $\Psi_{ij\cdots}^{ab\cdots}$ designates an excited Slater determinant constructed by moving electrons from occupied orbitals $i, j, k, \ldots$ to unoccupied orbitals $a, b, c, \ldots$. For example, Ψ_{ij}^{ab} denotes the doubly excited determinant formed by moving electrons from orbitals i and j to orbitals a and b, respectively. The energies of the ground and excited electronic states (eigenvalues) and corresponding wave functions (eigenvectors) are obtained by diagonalizing the Hamiltonian in this determinantal basis.

The size of this determinantal basis grows exponentially fast. For example, for H_2O (ten electrons, fifty-eight basis functions), the number of determinants in the FCI expansion is on the order of 10^{13}. Diagonalization of a matrix of this size via conventional techniques is clearly infeasible, because storing this matrix would require more than 10^{12} petabytes. Symmetry considerations enable the Hamiltonian matrix to be block diagonalized, reducing the size of the largest sub-block by about a factor of two. That is of little consolation. Clearly, practical chemical applications require more approximate methods, which are formulated by considering only a small number of possible excitations. This gives rise to hierarchical families of *electron correlation methods* abbreviated as

CISD, CISDT, CISDTQ, … ; CCSD, CCSDT,
CCSDTQ, … ; MP2, MP3, MP4, … ,

where CI = configuration interaction, CC = coupled cluster (method) (see, e.g., [9]), S = single (excitations), D = double (excitations), T = triple (excitations), Q = quadruple (excitations), MPn = Moller–Plesset (perturbation theory, n is the order of perturbation).

Having described several families of approximate methods to solve (9), we now turn to (11). Expanding V about $\boldsymbol{R}_{eq}$, we obtain

$$\begin{aligned} V &= V_0 + \sum_{i=1}^{3N_1} \left(\frac{\partial V}{\partial x_i}\right)_{R_{eq}} \boldsymbol{x}_i \\ &\quad + \sum_{i,j=1}^{3N_1} \left(\frac{\partial^2 V}{\partial x_i \partial x_j}\right)_{R_{eq}} \boldsymbol{x}_i \boldsymbol{x}_j + \cdots . \end{aligned} \tag{22}$$

Because $\boldsymbol{R}_{eq}$ is a stationary point on the potential energy surface, the first derivative term is zero. If we truncate (22) at the quadratic term (the harmonic approximation), the vibrational modes can be decoupled and (11) solved exactly. The effects of third- and fourth-order derivatives of the potential can then be treated perturbatively to predict anharmonic vibrational frequencies, which can be compared more directly with spectroscopic measurements.

In general, harmonic vibrational frequencies and associated normal modes can be obtained by diagonalizing a force constant matrix, $\boldsymbol{W}$, which has elements given by

$$W_{ij} = \frac{1}{\sqrt{m_i m_j}} \left(\frac{\partial^2 V}{\partial x_i \partial x_j} \right). \tag{23}$$

The square roots of the eigenvalues are the vibrational frequencies and the eigenfunctions the normal modes, Q_k. In general there will be three translational modes and three rotational modes. These correspond to eigenvalues of zero.

Remark 2. The ability to accurately predict the IR absorption profiles of small molecules can enable the *in silico* screening of chemicals before they are manufactured on large scale and introduced into the environment. For example, Bera, Francisco, and Lee [2] recently proposed a strategy to screen potential industrial chemicals based on their radiative efficiencies. In systematic studies of the absorption frequencies and IR intensities of series of hydrofluorocarbons (HFCs), which include popular refrigerants, [2] found that certain functional groups are associated with higher total absorption cross-sections within the atmospheric IR window. Consequently, these species are predicted to have higher potential for global warming and should be avoided in industrial applications. Computationally predicted absorption profiles can provide a means of evaluating future refrigerants, for example, to avoid molecular species that might pose undue long-term risks to the global climate. □

Remark 3. We mention the following items as information for mathematicians interested in physical chemistry:

(i) The National Research Council report [20] represents an important, first publication to draw mathematicians' attention to chemistry.

(ii) The Born–Oppenheimer separation we discussed in (6)–(10) constitutes a cornerstone for chemical physics. Concerning its mathematical justifications and limitations, we refer to Hagedorn and Joye [15] for a brief history, important literature, and some new progress.

(iii) For the mathematical analysis and convergence in the Hartree–Fock approximations, we refer to Cancès and Le Bris [5], for example. □

Concluding Remarks

Political and public debates are still ongoing about whether global warming is "real" or just scientific hype. Regardless, there is ample empirical evidence of a global warming trend. For example, Figure 12 shows photos taken twenty-six years apart of Qori Kalis Glacier in Peru. There is clear evidence that a large number of the well known glaciers are receding, and many scientists studying these put the blame on the increase of the molecules discussed in this article.

Figure 12. Retreat of the Qori Kalis Glacier in Peru. (Courtesy of Prof. Lonnie G. Thompson, The Ohio State University.)

In this article, we show the utility of mathematical studies regarding representations of finite groups, symmetries, and computational mathematics in the research on greenhouse gas and other general molecules through physical and computational chemistry. In fact, *quantification* by mathematics, chemistry, physics, and atmospheric science provides the only fact-finding way to confirm or dismiss the occurrence of anthropogenic global warming. This is exactly what Joseph Fourier initiated nearly two hundred years ago.

Most of the contemporary discussions on global warming seem to be aimed at the atmospheric concentration of anthropogenic carbon dioxide and the severity and harmful effects if and when that doubles. We have only touched lightly on the greenhouse gas *methane* here. But methane (the key ingredient of natural gas) could be a much more worrisome greenhouse gas as it can trap much more heat in the atmosphere than carbon dioxide does. For example, a recent publication by N. Shakhova et al. [22] in *Science* has reported a large release of methane from the warming up of the arctic permafrost. Will the Earth's ecosystems be able to adapt? The limits, costs, and consequences are very intimidating. We feel that the investment in alternative energy and carbon sequestration technologies is a safe and forward-thinking approach, which can at least buy us some time.

Acknowledgments

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Modelling the Journey from Elementary Word Problems to Mathematical Research

Chris Sangwin

This article argues that traditional "word problems" provide an important early encounter with, and form of, mathematical modelling. Through a number of examples we will illustrate difficulties with the modelling process, both for mathematical and psychological reasons. Hence we argue that word problems, as simple proto-modelling exercises, play an important part in mathematics education.

Mathematics first developed over four thousand years ago in ancient Iraq to solve practical problems associated with metrology and accounting [17]. Such problems motivated the need for the development from arithmetic to algebra, and some of these "word problems" survive today in a recognizable form as traditional algebra story problems. Other examples occur in geometry and later in mechanics.

Example 1. A ladder stands with one end on the floor and the other against a wall. It slides along the floor and down the wall. If a cat is sitting in the middle of the ladder, what curve does it move along? [8, Ex 0.1]

To model this situation we need to make many assumptions that are not explicitly stated—for example, a smooth vertical wall and horizontal floor. We also need to imagine a thin ladder and even a point-sized cat! Unfortunately these assumptions can appear contrived and even silly, although we argue that the confidence needed to be able to take the initiative when making such assumptions and the experience to do so successfully are important parts of mathematical enculturation. When modelling a situation such as this, we represent it mathematically, we operate on the mathematical abstractions, but we periodically check for a meaningful correspondence between the real situation and symbolic abstraction. This is all part of *making sense*.

Engineering and science, particularly physics, all make use of mathematics in this way. Pure mathematics is less concerned with "real world" problems than with patterns and consequences and connections. These patterns are important, indeed even beautiful and intriguing, but of less interest to us here. Applied mathematics sits between explicit applications and pure intellectual pursuits with no clear boundaries between them. Rather than attempt to distinguish between pure and applied mathematics, we highlight the difference between deductive and empirical justification for truth. The mathematician justifies his or her work deductively from stated hypotheses, whereas the experimental scientist looks for empirical evidence. Therefore the applied mathematician needs to use and acknowledge both: checking the validity of an approximation or modelling assumption against an observation of a physical system is an empirical process. However, even pure mathematicians often do not *discover* their theorems by working deductively from axioms.

Mathematics can be used very effectively for solving *certain specific types of problems*; but change something, even in apparently trivial ways, and the resulting equations can become very difficult indeed to solve, perhaps even requiring mathematical research. Hence the skill of effective mathematical modelling is to represent a system in a way which (i) is not too bad, ideally *exact*, and (ii) results in an equation/system *which is still solvable*. Based on this observation we will consider the following four kinds of situations.

(1) Exact models, with exact solutions.

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(2) Exact models with approximate solutions. Ideally the accuracy of the approximation can be established.

(3) Approximate models with exact solutions. Here we have few ways of knowing with any certainty how well those things we have modelled fit the real system. More seriously, we may not have modelled something which is important and don't know it. At least the mathematical solution is exact.

(4) Approximate models with approximate solutions. This situation combines the problems of (2) and (3) above.

The experienced mathematician may reserve the verb "to model" to describe situations in which relevant aspects of a complex problem are selected and other parts of the physics are ignored. For example, we might choose to ignore friction or assume rigidity of components. Under this interpretation, an "exact model" is an oxymoron. However, we take the word *modelling* in its broadest sense so that the techniques can be valued and included at all levels of the curriculum, from the research mathematician to the elementary school student.

To see why this discussion is needed, let us look at two examination questions from United Kingdom GCE Advanced Level examinations. Example 2 was set in a paper titled "Mathematics" in June 1971, while Example 3 was set in June 2003 in a paper titled "Mechanics 1".

Example 2. A projectile is fired with an initial velocity of magnitude V inclined at an angle α above the horizontal. Find the equation of the trajectory referred to the horizontal and vertical axes through the point of projection.

A projectile is fired horizontally from a point O, which is at the top of a cliff, so as to hit a fixed target in the water, and it is observed that the time of flight is T. It is found that, with the same initial speed, the target can also be hit by firing at an angle α above the horizontal. Show that the distance of the target from the point at sea level vertically below O is $\frac{1}{2}gT^2 \tan\alpha$.

Example 3. Air resistance should be neglected in this question.

A bottle of champagne is held with its cork 1.5m above a level floor. The cork leaves the bottle at 60^o to the horizontal. The cork has *vertical* component of velocity of 9ms^{-1}, as shown in Figure 1.

(1) Show that the initial horizontal component of velocity is 5.20ms^{-1}, correct to three significant figures. [2 marks]

(2) Find the maximum height above the floor reached by the cork. [3 marks]

(3) Write down an expression in terms of t for the height of the cork above the floor t seconds after projection. [2 marks]

After projection, the cork is in the air for T seconds before it hits the floor.

(4) Show that T satisfies the equation $49T^2 - 90T - 15 = 0$.
Hence show that the cork is in the air for 1.99s, correct to three significant figures.
Calculate the horizontal distance travelled by the cork before it hits the floor. [5 marks]

(5) Calculate the speed with which the cork hits the floor. [3 marks]

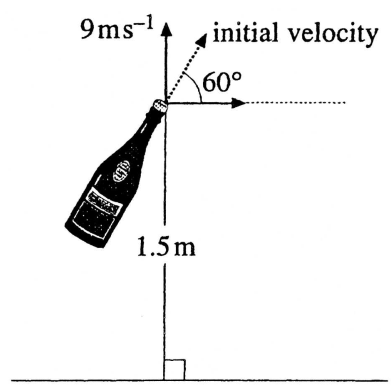

Figure 1.

These questions are on substantially the same topic, that of projectiles without air resistance. It is true they are quoted here without explaining any teaching context or the familiarity students have with the subject, but there is a striking difference. In Example 2, there is a modelling phase, and the student must read the question, draw and label an appropriate diagram, and link together multiple steps. All this is absent in Example 3: the diagram has been drawn and the question broken up into parts. For more comments on the contemporary habit of reducing problems to "single piece jigsaws", see [6]. There are other differences, such as the use of a specific and "nice" angle and the use of numerical approximations. But this is less important to our theme here than the total lack of the modelling phase in Example 3. We emphasize that these examples are from *final examinations*. Example 3 may be very useful at formative stages in providing some structure. Ultimately, eliminating the modelling stage reduces the activity to only imitation and practice, which is sterile and dull. More seriously, it is a short-term strategy for success in contrived examinations, which does not leave the student as an independent and confident mathematician.

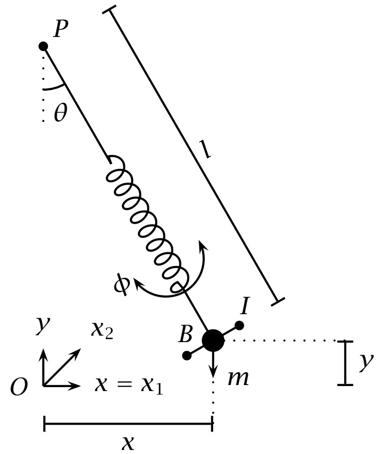

Figure 2. The general elastic pendulum.

Approximate Models with Exact Solutions

To motivate our discussion of more elementary problems, we shall examine, in some detail, the problem of modelling the pendulum. This iconic problem is solved with the help of various approximations, some of which are made *only* to ensure that exact solutions can be found to the resulting equations.

Consider the general *elastic pendulum* system shown in Figure 2. This consists of a heavy bob B attached to a point P by means of an elastic spring. The bob has a number of degrees of freedom relative to its rest position O: it may move horizontally ($x = x_1$) and vertically (y) in the plane of the page. It may perhaps also move in a direction perpendicular to the plane, (x_2). We describe any rotation along the axis PB by the coordinate ϕ. Although the full system is rather complex, we highlight these degrees of freedom to examine the modelling process.

The first modelling assumption is that the pendulum is rigid so that it does not vibrate vertically or rotate as a torsion spring and that all mass is concentrated at B. We further assume that the motion occurs only in one plane. This reduces the independent variables effectively to just the angle θ. Assuming that there is no air resistance or friction in the bearing at P, the only forces on the pendulum are contact forces at P and force due to gravity, assumed to act vertically downwards and to be constant, since the pendulum is small relative to the Earth. Resolving this perpendicular to PB gives a force of $-mg\sin(\theta)$ acting on B. Acceleration is given by $l\ddot{\theta}$, and by Newton's second law we have

$$(1)\quad ml\ddot{\theta} = -mg\sin(\theta), \quad \theta(0) = \theta^0,\ \dot{\theta}(0) = \dot{\theta}^0.$$

Even under all these assumptions we have a differential equation that does not yield an exact solution in terms of elementary functions! We shall make a further assumption that θ is small so that $\sin(\theta) \approx \theta$. Then $x = l\sin(\theta) \approx l\theta$, and (1) becomes

$$(2)\qquad \ddot{x}(t) = -\frac{g}{l}x(t),$$

the well-known equation for *simple harmonic motion*. The assumption that θ is small is made only so that the equation becomes linear and hence has an exact and explicit solution in terms of elementary functions. Taking $\omega_p = \sqrt{g/l}$, the unique solution of this equation is

$$(3)\qquad x(t) = a\cos(\omega_p t + \rho).$$

Here ω_p is the *natural frequency*. The *amplitude*, a, and *phase angle*, ρ, depend on the initial conditions. Notice the period of the oscillation is independent of the amplitude. However, for the full rigid pendulum, the period of the swing depends on the amplitude. So, how accurately does (3) approximate solutions to (1)? For a clock maker this is a key consideration. After all, there are $60 \times 60 \times 24 = 86,400$ seconds in a day. Any errors between the model and real system are cumulative, so that even a tiny error per oscillation may become significant. To the horologist this is known as *circular error*. To calculate the actual period of (1) we are quickly, and unavoidably, led to elliptic integrals of the first kind. The analysis needed to establish the accuracy of the approximation (3) to the true solution (1) is rather difficult. The search for a mechanism to avoid circular error motivated Huygens to investigate the path along which the pendulum bob would have to travel to ensure the period is independent of its amplitude. It is known as an *isochrone* (or *tautochrone*) curve and is first considered in [10].

Imagine you are teaching simple harmonic motion or applications of differential equations. We could argue that, to introduce simple harmonic motion, we can avoid the approximation $\sin(\theta) \approx \theta$ if we choose the vertically displaced mass and spring system. In terms of the situation shown in Figure 2, this corresponds to ignoring any horizontal movement but dropping the assumption of rigidity. Let us assume that the spring obeys Hooke's law, so that extension is proportional to the applied force. We shall continue to ignore damping, so for our system we derive

$$(4)\qquad m\ddot{y}(t) = -ky(t)$$

where k is the extension spring constant. We immediately recover (2) for $y(t)$ with natural frequency $\omega_s = \sqrt{k/m}$, and so we can apply standard techniques to solve the equation exactly. Here we have ignored some features, for example, air resistance, but under our assumptions the approximate model results in equation (4), which has an exact solution.

A very similar system is a *torsion spring*. Here there is no vertical or horizontal displacement; instead, the spring is twisted and released. The

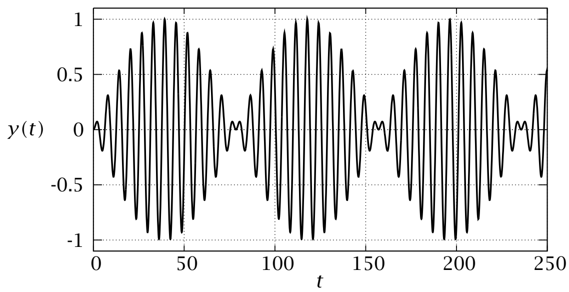

Figure 3. A time series for *y(t)* showing beating behavior.

displacement variable is ϕ, and now the moment of inertia, I, replaces the mass in the counterpart of (4). Such spiral springs are important in clocks and watches, but they are essentially linear models, as before.

When a spring is extended or compressed, there is a small winding or unwinding of the spring, and so we might expect the vertical compression/extension system to be weakly coupled to the torsion system. Can this be ignored? Including this within the model, the resulting system has *two degrees of freedom*: torsional (ϕ) and vertical (y) displacements. The resulting equations turn out to be weakly coupled linear ordinary differential equations, which can be solved exactly.

If the frequencies of the vertical and torsional modes are the same, then we have *resonance*, and the system will transfer its energy back and forth completely between the two modes of oscillation. This is quite a striking practical demonstration. One configuration is known as a Wilberforce spring, after its inventor Lionel Robert Wilberforce. See [23] or [7, pp. 205–212]. A time series for $y(t)$ showing beating behavior is illustrated in Figure 3. Notice how the amplitude changes over time. In practice the vertical oscillation appears to stop and start, out of phase with the twisting oscillation, which also appears to stop and start. With hindsight it was a mistake to ignore the torsion effect in the model in the special case of resonance. The exact solution to the full system is strikingly different from the exact solution to (4).

Approximate Models with Approximate Solutions

Our last class of systems is those in which approximate models are derived but only approximate solutions can be found. In fact, this is the usual situation in applied mathematics! Only very special cases can be solved explicitly. Most current work in fluid mechanics, for example, relies on numerical simulations to equations which approximate some of the physics. It is perhaps surprising quite how effective such approximations are in practice.

Even with the example illustrated in Figure 2 there are unexpected problems. We shall again ignore torsion, but this time consider the effect of horizontal displacement, resulting in y and $x = x_1$ coordinates. This allows a combination of a vertical spring mass and traditional planar pendulum to be considered. Let us assume we are trying to demonstrate (4) experimentally, with a traditional spring mass system. So we pull the mass vertically downwards and release it. In any experiment there will be a small horizontal displacement. What is its role? Perhaps, with intuitions guided by the independence of vertical and horizontal components during projectile motion, we might assume these motions are independent of each other? But if we pull the spring horizontally to one side, then an upward component of force is generated by the spring: they are not really decoupled.

Defining l_0 to be the unstretched length of the spring and l_1 to be the rest length of the spring under the mass m, we have

$$mg = k(l_1 - l_0), \quad \text{i.e.,} \quad l_1 = l_0 + \frac{mg}{k}. \tag{5}$$

For initial conditions close to those stated, that is, releasing from rest with $|x|$ small, the equations of motion relative to the rest position can [14] be *approximated by*

$$\ddot{x} = -\omega_p^2 x + \lambda x y, \tag{6}$$

$$\ddot{y} = -\omega_s^2 y + \frac{\lambda}{2} x^2, \tag{7}$$

where

$$\omega_p^2 = \frac{g}{l_1}, \quad \omega_s^2 = \frac{k}{m}, \quad \lambda = \omega_s^2 \frac{l_0}{l_1^2} = \frac{k}{m}\frac{l_0}{l_1^2}. \tag{8}$$

Note that if $x(0) = 0$, then from (6), $x(t) = 0$ for all time and (7) reduces to a simple harmonic oscillator with frequency ω_s, as expected. Although we are not free to choose λ, we note that if $\lambda = 0$, then we have decoupled linear oscillators. We might expect a strong coupling between these two equations to occur when the two frequencies resonate, and this really happens when $l_1 = \frac{4}{3} l_0$.

To illustrate a resonant system with $l_0 = 0.3$m, we release the mass from rest at $x = 0.0001$m (i.e., 0.1mm) and $y = 0.02$m. The resulting simulation is shown in Figure 4. The figure shows what appears, from long-term numerical simulations, to be one period of a periodic solution. Notice that for a significant initial period of time the system appears to be oscillating vertically, just as might be expected. However, for a brief period of time the vertical motion appears to all but stop, and a significant pendulum mode dominates. Literally, the mass stops going up and down and swings from side to side. Furthermore, the horizontal amplitude is double that of the vertical displacement.

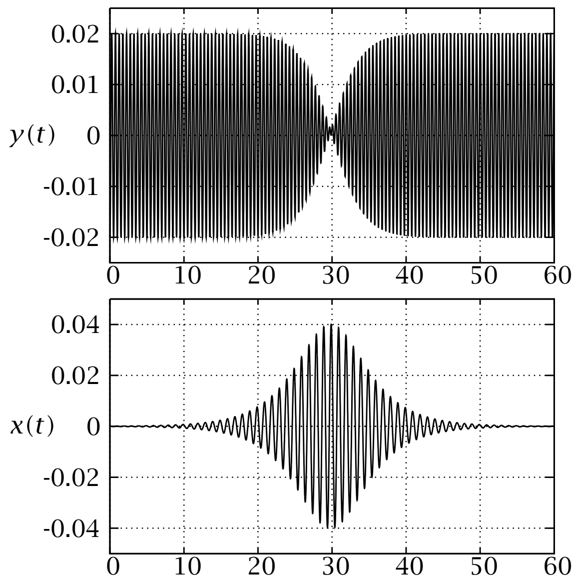

Figure 4. Time series *y(t)* (top) and *x(t)* (bottom) for the elastic pendulum at resonance.

Experimentally, this is difficult to achieve. The spring and mass have to be *very* carefully matched to see this nonlinear effect. Choosing random springs and masses, one is very unlikely to ever see this occur. And yet it happens and can be observed experimentally. You might just be unlucky, as Olsson in [14]:

> I first noticed the mode coupling in the swinging pendulum when I was a Teaching Assistant as a graduate student when (by chance) a student could not make the lab experiment work. This classical experiment, as you know, first measures the spring constant and then predicts the period. After stewing about this for a while I was able to work it out. (Olsson, M., 2008, private communication)

Unfortunately, the contemporary state of mathematical research only allows approximations to the equations to be derived, and these nonlinear differential equations cannot currently be solved explicitly; see [11].

By ignoring the horizontal component in the model (4), we failed to capture a significant and observable feature of the dynamics. In this case comparison of the mathematical solution with observations failed, and we were forced to include extra features in the model to accommodate the observations. In addition, when included, we only have approximate equations and numerical solutions. While the model assumes the motion to be planar, the oscillations are observed experimentally to occur in perpendicular horizontal directions alternately. It should also be possible to combine both the effects of the Wilberforce spring and misbehaving pendulums.

Whether a set of equations is tractable depends both on choosing the right type of model and on how this model is applied. In particular, a crucial aspect of modelling is the *ability to choose coordinates*. In many cases a particular choice of variables and coordinates significantly simplifies the resulting equations, ideally decoupling systems of equations so that they may be solved individually. Perhaps the most important example of the effect of the choice of coordinates is the way we view our solar system. If we put the Earth at the origin of our coordinate system, then describing the motion of the planets becomes terrible: they appear to "wander" on curves similar to epicycloids. If, however, the Sun is placed at the center, the resulting mathematical description becomes much simpler. The planets now all follow circles. Or the more accurate, but still approximate, model of Kepler is to fit the orbits to ellipses with the Sun at one focus. As another example, we turn to matrices which represent transformations. Matrices that can be "diagonalized" have a basis of eigenvectors. This basis is a coordinate system that reveals the structure of the transformation. The Jordan normal form does the same for more general matrices. The ability to choose a coordinate system which simplifies a problem requires sophistication, experience, and perhaps even some luck.

> For many years I used to give tutorials (and sometimes the lectures as well) on "Classical Mechanics" in the 2nd year at Oxford, largely about Lagrangian dynamics. The subject of "small oscillations" was always on the syllabus, and a typical question involved something like a double pendulum hanging down—find the (two) natural frequencies of oscillation and describe the type of motion involved in each. Such questions would *sometimes* end with "Find the normal coordinates", to which my reaction was often, "What's the point? What are the students going to do with them?"
>
> The point—whether clear to all the students or not—is that if you use normal coordinates, rather than the original "physical" ones, the linearized differential equations for small amplitude

> oscillations (two, in the double pendulum case) become *decoupled*.
>
> When I sat down to try and prove what turned out to be my upside-down pendulum theorem [1] on a rainy, windswept Sunday afternoon in November 1992, I therefore started out with the downward state, with N general pendulums, and wrote down N uncoupled equations using normal coordinates. I worried about what would happen when I turned my system upside-down, by changing the sign of g, and, even worse, how the equations would get vaguely Mathieu-like but hideously coupled when I added the effects of pivot vibration. I hadn't a clue what I would do in those circumstances.
>
> After a few minutes, and half a page of work, it suddenly became clear that with my particular system (and more or less that particular system only!) the equations *were never going to become coupled*, so I found myself dealing with N uncoupled Mathieu equations.
>
> The whole thing took 45 minutes, though my computer models of inverted 2 and 3 pendulum systems, generated over the previous few weeks, had set the scene. And so it was that I will always be grateful for hours of (otherwise pointless, I suspect) teaching of normal coordinates, over many years. (Acheson, D., 2008, private communication)

The pendulum is iconic. Renaissance science hypothesized "Divine order", and what better to represent this than the pendulum clock, studied through the geometrical physics of [10]? Small oscillations of a double pendulum, or the Wilberforce spring, are examined using the classical mechanics of the eighteenth and nineteenth centuries. Hysteresis and catastrophe theory can be demonstrated by the driven rigid pendulum. Chaos theory is aptly illustrated by the full double pendulum. Indeed, the pendulum in its many forms has served applied mathematics as the paradigmatic example for the concerns of each successive age. It continues to yield surprises, such as the pendulum theorem of [1]. There is every reason to suppose that it, and related systems, will continue to provide interesting mathematics. Here it illustrates the fact that selecting a mathematical model for the very simplest mechanical system is genuinely difficult. Under certain conditions the model may not be rich enough to illustrate significant qualitative features of the underlying physics/dynamics, that is, that the model is "bad" and that exact solutions are the exception rather than the norm. However, given a mathematical system, it is often far from clear to which of the following classes it belongs. Those for which

(1) a solution can be found exactly with a standard technique;
(2) a solution is known to exist but is difficult to establish explicitly, and hence it is approximated;
(3) a solution is known not to exist;
(4) the very question of existence of a solution is an open problem.

Mathematicians should, perhaps, be more honest about the range of problems that are solvable, but the culture of mathematics is very much to report only *theory* (1) and *counterexamples* (3). For the nonexpert, classes (2) and (4) remain shrouded in mystery. Perhaps this should be so: after all, innovations might be stifled if a student knows that others have struggled and failed.

Selection of Models by Students

We have set up a dichotomy between exact/approximate models and exact/approximate solutions to the resulting mathematical abstractions. The first class are problems that have exact models which can be solved exactly. While these are the exception in mechanics, they do occur in simpler problems, particularly in school in the form of word problems, e.g., [9].

Example 4. A dog starts in pursuit of a hare at a distance of thirty of his own leaps from her. He takes five leaps while she takes six but covers as much ground in two as she in three. In how many leaps of each will the hare be caught?

Or the ubiquitous

Example 5. A rectangle has length 8cm greater than its width. If it has an area of 33cm^2, find the dimensions of the rectangle.

Generally these word problems reduce to standard algebraic techniques, such as (i) a linear equation of a single variable, (ii) a quadratic equation in a single variable, (iii) simultaneous linear equations, or (iv) problems involving harmonic sums, for example, $\frac{1}{n} + \frac{1}{m}$. A systematic analysis of story problems is given in [13]. However, reducing problems to these forms is far from easy: problems involving rates are particularly difficult.

Example 6. Alice and Bob take two hours to dig a hole together. Bob and Chris take three hours to dig the hole. Chris and Alice would take four hours to dig the same hole. How long would all three of them take working together?

The temptation is to model the work of Alice and Bob incorrectly using literal transcription to rewrite "Alice and Bob take 2 hours" as $A + B = 2$. In one correct interpretation we use A to be Alice's *rate of work*, and so write $2A + 2B = 1$, where the right-hand side represents "one hole". This assumes rate of work is independent of any collaboration, which may or may not be realistic. The real psychological difficulty of these kinds of interpretations is well established, but the challenge of how to overcome them remains.

Example 7. Write an equation for the following statement: *"There are six times as many students as professors at this university."* Use S for the number of students and P for the number of professors.

When Clement et al. [4] gave the Students-and-Professors Problem to 150 calculus-level students, 37 percent answered incorrectly with $6S = P$, accounting for two-thirds of all errors. There are genuine difficulties in reaching a correct interpretation, and hence in moving from a word problem to a mathematical system which represents it. Similar conceptual difficulties occur with concentration and dilution problems. Consider a problem related to, but different from, Example 6, in which pairs of people "walk into town" rather than "dig a hole". We argue here that moving from such word problems to mathematical systems constitutes the beginning of mathematical modelling. We also note that such problems are available at all levels of mathematical sophistication, including the most elementary numeracy and algebra.

In Example 4, let l be the number of leaps taken by the dog. The problem can reduce to solving

$$l = \frac{6}{5} \times \frac{2}{3} l + 30.$$

This is a rather trivial linear equation, but it can only be arrived at by careful work on the part of the student. In Example 5 the student is free to choose either the length or width of the rectangle as a variable. Ignoring the particular letter used for the variable, this choice results in one of two different equations, that is, $x(x + 8) = 33$ or $x(x - 8) = 33$. One solution must be discarded as "unrealistic": a valuable critical judgment by the student.

Being able to select and correctly use standard techniques presupposes a certain level of fluency, which only comes with practice. This includes geometry puzzles for which a student builds up a range of approaches, "tricks", and useful general results. Seeing the practice of mathematics itself as solving real problems through modelling, and thus understanding the satisfaction of dispatching the routine steps accurately and efficiently, may act as motivation for students to undertake the repetitive work needed for the acquisition of skills.

Traditional word problems assume a certain level of cultural knowledge. For example, we take for granted that the problem solver is familiar with metrological systems or the rules of a sports game. This has always been the case since the first recorded use of such word problems in the mathematical training of scribes from around 2500 BCE in ancient Iraq [17]. In a mathematics class it is unlikely that the purpose of such problems is a test of such cultural knowledge, and this obviously has implications for how they are used with students, particularly if assessed. Problems need to be carefully selected for their intended students.

Given that modelling is, in the larger mathematical sense, very difficult, how are students supposed to solve problems? By "problems" we mean questions for which the modelling step is very much part of the process. For us something that is technically complex but that can be solved by a routine, well-established technique is not called a problem; rather we call it an *exercise*. For example,

$$\text{show that } \int_0^1 \frac{x^4(1-x)^4}{1+x^2} \mathrm{d}x = \frac{22}{7} - \pi$$

is an exercise in polynomial long division and basic integration. Here there is no modelling step. In fact it can be done automatically with a computer algebra system. For us, a problem requires the student to make choices about how to represent a problem in a mathematical form, even when there is a simple (e.g., linear) equation that represents the situation exactly. An excellent source of problems and puzzles for which there exists an exact model and exact solution is plane geometry, and many examples are given in [8].

The following example, proposed recently as a school project, illustrates how difficult apparently simple problems can actually be to design and solve.

Example 8. What is the greatest number of people who may comfortably stand on a soccer field?

Here choices have to be made. Soccer fields may vary in size (90–120m × 45–90m), so for argument's sake we might take Wembley Stadium at 105m × 68m. Such arguments can be excellent opportunities to waste valuable time discussing essentially arbitrary things. Similar problems arise with the size of the people, but much more seriously are those caused by their *shape*. Here the choices that are made have a direct bearing on the difficulty of the mathematics.

Let us assume that a person occupies a rectangle or square and that everybody occupies the same area. Then the problem becomes somewhat trivial: people tessellate the field in a natural way. There is an unambiguous correct answer to this problem under these modelling assumptions (i.e., exact model, exact solution). Given the minimum amount of space required by each person, we can find the maximum number of rectangles that fit inside the rectangular area. But a rectangular packing is not the most efficient. Choosing the shape occupied by people to be hexagons results in more people being packed into the area but without the distance between any two people being reduced. However, a more difficult analysis results, as hexagons tessellate an infinite plane but not a finite rectangle. This problem is still tractable, and results in an answer. What about modelling each person as occupying a circle? Furthermore, it is not true that everyone occupies the same space. Packing different-sized circles into a rectangle is a classical problem, and in general it is unsolvable. But how does the novice know in advance the consequences of choosing a particular model? What does the teacher say to a student who chooses a model the teacher knows will result in intractable mathematics? Arguing, or more ambitiously proving, a solution does not exist can require very advanced ideas. If the rectangular football field is replaced with an athletics track, a shape with half circles instead of the two short straight sides, then the whole problem becomes intractable. Even slight variations cause problems to fundamentally change in character.

For this reason problems for which an exact model results in an exact solution constitute safe territory for both the student and teacher. This does not make such problems easy for students to solve, of course. To prevent confusion and wasted time, learning in mathematics often proceeds in the following phases, which might be termed "traditional teaching".

Imitation

The student is shown something new. He or she then *imitates* this by solving exercises which differ only slightly from that shown and in ways which do not fundamentally matter. This imitation shows the student new examples. Here an example might be a new mathematical object, for example, rational function instead of polynomial, or an "example" could be a technique. The importance of this has been acknowledged by many writers, for example

> (1) Concepts of a higher order than those which a person already has cannot be communicated to him by definition, but only by arranging for him to encounter a suitable collection of examples.
> [. . .]
> (2) Since in mathematics these examples are almost invariably other concepts, it must first be ensured that these are already formed in the mind of the learner.
> [19, p. 32]

Problem Solving

Next the student is given a problem and must choose which technique should be used and how. This includes the modelling phase described above. In modelling the student has *autonomy*, and with this *responsibility* to pursue a line of thought to a conclusion. While there is some autonomy, normally the required techniques will involve those imitated most recently. This helps the students. It gives them confidence they do have the required techniques, if only they can recognize which ones to use and how.

> One of the fundamental contributions of modern *didactique* consists of showing the importance of the rôle played in the teaching process by the learning phases in which the student works almost alone on a problem or in a situation for which she assumes the maximum responsibility. [2, p. 229]

When the teacher proposes an activity in a mathematics class, the student trusts both that this will be interesting and will lead to mathematical insights. This is fundamental to what Brousseau [2] calls the *didactic contract*. The teacher has the responsibility of choosing problems that are sufficiently novel to be a worthwhile challenge but that students still have a realistic prospect of solving. The phrase *zone of proximal development* is used to refer to problem-solving processes that have not yet matured but are in the process of maturation. It is

> …the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more capable peers. [21, p. 86]

A fundamental part of the didactic contract is that a correct answer exists and that the problem is finished when this has been found and justified. This is why applied mathematics adopts standing assumptions such as "light inextensible strings" and "smooth frictionless pulleys". This characterizes the difference between mathematics and science, where experimental evidence suffices. Further work, such as generalizing the techniques used, abstracting, and so on form further separate problems. A student should have the satisfaction of recognizing this for him- or herself. For further comments on problem solving, see, for example, [18].

Practice

Practice of technique often follows. Ericsson et al. [5] stress that mere repetition is not sufficient to develop expertise; *deliberate practice* is required. Deliberate practice is *effortful* and can only be sustained for a limited period of time without leading to exhaustion.

> Drill in mere manipulation is necessary at every stage in school algebra. That this should be thorough, so far as it goes, will be admitted by all teachers, but it should in the main be given *after* its necessity in applications has been perceived by the pupil and not *before*; also, it should not be carried further than is needed to ensure facility in these applications. [20, p. 10]

Furthermore, carefully structured task design can help expose the *domains of variation* and *ranges of permissible change* within which a property holds or a technique remains valid; see [22]. Effective practice is far from just repetition. The teacher is essential in sequencing these tasks and monitoring performance to decide how to create a synopsis of what has just been done and when to introduce more complex and challenging topics. For this reason we are not proposing specific measures for particular stages of a student's education, but rather arguing for the importance of modelling in its broadest sense throughout. Many teachers do this already, but some examinations do not seem to reward it.

Some teachers omit the imitation phase and use problem solving to introduce new techniques; see, for example, [24] or [3]. Letting students work out solutions to problems themselves, without being given answers, is also the basis of the "Moore method" of teaching; see, for example, [15].

When trying to solve such problems we argue that *guess and check* or *trial and improvement* are only rarely appropriate techniques. Relevant details should be extracted to abstract the problem to a mathematical system of a known type in a conscious and deliberate way. Such questions may well provide choices of model, but the solution must involve a correctly applied standard technique, for example, solving a system of simultaneous linear equations or finding the roots of a quadratic equation. An important aspect of modelling is the selection of a particular model based on knowledge of how difficult it is going to be to solve. Once this is done, any solutions should be interpreted in the terms originally supplied by the problem. This modelling process immediately turns a rather trivial single-step mathematical exercise into a multistep chain of reasoning.

Conclusion

This article argues that word problems are a valuable form of mathematical modelling. Word problems are often seen as motivating a bridge from arithmetic to algebra, and in this role their use is controversial [12]. In arguing for their importance we do not trivialize the difficulties in teaching them, either as proto-algebra or proto-modelling. Other authors acknowledge the controversies but also assert their value.

> I hope I shall shock a few people in asserting that the most important single task of mathematical instruction in the secondary school is to teach the setting up of equations to solve word problems. [...] And so the future engineer, when he learns in the secondary school to set up equations to solve "word problems" has a first taste of, and has an opportunity to acquire the attitude essential to, his principal professional use of mathematics. [16, Vol. I, p. 59]

The point of applied mathematics is to solve real problems. And so, ultimately, one purpose of mathematics education is to allow students to solve problems independently and to appreciate the significance and accuracy of any solutions. Ideally, students should be in a position to criticize and compare various models, just as we have done in the case of the pendulum and Example 8. One purpose of practice and the acquisition of fluency in techniques is to recognize when a problem has a solution. This is crucial when moving from contrived word problems for which there is an exact model with exact solutions to approximations. Unless the student is intimately familiar with the *details of solving the standard problems*, he or she will simply not be able to make informed choices that result in an approximate model that (i) is within the desired level of accuracy and (ii) results in an equation/system *that is still solvable.*

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About the Cover

Home economics

This month's cover illustrates in rough graphical terms the Earth's energy transactions, in so far as they affect the Earth's temperature. The theme was suggested by Goong Chen, co-author of an article in this issue on the mathematics of greenhouse gases. The data used in the cover image and its basic design are taken from the article "Earth's annual global mean energy budget" in the *Bulletin of the American Meteorologial Society* of February 1997, written by J. T. Kiehl and Kevin Trenberth.

The heat transport represented in solid colors represents radiation, while the two dashed paths indicate the transfer of latent heat caused by water vapor condensing at altitude to form rain, and the direct transfer of heat by thermals rising from the Earth's surface. The ultimate radiation from the Earth is with the spectrum of a black body at 254° K, which is also what the Earth would emit if it had no atmosphere. The effect of the atmosphere, which contains molecules of water, carbon dioxide, and methane among others, is to absorb and emit low-frequency radiation, hence heat, back to the surface, whose temperature is therefore a considerably warmer 288° K.

Our first idea for a cover was something more specifically mathematical, but deciding which particular theme to illustrate reminded us of the proverbial blind men exploring the proverbial elephant. It didn't seem to do justice to the magnitude of the problems involved, whether in mathematics, physics, economics, or politics. So we decided to try to give an overall view.

The Wikipedia article on the greenhouse effect begins by telling you:

> *The greenhouse effect is a process by which thermal radiation from a planetary surface is absorbed by atmospheric greenhouse gases, and is re-radiated in all directions.*

This is true enough, but gives almost no idea of what is really going on, or how complex the issues are. There is a huge literature on the subject, but little of it seems to strike the right balance between clarity and accuracy. We have found the textbook *Principles of Planetary Climate* by Raymond Pierrehumbert enlightening, as well as *The Warming Papers*, edited by Pierrehumbert and David Archer. The Wikipedia article on black bodies is also instructive.

—Bill Casselman
Graphics editor
(`notices-covers@ams.org`)

The Establishment of Sampling as a Scientific Principle—A Striking Case of Multiple Discovery

Paulo J. S. G. Ferreira and Rowland Higgins

During the period 1928–1949, several engineers contributed to the establishment of a sampling principle. They did this virtually independently of each other in the context of communications theory and practice.

Five names stand out as being the main players in this drama: H. Nyquist, who laid the foundations for the minimal sampling rate; V. A. Kotel′nikov, C. E. Shannon, and I. Someya, whose treatment of the sampling principle was mathematical; and H. Raabe, who derived the minimal sampling rate and built hardware to reconstruct signals from samples taken at that rate. Their work is described below.

As Shannon recognized, the mathematical setting already existed. In fact, it was part of a tradition that can now be traced back to Cauchy, but its significance for application was simply not realized until Shannon's time.

Our notation is described in context, or else completely standard. For background and references to original work, see [1] and [3].

The Sampling Principle

I. Signal functions having finite energy and frequency content confined to a bounded set (e.g., $[-\pi w, \pi w]$, $w > 0$, the condition of *band-limitation*) are uniquely determined by countably many of their values, or samples, taken at a fast enough rate.

II. Such a function can be represented, usually in the form of a series, in terms of these samples.

The Sampling Principle asserts that *all the energy, indeed all the information, in this type of function is contained in only countably many samples.* Seen in this way, the principle is one of data compression and is basic in modern digital communications.

The minimum sampling rate is w samples/second, twice the highest frequency component (measured in radians/second). The rate is usually called the Nyquist sampling rate.

The Sampling Principle has found widespread applications in modern science and technology, where it has been greatly extended and generalized.

When an idea comes to fruition at the hands of two or more people independently and at about the same time, historians call it *multiple discovery*, or *multiple invention.* Here, *discovery* seems the more appropriate choice. The Sampling Principle furnishes an example; we shall see that it grew out of the need to respond to the limitations set by contemporary technology, limitations which had to be understood and overcome, and that by the early twentieth century motives such as these were being felt worldwide. This may help to answer the question: *Why was the Sampling*

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Principle the subject of multiple discovery, spread over three continents?

Before going on to explore this question, we shall give a proof of the Sampling Principle in the language of today. This proof is a prototype of the elegant treatment, in terms of Hilbert spaces and their bases, that can be given to many general sampling theorems.

We have the standard notion of Hilbert space as a complete inner product space, the inner product being denoted here by $\langle \cdot, \cdot \rangle$ and the norm derived from the inner product denoted by $\| \cdot \|$.

A sequence (φ_n) in a Hilbert space H is an *orthonormal basis for* H if it satisfies both the orthonormality property

$$\langle \varphi_m, \varphi_n \rangle = \delta_{mn}, \quad m, n \in \mathbb{Z},$$

and the property that whenever $f \in H$, the representation

$$f = \sum_{n \in \mathbb{Z}} c_n \varphi_n$$

holds, with convergence in norm and with unique coefficients $c_n = \langle f, \varphi_n \rangle$.

To give the Sampling Principle a mathematical description, two Hilbert spaces are involved. One is $L^2(-\pi w, \pi w)$ (by a slight abuse of notation this symbol will mean those members of $L^2(\mathbb{R})$ that are null outside $[-\pi w, \pi w]$); the other is the Paley-Wiener space, denoted by PW, of continuous and square integrable functions on $\mathbb{R}$ whose Fourier transform is null outside $[-\pi w, \pi w]$, the norm being that of $L^2(\mathbb{R})$. Here the Fourier transform is defined by

$$(\mathcal{F} f)(\omega) := \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} f(t) e^{-i\omega t} dt,$$

the integral taken in the L^2-sense.

It follows from the Plancherel theory of the L^2 Fourier transform that we can understand $f \in PW$ to be of the form

$$f(t) = \int_{-\pi w}^{\pi w} \varphi(\omega) e^{i\omega t} \, d\omega \tag{1}$$

for some $\varphi \in L^2(-\pi w, \pi w)$, and furthermore that the two Hilbert spaces are isometrically isomorphic under the transformation $\mathcal{F}$. In particular, an orthonormal basis for one space goes over into an orthonormal basis for the other.

Consider the orthonormal system

$$(2\pi w)^{-1/2} e^{-i\omega n/w}, \quad |\omega| \le \pi w, \ (n \in \mathbb{Z}), \tag{2}$$

the standard trigonometrical basis for $L^2(-\pi w, \pi w)$.

First, since a basis is, in particular, a complete set, we have

$$\int_{-\pi w}^{\pi w} \varphi(\omega) e^{i\omega n/w} \, d\omega = 0 \ \text{ for every } n$$

implies that φ is the null element of $L^2(-\pi w, \pi w)$. By (1) this means that $f(n/w) = 0$ for every n implies that φ is null. But if φ is null, so is f, and we have shown that $\{n/w\}$ *is a set of uniqueness for* PW. This accounts for part I.

Second, for Paley-Wiener functions, the classical sampling series is:

$$f(t) = \sum_{n \in \mathbb{Z}} f\left(\frac{n}{w}\right) \frac{\sin \pi(wt - n)}{\pi(wt - n)}. \tag{3}$$

To prove this we find, after a little calculation, that the (inverse) Fourier transform of the basis elements (2) (remembering that they are null outside $[-\pi w, \pi w]$) are proportional to the expansion functions in (3), and a little more calculation shows that the coefficients are indeed samples of f.

Thus the series (3) is an orthonormal expansion for $f \in PW$, and the corresponding Parseval relation is clearly

$$\|f\|_{L^2}^2 = \frac{1}{w} \sum_{n \in \mathbb{Z}} \left| f\left(\frac{n}{w}\right) \right|^2. \tag{4}$$

Convergence in (3) is in norm, but pointwise and uniform convergence can be obtained because the Cauchy-Schwarz inequality applies to (3), thanks to (4). This accounts for part II.

The Challenges of Communications Engineering

The answer to our question about multiple discovery must surely lie in the development of communications technology and theory during the interwar period.

Bandwidth limitation and its effect on the communication rate had been felt for the first time when telegraphy over submarine cables was attempted. The transmission speed was found to be severely limited because the cable acted like a capacitor, which had to charge and discharge before the signal could be received at the far end. This was experimentally shown by Faraday, in 1854. The delay reduced the transmission speed dramatically: the ninety-word message sent in 1858 across the Atlantic from Queen Victoria to President Buchanan took sixty-seven minutes to transmit. Rates of one or two words per minute were common.

Initially, this limitation was not fully understood. The Atlantic cable failed after only a few weeks, damaged by the high voltages used. Edward Orange Whitehouse, chief electrician with the Atlantic Telegraph Company, who had been convinced that only high voltages could deliver information across the Atlantic, was dismissed.

Kelvin's law of squares shed some light on the problem. It states that the maximum operating speed is proportional to $1/(RC\ell^2)$, where R and C are the cable's resistance and shunt capacitance per unit length and ℓ is its length. Since a simple RC circuit with resistance $R\ell$ and shunt capacitance $C\ell$ has bandwidth $\omega_0 = 1/(RC\ell^2)$, Kelvin's law states that *operating speed is proportional to bandwidth*.

The underlying theory was developed in 1854 in a correspondence between Stokes and Kelvin [4]. Assuming no inductance, Ohm's law is

$$-\frac{\partial V}{\partial x} = Ri,$$

where, as usual, V denotes the potential and i the current. A segment of unit length of cable at x accumulates charge at the rate $-\partial i/\partial x$. The potential therefore increases at the rate $-(1/C)\partial i/\partial x$, and so

$$C\frac{\partial V}{\partial t} = -\frac{\partial i}{\partial x}.$$

Eliminating i between these two equations, Kelvin obtained

$$RC\frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial x^2}, \tag{5}$$

which is similar to Fourier's equation for the propagation of heat. Kelvin knew Fourier's work very well and immediately recognized that it was "perfectly adapted" to the problem of the submarine cable. We outline two of the contributions found in the Kelvin–Stokes correspondence. First, the elementary solution of (5) is

$$V(x,t) = e^{-(RCn)^{1/2}x}\sin[2nt - (RCn)^{1/2}x]$$

and shows that harmonic terms of different frequencies are propagated at different velocities. The consequence is that no definite velocity of transmission is to be expected for more general signals, namely, linear combinations of such harmonic terms.

The second result, and the most important for us, is Kelvin's law of squares. To obtain it, he solved the diffusion equation for a unit step

$$V(0,t) = \begin{cases} 1, & t \geq 0, \\ 0, & t < 0, \end{cases}$$

and then computed the electrical current using

$$-\frac{\partial V}{\partial x} = Ri.$$

Kelvin determined that the current would reach a maximum after a certain time t_0, which he found by setting the derivative of the current to zero.

Kelvin's conclusion also follows from dimensional analysis: t_0 can depend only on ℓ, the length of the cable, and the product RC, the only parameter in the diffusion equation. The units into which the product RC can be expressed are

$$\frac{\text{unit of time}}{(\text{unit of length})^2}.$$

It follows that $RC\ell^2/t_0$ is invariant with respect to changes in the units of length and time. This means that it is a constant and so t_0 must be proportional to $RC\ell^2$, the law of squares.

Kelvin would apply Fourier's theory again in 1862, in a famous and controversial paper in which he used the equation of heat propagation to estimate the age of the Earth. It was in that paper that Kelvin referred to Fourier's work as a "great mathematical poem".

Heaviside, who considered a more complex heat propagation model and showed that the Earth could be much older than Kelvin had predicted, also improved Kelvin's telegraphy line model. He completed the model in 1887 by introducing series inductance. This decisive contribution allowed him to show that by carefully adding inductance to a cable its bandwidth could be increased. In fact, the attenuation of the cable could theoretically be made constant, that is, independent of the frequency.

Transmission speed was very important in telegraphy and the need for improvements globally felt. By 1896 there were about 160,000 nautical miles of cable, laid at a cost of $1,200 per mile, spread out all over the world, with London at the center. Due to theoretical progress and better instrumentation, the line between New York and the Azores Islands was being operated at four hundred words per minute in 1924.

Hartley and Nyquist made Kelvin's law precise. Between 1924 and 1928 they focused on abstract models of the channel rather than the physical properties of the cable. As a result, they captured in a precise way the interplay between transmission speed and bandwidth.

Progress in wireless telegraphy was also being made. Marconi's first transmissions across the Atlantic date from 1901. In Germany, Braun perfected the technology (and shared the 1909 Nobel Prize with Marconi). The discovery of the vacuum tube brought enormous progress, and a new form of bandwidth limitation was soon found: "the crowding of the ether", as Kotel′nikov put it. The wireless transmission of a signal required a certain bandwidth. The natural question was: How much bandwidth should be allocated to a certain signal? Conversely, given a certain bandwidth, how much information can be packed into it?

Telegraphy had exposed the effect of bandwidth limitations on the speed of transmission, leading to the results of Nyquist and Hartley. Telephony was exposing other forms of bandwidth limitation and raising new challenges.

Time-domain multiplexing had been used in telegraphy in order to allow the cables to transmit more than one signal simultaneously. The more complex multiplexing problem for telephony led Raabe in Germany to discover the minimum sampling rate for a given signal bandwidth.

Wireless transmission and frequency-domain multiplexing and the "crowding of the ether" pressed the engineers for precise answers regarding transmission rates, bandwidth, and the effect of noise. Answers appeared in Russia, the United States, and Japan by Kotel′nikov, Shannon, and Someya, respectively.

Shannon's work not only marks the birth of information theory, but it also represents a bridge between developments that can be traced back to telegraphy—Nyquist, Hartley, Kelvin—and mathematics—Fourier analysis and interpolation.

H. Nyquist

By 1928 Nyquist had identified the fundamental parameters that determine the rate at which information can be transmitted in a telegraph.

Nyquist observed that in telegraphy, time is divided into equal units. In each unit, the transmitted information identifies one symbol among m. He showed that the rate at which information can be transmitted increases linearly with both the number of symbols per second and the number of bits per symbol, $\log_2 m$. In 1928 Hartley reached similar conclusions independently, referring to the "considerable historical importance" of Kelvin's *RC* law.

Nyquist also clarified the connection between the transmission speed in telegraphy and bandwidth. This part of his work is closer in spirit to the sampling principle and led Shannon to coin the expression "Nyquist interval". The reciprocal of the Nyquist interval became known as the "Nyquist rate".

Nyquist considered the effect of transmitting periodic signals $f(t)$ and $f(2t)$ through two channels and realized that the channel used to transmit $f(2t)$ would have to deal with frequencies twice as large. He concluded that "frequency band is directly proportional to speed" and determined the proportionality constant by means of an argument involving Fourier series.

Nyquist's discoveries were motivated by concrete practical problems, but his focusing on the essential, abstract characteristics of telegraphy led him to general conclusions of lasting significance.

H. Raabe

By 1939 Raabe had built a multiplexing system for telephony, that is, a system to simultaneously transmit several signals over the same transmission line and recover each of them at the receiving end. The system worked by sampling each input signal in turn, at a certain fixed rate. The question that Raabe had to address was: At what rate should each signal be sampled?

In Raabe's system, each signal is multiplied by a square wave $s(t)$ that determines the sampling rate. He assumes that the input signal is periodic and that it can be expanded in a Fourier series. Multiplication of its components by the Fourier series of the square wave led Raabe to the answer: "distortionless transmission" is possible if the sampling frequency is at least twice the highest signal frequency.

To reach this conclusion, Raabe starts by writing the Fourier series of the square wave $s(t)$, translated to become an even function, as

$$s(t) = c + \sum_{n=1}^{\infty} \alpha_n \cos(n\omega_1 t), \tag{6}$$

where c is a nonzero constant—essentially, the average value of $s(t)$. The multiplexed signal is given by $r(t) = s(t)f(t)$. The question is of course whether $f(t)$ can be recovered from $r(t)$. To answer this question, Raabe expands $f(t)$ in a Fourier series. Instead of multiplying this Fourier series by that of $s(t)$, Raabe invokes superposition and investigates the multiplication of a single term of this Fourier series (one sinusoid, therefore) by that of $s(t)$. If this sinusoid is written as $\cos(m\omega_1 t)$, where m is a real number, the product becomes

$$r(t) = c\cos(m\omega_1 t) + \frac{1}{2}\sum_{n=1}^{\infty} \alpha_n \{\cos[(n-m)\omega_1 t] + \cos[(n+m)\omega_1 t]\}.$$

Of the frequencies involved in this equation, $m\omega_1$ is due to the input signal, and the remaining frequencies, $(n \pm m)\omega_1$, can be regarded as "noise frequencies". Raabe observes that if $m\omega_1$ is known to fall below the smallest noise frequency, which is $(1-m)\omega_1$, there will be no problem in separating noise frequencies from signal frequencies. This leads him to the condition for lossless recovery: $m\omega_1 < (1-m)\omega_1$, that is, $m < 1/2$. In other words, the input frequency, $m\omega_1$, must be below one half the sampling frequency.

Raabe also found that band-pass signals (band-limited signals with no low-frequency terms) can be sampled at a lower rate, the first time this had been noted. These are important theoretical contributions in a work that has a remarkably strong practical character.

Raabe's finding that there is a minimum sampling frequency for low-pass and band-pass signals that, in theory, allows distortionless transmission is another instance of a theoretical discovery prompted by practical needs; in this case, the multiplexing problem.

V. A. Kotel′nikov, C. E. Shannon, and I. Someya

These three engineers introduced the sampling series into communications engineering independently of each other; Kotel′nikov in 1933, Shannon in 1949, and Someya also in 1949. Their proofs differ, of course, and none of them is strictly rigorous, but all are directly appealing to the intuition. Some minor changes in the original notations have been made.

Kotel′nikov's hypotheses are that $f \in L^1(\mathbb{R})$, f satisfies Dirichlet's conditions and is band-limited to $[-\pi w, \pi w]$.

His proof is based on a Fourier inversion principle for such functions, quoted from the classical literature, to the effect that f satisfies (1) where

$$\varphi(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t}\, dt. \tag{7}$$

Next he writes down the Fourier series for φ, the coefficients being $(f(n/w))$ from (1). This Fourier series is now substituted into (1), and (3) results.

An interesting feature of the proof, not found in the other two, is that Kotel′nikov recognizes the presence of a converse to the sampling theorem, that is, that if a function f is represented by a series of the form (3), with $(f(n/w))$ replaced by a numerical sequence (D_n), then it must be band-limited. He argues that, since every term of the series is band-limited to $[-\pi w, \pi w]$ (by a special Fourier transform), the same must be true of its sum.

Shannon works with what we have called Paley–Wiener functions. An interesting feature of Shannon's argument, not found in the other two, is that he shows $(1/w)\mathbb{Z}$ to be a set of uniqueness for PW (this is Part I of the Sampling Principle). He does this by following through a chain of unique determinations: f is uniquely determined by its Fourier transform, which in turn is uniquely determined by its Fourier coefficients, which in turn are uniquely determined by samples of f at (scaled) integer time points. That is, f is uniquely determined by its samples.

As for (3), Shannon argues that the sum is band-limited, just as Kotel′nikov did. This sum coincides with f at the sample points (a simple calculation); therefore, by the uniqueness proved in the first part, the sum is f.

Someya works with functions that he designates as being merely "band-limited", nothing more. His proof is similar in outline to that of Shannon but differs in detail; in fact, it is obscure and unnecessarily lengthy, and it will not be feasible to give a complete account of it here (see [2] for an assessment of this proof). However, the important fact remains that Someya's contribution is a completely independent introduction of the sampling theorem in the engineering context.

Conclusion

We have asked a historical question, but history seldom provides us with clear-cut answers. However, the emergence of sampling in practice seems to be closely connected to the development of communications engineering. Bandwidth limitation was first felt in connection with submarine telegraphy and led to the results of Kelvin and Heaviside. By 1928 Hartley and Nyquist were taking a more general approach to telegraphy, linking transmission speed and bandwidth in a precise way.

As wireless telegraphy and telephony began to develop, new forms of bandwidth limitation were found. The two simplest ways of sharing a channel are time-division and frequency-division multiplexing. The former stimulated the work of Raabe. The latter, in which different signals are assigned different band-regions, raises the question of how much bandwidth needs to be allocated to a signal. We have seen that Kotel′nikov, Someya, and Shannon addressed the problem. The work of Shannon, in particular, established a bridge between developments that had their origin in telegraphy, multiplexing, mathematics, and signal analysis.

The recent growth in bandwidth usage due to the widespread use of mobile devices is raising new challenges. The "crowding of the ether" is a problem as pressing today as it was in Kotel′nikov's time. Bandwidth remains precious: half of the 108 MHz of prime spectrum freed thanks to the recent shift to digital television in the United States was auctioned by the U.S. Treasury and sold for $19 billion. Telecommunications, as a source of problems of theoretical interest and practical importance, has not yet been exhausted.

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Book Review

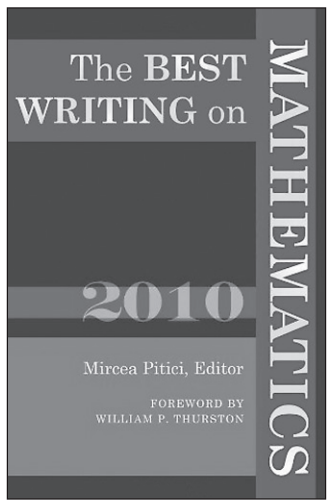
The Best Writing on Mathematics 2010

Reviewed by Gerald B. Folland

The Best Writing on Mathematics 2010
Edited by Mircea Pitici
Princeton University Press, 2010
Paperback US$19.95, 440 pages
ISBN-13: 978-0691148410

What sorts of pieces would you expect to find in an anthology titled *The Best Writing on Mathematics*? Whatever species of article you envision, you can probably find at least one specimen of it in this highly eclectic collection. It contains papers on a wide range of subjects, including current mathematical developments, mathematicians and the practice of mathematics, pedagogical theory, philosophy, and mathematics in the everyday world, taken from sources ranging from the *Bulletin of the AMS* to the *Boston Globe*; their only common feature is that they were originally published in 2009. It is not a book you would want to hand to the people who say "Oh, math was always my worst subject" in the hope of enlightening them about your profession. Most of the articles in it are directed to an audience with a certain amount of mathematical literacy, although none of them is really technical, and even the entertaining essays from the popular press about the mathematical resonances in zombie movies and love affairs are best appreciated by those who can recognize the mathematical humor.

I found a few of the pieces in this book to be superb, many more to be well done but of varying degrees of interest, and a couple that might better have been omitted. I suspect that others would have similar responses, although they might well rank the articles in a different order. (This is probably a normal reaction to most anthologies. I feel the same way about a "Best Short Stories" collection that I recently read, and a cookbook in which I find more than a couple of recipes to add to my permanent repertoire is a real gem.) In any case, this is a book for browsing rather than systematic study. If you feel free to proceed onward to the next article when the one you're reading is not to your taste, you will surely find quite a lot to enjoy.



One of my favorites among the offerings of this book is Freeman Dyson's "Birds and Frogs", which originally appeared in the *Notices*. The title refers to a dichotomy among great mathematicians between big-picture visionaries and master technicians, which Dyson uses as a launching pad for a very engaging essay that involves stories about famous mathematicians and reflections on various interesting areas of mathematics and physics. Readers might enjoy contemplating the interplay between Dyson's dichotomy and another one due to Mark Kac [2, p. xxv]: ordinary geniuses and magicians. An ordinary genius, says Kac, is "a fellow that you and I would be just as good as, if only we were many times better. …Once we understand

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what he has done, we feel certain that we, too, could have done it." Magicians, on the other hand, pull brilliantly original and unexpected ideas out of the inaccessible recesses of their minds. (Kac was making a point about the magician Richard Feynman.) Among nineteenth-century mathematicians, one might name Cauchy, with his cornucopia of useful, accessible results, and Riemann, who invented Riemannian manifolds fifty years before the world was ready for them, as exemplars of the two species. The game of categorizing the present-day giants of mathematics is left to the reader.

Judith Grabiner's "Why Did Lagrange 'Prove' the Parallel Postulate?" is another highlight of the collection. Her main aim is "to show how Lagrange and his contemporaries thought about mathematics, physics, and the universe", especially the way they regarded Euclidean geometry as a method for learning truths about the physical world rather than as an abstract, self-contained system. The story of the reconceptualization of mathematics from Lagrange's time (around 1800) to the present is a fascinating one. Readers whose interest is sufficiently piqued by Grabiner's article to delve more deeply into the story should turn to Jeremy Gray's illuminating and scholarly treatise *Plato's Ghost* [1].[1]

Some of the articles in the book under review are not about mathematics, properly speaking, but about mathematics education and the philosophy of mathematics, written by scholars from departments of education and philosophy. We in the mathematical community may find some of them to be of more interest as glimpses into the worlds of these disciplines than as commentaries on our own. They have their own concerns, their own literatures, and their own characteristic modes of expression, which may strike us as a bit peculiar and perhaps off-kilter. (After reading these articles, one might have more sympathy for the frustrations of scientists who read books and papers on mathematics that are relevant to their work but written by and for mathematicians. We also have our peculiarities of style and our internal concerns that are of limited interest to others.)

I shall cite one example. Appliers of mathematics (whether they call themselves mathematicians or scientists) often develop mathematical tools in a nonrigorous way that does not quite make sense when examined too closely. The most famous case is the infinitesimals of eighteenth-century calculus, but there are plenty of present-day examples. What is one to make of this? The appliers generally don't worry; they don't care if their mathematical methods are internally completely consistent as long as they are consistent with the real-world phenomena they are studying. Pure mathematicians regard the defects as unfinished business that calls for more work and better understanding. It would probably not occur to any of us to try to develop a formal logical framework, involving "para-consistent logics"—logics in which the proposition "(P and not-P) implies (Q)" is not a tautology—into which such half-baked mathematics can be inserted and studied on its own terms. But there are papers in the philosophical literature which do just that, and you can read one of them in this collection: Mark Colyvan's "Applying Inconsistent Mathematics".

I wish the editor had placed Chandler Davis's "The Role of the Untrue in Mathematics" (which is actually the first article in the collection) directly after Colyvan's, for they provide an instructive contrast. Davis gives a working mathematician's take on several topics, including approximate truths, counterfactual statements, and the pitfalls of what he calls "truth-fetishism", that is, "the tradition that mathematics is the truth, the whole truth, and nothing but the truth". His remarks are incisive, crisply phrased, and altogether a delight. In particular, he has the right response to those who would analyze the value of a piece of mathematics on purely internal grounds: "Truth—even truth understood in some new sophisticated way—is not the point," he says. "The point is pertinence. The point is relevance."

Right on, brother.

References

[1] J. Gray, *Plato's Ghost: The Modernist Transformation of Mathematics*, Princeton University Press, Princeton, NJ, 2008.

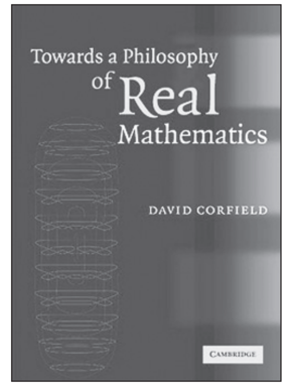
[2] M. Kac, *Enigmas of Chance: An Autobiography*, Harper and Row, New York, 1985.

[1] *Yuri Manin's review of* Plato's Ghost *in the February 2010 issue of the* Notices *is also illuminating.*

Book Review

Towards a Philosophy of Real Mathematics

Reviewed by E. Brian Davies



Towards a Philosophy of Real Mathematics
David Corfield
Cambridge University Press, 2003
US$105.00, 300 pages
ISBN-13: 9780521817226

David Corfield starts his interesting book with a radical rejection of much that has been written about the philosophy of mathematics. He has no interest in ontology, epistemology, logicism, Platonism, Kantianism, nominalism, fictionalism, and so forth and does not mention most of them. Taking his cue from Nietzsche, Lakatos, and a few others, he likens the traditional approach to the examination of a dead body (pp. 3–5). He criticizes the attitude that has led many philosophers of mathematics to imagine that everything of interest to their subject occurred between 1880 and 1930, contrasting this with a very different attitude among the philosophers of physics. We shall see that events since the publication of his book make it even more relevant now than it might have seemed in 2003.

One of Corfield's strongest arguments for turning away from the philosophy of dead mathematics is that it focuses on logical correctness and is powerless to explain why some topics are regarded as crucial to the subject while others are considered irrelevant, no more than technical games. He argues that if one wants to discuss *real* mathematics, one has to place the subject in its historical context and focus on the value judgments that mathematicians make and what lies behind them.

Chapter 1 is fairly demanding philosophically and is written to explain his rejection of the standard approach and to lay out his stall. Put briefly, he is interested in teasing out philosophical questions from the many ways in which real mathematics has been done. He castigates philosophers for presenting the subject as if it were a purely abstract entity that can and should be discussed in a historical and social vacuum. He has to avoid going to the other extreme, letting his book become no more than a study of the history of mathematics. But, being a philosopher, he succeeds in this task.

The first word of the title of the book is "Towards", and Corfield admits that he intends to cover a wide range of topics, each of which needs to be fleshed out in much more detail. Many of his examples come from the fields he knows best, but this is inevitable. Its range is wider than the expertise of most mathematicians, and Corfield knows what he is talking about, at least as far as I could tell. An important merit of the book is its large number of quotations. These help to keep the arguments anchored in the real world—to the extent that mathematicians live in the real world.

Before I continue, a small digression is in order. In most circumstances a reviewer should not read other reviews, in order to assure independence. I followed this procedure, but after I submitted the review, it was suggested that I might say something about the reaction the book had among traditional philosophers of mathematics. I then read a review by Timothy Bays, which appeared shortly after the book came out in 2003. I comment on this review in an appendix.

Corfield greatly admires Imre Lakatos, who was one of the pioneers in this approach to the philosophy of mathematics (see Chapter 7). However, Corfield emphasizes that Lakatos provided little in the way of examples and that he relied excessively on an analysis of the history of Euler's polyhedron theorem. Lakatos maintained that the most interesting part of a mathematical investigation was the struggle to find the best way of expressing one's intuitions about it and that the final formulation of the relevant axioms inhibited further advances. Corfield rejects this as a universal rule and demonstrates, by giving examples, that the development of mathematics can proceed in many different ways (p. 152). To illustrate this, he discusses the Eilenberg-Steenrod axioms at some length (p. 169). These put homological algebra on a much firmer foundation when they appeared in 1945 and were the starting point for later research. They led to a vigorous research program and needed few

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changes subsequently, except for the removal of the dimension axiom in some applications, such as K-theory.

One might add the classification of the finite simple groups, most of which was completed by 1983; see [2] for an account of some issues arising from the nature of the proof. Although it is still the largest ever collaborative project in pure mathematics, the axioms defining finite simple groups had been known for many decades before it started. These examples do not show that Lakatos was totally misguided, only that a developed philosophy of real mathematics must go far beyond the issues that he considered, which most mathematicians would recognize as embodying an important aspect of their work.

There are entire fields in which the production of new axiom systems seems to be largely irrelevant, provided one is willing to make a distinction between axiom systems and mere definitions. To the despair of those philosophers who seek precise meanings for words, there is a continuous scale between these two notions. Corfield mentions the formula of David Bailey, Peter Borwein, and Simon Plouffe in 1997 that enables one to calculate very remote digits in the expansion of π without having calculated all the earlier ones, provided one works base 16. While he states that this is not in itself a fundamental piece of mathematics (p. 66), some fields advance by the accumulations of large numbers of individually modest pieces. The cumulative effect over fifty years can be transformative. One instance is the progress in understanding—rather than just computing—the spectra of differential operators of the type that occur in nonrelativistic quantum mechanics.

Nobody could deny that Bayesian statistics is of considerable philosophical interest. A subject with eighteenth-century origins, it was widely considered disreputable, right up to the 1970s, because its dependence on human judgment was considered incompatible with the Baconian philosophy of science. In the formulation due to Jaynes, this problem was minimized by the use of maximum entropy priors, but in applications outside physics it is often necessary to use expert judgment to set the priors. Over the next thirty years Bayesian statistics was rehabilitated to such an extent that in 1996 Bill Gates could claim that "Microsoft's competitive advantage is in its expertise in Bayesian networks." Today it is fully recognized as the most appropriate technique for analyzing certain types of highly complex data sets. It would be interesting to explore the relationship, if any, between this and the demise of the Baconian philosophy of science under the onslaught of Karl Popper's fallibilism, now regarded by scientists as the natural philosophy of science. A posthumously published book by Jaynes appeared in 2002; see [8] for a review.

Corfield discusses the issues surrounding Bayesianism and points out that leading Bayesian statisticians have disagreed about the prospects of using its methods to study mathematical discovery, even in a qualitative manner (pp. 110–119). One of the problems emphasized by John Earman is the possibility of "non-Bayesian shifts", in which some minor observation, or even a dream, might cause one to completely reassess one's view about the nature of a problem. There is a well-established literature attesting to the frequency of such events, and many mathematicians have personal experience of them. Corfield goes on to ask what type of evidence persuades mathematicians to change their minds about the likely truth of a proposition. Checking some identity in a finite number of cases cannot be enough, because there are well-known examples of arithmetic identities whose first counterexample occurs for an extraordinarily large integer; Corfield mentions a certain bound on the number of primes less than x in terms of a function $li(x)$, for very large x (p. 127). One might also mention the Pólya conjecture for prime factorizations, which holds for all natural numbers up to, but not including, 906,150,257; in spite of its falsity, it appears that the conjecture may well be true for "most" numbers, in a certain well-recognized sense.

On the other hand, when John McKay noticed a very small number of coincidences between the dimensions of representations of the monster group and functions arising in the study of elliptic curves, this attracted great attention (pp. 125–126). Eventually Richard Borcherds found a logical thread connecting these two very different facts in 1992, an achievement for which he was awarded a Fields Medal in 1998. Corfield explains the compelling nature of the coincidences as based on the deep background knowledge of those in the field. The experts felt that it should be possible to use the links between the two subjects to provide a proof. It is not easy to see how intuitions about the significance of such a coincidence could be formalized, but there is no doubt that they exist and that they are important. To "explain" them as arising from mysterious Platonic perceptions of an ultimate reality is simply to refuse to address the issue seriously [7].

Chapter 3, on uncertainty in mathematics and science, discusses a range of interesting interconnections between the subjects. I will not spoil his tale by listing these, but Corfield has missed an opportunity in his brief discussion of quantum field theory (pp. 137–138). He is not referring to this field as it was understood in the 1970s, which has extremely impressive experimental support in spite of the fact that it is not coherent by the standards of pure mathematicians, but to the circle of ideas related to superstring theory, mirror manifolds, duality, and so forth. He should have drawn the reader's attention to the fact that a large number of physicists do not regard this as physics, because over a period of thirty years it has not produced any even remotely testable prediction. Even if evidence for supersymmetry appears in the current round of experiments at the Large Hadron Collider, this will not validate string theory itself. The fact that string theory has led to some extremely interesting mathematical conjectures, some of which have been proved in the accepted rigorous sense, does not say anything about its status *as physics*. These are very controversial issues, with some very big names on both sides. At stake is the very definition of science, as well as the question of how long one is justified in suspending judgment.

There are many other problems that merit the type of treatment that Corfield is interested in. One arises in the spectral theory of operators on Banach spaces. By the 1970s numerical analysts had already realized that the eigenvalues of medium-sized matrices could depend on the matrix entries in a very unstable manner. Nick Trefethen took up this issue around 1990 and introduced the notion of pseudospectra, which makes sense in infinite dimensions as well as for matrices [11]. This notion gathers under one roof a plethora of different elements, which had previously been studied largely independently. Some researchers reacted negatively to the subject, and, indeed, one can do without it, but it has been accepted by steadily increasing numbers of mathematicians over the last ten years because it helps to clarify the nature of problems that had been faced in various contexts. Although motivated by problems in numerical analysis, its influence has now penetrated to the theory of pseudodifferential operators. Perhaps it lies near the higher end of Corfield's scale from crucial theories to those that are "not worth a jot" (p. 11).

A second issue relating to analysis is its heavy dependence on inequalities. When an analyst states that an equality is merely a pair of inequalities that go in both directions (with different proofs in the two cases), this is precisely what makes many other mathematicians feel that analysis is an unnatural subject. Unfortunately for them, it shows no sign that it is about to abandon this "abhorrent" methodology. Analysts also seem to be more sympathetic to constructive mathematics, particularly the version of Errett Bishop [4], than are algebraists and geometers. Perhaps this is because analysts are closer to applied mathematics and see that an existence proof that relies on a nonconstructive element, such as a compactness argument, may carry very little useful information about the properties of the solution. These differences are partly aesthetic and partly driven by necessity, but they certainly create a barrier between analysts and many other pure mathematicians.

In 2004 Peter Swinnerton-Dyer made a distinction between structural statements such as the Riemann hypothesis and accidental statements such as Goldbach's conjecture [9, p. 2439]. In purely logical terms, the word "accidental" is meaningless in this context, but somehow one knows what he means. I suspect that his distinction involves a value judgment: far more people care about the truth of the Riemann hypothesis, and for reasons that a philosopher might wish to understand. But value judgments are dangerous; if either conjecture were to be proved false, it would suddenly seem much less interesting.

In 2001 Alain Connes, a committed Platonist, who has spent a lifetime working on C*-algebras and their applications, nevertheless excluded the theory of Jordan algebras from the Platonic world of mathematics [6]. How do mathematicians make such value judgments, and are their opinions more than prejudices? Corfield gives a lengthy account of another example, the theory of groupoids, which are still not nearly as well accepted as he thinks they should be, in spite of strong support from Alexander Grothendieck, Ronald Brown, Alan Weinstein, Jean Bellisard, Alain Connes, and others (pp. 208–214). Philosophers should try to understand the issues that underlie the acceptance or nonacceptance of such concepts. Category theory has an even stronger claim to being an essential part of mathematics, perhaps as an alternative to set theory, but many pure mathematicians have no interest in either subject and do not seem to suffer from that fact (p. 201). Corfield also points out that the apparently accepted set-theoretic definition of a function as a set of ordered pairs satisfying certain axioms is not well adapted to computation, where the older idea of a function as an algorithm that produces an output given some value of the input is more relevant. This reminds one of the warning of Lakatos that one should not confuse a set of axioms with the intuitions that gave rise to it (pp. 201–202).

Since Corfield's book was published, the ingress of computers into the preserve of mathematicians has proceeded apace, and a meeting of the Royal Society was convened in 2004 to discuss "The Nature of Mathematical Proof" [9]. Some of the issues that arose were anticipated by Corfield, but not all. One was Thomas Hales's solution of Kepler's sphere-packing problem. Robert MacPherson, the editor of the *Annals of Mathematics*, described why the editors of that journal had felt compelled to accept the paper even though a team of experts had eventually abandoned the effort to check all the details after several years of intense work. (Unfortunately MacPherson's contribution was not published.) Perhaps the notion that pure mathematics revolves around concepts that mathematicians can settle with certainty will one day be consigned to the dustbins of history.

In 1998 David Ruelle wrote that "human mathematics is a sort of dance around an unwritten formal text, which if written would be unreadable" [10]. In spite of this, the Royal Society meeting needed to discuss the slow but steady growth of formally verified mathematics. This arose from the development of computer languages and programs that can check every step of a purported proof for correctness and also fill in moderately small gaps in accordance with accepted formal rules. Applications of this process still require much effort, but a formal proof of the four-color theorem was announced by Georges Gonthier and Benjamin Werner in 2004. Such proofs are not intended to be read, and they do not provide the intuitive insights of traditional rigorous proofs. Formal proof techniques and other developments, such as the growing use of interval arithmetic in rigorous computer-assisted proofs, seem to be taking us across the Rubicon. This fate can be avoided if enough mathematicians insist that they will accept only proofs that can be understood without artificial aids, however the proofs might have been generated. There is no evidence that this will happen.

The task of a reviewer is to whet readers' appetites, not to milk the contents so thoroughly that nothing worthwhile is left. I have said little or nothing about Corfield's lengthy discussions of the role of analogy in mathematics, the mutual interactions between mathematics and physics, and higher-dimensional algebra. The book abounds with thought-provoking ideas and well repays the effort needed to understand the subtleties of its subject. Much remains to be done, and one can only hope that others will join Corfield in taking up the task.

Appendix

Corfield's book has been discussed in several places on the Web, but I will restrict attention to a review written by Timothy Bays in 2004 [3], which represents some of the reactions of traditional philosophers. Some of the differences between us can be traced to the fact that Bays is a professional philosopher whose specialty is formal logic and related fields, whereas I am an analyst whose specialties are operator and spectral theory. Bays describes Chapter 1 as polemical, whereas I describe Corfield as setting out his stall. It is indeed the case that, in doing so, Corfield argues that the other stall-holders are selling "dead mathematics", but one need not be put off by this rhetoric if he is selling something new and interesting, as I believe he is.

Bays is right when he states that Corfield dismisses mathematical foundationalism without exploring its faults or explaining why many philosophers have found it attractive. Corfield, however, is advocating a radical expansion of the range of philosophical debate, and this cannot be achieved by yet another discussion of old issues, even if they are important. Today, one might refer to the Royal Society meeting, for example the articles of Michael Aschbacher and Paul Cohen [2, 5], as evidence of the steadily increasing range of quasi-philosophical issues that mathematicians now have to cope with.

Bays does not consider that Corfield provides substantial new philosophical insights but accepts that there probably are new insights available to those who are prepared to do the work of fleshing out the program that Corfield is outlining [3, endnote 6]. Corfield covers too much ground to please many professional philosophers, who are well aware that attractive ideas may be very difficult to develop in detail. One might liken his approach to those of Popper, Lakatos, Kuhn, and others, who gave overviews of new areas that needed to be explored. Only if this is done with conviction will others start to travel over the unfamiliar ground that he is describing. The present review is written for mathematicians, and it seems appropriate to adopt their standards, whether or not they mesh with those of philosophers of mathematics.

Andrew Arana has written yet another review of Corfield's book [1]. The review discusses a number of further issues of great interest and is strongly recommended to readers.

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Remembering Constance Reid (1918–2010)

Gerald L. Alexanderson

Constance Reid will be remembered in mathematics for many years to come. She was not a mathematician, though she had close ties to important mathematicians, but what she did for mathematics may have more influence than the research of many professionals in the field. She would have a place in history if she had done nothing more than write her biography of David Hilbert.

Photograph by George Csicsery.

Constance Reid

Prior to Constance's appearing on the mathematical scene, there was rather little to read in mathematics, at least in English, beyond textbooks and monographs. Students wishing to learn of the culture of mathematics or amateurs who loved mathematics could go to *What Is Mathematics?* by Courant and Robbins, the unfortunately named *Men of Mathematics* by E. T. Bell, along with a few others. And then, in the mid-1950s, influenced by her brother-in-law, Raphael Robinson, a number theorist at Berkeley, and her mathematician sister, Julia Robinson, Constance wrote *From Zero to Infinity: What Makes Numbers Interesting* (Crowell, 1955) and *A Long Way from Euclid* (Crowell, 1963). Both were successful and filled an obvious need. Her next book was initially a bit more problematical. Constance actually started work on a successor volume to *Men of Mathematics* and had written several short biographies of post-Poincaré mathematicians when she encountered the achievements of Hilbert. She found his story to be so captivating that she concentrated her efforts on him and jettisoned the other subjects. She was often asked, "Why Hilbert?" to which she responded that his life was a wonderful narrative. And when her work was referred to as a biography, she demurred. She said she thought of it as a "life", in the Plutarchian sense.

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She was fond of telling at mathematics meetings of her experience when she approached her publisher with the proposal to write a biography of the most eminent early-twentieth-century mathematician, David Hilbert. His response was that the only thing that would sell worse than the biography of a mathematician would be a book about South America! But she persisted and found a publisher, Springer. It appeared in 1970 and was an instant classic, a book for the ages. Beautifully written and carefully researched—she discovered Hilbert's papers in Göttingen uncatalogued in boxes at the Mathematical Institute, thus quite possibly saving them from loss—her book was a sensation and resulted in many invitations to speak at regional and national meetings. Her earlier publisher probably came to regret his quick dismissal of Constance's proposal. Times have changed. Consider the flood of popular mathematics books now appearing every year: descriptions of the solution of classic problems, biographies, histories, problem collections, even plays and novels about mathematicians. It was unimaginable in the 1950s. And, in many ways, Constance was there first. And it was with the publication of the Hilbert biography that I became aware of her work. It led to a long friendship and even some collaborations along the way.

Constance, who was born in St. Louis in 1918, and her sister Julia both started college at San Diego State, Constance majoring in English, Julia in mathematics. But Julia moved on to Berkeley

Photograph by Ralph Bowman.

Constance and her sister Julia at the beach.

where, years later, she did essential work on Hilbert's Tenth problem, which was eventually solved by Yuri Matiyasevich. This work earned her a MacArthur Fellowship and the honor of being the first woman to be president of the American Mathematical Society. Constance's first book was *Slacks and Calluses: Our Summer in a Bomber Factory* (Longmans, Green; 1944), which told about her experiences working in a defense plant during World War II. Later she married Neil D. Reid, a San Francisco attorney, and they lived and raised a family in their charming craftsman-style house in the Ashbury Terrace section of San Francisco.

The Hilbert book was followed by two additional

Photograph by Ralph Bowman.

Constance (at right) and Julia.

biographies: *Courant in Göttingen and New York* (Springer, 1976), and *Neyman—from Life* (Springer, 1982). The first in some sense continued her work on the biography of Hilbert because Richard Courant, before moving to New York in 1937, had been director of the famous mathematical institute at Göttingen. For anyone interested in the migration of enormously talented mathematicians and scientists from Europe to the United States at that time, the Courant book is essential reading. The Neyman book chronicled the influence of the Polish statistician, Jerzy Neyman, on American science when he led the statistics department at Berkeley to eminence during the postwar period.

Her sister Julia had remarked on how influential E. T. Bell's book had been in convincing her to be a mathematician. When Don Albers and I found that Bell's only son, Taine Bell, was living close by, we contacted him, because we too had been motivated to go into mathematics by Bell's famous book. Because we had recently worked with Constance on *International Mathematical Congresses: An Illustrated History 1893–1986* (Springer, 1986) and *More Mathematical People* (Harcourt Brace Jovanovich, 1990), we managed to get Constance to take over a project we had planned but were too committed to other projects to do ourselves—a "life" of E. T. Bell. She took on the task with her usual enthusiasm, and a wonderful book it turned out to be. Much of Bell's life was known, but other aspects were not known, even to his family. After extensive sleuthing in California and Britain, she

Photograph by Neil Reid.

Constance with Russian mathematician Yuri Matiyasevich, January 1999.

published *The Search for E. T. Bell, Also Known as John Taine* (Mathematical Association of America, 1993).

Her last book was *Julia: A Life in Mathematics* (MAA, 1996). These books in a sense continued her work on twentieth-century mathematicians. When asked why she did not turn attention to earlier people such as Riemann, she replied that she preferred what Henry James called "the visitable past". During that period I recall that the three of us traveled to New York to attend the memorial service for Walter Kaufmann-Bühler, mathematics editor at Springer, who had urged us to do the book on the Congresses. Traveling with Constance was always rewarding. She seemed to know everyone who mattered in mathematics. On that trip we visited a brownstone in Chelsea, the home of Hermann Weyl's daughter, where a bronze bust of Weyl was in the library. On another occasion, I recall "doing the French Quarter in New Orleans" with Constance and a good friend of hers, Natasha Artin Brunswick, the mother of Michael Artin and one-time wife of Emil Artin, both extraordinary mathematicians.

Constance was a woman of great style and presence. She cared about clothes and wore breathtakingly elegant and occasionally dramatic jackets and accessories. She was an ardent fan of the ballet, reeling off names of dancers and descriptions of memorable performances with ease. When she entered a room people sat up and took notice.

Her husband is a pilot, and they sometimes flew to meetings and to family events in their own plane. She lived a full and active life. And she made lasting contributions to the culture of mathematics. She died October 14, 2010, in San Francisco after a two-year illness.

Assessment and Placement through Calculus I at the University of Illinois

Alison Ahlgren and Marc Harper

University students come from many geographic locations and types of secondary and post-secondary schools (including public, private, and preparatory schools) with very different mathematical backgrounds. This results in a diversity of mathematical knowledge, augmented by the fact that what constitutes precalculus varies with source institution and instructor preferences, confounding traditional indicators of student knowledge and maturity such as high school grades. Additionally, students and institutions have disparate expectations of sufficient preparation for higher-educational institutions.

Mathematics placement at the University of Illinois prior to 2007 was an assessment based on ACT math scores. Because of undesirably high proportions of students failing to successfully complete Calculus I, the mathematics department began searching for a more effective placement program. Data from the placement program indicated that the ACT math score was a poor measure of preparedness, while external research indicated that the online software product ALEKS can serve as a preparedness measure for calculus, as higher initial ALEKS scores correlated with higher final grades [3].

ALEKS[1] (**A**ssessment and **Le**arning in **K**nowledge **S**paces) is an artificially intelligent mathematics learning product commercially available to individual users and institutions for assessment and gain of knowledge. ALEKS mathematics courses are available for many grade levels and courses up to precalculus as well as for other subjects.[2]

In 2007 the mathematics department at the University of Illinois began a new placement program using ALEKS, which was chosen for the ability of students to be assessed remotely via the Internet with immediate reporting and for the ability of ALEKS to provide personalized remediation. The placement exam is an ALEKS assessment—an adaptive series of questions that determines the knowledge state of a student, which is a set of items such as the ability to plot an exponential function or to solve an equation involving rational expressions. The total number of items in the assessment is approximately 200, of which students are asked approximately thirty directly (performance on the remaining items is inferred from these responses). The initial assessment primes the learning module within ALEKS, and the organization of items within the knowledge space allows ALEKS to present to a particular student items that he or she is ready to learn or review. Students are automatically reassessed as they progress.

The remediation component of ALEKS complements the use of an assessment as a placement exam by allowing students to self-remediate before courses begin. At Illinois, the students must

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[1] *See* http://www.aleks.com.

[2] *For ALEKS course products, see* http://www.aleks.com/about_aleks/course_products.

achieve the minimum measure of readiness in the four months preceding the start of a course. This approach allows the absolute enforcement of the placement policy, avoiding the difficulties with static and not easily repeatable placement exams (e.g., paper and pencil exams given in large groups on few occasions). Moreover, the responsibility for review of prerequisite material is shifted from the instructors and course to the students. The learning mechanism allows students to refresh forgotten knowledge and distinguish themselves from students who are in a novel learning situation. The placement program and policy were specifically designed to be independent of uncontrollable variables such as instructor variation and textbook selection. The placement exam and syllabi can be modified to respond to variations in the student population and course goals while maintaining an objective standard of readiness.

All students complete an ALEKS assessment and are offered the opportunity to use the mechanisms within ALEKS (the learning module) to remediate if they do not place into the course they wish to take. Roughly 20 percent of the students being assessed independently use the learning module or take another assessment to improve their placement. ALEKS is also used in on-campus summer programs that reach out to certain subsets of the student population to help prepare them for college-level studies. A sufficient ALEKS score is the only access point to enrollment—grades from any prerequisite courses neither allow nor deny access to enrollment under the placement policy. The placement score is the highest cardinality achieved on an assessment within four months of the start of the course.

The underlying hypothesis of the placement program at the University of Illinois is that ALEKS effectively measures the current knowledge of students before beginning a course and that the initial knowledge should be indicative of student performance. Three years of data (over 20,000 assessments) support this hypothesis. In many of the courses and semesters examined, the data shows very high correlations between mean grades over small ALEKS score ranges and range midpoints. This greatly outperforms the former placement policy. Similar correlations for ACT math scores were generally much lower, less consistent year to year, and sometimes negative. (Similar correlations were found for the SAT in previous work by Baron and Norman [1].) ALEKS assessments also report scores for subcategories such as trigonometry and geometry, radical expressions, and exponentials and logarithms. Data analysis indicates that ALEKS scores and some subscores correlate well with final grades and that the ALEKS-based placement program lowered failure and withdrawal rates in nearly all the placement classes in each semester. The placement exam does not require mastery of every subcategory; nevertheless, a sufficient score for calculus indicates broad mathematical knowledge and maturity and near-mastery or mastery of some subcategories. For placement, only the total score was used; the subscore correlations are an interesting consequence of the data provided by an ALEKS assessment.

Using the subscore data in aggregate, Illinois is able to compare the knowledge states of students completing precalculus to those entering directly into calculus. This allows an objective measurement of the effectiveness of precalculus, as taught by the university, as preparation for calculus. If students want to take one of the placement courses in a future semester at Illinois, they must take another assessment so that the knowledge state is current. This allows the progress of some students to be followed through successive courses, enabling the measure of specific knowledge gains and aggregate knowledge gain in several subcategories. Results from these comparisons will be included in upcoming publications.

Departmental changes due to the use of ALEKS also include a reduction in advising staff in the mathematics department and increases in enrollment (perhaps because of the limited temporal validity of the assessments). Lower withdrawal percentages yield more stable rosters, which may allow for more effective course planning and management of instructional resources. The success of the placement program at the University of Illinois has led to several other institutions adopting similar placement mechanisms, including the University of Arizona, Arizona State, the University of Colorado, the University of Missouri, and others.

The authors believe that the success of the placement program is based largely on the accountability of a knowledge-based placement exam that is independent of grades in previous coursework and the active assumption of responsibility of preparation by students. The link between the placement exam and the personalized remediation mechanism strengthens the effectiveness of the placement program, allowing students to demonstrate preparation in a low-risk and high-reward setting. Finally, we believe the success of any placement program hinges on strict enforcement, which is university policy at Illinois. These properties and policies support a successful program that serves the student body and university well.

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Book Publishing for the Mathematician

Steven G. Krantz

Publishing a book is a way of planting your flag in your special field of study. Along with your research reputation, it can establish you as a "name" and an authority figure. But prospering in the book-publishing environment can be a challenge. With the advent of electronic media, print on demand, Amazon, e-readers, preprint servers, and other new technologies, the entire publishing process is dramatically, and sometimes bewilderingly, changing. In addition, the entire market for books has a new *gestalt*. If you are going to be a player in this brave new world, then you must become conversant with some of the details.

Eighty years ago publishers for advanced mathematics books were few. It was common for a mathematician to write a set of notes (based on lectures, for instance), have it typed up, and then mail it out to colleagues. No formal book was ever published. One of the treasures of the Princeton math library is a special room devoted to sets of notes going back to the time of Hilbert.

Fifty years ago the situation changed dramatically. That was the Sputnik era, and science was undergoing explosive growth. At that time, virtually every university mathematics library had a standing order with every major mathematics book series publisher. As a result, any math monograph that was published had guaranteed sales of 1,500 copies. This was a great situation for everyone: publishers were anxious to publish a great volume of work, and they could price the books reasonably because of the guaranteed sales. The very fact that the Springer Lecture Notes Series has over 2,000 volumes is moot testimony to the milieu that I have just described.

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Today life is closer to the bone. Library budgets are tight, and few academic libraries have standing orders. Libraries pick and choose which volumes they will buy. When an editor prices out a new book, he or she must estimate the sales; the estimate will generally be rather conservative—typically it runs in the hundreds. Furthermore, in the good old days, the royalty was a standard 15 percent. Today, some editors will sing a sob story to the author about decreased sales and increased costs and offer 12 percent or 10 percent (or even less). A lucky author will get a sliding scale. Other factors play a role today in the income cycle. For instance, licensing to Amazon and other third-party vendors contributes to the revenue stream.

A great many of the advanced mathematics publishers from the 1960s and 1970s have vanished from the math scene.[1] Now only a handful of publishers will even consider an advanced math book, and they are *very* picky.

There are, of course, positive, new dimensions to this picture. Because the big publishers offer their books in electronic bundles, a large number of books are available to a library for a discounted price. Electronic bundles also have many advantages for the library: many users can simultaneously access a book, books cannot be stolen, there are no overdue books, and books do not take shelf space.

[1] *These include Academic Press; Addison-Wesley; Allyn and Bacon; Benjamin; Blaisdell; Cepadues Press; Courant Institute Press; Dickenson; Gordon and Breach; Harvard University Press; Holt, Rinehart, and Winston; Interscience; Johns Hopkins Press; Kluwer; Macmillan; Marcel Dekker; McGraw-Hill; M.I.T. Press; Norton; Pergamon; Pitman; Prentice-Hall; Research Studies Press; Ungar; University of Chicago Press; Van Nostrand; Wadsworth; and John Wiley. This list is far from complete.*

As advanced mathematics publishers have dwindled, the process of getting a book to press has accelerated. My first book, published in 1982, was published according to a century-old paradigm. After the book was accepted for publication, I submitted a hardcopy manuscript to the publisher. The publisher copyedited my manuscript and I reviewed the result. Then the publisher produced *galley proofs*—typeset material that is not broken for pages and in which figures and other formatting features do not yet appear. The galleys contained many author-directed queries and edits, and I had to respond to them with a red pen. Then the publisher produced *page proofs*. At this point the book was broken up into pages, with all figures and footnotes and other displays in place. I was given the opportunity to do a final proofing. I also produced an index by means of a laborious process using index cards. Finally the book went to bed.

Today the process is streamlined. When I complete work on a book today, a typical scenario is this: I send (often by email) a `*.tex` and a `*.pdf` file to the publisher, together with individual files for each of the graphics. After a period of work, the publisher then remits to me the `*.pdf` file with electronic "sticky notes" telling me of edits and author queries. [Note that this `*.pdf` file will have all figures and displays in place just as they will appear in the final book.] *Then it is up to me* to implement the necessary changes in TeX code. After my part of the transaction, I submit a new collection of electronic files to the publisher. Note that the steps of galley proof and page proof no longer make any sense. LaTeX produces page proofs from the get-go. Of course there are LaTeX and Plain TeX utilities that make producing a detailed index efficient and easy. As you can see, the entire process has many fewer steps, and the author has much more control (and much more responsibility).

Some of the features of this new system are perhaps lamentable. Many of the tasks that the author used to delegate must now be performed by the author (or perhaps by a hired graduate student). These include the typing (typically you type in your own TeX code), the production of the figures (which you generally do with software), the typesetting, and the copyediting (which some publishers now downplay). Although there are still publishers with high standards who keep a strong hand in every step of the publishing process, there are a number of publishers today who take many shortcuts. One of the main things that a publisher *does* do for the author today—and which you, the author, cannot do for yourself—is to promote and market the book. Only a publisher can set up links to journals and to reference works and arrange conferences at which the book is promoted. The publisher knows how to get your book onto Amazon.

Dealing with a publisher means that there will be a publishing contract. Many mathematicians find this document to be disconcerting, as it is obviously written by the publisher's lawyers in a fashion that favors the publisher. You should read your contract carefully—check for due date, royalty rate, number of free copies, copyright provisions, option not to publish, and many other details. You may find it useful to get your own attorney/agent to negotiate the publishing contract for you.

When the book appears now, it does so simultaneously both in hardcopy form and in electronic form. It goes immediately to Amazon. Amazon often archives books in electronic form only and can produce a hard copy via the print-on-demand process. Or else Amazon can sell you (with the cooperation of your publisher, of course) an electronic version of the book for your Kindle. One of the truly marvelous features of today's book-publishing world is that there is no longer a concept of a book going out of print.

On the whole, publishing a book can be a rewarding experience. It is a great way to shape your subject and to put your mark on it. I have excellent relationships with all my publishers and editors, and these are parts of life that I value and that I can build on. The best publishers are working *for* you and will take good care of you. You can develop a good working relationship with such a publisher, one that will last many years.

Deciding to publish a book is a big step. It will take a great deal of your time and impact your life in profound ways. If you are going to get the most out of the process, then you must become informed of the many features that are involved. And you must be prepared to take steps to protect your interests. Nobody else will do it for you.

Mathematics People

Eisenbaum and Kaspi Awarded Itô Prize

Nathalie Eisenbaum of the University of Paris VI and Haya Kaspi of the Technion Israel Institute of Technology have been awarded the 2011 Itô Prize for their joint paper "On permanental processes", published in *Stochastic Processes and Applications* **119**, no. 5 (2009), pp. 1401–1415. The prize honors the memory and celebrates the legacy of Professor Kiyosi Itô and his vast and seminal contributions to probability theory. The winning article was selected by the editorial board of the journal and consists of a monetary award of US$5,000.

—Bernoulli Society announcement

Daubechies Awarded 2011 Kilby Medal

Ingrid Daubechies of Duke University has been awarded the 2011 Jack S. Kilby Signal Processing Medal by the Institute of Electrical and Electronics Engineers (IEEE). She was recognized "for her pioneering contributions to the theory and application of wavelet transforms, which have revolutionized audio, image, and video devices and communications systems."

—From a Duke University press release

MAA Awards Presented

The Mathematical Association of America (MAA) presented several awards at its Summer MathFest in Lexington, Kentucky, August 4–6, 2011.

The Carl B. Allendoerfer Awards are given for articles of expository excellence published in *Mathematics Magazine*. They carry a cash award of US$500. The awardees for 2011 are Gene Abrams and Jessica K. Sklar, "The graph menagerie: Abstract algebra and the mad veterinarian", *Mathematics Magazine* **83**, no. 3 (2010), pp. 168–179; and Curtis D. Bennett, Blake Mellor, and Patrick D. Shanahan, "Drawing a triangle on the Thurston model of hyperbolic space", *Mathematics Magazine* **83**, no. 2 (2010), pp. 83–99.

The Trevor Evans Awards are presented to authors of exceptional articles that are accessible to undergraduates and published in *Math Horizons*. The amount of the cash award is US$250. The awardee for 2011 is Lawrence Brenton, "The adventures of π-man: Measuring the universe", *Math Horizons* **17**, no. 4 (2010), pp. 12–15.

The Lester R. Ford Awards are given for articles of expository excellence published in *The American Mathematical Monthly*. The award carries a cash prize of US$250. The recipients for 2011 are: James T. Smith, "Definitions and nondefinability in geometry", *American Mathematical Monthly* **117**, no. 6 (2010), pp. 475–489; Marvin Jay Greenberg, "Old and new results in the foundations of elementary plane Euclidean and non-Euclidean geometries", *American Mathematical Monthly* **117**, no. 3 (2010), pp. 198–219; Mark A. Conger and Jason Howald, "A better way to deal the cards", *American Mathematical Monthly* **117**, no. 8 (2010), pp. 686–700; Alexander Borisov, Mark Dickinson, and Stuart Hastings, "A congruence problem for polyhedra", *American Mathematical Monthly* **117**, no. 3 (2010), pp. 232–249; Aaron Abrams and Skip Garibaldi, "Finding good bets in the lottery, and why you shouldn't take them", *American Mathematical Monthly* **117**, no. 1 (2010), pp. 3–26.

The George Pólya Award is given for articles of expository excellence published in the *College Mathematics Journal*. It carries a cash award of US$500. The 2011 honorees are Jonathan K. Hodge, Emily Marshall, and Geoff Patterson, "Gerrymandering and convexity", *The College Mathematics Journal* **41**, no. 4 (2010), pp. 312–324; and John Martin, "The Helen of geometry", *The College Mathematics Journal* **41**, no. 1 (2010), pp. 17–27.

The Merten M. Hasse Prize is designed to be an encouragement to younger mathematicians to take up the challenge of exposition and communication by recognizing a noteworthy expository paper appearing in an MAA publication. The prize carries a cash award of US$1,000. The 2011 awardees are Alissa S. Crans, Thomas M. Fiore, and Ramon Satyendra, "Musical actions of dihedral groups", *The American Mathematical Monthly* **116**, no. 6 (2009), pp. 479–495.

Henry L. Alder Awards for Distinguished Teaching by a Beginning College or University Mathematics Faculty Member carry a cash award of US$1,000. The 2011 awardees are Alissa Crans, Loyola Marymount University; Sarah Eichhorn, University of California Irvine; and Sam Vandervelde, St. Lawrence University.

—From an MAA announcement

O'Brien Named Jefferson Fellow

TIMOTHY O'BRIEN of the Department of Mathematics and Statistics, Loyola University, Chicago, has been named a Jefferson Science Fellow for 2011–2012. He obtained his Ph.D. in statistics from North Carolina State University. His current research focuses on robust optimal experimental designs whereby practitioners are provided the means to efficiently conduct their research studies with the lowest cost. He also teaches short courses worldwide on statistical consulting, statistical design, environmental and biomedical methods, and modeling of diverse systems and phenomena. He helped establish the undergraduate bioinformatics program at Loyola and has won numerous teaching awards. As a Peace Corps volunteer in Benin, he taught mathematics (in French), and he has twice received William J. Fulbright scholarships to consult, teach, and do research at Chiang Mai University in Thailand. Additionally, he has studied, taught, or conducted research in approximately thirty countries outside the United States and regularly provides technical assistance to researchers at U.S. and international universities and organizations, including Partners in Health, Statisticians WithOut Borders, and the Infectious Disease Institute in Kampala, Uganda.

The Jefferson Science Fellows (JSF) program at the U.S. Department of State is intended to involve the American academic science, technology, and engineering communities in the formulation and implementation of U.S. foreign policy. Each fellow spends one year at the U.S. Department of State or the U.S. Agency for International Development (USAID) for an on-site assignment in Washington, D.C., that may also involve extended stays at U.S. foreign embassies and/or missions. The JSF program is administered by the National Academies and supported through a partnership of the U.S. science, technology, and academic communities, professional scientific societies, and the U.S. Department of State.

—From a National Academies announcement

2011 Dirac Medals Awarded

EDOUARD BRÉZIN of the École Normale Supérieure, Paris; JOHN CARDY of the University of Oxford; and ALEXANDER ZAMOLODCHIKOV of Rutgers University have been awarded 2011 Dirac Medals by the Abdus Salam International Centre for Theoretical Physics (ICTP). According to the prize citation, they were honored "in recognition of their independent pioneering work on field theoretical methods to the study of critical phenomena and phase transitions; in particular for their significant contributions to conformal field theories and integrable systems. Their research and the physical implications of their formal developments have had important consequences in classical and quantum condensed matter systems and in string theory."

The ICTP awarded its first Dirac Medal in 1985. Given in honor of P. A. M. Dirac, the medal is awarded annually on Dirac's birthday, August 8, to an individual or individuals who have made significant contributions to theoretical physics and mathematics. The medalists also receive a prize of US$5,000. An international committee of distinguished scientists selects the winners from a list of nominated candidates. The Dirac Medal is not awarded to Nobel laureates, Fields Medalists, or Wolf Foundation Prize winners.

—From an ICTP announcement

China Girls' Olympiad

Eight high school girls from the United States have all won medals in the 2011 China Girls' Mathematical Olympiad (CGMO). The competition was held in Shenzhen in southern China's Guangdong Province. On the U.S. team, DANIELLE WANG of Campbell, California, and VICTORIA XIA of Vienna, Virginia, were awarded gold medals; JULIA HUANG of Saratoga, California, was awarded a silver medal; and REBECCA BURKS of Los Altos, California, CHRISTINA CHEN of Newton, Massachusetts, SARAH HERRMANN of La Jolla, California, ELAINE HOU of Seffner, Florida, and HAOTIAN (TIFFANY) WU of Sugar Land, Texas, were all awarded bronze medals. The students' participation was sponsored by the Mathematical Sciences Research Institute (MSRI) for the fifth consecutive year.

—From an MSRI announcement

Norrie Everitt (1924–2011)

Professor William Norrie Everitt, FRSE, died on July 17, 2011, at age eighty-seven. He will be remembered as a leading British mathematical analyst who contributed to differential equations, linear operators, spectral theory, inequalities, and special functions. He had been a member of the AMS since 1959.

Norrie was born June 10, 1924, in Birmingham. In 1944 he graduated with first-class honours in electrical engineering from the University of Birmingham. In 1947, while serving in the U.K. armed forces, he suffered a fractured spine. After being told he might never walk again, he climbed the Matterhorn at age twenty-five. He entered Oxford (Balliol College) in 1949 to study mathematics and received his D.Phil. under the supervision of E. C. Titchmarsh in 1955.

Norrie was an eminent authority on the spectral theory of differential equations. He generalized the Hardy-Littlewood-Pólya inequality to yield the HELP inequality (E for Everitt), which is intimately connected with spectral theory. Norrie helped set up the SLEIGN2 program, a computer code to calculate eigenvalues of Sturm-Liouville problems. He also edited the translation of Naimark's *Linear Differential Operators*, a book that has had a profound influence on Western mathematical analysis. These are only glimpses of his manifold contributions.

Norrie began his mathematical career at the Royal Military College of Science in Shrivenham (1954–1963). From 1963 to 1982 he was the Baxter Professor of Mathematics in the Department of Mathematical Sciences at the

University of Dundee, serving twice as head of the department (1963–1967, 1977–1980). It was during his Dundee years that he demonstrated his organizational skills in running the Dundee Conferences on Differential Equations. In 1982 Norrie returned home as Mason Chair and head of the Department of Mathematics at the University of Birmingham. He remained head until his retirement in 1989 and stayed as an honorary Senior Research Fellow until September 2009. Norrie was an excellent mentor during his career; he supervised thirteen Ph.D. students and guided many young mathematicians throughout the world.

Norrie was elected a Fellow of the Royal Society of Edinburgh (1966) and served as president of the Edinburgh Mathematical Society (1970–1971) and as vice president of the Royal Society of Edinburgh (1970–1973). In 1978 he was part of the U.K. delegation to the International Mathematical Union in Helsinki. He made several trips behind the Iron Curtain to ensure that the flow of mathematical ideas continued between the East and West. Norrie was a keen student of opera, British history, literature, poetry, trees, films, railroad history, and the American West and was an excellent after-dinner speaker. He had remarkable teaching and blackboard skills. Norrie began writing his well-prepared lectures in the upper left-hand corner of the board and ended his talks, on time, with his customary period (.) in the lower right-hand corner. As technology evolved, Norrie adapted and skillfully delivered Beamer-type presentations.

Norrie is survived by his wife, Kit; two sons, Charles (Father Gabriel, OSB) and Timothy; and two granddaughters, Sophie and Lucy. Norrie was a dear friend who will be greatly missed by all who knew him. A full obituary of Norrie Everitt is expected to appear in a forthcoming issue of the *Bulletin of the London Mathematical Society*.

—Desmond Evans (Cardiff University), Tomas Johansson (University of Birmingham), and Lance Littlejohn (Baylor University)

Mathematics Opportunities

American Mathematical Society Centennial Fellowships

Invitation for Applications for Awards for 2012–2013
Deadline December 1, 2011

Description: The AMS Centennial Research Fellowship Program makes awards annually to outstanding mathematicians to help further their careers in research. The number of fellowships to be awarded is small and depends on the amount of money contributed to the program. The Society supplements contributions as needed. One fellowship will be awarded for the 2012–2013 academic year. A list of previous fellowship winners can be found at `http://www.ams.org/profession/prizes-awards/ams-awards/centennial-fellow`.

Eligibility: The eligibility rules are as follows. The primary selection criterion for the Centennial Fellowship is the excellence of the candidate's research. Preference will be given to candidates who have not had extensive fellowship support in the past. Recipients may not hold the Centennial Fellowship concurrently with another research fellowship such as a Sloan or NSF Postdoctoral Fellowship. Under normal circumstances, the fellowship cannot be deferred. A recipient of the fellowship shall have held his or her doctoral degree for at least three years and not more than twelve years at the inception of the award (that is, received between September 1, 2000, and September 1, 2009). Applications will be accepted from those currently holding a tenured, tenure-track, postdoctoral, or comparable (at the discretion of the selection committee) position at an institution in North America. Applications should include a cogent plan indicating how the fellowship will be used. The plan should include travel to at least one other institution and should demonstrate that the fellowship will be used for more than reductions of teaching at the candidate's home institution. The selection committee will consider the plan in addition to the quality of the candidate's research and will try to award the fellowship to those for whom the award would make a real difference in the development of their research careers. Work in all areas of mathematics, including interdisciplinary work, is eligible.

Grant amount: The stipend for fellowships awarded for 2012–2013 is expected to be US$79,000, with an additional expense allowance of about US$7,900. Acceptance of the fellowship cannot be postponed.

Deadline: The deadline for receipt of applications is **December 1, 2011**. The award recipient will be announced in February 2012 or earlier if possible.

Application information: Find Centennial information and the application form via the Internet at `http://www.ams.org/ams-fellowships/`. For paper copies of the form, write to the Membership and Programs Department, American Mathematical Society, 201 Charles Street, Providence, RI 02904-2294; `prof-serv@ams.org`; 401-455-4105.

—AMS announcement

AMS Epsilon Fund

The AMS Epsilon Fund awards grants to summer mathematics programs that support and nurture mathematically talented high school students in the United States. The deadline to apply for funding for summer 2012 programs is **December 15, 2011**. Applications are now taken online at `MathPrograms.org` (`http://www.`

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Grant amount: The stipend for fellowships awarded for 2012–2013 is expected to be US$79,000, with an additional expense allowance of about US$7,900. Acceptance of the fellowship cannot be postponed.

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mathprograms.org). For more information about the program and updated application information, go to http://www.ams.org/programs/edu-support/epsilon/emp-epsilon. For more information contact the AMS Membership and Programs Department by email at prof-serv@ams.org or by telephone at 800-321-4267, ext. 4170.

—AMS announcement

Research Opportunities for U.S. Graduate Students in Asia and Australia

The National Science Foundation (NSF) is sponsoring a summer research program in Australia, China, Japan, Korea, Taiwan, New Zealand, and Singapore for U.S. graduate students during the summer of 2012. The East Asia and Pacific Summer Institutes (EAPSI) provide U.S. graduate students in science and engineering with firsthand research experience in Australia, China, Japan, Korea, Taiwan, New Zealand, or Singapore; an introduction to the science and science policy infrastructure of the respective location; and orientation to the culture and language. The primary goals of EAPSI are to introduce students to East Asian and Pacific science and engineering in the context of a research laboratory and to initiate personal relationships that will better enable them to collaborate with foreign counterparts in the future. The institutes last approximately eight weeks (ten weeks in Japan) from June to August and are administered in the United States by the NSF.

Applicants must be U.S. citizens or permanent residents. They must be enrolled at U.S. institutions in a research-oriented master's or Ph.D. program (including joint degree programs) in fields of science or engineering research and education that are supported by the NSF and that also are represented among the potential host institutions. International travel will be provided, and each awardee will receive an allowance of US$5,000.

The deadline for application materials to be postmarked is **November 9, 2011**. Proposers are required to prepare and submit all proposals for this announcement/solicitation through the FastLane system. Further information and detailed instructions are available at http://www.nsf.gov/pubs/2010/nsf10591/nsf10591.htm.

—From an NSF announcement

AMS Congressional Fellowship

The AMS in conjunction with the American Association for the Advancement of Science (AAAS) will sponsor a Congressional Fellow from September 2012 through August 2013. The fellow will spend the year working on the staff of a member of Congress or a congressional committee as a special legislative assistant in legislative and policy areas requiring scientific and technical input. The fellowship is designed to provide a unique public policy learning experience, to demonstrate the value of science-government interaction, and to bring a technical background and external perspective to the decision-making process in the Congress. The deadline for applications is **February 15, 2012**. Applicants must have a Ph.D. or an equivalent doctoral-level degree in the mathematical sciences by the application deadline. For further information, please consult the webpage at http://www.ams.org/programs/ams-fellowships/ams-aaas/ams-aaas-congressional-fellowship or contact the AMS Washington Office at 202-588-1100, email: amsdc@ams.org.

—AMS Washington Office

Jefferson Science Fellows Program

The Jefferson Science Fellows (JSF) program at the U.S. Department of State is intended to involve the American academic science, technology, and engineering communities in the formulation and implementation of U.S. foreign policy. Each fellow will spend one year at the U.S. Department of State or the U.S. Agency for International Development (USAID) for an on-site assignment in Washington, D.C., that may also involve extended stays at U.S. foreign embassies and/or missions. Each fellow will receive a stipend of up to US$50,000. Following the fellowship year, the Jefferson Science Fellow will return to his or her academic career but will remain available to the U.S. Department of State for short-term projects over the following five years. The JSF program is administered by the National Academies and is supported through a partnership of the U.S. science, technology, and academic communities, professional scientific societies, and the U.S. Department of State. The deadline for applications is **January 13, 2012**. For further information, email jsf@nas.edu, telephone 202-334-2643, or see the website http://sites.nationalacademies.org/PGA/Jefferson/PGA_046612.

—From a National Academies announcement

AAUW Educational Foundation Fellowships and Grants

The American Association of University Women (AAUW) awards Selected Professions Fellowships to women who intend to pursue a full-time course of study at accredited institutions during the fellowship year in a designated degree program in which women's participation has traditionally been low. All women who are candidates for the master of science (M.S.) degree in mathematics or statistics are eligible to apply.

Applications are now available for Master's and First Professional Awards, which carry cash awards of between US$5,000 and US$18,000. The deadline for applications to be postmarked is **January 10, 2012**. The fellowship year runs from July 1, 2012, to June 30, 2013. For more information, see the AAUW's website at http://www.aauw.org/fga/fellowships_grants/selected.cfm or contact AAUW Fellowships and Grants, 101 ACT Drive,

P. O. Box 4030, Iowa City, IA 52243-4030; telephone: 319-337-1716, ext. 60; e-mail: `aauw@act.org`.

—From an AAUW announcement

AMS Department Chairs Workshop

The annual workshop for department chairs will be held a day before the start of the Joint Mathematics Meetings in Boston, Massachusetts, on Tuesday, January 3, 2012, from 8:00 a.m. to 6:30 p.m. This one-day session for mathematical sciences department chairs is organized in a workshop format so as to stimulate discussion among attendees. Sharing ideas and experiences with peers creates an environment that enables attending chairs to address departmental challenges from new perspectives.

Past workshop sessions have focused on a range of issues facing departments, including planning and budgeting, personnel management, assessment, outreach, faculty development, communications, and departmental leadership.

There is a registration fee for the workshop, which is in addition to and separate from the Joint Meetings registration. An invitation to attend the workshop will be sent to department chairs this fall. Information will also be posted on the AMS website. For further information, please contact the AMS Washington Office at 202-588-1100 or `amsdc@ams.org`.

—AMS Washington Office

NRC-Ford Foundation Diversity Fellowships

The National Research Council (NRC) administers the Ford Foundation Diversity Fellowships program. The program seeks to promote the diversity of the nation's college and university faculties by increasing their ethnic and racial diversity, to maximize the educational benefits of diversity, and to increase the number of professors who can and will use diversity as a resource for enriching the education of all students. Predoctoral fellowships support study toward a Ph.D. or Sc.D.; dissertation fellowships offer support in the final year of writing the Ph.D. or Sc.D. thesis; postdoctoral fellowships offer one-year awards for Ph.D. recipients. Applicants must be U.S. citizens or nationals in research-based fields of study. Membership in one of the following groups will be considered a positive factor: Alaska Native (Eskimo, Aleut, or other indigenous peoples), Black/African American, Mexican American/Chicana/Chicano, Native American Indian, Native Pacific Islander (Hawaiian/Polynesian/Micronesian), or Puerto Rican.

Approximately sixty predoctoral fellowships will be awarded for 2012. The awards provide three years of support and are made to individuals who, in the judgment of the review panels, have demonstrated superior academic achievement, are committed to a career in teaching and research at the college or university level, show promise of future achievement as scholars and teachers, and are well prepared to use diversity as a resource for enriching the education of all students. The annual stipend is US$20,000, with an institutional allowance of US$2,000. The deadline for applying online is **November 14, 2011**.

Approximately thirty-five dissertation fellowships will be awarded for 2012 and will provide one year of support for study leading to a Ph.D. or D.Sc. degree. The stipend for one year is US$21,000. The deadline for applying online is **November 17, 2011**.

The postdoctoral fellowship program offers one year of postdoctoral support for individuals who have received their Ph.D.'s no earlier than November 30, 2004, and no later than November 17, 2011. The stipend is US$40,000, with an employing institution allowance of US$1,500. Approximately twenty postdoctoral fellowships will be awarded for 2012. The deadline for applying online is **November 17, 2011**.

More detailed information and applications are available at the website `http://sites.nationalacademies.org/PGA/FordFellowships/index.htm`. The postal address is: Fellowships Office, Keck 576, National Research Council, 500 Fifth Street, NW, Washington, DC 20001. The telephone number is 202-334-2872. The email address is `infofell@nas.edu`.

—From an NRC announcement

AWM Essay Contest

To increase awareness of women's ongoing contributions to the mathematical sciences, the Association for Women in Mathematics (AWM) is holding an essay contest for biographies of contemporary women mathematicians and statisticians in academic, industrial, and government careers.

The essays will be based primarily on interviews with women who are currently working in mathematical sciences careers. The contest is open to students in the following categories: 6th–8th grades, 9th–12th grades, and college undergraduates. At least one winning submission will be chosen from each category. Winners will receive a prize, and their essays will be published online at the AWM website. A grand prize winner will have his or her submission published in the *AWM Newsletter* as well. The deadline for entries is **January 31, 2012**.

In addition to student entries, organizers are currently seeking women mathematicians to volunteer as the subjects of these essays. For more information, see `http://www.awm-math.org/biographies/contest.html`.

—From an AWM announcement

Mathematics Research Communities 2012

The American Mathematical Society (AMS) invites mathematicians just beginning their research careers to become part of Mathematics Research Communities, a program to develop and sustain long-lasting cohorts for collaborative research projects in many areas of mathematics. Women and underrepresented minorities are especially encouraged to participate. The AMS will provide a structured program to engage and guide all participants as they start their careers. The program will include a one-week summer conference for each topic, special sessions at the national meeting, discussion networks by research topic, and a longitudinal study of early-career mathematicians.

The summer conferences of the Mathematics Research Communities will be held in Snowbird Resort, Utah, where participants can enjoy the natural beauty and a collegial atmosphere. The application deadline for summer 2012 is **March 1, 2012**. This program is supported by a grant from the National Science Foundation. Advanced graduate students and postdoctoral researchers are welcome to apply to be participants.

The topics, dates, and organizers of the 2012 conferences follow:

Week 1 (June 10–16, 2012): Discrete and Computational Geometry, Satyan Devadoss (Williams College), Vida Dujmovic (Carleton University), Joseph O'Rourke (Smith College), Yusu Wang (The Ohio State University).

Week 2 (June 17–23, 2012): Partial Differential Equations, Harmonic Analysis, Complex Analysis, and Geometric Measure Theory, Dorina Mitrea (University of Missouri at Columbia), Irina Mitrea (Temple University), Katharine Ott (University of Kentucky).

Week 3 (June 17–23, 2012): Geometry and Representation Theory Related to Geometric Complexity and Other Variants of P v. NP, Shrawan Kumar (University of North Carolina, Chapel Hill), J. M. Landsberg (Texas A&M University), Jerzy Weyman (Northeastern University).

Week 4 (June 24–30, 2012): Arithmetic Statistics, Brian Conrey (American Institute of Mathematics), Chantal David (Concordia University), Wei Ho (Columbia University), Michael Rubinstein (University of Waterloo), Nina Snaith (University of Bristol), William Stein (University of Washington).

Situated in a breathtakingly beautiful mountain setting, Snowbird Resort provides an extraordinary environment for the MRC. The atmosphere is comparable to the collegial gatherings at Oberwolfach and other conferences that combine peaceful natural ambience with stimulating meetings.

MRC participants have access to a range of activities, such as a tram ride to the top of the mountain, guided hikes, swimming, mountain bike tours, rock climbing, plus heated outdoor pools. More than a dozen walking and hiking trails head deep into the surrounding mountains. Participants can also enjoy the simpler pleasures of convening on the patios at the resort to read, work, and socialize with other participants. In the evenings colleagues enjoy informal gatherings to network and continue discussion of the day's sessions over refreshments.

Within a half hour of the University of Utah, Snowbird is easily accessible from the Salt Lake City International Airport. For more information about Snowbird Resort, see `http://www.snowbird.com`.

A report about the 2008 MRC conferences appears in the February 2009 issue of the *Notices* (`www.ams.org/notices/200902/rtx090200224p.pdf`). For information on applying for the 2012 program, please visit the website `http://www.ams.org/programs/research-communities/mrc-12`. For further information about the MRC program, please contact AMS associate executive director Ellen Maycock at `ejm@ams.org`.

Call for Nominations for 2012 Clay Research Fellows

The Clay Mathematics Institute (CMI) solicits nominations for its competition for the 2012 Clay Research Fellowships. Fellows are appointed for a period of two to five years. They may conduct their research at whatever institution or combination of institutions best suits their research. In addition to a generous salary, the fellow receives support for travel, collaboration, and other research expenses.

The selection criteria are the quality of the candidate's research and promise to make contributions of the highest level. At the time of their selection, most recent appointees were graduating Ph.D. students. However, mathematicians within three years of the Ph.D. are sometimes appointed. Selection decisions are made by CMI's Scientific Advisory Board: Jim Carlson, Simon Donaldson, Gregory Margulis, Richard Melrose, Yum-Tong Siu, and Andrew Wiles.

To nominate a candidate, please send the following items by **November 1, 2011**: (1) letter of nomination, (2) names and contact information of two other references, (3) curriculum vitae for the nominee, and (4) publication list for the nominee.

Nominations should be sent to the attention of Alagi Patel, Clay Mathematics Institute, One Bow Street, Cambridge, MA 02138. Electronic submissions are also accepted at `nominations@claymath.org`.

Information about the Clay Research Fellows is available on the CMI website at: `http://www.claymath.org/research_fellows`. Additional information may be obtained by calling Alagi Patel at 617-995-2602 or emailing her at `patel@claymath.org`.

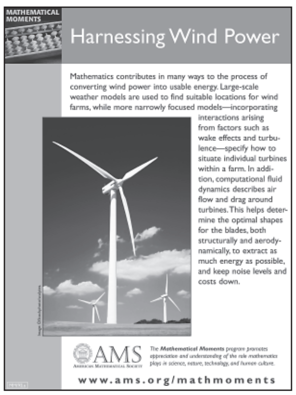
Current and alumni Clay Research Fellows: Mohammed Abouzaid, Spyridon Alexakis, Timothy Austin, Artur Avila, Roman Bezrukavnikov, Manjul Bhargava, Daniel Biss, Alexei Borodin, Maria Chudnovsky, Dennis Gaitsgory, Soren Galatius, Daniel Gottesman, Ben Green, Sergei Gukov, Adrian Ioana, Bo'az Klartag, Elon Lindenstrauss, Ciprian Manolescu, Davesh Maulik, Maryam Mirzakhani, Sophie Morel, Mircea Mustata, Sam Payne, Igor Rodnianski, Sucharit Sarkar, Peter Scholze, David Speyer, Terence Tao, Andras Vasy, Akshay Venkatesh, Teruyoshi Yoshida, Xinyi Yuan.

—Clay Mathematics Institute announcement

Inside the AMS

2011 Mathematical Moments

Mathematical Moments provide a chance for students and colleagues to see math's applications in their daily lives.



Recent topics are: Keeping Things in Focus (applications of conic sections), Sounding the Alarm (on tsunami warnings), Answering the Question and Vice Versa (on Watson, winner of the *Jeopardy!* Challenge), Harnessing Wind Power, Keeping the Beat (on math and the heart), Sustaining the Supply Chain (preparing distribution networks for disasters), and Putting Another Cork in It (on a snowboarding jump). Download these small posters and hear podcast interviews with experts at `http://www.ams.org/samplings/mathmoments/mathmoments`.

—Annette Emerson and Mike Breen
Public Awareness Officers
`paoffice@ams.org`

2011 Mathematics Research Communities

The 2011 Mathematics Research Communities (MRC) summer conferences, held at the Snowbird Resort in Utah, drew one hundred twenty early-career mathematicians. Here's the reaction of one participant: "The talks and group work exercises were informative and helpful. The accommodations and setting were first rate. Mostly, though, the people who participated in this workshop—the organizers as well as the participants—made it so wonderful. We truly were a mathematics research community in the best sense." See the sessions and organizers, more feedback from participants, and photos at `http://www.ams.org/programs/research-communities/mrc-11-highlights`.

—Annette Emerson and Mike Breen
Public Awareness Officers
`paoffice@ams.org`

Deaths of AMS Members

Robert D. M. Accola, of Providence, RI, died on May 15, 2011. Born on September 11, 1929, he was a member of the Society for 50 years.

Franz L. Alt, of New York, NY, died on July 21, 2011. Born on November 30, 1910, he was a member of the Society for 64 years.

Robert M. Baer, of San Anselmo, California, died on November 29, 2010. Born on May 15, 1925, he was a member of the Society for 61 years.

Bruce H. Barnes, of Leesburg, Virginia, died on June 1, 2011. Born on May 28, 1931, he was a member of the Society for 53 years.

Earl L. Bell, of Texas, died on April 13, 2011. Born on September 22, 1920, he was a member of the Society for 62 years.

Frank T. Birtel, professor at Tulane University, died on September 1, 2011. Born on April 4, 1932, he was a member of the Society for 54 years and AMS associate secretary 1977–1988.

Jianguo Cao, professor at the University of Notre Dame, died on June 23, 2011. Born on January 25, 1960, he was a member of the Society for 25 years.

Jal R. Choksi, professor at McGill University, died on March 30, 2011. Born on November 24, 1932, he was a member of the Society for 37 years.

Raoul A. De Villiers, of Trumbull, Connecticut, died on March 8, 2011. Born in April 1926, he was a member of the Society for 47 years.

W. Norrie Everitt, professor at the University of Birmingham, UK, died on July 17, 2011. Born on June 10, 1924, he was a member of the Society for 51 years.

Jay E. Folkert, professor at Hope College, died on August 9, 2011. Born on December 16, 1916, he was a member of the Society for 44 years.

Henry H. Glover, professor at The Ohio State University, died on May 31, 2011. Born on April 10, 1935, he was a member of the Society for 48 years.

James D. Halpern, of Belmont, California, died on July 16, 2011. Born in August 1934, he was a member of the Society for 54 years.

F. B. Hildebrand, of Halifax, Canada, died on November 29, 2002. Born on September 1, 1915, he was a member of the Society for 62 years.

Robert S. Johnson, professor at Washington & Lee University, died on August 13, 2011. Born in November 1937, he was a member of the Society for 51 years.

John Kahila, of Boston, Massachusetts, died on January 28, 2010. Born on April 24, 1949, he was a member of the Society for 11 years.

J. H. B. Kemperman, of East Brunswick, New Jersey, died on June 12, 2011. Born on July 16, 1924, he was a member of the Society for 59 years.

Claire E. Krukenberg, of Charleston, Illinois, died on June 20, 2011. Born on September 16, 1939, she was a member of the Society for 44 years.

Ray G. Langebartel, of Oak Park, Illinois, died on February 25, 2011. Born on April 27, 1921, he was a member of the Society for 67 years.

Manohar L. Madan, professor at The Ohio State University, died on April 3, 2011. Born on January 6, 1931, he was a member of the Society for 39 years.

Milan Mares, of the Czech Republic, died on July 25, 2011. Born on May 30, 1943, he was a member of the Society for 17 years.

Mikhail Valerevich Matveev, professor at George Mason University, died on May 17, 2011. Born on December 12, 1958, he was a member of the Society for 13 years.

John E. Maxfield, of Placerville, California, died on June 25, 2011. Born on March 17, 1927, he was a member of the Society for 57 years.

D. D. Miller, of Ormond Beach, Florida, died on June 5, 2011. Born on April 8, 1913, he was a member of the Society for 72 years.

Theodore Mitchell, of Atlantic City, New Jersey, died on February 27, 2011. Born in April 1926, he was a member of the Society for 39 years.

Barrett O'Neill, of Pacific Palisades, California, died on June 16, 2011. Born on March 26, 1924, he was a member of the Society for 61 years.

T. G. Ostrom, of Pullman, Washington, died on May 12, 2011. Born on January 4, 1916, he was a member of the Society for 64 years.

Joseph R. Shoenfield, of Durham, North Carolina, died on November 15, 2000. Born on May 1, 1927, he was a member of the Society for 48 years.

Yurij Ivanovich Petunin, professor at Kiev University, died on June 1, 2011. Born on September 30, 1937, he was a member of the Society for 34 years.

Joseph R. Shoenfield, of Durham, North Carolina, died on November 15, 2000. Born on May 1, 1927, he was a member of the Society for 48 years.

Rosemarie S. Stemmler, of Potomac, Maryland, died on July 18, 2011. Born on July 27, 1930, she was a member of the Society for 53 years.

Reference and Book List

*The **Reference** section of the* Notices *is intended to provide the reader with frequently sought information in an easily accessible manner. New information is printed as it becomes available and is referenced after the first printing. As soon as information is updated or otherwise changed, it will be noted in this section.*

Contacting the *Notices*

The preferred method for contacting the *Notices* is electronic mail. The editor is the person to whom to send articles and letters for consideration. Articles include feature articles, memorial articles, communications, opinion pieces, and book reviews. The editor is also the person to whom to send news of unusual interest about other people's mathematics research.

The managing editor is the person to whom to send items for "Mathematics People", "Mathematics Opportunities", "For Your Information", "Reference and Book List", and "Mathematics Calendar". Requests for permissions, as well as all other inquiries, go to the managing editor.

The electronic-mail addresses are `notices@math.wustl.edu` in the case of the editor and `notices@ams.org` in the case of the managing editor. The fax numbers are 314-935-6839 for the editor and 401-331-3842 for the managing editor. Postal addresses may be found in the masthead.

Upcoming Deadlines

October 19, 2011: Proposals for NSF Postdoctoral Research Fellowships. See `http://www.nsf.gov/funding/pgm_summ.jsp?pims_id=5301`.

October 30, 2011: Nominations for ICTP Ramanujan Prize. See `http://prizes.ictp.it/Ramanujan/`.

November 1, 2011: Nominations for Clay Research Fellowships. See "Mathematics Opportunities" in this issue.

November 1, 2011: Nominations for CRM-Fields-PIMS Prize. Submit nominations to `crm-fields-pims-prize@fields.utoronto.ca`.

November 1, 2011: Proposals for AIM Workshops. See `www.aimath.org`.

November 1, 2011: Applications for November review for National Academies Research Associateship Programs. See the National Academies website at `http://sites.nationalacademies.org/PGA/RAP/PGA_050491` or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email `rap@nas.edu`.

November 9, 2011: Applications for NSF East Asia and Pacific Summer Institutes (EAPSI). See "Mathematics Opportunities" in this issue.

November 14, 2011: Applications for NRC-Ford Foundation Predoctoral DiversityFellowships. **November 1, 2011:** Nominations for Clay Research Fellowships. See "Mathematics Opportunities" in this issue.

November 17, 2011: Applications for NRC-Ford Foundation Dissertation and Postdoctoral Diversity Fellowships. **November 1, 2011:**

John Kahila, of Boston, Massachusetts, died on January 28, 2010. Born on April 24, 1949, he was a member of the Society for 11 years.

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Nominations for Clay Research Fellowships. See "Mathematics Opportunities" in this issue.

December 1, 2011: Applications for AMS Centennial Fellowship Program. **November 1, 2011:** Nominations for Clay Research Fellowships. See "Mathematics Opportunities" in this issue.

December 1, 2011: Applications for PIMS postdoctoral fellowships. See `http://www.pims.math.ca/scientific/postdoctoral` or contact: `assistant.director@pims.math.ca`.

December 2, 2011: Entries for the 2012 Ferran Sunyer i Balaguer Prize. See the website `http://ffsb.iec.cat`.

December 15, 2011: Applications for AMS Epsilon Fund grants. See "Mathematics Opportunities" in this issue.

December 21, 2011: Nominations for the Schauder Medal. Contact Lech Gorniewicz, `tmna@mat.uni.torun.pl`.

December 31, 2011: Nominations for Otto Neugebauer Prize for the History of Mathematics. See `http://www.euro-math-soc.eu/node/995`.

January 1, 2012: Proposals for MSRI Hot Topic Workshops for 2012. See `http://www.msri.org/msri-htw`.

January 1, 2012: Proposals for MSRI Summer Graduate Schools for 2012. See `http://www.msri.org/msri-sgw`.

January 10, 2012: Applications for American Association of University Women (AAUW) Selected Professions Fellowships. See "Mathematics Opportunities" in this issue.

January 13, 2012: Applications for Jefferson Science Fellows (JSF) program. See "Mathematics Opportunities" in this issue.

January 31, 2012: Entries for the Association for Women in Mathematics (AWM) essay contest. See "Mathematics Opportunities" in this issue.

February 1, 2012: Applications for AWM Travel Grants and Mentoring Travel Grants. See `http://www.awm-math.org/travelgrants.html#standard`; telephone: 703-934-0163; or email: `awm@awm-math.org`; or contact Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030.

February 12, 2012: Applications for IPAM summer program, Research in Industrial Projects for Students (RIPS). See `www.ipam.ucla.edu`.

February 15, 2012: Applications for AMS Congressional Fellowship. See "Mathematics Opportunities" in this issue.

May 1, 2012: Applications for AWM Travel Grants. See `http://www.awm-math.org/travelgrants.html#standard`; telephone: 703-934-0163; or email: `awm@awm-math.org`; or contact Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030.

July 10, 2012: Full proposals for NSF Research Networks in the Mathematical Sciences. See `http://www.nsf.gov/pubs/2010/nsf10584/nsf10584.htm?WT.mc_id=USNSF_25&WT.mc_ev=click`.

October 1, 2012: Applications for AWM Travel Grants. See `http://www.awm-math.org/travelgrants.html#standard`; telephone: 703-934-0163; or email: `awm@awm-math.org`; or contact Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030.

Where to Find It

A brief index to information that appears in this and previous issues of the *Notices*.

NSF Division of Mathematical Sciences

Listed below are names, email addresses, and telephone numbers for the program directors for the present academic year in the Division of Mathematical Sciences (DMS) of the National Science Foundation. The postal address is: Division of Mathematical Sciences, National Science Foundation, Room 1025, 4201 Wilson Boulevard, Arlington, VA 22230. The DMS webpage is `http://www.nsf.gov/div/index.jsp?div=DMS`.

Algebra and Number Theory
Tie Luo
703-292-8448
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Program Support Manager
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Operations Specialist
Sharon J. Alston
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Division Secretary
Jennifer A. Connell
703-292-5301
jconnell@nsf.gov

Book List

The Book List highlights books that have mathematical themes and are aimed at a broad audience potentially including mathematicians, students, and the general public. When a book has been reviewed in the Notices, *a reference is given to the review. Generally the list will contain only books published within the last two years, though exceptions may be made in cases where current events (e.g., the death of a prominent mathematician, coverage of a certain piece of mathematics in the news) warrant drawing readers' attention to older books. Suggestions for books to include on the list may be sent to* notices-booklist@ams.org.

*Added to "Book List" since the list's last appearance.

The Adventure of Reason: Interplay between Philosophy of Mathematics and Mathematical Logic, 1900–1940, by Paolo Mancosu. Oxford University

Press, January 2011. ISBN-13: 978-01995-465-34.

At Home with André and Simone Weil, by Sylvie Weil. (Translation of *Chez les Weils*, translated by Benjamin Ivry.) Northwestern University Press, October 2010. ISBN-13: 978-08101-270-43. (Reviewed May 2011.)

The Autonomy of Mathematical Knowledge: Hilbert's Program Revisited, by Curtis Franks. Cambridge University Press, December 2010. ISBN-13: 978-05211-838-95.

The Beginning of Infinity: Explanations That Transform the World, by David Deutsch. Viking Adult, July 2011. ISBN-13: 978-06700-227-55.

The Best Writing on Mathematics: 2010, edited by Mircea Pitici. Princeton University Press, December 2010. ISBN-13: 978-06911-484-10. (Reviewed in this issue.)

The Big Questions: Mathematics, by Tony Crilly. Quercus, April 2011. ISBN-13: 978-18491-624-01.

The Black Swan: The Impact of the Highly Improbable, by Nassim Nicholas Taleb. Random House Trade Paperbacks, second edition, May 2010. ISBN-13: 978-08129-738-15. (First edition reviewed March 2011.)

The Blind Spot: Science and the Crisis of Uncertainty, by William Byers. Princeton University Press, April 2011. ISBN-13:978-06911-468-43.

The Calculus Diaries: How Math Can Help You Lose Weight, Win in Vegas, and Survive a Zombie Apocalypse, by Jennifer Ouellette. Penguin, reprint edition, August 2010. ISBN-13: 978-01431-173-77.

The Calculus of Selfishness, by Karl Sigmund. Princeton University Press, January 2010. ISBN-13: 978-06911-427-53.

Chasing Shadows: Mathematics, Astronomy, and the Early History of Eclipse Reckoning, by Clemency Montelle. Johns Hopkins University Press, April 2011. ISBN-13: 978-08018-969-10.

The Clockwork Universe: Isaac Newton, the Royal Society, and the Birth of the Modern World, by Edward Dolnick. Harper, February 2011. ISBN-13: 978-00617-195-16. (Reviewed April 2011.)

Complexity: A Guided Tour, by Melanie Mitchell. Oxford University Press, April 2009. ISBN-13: 978-01951-244-15. (Reviewed April 2011.)

Crafting by Concepts: Fiber Arts and Mathematics, by Sarah-Marie Belcastro and Carolyn Yackel. A K Peters/CRC Press, March 2011. ISBN-13: 978-15688-143-53.

Cycles of Time: An Extraordinary New View of the Universe, by Roger Penrose. Knopf, May 2011. ISBN-13: 978-03072-659-06.

An Early History of Recursive Functions and Computability from Gödel to Turing, by Rod Adams. Docent Press, May 2011. ISBN-13: 978-09837-004-01.

**Elegance with Substance*, by Thomas Colignatus. Dutch University Press, 2009. ISBN-13: 978-90361-013-87.

The Evolution of Logic, by W. D. Hart. Cambridge University Press, August 2010. ISBN-13: 978-0-521-74772-1

The Grand Design, by Stephen Hawking and Leonard Mlodinow. Bantam, September 2010. ISBN-13: 978-05538-053-76.

Hidden Harmonies (The Lives and Times of the Pythagorean Theorem), by Robert and Ellen Kaplan. Bloomsbury Press, January 2011. ISBN-13: 978-15969-152-20.

The History and Development of Nomography, by H. A. Evesham. Docent Press, December 2010. ISBN-13: 978-14564-796-26.

Hot X: Algebra Exposed, by Danica McKellar. Hudson Street Press, August 2010. ISBN-13: 978-15946-307-05.

I Want to Be a Mathematician: A Conversation with Paul Halmos. A film by George Csicsery. Mathematical Association of America, March 2009. ISBN-13:978-08838-590-94. (Reviewed June/July 2011.)

Le Operazioni del Calcolo Logico, by Ernst Schröder. Original German version of *Operationskreis des Logikkalkuls* and Italian translation with commentary and annotations by Davide Bondoni. LED Online, 2010. ISBN-13: 978-88-7916-474-0.

Loving + Hating Mathematics: Challenging the Myths of Mathematical Life, by Reuben Hersh and Vera John-Steiner. Princeton University Press, January 2011. ISBN-13: 978-06911-424-70.

**The Man of Numbers: Fibonacci's Arithmetic Revolution,* by Keith Devlin. Walker and Company, July 2011. ISBN-13: 978-08027-781-23.

**A Mathematical Nature Walk,* by John A. Adam. Princeton University Press, October 2011 (paperback edition). ISBN-13: 978-06911-526-53.

A Mathematician's Lament: How School Cheats Us Out of Our Most Fascinating and Imaginative Art Form, by Paul Lockhart. Bellevue Literary Press, April 2009. ISBN-13: 978-1-934137-17-8.

Mathematics and Reality, by Mary Leng. Oxford University Press, June 2010. ISBN-13: 978-01992-807-97.

Mathematics Education for a New Era: Video Games as a Medium for Learning, by Keith Devlin. A K Peters/CRC Press, February 2011. ISBN-13: 978-1-56881-431-5.

A Motif of Mathematics: History and Application of the Mediant and the Farey Sequence, by Scott B. Guthery. Docent Press, September 2010. ISBN-13: 978-4538-105-76.

Mysteries of the Equilateral Triangle, by Brian J. McCartin. Hikari, August 2010. ISBN-13: 978-954-91999-5-6. Electronic copies available for free at `http://www.m-hikari.com/mccartin-2.pdf`.

Newton and the Counterfeiter: The Unknown Detective Career of the World's Greatest Scientist, by Thomas Levenson. Houghton Mifflin Harcourt, June 2009. ISBN-13: 978-01510-127-87.

NIST Handbook of Mathematical Functions, Cambridge University Press, Edited by Frank W. J. Olver, Daniel W. Lozier, Ronald F. Boisvert, and Charles W. Clark. Cambridge University Press, May 2010. ISBN-13: 978-05211-922-55 (hardback plus CD-ROM); ISBN-13: 978-05211-406-38 (paperback plus CD-ROM). (Reviewed September 2011.)

Nonsense on Stilts: How to Tell Science from Bunk, by Massimo Pigliucci. University of Chicago Press, May 2010. ISBN-13: 978-02266-678-67. (Reviewed April 2011.)

Number Freak: From 1 to 200—The Hidden Language of Numbers Revealed, by Derrick Niederman. Perigee Trade, August 2009. ISBN-10: 03995-345-98.

Numbers: A Very Short Introduction, by Peter M. Higgins. Oxford University Press, February 2011. ISBN-13: 978-0-19-958405-5.

Numbers Rule: The Vexing Mathematics of Democracy, from Plato *to the Present*, by George G. Szpiro. Princeton University Press, April 2010. ISBN-13: 978-06911-399-44. (Reviewed January 2011.)

One, Two, Three: Absolutely Elementary Mathematics [Hardcover] David Berlinski. Pantheon, May 2011. ISBN-13: 978-03754-233-38.

Origami Inspirations, by Meenakshi Mukerji. A K Peters, September 2010. ISBN-13: 978-1568815848.

The Perfect Swarm: The Science of Complexity in Everyday Life, by Len Fisher. Basic Books, March 2011 (paperback). ISBN-13: 978-04650-202-49.

The Pleasures of Statistics: The Autobiography of Frederick Mosteller. Edited by Stephen E. Fienberg, David C. Hoaglin, and Judith M. Tanur. Springer, January 2010. ISBN-13: 978-03877-795-53.

Problem-Solving and Selected Topics in Number Theory in the Spirit of the Mathematical Olympiads, by Michael Th. Rassias. Springer, 2011. ISBN-13: 978-1-4419-0494-2.

Proofiness: The Dark Arts of Mathematical Deception, by Charles Seife. Viking, September 2010. ISBN-13: 978-06700-221-68.

The Quants: How a New Breed of Math Whizzes Conquered Wall Street and Nearly Destroyed It, by Scott Patterson. Crown Business, January 2011. ISBN-13: 978-03074-533-89. (Reviewed May 2011.)

Riot at the Calc Exam and Other Mathematically Bent Stories, by Colin Adams. AMS, July 2009. ISBN-13: 978-08218-481-73.

Roads to Infinity: The Mathematics of Truth and Proof, by John C. Stillwell. A K Peters/CRC Press, July 2010. ISBN-13: 978-15688-146-67.

The Shape of Inner Space: String Theory and the Geometry of the Universe's Hidden Dimensions, by Shing-Tung Yau (with Steve Nadis). Basic Books, September 2010. ISBN-13: 978-04650-202-32. (Reviewed February 2011.)

The Strangest Man, by Graham Farmelo. Basic Books, August 2009. ISBN-13: 978-04650-182-77. (Reviewed October 2011.)

Street-Fighting Mathematics: The Art of Educated Guessing and Opportunistic Problem Solving, by Sanjoy Mahajan. MIT Press, March 2010. ISBN-13: 978-0-262-51429-3. (Reviewed August 2011.)

Survival Guide for Outsiders: How to Protect Yourself from Politicians, Experts, and Other Insiders, by Sherman Stein. BookSurge Publishing, February 2010. ISBN-13: 978-14392-532-74.

The Theory That Would Not Die: How Bayes' Rule Cracked the Enigma Code, Hunted Down Russian Submarines, and Emerged Triumphant from Two Centuries of Controversy, by Sharon Bertsch McGrayne. Yale University Press, April 2011. ISBN-13: 978-03001-696-90.

Towards a Philosophy of Real Mathematics, by David Corfield. Oxford University Press, April 2003. ISBN 0-521-81722-6. (Reviewed in this issue.)

Train Your Brain: A Year's Worth of Puzzles, by George Grätzer. A K Peters/CRC Press, April 2011. ISBN-13: 978-15688-171-01.

Viewpoints: Mathematical Perspective and Fractal Geometry in Art, by Marc Frantz and Annalisa Crannell. Princeton University Press, August 2011. ISBN-13: 978-06911-259-23.

Visual Thinking in Mathematics, by Marcus Giaquinto. Oxford University Press, July 2011. ISBN-13: 978-01995-755-34.

Why Beliefs Matter: Reflections on the Nature of Science, by E. Brian Davies. Oxford University Press, June 2010. ISBN13: 978-01995-862-02.

Backlog of Mathematics Research Journals

Journal (Print and Electronic)	Number issues per Year	Approximate Number Pages per Year	2010 Median Time (in Months) from: Submission to Final Acceptance	Acceptance to Print	Acceptance to Electronic Posting	Editor's Current Estimate of Waiting Time between Submission and Publication (in Months): Print	Electronic
Abstr. Appl. Anal.	*	1500	3.6	3–9**	1.4	7–9**	4
Acta Inform.	8	600	7	3	1	10	8
Acta Math.	4	700	9	25	24	24	23
Adv. Math. Commun.	4	600	3	2	1	5	4
Algebr. Geom. Topol.	4–5	2500	10	6	1	12	11
Algebra Number Theory	8	1250	9	5	4	12	12
Algorithmica	12	1800	4	NA	0.7	NA	4.5
Amer. J. Math.	6	1728	NA	14	13	16–18	15–17
Anal. PDE	5	1200	9	7	6	14	14
Ann. Appl. Probab.	6	2400	9.75	9	8.5	20.5	20
Ann. Mat. Pura Appl. (4)	4	720	9	13	6.9	20.5	16.8
Ann. of Math. (2)	6	3800	19	17	17	11	11
Ann. Probab.	6	2500	9	8.5	8.5	18.5	18.5
Ann. Statist.	6	3860	5	7.5	7.5	14.25	14.25
Appl. Anal.	12	1968	3	4.5	5.5	8.5	4
Arch. Hist. Exact. Sci.	6	700	1.5	4	1.3	4.5	4
Arch. Math. Logic	8	800	11	2	0.6	16	11.5
Arch. Ration. Mech. Anal.	12	4320	6	10	6	5	1
Balkan J. Geom. Appl.	2	360	5	5	3	8	6
Bull. Lond. Math. Soc.	6	1248	7.5	6.5	3.4	14	10.9
Bull. Soc. Math. France	4	600	6	6	5	6	5
Calc. Var. Partial Differential Equations	12	1700	8.5	6.5	0.67	16.5	9.5
Canad. J. Math.	6	1440	7	23	17	19	15
Canad. Math. Bull.	4	768	9	23	22	30	13
Cent. Eur. J. Math.	6	1500	4.6	2.4	1.8	5.6	4.7
Combinatorica	6	750	6	6	3	6	3
Comm. Math. Phys.	24	7000	6	4.5	1	4	1
Commun. Appl. Math. Comput. Sci.	1–2	450	9	3	2	11	11
Commun. Pure Appl. Anal.	6	1800	5	6	5	11	10
Complex Var. Elliptic Equ.	12	1200	3	11	7.5	9	5
Compos. Math.	6	2016	7.5	10	8.8	15	12
Comput. Math. Appl.	24	1600	6	4	1	7	7

The Backlog of Research Journals is reported each year in the November issue of the *Notices*. The report covers journals of publishers who have agreed to participate and who continue to provide backlog information. Publishers whose journals are not currently included can request that their journals be added. Such requests should be made in email to Marcia Almeida, `backlogreport@ams.org`. To be eligible for inclusion in the backlog report, a journal must be on the list of journals receiving cover-to-cover treatment in *Mathematical Reviews* (`http://www.ams.org/msnhtml/serials.pdf`).

Once a publisher's journals are accepted for inclusion, the publisher must designate a contact person or persons to supply data about the journals to the AMS. While the AMS makes every effort to obtain the data from the designated contacts, if data about a journal is not supplied, then that journal will not appear in the backlog report.

Journal (Print and Electronic)	Number issues per Year	Approximate Number Pages per Year	2010 Median Time (in Months) from: Submission to Final Acceptance	Acceptance to Print	Acceptance to Electronic Posting	Editor's Current Estimate of Waiting Time between Submission and Publication (in Months): Print	Electronic
Comput. Methods Funct. Theory	2	700	6	7	2	8	6
Computing	12	980	6	2	1	5	4
Constr. Approx.	6	1000	7	16	5	10	13
Discrete Comput. Geom.	8	1800	8	10	2	14	9
Discrete Contin. Dyn. Syst.	12	5000	5	7	6	12	11
Discrete Contin. Dyn. Syst. Ser. B	8	2500	4	7	6	11	10
Duke Math. J.	15	3000	11	9	9	18	18
Dyn. Syst.	4	592	6	7	1.5	10	7.5
Found. Comput. Math.	6	750	11	7	1.5	18	12
Geom. Dedicata	6	1500	2.5	4	0.5	6–7	3
Geom. Topol.	4–5	2500	11	6	1	13	12
Houston J. Math.	4	1300	6	17	15	24	22
Illinois J. Math.	4	1300	6	12	10	18	16
Int. J. Math. Math. Sci.	*	1500	4.7	3–9**	1.6	7–9**	4.3
Int. J. Stoch. Anal.	*	200	3.9	9–10**	1.9	7–9**	3.8
Invent. Math.	12	2740	7.9	5.9	1.5	11.5	9.8
Inverse Probl. Imaging	4	800	3	3	2	6	5
Involve	5	650	9	4	3	11	11
Israel J. Math.	6	3000	7	19	19	25	16
J. Algebraic Geom.	4	800	10	11	2	15	12
J. Amer. Math. Soc.	4	1200	11.7	6.3	1.8	15.6	11.8
J. Anal. Math.	3	1180	20	15	16	18	18
J. Appl. Math.	*	400	4.3	9–10**	1.9	7–9**	3.9
J. Aust. Math. Soc.	6	864	9	6	4	9	6
J. Complexity	6	750	6	6	1	12	7
J. Comput. System Sci.	6	1200	12	4	NR	NR	NR
J. Convex Anal.	4	1200	10	6	1	16	10
J. Eur. Math. Soc. (JEMS)	6	1600	6	12	11	18	17
J. Geom. Anal.	4	1000	6.6	7.7	3.5	9	7
J. Ind. Manag. Optim.	4	1000	4	5	4	9	8
J. Integral Equations Appl.	4	600	12	20	18	18	16
J. Lie Theory	4	1000	6	2	0.5	8	6
J. Lond. Math. Soc. (2)	6	1632	7.8	6.2	3.7	14.3	11.3
J. Math. Biol.	12	1676	10	7.5	2.9	17.5	11.8
J. Math. Phys.	12	9430	3.7	2	1.8	5	4.5
J. Mod. Dyn.	4	800	3	3	2	6	5
J. Operator Theory	4	1000	7	26	23	36	24
J. Symbolic Logic	4	1470	10	11	9	18	16
J. Theoret. Probab.	4	1100	2.5	4	3	2	1.5
J. Topol.	4	1024	8.5	5.3	3.3	12.8	10.8
Kyoto J. Math.	4	900	6.8	5.9	5.9	14	14
Linear Algebra Appl.	24	6000	5	4	1	8	6
Linear Multilinear Algebra	12	1476	4.5	17	9.5	10	7.5
Lobachevskii J. Math.	4	400	3	5	3	4	3
Manuscripta Math.	12	1630	4.5	2.5	0.6	14	11
Math. Ann.	12	3000	10	5	4	15	13
Math. Comp.	4	2400	8.5	11.4	5.7	20.5	13.9
Math. Oper. Res.	4	1024	18	5	3	16	14
Math. Program.	10	2100	20	12	1	13	13

Journal (Print and Electronic)	Number issues per Year	Approximate Number Pages per Year	2010 Median Time (in Months) from: Submission to Final Acceptance	Acceptance to Print	Acceptance to Electronic Posting	Editor's Current Estimate of Waiting Time between Submission and Publication (in Months): Print	Electronic
Math. Social Sci.	6	840	12	7	1	18	12
Math. Z.	12	2900	12.5	13.9	3.3	25.1	15.3
Mathematika	2	384	6.2	5.4	3.8	12	10
Mem. Amer. Math. Soc.	6	3800	15.6	24.2	19.5	34	27.5
Michigan Math. J.	3	720	5	15	14	15	14
Monatsh. Math.	12	1440	5	16	1	19–20	6
Multiscale Model. Simul.	4	1750	8.5	6.9	2.7	13.1	11.1
Nagoya Math. J.	4	800	8.6	9.2	9.2	18	18
Notre Dame J. Form. Log.	4	500	3	6	6	12	12
Numer. Math.	12	2400	12.5	6	4	15	13.5
Pacific J. Math.	10–12	2550–3000	8	7	6	14	13
Probab. Theory Related Fields	12	1820	8.5	10.1	3.2	18.1	10.9
Proc. Amer. Math. Soc.	12	4200	4.7	7.3	3.4	15.1	8.9
Proc. Lond. Math. Soc. (3)	12	2304	7.4	8.8	2.2	16.5	9.6
Publ. Math. Inst. Hautes Etudes Sci.	2	500	12.5	4.1	2.8	15	12.8
Q. J. Math.	4	500	5	18	2	24	7
Quart. Appl. Math.	4	800	1.1	16.3	13.7	20.9	18.2
Rocky Mountain J. Math.	6	1400	10	25	23	27	25
Semigroup Forum	6	1100	8	5	1	13	9
SIAM J. Appl. Math.	6	1900	9.2	8.8	2.7	14	12
SIAM J. Comput.	6	2150	16	9.5	2.7	21	19
SIAM J. Control Optim.	6	2575	14.2	7.9	2.7	18.9	16.9
SIAM J. Discrete Math.	4	2200	12.8	11.9	2.6	18	15
SIAM J. Math. Anal.	6	3700	8.4	12	2.7	13.1	11.1
SIAM J. Matrix Anal. Appl.	4	1550	10.5	6.9	2.5	14.7	12.7
SIAM J. Numer. Anal.	6	3100	11.7	10.8	2.6	15.3	14.3
SIAM J. Optim.	4	2150	12.9	9	2.4	17.1	15.1
SIAM J. Sci. Comput.	6	3700	10.1	11.6	2.8	14.9	12.9
SIAM Rev.	4	800	9.4	11.7	10.7	21.1	20.1
Theory Comput. Syst.	8	1300	4	NA	0.7	NA	4.5
Trans. Amer. Math. Soc.	12	6600	8.4	18.6	14.1	28.4	23.7

Journal (Electronic)	Number of Articles Posted in 2010	2010 Median Time (in days) from: Submission to Final Acceptance	Acceptance to Posting	Format(s)
Acta Math. Acad. Paedagog. Nyházi. (N.S.) www.emis.de/journals/AMAPN/	32	201	150	pdf, ps
Appl. Math. E-Notes www.math.nthu.edu.tw/~sscheng/	34	180	180	pdf
Conform. Geom. Dyn. www.ams.org/ecgd	18	171	69	pdf
Differ. Geom. Dyn. Syst. www.mathem.pub.ro/dgds	20	100	180	pdf
Electron. J. Combin. www.combinatorics.org	215	198	12	pdf

Journal (Electronic)	Number of Articles Posted in 2010	2010 Median Time (in days) from: Submission to Final Acceptance	Acceptance to Posting	Format(s)
Electron. J. Differential Equations ejde.math.txstate.edu/	178	166	8	html, pdf, tex
Electron. J. Linear Algebra www.math.technion.ac.il/iic/ela/	66	231	24	pdf
Electron. J. Qual. Theory Differ. Equ. www.math.u-szeged.hu/ejqtde	79	150	10	pdf
Electron. Res. Announc. Math. Sci. www.math.psu.edu/era/	14	40	10	pdf
Electron. Trans. Numer. Anal. etna.mcs.kent.edu	26	217	139	html, pdf, ps
ESAIM Control Optim. Calc. Var. www.esaim-cocv.org	55	309	111	pdf
ESAIM Math. Model. Numer. Anal. www.esaim-m2an.org	71	373	117	pdf
ESAIM Probab. Stat. www.esaim-ps.org	21	345	210	pdf
Integers www.integers-ejcnt.org	63	224	115	pdf
J. Integer Seq. www.cs.uwaterloo.ca/journals/JIS/	71	106	3	html, pdf, ps, dvi, tex
LMS J. Comput. Math. www.lms.ac.uk/jcm	25	226	75	pdf
Math. Biosci. Eng. aimsciences.org/journals/mbe	52	60	60	pdf
Netw. Heterog. Media aimsciences.org/journals/NHM/index.htm	21	60	60	pdf
Represent. Theory www.ams.org/ert	21	639	111	pdf
Sém. Lothar. Combin. www.mat.univie.ac.at/~slc/	22	228	8	pdf, ps, dvi, tex
SIAM J. Appl. Dyn. Syst. epubs.siam.org/SIADS/siads_toc.html	47	228	82	pdf
SIAM J. Imaging Sci. epubs.siam.org/SIIMS/siims_toc.html	44	357	90	html, BibTeX
Theory Appl. Categ. www.tac.mta.ca/tac/	34	225	5	pdf, ps, dvi

NR means no response received. NA means not available or not applicable.
*Articles are published on an article-by-article basis.
**Print edition is printed on demand and in the form of an archival volume.

Reciprocity Agreements

Africa

Egyptian Mathematical Society (ETMS)*

Apply to: M. H. Fahmy, Department of Mathematics, Faculty of Science, Al-Azhar University, Nasr City 11884, Cairo, Egypt; email: Secretary_etms@yahoo.com; http://www.etms-web.org.

Dues: U.S. $15, payable to Egyptian Mathematical Society at the above address.

Privileges: Receive a 60% discount on the prices of ETMS publications, a 50% discount on the publication charge per printed page in *ETMS Journal*, and reduced charge for participating at ETMS conferences.

Officers: A.-S. F. Obada (President), E. H. Doha (Vice-President), F. F. Ghaleb (Treasurer), M. H. Fahmy (Secretary).

Nigerian Mathematical Society

Address for mail: Department of Pure and Applied Mathematics, Ladoke Akintola University of Technology, Ogbomoso, Nigeria; email: ayeni_ro@yahoo.com; http://www.nigerianmathematicalsociety.com.

Apply to: Franic I. Njoku (Secretary), Nigerian Mathematical Society, Department of Mathematics, University of Nigeria, Nsukka, Nigeria.

Dues: U.S. $60, payable to Samuel Segun Okoya (Treasurer), Department of Mathematics, Obafemi Awolowo University, Ile-Ife, Nigeria.

Privileges: *Journal of the Nigerian Mathematical Society* and *Notices of the Nigerian Mathematical Society*.

Officers: Reuben Olafenwa Ayeni (President), Abbas A. Tijjani (Vice-President), Samuel Oegun Okoya (Treasurer), Franic I. Njoku (Secretary).

South African Mathematical Society*

Address for mail: School of Mathematics, Witwatersrand University, Private Bag 3, Wits 2050, South Africa; email: clint.VanAlten@wits.ac.za.

Apply to: Erwin Brüning, School of Mathematical Sciences, Kwazulu-Natal University, Private Bag X54001, Durban 4000, South Africa.

Dues: R210.00 (Two hundred ten rands), payable to the South African Mathematical Society (SAMS), c/o Prof. Erwin Brüning (Treasurer) at the above address.

Privileges: The right to receive the *Notices of the SAMS* at no additional cost; reduced fees at SAMS meetings.

Officers: Nigel Bishop (President), Themba Dube (Vice-President), Erwin Brüning (Treasurer), Clint Van Alten (Secretary).

Tunisian Mathematical Society*

Apply to: Khalifa Trimèche, Faculty of Sciences of Tunis, Department of Mathematics, CAMPUS 2092, Tunis, Tunisia; email: TMS@tms.rnu.tn; http://www.tms.rnu.tn.

Dues: $20, payable to Lotfi Kamoun at the above address.

Privileges: Obtain the publications of the Society, and possible partial financial support to attend the annual colloquium of the Society.

Officers: Khalifa Trimèche (President); Mohamed Sifi (Vice-President); Lotfi Kamoun (Treasurer); Abderrazek Karoui (Secretary).

The Americas

Canadian Mathematical Society

Address for mail: Canadian Mathematical Society, 209-1725 St. Laurent Blvd., Ottawa, Ontario, Canada

The American Mathematical Society has "reciprocity agreements" with a number of mathematical organizations around the world. A current list appears here.

These reciprocity agreements provide for reduced dues for members of these organizations who choose to join the AMS and who reside outside of the U.S. and Canada. Reciprocally, members of the AMS who reside in the U.S. or Canada may join these organizations at a reduced rate. Summaries of the privileges available to AMS members who join under the terms of reciprocity agreements are given on the following pages. Members of these organizations who join the AMS as reciprocity members enjoy all the privileges available to ordinary members of the Society. AMS dues for reciprocity members are $84 for 2011 and $86 for 2012. Each organization was asked to review and update its listing in the spring. An asterisk (∗) after the name of an organization indicates that no response to this request had been received when the November *Notices* went to press.

K1G 3V4; email: office@cms.math.ca; http://www.cms.math.ca/.

Apply to: Membership Department at the above address.

Dues: 50% off regular rates, payable in U.S. funds to the Canadian Mathematical Society at the above address.

Privileges: *CMS Notes*; access to members section on website; reductions on all CMS publications, books, promotional items, and meeting registration (biannual meetings).

Officers: Jacques Hurtubise (President), Catharine Baker, V. Kumar Murty, Pengfei Guan, Michael Lamoureux (Vice-Presidents), David Rodgers (Treasurer), Johan Rudnik (Executive Director/Secretary).

Sociedad Colombiana de Matemáticas*

Address for mail: Apartado Aereo 2521, Bogotá, Colombia; email: scm@scm.org.co; http://www.scm.org.co.

Apply to: Carlos H. Montenegro E., Apartado Aereo 2521, Bogotá, Colombia.

Dues: U.S. $27, payable to Sociedad Colombiana de Matemáticas.

Privileges: Subscription to one of the publications of the Society (*Revista Colombiana de Matemáticas or Lecturas Matematicas*), discounts for participation in Society activities, and e-mail in the scm.org.co domain.

Officers: Carlos H. Montenegro E. (President), Jose Ricardo Arteaga (Vice-President).

Sociedad de Matemática de Chile*

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Dues: U.S. $50, payable to Sociedad de Matemática de Chile.

Privileges: Receive *Gaceta de la Sociedad de Matemática*, *Notas de la Sociedad de Matemática de Chile*.

Officers: Rubí E. Rodríguez (President), Ana Cecilia De La Maza (Vice-President), Hernán Burgos (Treasurer), Andrés Navas (Secretary).

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Apply to: Isidro Rodríguez, Sociedad Matemática de la República Dominicana, Apartado 797-2, Santo Domingo, República Dominicana.

Dues: U.S. $10, payable to Amado Reyes at the above address.

Privileges: Right to receive *Notimat* (bimonthly newsletter) and *Revista Matemática Dominicana* (twice a year).

Officers: Isidro Rodríguez (President), Mariana Morales (Vice-President), Amado Reyes (Treasurer), Eliseo Cabrera (Secretary).

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Dues: U.S. $25, payable to Sociedad Matemática Mexicana.

Privileges: Reciprocity members pay reduced membership fees and receive the newsletter, *Bulletin of the Mexican Mathematical Society*, or *Miscelánea Matemática*.

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Apply to: José L. Vieitez (Presidente de SUME), at the above address.

Dues: U.S. $100, payable to Jorge Blanco at the above address.

Privileges: Receive PMU series and Predat series free.

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Address for mail: Estrada Dona Castorina–110, 22460-320 Rio de Janeiro, Brazil; email: presidente@sbm.org.br; http://www.sbm.org.br.

Apply to: secretaria@sbm.org.br.

Dues: BRL 40,00 (50% discount); email: secretaria@sbm.org.br.

Privileges: *Revista Matématica Universitária* (free subscription) and 25% discount on SBM books.

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Apply to: Andrea Alves Ribeiro, SBMAC/ICMC, Caixa Postal 668, Av., 13560-970 São Carlos–SP, Brazil; email: sbmac@icmc.usp.br; http://www.sbmac.org.br.

Dues: U.S. $50, payable to Sociedade Brasileira de Matemática Aplicada e Computacional at the above address.

Privileges: Free copies of SBMAC journals.

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Dues: U.S. $20, payable to Marcelo Escudiero Hernandes at the above address.

Privileges: A free subscription to the print version of *Boletim da Sociedade Paranaense de Matemática* (two issues per year).

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Apply to: Beatriz Marrón, Departamente de Matemática, Universidad Nacional del Sur, Av. Alem 1253, 8000 Bahía Blanca, Provincia de Buenos Aires, Argentina; email: `uma@union-matematica.org.ar`; `http://www.union-matematica.org.ar`.

Dues: U.S. $60, payable to Rosana Entizne at the above address.

Privileges: Same as those granted to UMA members.

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Asia

Allahabad Mathematical Society

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Dues: U.S. $30 for annual members, payable to Allahabad Mathematical Society at the above address.

Privileges: All journals/publications at discounted prices.

Officers: D. P. Gupta (President), S. P. Singh and S. L. Singh (Vice-Presidents), S. Srivastava (Treasurer), M. Khare (Secretary).

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Apply to: M. R. Adhikari, Secretary, Calcutta Mathematical Society, AE-374, Sector-1, Salt Lake City, Calcutta 700 064, India; telephone: (033) 2337-8882; Fax: (0091) 33-2337-6290; email: `cms@cal2.vsnl.net.in`.

Dues: U.S. $40, payable to Secretary, Calcutta Mathematical Society, at the above address.

Privileges: *Bulletin of the Calcutta Mathematical Society*; *News Bulletin of the Calcutta Mathematical Society*; *Review Bulletin of the Calcutta Mathematical Society*; library; seminars/symposia, summer and winter schools; workshops, popular lectures, etc.

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Privileges: Complimentary copy of the *The Mathematics Student*.

Officers: R. Sridharan (President), S. K. Nimbhorkar (Treasurer), S. B. Nimse (Administrative Secretary), Satya Deo (Academic Secretary), V. M. Shah (General Secretary).

Indonesian Mathematical Society (IndoMS)*

Apply to: Indonesian Mathematical Society, c/o Dr. Hilda Assiyatun, Department of Mathematics, Institut Teknologi Bandung (ITB), Jalan Ganesa 10 Bandung, Indonesia; `http://www.indoms-center.org`.

Dues: U.S. $15, payable to Dr. Hilda Assiyatun (Treasurer) at the above address.

Privileges: Reduced registration at conferences sponsored by The IndoMS and reduced price for any publications.

Officers: Edy Tri Baskoro (President), Widodo, Stevanus Budi Waluya, Angie Siti Anggari (Vice-Presidents), Hilda Assiyatun (Treasurer), Budi Nurani (Secretary).

Korean Mathematical Society

Address for mail: Korean Mathematical Society, Korea Science and Technology Center 202, 635-4 Yeoksam-dong, Kangnam-gu, Seoul 135-703, Korea; email: `kms@kms.or.kr`; `http://www.kms.or.kr/`.

Apply to: Dong-Kwan Shin (Vice-President), at the above address.

Dues: U.S. $40, payable to Korean Mathematical Society, at the above address.

Privileges: Members will receive six issues of *Journal of the KMS* and six issues of *Bulletin of the KMS*.

Officers: Dong Youp Suh (President), Jong Hae Keum, Yongjin Song, Dong-Kwan Shin (Vice-Presidents), Seonja Kim (Treasurer), Chang-Ock Lee and Yong-Kum Cho (Secretaries).

Mathematical Society of Japan*

Apply to: Yukino Ueno, Secretary, Mathematical Society of Japan, 34-8, Taito 1 chome, Taito-ku, Tokyo 110-0016, Japan; `http://wwwsoc.nii.ac.jp/msj6/math`.

Dues: Category I: 9,000 yen; Category II: 10,800 yen, payable to Mathematical Society of Japan at the above address.

Privileges: Category I: *Journal of the Mathematical Society of Japan*, *Sugaku-Tsusin* (2 issues); Category II: *Journal of the Mathematical Society of Japan*, *Sugaku* (in Japanese), *Sugaku-Tsushin* (4 issues).

Officers: Takashi Tsuboi (President), Liang Zhang (Treasurer), Yukino Ueno (Secretary).

Mathematical Society of the Philippines*

Address for mail: Mathematical Society of the Philippines, Institute of Mathematics, University of the Philippines, Diliman, Quezon City, Philippines 1101; email: mathsoc@mathsocietyphil.org; http://www.mathsocietyphil.org.

Apply to: Reginaldo Marcelo, Mathematics Department, Ateneo de Manila University, P.O. Box 154, Manila, Philippines.

Dues: U.S. $7, payable to Mathematical Society of the Philippines.

Privileges: Publications of the Mathematical Society of the Philippines; discount on conference fees.

Officers: Fidel Nemenzo (President), Jumela Sarmiento (Vice-President), Marian Roque (Treasurer), Ederlina Nocon (Secretary).

Mathematical Society of the Republic of China*

Address for mail: c/o Department of Mathematics, National Taiwan University No. 1, Roosevelt Road Section 4, Taipei 10617, Taiwan; email: tms@math.ntu.edu.tw; http://www.taiwanmathsoc.org.tw.

Dues: U.S. $45, payable to Mathematical Society of the Republic of China at the above address.

Privileges: One-year free subscription to the *Taiwanese Journal of Mathematics*.

Officers: Sze-Bi Hsu (President), Gerard Jennhwa Chang (Vice-President), Hui-Wen Lin (Treasurer), Jenn-Nan Wang (Secretary).

Mongolian Mathematical Society

Apply to: A. Galtbayar, Mongolian Mathematical Society, P. O. Box 187, Post Office 46A, Ulaanbaatar, Mongolia; email: galtbayar@num.edu.mn.

Dues: U.S. $50, payable to A. Galtbayar at the above address.

Privileges: Receive the *Mongolian Mathematical Journal* for free.

Officers: A. Mekei (President), B. Battsengel (Vice-President), A. Galtbayar and D. Purevsuren (Secretaries).

Nepal Mathematical Society

Apply to: Chet Raj Bhatta, Secretary, Nepal Mathematical Society, Central Department of Mathematics, Tribhuvan University, Kirtipur, Kathmandu, Nepal; email: crbhatta@yahoo.com.

Dues: No dues.

Privileges: All privileges enjoyed by an ordinary member, which includes purchasing NMS publications and participation in seminars at concessional rates.

Officers: Bhadra Man Tuladhar (President), Sharada Shrestha (Vice-President), Kabita Luitel (Treasurer), Chet Raj Bhatta (Secretary).

Persatuan Sains Matematik Malaysia*

Address for mail: Pusat Pengajian Sains Matematik, Universiti Kebangsaan Malaysia, 43600 Bangi, Selangor, Malaysia; email: maslina@pkrisc.cc.ukm.my; http://www.tmsk.uitm.edu.my/~persama.

Apply to: Dr. Maslina at the above address.

Dues: U.S. $7.50, payable to Bendahari, PERSAMA, at the above address.

Privileges: *Warkah Berita PERSAMA* (two issues per year), *Bulletin of the Malaysian Mathematical Society* (two issues per year), *Menemui Matematik* (two issues per year).

Officers: Mohd Salmi Md Noorani (President), Husna Hassan and Arsmah Ibrahim (Vice-Presidents), Wan Rosmanira Ismail (Treasurer), Maslina Darus (Secretary).

Punjab Mathematical Society*

Address for mail: Department of Mathematics, University of the Punjab, Quaid-i-Azam Campus, Lahore, Pakistan; email: mathdept@paknet.ptc.pk.

Apply to: Zia ul Haq, Secretary, Punjab Mathematical Society, Department of Maths., University of the Punjab, Lahore, Pakistan.

Dues: U.S. $30 for life membership, payable to Umar Farooq Qureshi, Treasurer, P.M.S.

Officers: G. Mustafa Habibullah (President), Zia Ullah Randhawa and Munir Ahmad Ch. (Vice-Presidents), Umar Farooq Qureshi (Treasurer), Nawazish Ali Shah (Secretary).

Ramanujan Mathematical Society*

Apply to: Professor V. Thangaraj, Secretary, Ramanujan Institute for Advanced Study in Mathematics, University of Madras, Chennai-600005, India; email: riasm@md3.vsnl.net.in; http://rms.enmail.com/.

Dues: U.S. $20 (annual), U.S. $200 (life), payable to Professor V. Thangaraj at the above address.

Privileges: Complimentary copy of the *Journal of the Ramanujan Mathematical Society*.

Officers: Phoolan Prasad (President), S. Sri Bala (Vice-President), P. Paulraja (Treasurer), V. Thangaraj (Secretary).

Singapore Mathematical Society

Apply to: Chan Lai Chee, Singapore Mathematical Society, c/o Department of Mathematics, National University of Singapore, 10 Lower Kent Ridge Road, Singapore 119076; email: smsuser@nus.edu.sg; http://sms.math.nus.edu.sg.

Dues: 10 Singapore dollars, payable to Singapore Mathematical Society at the above address.

Privileges: Reduced membership fee of S$10 (usual S$15)

Officers: Chengbo Zhu (President), Kim Hoo Hang, San Ling, Helmer Aslaksen (Vice-Presidents), Victor Tan (Treasurer), Kah Loon Ng (Secretary).

Southeast Asian Mathematical Society*

Apply to: c/o School of Mathematical Sciences, Universiti Sains Malaysia, 11800 USM Penang, Malaysia; email: rosihan@cs.usm.my; http://seams.math.nus.edu.sg.

Dues: None, membership by society of SEAMS only.

Officers: Rosihan M. Ali, Dato (President), Le Tuan Hoa and Fidel Nemenzo (Vice-Presidents), Saiful Hafizah Jaaman (Treasurer), Masllina Darus (Secretary).

Vietnam Mathematical Society

Address for mail: c/o 18 Hoang Quoc Viet Road, CauGiay District, 10307, Hanoi, Vietnam; email: vms@vms.org.vn; http://www.vms.org.vn.

Apply to: Phung Ho Hai at above address.

Dues: Hoi Toan Hoc Viet Nam, Account Number: 0491371684139, Vietcombank Thang Long, SWIFT: BFTVVNVX 049, 98 Hoang Quoc Viet, Cau Giay, Hanoi, Vietnam.

Privileges: Pay conference fees at the same rate as individual VMS members in any conferences organized or jointly organized by the VMS; buy (a) *Notices of the VMS* (in Vietnamese), 4 issues/year: U.S. $15 per year, or (b) *Vietnam Journal of Mathematics* (in English), 4 issues/year: U.S. $60 per year.

Officers: Le Tuan Hoa (President), Phung Ho Hai (Treasurer), Nguyen Huu Du (Secretary)

Vijnana Parishad of India*

Apply to: R. C. Singh Chandel, Secretary, Vijnana Parishad of India, D. V. Postgraduate College, Orai-285001, U.P., India; email: rc_chandel@yahoo.com.

Dues: U.S. $10, payable to Vijnana Parishad of India, D. V. Postgraduate College, Orai-285001, U.P., India.

Privileges: *Jñānābha* (an interdisciplinary mathematical journal currently published once a year); back volumes available at 25% discount.

Officers: G. C. Sharma (President), S. L. Singh, A. P. Singh, and A. K. Srivastava (Vice-Presidents), R. C. Singh Chandel (Secretary-Treasurer), H. M. Srivastava (Foreign Secretary).

Europe

Azerbaijan Mathematical Society*

Apply to: A. Ali Novruzov, Department of Mechanics and Mathematics, Baku State University, Baku, Azerbaijan, 370145.

Dues: U.S. $10, payable to Azerbaijan Mathematical Society.

Privileges: All privileges of ordinary members plus 50% discount on all AzMS publications.

Officers: O. A. Veliev (President), F. A. Abdullaev (Treasurer), V. A. Gasimov (Secretary).

Balkan Society of Geometers*

Apply to: Constantin Udriste, Treasurer, Department of Mathematics-Informatics I, University Politehnica of Bucharest, Splaiul Independentei 313, Bucharest 060042, Romania; email: udriste@mathem.pub.ro; http://www.mathem.pub.ro.

Dues: 30 euros (except persons from countries with financial difficulties, 10 euros), payable to the Balkan Society of Geometers at the above address.

Privileges: Participation in meetings and all other privileges enjoyed by an ordinary member; discounts (at least 10%) on the prices of BSG publications.

Officers: Constantin Udriste (President), Mihai Anastasiei, Gabriel Pripoaie, Vladimir Balan (Vice-Presidents), Constantin Udriste (Treasurer), Vasile Iftode (Secretary).

Belgian Mathematical Society*

Apply to: Jan van Casteren, Secretary, University of Antwerp, Department of Mathematics, Middelheimlaan 1, B-2020 Antwerp, Belgium; email: bms@ulb.ac.be; email: jan.vancasteren@ua.ac.be; http://bms.ulb.ac.be.

Dues: 18 euros, payable to Belgian Mathematical Society, Campus Plaine, CP 218/01, Bld. du Triomphe, B-1050 Brussels, Belgium. Account number: 000-0641030-54 (IBAN : BE 42 0000 6410 3054, BIC : BPOTBEB1).

Privileges: Membership includes a subscription to *Bulletin of the Belgian Mathematical Society—Simon Stevin*; newsletter.

Officers: Cathérine Finet (President), Stefaan Caenepeel (Vice-President), Guy Van Steen (Treasurer), Jan van Casteren (Secretary).

Berliner Mathematische Gesellschaft e. V.

Apply to: Wolfgang Volk, Secretary, Berliner Mathematische Gesellschaft, Schriftführer, Freie Universität Berlin, Institut für Mathematik, Arnimallee 3, 14195 Berlin, Germany; email: wolfgang.volk@berlin.de; http://www.berlmathges.de.

Dues: 30 euros, payable to Berliner Mathematische Gesellschaft e. V. at the above address. IBAN: DE48100708480472885300, BIC : DEUTDEDB110.

Privileges: *Sitzungsberichte der BMG* and quarterly journal *BMG-Forum*.

Officers: Gerhard Preuss (President), Rudolf Baierl (Vice-President), Michael E. Klews (Treasurer), Wolfgang Volk (Secretary).

Croatian Mathematical Society

Apply to: Renata Svedrec, Secretary, HMD, Department of Mathematics, Bijenička 30, 10000 Zagreb, Croatia; email: `hmd@math.hr`; `http://www.matematika.hr`.

Dues: U.S. $10, payable to HMD, Zagrebačka banka d.d. Zagreb, 2500-03688780–IBAN: HR442360000-1101530802.

Privileges: *Vjesnik HMD* (in Croatian) and one of five journals edited by CMS, free of charge. All publications of the CMS and all fees reduced by at least 25%.

Officers: Hrvoje Kraljević (President), Mirko Primc (Vice-President), Ivana Kokić (Treasurer), Renata Svedrec (Secretary).

Cyprus Mathematical Society

Apply to: Chrysanthi Stavrou (Secretary), 36 Stasinou Street, Suite 102, Strovolos 2003, Nicosia, Cyprus; email: `cms@cms.org.cy`; `http://www.cms.org.cy`.

Dues: 25 euros,, payable to Cyprus Mathematical Society at the above address.

Privileges: Members' fee for conferences and events organized by the CMS.

Officers: Gregory Makrides (President), Athanasios Gagatsis (Vice-President), Theoklitos Paragiou (Treasurer), Andreas Philippou (Secretary).

Dansk Matematisk Forening (Danish Mathematical Society)*

Address for mail: c/o President Vagn L. Hansen, Department of Mathematics, Building 303 S, Technical University of Denmark, DK-2800 Kongens Lyngby, Denmark; email: `dmf@mathematics.dk/`; `http://www.mathematics.dk/`.

Apply to: Please use the electronic form at `http://www.mathematics.dk/`.

Dues: DKK 155, payable to Carsten L. Petersen, Treasurer, Department of Science, NSM, Roskilde University (RUC), Building 27.2, Universitetsvej 1, Postbox 260, DK-4000 Roskilde, Denmark.

Privileges: *Mathematica Scandinavica* (750 DKK per year), *Nord. Mat. Tidss.* (*Normat*) (320 SEK per year). Members of the American Mathematical Society do not have to join Dansk Matematisk Forening to obtain the journals. Subscription orders should be sent directly to the journals: *Normat*, NCM Göteborgs Université, Box 160, SE-405 30 Gothenburg, Sweden; *Mathematica Scandinavica*, Matematisk Institut, Aarhus Universitet, 8000 Aarhus C, Denmark. Members of the American Mathematical Society who join the Danish Mathematical Society as reciprocity members will receive the newsletter *Matilde*.

Officers: Vagn Lundsgaard Hansen (President), Poul Hjorth (Vice-President), Carsten Lunde Peterson (Treasurer), Poul Hjorth (Secretary).

Deutsche Mathematiker-Vereinigung e.V. (DMV) (German Mathematical Society)*

Apply to: Roswitha Jahnke, DMV-Office, c/o WIAS, Mohrenstr. 39, 10117 Berlin, Germany; email: `dmv@wias-berlin.de`; `http://dmv.mathematik.de`.

Dues: 50 euros, payable to Deutsche Mathematiker-Vereinigung e.V., Volksbank Freiburg Konto: 6955 002, BLZ: 680 900 00, IBAN: DE 66 6809 0000 0006 9550 02, BIC: GENODE61FR1.

Privileges: Free subscription to *Jahresbericht der DMV* and one of these publications: *Math. Semesterberichte* or *Journal für Mathematik-didaktik*.

Officers: Wolfgang Lueck (President), Christian Baer (Vice-President), Juerg Kramer (Treasurer), Guenther Toerner (Secretary).

Edinburgh Mathematical Society*

Apply to: A. D. Gilbert, Honorary Secretary, Edinburgh Mathematical Society, James Clerk Maxwell Building, King's Buildings, Mayfield Road, Edinburgh EH9 3JZ, Scotland; email: `edmathsoc@ed.ac.uk`; `http://www.maths.ed.ac.uk`.

Dues: U.S. $20 (£10 sterling) without Society's proceedings.

Privileges: The Society's proceedings are available at a concessionary rate directly from Cambridge University Press (journals@cambridge.org): Print only: U.S. $23 (£15 sterling); Print and online: U.S.$28 (£18 sterling).

Officers: P. J. Davies (President), R. J. Archbold (Vice-President), M. A. Youngson (Treasurer), A. D. Gilbert and T. H. Lenagan (Secretaries).

European Mathematical Society

Apply to: Terhi Hautala, Department of Mathematics and Statistics, P.O. Box 68, 00014 University of Helsinki, Helsinki, Finland. email: `ems-office@helsinki.fi`; `http://www.euro-math-soc.eu`.

Dues: 34 euros, payable to Terhi Hautala at the above address.

Privileges: Same benefits as regular EMS individual members; benefits are listed on our website.

Officers: Marta Sanz-Solé (President), Mireille Martin-Deschamps and Martin Raussen (Vice-Presidents), Jouko Väänänen (Treasurer), Stephen Huggett (Secretary)

Finnish Mathematical Society

Apply to: Tadeas Priklopil (Secretary), Department of Mathematics and Statistics, P. O. Box 68, 00014 University of Helsinki, Finland email: `tadeas.priklopil@helsinki.fi`; `http://www.math.helsinki.fi`.

Dues: 16 euros, payable to Tadeas Priklopil at the above address.

Privileges: *Arkhimedes* (six issues per year) and *Eukleides* (newsletter), *Mathematica Scandinavica* at reduced price.

Officers: Matti Lassas (President), Maarit Jārvenpāc (Vice-President), Pekka Nieminen (Treasurer), Tadeas Priklopil (Secretary)

Gesellschaft für Angewandte Mathematik und Mechanik e.V. (GAMM)*

Apply to: GAMM, M. Kaliske, Institut für Statik und Dynamik der Tragwerke, Technische Universität Dresden, 01062 Dresden, Germany; email: gamm@mailbox.tu-dresden.de; http://www.gamm-eV.de.

Dues: 60 euros, payable to GAMM, Deutsche Bank 24 Wuppertal, BLZ 330 700 24, Konto-Nr. 2220911, GAMM, IBAN: DE09 3307 0024 0222 0911 00, BIC: DEUTDEDBWUP.

Privileges: Regular publications of GAMM and participation in scientific meetings at a reduced rate.

Officers: Volker Mehrmann (President), Peter Wriggers (Vice-President), Michael Günther (Treasurer), Michael Kaliske (Secretary).

Glasgow Mathematical Association*

Apply to: Frances Goldman, Treasurer, Glasgow Mathematical Association, Department of Mathematics, University of Glasgow, Glasgow G12 8QW, United Kingdom; email: fhg@maths.gla.ac.uk; http://www.maths.gla.ac.uk/.

Dues: £7, payable to Glasgow Mathematical Association, at the above address.

Privileges: *Glasgow Mathematical Journal* at reduced rate (£45).

Officers: A. Craw (President), F. Goldman (Treasurer), L. Moon (Secretary).

Hellenic (Greek) Mathematical Society*

Apply to: Hellenic Mathematical Society, 34, Panepistimiou Street, 106 79 Athens, Greece; email: info@hms.gr; http://www.hms.gr/.

Dues: U.S. $20 payable to Hellenic Mathematical Society at the above address.

Privileges: The *Bulletin of HMS*, News-Bulletin (Enimerosi), discounts that are available to all members.

Officers: Nikolaos Alexandris (President), George Dimakos and Dionysios Anapolitanos (Vice-Presidents), Evaggelos Eustathiou (Treasurer), Ioannis Tyrlis (Secretary).

Icelandic Mathematical Society*

Apply to: Ragnar Sigurdsson, Icelandic Mathematical Society, Raunvisindastofnun Haskolans, Dunhaga 3, IS-107 Reykjavik, Iceland; email: islenska.staerdfraedafelagid@gmail.com; http://www.staefelag.raunvis.hi.is.

Dues: U.S. $12, payable to Jóhann Sigurdsson at the above address.

Privileges: Reduced subscription rate on *Mathematica Scandinavia* and *Nordisk matematisk Tidskrift* (*Normat*); subscription orders should be sent directly to the journals.

Officers: Ragnar Sigurdsson (President), Jóhann Sigurdsson (Treasurer), Júlíana Rún Indridadóttir (Secretary).

Irish Mathematical Society*

Address for mail: Shane O'Rourke, Department of Mathematics, Cork Institute of Technology, Rossa Avenue, Bishopstown, Cork, Ireland; email: Shane.ORourke@cit.ie.

Apply to: Sinéad Breen, Department of Mathematics, St. Patrick's College, Drumcondra, Dublin 9, Ireland; email: sinead.breen@spd.dcu.ie.

Dues: U.S. $15 payable to Irish Mathematical Society, at the above address.

Privileges: Free copy of the *Bulletin of the Irish Mathematical Society* (two times per year); free registration at IMS annual conference (September).

Officers: James Cruickshank (President), Stephen Wills (Vice-President), Sinéad Breen (Treasurer), Shane O'Rourke (Secretary).

János Bolyai Mathematical Society*

Apply to: Ildikó Rákóczi, Executive Director, János Bolyai Mathematical Society, Fö utca 68, H-1027 Budapest, Hungary; email: bjmt@renyi.hu.

Dues: Are voluntary but should minimally cover duplication and mailing costs; for reciprocity members (residing outside Hungary) suggested fee is 1/8 of 1 percent of the member's net income, payable to Kereskedelmi ES Hitelbank P.T., Account Number 10200830-32310243. Sponsoring members pay at least U.S. $180 or equivalent per year.

Privileges: Upon request, *Matematikai Lapok* (twice a year), *Középiskolai Matematikai Lapok* (monthly). If sufficient interest is expressed, a bulletin in English will be available. In addition, the JBMS is negotiating to obtain discounts for its reciprocity and sponsoring members on several serial publications and periodicals appearing in Hungary. Contact the JBMS secretary for more information regarding this and other privileges of membership.

Officers: Gyula Katona (President), Ildikó Rákóczi (Executive Director), György Lippner (Treasurer), András Recski (Secretary General), Tibor Jordán (Vice Secretary General).

Jednota českých matematiků a fyziků (Union of Czech Mathematicians and Physicists)*

Apply to: Jan Kratochvíl, Union of Czech Mathematicians and Physicists, Žitná 25, 117 10 Praha 1, Czech Republic; email: jcmf@math.cas.cz; http://www.jcmf.cz.

Dues: U.S. $20, payable to Jan Obdržálek at the above address.

Privileges: (i) A discount of 20% in the conference fees for conferences, symposia, summer schools, and similar events organized (or coorganized) by the JČMF; (ii) newsletter.

Officers: Štefan Zajac (President), Eduard Fuchs and Oldřich Lepil (Vice-Presidents), Jan Obdržálek (Treasurer), Petr Řepa (Secretary).

Jednota slovenských matematikov a fyzikov (JSMF) (Union of Slovak Mathematicians and Physicists)*

Address for mail: Secretary of JSMF, FMFI UK Pavilon F1, Mlynská dolina, 842 48 Bratislava, Slovak Republic; email: JSMF@CENTER.FMPH.UNIBA.SK; http://www.uniba.sk/~jsmf.

Apply to: Hilda Draškovičová, FMFI UK, KATC, Mlynská dolina, 842 48 Bratislava, Slovak Republic.

Dues: U.S. $20, payable to Slovenská sporitel'ňa, Záhradnicka 93, 8000 Bratislava, Slovak Republic; č.u.: 101848-019/0900 IČO: 178705.

Privileges: A discount of 20% in conference fees for conferences, symposia, summer schools, and similar events organized by the JSMF.

Officers: Victor Bezak (President), Hilda Draškovičová (Vice-President), Edmund Dobročka (Treasurer), Imrich Morva (Secretary).

Koninklijk Wiskundig Genootschap*

Apply to: Rob van der Mei, CWI, P. O. Box 94079, 1090 GB Amsterdam, The Netherlands; email: r.d.van.der.mei@cwi.nl; http://www.wiskgenoot.nl.

Dues: 50 euros.

Privileges: Free periodical *Nieuw Archief voor Wiskunde*.

Officers: G. W. Vegter (President), S. Bhulai (Treasurer), R. van der Mei (Secretary).

London Mathematical Society

Apply to: Membership Department, London Mathematical Society, De Morgan House, 57–58 Russell Square, London
WC1B 4HS, United Kingdom; email: membership@lms.ac.uk; http://www.lms.ac.uk/.

Dues: £13.50. See: http://www.lms.ac.uk/content/membership.

Privileges: See above URL.

Officers: G. Segal (President), J. Greenlees and K. Brown (Vice-Presidents), R. Curtis (Treasurer), M. Hyland (Secretary).

Mathematical Society of Serbia*

Apply to: Milica Babić, Mathematical Society of Serbia, Knez Mihailova 35/IV, p.p. 355, 11000 Belgrade, Serbia; email: info@dms.org.yu; http://www.dms.org.yu.

Dues: U.S. $12, payable to DRUŠTVO MATEMATIČARA SRBIJE Acct. No. 250-6498-06, NACIONALNA ŠTEDIONICA.

Privileges: *Matematički Vesnik*, *Teaching of Mathematics*.

Officers: Branislav Popović (President), Zoran Kadelburg (Vice-President), Milica Babić (Treasurer), Biljana Babić (Secretary).

Norsk Matematisk Forening (Norwegian Mathematical Society)

Apply to: Marius Irgens, Norsk Matematisk Forening, Department of Mathematical Sciences, NTNU, No-7491, Trondheim, Norway; email: nmf@math.ntnu.no; http://matematikkforeningen.no.

Dues: NOK 100, payable to Norsk Matematisk Forening at the above address.

Privileges: All regular membership privileges, including the monthly newsletter *Infomat*.

Officers: Sigmund Selberg (President), Aslak B. Buan (Vice-President), Marius Irgens (Treasurer and Secretary).

Österreichische Mathematische Gesellschaft (OMG)*

Apply to: Robert F. Tichy, Institut für Mathematik, Technische Universität Graz, Steyrergasse 30, A-8010 Graz, Austria; email: oemg@oemg.ac.at; http://www.oemg.ac.at/.

Dues: 20 euros, payable to ÖMG, Wiedner Hauptstr. 8, A-1040 Wien, Bank Austria-Creditanstalt, IBAN: AT 83 12000229 10389200, BIC: BKAUATWW.

Privileges: *Internationale Mathematische Nachrichten* (IMN), reduction of fees at our congresses and meetings.

Officers: Robert F. Tichy (President), Michael Drmota (Vice-President), Helmut Pottmann (Treasurer), Michael Oberguggenberger (Secretary).

Polskie Towarzystwo Matematyczne

Apply to: Krystyna Jaworska, ul. Śniadeckich 8, 00-956 Warszawa, Poland; email: zgptm@ptm.org.pl; http://www.ptm.org.pl.

Dues: U.S. $22, payable to Polskie Towarzystwo Matematyczne, ul. Śniadeckich 8, 00-956 Warszawa, Poland, KREDYT BANK S.A. IBAN: PL 98 1500 1777 1217 7008 4349 0000, BIC/SWIFT: KRDBPL.

Privileges: Members receive one of the following series of the publication *Annales Societatis Mathematicae Polonae*: *Commentationes Mathematicae*, *Wiadomości Matematyczne* (in Polish).

Officers: Stefan Jackowski (President), Wacław Marzantowicz and Paweł Walczak (Vice-Presidents), Krystyna Jaworska (Treasurer and Secretary).

Real Sociedad Matemática Española

Apply to: Henar Herrero, Secretaria de la Real Sociedad Matemática Española, Universidad Complutense de Madrid, Facultad de Matematicas, Despacho 525, 28040 Madrid, Spain; email: `secretaria@rsme.es`; `http://www.rsme.es`.

Dues: 25 euros, payable to Real Sociedad Matemática Española at the above address.

Privileges: *La Gaceta de la RSME* and *Descoentos en Congresos Organizados por la RSME.*

Officers: Antonio Campillo (President), Santos González and Luis Vega (Vice-Presidents), Julio Bernués (Treasurer), Henar Herrero (Secretary).

Sociedad Española de Matemática Aplicada (SEMA)

Apply to: Julio Moro (Secretary), Despacho 520, Facultad de Matematicas, Universidad Complutense, 28040 Madrid, Spain; email: `info@sema.org.es`; `http://www.sema.org.es` .

Dues: 15 euros, payable to SEMA at the above address.

Privileges: Information concerning applied mathematics in Spain through *Boletín de la SEMA*; reduced registration fees for activities sponsored by SEMA.

Officers: Pablo Pedregal (President), Rosa Donat (Vice-President), Juan Belmonte (Treasurer), Julio Moro (Secretary).

Sociedade Portuguesa de Matemática

Apply to: Filipe Oliveira, Sociedade Portuguesa de Matemática, Av. da República 45–3°, 1050-187 Lisboa, Portugal; email: `spm@spm.pt`; `http://www.spm.pt`.

Dues: 20 euros, payable to the Sociedade Portuguesa de Matemática, at the address above.

Privileges: Each reciprocity member receives all publications that SPM members receive and pays half the SPM membership fees.

Officers: Miguel Abreu (President), Filipe Oliveira and Joana Teles (Vice-Presidents), Silvia Anjos (Treasurer).

Societat Catalana de Matemàtiques

Apply to: Joan de Solà-Morales, President, Societat Catalana de Matemàtiques, Carrer del Carme 47, 08001, Barcelona, Spain; email: `scm@iec.cat`; `http://scm.iec.cat`.

Dues: 18 euros, payable to the Societat Catalana de Matemàtiques.

Privileges: *Butlletí de la Societat Catalana de Matemàtiques* (two times per year) and *SCM/Notices* (two times per year).

Officers: Joan de Solà-Morales (President), Joaquim Ortega-Cerdà (Vice-President), Mariona Petit-Vilà (Treasurer), Mercè Farré-Cervelló (Secretary).

Societatea Matematicienilor din Romania*

Apply to: Horia I. Ene, Calea Grivitei 21, P. O. Box 1-764, 70700 Bucvharest, Romania.

Dues: U.S. $10, payable to Societatea Matematicienilor din Romania at the address above.

Privileges: Reduced rates for participation in scientific conferences organized by SMR, *Bulletin Mathématiques* (four times per year) free.

Officers: Horia I. Ene (President), Nicolae Popa (Vice-President), Serban Barcanescu (Treasurer), Radu Purice (Secretary).

Societatea de Ştiinţe Matematice din România

Apply to: Radu Gologan, President, Str. Academiei, NR. 14, Sector 1, 010014, Bucureşti, Rômania; email: `office@rms.unibuc.ro`; `http://www.rms.unibuc.ro`.

Dues: U.S. $15/$30 (see privileges below), payable to Societatea de Ştiinţe Matematice din România, Account R008 RNCB 0076 0043 5732 0002, Banca Comercială Română, Filiala Sector 5, Bucureşti, România. SWIFT Code: RNCB ROBU B5O.

Privileges: For membership dues of U.S. $30, free subscription to one of the Society's journals. When participating in the annual meetings of the Society, all AMS members are exempt from taxes.

Officers: Radu Gologan (President), Doru Ştefănescu (Vice-President), Cristina Luţu (Treasurer), Mircea Trifu (Secretary).

Société Mathématique de France

Apply to: C. Ropartz, Société Mathématique de France, Institut Henri Poincaré, 11 Rue Pierre et Marie Curie, F-75231 Paris cedex 05, France; email: `smf@dma.ens.fr`; `http://smf.emath.fr/`.

Dues: U. S. $40, payable to the American Mathematical Society for SMF.

Privileges: *Bulletin*, U.S. $176; *Memoires*, U.S. $139; *Bulletin* and *Mémoires*, U.S. $315; *Bulletin* (electronic-only version), U.S. $135; *Astérisque*, U.S. $560; *Histoire des Mathématiques*, U.S. $84; *Histoire des Mathématiques* (electronic-only version), U.S. $64; *Panoramas et Synthèses*, U.S. $100; *Annales scientifiques de l'ENS*, U.S. $534.

Officers: Bernard Helffer (President), Yves Aubry, Olivier Ramaré, Aviva Szpirglas, Nalini Anantharaman (Vice-Presidents), Micheline Vigué (Treasurer), Clotilde Fermanian (Adjoint Treasurer), Emmanuel Russ (Secretary).

Société Mathématique du Luxembourg*

Apply to: Martin Schlichenmaier, Société Mathématique du Luxembourg, Université du Luxembourg, Mathematics Research Unit, FSTC, Campus Kirchberg 6, rue Coudenhove-Kalergi, L-1359 Luxembourg-Kirchberg; email: martin.schlichenmaier@uni.lu; http://math.uni.lu/sml.

Dues: 20 euros payable to Société Mathématique du Luxembourg at the above address.

Privileges: Discount on membership dues.

Officers: Martin Schlichenmaier (President), Martin Olbrich (Vice-President), Jean Schiltz (Treasurer), Jean-Luc Marichal (Secretary).

Société de Mathématiques Appliquées et Industrielles (SMAI)

Apply to: Société de Mathématiques Appliquées et Industrielles (SMAI), Institut Henri Poincaré, 11 rue Pierre et Marie Curie, 75231 Paris cedex 05, France; email: smai@emath.fr; http://smai.emath.fr/.

Dues: 35 euros, payable to Société de Mathématiques Appliquées et Industrielles at the above address.

Privileges: Reduced fees.

Officers: M. J. Esteban (President), F. Murat (Vice-President), F. Lagoutière (Treasurer), A. Lejay (Secretary).

Society of Associations of Mathematicians and Computer Scientists of Macedonia*

Apply to: Boro Piperevski, President SAMCSM, Pirinska B.B., 91000 Skopje, Macedonia.

Dues: $5, payable to SDMI na MAKEDONIA, acct. 40120-678-10217, Pirinska B.B., 91000 Skopje, Macedonia.

Privileges: Receiving the *Bulletin of SAMCSM* and taking part in SAMCSM activities.

Officers: Boro Piperevski (President), Borko Ilievski (Vice-President), Kosta Miševski (Treasurer), Vasile Marčevski (Secretary).

Society of Mathematicians, Physicists, and Astronomers of Slovenia*

Address for mail: DMFA, P.P. 2964, 1000 Ljubljana, Slovenia; email: tomaz.pisanski@fmf.uni-lj.si; http://www.dmfa.si/.

Apply to: Tomaž Pisanski at the above address.

Dues: SKB Banka D. D., Ajdovscina 4, SWIFT (BIC): SKBASI2X, IBAN: SI56 0310 0100 0018 78

Privileges: Subscription to *Obzornik za matematiko in fiziko* (surface mail).

Officers: Zvonko Trontelj (President), Nada Razpet (Vice-President), Andreja Jaklič (Treasurer), Janez Krušič (Secretary).

Svenska Matematikersamfundet (Swedish Mathematical Society)*

Address for mail: Tobias Ekholm, Matematikcentrum, Lund University, Box 118, SE-221 00 Lund, Sweden; email: treasurer@swe-math.soc-se; http://www.matematikersamfundet.org.se.

Apply to: Milagros I. Barrios, MAI, Linköping University, SE-581 83 Linköping, Sweden.

Dues: 100 Swedish crowns, payable to Milagros I. Barrios at above address.

Privileges: *Mathematica Scandinavia* and *Nordisk Matematisk Tidskrift* at reduced rates. Newsletter (*Utskicket*) about the activities and meetings of the Society.

Officers: Tobias Ekholm (President), Mikael Passare (Vice-President), Milagros I. Barrios (Treasurer), Warwick Tucker (Secretary).

Swiss Mathematical Society

Address for mail: Claudia Kolly, Swiss Mathematical Society, Université de Fribourg, Perolles, Mathematics Department, Chemin du musee 23, CH-1700 Fribourg, Switzerland; email: claudia.kolly@unifr.ch; http://www.math.ch.

Apply to: Norbert Hungerbühler, email: hungerbuehler@math.ethz.ch.

Dues: 25 CHF, payable to Swiss Mathematical Society, IBAN: CH35 0900 0000 8001 6483 5.

Privileges: Membership fee half of ordinary fee.

Officers: Bruno Colbois (President), Christine Riedtmann (Vice-President), Nicolas Monod (Secretary-Treasurer).

Ukrainian Mathematical Society*

Apply to: A. S. Serdyuk, Institute of Mathematics, National Academy of Sciences, Ukraine, Tereschenkivskaja str., 3, 01601 Kyiv-4, Ukraine; email: sam@imath.kiev.ua.

Dues: U.S. $30, payable to N. A. Nazarenko at the above address.

Privileges: All privileges of a normal individual UMS member.

Officers: A. M. Samoilenko (President), M. L. Gorbachuk (Vice-President), N. A. Nazarenko (Treasurer), A. S. Serdyuk (Secretary).

Union of Bulgarian Mathematicians*

Apply to: Sava Ivanov Grozdev, Secretary, Union of Bulgarian Mathematicians, Acad. G. Bonchev Str., Block 8, BG-1113 Sofia, Bulgaria.

Dues: 20 USD, payable to Union of Bulgarian Mathematicians, Account #1100366612, BULBANK AD Central office, code 62196214.

Privileges: The right to attend all events organized by the UBM at reduced rate and to present papers at them, the right to attend other events in Bulgaria at a reduced rate, and the right to purchase all UMB editions at a reduced rate.

Officers: St. Dodunekov (President), I. Tonov, O. Mushkarov, R. Nikolaev (Vice Presidents).

Unione Matematica Italiana*

Apply to: Giuseppe Anichini, Unione Matematica Italiana (U.M.I.), Piazza Porta San Donato, 5, 40126 Bologna, Italy; email: `umi@dm.unibo.it`; `http://umi.dm.unibo.it/`.

Dues: 50 euros, payable to Unione Matematica Italiana.

Privileges: Free *Notiziario dell'UMI* (10 issues a year), *Rivista la Matematica nella Societa e nella cultura* (4 issues per year).

Officers: Franco Brezzi (President), Graziano Gentili (Vice-President), Barbara Lazzari (Treasurer), Giuseppe Anichini (Secretary).

Middle East

Iranian Mathematical Society*

Apply to: M. Shokouhi, Iranian Mathematical Society, P.O. Box 13145-418, Tehran, Iran; email: `iranmath@ims.ir`; `http://www.ims.ir`.

Dues: U.S. $45 payable to Iranian Mathematical Society at the above address.

Privileges: *Bulletin of the Iranian Mathematical Society* (two issues per year in English), *Farhang va Andisheh Riazi* (two issues per year in Persian), *Khabarnameh* and *Gozaresh* (8 issues per year in Persian), and reduced rate for participation in the conferences and seminars organized by IMS.

Officers: A. R. Medghalchi (President), M. J. Mamayhani (Treasurer).

Israel Mathematical Union (IMU)

Apply to: Uzi Vishne (Secretary), Israel Mathematical Union, Department of Mathematics, Bar-Ilan University, Ramat-Gan 52900, Israel; email: `imu@imu.org.il`; `http://www.imu.org.il/`.

Dues: U. S. $50, payable to Israel Mathematical Union.

Privileges: Participation in meetings and all other privileges enjoyed by an ordinary member.

Officers: Louis Rowen (President), Uzi Vishne (Treasurer), Tahl Nowik (Secretary).

Palestinian Society for Mathematical Sciences*

Address for mail: Mathematics Department, Birzeit University, P. O. Box 14, West Bank, Palestine.

Apply to: Fawzi Yagoub, Department of Mathematics and Computer Science, SUNY College at Fredonia, Fredonia, NY 14063.

Dues: U.S. $30, payable to Fawzi Yagoub; see address above.

Privileges: Free issues of the *PSMS Newsletter*, 50% reduction on all PSMS conference fees, 50% reduction on all PSMS publications.

Officers: Mohammad Al-Amleh (President); Mohammad Saleh, Tahseen Mughrabi (Vice-Presidents); Raghib Abu Saris, Nur edden Rabei, Mohammad El-Atrash, Taha Abu Kaf, Saber Elaydi (Members).

Saudi Association for Mathematical Sciences*

Apply to: M. A. Alabdullatif, President, King Saud University, College of Science, P. O. Box 2455, Riyadh 11451, Saudi Arabia.

Dues: U.S. $30, payable to Saudi Association for Mathematical Sciences at the above address.

Privileges: Reduction in membership fee from U.S. $40 to U.S. $30; proceedings of conferences, symposia, and seminars arranged by the Association.

Officers: M. A. Alabdullatif (President), A. Alshihah (Vice-President), M. A. Aseerj (Treasurer), M. S. Qutaifan (Secretary).

Turkish Mathematical Society

Apply to: Cem Güneri, Sabanci Üniversitesi İletişim Merkezi, Bankalar caddesi 2, Karaköy 80020 Istanbul, Turkey; email: `tmd@tmd.org.tr`; `http://www.tmd.org.tf`.

Dues: US $40, payable to Türk Matematik Dernegi, IBAN: TR24 0006 2000 0670 0009 0889 51.

Privileges: 50% discount for the National Symposium fee; membership for the journal *Matematik Dünyasi*.

Officers: Betül Tanbay (President); Hüsnü Erbay (Vice-President); Bariş Coşkünuzer (Treasurer); Cem Güneri (Secretary).

South Pacific

Australian Mathematical Society Inc.

Apply to: The Business Manager, AustMS Business Office, Department of Mathematics, Australian National University, Canberra, ACT 0200, Australia; email: `austms@anu.edu.au`; `http://www.austms.org.au/`.

Dues: $AUD 59, payable to the Australian Mathematical Society, c/o The Business Manager, at the above address.

Privileges: Complimentary issues of *The Gazette*. Reduced prices for the *Journal of the AustMS* ($AUD 80), the *ANZIAM Journal* ($AUD 68), *Bulletin of the AustMS* ($AUD 74), and for volumes in the AustMS Lecture Series.

Reduced registration fees at conferences supported by AustMS.

Officers: P. G. Taylor (President), T. R. Marchant and A. P. Mathas (Vice-Presidents), A. Howe (Treasurer), P. J. Stacey (Secretary).

New Zealand Mathematical Society*

Address for mail: New Zealand Mathematical Society, c/o Alex James (NZMS Secretary), Institute of Information and Mathematical Sciences, Massey University at Albany, Private Bag 102904, North Shore 0745, Auckland, New Zealand; email: `jshanks@maths.otago.ac.nz`; `http://www.math.waikato.ac.nz/NZMS/NZMS.html`.

Apply to: John Shanks, Department of Mathematics and Statistics, University of Otago, P.O. Box 56, Dunedin 9054, New Zealand.

Dues: NZ$25 payable to John Shanks at above address.

Privileges: 50% discount on membership fees.

Officers: Charles Semple (President), Peter Donelan (Treasurer), Alex James (Secretary).

Mathematics Calendar

Please submit conference information for the Mathematics Calendar through the Mathematics Calendar submission form at http://www.ams.org/cgi-bin/mathcal-submit.pl.
The most comprehensive and up-to-date Mathematics Calendar information is available on the AMS website at http://www.ams.org/mathcal/.

November 2011

* 10–11 **International Conference on Web Information System and Computing Education**, Hotel Sleep Withinn, Bangkok, Thailand.
Description: ICWISCE 2011 focuses on real world applications; therefore authors should highlight the benefits of Web Information Systems and Computing Education for industry and services, in addition to academic applications. Ideas on how to solve business problems, using Web-based information systems and technologies will arise from the conference. Papers describing advanced prototypes, systems, tools and techniques and general survey papers indicating future directions are also encouraged. Both technological and social-oriented papers are accepted. All papers must describe original work, not previously published or submitted to another conference. Accepted papers, presented at the conference by one of the authors, will be published in the Proceedings of ICWISCE. Acceptance will be based on quality, relevance and originality. Both full research reports and work-in-progress reports are welcome. There will be both oral and poster sessions.
Information: http://www.icwisce.com.

* 28–29 **International Conference on Information Systems, Computer Engineering & Application—ICISCEA 2011**, Hotel Meritus, Singapore, Singapore.
Description: To bring together researchers, engineers and practitioners interested in the technological advances and business applications of information systems. The conference also serves as forum to share research solutions to problems of today's information society and to identify new issues and directions for future research and development work. The conference is organized by Department of CSE, Open Learning Society.
Information: http://www.cfisws.com.

* 29–December 2 **International Conference on Design and Modeling in Science, Education, and Technology: DeMset 2011**, Orlando, Florida.
Description: The purpose of organizing the international conference on Design and Modeling in Science, Education, and Technology: DeMSET 2011 is to promote intra- and inter-disciplinary communication by means of common concepts, methods, and tools used in different sub-disciplines and disciplines. Formal presentations might be made in disciplinary and interdisciplinary terms, while informal inter-disciplinary communication could produce insights and analogical thinking.
Suggested Types of Submissions: Articles related to Models, Design, and/or Reflective Practice in Modeling in Science, Education and/or Engineering . Articles related to specific models and designs presented as case studies, where the model, or the design, is presented with a reflection on the modeling or designing method followed, and its potential applicability in other disciplines or areas. Articles related to designing and modeling methods and methodologies. etc.
Information: http://www.iiis2011.org/demset/Website/home.asp?vc=47.

This section contains announcements of meetings and conferences of interest to some segment of the mathematical public, including ad hoc, local, or regional meetings, and meetings and symposia devoted to specialized topics, as well as announcements of regularly scheduled meetings of national or international mathematical organizations. A complete list of meetings of the Society can be found on the last page of each issue.
An announcement will be published in the *Notices* if it contains a call for papers and specifies the place, date, subject (when applicable), and the speakers; a second announcement will be published only if there are changes or necessary additional information. Once an announcement has appeared, the event will be briefly noted in every third issue until it has been held and a reference will be given in parentheses to the month, year, and page of the issue in which the complete information appeared. Asterisks (*) mark those announcements containing new or revised information.
In general, announcements of meetings and conferences carry only the date, title of meeting, place of meeting, names of speakers (or sometimes a general statement on the program), deadlines for abstracts or contributed papers, and source of further information. If there is any application deadline with respect to participation in the meeting, this fact should be noted. All communications on meetings and conferences in the mathematical sciences should be sent to the Editor of the *Notices* in care of the American Mathematical Society in Providence or electronically to notices@ams.org or mathcal@ams.org.
In order to allow participants to arrange their travel plans, organizers of meetings are urged to submit information for these listings early enough to allow them to appear in more than one issue of the *Notices* prior to the meeting in question. To achieve this, listings should be received in Providence **eight months** prior to the scheduled date of the meeting.
The complete listing of the Mathematics Calendar will be published only in the September issue of the *Notices*. The March, June/July, and December issues will include, along with new announcements, references to any previously announced meetings and conferences occurring within the twelve-month period following the month of those issues. New information about meetings and conferences that will occur later than the twelve-month period will be announced once in full and will not be repeated until the date of the conference or meeting falls within the twelve-month period.
The Mathematics Calendar, as well as Meetings and Conferences of the AMS, is now available electronically through the AMS website on the World Wide Web. To access the AMS website, use the URL: http://www.ams.org/.

February 2012

* 6–10 **AIM Workshop: Systems approaches to drug discovery and development in oncology**, American Institute of Mathematics, Palo Alto, California.
Description: This workshop, sponsored by AIM and the NSF, will bring leading experts and new researchers from pharmaceutical companies and academic institutions to share their perspectives on current mechanism-based systems approaches in drug discovery and development programs in oncology.
Information: `http://www.aimath.org/ARCC/workshops/systemsoncology.html`.

* 27–28 **4th International Conference on Wireless Information Networks and Business Information System (WINBIS 2012)**, Hotel Marshyandi, Kathmandu, Nepal.
Description: On behalf of the organizing committee, it is our pleasure to invite you to this international meeting. Since 2009, Open Learning Society is organizing WINBIS and it's series every year.
Information: `http://www.win-bis.com`.

April 2012

* 9–13 **AIM Workshop: Nonlinear solvers for high-intensity focused ultrasound with application to cancer treatment**, American Institute of Mathematics, Palo Alto, California.
Description: This workshop, sponsored by AIM and the NSF, will focus on numerical techniques for the solution of nonlinear acoustic problems with application to cancer treatment by means of high-intensity focused ultrasound.
Information: `http://www.aimath.org/ARCC/workshops/hifucdi.html`.

* 23–27 **Spring School in Probability**, Inter-University Center, Dubrovnik, Croatia.
Description: The school is primarily aimed at advanced doctoral students and early postdocs in probability theory and random processes. There will be five intensive courses broadly focused on random walks and jump processes and their applications to real-world problems. More specifically, the topics covered will include the coupling of Lévy processes, probabilistic potential theory of jump processes and heat kernel estimates, random walks on graphs and disordered media and their scaling limits, reinforced random walks, and discretization of jump processes and Lévy driven stochastic differential equations. Participants will have a chance to present their research results.
Lecturers: P. Kim, T. Kumagai, V. Limić, R. L. Schilling, P. Tankov.
Organizers: R. L. Schilling, R. Song, Z. Vondraček.
Information: `http://web.math.hr/ssp-iuc`.

* 29–May 2 **4th International Interdisciplinary Chaos Symposium on "Chaos and Complex Systems"**, Antalya, Turkey.
Description: The aim of the symposium is to convene the scientists and social scientists from various branches and to discuss the latest improvements in "Chaos and Complex Systems". Since the topic of chaos is a relatively new branch of science which is applicable to various disciplines, it is an attractor for researchers. Therefore, in order to convene regularly the scientists who work on and/or are interested in this area, the "Istanbul Kultur University Chaos Symposia" are organized. The contents of the symposium have been widened in an "interdisciplinary" manner, so as to allow all participants to present their works.
Information: `http://www.ccs2012.org`.

June 2012

* 7–9 **Quantum invariants of 3-manifolds**, Institut de Recherche Mathématiquée Avancé, University of Strasbourg, France.
Description: The theme is "Quantum Invariants of 3-Manifolds in Mathematics and in Physics".
Organizers: Athanase Papadopoulos and Vladimir Turaev.
Invited speakers: Norbert A.'Campo (Basel), Joergen Ellegaard Andersen (Aarhus), Anna Baliakova (Zürich), Francesco Constantino (Strasbourg), Etera Livine (ENS Lyon), Julien Marché (Ecole Polytechnique), Rinat Kashaev (Genève), Thomas Krajewski (CPT Marseille), Vladimir Matveev (Cheliabinsk), Gwenael Massuyeau (Strasbourg), Majid Narimannejad (Basel).
Language: English. Some of the talks will be survey talks intended for a general audience. Graduate students and young mathematicians are welcome.
Contact: email: `athanase.papadopoulos@math.unistra.fr`; and email: `vladimir.turaev@math.unistra.fr`.
Information: `http://www-irma.u-strasbg.fr/article1172.html`.

* 12–15 **5th Chaotic Modeling and Simulation International Conference (CHAOS2012)**, Athens, Greece.
Description: The general topics and the special sessions proposed for the Conference include but are not limited to: Chaos and Nonlinear Dynamics, Stochastic Chaos, Chemical Chaos, Data Analysis and Chaos, Hydrodynamics, Turbulence and Plasmas, Optics and Chaos, Chaotic Oscillations and Circuits, Chaos in Climate Dynamics, Geophysical Flows, Biology and Chaos, Neurophysiology and Chaos, Hamiltonian Systems, Chaos in Astronomy and Astrophysics, Chaos and Solitons, Micro- and Nano- Electro-Mechanical Systems, Neural Networks and Chaos, Ecology and Economy. The publications of the conference include: 1. The Book of Abstracts in Electronic and in Paper form; 2. Electronic Proceedings in CD and in the web in a permanent website; 3. Publication of the best papers in the *Journal of Chaotic Modeling and Simulation*.
Information: For more information and Abstract/Paper submission and Special Session Proposals please visit the conference website at: `http://www.cmsim.org`.

* 25–29 **AIM Workshop: Hypergeometric motives**, International Centre for Theoretical Physics, Trieste, Italy.
Description: This workshop, sponsored by AIM, NSF, and ICTP, will focus on the L-functions of arithmetic geometry whose Euler factors are generically of degree higher than two.
Information: `http://aimath.org/ARCC/workshops/hypermotives.html`.

* 25–29 **3rd European Seminar on Computing (ESCO 2012)**, Pilsen, Czech Republic.
Description: ESCO 2012 is the 3rd event in a successful series of interdiscipilineary meetings dedicated to modern methods and practices of scientific computing. Main thematic areas include: Multiphysics coupled problems, Higher-order computational methods, Computing with Python, GPU computing, and Cloud computing. Theoretical results as well as applications are welcome. Application areas include, but are not limited to: Computational electromagnetics, Civil engineering, Nuclear engineering, Mechanical engineering, Computational fluid dynamics, Computational geophysics, Geomechanics and rock mechanics, Computational hydrology, Subsurface modeling, Biomechanics, Computational chemistry, Climate and weather modeling, Wave propagation, Acoustics, Stochastic differential equations, and Uncertainty quantification. Minisymposia proposals are welcome. A minisymposium should consist of at least four lectures related to the same topic. Current list of minisymposia can be found on conference webpage.
Information: `http://esco2012.femhub.com`.

July 2012

* 1–7 **Workshop on geometric structures on manifolds and their applications**, Castle Rauischholzhausen near Marburg, Germany.
Description: The workshop takes places at Castle Rauischholzhausen near Marburg, 100 km north of Frankfurt. Its aim is to bring together in a casual and friendly atmosphere scientists from mathematics and theoretical physics working on topics like: Einstein

metrics, Sasakian geometry, almost Hermitian geometry, G_2 structures, holonomy theory, Dirac operators, spin geometry, generalized Kähler geometry, calibrated geometry, applications to super strings and mathematical physics.
Organizers: Ilka Agricola (Marburg, Germany), Anna Fino (Torino, Italy), Andrew Dancer (Oxford, UK), Nigel Hitchin (Oxford, UK).
Information: `http://www.mathematik.uni-marburg.de/~agricola/rauisch2012/`.

* 2–6 **24th Conference in Operator Theory**, West University, Timisoara, Romania.
Description: The conference is devoted to operator theory, operator algebras and their applications (differential operators, complex functions, mathematical physics, matrix analysis, system theory, etc.). The editors of *Journal of Operator Theory* (K. R. Davidson, N. K. Nikolski, G. Pisier, S. Stratila, F.-H. Vasilescu) form the Steering Committee of the conference.
Information: `http://www.imar.ro/~ot/`.

* 16–18 **SING8: 8th Spain-Italy-Netherlands Meeting on Game Theory**, Institute of Economics, Hungarian Academy of Sciences and Corvinus University, Budapest, Hungary.
Description: SING8 is the 8th conference in the Spain-Italy-Netherlands series of meetings on Game Theory and the first organised in Hungary. While many of the participants come from the founding countries or other European countries, the conference is open to all and covers all areas and aspects of game theory.
Organizers: Organized jointly by the Institute of Economics, Hungarian Academy of Sciences, and Corvinus University Budapest.
Invited speakers: Francis Bloch (École Polytechnique), Péter Csermely (Semmelweis University), Aviad Heifetz (Open University of Israel), Jean-Jacques Herings (Maastricht University), Hamid Sabourian (University of Cambridge).
Important dates: Invited sessions summary: January 12, 2012. Abstracts submission: February 12, 2012. Notification of acceptance: March 12, 2012. Early registration: April 12, 2012.
Information: `http://sing8.iehas.hu/`.

* 30–August 3 **The 24th International Conference on Formal Power Series and Algebraic Combinatorics (FPSAC.12)**, Nagoya University, Nagoya, Japan.
Topics: Include all aspects of combinatorics and their relations with other parts of mathematics, physics, computer science, and biology. The conference will feature invited lectures, contributed presentations, poster session, problem session, and software demonstrations. As usual, there will be no parallel sessions.
Languages: English and French.
Invited Speakers: Alexei Borodin (M.I.T., U.S.A.), Gil Kalai (Hebrew Univ., Israel), Manuel Kauers (Johannes Kepler Univ., Austria), Rinat Kedem (Univ. Illinois, Urbana-Champaign, U.S.A.), Thomas Lam (Univ. Michigan, U.S.A.), Jennifer Morse (Drexel Univ., U.S.A.), Hjalmar Rosengren (Chalmers Univ. and Univ. Gothenburg, Sweden), Catharina Stroppel (Univ. Bonn, Germany), Hiroaki Terao (Hokkaido Univ., Japan).
Submission: Authors are invited to submit extended abstracts of at most twelve pages by November 30, 2011.
Information: `http://www.math.nagoya-u.ac.jp/fpsac12`.

September 2012

* 20–22 **Lie and Klein; the Erlangen program and its impact on mathematics and physics**, Institut de Recherche Mathématiqué Avancé, University of Strasbourg, France.
Description: The theme is "Lie and Klein; the Erlangen program and its impact on mathematics and physics".
Organizers: Lizhen Ji and Athanase Papadopoulos.
The invited speakers are: Norbert A.'Campo (Basel), Gérard Besson (Grenoble), Pierre Cartier (IHES), Hubert Goenner (Göttingen), Misha Gromov (IHES), to be confirmed, Frances Kirwan (Oxford), to be confirmed, Vladimir Matveev (Jena), Catherine Meusburger (Erlangen), Pierre Py (Strasbourg), Jean-Marc Schlenker (Toulouse), Alexei Sossinsky (Moscow), Anna Wienhard (Princeton).
Language: English. Graduate students and young mathematicians are welcome.
Reservations: Hotel booking can be asked for through the registration link. Contact: email: `lji@umich.edu` and `athanase.papadopoulos@math.unistra.fr`.
Information: `http://www-irma.u-strasbg.fr/article1173.html`.

The following new announcements will not be repeated until the criteria in the next to the last paragraph at the bottom of the first page of this section are met.

December 2012

* 16–22 **Commutative rings, Integer-valued polynomials and Polynomial functions**, Graz University of Technology, Graz, Austria.
Confirmed speakers: Jean-Luc Chabert, Sarah Glaz, Byung Gyun Kang, Alan Loper, Irena Swanson.
Description: The conference will be preceded by different mini-courses, December 16–18, 2012, taught by the main speakers of the conference.
Information: Contact: email: `commring@tugraz.at`; `http://integer-valued.org/conf2012/`.

* 17–20 **9th IMA International Conference on Mathematics in Signal Processing**, Austin Court, Birmingham, United Kingdom.
Description: Signal processing constitutes an important area for the application of mathematical concepts and techniques fuelled, for example, by developments in mobile communications, multimedia systems and digital TV. The subject is still advancing rapidly in areas such as non-linear/non-Gaussian/non-stationary signal processing, compressive sampling, digital communication systems, iterative estimation (e.g. ,turbo codes), blind deconvolution/signal separation and broadband systems. The last IMA conference on this subject was held in December 2008 and in response to popular demand, the next one will be held in December 2012. The aim of the conference is to bring together mathematicians, statisticians, and engineers with a view to exploring recent developments and identifying fruitful avenues for further research. It is hoped that the meeting will help to attract more mathematicians into this important and challenging field.
Information: `http://www.ima.org.uk/conferences/conferences_calendar/mathematics_in_signal_processing.cfm`.

* 27–30 **Eighth International Triennial Calcutta Symposium on Probability and Statistics**, Department of Statistics, Calcutta University, Kolkata, West Bengal, India.
Description: The Eighth International Triennial Calcutta Symposium following the foot steps of the previous seven symposia held in the years 1991, 1994, 1997, 2000, 2003, 2006 and 2009 plans to bring together researchers engaged in theoretical, methodological, and applied aspects of Statistics and Probability on a common platform to exchange ideas and facilitate discussions. A fairly large number of eminent researchers from all over the world are expected to attend. The symposium will feature invited and contributory sessions on theoretical and applied statistics and probability. There will also be poster sessions for students and young researchers. As in the past, a special session on Design of Experiments and Related Combinatorial Aspects will be organized in the memory of late Prof. Raj Chandra Bose.
Information: `http://triennial.calcuttastatisticalassociation.org/sympBrochure.php`.

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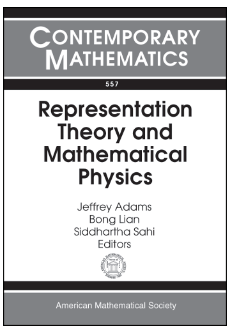
Algebra and Algebraic Geometry



Representation Theory and Mathematical Physics

Jeffrey Adams, *University of Maryland, College Park, MD*, **Bong Lian**, *Brandeis University, Waltham, MA*, and **Siddhartha Sahi**, *Rutgers University, Piscataway, NJ*, Editors

This volume contains the proceedings of the conference on Representation Theory and Mathematical Physics, in honor of Gregg Zuckerman's 60th birthday, held October 24–27, 2009, at Yale University.

Lie groups and their representations play a fundamental role in mathematics, in particular because of connections to geometry, topology, number theory, physics, combinatorics, and many other areas. Representation theory is one of the cornerstones of the Langlands program in number theory, dating to the 1970s. Zuckerman's work on derived functors, the translation principle, and coherent continuation lie at the heart of the modern theory of representations of Lie groups. One of the major unsolved problems in representation theory is that of the unitary dual. The fact that there is, in principle, a finite algorithm for computing the unitary dual relies heavily on Zuckerman's work.

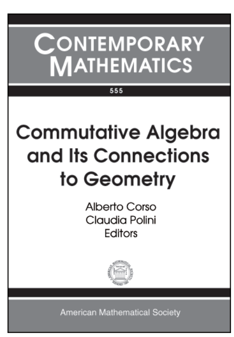
In recent years there has been a fruitful interplay between mathematics and physics, in geometric representation theory, string theory, and other areas. New developments on chiral algebras, representation theory of affine Kac-Moody algebras, and the geometric Langlands correspondence are some of the focal points of this volume. Recent developments in the geometric Langlands program point to exciting connections between certain automorphic representations and dual fibrations in geometric mirror symmetry.

This item will also be of interest to those working in number theory and mathematical physics.

Contents: *Expository papers:* **R. A. Herb** and **P. J. Sally, Jr.**, The Plancherel formula, the Plancherel theorem, and the Fourier transform of orbital integrals; **T. Kobayashi**, Branching problems of Zuckerman derived functor modules; **B. H. Lian**, **A. R. Linshaw**, and **B. Song**, Chiral equivariant cohomology of spheres; *Research papers:* **J. Adams**, Computing global characters; **D. M. Barbasch** and **P. E. Trapa**, Stable combinations of special unipotent representations; **E. Dan-Cohen** and **I. Penkov**, Levi components of parabolic subalgebras of finitary Lie algebras; **H. Garland**, On extending the Langlands-Shahidi method to arithmetic quotients of loop groups; **M. W. Hero**, **J. F. Willenbring**, and **L. K. Williams**, The measurement of quantum entanglement and enumeration of graph coverings; **D. Lu** and **R. Howe**, The dual pair $(O_{p,q}, O\widetilde{Sp}_{2,2})$ and Zuckerman translation; **B. Kostant** and **N. Wallach**, On the algebraic set of singular elements in a complex simple Lie algebra; **A. G. Lisi**, An explicit embedding of gravity and the standard model in E_8; **G. Lusztig**, From groups to symmetric spaces; **G. Lusztig**, Study of antiorbital complexes; **S. D. Miller** and **W. Schmid**, Adelization of automorphic distributions and mirabolic Eisenstein series; **I. Penkov** and **V. Serganova**, Categories of integrable $sl(\infty)-$, $o(\infty)-$, $sp(\infty)$-modules; **S. Sahi**, Binomial coefficients and Littlewood-Richardson coefficients for interpolation polynomials and Macdonald polynomials; **B. Speh**, Restriction of some representations of $U(p,q)$ to a symmetric subgroup.

Contemporary Mathematics, Volume 557

December 2011, 388 pages, Softcover, ISBN: 978-0-8218-5246-0, 2010 *Mathematics Subject Classification:* 22E45, 22E46, 22E47, 17B65, 17B68, 17B69, 33D52, **AMS members US$100**, List US$125, Order code CONM/557



Commutative Algebra and Its Connections to Geometry

Alberto Corso, *University of Kentucky, Lexington, KY*, and **Claudia Polini**, *University of Notre Dame, IN*, Editors

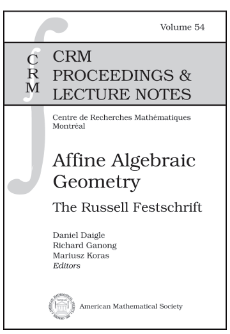
This volume contains papers based on presentations given at the Pan-American Advanced Studies Institute (PASI) on commutative algebra and its connections to geometry, which was held August 3–14, 2009, at the Universidade Federal de Pernambuco in Olinda, Brazil.

The main goal of the program was to detail recent developments in commutative algebra and interactions with such areas as algebraic geometry, combinatorics and computer algebra. The articles in this volume concentrate on topics central to modern commutative algebra: the homological conjectures, problems in positive and mixed characteristic, tight closure and its interaction with birational geometry, integral dependence and blowup algebras, equisingularity theory, Hilbert functions and multiplicities, combinatorial commutative algebra, Gröbner bases and computational algebra.

Contents: J. Martínez-Bernal, **C. Rentería-Márquez**, and **R. H. Villarreal**, Combinatorics of symbolic Rees algebras of edge ideals of clutters; **W. Bruns**, **C. Krattenthaler**, and **J. Uliczka**, Hilbert depth of powers of the maximal ideal; **C-Y.J. Chan** and **J.-C. Liu**, A note on reductions of monomial ideals in $k[x,y]_{(x,y)}$; **C. Ciliberto** and **V. Di Gennaro**, Plücker-Clebsch formula in higher dimension; **E. De Negri** and **E. Gorla**, Invariants of ideals generated by pfaffians; **J. Elias** and **J. Martínez-Borruel**, Hilbert polynomial and the intersection of ideals; **V. Ferrer** and **I. Vainsencher**, Polynomial vector fields with algebraic trajectories; **P. Gimenez**, **I. Sengupta**, and **H. Srinivasan**, Minimal free resolutions for certain affine monomial curves; **S. Goto** and **K. Ozeki**, Uniform bounds for Hilbert coefficients of parameters; **C. Huneke**, Absolute integral closure; **G. Lyubeznik**, **W. Zhang**, and **Y. Zhang**, A property of the Frobenius map of a polynomial ring; **M. Majidi-Zolbanin** and **B. Snapp**, A note on the variety of pairs of matrices whose product is symmetric; **P. Roberts** and **A. K. Singh**, Reconciling Riemann-Roch results; **M. E. Rossi**, Hilbert functions of Cohen-Macaulay local rings; **J. Striuli** and **A. Vraciu**, Some homological properties of almost Gorenstein rings.

Contemporary Mathematics, Volume 555

November 2011, approximately 224 pages, Softcover, ISBN: 978-0-8218-4959-0, 2010 *Mathematics Subject Classification:* 05C90, 13C40, 13D02, 13D07, 13D40, 14C05, 14J60, 14M12, 14N05, **AMS members US$63.20**, List US$79, Order code CONM/555



Affine Algebraic Geometry

The Russell Festschrift

Daniel Daigle, *University of Ottawa, ON, Canada*, **Richard Ganong**, *York University, Toronto, ON, Canada*, and **Mariusz Koras**, *University of Warsaw, Poland*

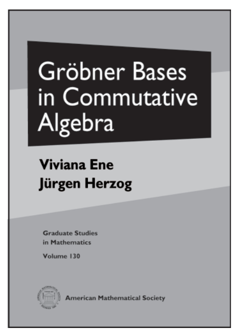
This volume grew out of an international conference which was held in June 2009 at McGill University, in honour of Professor Peter Russell, on the occasion of his 70th birthday and his retirement from McGill. It contains 19 refereed articles, essentially all in the area of affine algebraic geometry and, more specifically, in the following subjects: automorphisms and group actions, surfaces, embeddings of certain rational curves in the affine plane, and problems in positive characteristic geometry. These are also some of the themes running through the very substantial body of work done by Professor Russell in this relatively young branch of algebraic geometry. The volume also includes a foreword, in which Professor Russell shares some personal reminiscences on the development of affine algebraic geometry, a field he describes as "loosely speaking, the study of affine spaces and algebraic varieties closely resembling them."

Titles in this series are co-published with the Centre de Recherches Mathématiques.

Contents: P. Cassou-Noguès, Newton trees at infinity of algebraic curves; **D. Daigle**, Polynomials $f(X,Y,Z)$ of low LND-degree; **H. Flenner**, **S. Kaliman**, and **M. Zaidenberg**, Corrigendum to our paper "Birational transformations of weighted graphs"; **G. Freudenburg**, Bivariate analogues of Chebyshev polynomials with application to embeddings of affine spaces; **R. Ganong**, The pencil of translates of a line in the plane; **R. V. Gurjar**, A geometric proof of Boutot's result on singularities of quotients; **S. Kaliman** and **F. Kutzschebauch**, On the present state of the Andersén–Lempert theory; **T. Kishimoto**, **Y. Prokhorov**, and **M. Zaidenberg**, Group actions on affine cones; **M. Koras**, $\mathbb{C}^*$ in $\mathbb{C}^2$ is birationally equivalent to a line; **S. Kuroda**, Initial algebras and the Jung–van der Kulk theorem; **S. S.-Y. Lu**, Holomorphic curves on irregular varieties of general type starting from surfaces; **L. Makar-Limanov**, Locally nilpotent derivations of affine domains; **K. Masuda**, Equivariant derivations and additive group actions; **M. Miyanishi**, Frobenius sandwiches of affine algebraic surfaces; **L. Moser-Jauslin**, Automorphism groups of Koras–Russell threefolds of the first kind; **K. Palka**, Recent progress in the geometry of $\mathbb{Q}$-acyclic surfaces; **V. L. Popov**, On the Makar-Limanov, Derksen invariants, and finite automorphism groups of algebraic varieties; **A. Sathaye**, Embeddings of hyperbolas; **Y. Takeda**, Groups of Russell type and tango structures.

CRM Proceedings & Lecture Notes, Volume 54

November 2011, 335 pages, Softcover, ISBN: 978-0-8218-7283-3, LC 2011030116, 2010 *Mathematics Subject Classification:* 14Rxx, 13-XX, **AMS members US$100**, List US$125, Order code CRMP/54



Gröbner Bases in Commutative Algebra

Viviana Ene, *Ovidius University, Constanta, Romania*, and **Jürgen Herzog**, *Universität Duisburg-Essen, Germany*

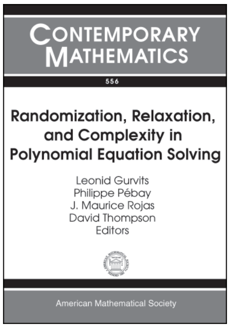
This book provides a concise yet comprehensive and self-contained introduction to Gröbner basis theory and its applications to various current research topics in commutative algebra. It especially aims to help young researchers become acquainted with fundamental tools and techniques related to Gröbner bases which are used in commutative algebra and to arouse their interest in exploring further topics such as toric rings, Koszul and Rees algebras, determinantal ideal theory, binomial edge ideals, and their applications to statistics.

The book can be used for graduate courses and self-study. More than 100 problems will help the readers to better understand the main theoretical results and will inspire them to further investigate the topics studied in this book.

Contents: Polynomial rings and ideals; Gröbner bases; First applications; Gröbner bases for modules; Gröbner bases of toric ideals; Selected applications in commutative algebra and combinatorics; Bibliography; Index.

Graduate Studies in Mathematics, Volume 130

January 2012, approximately 173 pages, Hardcover, ISBN: 978-0-8218-7287-1, LC 211032432, 2010 *Mathematics Subject Classification:* 13-01, 13A15, 13D02, 13H10, 13P10, **AMS members US$42.40**, List US$53, Order code GSM/130



Randomization, Relaxation, and Complexity in Polynomial Equation Solving

Leonid Gurvits, *Los Alamos National Laboratory, NM*, **Philippe Pébay**, *Sandia National Laboratories, Livermore, CA*, **J. Maurice Rojas**, *Texas A&M University, College Station, TX*, and **David Thompson**, *Sandia National Laboratories, Livermore, CA*, Editors

This volume corresponds to the Banff International Research Station Workshop on Randomization, Relaxation, and Complexity, held from February 28–March 5, 2010 in Banff, Ontario, Canada.

This volume contains a sample of advanced algorithmic techniques underpinning the solution of systems of polynomial equations. The papers are written by leading experts in algorithmic algebraic geometry and touch upon core topics such as homotopy methods for approximating complex solutions, robust floating point methods for clusters of roots, and speed-ups for counting real solutions. Vital related topics such as circuit complexity, random polynomials over local fields, tropical geometry, and the theory of fewnomials, amoebae, and coamoebae are treated as well. Recent advances on Smale's 17th Problem, which deals with numerical algorithms that approximate a single complex solution in average-case polynomial time, are also surveyed.

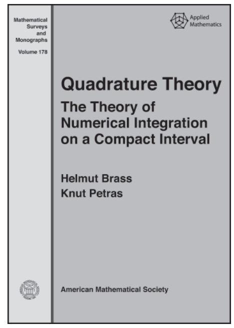
This item will also be of interest to those working in applications.

Contents: **M. Avendaño** and **A. Ibrahim**, Multivariate ultrametric root counting; **D. J. Bates**, **J. D. Hauenstein**, and **A. J. Sommese**, A parallel endgame; **C. Beltrán** and **L. M. Pardo**, Efficient polynomial system solving by numerical methods; **B. Grenet**, **E. L. Kaltofen**, **P. Koiran**, and **N. Portier**, Symmetric determinantal representation of formulas and weakly skew circuits; **T.-L. Lee** and **T.-Y. Li**, Mixed volume computation in solving polynomial systems; **A. Leykin**, A search for an optimal start system for numerical homotopy continuation; **M. Nisse**, Complex tropical localization, and coamoebas of complex algebraic hypersurfaces; **O. Bastani**, **C. J. Hillar**, **D. Popov**, and **J. M. Rojas**, Randomization, sums of squares, near-circuits, and faster real root counting; **K. Rusek**, **J. Shakalli**, and **F. Sottile**, Dense fewnomials; **Z. Zeng**, The numerical greatest common divisor of univariate polynomials.

Contemporary Mathematics, Volume 556

November 2011, 217 pages, Softcover, ISBN: 978-0-8218-5228-6, LC 2011029869, 2010 *Mathematics Subject Classification:* 11Y16, 12Y05, 14M25, 14P25, 14Q20, 14T05, 52B55, 65H04, 65Y20, **AMS members US$63.20**, List US$79, Order code CONM/556

Analysis



Quadrature Theory

The Theory of Numerical Integration on a Compact Interval

Helmut Brass and **Knut Petras**, *Technische Universität Braunschweig, Germany*

Every book on numerical analysis covers methods for the approximate calculation of definite integrals. The authors of this book provide a complementary treatment of the topic by presenting a coherent theory of quadrature methods that encompasses many deep and elegant results as well as a large number of interesting (solved and open) problems.

The inclusion of the word "theory" in the title highlights the authors' emphasis on analytical questions, such as the existence and structure of quadrature methods and selection criteria based on strict error bounds for quadrature rules. Systematic analyses of this kind rely on certain properties of the integrand, called "co-observations," which form the central organizing principle for the authors' theory, and distinguish their book from other texts on numerical integration. A wide variety of co-observations are examined, as a detailed understanding of these is useful for solving problems in practical contexts.

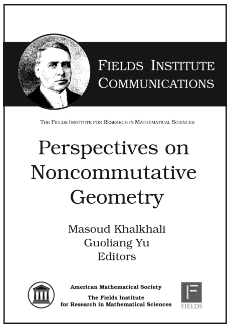
While quadrature theory is often viewed as a branch of numerical analysis, its influence extends much further. It has been the starting point of many far-reaching generalizations in various directions, as well as a testing ground for new ideas and concepts. The material in this book should be accessible to anyone who has taken the standard undergraduate courses in linear algebra, advanced calculus, and real analysis.

This item will also be of interest to those working in applications.

Contents: Introduction; The abstract framework; Norm and kernel of the remainder functional; Co-observations; Quadrature rules of interpolatory type; Gaussian quadrature; Quadrature rules with equidistant nodes; Periodic integrands; Variance and Chebyshev-type rules; Problems; Orthogonal polynomials; Bernoulli polynomials; Validation of co-observations; Bibliography; Symbols; Index.

Mathematical Surveys and Monographs, Volume 178

November 2011, 363 pages, Hardcover, ISBN: 978-0-8218-5361-0, LC 2011030201, 2010 *Mathematics Subject Classification:* 41A55; 65D30, 65D32, **AMS members US$75.20**, List US$94, Order code SURV/178



Perspectives on Noncommutative Geometry

Masoud Khalkhali, *University of Western Ontario, London, ON, Canada*, and **Guoliang Yu**, *Vanderbilt University, Nashville, TN*, Editors

This volume represents the proceedings of the Noncommutative Geometry Workshop that was held as part of the thematic program on operator algebras at the Fields Institute in May 2008.

Pioneered by Alain Connes starting in the late 1970s, noncommutative geometry was originally inspired by global analysis, topology, operator algebras, and quantum physics. Its main applications were to settle some long-standing conjectures, such as the Novikov conjecture and the Baum-Connes conjecture.

Next came the impact of spectral geometry and the way the spectrum of a geometric operator, like the Laplacian, holds information about the geometry and topology of a manifold, as in the celebrated Weyl law. This has now been vastly generalized through Connes' notion of spectral triples.

Finally, recent years have witnessed the impact of number theory, algebraic geometry and the theory of motives, and quantum field theory on noncommutative geometry. Almost all of these aspects are touched upon with new results in the papers of this volume.

This book is intended for graduate students and researchers in both mathematics and theoretical physics who are interested in noncommutative geometry and its applications.

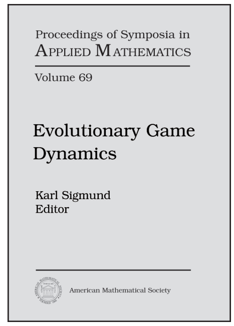
Titles in this series are co-published with the Fields Institute for Research in Mathematical Sciences (Toronto, Ontario, Canada).

Contents: M.-T. Benameur and **A. Gorokhovsky**, Local index theorem for projective families; **A. L. Carey**, **J. Phillips**, **I. F. Putnam**, and **A. Rennie**, Type III KMS states on a class of C^*-algebras containing O_n and $\mathcal{Q}_N$ and their modular index; **H. Emerson**, Duality, correspondences and the Lefschetz map in equivariant KK-theory: A survey; **F. Fathizadeh** and **M. Khalkhali**, Twisted spectral triples and Connes' character formula; **B. Nica**, Spectral morphisms, K-theory, and stable ranks; **A. Pourkia**, A survey of braided Hopf cyclic cohomology; **R. Rochberg**, **X. Tang**, and **Y.-J. Yao**, A survey of Rankin-Cohen deformations; **O. Uuye**, Pseudo-differential operators and regularity of spectral triples.

Fields Institute Communications, Volume 61

November 2011, 163 pages, Hardcover, ISBN: 978-0-8218-4849-4, LC 2011032554, 2010 *Mathematics Subject Classification:* 58B34; 19D55, 16T05, 18G30, **AMS members US$71.20**, List US$89, Order code FIC/61

Applications



Evolutionary Game Dynamics

Karl Sigmund, *University of Vienna, Austria*, Editor

This volume is based on lectures delivered at the 2011 AMS Short Course on Evolutionary Game Dynamics, held January 4–5, 2011 in New Orleans, Louisiana.

Evolutionary game theory studies basic types of social interactions in populations of players. It combines the strategic viewpoint of classical game theory (independent rational players trying to outguess each other) with population dynamics (successful strategies increase their frequencies). A substantial part of the appeal of evolutionary game theory comes from its highly diverse applications such as social dilemmas, the evolution of language, or mating behaviour in animals. Moreover, its methods are becoming increasingly popular in computer science, engineering, and control theory. They help to design and control multi-agent systems, often with a large number of agents (for instance, when routing drivers over highway networks or data packets over the Internet).

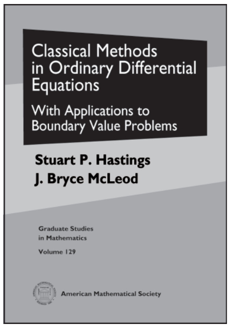
While these fields have traditionally used a top down approach by directly controlling the behaviour of each agent in the system, attention has recently turned to an indirect approach allowing the agents to function independently while providing incentives that lead them to behave in the desired way. Instead of the traditional assumption of equilibrium behaviour, researchers opt increasingly for the evolutionary paradigm and consider the dynamics of behaviour in populations of agents employing simple, myopic decision rules.

Contents: K. Sigmund, Introduction to evolutionary game theory; **R. Cressman**, Beyond the symmetric normal form: Extensive form games, asymmetric games and games with continuous strategy spaces; **J. Hofbauer**, Deterministic evolutionary game dynamics; **S. Sorin**, On some global and unilateral adaptive dynamics; **W. H. Sandholm**, Stochastic evolutionary game dynamics: Foundations, deterministic approximation, and equilibrium selection; **S. Lessard**, Evolution of cooperation in finite populations; Index.

Proceedings of Symposia in Applied Mathematics, Volume 69

November 2011, 171 pages, Hardcover, ISBN: 978-0-8218-5326-9, LC 2011028869, 2010 *Mathematics Subject Classification:* 91A22, **AMS members US$41.60**, List US$52, Order code PSAPM/69

Differential Equations



Classical Methods in Ordinary Differential Equations

With Applications to Boundary Value Problems

Stuart P. Hastings, *University of Pittsburgh, PA*, and **J. Bryce McLeod**, *Oxford University, England and University of Pittsburgh, PA*

This text emphasizes rigorous mathematical techniques for the analysis of boundary value problems for ODEs arising in applications. The emphasis is on proving existence of solutions, but there is also a substantial chapter on uniqueness and multiplicity questions and several chapters which deal with the asymptotic behavior of solutions with respect to either the independent variable or some parameter. These equations may give special solutions of important PDEs, such as steady state or traveling wave solutions. Often two, or even three, approaches to the same problem are described. The advantages and disadvantages of different methods are discussed.

This book gives complete classical proofs, while also emphasizing the importance of modern methods, especially when extensions to infinite dimensional settings are needed. There are some new results as well as new and improved proofs of known theorems. The final chapter presents three unsolved problems which have received much attention over the years.

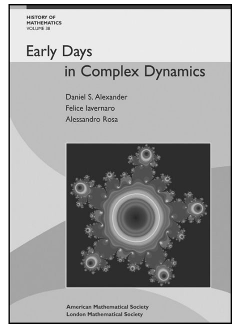
Both graduate students and more experienced researchers will be interested in the power of classical methods for problems which have also been studied with more abstract techniques. The presentation should be more accessible to mathematically inclined researchers from other areas of science and engineering than most graduate texts in mathematics.

Contents: Introduction; An introduction to shooting methods; Some boundary value problems for the Painlevé transcendents; Periodic solutions of a higher order system; A linear example; Homoclinic orbits of the FitzHugh-Nagumo equations; Singular perturbation problems—rigorous matching; Asymptotics beyond all orders; Some solutions of the Falkner-Skan equation; Poiseuille flow: Perturbation and decay; Bending of a tapered rod; variational methods and shooting; Uniqueness and multiplicity; Shooting with more parameters; Some problems of A. C. Lazer; Chaotic motion of a pendulum; Layers and spikes in reaction-diffusion equations, I; Uniform expansions for a class of second order problems; Layers and spikes in reaction-diffusion equations, II; Three unsolved problems; Bibliography; Index.

Graduate Studies in Mathematics, Volume 129

January 2012, approximately 373 pages, Hardcover, ISBN: 978-0-8218-4694-0, LC 2011029730, 2010 *Mathematics Subject Classification:* 34B15, 34B16, 34C28, 34C37, 34E05, 35A24, 35C07, 37C29, 34D45, **AMS members US$50.40**, List US$63, Order code GSM/129

General Interest



Early Days in Complex Dynamics

A history of complex dynamics in one variable during 1906–1942

Daniel S. Alexander, *Drake University, Des Moines, IA*, **Felice Iavernaro**, *Università di Bari, Italy*, and **Alessandro Rosa**

The theory of complex dynamics, whose roots lie in 19th-century studies of the iteration of complex function conducted by Kœnigs, Schöder, and others, flourished remarkably during the first half of the 20th century, when many of the central ideas and techniques of the subject developed. This book by Alexander, Iavernaro, and Rosa paints a robust picture of the field of complex dynamics between 1906 and 1942 through detailed discussions of the work of Fatou, Julia, Siegel, and several others.

A recurrent theme of the authors' treatment is the center problem in complex dynamics. They present its complete history during this period and, in so doing, bring out analogies between complex dynamics and the study of differential equations, in particular, the problem of stability in Hamiltonian systems. Among these analogies are the use of iteration and problems involving small divisors which the authors examine in the work of Poincaré and others, linking them to complex dynamics, principally via the work of Samuel Lattès, in the early 1900s, and Jürgen Moser, in the 1960s.

Many details will be new to the reader, such as a history of Lattès functions (functions whose Julia set equals the Riemann sphere), complex dynamics in the United States around the time of World War I, a survey of complex dynamics around the world in the 1920s and 1930s, a discussion of the dynamical programs of Fatou and Julia during the 1920s, and biographical material on several key figures. The book contains graphical renderings of many of the mathematical objects the authors discuss, including some of the intriguing fractals Fatou and Julia studied, and concludes with several appendices by current researchers in complex dynamics which collectively attest to the impact of the work of Fatou, Julia, and others upon the present-day study.

This item will also be of interest to those working in analysis.

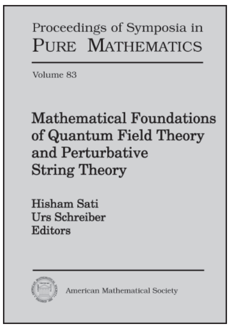
Co-published with the London Mathematical Society beginning with Volume 4. Members of the LMS may order directly from the AMS at the AMS member price. The LMS is registered with the Charity Commissioners.

Contents: *Preliminaries:* A complex dynamics primer; Introduction: Dynamics of a complex history; Iteration and differential equations I: The Poincaré connection; Iteration and differential equations II: Small divisors; *The core (1906–1920):* Early overseas results: The United States; The road to the *Grand Prix des Sciences Mathématiques*; Works written for the *Grand Prix*; Iteration in Italy; The giants fall; *After-maths (1920–1942):* Branching out: Fatou and Julia in the 1920s; The German wave; Siegel, the center problem, and KAM theory; Iteratin' around the globe; *Tying the future to the past:* Report on the *Grand Prix des Sciences Mathématiques* in 1918; A history of normal families; Singular lines of analytic functions; Kleinian groups; Curves of Julia; Progress in Julia's extension of Schwarz's lemma; The Denjoy-Wolff theorem; Dynamics of self-maps of the unit disc; Koebe and uniformization; Permutable maps in the 1920s; The last 60 years in permutable maps; Understanding Julia sets of entire maps; Fatou: A biographical sketch; Gaston Julia: A biographical sketch; Selected biographies; Remarks on computer graphics; Glossary; Bibliography; Index.

History of Mathematics, Volume 38

December 2011, approximately 442 pages, Hardcover, ISBN: 978-0-8218-4464-9, LC 2011014903, 2010 *Mathematics Subject Classification:* 01A55, 30-03, 01A60, **AMS members US$79.20**, List US$99, Order code HMATH/38

Mathematical Physics



Mathematical Foundations of Quantum Field Theory and Perturbative String Theory

Hisham Sati, *University of Pittsburgh, PA*, and **Urs Schreiber**, *Utrecht University, The Netherlands*, Editors

Conceptual progress in fundamental theoretical physics is linked with the search for the suitable mathematical structures that model the physical systems. Quantum field theory (QFT) has proven to be a rich source of ideas for mathematics for a long time. However, fundamental questions such as "What is a QFT?" did not have satisfactory mathematical answers, especially on spaces with arbitrary topology, fundamental for the formulation of perturbative string theory.

This book contains a collection of papers highlighting the mathematical foundations of QFT and its relevance to perturbative string theory as well as the deep techniques that have been emerging in the last few years.

The papers are organized under three main chapters: Foundations for Quantum Field Theory, Quantization of Field Theories, and Two-Dimensional Quantum Field Theories. An introduction, written by the editors, provides an overview of the main underlying themes that bind together the papers in the volume.

Contents: **H. Sati** and **U. Schreiber**, Introduction; *Foundations for quantum field theory:* **J. E. Bergner**, Models for (∞, n)-categories and the cobordism hypothesis; **I. Weiss**, From operads to dendroidal sets; **A. Davydov**, **L. Kong**, and **I. Runkel**, Field theories with defects and the centre functor; *Quantization of field theories:* **F. Paugam**, Homotopical Poisson reduction of gauge theories; **J. Distler**, **D. S. Freed**, and **G. W. Moore**, Orientifold précis; *Two-dimensional quantum field theories:* **A. Kapustin** and **N. Saulina**, Surface operators in 3d topological field theory and 2d rational conformal field theory; **L. Kong**, Conformal field theory and a new geometry; **Y. Soibelman**, Collapsing conformal field theories, spaces with non-negative Ricci curvature and non-commutative geometry; **S. Stolz** and **P. Teichner**, Supersymmetric field theories and generalized cohomology; **C. L. Douglas** and **A. G. Henriques**, Topological modular forms and conformal nets.

Proceedings of Symposia in Pure Mathematics, Volume 83

December 2011, approximately 357 pages, Hardcover, ISBN: 978-0-8218-5195-1, LC 2011030793, 2010 *Mathematics Subject Classification:* 81T45, 81T40, 81T30, 81T05, 81T60, 70S05, 18D05, 18D50, 55N34, 55U40, **AMS members US$76**, List US$95, Order code PSPUM/83

New AMS-Distributed Publications

General Interest

Collected Papers of C. S. Seshadri

Volumes 1 and 2

Vikraman Balaji, *Chennai Mathematical Institute, India*, **V. Lakshmibai**, *Northeastern University, Boston, MA*, and **M. Pavaman Murthy** and **Madhav V. Nori**, *University of Chicago, Illinois*, Editors

For the past fifty years, C. S. Seshadri has been a towering figure in the mathematical world, and his contributions have been central to the development of moduli problems and geometric invariant theory as well as representation theory of algebraic groups. These two volumes of his collected papers have been organized in accordance with the subject matter, faithfully reflecting the diversity of his mathematical contributions.

These volumes will inspire future generations of mathematicians and provide insights into the unique mathematical personality of C. S. Seshadri.

This item will also be of interest to those working in algebra and algebraic geometry.

A publication of Hindustan Book Agency. Distributed on an exclusive basis by the AMS in North America. Online bookstore rights worldwide.

Contents: Volume 1: Vector Bundles and Invariant Theory; Volume 2: Schubert Geometry and Representation Theory.

Hindustan Book Agency

September 2011, 1632 pages, Hardcover, ISBN: 978-93-80250-17-5, 2010 *Mathematics Subject Classification:* 14H60, 14-XX, 20G15, 14C30, 13A35, 13D02, 16S37, 17-XX, 20-XX, **AMS members US$156**, List US$195, Order code HIN/50

Classified Advertisements

Positions available, items for sale, services available, and more

CALIFORNIA

UNIVERSITY OF CALIFORNIA, BERKELEY
Department of Mathematics

We invite applications for the following positions: 1. One or more tenure-track positions 2. Charles B. Morrey, Jr. Assistant Professorship 3. Visiting Assistant Professorships (partially funded by the NSF) These positions begin July 1, 2012. For more information on these positions and how to apply for them, please go to: `http://math.berkeley.edu/employment_academic.html`.

000042

MATHEMATICAL SCIENCES RESEARCH INSTITUTE (MSRI),
Position Available: Associate Director

The MATHEMATICAL SCIENCES RESEARCH INSTITUTE (MSRI), in Berkeley, seeks an ASSOCIATE DIRECTOR to serve for two to three years beginning in August 1, 2012.

About MSRI: MSRI was founded in 1982 as an independent non-profit corporation located on the campus of the University of California at Berkeley. Its fundamental purpose is to further research in the mathematical sciences through major programs of a semester or a year, through workshops, and through postdoctoral training. MSRI has about 2,000 visitors to its programs during the year and has an average of about 15 postdocs and 60 more senior mathematicians in residence at any time.

Job Description and Qualifications: MSRI engages in outreach to other sciences, to the public, and to various efforts in education, and it is active in encouraging diversity in the research population. The Associate Director oversees these outreach projects, which include workshops, Math Circles coordination, and corporate-affiliate activities. He or she has the responsibility to administer the current projects as well as oversee the creation of new ones (when appropriate). The Associate Director reports to the Deputy Director, works with the Director and the Deputy Director on all phases of Institute activity, and helps to formulate Institute policy. The Associate Director should have a broad understanding of mathematical culture, demonstrate an interest in outreach activities, and have substantial administrative experience. Application Process: An application must include a CV, a list of at least three references, and a statement of interest. The latter should tell why the applicant is interested in coming to MSRI and what he or she would bring to the position. For more information, or to submit an application, contact ADsearch AT msri DOT org or write to:

Associate Director Search Committee
Mathematical Sciences Research Institute
17 Gauss Way
Shiing-Shen Chern Hall
Berkeley, CA 94720-5070

Consideration of applications will begin Monday, December 5, 2011, and will continue until the position is filled. MSRI is an Equal Opportunity Employer and an NSF-Funded Institute.

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UNIVERSITY OF CALIFORNIA, DAVIS
Arthur J. Krener Assistant Professor Positions in Mathematics

The Department of Mathematics at the University of California, Davis, is soliciting applications for several Arthur J. Krener positions starting July 1, 2012.

The department seeks applicants with excellent research potential in areas of faculty interest and effective teaching skills. Applicants are required to have completed their Ph.D. by the time of their appointment, but no earlier than July 1, 2008. The annual salary is $56,400. The teaching load is 3 to 4 quarter-long courses. Krener appointments are renewable for a total of up to three years, assuming satisfactory performance in research and teaching.

Additional information about the department may be found at: `http://math.ucdavis.edu/`. Our postal address is Department of Mathematics, University of California, One Shields Avenue, Davis, CA 95616-8633. Applications will be accepted until the positions are filled. For full consideration, the application should be received by November 30, 2011. To apply: submit the AMS Cover Sheet and supporting documentation electronically through `http://www.mathjobs.org/`. The University of California is an Affirmative Action/Equal Opportunity Employer.

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Suggested uses for classified advertising are positions available, books or lecture notes for sale, books being sought, exchange or rental of houses, and typing services.

The 2011 rate is $3.25 per word. No discounts for multiple ads or the same ad in consecutive issues. For an additional $10 charge, announcements can be placed anonymously. Correspondence will be forwarded.

Advertisements in the "Positions Available" classified section will be set with a minimum one-line headline, consisting of the institution name above body copy, unless additional headline copy is specified by the advertiser. Headlines will be centered in boldface at no extra charge. Ads will appear in the language in which they are submitted.

There are no member discounts for classified ads. Dictation over the telephone will not be accepted for classified ads.

Upcoming deadlines for classified advertising are as follows: December 2011 issue–September 28, 2011; January 2012 issue–October 31, 2011; February 2012 issue–November 28, 2011; March 2012 issue–December 28, 2011; April 2012 issue–January 30, 2012; May 2012 issue–February 28, 2012.

U.S. laws prohibit discrimination in employment on the basis of color, age, sex, race, religion, or national origin. "Positions Available" advertisements from institutions outside the U.S. cannot be published unless they are accompanied by a statement that the institution does not discriminate on these grounds whether or not it is subject to U.S. laws. Details and specific wording may be found on page 667 (vol. 56).

Situations wanted advertisements from involuntarily unemployed mathematicians are accepted under certain conditions for free publication. Call toll-free 800-321-4AMS (321-4267) in the U.S. and Canada or 401-455-4084 worldwide for further information.

Submission: Promotions Department, AMS, P.O. Box 6248, Providence, Rhode Island 02940; or via fax: 401-331-3842; or send email to `classads@ams.org`. AMS location for express delivery packages is 201 Charles Street, Providence, Rhode Island 20904. Advertisers will be billed upon publication.

UNIVERSITY OF CALIFORNIA, IRVINE
Department of Mathematics
Irvine, CA 92697-3875
Job #5515. Assistant Professor position in Mathematics

The Department of Mathematics at the University of California, Irvine, is seeking outstanding candidates to fill one tenure-track position to start July 1, 2012. Applicants must hold a Ph.D. and should have demonstrated excellence in research and teaching. We encourage applications from any area in pure and applied mathematics. The level of appointment will be commensurate with qualifications and experience. Applications are welcome at any time. The review process starts December 1, 2011, and will continue until the position is filled.

Completed applications must be submitted through www.mathjobs.org and must contain

(1) AMS cover sheet
(2) Curriculum Vitae
(3) Cover letter
(4) Research statement
(5) Teaching statement
(6) Selected reprints and/or preprints
(7) Three reference letters sent electronically through http://www.mathjobs.org. Instructions for the electronic application process can be found at: http://www.mathjobs.org

Indicate your area of mathematical specialization in field labeled: Area of Specialization. Example: Algebra.

UCI is an Equal Opportunity Employer committed to excellence through diversity and strongly encourages applications from all qualified applicants, including women and minorities.

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UCLA
Department of Mathematics

The Department of Mathematics invites applications for a tenure-track/tenure faculty position. Salary is commensurate with the level of experience.

We also plan to make temporary and visiting appointments in the categories 1-4 below. Depending on the level, candidates must give evidence of potential or demonstrated distinction in scholarship and teaching.

Temporary Positions: (1) E.R. Hedrick Assistant Professorships. Salary is $61,200, and appointments are for three years. The teaching load is four one-quarter courses per year.

(2) Computational and Applied Mathematics (CAM) Assistant Professorships. Salary is $61,200, and appointments are for three years. The teaching load is normally reduced by research funding to two quarter courses per year.

(3) Program in Computing (PIC) Assistant Adjunct Professorships. Salary is $65,500. Applicants for these positions must show very strong promise in teaching and research in an area related to computing. The teaching load is four one-quarter programming courses each year and one additional course every two years. Initial appointments are for one year and possibly longer, up to a maximum service of four years.

(4) Assistant Adjunct Professorships and Research Postdocs. Normally appointments are for one year, with the possibility of renewal. Strong research and teaching background required. The salary range is $53,200-$59,500. The teaching load for adjuncts is six one-quarter courses per year.

Applications and supporting documentation must be submitted electronically via http://www.mathjobs.org. All letters of evaluation are subject to UCLA campus policies on confidentiality. Refer potential reviewers to the UCLA Statement of Confidentiality at http://www.math.ucla.edu/people/confidentiality.pdf.

For fullest consideration, all application materials should be submitted on or before December 9, 2011. A Ph.D. is required for all positions.

The university is an Equal Opportunity/Affirmative Action Employer. UCLA and the Department of Mathematics have a strong commitment to the achievement of excellence in teaching and research and diversity among its faculty and staff. The University of California asks that applicants complete the Equal Opportunity Employer survey for Letters and Science at the following URL: http://cis.ucla.edu/facultysurvey. Under Federal law, the University of California may employ only individuals who are legally authorized to work in the United States as established by providing documents specified in the Immigration Reform and Control Act of 1986.

000055

UNIVERSITY OF CALIFORNIA, SAN DIEGO
Faculty Positions in the Department of Mathematics 2011-2012

The Department of Mathematics, within the Division of Physical Sciences at the University of California, San Diego (http://www.math.ucsd.edu) is committed to academic excellence and diversity within its faculty, staff, and student body. The department is seeking applications for 4 tenured/tenure-track positions from outstanding candidates in all areas of pure and applied mathematics, and statistics. Two of the positions are expected to be filled at the Assistant Professor level, and two at the Associate/Full Professor level. One position is available for a distinguished mathematician with an exceptional research record of the highest caliber. One of the positions will be filled in analysis/PDE (including areas of mathematical physics), one in the areas of representation theory or combinatorics, and one in applied mathematics with potential interdisciplinary implications. One position is open to applicants from all areas.

Successful candidates will be evaluated on teaching and research accomplishments, as well as on a demonstrated commitment to diversity, equity and inclusion in higher education. Candidates must receive their Ph.D. prior to their first quarter of teaching. Salary is commensurate with qualifications and based on UC pay scales. The starting date for the positions, pending funding approval, will be July 1, 2012. To receive full consideration, applications should be submitted online through http://www.mathjobs.org/ by November 1, 2011. For instructions on the application procedure, see http://www.math.ucsd.edu/about/employment/faculty.

In compliance with the Immigration Reform and Control Act of 1986, individuals offered employment by the University of California will be required to show documentation to prove identity and authorization to work in the United States before hiring can occur. For applicants interested in spousal/partner employment, please visit the UCSD Partner Opportunities Program at: http://academicaffairs.ucsd.edu/offices/partneropp/. UCSD is an Equal Opportunity/Affirmative Action Employer with a strong institutional commitment to excellence and diversity (http://diversity.ucsd.edu/).

000084

CONNECTICUT

FAIRFIELD UNIVERSITY
Assistant Professor in Mathematics
Tenure-Track position

The Department of Mathematics and Computer Science at Fairfield University invites applications for one tenure-track position in mathematics, at the rank of assistant professor, to begin in September 2012. We seek a highly qualified candidate with a commitment to and demonstrated excellence in teaching, and strong evidence of research potential. A doctorate in mathematics or a related field is required. The teaching load is 3 courses/9 credit hours per semester and consists primarily of courses at the undergraduate level. The successful candidate will be expected to teach a wide variety of courses from elementary calculus and statistics to graduate level courses; in particular, Fairfield University's core curriculum includes two semesters of mathematics for all undergraduates.

Special consideration will be given to candidates in applied areas of mathematics, or to those who are interested in playing an active role in our statistics curriculum.

Fairfield University is a Catholic Jesuit institution and is consistently ranked as a top comprehensive university in New England. It is located in the scenic shoreline community of Fairfield, CT, one hour from New York City along Long Island Sound. Our six colleges and professional schools enroll approximately 3,500 undergraduate and 1,200 graduate students with a strong emphasis on liberal arts education. The department of Mathematics and Computer Science has an active faculty of 15 full-time tenured or tenure-track members. We offer a BS and an MS in mathematics, as well as a BS in computer science. The MS program is an evening program and attracts students from various walks of life—secondary school teachers, eventual Ph.D. candidates, and people working in industry or business, among others.

Fairfield offers competitive salaries and compensation benefits. Fairfield is an Affirmative Action/Equal Opportunity Employer.

How to Apply: Applicants are required to apply electronically through `http://MathJobs.org`. For full consideration, please submit an application with all supporting materials by the deadline stated below. Applications must include the following: a curriculum vitae, teaching and research statements, and three letters of recommendation commenting on the applicant's experience and promise as a teacher and scholar. Reference letter writers should be asked to submit their letters online through `http://MathJobs.org`. If they are unable to do so, they may send their letters to the following address: Matt Coleman, Chair of the Department of Mathematics and Computer Science, Fairfield University, 1073 N. Benson Rd., Fairfield CT 06824-5195. Full consideration will be given to complete applications received by December 9, 2011. We will be interviewing at the Joint Mathematics Meetings in Boston, January 4-7, 2012. Please let us know if you will be attending.

000056

YALE UNIVERSITY
J. Willard Gibbs Assistant Professorships in Mathematics 2012-13

The Gibbs Assistant Professorships are intended primarily for men and women who received the Ph.D. degree and show definite promise in research in pure or applied mathematics. Appointments are for three years. The salary will be at least $71,000. Each recipient of a Gibbs Assistant Professorship will be given a moving allowance based on the distance to be moved.

The teaching load for Gibbs Assistant Professors will be kept light, so as to allow ample time for research. This will consist of three one-semester courses per year. Part of the duties may consist of a one-semester course at the graduate level in the general area of the instructor's research. Yale is an Affirmative Action/Equal Opportunity Employer. Qualified women and members of minority groups are encouraged to apply. Submit applications and supporting material through `MathJobs.org` by January 1, 2012. Submit inquiries to `gibbs.committee@yale.edu`. Offers expected to be made in early February 2012.

000060

DISTRICT OF COLUMBIA

THE GEORGE WASHINGTON UNIVERSITY
Department of Mathematics

The Department of Mathematics of The George Washington University invites applications to a tenure-track assistant professor position. Fields of interest include applied, geometric, or stochastic analysis, but truly exceptional candidates in other areas may also be considered.

Applicants must possess a Ph.D. in mathematics or applied mathematics. For additional information on the position, including a detailed description of the basic and preferred qualifications, and the application procedure, please see full position announcement, and apply online at: `http://www.mathjobs.org`. The George Washington University is an Equal Opportunity/Affirmative Action Employer.

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HAWAII

UNIVERSITY OF HAWAII
Department of Meteorology

The University of Hawaii at Manoa, Department of Meteorology, invites applications for two full-time tenure-track positions supported by general funds at the assistant professor level (appointment at the associate professor level may be considered in cases where the applicant has a proven record of outstanding research and teaching), to begin approximately January 1, 2012, subject to position clearance. The department seeks candidates with demonstrated research expertise preferably in the areas of physical, satellite, boundary layer or tropical meteorology. Candidates with outstanding track records in other areas are also encouraged to apply. In addition, candidates will be expected to teach undergraduate and graduate courses preferably in one or more of the areas of radiative transfer, cloud physics, satellite meteorology, meteorological instrumentation, air-sea interaction, synoptic, tropical, and boundary layer meteorology. A Ph.D. earned no later than January 1, 2012, in the area of meteorology or a closely related field is required, as are excellent communication skills; a demonstrated capability for creative, high quality research; and evidence of effectively teaching and mentoring undergraduate and graduate students. For the associate professor level, a minimum of four years teaching and research experience is also required. Applicants should submit curriculum vitae; detailed statement of research and teaching interests; and the names, addresses, phone numbers, and email addresses of at least three references to: Prof. Bin Wang, Chair, Department of Meteorology, University of Hawaii, 2525 Correa Rd., Hawaii Institute of Geophysics Building, Room 350, Honolulu, HI 96822. Inquiries: phone (808) 956-8775; fax (808) 956-2877; email: `metdept@hawaii.edu`. Review of applications will begin August 15, 2011, and will continue until the positions are filled. The University of Hawaii is an Equal Employment Opportunity/Affirmative Action Institution.

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ILLINOIS

UNIVERSITY OF CHICAGO
Department of Mathematics

The University of Chicago Department of Mathematics invites applications for the following positions:

1. L.E. Dickson Instructor: This is open to mathematicians who have recently completed or will soon complete a doctorate in mathematics or a closely related field, and whose work shows remarkable promise in mathematical research and teaching. The appointment typically is for two years, with the possibility of renewal for a third year. The teaching obligation is up to four one-quarter courses per year.

2. Simons Fellow (at the rank of Dickson Instructor): This is open to candidates who receive the Ph.D. within the period September 1, 2011, through August 31, 2012. The duration of the fellowship is three years, and the teaching obligation is four one-quarter courses during the three-year fellowship. This appointment would be at the University's rank of Dickson Instructor but would also carry the title of Simons Fellow within the Department of Mathematics.

3. Assistant Professor: This is open to mathematicians who are further along in their careers, typically two or three years past the doctorate. These positions are intended for mathematicians whose work has been of outstandingly high caliber. Appointees are expected to have the potential to become leading figures in their fields. The appointment is generally for

three years, with a teaching obligation of three one-quarter courses per year.

Applicants will be considered for any of the positions above which seem appropriate. Complete applications consist of (a) a cover letter, (b) a curriculum vitae, (c) three or more letters of reference, at least one of which addresses teaching ability, and (d) a description of previous research and plans for future mathematical research. Applicants are strongly encouraged to include information related to their teaching experience, such as a teaching statement or evaluations from courses previously taught, as well as an AMS cover sheet. If you have applied for an NSF Mathematical Sciences Postdoctoral Fellowship, please include that information in your application, and let us know how you plan to use it if awarded. If you are eligible for the Simons Fellowship and wish to be considered for it, please indicate this in your cover letter. Applications must be submitted online through http://www.mathjobs.org. Questions may be directed to apptsec@math.uchicago.edu. We will begin screening applications on December 1, 2011. Screening will continue until all available positions are filled. The University of Chicago is an Affirmative Action / Equal Opportunity Employer.

000050

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Mathematics
Tenure-Track Position

Applications are invited for a full time faculty position to commence approximately August 16, 2012, at the tenure-track (assistant professor) level. Appointees will be expected to pursue a vigorous research program and to teach graduate as well as undergraduate students. The department is interested in applicants in all areas of mathematics. Salary and teaching load are competitive.

Applicants must have a Ph.D. (or equivalent) in hand or show clear evidence of completion by the time the appointment begins, and are expected to present evidence of excellence in research and teaching. Applications should be submitted electronically through http://mathjobs.org/. A complete application must include the AMS Standard Cover Sheet for Academic Employment, curriculum vitae including email address, a publication list, a research statement, and the names and contact information for three professional references. An additional reference addressing teaching is strongly recommended. It is strongly suggested that the reference letters are uploaded before the deadline. Reference letter writers should submit their letters online through http://mathjobs.org/. If they are unable to do so, they may send their letters to the following address: Search, Department of Mathematics, University of Illinois at Urbana-Champaign, 1409 West Green Street, Urbana, IL 61801, USA; tel: (217) 333-33; search@math.uiuc.edu.

Complete applications must be received by November 14, 2011. Late applications cannot be considered. Applicants may be interviewed before the closing date; however, no hiring decision will be made until after the closing date of November 14, 2011.

Illinois is an Affirmative Action /Equal Opportunity Employer and welcomes individuals with diverse backgrounds, experiences, and ideas who embrace and value diversity and inclusivity. (inclusiveillinois.illinois.edu).

000087

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Mathematics
Tenure-Track Assistant Professor Position in Actuarial Science

Applications are invited for a full-time tenure-track assistant professor faculty position in Actuarial Science to commence approximately August 16, 2012. Appointees will be expected to teach and advise graduate and undergraduate students and to pursue research in actuarial science or related areas. Salary and teaching load are competitive.

Applicants must have completed a Ph.D. (or equivalent) in actuarial science or related areas such as risk management and insurance, finance, or another subject appropriate for actuarial science, by the time the appointment begins. Preference will be given to applicants who are associates or fellows of one of the professional actuarial societies, or who have made exam progress toward membership. A strong commitment to teaching is essential. Applications should be submitted electronically through http://mathjobs.org/. A complete application must include the AMS Standard Cover Sheet for Academic Employment, curriculum vitae including email address, a publication list, a research statement, and the names and contact information for three professional references. An additional reference addressing teaching is strongly recommended. It is strongly suggested that the reference letters are uploaded before the deadline. Reference letter writers should submit their letters online through http://mathjobs.org/. If they are unable to do so, they may send their letters to the following address: Search, Department of Mathematics, University of Illinois at Urbana-Champaign, 1409 West Green Street, Urbana, IL 61801, USA; tel: (217) 333-3351; search@math.uiuc.edu.

Complete applications must be received by November 14, 2011. Late applications cannot be considered. Applicants may be interviewed before the closing date; however, no hiring decision will be made until after the closing date of November 14, 2011.

Illinois is an Affirmative Action /Equal Opportunity Employer and welcomes individuals with diverse backgrounds, experiences, and ideas who embrace and value diversity and inclusivity. (http://www.inclusiveillinois.illinois.edu).

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INDIANA

INDIANA UNIVERSITY BLOOMINGTON
Department of Mathematics
Assistant/Associate Professor

The Department of Mathematics seeks applications for a tenure-track position, with appointment beginning in the fall of 2012. Exceptionally well-qualified applicants may be considered also at the tenured associate professor level. Outstanding candidates with a Ph.D. in any area of pure or applied mathematics and with postdoctoral or faculty-level experience are encouraged to apply; a minimum requirement is a Ph.D. in mathematics. Salary will be commensurate with qualifications and the level at which the position is filled. The base teaching load for research-active faculty is three courses per year. The department maintains strong research groups in all of the principal fields of mathematics. Bloomington is located in the forested hills of southern Indiana and offers a rich variety of musical and cultural attractions.

Applicants should submit an AMS cover sheet, curriculum vitae, a research statement, and a teaching statement using the online service provided by the AMS at: http://www.mathjobs.org. Applicants should arrange for four letters of recommendation, including one evaluating teaching experience. Where applicable, please ask reference writers to submit their letters electronically through http://www.mathjobs.org. If applicants or letter writers are unable to submit materials online, they may submit them alternatively to the following address: Search Committee, Department of Mathematics, Indiana University, 831 East 3rd Street, Rawles Hall, Bloomington, IN 47405-7106. Applications should be received by November 15, 2011, but will continue to be accepted until the search is filled.

Indiana University is supportive of the needs of dual career couples and is an Equal Opportunity/Affirmative Action Employer.

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INDIANA UNIVERSITY BLOOMINGTON
Department of Mathematics
Zorn Research Postdoctoral Fellowships

The Department of Mathematics seeks applications for two Zorn Research Postdoctoral Fellowships beginning in the fall of 2012. These are three-year, non-tenure-track positions with reduced teaching loads. Outstanding candidates with a recent Ph.D. in any area of pure or applied mathematics are encouraged to apply; a minimum requirement is a Ph.D. in mathematics. Zorn fellows are paired with mentors with whom they have compatible research interests. The department maintains strong research groups in all of the principal fields of mathematics. Bloomington is located in the forested hills of southern Indiana and offers a rich variety of musical and cultural attractions.

Applicants should submit an AMS cover sheet, a curriculum vitae, a research statement, and a teaching statement using the online service provided by the AMS at: `http://www.mathjobs.org`. If unable to do so, send application materials to the address below. Applicants should arrange for four letters of recommendation, including one evaluating teaching experience. Please ask reference writers to submit their letters electronically through `http://www.mathjobs.org`. If they are unable to do so, they may also send their letters to the following address: Zorn Postdoctoral Fellowships Search Committee, Department of Mathematics, Indiana University, 831 East 3rd Street, Rawles Hall, Bloomington, IN 47405-7106. Applications should be received by December 15, 2011. Indiana University is an Equal Opportunity/Affirmative Action Employer.

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MARYLAND

JOHNS HOPKINS UNIVERSITY
Department of Mathematics

Subject to availability of resources and administrative approval, the Department of Mathematics invites applications for one or more positions at the tenure-track or tenured positions at the Associate Professor or Full Professor level beginning fall 2012 or later. Candidates in all areas of pure mathematics are encouraged to apply. To submit your applications go to: `http://www.mathjobs.org/jobs/jhu`. Applicants are strongly advised to submit their other materials electronically at this site. Submit the AMS cover sheet, a curriculum vitae, a list of publications, and the names and addresses of three references. Applicants should indicate whether they are applying for an associate professor or a full professor position. The department will assume responsibility to solicit letters of evaluation and will provide evaluators with a copy of the summary of policies on confidentiality of letters of evaluation. If you do not have computer access, you may mail your application to: Appointments Committee, Department of Mathematics, Johns Hopkins University, 404 Krieger Hall, Baltimore, MD 21218. Write to: `cpoole@jhu.edu` for questions concerning these positions. Applications received by December 1, 2011, will be given priority. The Johns Hopkins University is an Affirmative Action/Equal Opportunity Employer. Minorities and women candidates are encouraged to apply.

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JOHNS HOPKINS UNIVERSITY
Non-Tenure-Track
J.J. Sylvester Assistant Professor

Subject to availability of resources and administrative approval, the Department of Mathematics solicits applications for non-tenure-track Assistant Professor positions beginning fall 2012. The J.J. Sylvester Assistant Professorship is a three-year position offered to recent Ph.D.s with outstanding research potential. Candidates in all areas of pure mathematics, including analysis, mathematical physics, geometric analysis, complex and algebraic geometry, number theory, and topology are encouraged to apply. The teaching load is three courses per academic year. To submit your applications go to: `http://www.mathjobs.org/jobs/jhu`. Applicants are strongly advised to submit their other materials electronically at this site. If you do not have computer access, you may mail your application to: Appointments Committee, Department of Mathematics, Johns Hopkins University, 404 Krieger Hall, Baltimore, MD 21218. Application should include a vita, at least four letters of recommendation of which one specifically comments on teaching, and a description of current and planned research. Write to: `cpoole@jhu.edu` for questions concerning these positions. Applications received by December 1, 2011, will be given priority. The Johns Hopkins University is an Affirmative Action/Equal Opportunity Employer. Minorities and women candidates are encouraged to apply.

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MASSACHUSETTS

BOSTON COLLEGE
Department of Mathematics
Tenure-Track Positions

The Department of Mathematics at Boston College invites applications for two tenure-track positions at the level of Assistant Professor beginning in September 2012, one in Algebraic Geometry or related areas, and a second in Number Theory, Representation Theory, Algebraic Geometry, Geometry, Topology, or related areas. In exceptional cases, a higher level appointment may be considered. The teaching load for each position is three semester courses per year.

Requirements include a Ph.D. or equivalent in mathematics awarded in 2010 or earlier, a record of very strong research combined with outstanding research potential, and demonstrated excellence in teaching mathematics.

A completed application should contain a cover letter, a description of research plans, a statement of teaching philosophy, curriculum vitae, and at least four letters of recommendation. One or more of the letters of recommendation should directly comment on the candidate's teaching credentials.

Applications completed no later than December 1, 2011, will be assured our fullest consideration. Please submit all application materials through: `http://MathJobs.org`.

Applicants may learn more about the department, its faculty and its programs, and about Boston College at: `http://www.bc.edu/math`. Electronic inquiries concerning these positions may be directed to: email: `math-search@bc.edu`. Boston College is an Affirmative Action/Equal Opportunity Employer. Applications from women, minorities, and individuals with disabilities are encouraged.

000081

BOSTON COLLEGE
Department of Mathematics
Postdoctoral Position

The Department of Mathematics at Boston College invites applications for a postdoctoral position beginning September 2012. This position is intended for a new or recent Ph.D. with outstanding potential in research and excellent teaching. This is a 3-year Visiting Assistant Professor position, and carries a 2-1 annual teaching load. Research interests should lie within Geometry or Topology or related areas. Candidates should expect to receive their Ph.D. prior to the start of the position and have received the Ph.D. no earlier than spring 2011.

Applications must include a cover letter, description of research plans, curriculum vitae, and four letters of recommendation, with one addressing the candidate's teaching qualifications. Applications received no later than January 1, 2012, will be assured our fullest consideration. Please submit all application materials through `http://MathJobs.org`.

Applicants may learn more about the department, its faculty and its programs and about Boston College at: `http://www.bc.edu/math`. Email inquiries concerning this position may be directed to: `postdoc-search@bc.edu`. Boston College is an Affirmative Action/Equal Opportunity Employer. Applications from women, minorities and individuals with disabilities are encouraged.

000082

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Mathematics

The Mathematics Department at MIT is seeking to fill positions in Pure and Applied Mathematics, Statistics, and Applied Probability at the level of Instructor, Assistant Professor or higher beginning September 2012. At the same time, the department seeks candidates for the positions of Simons Postdoctoral Fellow, Schramm Postdoctoral Fellow, and ENSW21 Postdoctoral Fellow. Appointments are based primarily on exceptional research qualifications. Appointees will be expected to fulfill teaching duties and to pursue their own research program. Ph.D. required by employment start date.

For more information, and to apply, please visit: `http://www.mathjobs.org`.

To receive full consideration, please submit applications by December 1, 2011. Recommendations should be submitted through `mathjobs.org`, but may also be sent as PDF attachments to `http://hiring@math.mit.edu`, or as paper copies mailed to: Mathematics Search Committee, Room 2-345, Department of Mathematics, MIT, 77 Massachusetts Ave., Cambridge, MA 02139-4307.

Please do not mail or email duplicates of items already submitted via `mathjobs`.

MIT is an Equal Opportunity, Affirmative Action Employer.

000054

MICHIGAN

UNIVERSITY OF MICHIGAN
Faculty Position in Atmospheric, Oceanic and Space Sciences

The Atmospheric, Oceanic and Space Sciences (AOSS) Department at the University of Michigan (UM) is seeking tenured or tenure-track applicants at the assistant professor or higher level in the general areas of Climate Modeling and Integrated Assessment for Engineered Designs.

We seek applicants who are interested in the climate prediction modules for global, regional- or finer-scale application to assess the impact of global change and energy choices on economics and policy for a region or a country like the U.S. For this, a candidate with experience in climate systems modeling, but with the skills and interest needed to integrate this expertise with energy systems and economics will be desired. This candidate will need to develop approaches to integrate climate information into regional- and fine-scale Integrated Assessment models, for which a crucial issue will be to address the uncertainty of factors that influence the expected outcomes and the actions taken in decision-making on climate change. Interest in, for example, abrupt climate change and extreme weather events for policy relevant science is included. Uncovering methods to assess the confidence of analyses will likely be important. So, too, would thorough investigations of the connections between monitoring, prediction, and the likelihood of crossing critical thresholds. Focusing attention on the distributions of critical climate-related parameters, such as the climate sensitivity, could also be important.

This position was awarded to AOSS under UM's Interdisciplinary Faculty Cluster Hire initiative. The Cluster Hire includes one other position in the School of Natural Resources and the Environment with a focus on Modeling and Assessment of Climate Change and Impacts. Successful candidates will work in their primary disciplinary area but will ideally also contribute to the development of greater cohesion among the diverse group of faculty across UM interested in Integrated Assessment.

The successful candidate will be expected to (1) develop a widely recognized research program, (2) attract external funding, (3) mentor graduate students, and (4) participate in the teaching mission of the AOSS department and the College at both the graduate and undergraduate level. Candidates capable of developing collaborative relationships in research and teaching are of particular interest. A Ph.D. in atmospheric science or another relevant discipline is required. Applications should include a cover letter, CV, a concise personal statement describing the candidate's vision for research and education, and a list of at least four references.

For full consideration, applications should be received before November 1, 2011. Questions concerning this position and applications (in a single pdf file) should be directed to email: `UMIA-AOSS@umich.edu`.

The college is especially interested in qualified candidates who can contribute, through their research, teaching, and/or service, to the diversity and excellence of the academic community. Women, minorities, individuals with disabilities, and veterans are encouraged to apply. The university is also responsive to the needs of dual career couples.

000089

NEBRASKA

UNIVERSITY OF NEBRASKA-LINCOLN
Department of Mathematics

Applications are invited for three tenure-track and three postdoctoral positions in mathematics, starting in August, 2012, as follows:

1. Research Assistant Professors. (Requisition #110610). Three three-year (non-tenure-track) positions in mathematics.

2. Algebra. (Requisition #110585). One tenure-track assistant professor position in algebra.

3. Mathematical Biology. (Requisition #110586) One tenure-track assistant professor position in mathematical biology.

4. Mathematical Education of Teachers. (Requisition #110592). One tenure-track associate or full professor position in mathematics with emphasis on the mathematical education of teachers.

Applicants for all positions are encouraged to use the AMS application cover sheet and to submit their applications via `http://mathjobs.org`. First review of applications will begin on November 30, 2011, (December 12, 2011, for the mathematical biology position). Successful candidates for each position should have a Ph.D. in the mathematical sciences and potential for research and teaching in mathematics at a research university. Applicants should submit a letter of application, a CV, statements addressing their research and teaching, and at least three letters of reference, at least one of which should address teaching, via `http://mathjobs.org` or to: Search Committee Chair (position title), Department of Mathematics, University of Nebraska-Lincoln, Lincoln, NE 68588-0130. To be considered for the position, applicants must complete the Faculty/Administrative Information Form at: `http://employment.unl.edu`, (appropriate requisition #). For more information about the positions see the department's website: `http://www.math.unl.edu`.

The University of Nebraska has an active National Science Foundation ADVANCE gender equity program, and is committed to a pluralistic campus community through Affirmative Action, Equal Opportunity, work-life balance, and dual careers.

000076

NEW JERSEY

INSTITUTE FOR ADVANCED STUDY
School of Mathematics

The School of Mathematics has a limited number of memberships with financial support for research in mathematics and computer science at the Institute during the 2012-13 academic year. During the 2012-13 academic year, the school will have a special program on Univalent Foundations of Mathematics. The program will be organized by Steve Awodey of Carnegie Mellon University, Thierry Coquand of the University of Gothenburg, and Vladimir Voevodsky of the Institute. The main goal of the program is to make available to a wider mathematical audience the recent advances which may finally make it practical for pure mathematicians to use "proof assistants" in their work. More information about the special program can be found at "special years" on the school's home page at: http://www.math.ias.edu. Several years ago the school established the von Neumann Fellowships, and up to 6 of these fellowships will be available for the 2012-13 year. To be eligible for a von Neumann Fellowship, applicants should be at least 5, but no more than 15, years following the receipt of their Ph.D. The Veblen Research Instructorship is a 3-year position which the School of Mathematics and the Department of Mathematics at Princeton University established in 1998. Three-year instructorships will be offered each year to candidates in pure and applied mathematics who have received their Ph.D. within the last 3 years. The first and third year of the instructorship will be spent at Princeton University and will carry regular teaching responsibilities. The second year will be spent at the Institute and dedicated to independent research of the instructor's choice. Candidates must have given evidence of ability in research comparable at least with that expected for the Ph.D. degree. Application materials may be requested from Applications, School of Mathematics, Institute for Advanced Study, Einstein Drive, Princeton, NJ 08540; email: applications@math.ias.edu. Postdoctoral computer science and discrete mathematics applicants may be interested in applying for a joint (2-year) position with one of the following: The Department of Computer Science at Princeton University, http://www.cs.princeton.edu; DIMACS at Rutgers, The State University of New Jersey, http://www.dimacs.rutgers.edu; or the Intractability Center, http://intractability.princeton.edu. For a joint appointment, applicants should apply to the School of Mathematics as well as to the above noting their interest in a joint appointment. Applications may be made online at: http://applications.ias.edu. The deadline for all applications is December 1, 2011. The Institute for Advanced Study is committed to diversity and strongly encourages applications from women and minorities.

000045

NEW HAMPSHIRE

DARTMOUTH COLLEGE
Department of Mathematics

Instructorships in Applied and Computational Mathematics, 2-3 years, new or recent Ph.D. graduates with research interest in applied and computational mathematics. Teach 3 ten-week courses spread over 3 terms. Appointment for 26 months, with possible 12 month renewal; monthly salary of $4,950, including two-month research stipend for instructors in residence during 2 of 3 summer months; if not in residence, salary adjusted accordingly. To initiate an application go to: http://www.mathjobs.org. Position ID: IACM #2948. You can also access the application through a link at: http://www.math.dartmouth.edu/activities/recruiting/. General inquiries can be directed to Stephanie Kvam, Administrative Assistant, Department of Mathematics: stephanie.kvam@dartmouth.edu or 603-646-2415. Applications completed by January 5, 2012, considered first. Dartmouth College is committed to diversity and strongly encourages applications from women and minorities.

000069

DARTMOUTH COLLEGE
Department of Mathematics

John Wesley Young Research Instructorships, 2-3 years, new or recent Ph.D. graduates whose research overlaps a department member's. Teach 3 ten-week courses spread over 3 terms. Appointment for 26 months, with possible 12 month renewal; monthly salary of $4,950, including two-month research stipend for Instructors in residence during 2 of 3 summer months; if not in residence, salary adjusted accordingly. To initiate an application go to: http://www.mathjobs.org. Position ID: JWY #2935. You can also access the application through a link at: http://www.math.dartmouth.edu/activities/recruiting/. General inquiries can be directed to Stephanie Kvam, Administrative Assistant, Department of Mathematics, stephanie.kvam@dartmouth.edu or 603-646-2415. Applications completed by January 5, 2012, considered first. Dartmouth College is committed to diversity and strongly encourages applications from women and minorities.

000070

NEW YORK

CORNELL UNIVERSITY
Department of Mathematics

The Department of Mathematics at Cornell University invites applications for two tenure-track assistant professor positions, or higher rank, pending administrative approval, starting July 1, 2012. The searches are open to all areas of mathematics with an emphasis on the areas of probability, number theory, and PDE. The department actively encourages applications from women and minority candidates.

Applicants must apply electronically at: http://www.mathjobs.org.

For information about our positions and application instructions see: http://www.math.cornell.edu/Positions/facpositions.html. Applicants will be automatically considered for all eligible positions. Deadline: November 1, 2011. Early applications will be regarded favorably. Cornell University is an Affirmative Action/Equal Opportunity Employer and Educator.

000040

CORNELL UNIVERSITY
H.C. Wang Assistant Professor

The Department of Mathematics at Cornell University invites applications for two H.C. Wang Assistant Professors, non-renewable, 3-year position beginning July 1, 2012. Successful candidates are expected to pursue independent research at Cornell and teach three courses per year. A Ph.D. in mathematics is required. The department actively encourages applications from women and minority candidates.

Applicants must apply electronically at http://www.mathjobs.org.

For information about our positions and application instructions, see: http://www.math.cornell.edu/Positions/positions.html.

Applicants will be automatically considered for all eligible positions. Deadline December 1, 2011. Early applications will be regarded favorably. Cornell University is an Affirmative Action/Equal Opportunity Employer and Educator.

000048

NORTH CAROLINA

NORTH CAROLINA STATE UNIVERSITY
Statistical and Applied Mathematical Sciences Institute (SAMSI)

The Statistical and Applied Mathematical Sciences Institute (SAMSI) invites applications for the position of Deputy Director for a term of five years beginning July 1, 2012.

SAMSI is one of eight mathematical sciences institutes funded by the National Science Foundation. The Deputy Director will be a distinguished researcher who will provide academic direction and oversight of the SAMSI grant, and who will work closely with the Director on all aspects of the Institute's oversight and program activities. The Deputy Director will also be strongly encouraged to pursue his or her personal research in conjunction with the SAMSI programs or independently.

SAMSI is managed by a directorate which comprises five members: the Director, the Deputy Director and three part-time Associate Directors. The Director and Deputy Director form the executive side of the directorate and are responsible for the administration of programs, human resources and personnel issues, financial operation and infrastructure. Together with the other members of the directorate, they also share the responsibilities of the selection, development and implementation of SAMSI programs.

The appointment will be made as a member of the research faculty at North

Carolina State University. Rank and salary will be commensurate with the candidate's experience and qualifications.

Education Requirements: Candidate must have a minimum of a Ph.D. in mathematics or statistics or equivalent.

Qualifications and Experience: Qualified candidates should be mathematicians or statisticians with excellent management skills and research record. Proven administrative experience is an asset. They should have a strong interest in developing the programs of the Institute.

To submit your application materials, go to: `http://www.mathjobs.org/jobs/ncsu`. Include a vita, a letter of application, and three letters of recommendation. To be considered for this position please also go to: `https://jobs.ncsu.edu` and reference Position number 102319 to complete a Faculty Profile. Applications received by November 30, 2011, will be given priority. Write to: `math-jobs@math.ncsu.edu` for questions concerning this position.

AA/EOE. In addition, NC State welcomes all persons without regard to sexual orientation. The College of Physical and Mathematical Sciences welcomes the opportunity to work with candidates to identify suitable employment opportunities for spouses or partners. For ADA accommodations, please contact Human Resources by email at: `employment@ncsu.edu` or by calling (919) 515-2135.

000079

NORTH CAROLINA STATE UNIVERSITY
Department of Mathematics

The Mathematics Department at North Carolina State University invites applications for one or more tenure-track position beginning fall 2012, depending on the availability of funding. We are seeking exceptionally well-qualified individuals with research interests compatible with those in the department. All areas of pure and applied mathematics will be considered. Candidates must have a Ph.D. in the mathematical sciences, an outstanding research program, a commitment to effective teaching at the undergraduate and graduate levels, and demonstrated potential for excellence in both research and teaching. She or he will likely have had successful postdoctoral experience. The Department of Mathematics has strong research programs in both pure and applied mathematics. Many members of the department participate in interdisciplinary programs and research groups on campus and in the broader Research Triangle community. More information about the department can be found at: `http://www.math.ncsu.edu`.

To submit your application materials, go to: `http://www.mathjobs.org/jobs/ncsu`. Include a vita, at least three letters of recommendation, and a description of current and planned research. To be considered for this position please also go to `http://jobs.ncsu.edu/applicants/Central?quickFind=91522` and complete a Faculty Profile for the position.

Write to: `math-jobs@math.ncsu.edu` for questions concerning this position.

AA/EOE. In addition, NC State welcomes all persons without regard to sexual orientation. The College of Physical and Mathematical Sciences welcomes the opportunity to work with candidates to identify suitable employment opportunities for spouses or partners. For ADA accommodations, please contact Human Resources by email at: `employment@ncsu.edu` or by calling (919) 515-2135. Applications received by November 15, 2011, will be given priority.

000073

ALLIANCE FOR BUILDING FACULTY DIVERSITY IN THE MATHEMATICAL SCIENCES
NSF-funded Postdoctoral Fellowships
Postdoctoral Fellowships

The Alliance for Building Faculty Diversity in the Mathematical Sciences aims to increase the access of U.S. underrepresented minority groups to academic tenure-track positions. The Alliance offers six NSF funded Postdoctoral Fellowships beginning Fall 2012 targeted at new or recent minority Ph.D.s. The alliance is comprised of NSF Mathematical Sciences Institutes and seven major research universities with a good record of mentoring underrepresented mathematics graduate students. Successful applicants will show strong research potential and be interested in continuing in a career at a research university.

The postdoctoral fellowships are for three years. A typical 3-year postdoctoral fellow will spend 2 years at one of the alliance universities and up to a year at a national institute if there is a suitable program. Each postdoc will be matched with a research mentor at the host university. The Fellowship salary will be $60,000 per year plus benefits. In addition the fellows will receive a travel and research allowance. For more information see: `http://www.math.ncsu.edu/alliance`.

Eligibility. Applicants must be U.S. citizens or permanent pesidents who have obtained a Ph.D. in mathematics within the last 5 years. Particular attention will be given to U.S. underrepresented minority (U.S. URM) candidates.

The Alliance universities are (listed alphabetically): Arizona State University, Howard University, Iowa State University, North Carolina State University, University of Arizona, University of Iowa, and University of Nebraska. The Alliance NSF Mathematical Sciences Institutes are (listed alphabetically): American Institute of Mathematics (AIM), Institute for Computational and Experimental Research in Mathematics (ICERM), Institute for Mathematics and its Applications (IMA), Institute For Pure and Applied Mathematics (IPAM), Mathematical Biosciences Institute (MBI), Mathematical Sciences Research Institute (MSRI), National Institute for Mathematical and Biological Synthesis (NIMBioS), Park City Mathematics Institute (PCMI/IAS), and Statistical and Applied Mathematical Sciences Institute (SAMSI).

To submit your application materials, go to: `http://www.mathjobs.org/jobs/alliance`. Include a vita, at least three letters of recommendation, and a description of current and planned research. Please also include a short statement to address how your plans fit with the priorities of this program. Applicants are encouraged to consult the Alliance website `http://www.math.ncsu.edu/alliance` and provide a list of potential mentors and preferred institutions.

Write to `alliance@math.ncsu.edu` for questions concerning this position. Applications received by December 15, 2011, will be given priority.

000065

WAKE FOREST UNIVERSITY
Department of Mathematics

Applications are invited for one tenure-track position in statistics at the assistant professor level beginning July 2012. Hiring at the associate professor level may be considered. We seek highlyqualified candidates who have a commitment to excellence in both teaching and research. A Ph.D. in statistics or a related area is required. The department has twenty members and offers both a B.A. and a B.S. in mathematics, with an optional concentration in statistics, a B.S. in interdisciplinary mathematics, and a B.S. in each of mathematical business and mathematical economics. The department has a graduate program offering an M.A. in mathematics. A complete application will include a letter of application, curriculum vitae, teaching statement, research statement, graduate transcripts, and three letters of recommendation. The application deadline is December 1, 2011. Applicants are encouraged to post materials electronically at `http://www.mathjobs.org`. Hard copy can be sent to Edward Allen, Wake Forest University, Department of Mathematics, P.O. Box 7388, Winston-Salem, NC 27109. AA/EO Employer.

000047

OHIO

THE OHIO STATE UNIVERSITY
Columbus Ohio
Department of Mathematics
Tenure-Track Assistant Professor

The Department of Mathematics in the College of Arts and Sciences at The Ohio State University anticipates having a tenure-track assistant professor position available in Analysis, effective Autumn Quarter 2012. Further information about the department can be found at http://www.math.ohio-state.edu. Candidates are expected to have a Ph.D. in mathematics (or related area) and to present evidence of excellence in teaching and research. Applications will be considered on a continuing basis, but the annual review process begins November 14, 2011. Applications should be submitted online at: http://www.mathjobs.org. If you cannot apply online, please contact facultysearch@math.ohio-state.edu or write to: Hiring Committee, Department of Mathematics, The Ohio State University, 231 W. 18th Avenue, Columbus, OH 43210. To build a diverse workforce Ohio State encourages applications from individuals with disabilities, veterans and women. EEO/AA employer.

000066

THE OHIO STATE UNIVERSITY
Columbus Ohio
Department of Mathematics
Tenure-Track (Rank Open)

The Department of Mathematics in the College of Arts and Sciences at The Ohio State University anticipates having a tenure-track position available in Probability, rank open, effective Autumn Quarter 2012. Further information about the department can be found at: http://www.math.ohio-state.edu. Candidates are expected to have a Ph.D. in mathematics (or related area) and to present evidence of excellence in teaching and research. Applications will be considered on a continuing basis, but the annual review process begins November 14, 2011. Applications should be submitted online at: http://www.mathjobs.org. If you cannot apply online, please contact facultysearch@math.ohio-state.edu or write to: Hiring Committee, Department of Mathematics, The Ohio State University, 231 W. 18th Avenue, Columbus, OH 43210. To build a diverse workforce Ohio State encourages applications from individuals with disabilities, veterans and women. EEO/AA employer.

000067

UNIVERSITY OF CINCINNATI
Department of Mathematical Sciences

Applications are invited for two tenure track Assistant Professorships, beginning August 1, 2012. One position is in Financial Mathematics and one is in Probability. A requirement for both positions is a Ph.D. in Mathematical Sciences or equivalent prior to July 31, 2012. Applicants should have expertise in Probability or Financial Mathematics. Successful candidates are expected to have a very active research program, to constantly seek external funding, to mentor graduate students, and to teach graduate and undergraduate courses in mathematical sciences. The normal teaching load for research active faculty is 6 credit hours per semester with a 25% reduction for new faculty each of the first 2 years and with a reduced service role. Complete applications, consisting of a cover letter, vita, description of research program, description of teaching experience, and three letters of recommendation, should be submitted to: MathJobs.org (https://www.mathjobs.org/jobs/jobs/2974). In addition, candidates must submit a vita and a cover letter at http://www.jobsatuc.com—Probability, 211UC1559 (http://www.jobsatuc.com/applicants/Central?quickFind=79539) or Financial Mathematics, 211UC1561 (http://www.jobsatuc.com/applicants/Central?quickFind=79541). Review of applicants will begin on December 15, 2011. Applications will be accepted until both positions are filled. The Department of Mathematical Sciences is dedicated to excellence in both research and teaching. The University of Cincinnati is an Equal Opportunity/Affirmative Action Employer. Women, people of color, people with disability and veterans are encouraged to apply.

000080

RHODE ISLAND

BROWN UNIVERSITY
J. D. Tamarkin Assistant Professorship

Two three-year non-tenured non-renewable appointments, beginning July 1, 2012. The teaching load is one course one semester, and two courses the other semester and consists of courses of more than routine interest. Candidates are required to have received a Ph.D. degree or equivalent by the start of their appointment, and they may have up to three years of prior academic and/or postdoctoral research experience.

Applicants should have strong research potential and a commitment to teaching. Field of research should be consonant with the current research interests of the department.

For full consideration, applicants must submit a curriculum vitae, an AMS Standard Cover Sheet, and three letters of recommendation by December 1, 2011. Please submit all application materials on line at: http://www.mathjobs.org. Email inquiries should be addressed to: juniorsearch@math.brown.edu.

Brown University is an Equal Opportunity/Affirmative Action Employer and encourages applications from women and minorities.

000075

SOUTH CAROLINA

UNIVERSITY OF SOUTH CAROLINA
Department of Mathematics
Tenure-track Assistant Professor

Applications are invited for a tenure-track Assistant Professor position in the broad area of Algebra. Areas of particular, but not exclusive, interest include algebraic combinatorics, algebraic geometry, algebraic number theory, commutative algebra, Lie theory, and representation theory.

Candidates must have a Ph.D. in Mathematics, an outstanding research program, a commitment to effective teaching at the undergraduate and graduate levels, and a demonstrated potential for excellence in both research and teaching.

Applicants must apply electronically at: http://www.mathjobs.org. A completed application should contain a cover letter, standard AMS cover sheet, curriculum vitae, description of research plans, statement of teaching philosophy, and four letters of recommendation. One of the letters should appraise the candidate's teaching ability.

The beginning date for the position will be August 16, 2012. Review of applications will begin on December 1, 2011, and continue until the position is filled. To ensure consideration, applications should be received by January 10, 2012. Please address inquiries to: hiring@math.sc.edu.

The University of South Carolina is an Affirmative Action, Equal Opportunity Employer. Women and minorities are encouraged to apply.

000068

TENNESSEE

VANDERBILT UNIVERSITY
Department of Mathematics
Nashville, Tennessee
Non-Tenure-Track Assistant Professor Positions

We invite applications for several visiting and non-tenure-track assistant professor positions in the research areas of the Mathematics Department beginning fall 2012. These positions will have variable terms and teaching loads but most will be three-year appointments with a 2-2 teaching load. We anticipate that some of these appointments will carry a 1-1 teaching load and provide a stipend to support research.

We are looking for individuals with outstanding research potential and a strong commitment to excellence in teaching. Preference will be given to recent doctorates. Submit your application and supporting materials electronically through the AMS website Mathjobs.org via the url: http://www.mathjobs.org/jobs. Alternatively, application materials may be sent to: NTT Appointments

Committee, Vanderbilt University, Department of Mathematics, 1326 Stevenson Center, Nashville, TN 37240. These materials should include a letter of application, a curriculum vitae, a publication list, a research statement, four letters of recommendation and the AMS Cover Sheet. One of the letters must discuss the applicant's teaching qualifications. Reference letter writers should be asked to submit their letters online through Mathjobs.org. Evaluation of the applications will commence on December 1, 2011, and continue until the positions are filled. For information about the Department of Mathematics at Vanderbilt University, please consult the web at: http://www.math.vanderbilt.edu/.

Vanderbilt is an Equal Employment Opportunity/Affirmative Action employer. Women and minorities are especially invited to apply.

000061

UNIVERSITY OF TENNESSEE
Position Advertisement

The Department of Mathematics of The University of Tennessee seeks to fill one tenure-track assistant professor position in theoretical differential equations. A Ph.D. or equivalent degree is required. Outstanding research promise and dedication to excellent teaching are paramount. Some postdoctoral experience is desirable, though not required. Employment begins August 1, 2012.

Applicants should arrange to have submitted a curriculum vita, at least three letters of recommendation, a research statement (including future plans and abstracts of finished papers), and evidence of quality teaching. These documents can be submitted any of the following ways: (1) electronically at: http://www.mathjobs.org/jobs (preferred), (2) by email to: diffeqs@math.utk.edu, (3) by mail to Differential Equations Search, Department of Mathematics, The University of Tennessee, Knoxville, TN 37996-0612. Review of applications will begin December 1, 2011 and will continue until the position is filled.

The Knoxville campus of the University of Tennessee is seeking candidates who have the ability to contribute in meaningful ways to the diversity and intercultural goals of the university. For more information about the University and Department of Mathematics please see our websites at: http://www.utk.edu and http://www.math.utk.edu.

The University of Tennessee is an EEO/AA/Title VI/Title IX/Section 504/ADA/ADEA institution in the provision of its education and employment programs and services. All qualified applicants will receive equal consideration for employment without regard to race, color, national origin, religion, sex, pregnancy, marital status, sexual orientation, gender identity, age, physical or mental disability, or covered veteran status.

000062

TEXAS

BAYLOR UNIVERSITY
Positions in Department of Mathematics

The Department of Mathematics at Baylor University seeks to fill the following positions, all starting in August 2012:

(1) Tenure-Track (Tracking ID: BQ11964) at the Associate/Full Professor level.

(2) Tenure-Track (Tracking ID: BQ11965) at the Assistant Professor level.

(3) Post-Doctorate (Tracking ID: BQ11966)

For all three positions, we seek to hire strong applicants with research interests compatible with current faculty; these areas include algebra, algebraic geometry, analysis, computational mathematics, differential equations, mathematical physics, numerical analysis, representation theory, and topology. However, exceptional scholars in all mathematical areas of specialization are encouraged to apply.

Excellence in teaching and research is essential. Strong record/potential for obtaining external funding is desirable. Candidates should possess an earned doctorate in the appropriate field of study (for the post-doctorate position, candidates must have earned this doctorate degree by August, 2012). Salary and benefits are competitive.

The Baylor mathematics department has 30 faculty members and offers B.A., B.S., M.S., and Ph.D. degrees. To learn more about the above position, please visit the websites: http://www.baylor.edu/math and http://www.baylor.edu/comp_benefits/.

Chartered in 1845 by the Republic of Texas, Baylor University has approximately 14,000 students and is the oldest university in Texas and the world's largest Baptist University. Baylor's mission is to educate men and women for worldwide leadership and service by integrating academic excellence and Christian commitment within a caring community.

Each applicant is asked to provide a cover letter of application, at least three letters of reference, a current curriculum vitae, a copy of an official doctoral transcript, a research statement, a teaching statement, a publication list, and a religious statement. All applications must be uploaded to: MathJobs.org, where further details are given, via the URL http://www.mathjobs.org/jobs. Please include the appropriate tracking ID in your application. To ensure full consideration, an application should be received by December 1, 2011, but applications will be accepted until the position is filled or the search is terminated.

Baylor is a Baptist university affiliated with the Baptist General Convention of Texas. As an Affirmative Action/Equal Employment Opportunity Employer, Baylor encourages minorities, women, veterans, and persons with disabilities to apply.

000071

RICE UNIVERSITY
Department of Mathematics
Tenure-Track Position

The Department of Mathematics of Rice University anticipates filling at least one position at the rank of tenure-track Assistant Professor, contingent on budgetary approval. Candidates who could make an extraordinary contribution may be considered at other ranks. Applicants should have extremely strong research potential and provide evidence of success in teaching. All application materials, including at least three letters of reference, should be submitted via MathJobs (http://www.mathjobs.org/jobs). For more information, see http://www.math.rice.edu/About/jobtt.html. Applications should be received by November 15, 2011, to receive full consideration; early application is advisable. Rice University is an Equal Opportunity/Affirmative Action Employer and strongly encourages applications from women and members of under-represented minority groups.

000083

TEXAS A&M UNIVERSITY
Department of Mathematics

The Department of Mathematics anticipates several openings for postdoctoral positions at the level of visiting assistant professor, subject to budgetary approval. Our visiting assistant professor positions are three-year appointments and carry a three-course-per-year teaching load. They are intended for those who have recently received their Ph.D. and preference will be given to mathematicians whose research interests are close to those of our regular faculty members. We also anticipate several short-term (semester or year-long) visiting positions at various ranks, depending on budget. A complete dossier should be received by December 15, 2011. Early applications are encouraged since the department will start the review process in October 2011. Applicants should send the completed "AMS Application Cover Sheet", a vita, a summary statement of research and teaching experience, and arrange to have letters of recommendation sent to: Faculty Hiring, Department of Mathematics, Texas A&M University, 3368 TAMU, College Station, Texas 77843-3368.

Further information can be obtained from: http://www.math.tamu.edu/hiring.

Texas A&M University is an Equal Opportunity Employer. The university is dedicated to the goal of building a culturally diverse and pluralistic faculty and staff committed to teaching and working in a multicultural environment and strongly encourages applications from women, minorities, individuals with disabilities, and veterans. The university is responsive to the needs of dual career couples.

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UTAH

UNIVERSITY OF UTAH
Department of Mathematics

The Department of Mathematics at the University of Utah invites applications for the following positions:

Full-time tenure-track or tenured appointments at the level of assistant, associate, or full professor in all areas of mathematics and statistics.

Three-year Wylie and Burgess Assistant Professorships. Please see our website at: http://www.math.utah.edu/positions for information regarding available positions, application requirements, and deadlines. Applications must be completed through the website http://www.mathjobs.org.

The University of Utah is an Equal Opportunity/Affirmative Action Employer and Educator. Minorities, women, and persons with disabilities are strongly encouraged to apply. Veterans preference. Reasonable accommodations provided. For additional information: http://www.regulations.utah.edu/humanResources/5-106.html.

The University of Utah values candidates who have experience working in settings with students from diverse backgrounds, and possess a strong commitment to improving access to higher education for historically underrepresented students.

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WASHINGTON

UNIVERSITY OF WASHINGTON
Department of Mathematics

Applications are invited for a non-tenure-track Acting Assistant Professor position. The appointment is for a period of up to three years to begin in September 2012. Applicants are required to have a Ph.D. by the starting date, and to be highly qualified for undergraduate and graduate teaching and independent research.

Applications should include should include the American Mathematical Society's Cover Sheet for Academic Employment, a curriculum vitae, statements of research and teaching interests, and three letters of recommendation. We prefer applications and supporting materials to be submitted electronically via www.mathjobs.org. Application materials may also be mailed to: Appointments Committee Chair (AAP position), Department of Mathematics, Box 354350, University of Washington, Seattle, WA 98195-4350. Priority will be given to applicants whose complete applications, including recommendations, are received by December 15, 2011.

The University of Washington is building a culturally diverse faculty and strongly encourages applications from female and minority candidates. The University is an Equal Opportunity/Affirmative Action Employer.

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SINGAPORE

NATIONAL UNIVERSITY OF SINGAPORE (NUS)
Department of Mathematics

The Department of Mathematics at the National University of Singapore (NUS) invites applications for tenured, tenure-track and visiting positions at all levels, beginning in August 2012.

NUS is a research-intensive university that provides quality undergraduate and graduate education. The Department of Mathematics has about 65 faculty members and teaching staff whose expertise cover major areas of contemporary mathematical research.

We seek promising scholars and established mathematicians with outstanding track records in any field of pure and applied mathematics. The department, housed in a newly renovated building equipped with state-of-the-art facilities, offers internationally competitive salary with start-up research grants, as well as an environment conducive to active research, with ample opportunities for career development. The teaching load for junior faculty is kept especially light.

The department is particularly interested in, but not restricted to, considering applicants specializing in any of the following areas:

Analysis, Probability, and Ergodic Theory
Number Theory, Arithmetic Geometry
Computational Science, including but not restricted to,
Computational Biology, Medical Imaging, Computational
Materials Science and Nanoscience
Financial Mathematics
Operations Research

Application materials should be sent to the Search Committee via email (as PDF files): search@math.nus.edu.sg.

Please include the following supporting documentation in the application:

1. an American Mathematical Society Standard Cover Sheet;
2. a detailed CV including publications list;
3. a statement (max. of 3 pages) of research accomplishments and plan;
4. a statement (max. of 2 pages) of teaching philosophy and methodology. Please attach evaluation on teaching from faculty members or students of your current institution, where applicable; 5. at least three letters of recommendation including one which indicates the candidate's effectiveness and commitment in teaching. Please ask your referees to send their letters directly to: search@math.nus.edu.sg.

Enquiries may also be sent to this email address. Review process will begin on 15 October, and will continue until positions are filled.

For further information about the department, please visit http://www.math.nus.edu.sg.

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TAIWAN

TAIWAN
National Central University
Department of Mathematics

The Department of Mathematics invites applications for the following positions: regular positions at all levels and visiting positions (for a period of up to two years) at the level of assistant professor. Starting date is August 1, 2012. All application materials should be sent to Chair via email (as pdf files) to: ncu5100@ncu.edu.tw or mail to Chair, Department of Mathematics National Central University No. 300, Jhongda Rd., Jhongli City, Taoyuan County 32001 Taiwan (R. O. C.) Please include the following supporting documentation in the application:

1. Cover letter;
2. Curriculum Vitae;
3. Publication List;
4. Three Reference Letters (submitted directly by writers). In order to ensure full consideration, applications should be received by December 20, 2011. For more information, visit: http://www.math.ncu.edu.tw.

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Meetings & Conferences of the AMS

IMPORTANT INFORMATION REGARDING MEETINGS PROGRAMS: AMS Sectional Meeting programs do not appear in the print version of the *Notices*. However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on the AMS website. See `http://www.ams.org/meetings/`. Final programs for Sectional Meetings will be archived on the AMS website accessible from the stated URL and in an electronic issue of the *Notices* as noted below for each meeting.

Salt Lake City, Utah

University of Utah

October 22–23, 2011

Saturday – Sunday

Meeting #1075

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: August 2011
Program first available on AMS website: September 8, 2011
Program issue of electronic *Notices*: October 2011
Issue of *Abstracts*: Volume 32, Issue 4

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Graeme Walter Milton, The University of Utah, *Metamaterials: High contrast composites with unusual properties.*

Lei Ni, University of California San Diego, *Gap theorems on Kähler manifolds.*

Igor Pak, University of California Los Angeles, *The future of combinatorial bijections.*

Monica Visan, UCLA, *Dispersive PDE at critical regularity.*

Special Sessions

Algebraic Geometry, **Tommaso de Fernex** and **Christopher Hacon**, University of Utah.

Applied Analysis, **Marian Bocea**, North Dakota State University, and **Mihai Mihailescu**, University of Craiova Romania.

Category Theory in Graphs, Geometry and Inverse Problems, **Robert Owczarek**, Enfitek, Inc., and **Hanna Makaruk**, Los Alamos National Laboratory NM.

Celestial and Geometric Mechanics, **Lennard Bakker** and **Tiancheng Ouyang**, Brigham Young University.

Commutative Algebra, **Chin-Yi Jean Chan**, Central Michigan University, and **Lance E. Miller** and **Anurag K. Singh**, University of Utah.

Computational and Algorithmic Algebraic Geometry, **Zach Teitler**, Boise State University, and **Jim Wolper**, Idaho State University.

Electromagnetic Wave Propagation in Complex and Random Environments, **David Dobson**, University of Utah, and **Peijun Li**, Purdue University.

Geometric Evolution Equations and Related Topics, **Andrejs Treibergs**, University of Utah Salt Lake City, **Lei Ni**, University of California San Diego, and **Brett Kotschwar**, Arizona State University.

Geometric, Combinatorial, and Computational Group Theory, **Eric Freden**, Southern Utah University, and **Eric Swenson**, Brigham Young University.

Harmonic Analysis and Dispersive Partial Differential Equations, **Xiaoyi Zhang**, University of Iowa, and **Monica Visan** and **Betsy Stovall**, University of California Los Angeles.

Hypergeometric Functions and Differential Equations, **Laura F. Matusevich**, Texas A&M University, and **Christine Berkesch**, Stockholm University.

Inverse Problems and Homogenization, **Elena Cherkaev** and **Fernando Guevara Vasquez**, University of Utah.

Noncommutative Geometry and Algebra, **Kenneth R. Goodearl**, University of California Santa Barbara, and **Milen Yakimov**, Louisiana State University.

Nonlinear Waves, **Zhi-Qiang Wang** and **Nghiem Nguyen**, Utah State University.

Recent Progress in Numerical Partial Differential Equations, **Jichun Li**, University of Nevada, Las Vegas, and **Shue-Sum Chow**, Brigham Young University.

Reductive Groups and Hecke Algebras, **Dan Ciubotaru**, University of Utah, **Cathy Kriloff**, Idaho State University, and **Peter Trapa**, University of Utah.

Understanding Bio-fluids via Modeling, Simulation and Analysis, **Christel Hohenegger**, University of Utah.

Port Elizabeth, Republic of South Africa

Nelson Mandela Metropolitan University

November 29 – December 3, 2011

Tuesday – Saturday

Meeting #1076

First Joint International Meeting between the AMS and the South African Mathematical Society.
Associate secretary: Matthew Miller
Announcement issue of *Notices*: July 2011
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: Not applicable

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/internmtgs.html`.

Invited Addresses

Mark J. Ablowitz, University of Colorado at Boulder, *Nonlinear systems—From oceans to number theory*.

Zoltan Furedi, University of Illinois, Urbana-Champaign, *Superimposed codes and hypergraphs containing no grids*.

Mikhail Petrov, University of Swaziland, *Asymptotic methods in thin shells dynamics*.

James Raftery, University of Kwazulu Natal, *Bridges between algebra and logic: Some old and some new*.

Daya Reddy, University of Cape Town, *Mixed finite element approximations*.

Peter Sarnak, Princeton University, *Mobius randomness and dynamics*.

Lindi Tshabalala, Thuthuzekani Primary School, *Title to be announced*.

Amanda Weltman, University of Cape Town, *Title to be announced*.

Special Sessions

Combinatorial and Computational Group Theory with Applications, **Gilbert Baumslag**, City College of New York, **Mark Berman**, University of Cape Town, and **Vladimir Shpilrain**, City College of New York.

Combinatorics and Graph Theory, **Michael Henning**, University of Johannesburg, **Robin Thomas**, Georgia Institute of Technology, and **Jacques Verstraete**, University of California, San Diego.

Computer Vision, High Performance Computing, and Imaging, **Steve Damelin**, Georgia Southern University and University of the Witswatersrand, and **Hari Kumar**, University of the Witswatersrand.

Finite Groups and Combinatorial Structures, **Jashmid Moori**, North-West University, Mafikeng, and **B. Rodrigues**, University of Kwazulu-Natal, Westville.

Geometry and Differential Equations, **Jesse Ratzkin**, University of Cape Town.

High Performance Computing and Imaging, **Steven B. Damelin**, Georgia Southern University and University of the Witswatersrand, and **Hari Kumar**, University of the Witswatersrand.

Mathematical Inequalities and Applications, **Saver S. Dragomir**, University of the Witwatersrand and Victoria University, Australia.

Nonlinear Waves and Integrable Systems, **Mark Ablowitz**, University of Colorado at Boulder, and **Barbara Prinari**, University of Colorado at Colorado Springs.

Operator and Banach Algebras, and Noncommutative Analysis, **David Blecher**, University of Houston, **Garth Dales**, University of Leeds, **Louis Labuschagne**, North-West University, Potchefstroom Campus, and **Anton Stroh**, University of Pretoria.

Recent Advances in Computational Methods for Partial Differential Equations, **Kailash C. Patidar**, University of the Western Cape.

Theoretical and Numerical Aspects of Dynamical Systems, Partial Differential Equations, and Inequalities, Arising in Applications, **J. M.-S. Lubuma**, University of Pretoria, and **B. D. Reddy**, University of Cape Town.

Topology and Categories, **Hans-Peter Kuenzi**, University of Cape Town.

Boston, Massachusetts

John B. Hynes Veterans Memorial Convention Center, Boston Marriott Hotel, and Boston Sheraton Hotel

January 4–7, 2012

Wednesday – Saturday

Meeting #1077

Joint Mathematics Meetings, including the 118th Annual Meeting of the AMS, 95th Annual Meeting of the Mathematical Association of America, annual meetings of the

Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: October 2011
Program first available on AMS website: November 1, 2011
Program issue of electronic *Notices*: January 2012
Issue of *Abstracts*: Volume 33, Issue 1

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: Expired

MAA Program Updates

Are We Selling Mathematics As a Major?, organized by **Steve Deckelman**, University of Wisconsin Stout, and **Mary Kay Abbey**, Montgomery College Maryland; Thursday, 9:00– 0:20 a.m. Are we selling mathematics as a major? What brings a student to the mathematics major? Why do some schools have a large number of majors while others are service departments to the sciences and business? We mathematicians know about the ten best occupations website but do we share this with our students? Panelists **Michael Dorf**, Brigham Young University; **Sandy Ganzell**, St. Mary's College Maryland; **Daniel Kaplan**, Macalester College; and **Theresa Anderson**, Brown University; will highlight programs that have seen recent growth and will explain how minor changes such as renaming or linking courses, changes in the culture of departmental pedagogy, and letting students know they are important to us can bring more majors to the department. The faculty panelists have tried many ideas and have succeeded—perhaps not with all—and will share their stories. Finally, a graduate student will tell us what attracted her—maybe not what we thought!

Demonstration Math Circles, organized by **James Tanton**, St. Mark's Institute of Mathematics, and **Tatiana Shubin**, San Jose State University. A math circle is broadly defined as a semi-formal, sustained enrichment experience that brings mathematics professionals in direct contact with precollege students and/or their teachers. Circles foster passion and excitement for deep mathematics. These three demonstration sessions offer the opportunity for JMM2012 participants, and others, to observe and take part in Math Circle experiences and enjoy the thrill of the organic and creative process the conversational style of learning Circles offer. Seeing a circle in action, we believe, is the best way to generate enthusiasm to start one of your own. Come see why! Session for JMM participants, Thursday, 3:00 p.m–4:00 p.m.; Session for JMM participants, teachers, and other mathematical enthusiasts, Saturday, 9:00 a.m–9:50 a.m.; and Session for undergraduate students, Saturday, 10:00 a.m.–11:00 a.m. Sponsored by SIGMAA for Math Circles for Students and Teachers (SIGMAA MCST).

Helping Actuarial Science Students Prepare for Career Milestones and Choices, organized by **Patrick Brewer,** Lebanon Valley College; **Robert Buck**, Slippery Rock University; **Bettye Anne Case,** Florida State University; **Kevin Charlwood**, Washburn University; and **Steve Paris**, Florida State University; Friday, 5:00–7:00 p.m.

Undergraduate Poster Session (updated information, Friday, 3:30 p.m.–5:30 p.m., organized by **Joyati Debnath**, Wayne State University. The session is reserved to undergraduates and first-year graduate students submitting posters on work done while undergraduates. Abstracts are accepted on a first-come, first-served basis. Space is limited and students are encouraged to apply early. See `http://www.maa.org/students/undergrad/jmmposterindex.html` for pertinent details, including a link to the abstract submission form. Examples of poster topics include a new result, a different proof of a known theorem, an innovative solution of a Putnam problem, a new mathematical model, or method of solution of an applied problem. Purely expository posters cannot be accepted. Posters will be judged and certificates will be awarded to posters with the highest scores. Trifold, self-standing 48" by 36" tabletop posterboards will be provided. Additional material or equipment is the responsibility of the presenters. Participants must be available for setting up their posters from 2:30-3:30 p.m. for setup and then from 3:30 p.m. to 5:30 p.m. to answer questions from the judges and the general public. Questions regarding this session should be directed to the session organizer at `jdebnath@winona.edu`. The deadline for abstract submissions is **October 28, 2011**. Notifications of acceptance/rejection will be sent soon after that.

Activities of Other Organizations

The **NAM Claytor-Woodward Lecture** will be given on Friday night after the banquet by **Aderemi Oluyomi Kuku**, Grambling State University, *Profinite (continuous) equivariant higher algebraic K-theory for the action of algebraic groups*.

Honolulu, Hawaii

University of Hawaii at Manoa

March 3–4, 2012

Saturday – Sunday

Meeting #1078

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: December 2011
Program first available on AMS website: January 26, 2012
Program issue of electronic *Notices*: March 2012
Issue of *Abstracts*: Volume 33, Issue 2

Deadlines

For organizers: Expired

For consideration of contributed papers in Special Sessions: November 22, 2011
For abstracts: December 13, 2011

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Zhiqin Lu, University of California Irvine, *Title to be announced.*

Peter Schroder, California Institute of Technology, *Title to be announced.*

Pham Tiep, University of Arizona, Tucson, *Title to be announced.*

Lauren Williams, University of California Berkeley, *Title to be announced.*

Special Sessions

Algebraic Combinatorics (Code: SS 24A), **Federico Ardila**, San Francisco State University, **Sara Billey**, University of Washington, and **Kelli Talaska** and **Lauren Williams**, University of California, Berkeley.

Algebraic Geometry: Singularities and Moduli (Code: SS 22A), **Jim Bryan**, University of British Columbia, and **Jonathan Wise**, Stanford University.

Algebraic Number Theory, Diophantine Equations and Related Topics (Code: SS 6A), **Claude Levesque**, Université de Laval, Quebec, Canada.

Applications of Nonstandard Analysis (Code: SS 14A), **Tom Lindstrom**, University of Oslo, Norway, **Peter Loeb**, University of Illinois at Urbana-Champaign, and **David Ross**, University of Hawaii, Honolulu.

Arithmetic Geometry (Code: SS 5A), **Xander Faber**, **Michelle Manes**, and **Gretel Sia**, University of Hawaii.

Asymptotic Group Theory (Code: SS 12A), **Tara Davis**, Hawaii Pacific University, **Erik Guentner**, University of Hawaii, and **Michael Hull** and **Mark Sapir**, Vanderbilt University.

Automorphic and Modular Forms (Code: SS 4A), **Pavel Guerzhoy**, University of Hawaii, and **Zachary A. Kent**, Emory University.

C-algebras and Index Theory* (Code: SS 18A), **Erik Guentner**, University of Hawaii at Manoa, **Efren Ruiz**, University of Hawaii at Hilo, and **Erik Van Erp** and **Rufus Willett**, University of Hawaii at Manoa.

Computability and Complexity (Code: SS 23A), **Cameron E. Freer**, Massachusetts Institute of Technology, and **Bjorn Kjos-Hanssen**, University of Hawaii at Manoa.

Geometry and Analysis on Fractal Spaces (Code: SS 3A), **Michel Lapidus**, University of California, Riverside, **Hung Lu**, Hawaii Pacific University, **John A. Rock**, California State Polytechnic University, Pomona, and **Machiel van Frankenhuijsen**, Utah Valley University.

Holomorphic Spaces (Code: SS 8A), **Hyungwoon Koo**, Korea University, and **Wayne Smith**, University of Hawaii.

Kaehler Geometry and Its Applications (Code: SS 1A), **Zhiqin Lu**, University of California Irvine, **Jeff Streets**, Princeton University, **Li-Sheng Tseng**, Harvard University, and **Ben Weinkove**, University of California San Diego.

Kernel Methods for Applications on the Sphere and Other Manifolds (Code: SS 21A), **Thomas Hangelbroek**, University of Hawaii at Manoa.

Knotting in Linear and Ring Polymer Models (Code: SS 19A), **Tetsuo Deguchi**, Ochanomizu University, **Kenneth Millett**, University of California, Santa Barbara, **Eric Rawdon**, University of St. Thomas, and **Mariel Vazquez**, San Francisco State University.

Linear and Permutation Representations (Code: SS 2A), **Robert Guralnick**, University of Southern California, and **Pham Huu Tiep**, University of Arizona.

Mathematical Coding Theory and its Industrial Applications (Code: SS 13A), **J. B. Nation**, University of Hawaii, and **Manabu Hagiwara**, National Institute of Advanced Industrial Science and Technology, Japan.

Mathematical Teacher Preparation (Code: SS 17A), **Diane Barrett** and **Roberto Pelayo**, University of Hawaii at Hilo.

Model Theory (Code: SS 11A), **Isaac Goldbring**, University of California Los Angeles, and **Alice Medvedev**, University of California Berkeley.

New Techniques and Results in Integrable and Near-Integrable Nonlinear Waves (Code: SS 16A), **Jeffrey DiFranco**, Seattle University, and **Peter Miller**, University of Michigan.

Noncommutative Algebra and Geometry (Code: SS 15A), **Jason Bell**, Simon Fraser University, and **James Zhang**, University of Washington.

Nonlinear Partial Differential Equations at the Common Interface of Waves and Fluids (Code: SS 9A), **Ioan Bejenaru** and **Vlad Vicol**, University of Chicago.

Nonlinear Partial Differential Equations of Fluid and Gas Dynamics (Code: SS 7A), **Elaine Cozzi**, Oregon State University, and **Juhi Jang** and **Jim Kelliher**, University of California Riverside.

Singularities, Stratifications and Their Applications (Code: SS 20A), **Terence Gaffney**, Northeastern University, **David Trotman**, Université de Provence, and **Leslie Charles Wilson**, University of Hawaii at Manoa.

Transformation Groups in Topology (Code: SS 25A), **Karl Heinz Dovermann**, University of Hawaii at Manoa, and **Daniel Ramras**, New Mexico State University.

Universal Algebra and Lattice Theory (Code: SS 10A), **Ralph Freese**, **William Lampe**, and **J. B. Nation**, University of Hawaii.

Tampa, Florida

University of South Florida

March 10–11, 2012

Saturday – Sunday

Meeting #1079

Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: January
Program first available on AMS website: February 2, 2012
Program issue of electronic *Notices*: March 2012
Issue of *Abstracts*: Volume 33, Issue 2

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: November 29, 2011
For abstracts: January 18, 2012

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Anne Condon, University of British Columbia, *Some why's and how's of programming DNA molecules*.

Mark Ellingham, Vanderbilt University, *Beyond the map color theorem*.

Mauro Maggioni, Duke University, *Digital data sets: Geometry, random walks, multiscale analysis, and applications*.

Weiqiang Wang, University of Virginia, *Title to be announced*.

Special Sessions

Algebraic and Combinatorial Structures in Knot Theory (Code: SS 2A), **J. Scott Carter**, University of South Alabama, and **Mohamed Elhamdadi** and **Masahico Saito**, University of South Florida.

Analysis in Metric Spaces (Code: SS 3A), **Thomas Bieske**, University of South Florida, and **Jason Gong**, University of Pittsburgh.

Applications of Complex Analysis in Mathematical Physics (Code: SS 9A), **Razvan Teodorescu**, University of South Florida, **Mihai Putinar**, University of California, Santa Barbara, and **Pavel Bleher**, Indiana University-Purdue University Indianapolis.

Asymptotic Properties of Groups (Code: SS 20A), **Alexander Dranishnikov**, University of Florida, and **Mark Sapir**, Vanderbilt University.

Combinatorics: Algebraic and Geometric (Code: SS 23A), **Drew Armstrong**, University of Miami, and **Benjamin Braun**, University of Kentucky.

Complex Analysis and Operator Theory (Code: SS 8A), **Sherwin Kouchekian**, University of South Florida, and **William Ross**, University of Richmond.

Computational Algebraic Geometry and Applications (Code: SS 25A), **Tony Shaska**, Oakland University, and **Artur Elezi**, American University.

Dirac Analysis (Code: SS 18A), **Craig Nolder**, Florida State University, and **John Ryan**, University of Arkansas.

Discrete Mathematics and Geometry (Code: SS 17A), **Eunjeong Yi** and **Cong X. Kang**, Texas A&M University Galveston.

Discrete Models in Molecular Biology (Code: SS 1A), **Alessandra Carbone**, Université Pierre et Marie Curie and Laboratory of Microorganisms Genomics, **Natasha Jonoska**, University of South Florida, and **Reidun Twarock**, University of York.

Extremal Combinatorics (Code: SS 13A), **Brendan Nagle**, University of South Florida.

Finite Fields and Their Applications (Code: SS 15A), **Xiang-dong Hou**, University of South Florida, and **Gary Mullen**, Pennsylvania State University.

Graph Theory (Code: SS 14A), **Mark Ellingham**, Vanderbilt University, and **Xiaoya Zha**, Middle Tennessee State University.

Hopf Algebras and Galois Module Theory (Code: SS 7A), **James Carter**, College of Charleston, and **Robert Underwood**, Auburn University Montgomery.

Interaction between Algebraic Combinatorics and Representation Theory (Code: SS 4A), **Mahir Can**, Tulane University, and **Weiqiang Wang**, University of Virginia.

Inverse Problems in Partial Differential Equations (Code: SS 24A), **Xiaosheng Li**, Florida International University, and **Alexandru Tamasan**, University of Central Florida.

Low-Dimensional Topology (Code: SS 22A), **Peter Horn**, Columbia University, and **Constance Leidy**, Wesleyan University.

Modeling Crystalline and Quasi-Crystalline Materials (Code: SS 5A), **Mile Krajcevski** and **Gregory McColm**, University of South Florida.

Nonlinear Partial Differential Equations and Applications (Code: SS 19A), **Netra Khanal**, University of Tampa.

Recent Developments of Finite Element Methods for Partial Differential Equations (Code: SS 21A), **Bo Dong**, Drexel University, and **Wei Wang**, Florida International University.

Representations of Algebraic Groups and Related Structures (Code: SS 12A), **Joerg Feldvoss** and **Cornelius Pillen**, University of South Alabama.

Solvability and Integrability of Nonlinear Evolution Equations (Code: SS 6A), **Wen-Xiu Ma**, University of South Florida, and **Ahmet Yildirim**, Ege University and University of South Florida.

Spectral Theory (Code: SS 11A), **Anna Skripka** and **Maxim Zinchenko**, University of Central Florida.

Stochastic Analysis and Applications (Code: SS 16A), **Sivapragasam Sathananthan**, Tennessee State University, and **Gangaram Ladde**, University of South Florida.

Stochastic Partial Differential Equations and Random Global Dynamics (Code: SS 10A), **Yuncheng You**, University of South Florida, and **Shanjian Tang**, Fudan University.

Washington, District of Columbia

George Washington University

March 17–18, 2012

Saturday – Sunday

Meeting #1080

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: January 2012
Program first available on AMS website: February 9, 2012
Program issue of electronic *Notices*: March 2012
Issue of *Abstracts*: Volume 33, Issue 2

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: December 6, 2011
For abstracts: January 31, 2012

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Jim Geelen, University of Waterloo, *Title to be announced.*

Boris Solomyak, University of Washington, *Title to be announced.*

Gunther Uhlmann, University of Washington, *Title to be announced* (Einstein Public Lecture in Mathematics).

Anna Wienhard, Princeton University, *Title to be announced.*

Special Sessions

Analysis of Wavelets, Frames, and Fractals (Code: SS 11A), **Keri Kornelson**, University of Oklahoma, and **Judy Packer**, University of Colorado Boulder.

Computable Mathematics (in honor of Alan Turing) (Code: SS 8A), **Douglas Cenzer**, University of Florida, **Valentina Harizanov**, George Washington University, and **Russell Miller**, Queens College and Graduate Center–CUNY.

Convex and Discrete Geometry (Code: SS 9A), **Jim Lawrence** and **Valeriu Soltan**, George Mason University.

Dynamics of Complex Networks (Code: SS 7A), **Yongwu Rong**, **Guanyu Wang**, and **Chen Zeng**, George Washington University.

Homology Theories Motivated by Knot Theory (Code: SS 3A), **Jozef H. Przytycki**, George Washington University, **Radmila Sazdanovic**, University of Pennsylvania, and **Alexander N. Shumakovitch** and **Hao Wu**, George Washington University.

Mathematical Methods in Disease Modeling (Code: SS 15A), **Sivan Leviyang**, Georgetown University, and **Shweta Bansal**, Georgetown University and National Institutes of Health.

Mathematics Applied in the Sciences: From Statistics to Topology (Code: SS 12A), **James Carroll** and **Hanna Makaruk**, Los Alamos National Laboratory, and **Robert Owczarek**, Enfitek, Inc. and University of New Mexico.

Matroid Theory (Code: SS 1A), **Joseph E. Bonin**, George Washington University, and **Sandra Kingan**, Brooklyn College.

Nonlinear Dispersive Equations (Code: SS 10A), **Manoussos Grillakis**, University of Maryland, **Justin Holmer**, Brown University, and **Svetlana Roudenko**, George Washington University.

Optimization: Theory and Applications (Code: SS 2A), **Roman Sznajder**, Bowie State University.

Relations between the History and Pedagogy of Mathematics (Code: SS 14A), **David L. Roberts**, Prince George's Community College, and **Kathleen M. Clark**, Florida State University.

Self-organization Phenomena in Reaction Diffusion Equations (Code: SS 5A), **Xiaofeng Ren**, George Washington University, and **Junping Shi**, College of William and Mary.

Structural and Extremal Problems in Graph Theory (Code: SS 4A), **Daniel Cranston**, Virginia Commonwealth University, and **Gexin Yu**, College of William & Mary.

Symmetric Functions, Quasisymmetric Functions, and the Associated Combinatorics (Code: SS 16A), **Nicholas Loehr**, Virginia Tech, and **Elizabeth Niese**, Marshall University.

The Legacy of Goedel's Second Incompleteness Theorem for the Foundations of Mathematics (Code: SS 17A), **Karim J. Mourad**, Georgetown University.

Tilings, Substitutions, and Bratteli-Vershik Transformations (Code: SS 6A), **E. Arthur Robinson**, George Washington University, and **Boris Solomyak**, University of Washington.

Topics in Geometric Analysis and Complex Analysis (Code: SS 13A), **Zheng Huang** and **Marcello Lucia**, City University of New York, Staten Island.

Lawrence, Kansas

University of Kansas

March 30 – April 1, 2012

Friday – Sunday

Meeting #1081

Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: February 2012
Program first available on AMS website: March 8, 2012
Program issue of electronic *Notices*: March 2012
Issue of *Abstracts*: Volume 33, Issue 2

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: December 20, 2011
For abstracts: February 14, 2012

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Frank Calegari, Northwestern University, *Title to be announced.*

Christopher Leininger, University of Illinois at Urbana-Champaign, *Title to be announced.*

Alina Marian, University of Illinois at Chicago, *Title to be announced.*

Catherine Yan, Texas A&M University, *Title to be announced.*

Special Sessions

Algebraic Geometry and its Applications (Code: SS 9A), **Yasuyuki Kachi** and **B. P. Purnaprajna**, University of Kansas.

Combinatorial Commutative Algebra (Code: SS 1A), **Christopher Francisco** and **Jeffrey Mermin**, Oklahoma State University, and **Jay Schweig**, University of Kansas.

Dynamics and Stability of Nonlinear Waves (Code: SS 12A), **Mat Johnson** and **Myunghyun Oh**, University of Kansas.

Enumerative and Geometric Combinatorics (Code: SS 5A), **Margaret Bayer**, University of Kansas, **Joseph P. King**, University of North Texas, **Svetlana Poznanovik**, Georgia Institute of Technology, and **Catherine Yan**, Texas A&M University.

Geometric Representation Theory (Code: SS 4A), **Zongzhu Lin**, Kansas State University, and **Zhiwei Yun**, Massachusetts Institute of Technology.

Geometric Topology and Group Theory (Code: SS 16A), **Richard P. Kent IV**, University of Wisconsin-Madison, **Christopher J. Leininger**, University of Illinois Urbana-Champaign, and **Kasra Rafi**, University of Oklahoma.

Geometry of Moduli Spaces of Sheaves (Code: SS 17A), **Alina Marian**, University of Illinois at Chicago, and **Dragos Oprea**, University of California San Diego.

Harmonic Analysis and Applications (Code: SS 6A), **Arpad Benyi**, Western Washington University, **David Cruz-Uribe**, Trinity College, and **Rodolfo Torres**, University of Kansas.

Interplay between Geometry and Partial Differential Equations in Several Complex Variables (Code: SS 13A), **Jennifer Halfpap**, University of Montana, and **Phil Harrington**, University of Arkansas.

Invariants of Knots (Code: SS 3A), **Heather A. Dye**, McKendree University, and **Aaron Kaestner** and **Louis H. Kauffman**, University of Illinois at Chicago.

Mathematical Statistics (Code: SS 14A), **Zsolt Talata**, University of Kansas.

Mathematics of Ion Channels: Life's Transistors (Code: SS 15A), **Bob Eisenberg**, Rush Medical Center at Chicago, **Chun Liu**, Penn State University, and **Weishi Liu**, University of Kansas.

Nonlinear Dynamical Systems and Applications (Code: SS 11A), **Weishi Liu** and **Erik Van Vleck**, University of Kansas.

Numerical Analysis and Scientific Computing (Code: SS 10A), **Weizhang Huang**, **Xuemin Tu**, **Erik Van Vleck**, and **Honggou Xu**, University of Kansas.

Partial Differential Equations (Code: SS 2A), **Milena Stanislavova** and **Atanas Stefanov**, University of Kansas.

Singularities in Commutative Algebra and Algebraic Geometry (Code: SS 7A), **Hailong Dao**, University of Kansas, **Lance E. Miller**, University of Utah, and **Karl Schwede**, Pennsylvania State University.

Stochastic Analysis (Code: SS 18A), **Jin Feng**, **Yaozhong Hu**, and **David Hualart**, University of Kansas.

Topics in Commutative Algebra (Code: SS 8A), **Hailong Dao**, **Craig Huneke**, and **Daniel Katz**, University of Kansas.

Rochester, New York

Rochester Institute of Technology

September 22–23, 2012

Saturday – Sunday

Meeting #1082

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: May 2012
Program first available on AMS website: July 19, 2012
Program issue of electronic *Notices*: September 2012
Issue of *Abstracts*: Volume 33, Issue 3

Deadlines

For organizers: February 22, 2012
For consideration of contributed papers in Special Sessions: May 15, 2012
For abstracts: July 10, 2012

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Steve Gonek, University of Rochester, *Title to be announced.*

James Keener, University of Utah, *Title to be announced.*

Dusa McDuff, Barnard College, *Title to be announced.*

Peter Winkler, Dartmouth College, *Title to be announced.*

New Orleans, Louisiana

Tulane University

October 13–14, 2012

Saturday – Sunday

Meeting #1083

Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: June 2012
Program first available on AMS website: September 6, 2012
Program issue of electronic *Notices*: October 2012
Issue of *Abstracts*: Volume 33, Issue 3

Deadlines

For organizers: March 13, 2012
For consideration of contributed papers in Special Sessions: July 3, 2012
For abstracts: August 28, 2012

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Anita Layton, Duke University, *Title to be announced.*

Lenhard Ng, Duke University, *Title to be announced.*

Henry K. Schenck, University of Illinois at Urbana-Champaign, *From approximation theory to algebraic geometry: The ubiquity of splines.*

Milen Yakimov, Louisiana State University, *Title to be announced.*

Akron, Ohio

University of Akron

October 20–21, 2012

Saturday – Sunday

Meeting #1084

Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: August 2012
Program first available on AMS website: September 27, 2012
Program issue of electronic *Notices*: October 2012
Issue of *Abstracts*: Volume 33, Issue 4

Deadlines

For organizers: March 22, 2012
For consideration of contributed papers in Special Sessions: July 10, 2012
For abstracts: September 4, 2012

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Tanya Christiansen, University of Missouri, *Title to be announced.*

Tim Cochran, Rice University, *Title to be announced.*

Ronald Solomon, Ohio State University, *Title to be announced.*

Ben Weinkove, University of California San Diego, *Title to be announced.*

Tucson, Arizona

University of Arizona, Tucson

October 27–28, 2012

Saturday – Sunday

Meeting #1085

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: August 2012
Program first available on AMS website: October 4, 2012
Program issue of electronic *Notices*: October 2012
Issue of *Abstracts*: Volume 33, Issue 4

Deadlines

For organizers: March 27, 2012
For consideration of contributed papers in Special Sessions: July 17, 2012
For abstracts: September 11, 2012

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Michael Hutchings, University of California Berkeley, *Title to be announced.*

Kenneth McLaughlin, University of Arizona, Tucson, *Title to be announced.*

Ken Ono, Emory University, *Title to be announced* (Erdős Memorial Lecture).

Jacob Sterbenz, University of California San Diego, *Title to be announced.*

Goufang Wei, University of California, Santa Barbara, *Title to be announced.*

Special Sessions

Dispersion in Heterogeneous and/or Random Environments (Code: SS 2A), **Rabi Bhattacharya**, Oregon State University, Corvallis, and **Edward Waymire**, University of Arizona, Tucson.

Harmonic Maass Forms and q-Series (Code: SS 1A), **Ken Ono**, Emory University, **Amanda Folsom**, Yale University, and **Zachary Kent**, Emory University.

San Diego, California

San Diego Convention Center and San Diego Marriott Hotel and Marina

January 9–12, 2013

Wednesday – Saturday

Meeting #1086

Joint Mathematics Meetings, including the 119th Annual Meeting of the AMS, 96th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: October 2012
Program first available on AMS website: November 1, 2012
Program issue of electronic *Notices*: January 2012
Issue of *Abstracts*: Volume 34, Issue 1

Deadlines

For organizers: April 1, 2012

For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Oxford, Mississippi

University of Mississippi

March 1–3, 2013

Friday – Sunday
Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: August 1, 2012
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Chestnut Hill, Massachusetts

Boston College

April 6–7, 2013

Saturday – Sunday
Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 6, 2012
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Ames, Iowa

Iowa State University

April 27–28, 2013

Saturday – Sunday
Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: April 2013
Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 27, 2012
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Special Sessions

Operator Algebras and Topological Dynamics (Code: SS 1A), **Ken Ono**, Emory University, **Amanda Folsom**, Yale University, and **Zachary Kent**, Emory University.

Alba Iulia, Romania

June 27–30, 2013

Thursday – Sunday
First Joint International Meeting of the AMS and the Romanian Mathematical Society, in partnership with the "Simion Stoilow" Institute of Mathematics of the Romanian Academy.
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: Not applicable
Program issue of electronic *Notices*: Not applicable
Issue of *Abstracts*: Not applicable

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Louisville, Kentucky

University of Louisville

October 5–6, 2013

Saturday – Sunday
Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 5, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

St. Louis, Missouri

Washington University

October 18–20, 2013

Friday – Sunday
Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 20, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Riverside, California

University of California Riverside

November 2–3, 2013

Saturday – Sunday
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 2, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Baltimore, Maryland

Baltimore Convention Center, Baltimore Hilton, and Marriott Inner Harbor

January 15–18, 2014

Wednesday – Saturday
Joint Mathematics Meetings, including the 120th Annual Meeting of the AMS, 97th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Matthew Miller
Announcement issue of *Notices*: October 2013
Program first available on AMS website: November 1, 2013
Program issue of electronic *Notices*: January 2013
Issue of *Abstracts*: Volume 35, Issue 1

Deadlines

For organizers: April 1, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Tel Aviv, Israel

Bar-Ilan University, Ramat-Gan and Tel-Aviv University, Ramat-Aviv

June 16–19, 2014

Monday – Thursday
The 2nd Joint International Meeting between the AMS and the Israel Mathematical Union.
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

San Antonio, Texas

Henry B. Gonzalez Convention Center and Grand Hyatt San Antonio

January 10–13, 2015

Saturday – Tuesday
Joint Mathematics Meetings, including the 121st Annual Meeting of the AMS, 98th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: October 2014
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2015
Issue of *Abstracts*: Volume 36, Issue 1

Deadlines

For organizers: April 1, 2014
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Porto, Portugal

University of Porto

June 11–14, 2015

Thursday – Sunday
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: Not applicable

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Seattle, Washington

Washington State Convention Center and the Sheraton Seattle Hotel

January 6–9, 2016

Wednesday – Saturday
Joint Mathematics Meetings, including the 122nd Annual Meeting of the AMS, 99th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: October 2015
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2016
Issue of *Abstracts*: Volume 37, Issue 1

Deadlines

For organizers: April 1, 2015
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Atlanta, Georgia

Hyatt Regency Atlanta and Marriott Atlanta Marquis

January 4–7, 2017

Wednesday – Saturday
Joint Mathematics Meetings, including the 123rd Annual Meeting of the AMS, 100th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: October 2016
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2017
Issue of *Abstracts*: Volume 38, Issue 1

Deadlines

For organizers: April 1, 2016
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

San Diego, California

San Diego Convention Center and San Diego Marriott Hotel and Marina

January 10–13, 2018

Wednesday – Saturday
Joint Mathematics Meetings, including the 124th Annual Meeting of the AMS, 101st Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Matthew Miller
Announcement issue of *Notices*: October 2017
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 1, 2017
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

2012 Joint Mathematics Meetings Advance Registration/Housing Form

Name ______________________________
(please write name as you would like it to appear on your badge)

Mailing Address ______________________________

____

Telephone ____________________ Fax: ____________________

In case of emergency (for you) at the meeting, call: Day #____________ Evening #: ____________

E-mail Address ______________________________

Acknowledgment of this registration and any hotel reservations will be sent to the e-mail address given here, unless you check this box: *Send by U.S. Mail* ☐

Affiliation for badge ____________________ (company/university) Nonmathematician guest badge name: ____________________ (Note fee of US$15)

☐ **I DO NOT want my program and badge to be mailed to me on 12/12/11. (Materials will be mailed to the address listed above unless you check this box.)**

Registration Fees

Membership ✓ all that apply. First row is eligible to register as a JMM member.

❑ AMS ❑ MAA ❑ ASL ❑ CMS ❑ SIAM

❑ ASA ❑ AWM ❑ NAM ❑ YMN

Joint Meetings	**by Dec 15**	**at mtg**	**Subtotal**
❑ Member AMS, MAA, ASL, CMS, SIAM	US$228	US$ 300	
❑ Nonmember	US$356	US$ 462	
❑ Graduate Student (Member of AMS or MAA)	US$ 50	US$ 60	
❑ Graduate Student (Nonmember)	US$ 78	US$ 88	
❑ Undergraduate Student	US$ 50	US$ 60	
❑ High School Student	US$ 5	US$ 10	
❑ Unemployed	US$ 50	US$ 60	
❑ Temporarily Employed	US$185	US$214	
❑ Developing Countries Special Rate	US$ 50	US$ 60	
❑ Emeritus Member of AMS or MAA	US$ 50	US$ 60	
❑ High School Teacher	US$ 50	US$ 60	
❑ Librarian	US$ 50	US$ 60	
❑ Press	US$ 0	US$ 0	
❑ Nonmathematician Guest	US$ 15	US$ 15	
			$________

AMS Short Course 1: *Random Fields and Random Geometry(1/2–1/3)*

❑ Member of AMS or MAA	US$102	US$136	
❑ Nonmember	US$145	US$175	
❑ Student, Unemployed, Emeritus	US$ 50	US$ 71	
			$________

AMS Short Course 2: *Computing with Elliptic Curves Using Sage (1/2–1/3)*

❑ Member of AMS or MAA	US$102	US$136	
❑ Nonmember	US$145	US$175	
❑ Student, Unemployed, Emeritus	US$ 50	US$ 71	
			$________

MAA Short Course: *Discrete and Computational Geometry (1/2–1/3)*

❑ Member of MAA or AMS	US$153	US$163	
❑ Nonmember	US$204	US$214	
❑ Student, Unemployed, Emeritus	US$ 77	US$ 87	
			$________

MAA Minicourses (see listing in text)

I would like to attend: ❑ One Minicourse ❑ Two Minicourses

Please enroll me in MAA Minicourse(s) #______ and/or #______

In order of preference, my alternatives are: #______ and/or #______

Price: US$77 for each minicourse.

(For more than 2 minicourses call or email the MMSB.) $________

Graduate School Fair

❑ Graduate Program Table US$ 65 US$ 65

$________

Employment Center *Please go to http://www.ams.org/emp-reg to register. For further information, contact Steve Ferrucci at emp-info@ams.org.*

Events with Tickets

❑ Graduate Student/First Time Attendee Reception (1/4) (no charge)

MER/IME Banquet (1/5) US$ 60 # ___Chicken #___Fish #___Kosher #___Vegan

NAM Banquet (1/6) US$ 58 #___Chicken #___Fish #___Kosher #___Vegan

AMS Banquet (1/7) US$ 58 #___Chicken #___Fish #___Kosher #___Vegan

Reception for Tina Straley (1/6) US$ 28 #___tickets

(Additional fees may apply for Kosher meals.) $________

Total for Registrations and Events $________

Payment

Registration & Event Total (total from column on left) $ ____________

Hotel Deposit (only if paying by check) $ ____________

Total Amount To Be Paid **$** ____________

(Note: A US$5 processing fee will be charged for each returned check or invalid credit card. Debit cards cannot be accepted.)

Method of Payment

❑ **Check**. Make checks payable to the AMS. Checks drawn on foreign banks must be in equivalent foreign currency at current exchange rates. For all check payments, please keep a copy of this form for your records.

❑ **Credit Card**. All major credit cards accepted. For your security, we do not accept credit card numbers by postal mail, e-mail or fax. If the MMSB receives your registration form by fax or postal mail, we will contact you at the phone number provided on this form. For questions, contact the MMSB at mmsb@ams.org.

Signature: ______________________________

❑ **Purchase Order** #____________________ (please enclose copy)

Other Information

Mathematical Reviews field of interest # ____________________

How did you hear about this meeting? Check one:

❑ Colleague(s) ❑ Internet ❑ *Notices* ❑ *Focus* ❑ Other _________

❑ This is my first Joint Mathematics Meetings.

❑ I am a mathematics department chair.

❑ For planning purposes for the MAA Two-year College Reception, please check if you are a faculty member at a two-year college.

❑ I would like to receive promotions for future JMM meetings.

❑ Please do not include my name on any promotional mailing lists.

❑ Please ✓ this box if you have a disability requiring special services.

Mailing Address/Contact:

Mathematics Meetings Service Bureau (MMSB)

P. O. Box 6887

Providence, RI 02940-6887 Fax: 401-455-4004; **E-mail:** mmsb@ams.org

Telephone: 401-455-4143 or 1-800-321-4267 x4143 or x4144

Deadlines

To be eligible for the complimentary room drawing:	**Nov. 4, 2011**
For housing reservations, badges/programs mailed:	**Nov. 18, 2011**
For housing changes/cancellations through MMSB:	**Dec. 6, 2011**
For advance registration for the Joint Meetings, short courses, minicourses, and tickets:	**Dec. 15, 2011**
For 50% refund on banquets, cancel by:	**Dec. 27, 2011***
For 50% refund on advance registration, minicourses & short courses, cancel by:	**Dec. 30, 2011***

***no refunds after this date**

Registration for the Joint Meetings is not required for the short courses but it is required for the minicourses and the Employment Center.

2012 Joint Mathematics Meetings Hotel Reservations – Boston, MA

Please see the hotel page in the announcement or on the web for detailed information on each hotel. To ensure accurate assignments, please rank hotels in order of preference by writing 1, 2, 3, etc. in the column on the left and by circling the requested room type and rate. If the rate or the hotel requested is no longer available, you will be assigned a room at the next available comparable rate. Please call the MMSB for details on suite configurations, sizes, availability, etc. All reservations, including suite reservations, must be made through the MMSB to receive the JMM rates. Reservations made directly with the hotels before **December 14** may be changed to a higher rate. All rates are subject to a 14.45% sales/occupancy tax. **Guarantee requirements: First night deposit by check (add to payment on reverse of form) or a credit card guarantee.**

❏ Deposit enclosed (see front of form)

❏ Hold with my credit card. **For your security, we do not accept credit card numbers by postal mail, e-mail or fax.** If the MMSB receives your registration form by postal mail or fax, we will contact you at the phone number provided on the reverse of this form.

Date and Time of Arrival ________________ **Date and Time of Departure** ________________

Name of Other Room Occupant ________________ **Arrival Date** __________ **Departure Date** __________ **Child (give age(s)** __________

Order of choice	Hotel	Single	Double 1 bed	Double 2 beds	Triple 2 beds	Triple 2 beds w/cot	Triple - king or queen w/cot	Quad 2 beds	Quad 2 beds w/cot	Suites Starting rates
	Marriott Copley Place (co-headquarters)									
	Regular Rate	US$ 159	US$ 159	US$ 159	US$ 179	US$ 179	US$ 179	US$ 199	US$ 199	US$ 800
	Student Rate	US$ 149	US$ 149	US$ 149	US$ 169	US$ 169	US$ 169	US$ 189	US$ 189	N/A
	Sheraton Boston (Co-headquarters)									
	Regular Rate	US$ 159	US$ 159	US$ 159	US$ 169	N/A	US$ 169	US$ 179	N/A	US$ 219
	Student Rate	US$ 149	US$ 149	US$ 149	US$ 159	N/A	US$ 159	US$ 169	N/A	N/A
	Boston Omni Parker House									
	Regular Rate	US$ 149	US$ 149	US$ 149	US$ 179	US $179	US$ 179	US$ 209	US$ 209	N/A
	Student Rate	US$ 129	US$ 129	US$ 129	N/A	N/A	N/A	N/A	N/A	N/A
	Hilton Back Bay									
	Regular Rate	US$ 129	US$ 129	US$ 129	US$ 149	US $174	N/A	US$ 169	US$ 194	US$ 429
	Student Rate	US$ 119	US$ 119	US$ 119	US$ 139	US $164	N/A	US$ 159	US$ 184	N/A
	The Colonnade Hotel									
	Regular Rate	US$ 124	US$ 124	US$ 124	US$ 154	US $154	US$ 154	US$ 184	US$ 184	US$ 424
	Student Rate	US$ 114	US$ 114	US$ 114	US$ 144	US$ 144	US$ 144	US$ 174	US$ 174	N/A
	Boston Park Plaza and Towers									
	Regular Rate	US$ 122	US$ 122	US$ 122	US$ 132	N/A	US$ 132*	US$ 142	N/A	US$ 272
	Student Rate	US$ 112	US$ 112	US$ 112	US$ 122	N/A	US$ 122*	US$ 132	N/A	N/A

* Boston Park Plaza charges an additional US$25 per stay for a rollaway cot

Special Housing Requests:

❏ I have disabilities as defined by the ADA that require a sleeping room that is accessible to the physically challenged. My needs are: ________________

❏ Other requests: ________________

❏ I am a member of a hotel frequent-travel club and would like to receive appropriate credit.
The hotel chain and card number are: ________________

E-mail confirmations (no paper) will be sent by all hotels if an e-mail address is provided.

If you are not making a reservation, please check one of the following:

❏ I plan to make a reservation at a later date.

❏ I will be making my own reservations at a hotel not listed. Name of hotel: ________________

❏ I live in the area or will be staying privately with family or friends.

❏ I plan to share a room with ________________, who is making the reservations.

Meetings and Conferences of the AMS

Associate Secretaries of the AMS

Western Section: Michel L. Lapidus, Department of Mathematics, University of California, Surge Bldg., Riverside, CA 92521-0135; e-mail: lapidus@math.ucr.edu; telephone: 951-827-5910.

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The Meetings and Conferences section of the *Notices* gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited in the table of contents on this page for more detailed information on each event. Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. **Information in this issue may be dated. Up-to-date meeting and conference information can be found at** www.ams.org/meetings/.

Meetings:

Important Information Regarding AMS Meetings

Potential organizers, speakers, and hosts should refer to page 100 in the January 2011 issue of the *Notices* for general information regarding participation in AMS meetings and conferences.

Abstracts

Speakers should submit abstracts on the easy-to-use interactive Web form. No knowledge of LaTeX is necessary to submit an electronic form, although those who use LaTeX may submit abstracts with such coding, and all math displays and similarily coded material (such as accent marks in text) must be typeset in LaTeX. Visit http://www.ams.org/cgi-bin/abstracts/abstract.pl. Questions about abstracts may be sent to abs-info@ams.org. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

Conferences: (see http://www.ams.org/meetings/ for the most up-to-date information on these conferences.)

March 11–14, 2012: Fourth International Conference on Mathematical Sciences, United Arab Emirates (held in cooperation with the AMS). Please see http://icm.uaeu.ac.ae/ for more information.

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